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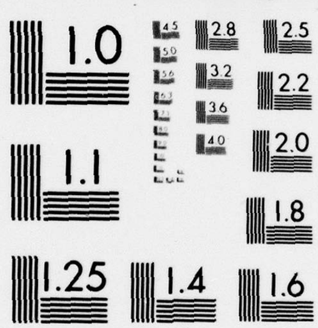

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(6) HOW TOLLS REDISTRIBUTE CUSTOMER BENEFITS IN A M/M/1 QUEUING SYSTEM.

by

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DEPARTMENT OF OPERATIONS RESEARCH
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How Tolls Redistribute Customer Benefits in an M/M/1 Queuing System

Abstract

In a FIFO M/M/1 queuing system an arriving customer chooses to enter the system if his reward for completing service is not less than a constant entrance toll plus expected waiting charge (accumulated at a constant rate). Customers are partitioned into K classes according to their rewards. The long run average benefit (reward minus waiting charge) per arriving customer is computed for each class.

Although a constant toll is fixed for all entering customers regardless of class or state of the system, the value of the toll can profoundly impact each classes' long run average benefit per arriving customer. When the value of the toll is increased, the change in benefit per arriving customer is investigated for each customer class. These changes are a monotone increasing function of customer reward. Thus a toll increase is most valuable for those customers who obtain the highest reward from service completion. Results obtained assume that (1) all rewards and tolls can be expressed as an integer multiple of a customer's expected cost of waiting through his own service, and (2) tolls are not subtracted in computing a customer's benefit from entering the system.

Note to the reader:

The layout of this paper reflects the fact that all but its appendix has been submitted for publication. Thus reference [2] where it appears in the text should direct the interested reader to the appendix. The appendix is provided to tackle issues that are peripheral to the main thrust of the paper and the appendix is not intended for publication.

How Tolls Redistribute Customer Benefits in an M/M/1 Queuing System

Following the pioneering effort of Naor [6], many authors (e.g. [1], [3], [4] [5], [7], [8]) have compared a queuing system operated so that customers are free to act in their own interest (individual optimization) with the same system managed to maximize some social benefit (social optimization). Regardless of the specific assumptions, these authors unanimously conclude that a customer acting in his own interest will choose to enter the system on any occasion when he would enter under social control. Typically, a self-interested customer would enter the system when it is more congested than under social optimization because he fails to account for the inconvenience that he would cause to future arrivals. A natural suggestion is to attempt to achieve social optimization through imposing a set of tolls whose effect is to encourage the self-interested customer to act in a socially optimal way.

Our purpose here is to investigate how benefits to individuals are redistributed through charging a toll $T \geq 0$ (independent of the number of customers present) to any customer who chooses to enter an M/M/1 system. Customers are partitioned into K classes according to the reward that they receive for completing service; customers of class k arrive at the facility at rate σ_k and receive a reward R_k if they choose to enter the system. We assume $R_1 > R_2 > \cdots > R_K$. Here we consider long run average measures of system performance; without discounting it does not matter when a customer's reward is paid. The server provides independent identically distributed services at rate μ on a first-come first-served basis and any customer pays a holding charge h per unit time while in the system. A customer arriving at the facility is informed of how many others are present and will choose to enter if and only if his toll plus expected holding costs does not exceed his reward. (We could obtain the results assuming that a customer will enter only if his expected holding cost plus toll is strictly less than his reward.) Without loss of generality we assume that time and money are measured in units so that $\mu=1$ and $h=1$.

We also make the restrictive assumption that T and all R_k's are integers. Any reward or toll must exactly equal the expected holding cost incurred during the complete service times of an integer number of customers. Otherwise, imposing a positive toll could further restrict one class from entering the system without restricting another. For example, with $R_1=3.1$, $R_2=2.9$ and $T=0$ vs. $T=0.2$, customers of type 1 enter with 2 or fewer, 1 or fewer present respectively; 2-customers enter with 0 or 1 present in either case.

Increasing T by 1 decreases by 1 for each k the maximum number of customers present under which k-customers will enter. This degree of "fairness" is necessary for our main result; a counterexample for non-integer T is included in [2].

With a fixed integer toll T we have a birth-death process with stationary distribution $\{\pi_i, i=0,1,2,\ldots\}$ easily computed. For class k the long run average benefit per unit time is defined as

$$B_k(T) \equiv \sigma_k \sum_{i=0}^{R_k-1-T} \pi_i(R_k-i-1)$$

and the long run average benefit per arrival as

$$A_k(T) = \sum_{i=0}^{R_k-1-T} \pi_i(R_k-i-1)$$

Assuming $\sigma_k>0$, $A_k(T) = B_k(T)/\sigma_k$. Even if $\sigma_k=0$, $A_k(T)$ is well-defined. Both $B_k(T)$ and $A_k(T)$ are defined to be 0 if R_k-1-T is negative.

Our main result is that for $T_1 > T_0 \geq 0$, $A_k(T_1) - A_k(T_0)$ is decreasing in k over all classes k whose customers opt for entering an empty system with toll T_1. Thus increasing (or imposing) the toll causes an increase in benefit to a typical 1-customer that exceeds the increase in benefit to a 2-customer, etc. Increasing the toll increases the attractiveness of the system more for customers who more highly value service. In particular, if T is chosen to maximize $\sum_{k=1}^{K} B_k(T)$ and $T>0$ then $A_1(T) > A_1(0)$ whereas some classes may be denied admission altogether and among those classes $k \geq 2$ who still choose to enter an empty system, $A_k(T) - A_k(0) < A_1(T) - A_1(0)$.

Notice that T is assumed to influence a customer's decision to enter the system but T is not counted in $B_k(T)$ or $A_k(T)$. If the toll were subtracted from net benefits, our main result does not hold (see [2]).

The assumption that tolls are not subtracted from net benefits may be reasonable if toll revenues collected from class k are redistributed to all class k customers regardless of whether or not they enter the system. An alternative context in which this assumption is plausible is one in which a private contractor classifies customers into K classes according to the revenue $(R_1 > R_2 > \cdots > R_K)$ he receives from a service completion. The contractor must pay a holding cost accumulated at a constant rate for each customer in the system and he can limit congestion by denying entrance to anybody. He chooses the operating policy "accept a customer only if the expected profit (i.e. revenue minus expected holding cost) is at least T." Then an increase in T implies that the contractor's average profit per arrival increases more for high revenue customers than for low revenue customers. The next section provides our main result.

1. Main Result

Lemma 1 is useful in comparing the stationary distributions of birth-death processes before and after a toll increase.

Lemma 1: If birth-death processes 1 and 2 both have death rates of 1 in every state and process 1 has birth rates $\lambda_i, i=0,1,2,\ldots$ and stationary distribution $\{\pi_i\}$ while process 2 has birth rates $\lambda'_i, i=0,1,2,\ldots$ with $\lambda'_i \le \lambda_i$ and stationary distribution $\{\pi'_i\}$ then for any M, $\sum_{i=0}^{M} \pi'_i \ge \sum_{i=0}^{M} \pi_i$.

Proof: For $i \ge 1$, let $W_i = \prod_{n=0}^{i-1} \lambda_n$ and $W'_i = \prod_{n=0}^{i-1} \lambda'_n$ and let $W_0 = W'_0 = 1$. Clearly $\sum_{i=0}^{M} \pi_i = 1$ implies $\sum_{i=0}^{M} \pi'_i = 1$; we need only consider M with both sums less than 1 and thus W_M and W'_M both positive. Then

$$\sum_{i=0}^{M} \pi_i = \sum_{i=0}^{M} W_i / \sum_{i=0}^{\infty} W_i = \sum_{i=0}^{M} W_i / [\sum_{i=0}^{M} W_i + W_M \sum_{i=M+1}^{\infty} W_i/W_M] = 1/[1 + (\sum_{i=M+1}^{\infty} W_i/W_M)(W_M/\sum_{i=0}^{M} W_i)]$$

$\sum_{i=0}^{M} \pi'_i$ is similarly defined. The lemma will be proved if we can demonstrate both: (1) $\sum_{i=M+1}^{\infty} W'_i/W'_M \le \sum_{i=M+1}^{\infty} W_i/W_M$

and (2) $W'_M / \sum_{i=0}^{M} W'_i \le W_M / \sum_{i=0}^{M} W_i$. The first is obvious from a term by term comparison since $W_i/W_M = \prod_{n=M}^{i-1} \lambda_n \ge \prod_{n=M}^{i-1} \lambda'_n = W'_i/W'_M$. For the second observe that

$W'_M / \sum_{i=0}^{M} W'_i = W_M / \sum_{i=0}^{M} \left(\frac{W_M}{W'_M}\right)(W'_i)$ and $\left(\frac{W_M}{W'_M}\right) W'_i \ge W_i$ for $i=0,1,\ldots,M$. Thus

$W'_M / \sum_{i=0}^{M} W'_i \le W_M / \sum_{i=0}^{M} W_i$. Q.E.D.

More specifically if we consider birth-death processes 1 and 2 before and after increasing the toll by 1 unit then $\lambda_i' = \lambda_{i+1}$ for $i=0,1,2,\ldots.$ Lemma 2, proved in [2], follows quickly.

Lemma 2: If birth-death processes 1 and 2 as defined in Lemma 1 represent a system before and after increasing the toll by 1 then $\pi_i' \equiv \pi_{i+1}/(1-\pi_0)$, $i=0,1,2,\ldots.$

Theorem 1, the main result, is stated for two classes k and k+1 whose rewards differ by 1. It can (and will) be easily generalized to any two classes.

Theorem 1: If classes k and k+1 have rewards R_k, R_{k+1} with $R_k-R_{k+1}=1$ then for any integer T, $A_k(T+1) - A_k(T) > A_{k+1}(T+1) - A_{k+1}(T)$ provided that class k+1 customers will enter an empty system with toll T+1 (i.e. $R_{k+1}-(T+1) \geq 1$).

Proof: Let $\{\pi_i\}$, $\{\pi_i'\}$ denote stationary distributions with tolls T, T+1 respectively.

Then
$$A_k(T+1) = \sum_{i=0}^{R_k-T-2} \pi_i'(R_k-i-1); \quad A_k(T) = \sum_{i=0}^{R_k-T-1} \pi_i(R_k-i-1)$$

$$A_{k+1}(T+1) = \sum_{i=0}^{R_{k+1}-T-2} \pi_i' (R_{k+1}-i-1); \quad A_{k+1}(T) = \sum_{i=0}^{R_{k+1}-T-1} \pi_i(R_{k+1}-i-1)$$

$$A_k(T+1) - A_k(T) = \sum_{i=0}^{R_k-T-2}(\pi_i' - \pi_i)(R_k-i-1) - T\pi_{R_k-T-1} \tag{1}$$

$$A_{k+1}(T+1) - A_{k+1}(T) = \sum_{i=0}^{R_{k+1}-T-2} (\pi_i' - \pi_i)(R_{k+1}-i-1) - T\pi_{R_{k+1}-T-1} \tag{2}$$

Subtracting (2) from (1) yields

$$A_k(T+1) - A_k(T) - \{A_{k+1}(T+1) - A_{k+1}(T)\} = \sum_{i=0}^{R_{k+1}-T-2}(\pi'_i - \pi_i) + (\pi'_{R_k-T-2} - \pi_{R_k-T-2})(T+1)$$

$$- T\{\pi_{R_k-T-1} - \pi_{R_k-T-2}\}$$

$$= \sum_{i=0}^{R_{k+1}-T-2}(\pi'_i - \pi_i) + \pi'_{R_k-T-2}(T+1) - T\{\pi_{R_k-T-1}$$

$$- \pi_{R_k-T-2}\}$$

$$= \sum_{i=0}^{R_k-T-2}(\pi'_i - \pi_i) + T\{\pi'_{R_k-T-2} - \pi_{R_k-T-1}\} \qquad (3)$$

Lemma 1 implies that the first term on the right of (3) is positive while lemma 2 guarantees that the second term is non-negative (positive for $T>0$). Q.E.D.

Since it was not necessary to assume σ_k and/or σ_{k+1} positive in the proof of Theorem 1, the result can be extended to arbitrary classes k and k+1 with $R_k - R_{k+1} > 1$ by establishing additional classes with rewards R_k-1, $R_k-2 \ldots, R_{k+1}+1$ and arrival rates all 0. Theorem 1 can then be used to compare average benefits per arrival for all pairs of classes whose rewards differ by exactly 1 and these results when combined will suffice to extend Theorem 1 to apply to classes k and k+1 with arbitrary $R_k - R_{k+1}$.

In [2], complications introduced by assuming non-integer R_k's are discussed. Our impression is that in most practical examples $A_k(T+1)-A_k(T)$ would exceed $A_{k+1}(T+1) - A_{k+1}(T)$ for all pairs of classes k, k+1. Pathologies discussed in [2] for non-integer R_k's are likely to be relevant only when R_k and R_{k+1} are particularly close. The result that customers who value service more also benefit more from the imposition of tolls is an accurate statement if all R_k's and T are integers and is likely to be accurate if T is an integer but some R_k's are not.

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Appendix

Here we include three results omitted from the paper:

(1) a counterexample to the main theorem when tolls and/or rewards are not integer-valued, and a discussion of the impact of non-integer R_k's and/or T.

(2) a counterexample to the main theorem when the toll is substracted from a customer's benefit and

(3) a proof of Lemma 2.

First, if tolls and rewards are both not integers, it is possible that an increase in the toll can further truncate the arrival process for some customer classes but not for others. For example, if $R_1=3.1$, $R_2=2.9$, $T=0$ then 1-customers enter with 0,1 or 2 present; 2-customers enter with 0 or 1 present. If T is increased to 0.2 then 1-customers enter only with 0 or 1 present; 2-customers still enter with 0 or 1 present. In this example, if $\sigma_1=\sigma_2=0.2$ then with T=0, $\pi_0 = .62814$, $\pi_1 = .25126$, $\pi_2 = .10050$, $\pi_3=.02010$ and average benefits per arriving customer $A_1(0) = 1.60553$, $A_2(0) = 1.41960$. If T=0.2 then $\pi_0=.64103$, $\pi_1 = .25641$, $\pi_2 = .10256$ and $A_1(0) = 1.62787$, $A_2(0) = 1.44872$. The differences $A_1(.2) - A_1(0) = 0.02234$ and $A_2(.2) - A_2(0) = 0.02912$ provide a counterexample to our main result when both T and the R_k's are non-integer.

If T is integer-valued but the R_k's are not, our main result still doesn't hold. We can prove two results, Theorem A1 and A2 below, but they can not be combined to reach a general conclusion about $A_k(T+1)-A_k(T)-\{A_{k+1}(T+1)-A_{k+1}(T)\}$. The proof of Theorem A1 parallels that of Theorem 1 but is slightly more complicated. Throughout [x] denotes the greatest integer in x.

<u>Theorem A1</u>: If classes k and k+1 have rewards R_k, R_{k+1} with $R_k-R_{k+1} \leq 1$ and $[R_k]-[R_{k+1}] = 1$ then for any integer T, $A_k(T+1)-A_k(T) > A_{k+1}(T+1) - A_{k+1}(T)$ provided that class k+1 customers will enter an empty system with toll T+1 (i.e. $R_{k+1}-(T+1) \geq 1$).

<u>Proof</u>: $A_k(T+1) = \sum_{i=0}^{[R_k-T-2]} \pi'_i (R_k-i-1)$; $A_k(T) = \sum_{i=0}^{[R_k-T-1]} \pi_i(R_k-i-1)$

$$A_{k+1}(T+1) = \sum_{i=0}^{[R_{k+1}-T-2]} \pi'_i (R_{k+1}-i-1); \; A_{k+1}(T) = \sum_{i=0}^{[R_{k+1}-T-1]} \pi_i(R_{k+1}-i-1)$$

The analogues of (1) and (2) are then

$$A_k(T+1)-A_k(T) = \sum_{i=0}^{[R_k-T-2]} (\pi'_i - \pi_i)(R_k-i-1) - \pi_{[R_k-T-1]}(R_k-[R_k]+T) \quad \text{(A1)}$$

$$A_{k+1}(T+1)-A_{k+1}(T) = \sum_{i=0}^{[R_{k+1}-T-2]} (\pi'_i - \pi_i)(R_{k+1}-i-1) - \pi_{[R_{k+1}-T-1]}(R_{k+1}-[R_{k+1}]+T) \quad \text{(A2)}$$

Subtracting (A2) from (A1) yields:

$$\begin{aligned}
A_k(T+1)-A_k(T)-\{A_{k+1}(T+1)-A_{k+1}(T)\} &= \sum_{i=0}^{[R_{k+1}-T-2]} (\pi'_i - \pi_i)(R_k-R_{k+1})+(\pi'_{[R_k-T-2]}-\pi_{[R_k-T-2]})(R_k-[R_k]+T+1) \\
&\quad - \pi_{[R_k-T-1]}(R_k-[R_k]+T)+\pi_{[R_{k+1}-T-1]}(R_{k+1}-[R_{k+1}]+T) \\
&= \sum_{i=0}^{[R_k-T-2]} (\pi'_i - \pi_i)(R_k-R_{k+1})+ \pi'_{[R_k-T-2]}(R_k-[R_k]+T+1-R_k+R_{k+1}) \\
&\quad - \pi_{[R_k-T-2]}(R_k-[R_k]+T+1-R_k+R_{k+1}-R_{k+1}+[R_{k+1}]-T) \\
&\quad - \pi_{[R_k-T-1]}(R_k-[R_k]+T) \\
&= \sum_{i=0}^{[R_k-T-2]} (\pi'_i - \pi_i)(R_k-R_{k+1})+ \{\pi'_{[R_k-T-2]}(R_{k+1}-[R_{k+1}]+T) \\
&\quad -\pi_{[R_k-T-1]}(R_k-[R_k]+T)\}
\end{aligned} \quad \text{(A3)}$$

Lemma 1 implies that the first term on the right of (A3) is positive. Lemma 2 implies $\pi'_{[R_k-T-2]} > \pi_{[R_k-T-1]}$ Also $R_k-R_{k+1} \leq 1$ with $[R_k]-[R_{k+1}] = 1$ implies

$(R_{k+1}-[R_{k+1}]+T) \geq (R_k-[R_k]+T)$. The second term on the right of (3) is also positive. Q.E.D.

Unfortunately, Theorem A2 below is a result contrary to that needed to extend Theorem 1 to non-integer R_k's.

Theorem A2: If classes k, k+1 have rewards R_k, R_{k+1} with $[R_k]=[R_{k+1}]$ then $A_k(T+1) - A_k(T) \leq A_{k+1}(T+1) - A_{k+1}(T)$ provided that class k+1 customers will enter an empty system with toll T+1.

Proof: Subtracting (A2) from (A1) and noting that $[R_k-T-2] = [R_{k+1}-T-2]$ yields:

$$A_k(T+1) - A_k(T) - \left\{A_{k+1}(T+1) - A_{k+1}(T)\right\} = \sum_{i=0}^{[R_k-T-2]} (\pi_i' - \pi_i)(R_k-R_{k+1}) - \pi_{[R_k-T-1]}(R_k-R_{k+1})$$

$$= \left\{\sum_{i=0}^{[R_k-T-2]} \pi_i' - \sum_{i=0}^{[R_k-T-1]} \pi_i\right\}(R_k-R_{k+1})$$

$$= \left\{\sum_{i=[R_k-T]}^{\infty} \pi_i - \sum_{i=[R_k-T-1]}^{\infty} \pi_i'\right\}(R_k-R_{k+1}) \qquad \text{(A4)}$$

$$= \left\{\sum_{i=[R_k-T]}^{\infty} \pi_i\right\}\left\{1 - 1/(1-\pi_0)\right\}(R_k-R_{k+1}) \qquad \text{(A5)}$$

The transition from (A4) to (A5) follows from Lemma 2. The right of (A5) is non-positive. Q.E.D.

Theorems (A1) and A(2) allow only the following very slight generalization in conditions for which Theorem 1 holds. Rather than insisting that all R_k's be integers we can allow for any set of R_k's with (1) all $[R_k]$ different and (2) the fractional parts $R_k-[R_k]$ non-decreasing in k. For example, $R_1 = 5.3$, $R_2 = 4.3$, $R_3 = 3.4$, $R_4 = 2.6$ is suitable. Theorem (A1) can then be used to successfully compare classes with arrival rates 5.3, 4.3, 3.4, 2.6.

The following numerical example demonstrates that Theorem 1 does not extend to cover the case where tolls are subtracted from net benefits per arriving customer. Let $\sigma_1 = \sigma_2 = \sigma_3 = .3$, $R_1=10$, $R_2=9$, $R_3=2$. For T=0 we have:

	entrance rate	average benefit per arrival	average benefit minus toll per arrival
class 1	.29948	7.03539	7.03539
2	.29773	6.04297	6.04297
3	.14624	0.25654	0.25654

For T=1:

	entrance rate	average benefit per arrival	average benefit minus toll per arrival
class 1	.29930	7.35508	6.35741
2	.29695	6.35744	5.36760
3	.09317	0.31059	0.00003

Comparision of the two last columns for any pair of classes demonstrates a counter-example.

Finally we prove Lemma 2.

Lemma 2: If birth-death processes 1 and 2 as defined in Lemma 1 represent a system before and after increasing the toll by 1 then $\pi_i' = \pi_{i+1}/(1-\pi_0)$, i=0,1,2,... .

Proof: $\lambda_i' = \lambda_{i+1}$ for i=0,1,2,... since increasing a toll by 1 decreases by 1 the maximum number of customers present such that class k customers will enter (for all k who will still enter an empty system after the toll increase).

Then $W_i' = W_{i+1}/\lambda_0$ for $i = 0,1,2,\ldots$. (since $W_1 = \lambda_0$)

and $$\sum_{i=0}^{\infty} W_i' = \sum_{i=0}^{\infty} W_{i+1}/\lambda_0$$

$$= \sum_{i=0}^{\infty} W_i/\lambda_0 - 1/\lambda_0$$

and $$\frac{W_n'}{\sum\limits_{i=0}^{\infty} W_i'} = \frac{W_{n+1}/\lambda_0}{\sum\limits_{i=0}^{\infty} W_i/\lambda_0 - 1/\lambda_0} = \frac{W_{n+1}}{\sum\limits_{i=1}^{\infty} W_i}$$

i.e. $$\pi_n' = \pi_{n+1} \frac{\sum\limits_{i=0}^{\infty} W_i}{\sum\limits_{i=1}^{\infty} W_i} = \pi_{n+1}/(1-\pi_0).$$ Q.E.D.

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Abstract

In a FIFO M/M/1 queuing system an arriving customer chooses to enter the system if his reward for completing service is not less than a constant entrance toll plus expected waiting charge (accumulated at a constant rate). Customers are partitioned into K classes according to their rewards. The long run average benefit (reward minus waiting charge) per arriving customer is computed for each class.

Although a constant toll is fixed for all entering customers regardless of class or state of the system, the value of the toll can profoundly impact each classes' long run average benefit per arriving customer. When the value of the toll is increased, the change in benefit per arriving customer is investigated for each customer class. These changes are a monotone increasing function of customer reward. Thus a toll increase is most valuable for those customers who obtain the highest reward from service completion. Results obtained assume that (1) all rewards and tolls can be expressed as an integer multiple of a customer's expected cost of waiting through his own service, and (2) tolls are not subtracted in computing a customer's benefit from entering the system.