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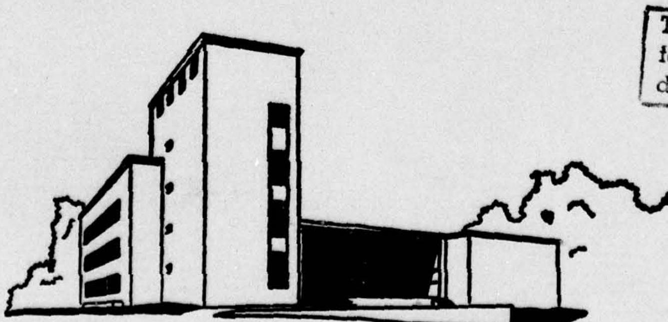

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(6) PARETO OPTIMAL DETERMINISTIC MODELS FOR BID AND OFFER AUCTIONS.

by

(10) Gerald L. Thompson
Carnegie-Mellon University

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Management Sciences Research Group
Graduate School of Industrial Administration
Carnegie-Mellon University
Pittsburgh, Pennsylvania 15213

PARETO OPTIMAL DETERMINISTIC MODELS FOR BID AND OFFER AUCTIONS

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Gerald L. Thompson
Carnegie-Mellon University

Abstract

This paper generalizes the Barr-Shaftel auctioning model in several ways. (a) It shows that the dual solutions they choose for prices and buyer surpluses is also the maximum buyer surplus solution. (b) It shows that this solution can easily be found by means of a perturbation technique and relates the solution to core solutions of assignment market games, in the sense of Shapley and Shubik. (c) It extends these models and theoretical results from an assignment to a transportation model. (d) By adding seller reservation bids the symmetry of the auction process is increased in that it becomes pareto optimal for sellers as well as buyers. (e) It proposes a "fair bid" auctioning process which has pareto optimality properties for both buyers and sellers, and which can be solved rapidly for problems having hundreds or thousands of buyers and sellers. (f) Finally, it suggests that practical applications of these models to real auctioning situations is possible. Such applications could reduce the transaction costs and improve the speed of auction processes.

1. Introduction

Each year trillions of dollars worth of goods are exchanged by auctions procedures throughout the world. The number of different kinds of auctions and the variety of settings in which they occur is

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enormous. Many auction mechanisms are inefficient and costly to run; for instance those that require bringing together to the same physical location the buyers, the sellers and the objects for sale. Although auctions are undeniably economic processes and huge in magnitude, no mention of them is made in current textbooks, and there are but few theoretical treatments in the economics literature.

In the introductory paper of a recent volume [1] of collected papers on auctions and bidding, A. Schotter [9] speculates on the reasons for this omission. He suggests the following description: "Auctions are exchange mechanisms without a tatônnement or recontracting provision in which the seller is relatively passive and goods are often indivisible." He then notes that each phrase of this definition contradicts a well-known assumption of economic theory, and hence concludes that auctions "are not theoretically convenient to study in terms of traditional economic processes."

An early paper, Vickery [15] (see also [16]) proposed a pareto optimal model of a dutch auction which he believed was limited to the case of a single seller offering a single unit of a commodity. In [2] Barr and Shaftel showed that the model could be extended to the sale of several different kinds of goods and several buyers. They constructed a model in which a single seller (the auctioneer) offers one unit each of several different commodities to a group of bidders each of whom wishes to buy one unit. Each bidder makes a sealed bid on each of the objects being auctioned. The auction mechanism used by Barr and Shaftel was essentially that of finding a basic optimal solution of a maximizing assignment problem whose cost entries were the bids of the buyers for the objects being sold. The primal solution determines the actual sales of goods to the buyers, while a certain special solution of the dual problem was selected to determine selling prices and buyer surpluses.

After reading [2] I became interested in extending the Barr-Shaftel model and noted that their rather clumsy mechanism for selecting the dual solution could be replaced by the perturbation (P1) described in Section 3 below. I also noticed that their model was formally equivalent to an assignment market game, in the sense of Shapley and Shubik [10]. It can be said that the present paper is based on these two excellent papers.

One of the results in [10] is that the core of an assignment mar-game has two distinguished points, the "maximum buyer surplus" and the "maximum seller surplus" points. In [14] I generalized this observation to transportation market games, showed that these two points could be computed by applying perturbations (P1) and (P2) (see the definitions in Sections 3 and 6.). I also gave an algorithm for finding the skeleton of core of the market game. I also noted, at that time that the Barr-Shaftel solution involved the price mechanism determined by the "maximum buyer surplus" point of the core.

The present paper offers the following improvements to and extensions of the Barr-Shaftel auction model. First, the use of the perturbation technique (P1) makes the determination of the prices and buyer surpluses much easier than the technique given in their paper. Second, the process is extended to a transportation model so that it is possible to consider auctions in which sellers and buyers can offer and buy multiple units of each kind of good. Third, by explicitly permitting sellers to also be buyers so that they can enter reservation bids, the symmetry of the auction procedure is improved and also it can be shown that it is pareto optimal for sellers to enter as reservation bids their true evaluations of the goods they offer for sale. Fourth, we show that with these extensions the price mechanism is that the "highest unsatisfied bidder" determines the selling price. Finally, we note that with modern computers and codes for the transportation problem, the possible use of these models for solving large auctions is now a practical reality. Such an application would be able to reduce transactions costs and decrease the time required to complete the auction process.

2. Notation and Problem Statement

The index set of the sellers is denoted by

$$I' = \{1,2,\ldots,m\} \tag{1}$$

and the index set of buyers is denoted by

$$J' = \{1,2,\ldots,n\} \quad . \tag{2}$$

We assume that seller $i \in I'$ has

$$a_i > 0 \tag{3}$$

(integer) units of a good to sell, and that buyer $j \epsilon J'$ wants to buy

$$b_j > 0 \tag{4}$$

(integer) units of that good. We assume buyer i bids

$$c_{ij} \geq 0 \tag{5}$$

for one unit of seller j's goods, i.e., we are considering a <u>bid auction</u> in which the buyer is active and the seller is passive. The nonnegativity requirement means that seller i can dispose of his goods without charge in case no one bids a positive amount for them.

In order to account for the possibility that the total amount offered may be unequal to the total amount demanded we introduce a dummy seller, m+1, and a dummy buyer, n+1. Our index sets become

$$I = I' \cup \{m+1\} = \{1,2,\ldots,m,m+1\} \tag{6}$$

$$J = J' \cup \{n+1\} = \{1,2,\ldots,n,n+1\} \quad . \tag{7}$$

We define bids for these dummy persons as

$$c_{m+1,j} = 0 \quad \text{for} \quad j \epsilon J \tag{8}$$

$$c_{i,n+1} = 0 \quad \text{for} \quad i \epsilon I \tag{9}$$

and note that (8) can be interpret (8) as a "free gift" option for the buyers, and (9) as a "free disposal" option for the sellers. We also define

$$S = \sum_{i \epsilon I'} a_i \quad \text{and} \quad T = \sum_{j \epsilon J'} b_j \tag{10}$$

and use them in turn to define the dummy sales and demands as

$$a_{m+1} = \frac{1}{2} [|T-S| + (T-S)] \tag{11}$$

$$b_{n+1} = \frac{1}{2} [|S-T| + (S-T)] \quad . \tag{12}$$

It is easy to see that at least one (and possibly both) of a_{m+1} and b_{n+1} is 0. Also if the a_i's and b_j's are integers then so will be a_{m+1} and b_{n+1}.

We now use the above definitions to state a transportation problem whose solution will be used to determine the outcome of the auction

$$\text{Maximize} \sum_{i \epsilon I} \sum_{j \epsilon J} x_{ij} c_{ij} \tag{13}$$

Subject to

$$\sum_{j \epsilon J} x_{ij} = a_i \quad \text{for} \quad i \epsilon I \tag{14}$$

$$\sum_{i \epsilon I} x_{ij} = b_j \quad \text{for} \quad j \epsilon J \tag{15}$$

$$x_{ij} \geq 0. \tag{16}$$

Because of definitions (11) and (12) it can be shown that solutions to this maximization problem always exist. If $x_i > 0$ we say seller i sells x_{ij} units to buyer j. Note that (13) says that the total value of all goods exchanged should be maximum; (14) says that all seller i's goods will be sold (to the dummy buyer, n+1, if no one else); (15) says that all buyer j's demands will be met (by the dummy seller, m+1, if no one else); and (16) says that all exchanges are from seller to buyer. Recall that if the a_i's and b_j's are integers, then so will be the x_{ij}'s; hence all property exchanges are in discrete numbers of units.

The dual transportation problem to (13)-(16) is

$$\text{Minimize } \left(\sum_{i \in I} u_i a_i + \sum_{j \in J} v_j b_j \right) \tag{17}$$

Subject to

$$u_i + v_j \geq c_{ij} \quad \text{for } i \in I,\ j \in J\ . \tag{18}$$

We will also add the nonnegativity requirements

$$u_i \geq 0 \text{ for } i \in I \text{ and } v_j \geq 0 \text{ for } j \in J\ . \tag{19}$$

It can be shown, see [14], that solutions to (17)-(19) always exist.

Our solution to a bidding and auctioning problem proceeds as follows: First collect the data as outlined by (1)-(5). Then use the definitions in (6)-(12) to set up the transportation problem in (13)-(16) and its dual in (17) and (18). Solve this problem. The primal solution, the x_{ij}'s, will determine the exchanges of goods between sellers and buyers. The dual solutions u_i and v_j, which can be shown to satisfy (19), determine the <u>unit prices</u>, u_i for seller i, and the <u>unit buyer surpluses</u>, v_j for buyer j. If $x_{ij} > 0$, that is, seller i sells to buyer j, then it is well-known by the complementary slackness condition that (18) is tight, so that

$$v_j = c_{ij} - u_i\ . \tag{20}$$

In other words, if buyer j actually buys from seller i, then his unit surplus is the difference between his evaluation of seller j's good and the price he has to pay for it. This makes economic good sense.

All of this sounds quite straightforward. However, there are difficulties with the approach due to possible degeneracies in the problem data.

First of all, the problem may be <u>dual degenerate</u>, that is (18) may be tight for nonbasis cells, indicating the possibility of alternate optimal primal solutions to the problem, and hence alternate ways of exchanging goods. We shall, for purposes of this paper, assume away this difficulty by making the assumption that the bids, the c_{ij}'s, are such that the problem is not dual degenerate; in other words, the primal solution is unique. It is well known that a slight perturbation of the bids is sufficient to rule out dual degeneracy.

The second difficulty is that the problem may be primal degenerate, that is, some $x_{ij} = 0$ when (i,j) is a basis cell. This difficulty cannot be assumed away because it depends on the "rim conditions" (3) and (4). Suppose, for example, $m = n$, $a_i = 1$ for $i \in I$, and $b_j = 1$ for $j \in J$; then the transportation problem in (13)-(18) is an assignment problem which is known to have massive primal degeneracy.

The problem stated above is called a market game, see [10] and [14]. The set of all dual solutions to (17)-(19) is called the core of the market game. An algorithm for determining all points in the core was given by the author in [14]. We shall get around the problem of primal degeneracy by selecting one of the extreme points of the core, the so-called maximum buyer surplus point, and use it to define our "recommended" or "fair bid" solution to the auctioning problem. This generalizes a similar solution concept for assignment auctions proposal by Barr and Shaftel in [2].

3. Mathematical Results

We now state some mathematical theorems which will be needed for characterizing the solution to the bid auction. All but one (Theorem 3) of these theorems were proved in [14].

By the auction core we shall mean the set of all nonnegative solutions to the dual transportation problem of the preceding section; that is, the auction core is the set of all solutions to (17), (18), and (19).

Theorem 1. The auction core is a non-empty bounded, convex, polyhedral set. All core solutions satisfy $u_{m+1} = v_{n+1} = 0$.

Proof. See [14].

We shall denote the auction core by $C = (C(U),C(V))$ where $C(U)$ is the set of row dual solutions which we call the seller core, and $C(V)$ is the set of all column dual solutions which we call the buyer core.

Let us apply the following perturbation to the transportation problem and its dual given in (13)-(18):

$$(\mathrm{P1})\quad \begin{cases} a_i \to a_i + \epsilon \text{ for } i\epsilon I', \; a_{m+1} \to a_{m+1} \\ b_j \to b_j \text{ for } j\epsilon J', \; b_{n+1} \to b_{n+1} + m\epsilon \\ \text{where } 0 < \epsilon < \dfrac{1}{2(m+1)} \quad . \end{cases}$$

As is well-known the perturbed problem is primal non-degenerate; also the primal solution $X(\epsilon)$ to the perturbed problem, when scientifically rounded, yields an optimal integer primal solution to the original problem.

<u>Theorem 2</u>. The dual solutions (u^o, v^o) to (17)-(19) after the perturbation (P1) has been applied belong to the auction core and also satisfy

$$u_i^o = \min_{U\epsilon(U)} u_i \tag{21}$$

$$v_j^o = \max_{V\epsilon(V)} v_j \quad . \tag{22}$$

Proof. See [14].

Because of (22) we shall call this solution the <u>maximum buyer surplus solution</u>.

Consider the solution of the primal transportation problem (13)-(16) after perturbation (P1) has been applied. Let $X(\epsilon)$ be the optimal solution to the perturbed problem, and let x_{ij} be the optimal shipping amount for cell (i,j). Define $R(x_{ij})$ to be an integer equal to the scientifically rounded value of x_{ij} and define

$$\epsilon_{ij} = x_{ij} - R(x_{ij}) \quad . \tag{23}$$

If $\epsilon_{ij} > 0$ for any i and j, we shall say that cell (i,j) is an <u>unsatisfied cell</u>, since the integer solution obtained by scientifically rounding $X(\epsilon)$ gives cell (i,j) less than x_{ij}. Otherwise cell (i,j) is a <u>satisfied</u> cell.

<u>Theorem 3</u>. Let $X(\epsilon)$ be the optimal solution to (13)-(16) after perturbation (P1) has been applied; then in each row $i\epsilon I'$ there is exactly one unsatisfied cell; moreover, price u_i, which is equal to

$$u_i = c_{ij} - v_j \tag{24}$$

is determined from (24) by the bid c_{ij} and buyer surplus v_j at the unsatisfied cell.

<u>Proof</u>. Let B be the optimal basis for $X(\epsilon)$, i.e., B is a set of

m+n+1 cells of the transportation tableau, and $G = (I \cup J, B)$ is a tree (see [5,6]). A basis cell which is unique in its column or its row corresponds to a pendant node of G. Every tree having more than one node has a pendant node, that is a node having exactly one edge of the tree adjacent to it. Consider a basis cell which is unique in its column, so that the column is a pendant node. Since b_j is an integer we must have $R(x_{ij}) = b_j = x_{ij}$ so that (i,j) is a satisfied cell. Replace a_i by $a_i - x_{ij}$ and b_j by $b_j - x_{ij}$ and cross out column j. The result is a transportation problem with one fewer column and whose solution indicated by the remaining basis cells still optimal. Repeat this process until no columns remain having only one basis cell. Now either all rows and columns of the problem have been crossed out or else there is a row with a unique basis cell (i,j), i.e., i is a pendant node. It is easy to see that if $i \in I'$ then cell (i,j) is an unsatisfied because of the form of the original perturbation. In the latter case replace a_i by $a_i - x_{ij}$ and b_j by $b_j - x_{ij}$ and cross out row i. Continue this process until all rows and columns of the problem have been crossed out, then stop. Since every row and every column is crossed out exactly once and since all pendant columns correspond to satisfied cells, and each row $i \in I'$ corresponds to an unsatisfied cell, we see that in each row $i \in I'$ there is exactly one unsatisfied cell. The fact that the price u_i is determined by the unsatisfied cell in row i follows from the fact that in the tree $G = (I \cup J, B)$ the path from row m+1 for which $u_{m+1} = 0$ and row i includes the unsatisfied cell (i,j) and no other basis cell in row i. This completes the proof.

If we say that j is an unsatisfied buyer provided cell (i,j) is an unsatisfied cell for any i, we can interpret Theorem 3 as follows: Each seller sells to some satisfied buyers and exactly one unsatisfied buyer; moreover, each seller's price is determined by net bid $(c_{ij}-v_j)$ of his unsatisfied buyer.

4. The Fair Bid Auction

As we have seen, the solution to the primal transportation problem (13)-(16) determines the exchanges of goods in the auctioning procedure described above. For purposes of expository simplicity we assume the primal solution is unique, but this assumption is not essential. To find the actual exchange prices we must select one of a possibly huge number of solutions to the dual problem (17)-(19).

In the present section we shall propose an easily computable procedure which yields a solution to an auction having a number of desirable properties from both the seller's and the buyer's points of view. It generalizes in several ways the Barr-Shaftel auction model presented in [2]. Although it is an ad hoc procedure in many respects, an effort was made in its design to balance each advantage of a buyer by one of a seller, and vice versa. These advantages are listed after the description of the auction procedure is given.

The Fair Bid Auction Procedure.

(1) Determine the set of sellers I' and the set of buyers J', where it is assumed that every seller is also listed as a buyer in order that each seller can make reservation bids on the goods he has for sale.

(2) Determine the amounts for sale, a_i for $i \epsilon I'$, the amounts demanded, b_j for $j \epsilon J'$, and the bids c_{ij} by buyer $j \epsilon J'$ for the goods seller $i \epsilon I'$ has for sale.

(3) Set up the transportation problem (13)-(16) with costs c_{ij} and rims a_i and b_j; add the slack row and column; perform perturbation (P1) and solve using any transportation algorithm such as the MODI method ([5,6]).

(4) Announce the solution:

  (a) For each $i \epsilon I, j \epsilon J$ such that $x_{ij} > 0$ announce $R(x_{ij})$ which gives the exchanges of goods between sellr i and buyer j; if seller i and buyer j are the same person, then the seller has "sold" himself his own goods for his reservation bid; i.e. he keeps his goods; otherwise there is an actual physical exchange of goods.

  (b) Announce the selling prices u_i which the successful buyer must pay the seller.

  (c) In order to show that the auction mechanism is "honest" it may also be desirable to announce the "highest unsatisfied buyer" for each seller's goods, and have him verify the correctness of the selling price.

  (d) Sometimes, but probably not usually, it may also be desirable to announce the buyer and seller surpluses.

The first remark concerning the fair bid auction procedure just described is that it is a practical procedure for auctions having hundreds or thousands of buyers and sellers. Once the data stated in (2) has been collected the solution of the transportation problem (13)-(16) can be carried out in a few seconds or minutes using one of the recent fast codes [4,7,12] and a modern computer.

The important characteristics of the fair bid auction are listed in Figure 1. Note that buyers and sellers have parallel characteristics. The correctness of properties (b), (c), and (d) for both was discussed in Section 3. However, the pareto optimal properties listed in (a) of both columns in Figure 1 deserve further comment.

Let us expand on the assertion that buyer i should make the quantity c_{ij} be his true evaluation of good j. The argument is contrapositive; for suppose buyer i's true evaluation is c^*_{ij} and he bids $c_{ij} < c^*_{ij}$.

Buyers	Sellers
(a) It is pareto optimal for each buyer to bid his true evaluation of each good offered for sale.	(a) It is pareto optimal for each seller to enter his true reservation price for each good he sells.
(b) The solution to the fair bid auction maximize the total value of all buyer surpluses.	(b) The solution to the fair bid auction maximizes the total valuation of all objects sold.
(c) No buyer pays more than his evaluation for any good he buys.	(c) No seller sells a good for less than his reservation price for that object.
(d) Some buyers don't buy all that they want.	(d) Some sellers don't sell all that they offer.

Figure 1. Properties of the Fair Bid Auction

It is then possible for another buyer, say buyer k, to bid c_{ik} for good i, where $c_{ij} < c_{ik} < c^*_{ij}$, and buy good i at a price $u_i \leq c_{ik} < c^*_{ij}$ which is less than buyer j's true evaluation for good i. A similar argument holds if buyer j bids $c_{ij} > c^*_{ij}$, except that in this case he may be forced to pay more than his true evaluation of good i.

Although no organization presently exists to carry out auctions by the fair bid method, it is possible that some day one will be set up to do so. The purpose of this paper is to point out that it is currently practical and it may be desirable to do so.

5. Example

We illustrate the auction mechanism of the previous section by solving the "house buying" example of Shapley and Shubik [10]. The problem concerns three sellers and three buyers of houses. Their respective valuations of each house are shown in Figure 2.

Houses	Seller's Evaluation	Buyer's Evaluations		
		Buyer 1	Buyer 2	Buyer 2
Seller 1	\$18,000	\$23,000	\$26,000	\$20,000
Seller 2	15,000	22,000	24,000	21,000
Seller 3	19,000	21,000	22,000	17,000

Figure 2. Buyer and seller evaluations of houses for the Shapley-Shubik example in [10].

The corresponding transportation problem, with reservation bids by the sellers and perturbation (P1) applied with $\epsilon = .01$, is shown in Figure 3. Note that the optimal basis cells with corresponding shipping

		Buyers			Sellers			Slack	
		$v_1=2$	$v_2=5$	$v_3=1$	$v_4=0$	$v_5=0$	$v_6=0$	$v_7=0$	
Sellers	$u_1=21$	$23^{.01}$	26^{1}	20	18	0	0	0	1.01
	$u_2=20$	$22^{.01}$	24	21^{1}	0	15	0	0	1.01
	$u_3=19$	$21^{.98}$	22	17	0	0	$19^{.03}$	0	1.01
Slack	$u_4=0$	0	0	0	0^{1}	0^{1}	$0^{.97}$	$0^{.03}$	3
		1	1	1	1	1	1	.03	

Figure 3. Transportation tableau for house buying example.

amounts are marked on the tableau. The optimal exchange is: seller 1 sells to buyer 2 for a price of \$21,000 giving seller 1 a surplus of \$3,000 and buyer 2 a surplus of \$5,000; seller 2 sells to buyer 3 giving seller 2 a surplus of \$5,000 and buyer 3 a surplus of \$1,000; and seller 3 sells to buyer 1 giving seller 3 a surplus of \$0 and buyer 1 a surplus of \$2,000. Note that buyer 1 is the unsatisfied buyer for sellers 1 and 2, while seller 3, by means of his own reservation bid, is his own unsatisfied buyer!

The reader may wish to change the valuations by various buyers and sellers and see how the solution changes. It is particularly interesting to raise one or more of the seller's reservation bids so high that he is unable to sell his house.

6. Other Auction Mechanisms

The "Fair Bid" auction mechanism is only one of a huge number of price mechanisms which is consistent with the transportation auction model. The convex set of all extreme points of the auction core can be generated by the algorithm given by the author in [14]. For instance, the auction core for the example in Section 5 has 6 extreme points [10, 14], only one of which is shown in Figure 2.

Another auction core solution that may be preferred by the sellers is the maximum seller surplus solution which can be obtained by applying the following perturbation to the transportation problem in (13)-(16)

$$(P2)\begin{cases} a_i \to a_i \text{ for } i\epsilon I', \; a_{m+1} \to a_{m+1} + n\epsilon \\ b_j \to b_j + \epsilon \text{ for } j\epsilon J', \; b_{n+1} \to b_{n+1} \\ \text{where } 0 < \epsilon < \dfrac{1}{2(n+1)} \end{cases}$$

Properties similar to those derived in Section 3 can be shown to exist here, see [14]. Moreover, this solution would be particularly applicable to an "offer" auction in which the c_{ij}'s in (5) are offers by the seller to the buyer. Here the buyer is passive and the seller is active. Such "auctions" occur frequently in the bidding for construction projects.

Clearly if the auction is being managed by the sellers, they may wish to impose the maximum seller surplus solution, while an auction being run by buyers may wish to impose the maximum buyer surplus solution.

Some auctions are run by governments, and they may wish to avoid either of these two extreme solutions in order not to antagonize any of their constituency. They could, for instance, choose the mid point of the line segment connecting the maximum buyer and seller solutions; and many other choices are also possible.

The problem here is not in showing that auction solutions exist, but rather in the selection, from the many possible such solutions, one that satisfies other desirable criteria.

7. Applications

Since computational techniques exist (see [4,7,12]) for solving very large transportation problems very quickly, the auction mechanisms proposed in this paper are immediately applicable to a large variety of auctioning situations where they are not presently being used. For instance, it would be possible to develop specific auction procedures for exchanging stocks, bonds, real estate, commodities, etc. The difficulty in making such an application is that of getting the buyers and sellers to agree on the use of this exchange mechanism. The advantages of doing so are many and include rapidity of exchange, pareto optimal solutions in some cases, low cost of operation of the auctioning mechanism, etc.

Another problem also exists to determine whether any existing auction mechanisms are, in fact, one of those discussed in this paper. This is a descriptive rather than a normative question. One candidate for such an auction is the Dutch flower market where single lots of flowers are sold by a "clock" that starts at a high price and works its way lower. The first rejected bid price mechanism is used.

Given the fact that the Canadian government already uses a highly mechanised system for selling hogs ([8]), it is entirely possible that the auction models proposed here may some day be put into practical use in a normative as well as a descriptive sense.

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This paper generalizes the Barr-Shaftel auctioning model in several ways. ~~(a)~~ It shows that the dual solutions they choose for prices and buyer surpluses is also the maximum buyer surplus solution. ~~(b)~~ It shows that this solution can easily be found by means of a perturbation technique and relates the solution to core solutions of assignment market games, in the sense of Shapley and Shubik. (c) It extends these models and theoretical results from an assignment to a transportation model. (d) By adding seller reservation bids the symmetry of the auction process is increased in that it becomes (over)

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pareto optimal for sellers as well as buyers. (e) It proposes a "fair bid" auctioning process which has pareto optimality properties for both buyers and sellers, and which can be solved rapidly for problems having hundreds or thousands of buyers and sellers. (f) Finally, it suggests that practical applications of these models to real auctioning situations is possible. Such applications could reduce the transaction costs and improve the speed of auction processes.