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THE IMPACTION FORCE OF AIRBORNE PARTICLES ON SPHERES AND CYLINDERS (U)

by

Stanley B. Mellsen

Project No. 20-90-03

September 1978

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ABSTRACT

The effect of dust on aerodynamic drag of spheres and cylinders was calculated by using a mathematical model developed for this purpose. The results were compared to experiments previously done by other workers. The calculated and experimental results agree favourably, showing that the mathematical model is satisfactory. Impaction efficiencies and drag coefficients due to dust alone were then obtained using the model for a wide range of the inertia parameter and the results are presented graphically. The model can also be used for calculating velocity distributions and points of impact for a stream of airborne particles flowing over a sphere or cylinder.

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NOTATION

A	frontal area of target, cm^2
A_d	far upstream cross sectional area of the envelope of particles which eventually hit the sphere or cylinder, cm^2
a	tube radius, cm
C_D	dimensionless drag coefficient for spheres or cylinders in air
C_{DC} or C_{DS}	drag coefficient of cylinder or sphere due to stream of particles alone
C_{De}	total effective drag coefficient for spheres or cylinders due to particles and air together
C_{DL}	drag coefficient for spheres in tubes adjusted to a sphere in free space by the Landenburg correction
C_{DpU}	drag coefficient for particles alone adjusted from v_{rms} to U
d	particle diameter, cm
E_m	impaction efficiency of particles on a target
F	force exerted on a target by the air and total stream of particles hitting it, dynes
F_a	force exerted on a target by the air alone, dynes
F_s or F_c	force exerted on the sphere or cylinder by the total stream of particles which hit it, dynes
F_p	force exerted on a target by the particles alone travelling in air, dynes
L	cylinder or sphere radius, cm
m'	total mass of particles which had its momentum changed in unit time, $g\ s^{-1}$
p_s or p_c	drag pressure on sphere or cylinder due to stream of particles alone, dynes cm^{-2}
r	distance from any point to the origin, cm
t	time, seconds
U	free-stream velocity, cm s^{-1}
u	local fluid velocity, cm s^{-1}

u_r	radial component of fluid velocity, cm s^{-1}
u_θ	circumferential component of fluid velocity, cm s^{-1}
v	local particle velocity, cm s^{-1}
v_{rms}	root-mean-square free stream particle speed upstream near target position, cm s^{-1}
v_x	component of particle velocity parallel to free field flow direction immediately before impact, cm s^{-1}
v_x'	component of particle velocity parallel to flow direction immediately after impact and reflection, cm s^{-1}
v_{xo}	far upstream particle velocity, cm s^{-1}
v_y	component of particle velocity perpendicular to free field flow direction immediately before impact, cm s^{-1}
Δv_x	change in the component of particle velocity parallel to the free stream flow direction caused by reflection from the sphere or cylinder, cm s^{-1}
x	co-ordinate (origin at centre of sphere or cylinder) of particle position parallel to free stream flow direction, cm
y	transverse co-ordinate of particle position, cm
y'	off-axis distance of a particle at point of impact with the sphere or cylinder, cm
y_c	far upstream transverse co-ordinate of the envelope of particles which eventually hit the sphere or cylinder, cm
α	angle of incidence of a particle against the sphere or cylinder, radians
β	angle between particle direction and free field flow direction just before the particle hits the sphere or cylinder, radians
γ	total angle change of a particle from its far upstream direction to its direction after reflection from the cylinder or sphere, radians
θ	polar angle between x axis and radius vector to particle position, radians
μ	absolute velocity of fluid, poise
ρ	fluid density, g cm^{-3}
ρ^*	total mass of particles and air per unit volume of space, g cm^{-3}
ρ_p	uniform particle density per unit volume of air far upstream, g cm^{-3}

σ	particle density, g cm^{-3}

The following were dimensionless:

$f_{(\bar{y})}$	change in component of particle velocity due to reflection from sphere or cylinder as a function of off-axis distance far upstream
K	particle inertia parameter
Re	spherical particle Reynolds number in flow influenced by presence of sphere or cylinder
Re_o	spherical particle Reynolds number in free stream
$\bar{r}$	$\frac{r}{L}$, distance from any point to the origin
T	time $\frac{tU}{L}$
$\bar{u}$	local fluid velocity $\frac{u}{U}$, $\bar{u}_x$ and $\bar{u}_y$ are x and y components
$\bar{v}$	local particle velocity $\frac{v}{U}$, $\bar{v}_x$ and $\bar{v}_y$ are x and y components
$\bar{v}_x$	$\frac{d\bar{x}}{dT}$, parallel component of particle velocity
$\bar{v}_y$	$\frac{d\bar{y}}{dT}$, transverse component of particle velocity
$\bar{x}$	parallel co-ordinate $\frac{x}{L}$
$\bar{y}$	transverse co-ordinate $\frac{y}{L}$
$\bar{y}_d$	far upstream transverse co-ordinate of the envelope of particles which eventually hit the cylinder
$\bar{y}_i$	co-ordinate of particle used in discretization for integration; y_i increases with i from $\bar{y}_i = o$ to $\bar{y}_n = \bar{y}_d$
ϕ	dimensionless group independent of drop size formed by combining Re_o and K

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1. INTRODUCTION

Extensive studies have been made of the drag exerted on spheres and transverse cylinders by moving air (Schlicting, 1960) and attempts have been made to determine the effect of introducing dust into the air stream (Gillespie and Gunter, 1957; 1959). Also, the trajectories of water drops moving in the neighborhood of a circular cylinder placed in a uniform stream of air have been calculated by numerical solution of the equations of motion (Glauert, 1940) and with a mechanical analog (Brun et al., 1953). The latter method has been used in conjunction with flight instruments used to study droplet size and distribution in icing clouds (Brun and Mergler, 1953). Measurements of aerodynamic drag on circular cylinders due to the blast wave from large TNT bursts have been done at DRES (Mellsen, 1974). The results of these tests have been used to provide blast loading input for structural analysis of lattice type masts also tested in the trials. On some of the trials there have been visible quantities of dust associated with the blast wave. In general the measured strains in the masts have agreed favorably in an individual field trial with the ones predicted from theoretical analysis based upon the blast loading input from the drag data averaged over several blast trials (Laidlaw, 1977). The influence of dust is difficult to determine and generally considered negligible.

Another area of interest in connection with moving particles in an air stream is the impingement of liquid drops on protective clothing and military equipment. The impaction efficiency and force, and the droplet distribution are all of interest. The study of these effects on transverse cylinders and spheres is useful in this application.

The purpose of this report is to describe numerical solutions to the equations of motion of particles in a moving air stream, with possible applications to both the effect on aerodynamic drag from the point of view of blast loading of structures and the interaction of droplets with spheres and cylinders from a chemical defence point of view. The mathematical techniques along with their associated computer programs are described herein. Also, the results are compared with experiments done at DRES (Gillespie and Gunter, 1957).

2. DEFINITION OF THE PROBLEM

A spherical particle flowing at free stream velocity far upstream of a spherical or transverse cylindrical solid target will not necessarily follow a streamline in the vicinity of the target where the radial and axial velocity components of the fluid may be changing markedly. A particle that just impinges on the outer circumference of the target was considered. All particles of the same diameter within an envelope generated by this limiting particle trajectory would hit the target, assuming that no particles collide with other particles reflected from the target.

All particles in the steady flow field far upstream were assumed to be travelling at the velocity of the fluid but the velocities of the particles will differ from the fluid velocity as the particles approach the target. This is due to inertial effects of the particles in the flow field near the object. The velocity of a particle immediately before contact, and also the point of contact with the target, depend upon the position of the particle within the limiting envelope far upstream from which it travels to the target. These particles all have their momentum changed when they collide with the target and in the case of solid particles are deflected in a new direction. The total change in momentum is balanced by the drag force exerted by the particles on the target.

The basic problem was, therefore, to determine the envelope of particles hitting the target and the change in particle motion due to the presence of the target in the flow. Then the drag coefficient due to particles alone could be obtained.

3. EQUATIONS OF MOTION

The motion of an individual spherical particle has been shown (Batchelor, 1967) to be determined by the following ordinary differential equations:

$$\frac{d\bar{v}_y}{dT} = \frac{C_D Re(\bar{u}_y - \bar{v}_y)}{24K} \qquad \text{(Eq. 1)}$$

$$\frac{d\bar{v}_x}{dT} = \frac{C_D Re(\bar{u}_x - \bar{v}_x)}{24K} \qquad \text{(Eq. 2)}$$

where

$$Re = Re_o[(\bar{u}_y - \bar{v}_y)^2 + (\bar{u}_x - \bar{v}_x)^2]^{\frac{1}{2}} \qquad \text{(Eq. 3)}$$

$$K = \frac{\sigma d^2 U}{18\mu L} \text{ particle inertia parameter} \qquad \text{(Eq. 4)}$$

$$Re_o = \frac{Ud\rho}{\mu} \text{ free stream Reynolds number} \qquad \text{(Eq. 5)}$$

The symbols are defined in the notation section near the front of this report and the basic geometry of the flow system is illustrated in Fig. 1.

Several assumptions were inherent in the development of Eq. 1 and Eq. 2 for calculating impaction efficiency and force due to a stream of particles, including,

(a) uniform particle distribution,

(b) no gravitational or electrostatic forces of consequence,

(c) monodisperse spherical particles with diameter very small in relation to the target sphere or cylinder diameter, and

(d) free stream flow that was steady, incompressible and irrotational.

The drag coefficient is a function of Reynolds number and was available in the form of definitive empirical equations (Davies, 1945). These equations are stated as follows:

$$Re = \frac{C_D Re^2}{24} - 2.3363 \times 10^{-4}(C_D Re^2)^2 + 2.0154 \times 10^{-6}(C_D Re^2)^3 - 6.9105 \times 10^{-9}(C_D Re^2)^4 \qquad \text{(Eq. 6)}$$

for $Re < 4$ or $C_D Re^2 < 140$

$$\log_{10} Re = -1.29536 + 9.86 \times 10^{-1} (\log_{10} C_D Re^2) - 4.6677 \times 10^{-2}(\log_{10} C_D Re^2)^3 + 1.1235 \times 10^{-3} (\log_{10} C_D Re^2)^3 \qquad \text{(Eq. 7)}$$

for $3 < Re < 10^4$ or $100 < C_D Re^2 < 4.5 \times 10^7$

4. AIR FLOW FIELD

The assumption was made that the air flow in front of the cylinder or sphere is given by classical hydrodynamics theory.

a) Cylinder

The equations of fluid velocity were derived from the stream function for ideal flow around a cylinder (Milne-Thomson, 1960). These were normalized to the free stream velocity and cylinder diameter and written as follows:

$$\bar{u}_x = 1 - \frac{\bar{x}^2 - \bar{y}^2}{\bar{r}^4} \qquad \text{(Eq. 8)}$$

$$\bar{u}_y = 1 - \frac{\bar{x}\,\bar{y}}{\bar{r}^4} \qquad \text{(Eq. 9)}$$

b) Sphere

For purposes of comparison of the results of the theoretical work reported herein to existing experimental data the flow field about a sphere on the axis of a circular tube was applied. The flow about a sphere in the absence of the tube was then readily available from this as a limiting case.

The flow velocity was derived (Mellsen et al., 1966) from the vector potential (Smythe, 1964). The velocity components are given in terms of a series solution. For a sphere diameter of 0.8 cm and tube diameter of 2.54 cm used in the experiments (Gillespie and Gunter, 1957) sufficient accuracy (Mellsen et al., 1966) was obtained by retaining three terms of the series.

The equations were

$$\frac{u_r}{U} = - \cos \Theta \left[1 - \frac{2C_0}{3} \frac{C}{r}^{3} - \frac{2A_0}{a} + \cdots \right] \qquad \text{(Eq. 10)}$$

$$\frac{u_\theta}{U} = \sin \Theta \left[1 + \frac{C_0}{3} \frac{C}{r}^{3} - \frac{2A_0}{a} + \cdots \right] \qquad \text{(Eq. 11)}$$

The constants C_0, C and A_0 which are dependent upon the sphere and tube diameters (Smythe, 1964) are given as follows:

$C_0 = 1.5401075$

$C = 1.0$ (normalized sphere radius)

$A_0 = -C\left(\frac{C}{a}\right)^2 \frac{I(2)C_0}{9\pi}$, $I(2) = 7.5098907$

$a = \frac{2.54}{0.8} C$ (normalized tube radius)

u_θ and u_r are shown in Fig. 2.

Converting to rectangular co-ordinates gives

$$\bar{u}_x = - \frac{u_r}{U} \cos \Theta + \frac{u_\theta}{U} \sin \Theta \qquad \text{(Eq. 12)}$$

$$\bar{u}_y = \frac{u_r}{U} \sin \Theta + \frac{u_\theta}{U} \cos \Theta \qquad \text{(Eq. 13)}$$

The equation for flow around a sphere in the absence of the tube is given by letting the value of "a" approach infinity.

For $a \rightarrow \infty$, $C_0 \rightarrow 1.5$ (Smythe, 1964)

Eq. 10 becomes $$\frac{u_r}{U} = -\cos \Theta \left[1 - \left(\frac{L}{r}\right)^3\right] \qquad \text{(Eq. 14)}$$

Eq. 11 becomes $$\frac{u_\theta}{U} = \sin \Theta \left[1 + \frac{1}{2}\left(\frac{L}{r}\right)^3\right] \qquad \text{(Eq. 15)}$$

Eqs. 14 and 15 are well known in ideal flow theory (Batchelor, 1967) and therefore serve as a partial check on the correctness of Eqs. 10 and 11.

5. VELOCITY CHANGE OF AN INDIVIDUAL PARTICLE

To calculate the drag force due to airborne particles, the velocity change of an individual particle due to the presence of the cylinder or sphere in the flow was first considered. The velocity change determined applied to both the cylindrical and spherical targets because the same geometry could be used for both (Fig. 3).

The motion considered concerned a spherical particle which starts far upstream within the envelope of particles which travel to the target, eventually hit it and are reflected by it in a new direction. The total change in direction of a particle which comes from a far upstream off-axis position y, and hits the target at a point defined by y' (Fig. 3), is represented by γ. For a spherical particle the angle of incidence, α, is equal to the angle of reflection. Then

$$\gamma = \left(\frac{\pi}{2} - \Theta\right) + \alpha \qquad \text{(Eq. 16)}$$

where

$$\Theta = \sin^{-1}\left(\frac{y'}{L}\right) = \tan^{-1}\left(\frac{y'}{\sqrt{L^2 - (y')^2}}\right) \qquad \text{(Eq. 17)}$$

and $$\alpha + \beta = \frac{\pi}{2} - \Theta \qquad \text{(Eq. 18)}$$

where $$\beta = \tan^{-1}\left(\frac{v_y}{v_x}\right) \qquad \text{(Eq. 19)}$$

and v_x and v_y are the components of particle velocity just before impact with the target. Substituting the value of α obtained from Eq. 18 into Eq. 16 gives

$$\gamma = \pi - 2\Theta - \beta \qquad \text{(Eq. 20)}$$

Substituting Eqs. 17 and 19 into Eq. 20 gives an equation of γ suitable for computation. Thus:

$$\gamma = \pi - 2\tan^{-1}\left(\frac{y'}{\sqrt{1 - y'^2}}\right) - \tan^{-1}\left(\frac{v_y}{v_x}\right) \qquad \text{(Eq. 21)}$$

or in non-dimensional terms

$$\gamma = \pi - 2\tan^{-1}\left(\frac{\bar{y}'}{\sqrt{1 - \bar{y}'^2}}\right) - \tan^{-1}\left(\frac{\bar{v}_y}{\bar{v}_x}\right) \qquad \text{(Eq. 21a)}$$

The component of velocity in the direction of free field flow immediately after particle reflection is

$$v_x' = v \cos \gamma \qquad \text{(Eq. 22)}$$

where $$v = \sqrt{v_x^2 + v_y^2} \qquad \text{(Eq. 23)}$$

or in non-dimensional terms $\bar{v}_x' = \bar{v} \cos \gamma$ (Eq. 22a)

and $$\bar{v} = \sqrt{\bar{v}_x^2 + \bar{v}_y^2} \qquad \text{(Eq. 23a)}$$

The total change of the component of velocity in the direction of free stream flow due to reflection from the target is then given by the following relationship:

$$\Delta v_x = v_x - v \cos \gamma \qquad \text{(Eq. 24)}$$

6. THE DRAG FORCE DUE TO PARTICLES ALONE

The force F_p of a stream of particles hitting a cylinder or sphere is related to the change in momentum which in general is given by the following relationship:

$$F_p = m'\Delta v_x \qquad \text{(Eq. 25)}$$

where m' is the total mass of particles having its momentum changed per unit time and Δv_x is its change of velocity in the free stream flow direction.

The quantity m' can be calculated using the following equation:

$$m' = \rho_p A_d v_{xo} \qquad \text{(Eq. 26)}$$

where ρ_p is the uniform particle density per unit volume of air far upstream, A_d is the far upstream cross sectional area of the envelope of particles which eventually hits the sphere or cylinder, and v_{xo} is the far upstream particle velocity which is assumed equal to the fluid velocity. Combining Eqs. 25 and 26 gives

$$F = \rho_p A_d v_{xo} \Delta v_x \qquad \text{(Eq. 27)}$$

Δv_x was not constant but was shown in the previous section to be a function of the off-axis, upstream starting location of an individual particle. This can be accounted for by integrating the velocity change over the range of starting locations across the particle envelope. Then the equation of force becomes

$$F = \rho_p v_{xo} \int_0^{A_d} \Delta v_x dA \qquad \text{(Eq. 28)}$$

where dA is an element of cross sectional area at location y.

Assuming that the particles do not interact with each other, the equation of drag force due to particles alone is then obtained in usable form by substituting Eq. 24 into Eq. 28 which gives the following equation:

$$F_p = \rho_p v_{xo} \int_0^{A_d} (v_x - v \cos \gamma) dA \qquad \text{(Eq. 29)}$$

7. THE DRAG COEFFICIENT

(a) Cylinder

For a transverse cylinder of unit length, Eq. 29 becomes

$$F_s = 2\rho_p v_{xo}\int_0^{y_c} (v_x - v\cos\gamma)dy \qquad \text{(Eq. 30)}$$

In non-dimensional terms this is written

$$\frac{F_s}{\rho_p U^2 L} = 2\bar{v}_{xo}\int_0^{\bar{y}_d} (\bar{v}_x - \bar{v}\cos\gamma)d\bar{y} \qquad \text{(Eq. 31)}$$

The drag coefficient is defined as follows:

$$C_D = \frac{p}{\frac{1}{2}\rho_p U^2} \qquad \text{(Eq. 32)}$$

where p is the average drag pressure (Schlicting, 1960).

The average drag pressure for a cylinder of unit length is given by the following equation:

$$p_c = \frac{F}{2L} \qquad \text{(Eq. 33)}$$

Substituting Eq. 33 into Eq. 32 and the resulting equation into Eq. 32 gives

$$C_{DC} = 2\bar{v}_{xo}\int_0^{\bar{y}_d} (\bar{v}_x - \bar{v}\cos\gamma)d\bar{y} \qquad \text{(Eq. 34)}$$

b) Sphere

For a sphere Eq. 29 becomes

$$F_s = 2\pi\rho_o v_{xo}\int_0^{y_c} (v_x - v\cos\gamma)y\,dy \qquad \text{(Eq. 35)}$$

In non-dimensional terms this is written

$$\frac{F_s}{\rho_p U^2 L^2} = 2\pi\bar{v}_{xo}\int_0^{\bar{y}_d} (\bar{v}_x - \bar{v}\cos\gamma)\bar{y}\,d\bar{y} \qquad \text{(Eq. 36)}$$

The drag pressure on the sphere is given by

$$p_s = \frac{F_s}{\pi L^2} \qquad \text{(Eq. 37)}$$

Substituting Eq. 36 into Eq. 37 and the resulting equation into Eq. 32 gives the drag coefficient for a sphere as follows:

$$C_{Ds} = 4\bar{v}_{xo}\int_{o}^{\bar{y}_d}(\bar{v}_x - \bar{v}' \cos \gamma)\bar{y}\, d\bar{y} \qquad \text{(Eq. 38)}$$

8. INTEGRATION METHOD FOR THE DRAG COEFFICIENT EQUATIONS

a) Cylinder

Let $f(\bar{y}) = \bar{v}_x - \bar{v} \cos \gamma$ (Eq. 39)

Then Eq. 33 can be written

$$C_D = 2\bar{v}_{xo}\int_{o}^{\bar{y}_d} f(\bar{y})\, d\bar{y} \qquad \text{(Eq. 40)}$$

$\int_{o}^{\bar{y}_d} f(\bar{y})\, d\bar{y}$ was integrated numerically by dividing the upstream cross sectional area into strips of equal width Δy, and calculating the contribution from each strip. These contributions were then summed as follows:

$$\int_{o}^{\bar{y}_d} f(\bar{y})\, d\bar{y} = \frac{f(\bar{y}_1) + f(\bar{y}_n)}{2} + \sum_{i=2}^{i=n-1} f(\bar{y}_i)\Delta\bar{y} \qquad \text{(Eq. 41)}$$

where y_i increases with i from $\bar{y}' = o$ to $\bar{y}_n = \bar{y}_d$.

b) Sphere

Using Eq. 39 in Eq. 38 gives the following equation for the drag coefficient of a sphere

$$C_{Ds} = 4\bar{v}_{xo}\int_{o}^{\bar{y}_d} f(\bar{y})\, \bar{y}\, d\bar{y} \qquad \text{(Eq. 42)}$$

$\int_0^{\bar{y}_d} f(\bar{y})\, \bar{y}\, d\bar{y}$ was integrated numerically by dividing the upstream cross sectional area into concentric annuli of equal width $\Delta\bar{y}$ and calculating the contribution from each ring. These contributions were then summed as follows:

$$\int_0^{\bar{y}_d} f(\bar{y})\, d\bar{y} = \sum_{i=1}^{n} i\ f(\bar{y}_i)\ \bar{y}_i\ \Delta\bar{y} \qquad \text{(Eq. 43)}$$

where y_i increases with i from $\bar{y}_i = \Delta\bar{y}$ to $\bar{y}_n = \bar{y}_d$

9. SOLUTION OF THE DRAG COEFFICIENT EQUATIONS

To solve Eq. 34 or Eq. 38 for the drag coefficient it was necessary to know the values of $\bar{v}_x$, $\bar{v}'$ and γ in the range $0 \le \bar{y} \le \bar{y}_d$.

The first step was to find the value of $\bar{y}_d$ which was the upper limit of the integral. This was done by an iterative procedure called the half interval method (Carnahan et al., 1969). The value of $\bar{y}$ for the critical particle was estimated far upstream and the path followed to the target. The difference between the ordinate of the target path and the ordinate of the point on the actual path parallel to the tangent path was the miss criterion used. The direction of the tangent path was not known a priori but was assumed parallel to the actual path. The half interval method previously mentioned was applied to determine a better initial estimate. Then the path was followed to the target again for another calculation of miss distance. This process was repeated several times until sufficient accuracy was achieved. The plane of initial position of particles which was perpendicular to the flow direction was located far enough from the target so that free stream conditions prevailed. A distance of five target radii upstream of the target centre was considered adequate (Batchelor, 1967).

The path of an individual particle was determined step by step by applying a fourth order Runge-Kutta method (Carnahan et al., 1969) to the equations of motion (Eqs. 1 and 2). The values of Re and K in these equations were easily determined for each new step by direct substitution of previously determined values into Eqs. 3, 4 and 5, but the value of C_DRe in Eqs. 1 and 2 had to be calculated in each step by numerical solution of the definitive empirical equations (Eqs. 6 and 7). This was done using Newton's Method (Carnahan et al., 1969) for finding the zeros of a function. The values of $\bar{u}_x$ and $\bar{v}_y$ in Eqs. 1, 2 and 3 were calculated from Eqs. 8 and 9 for a cylindrical target and Eqs. 10, 11, 12 and 13 for a spherical target.

Once the value of $\bar{y}_d$ had been determined the integration procedures described in Section 8 of this report were applied. The values of $f(\bar{y}_i)$ used in Eqs. 41 and 43 were obtained from Eq. 39. The quantities $\bar{V}$ and γ used in the latter equation were calculated from Eqs. 23a and 21a respectively. The values of $\bar{v}_x$, $\bar{v}_y$ and $\bar{y}'$ used in the latter two equations were calculated by following a particle from its initial position to the cylindrical or spherical target using the previously described step by step procedure applied in determining the value of $\bar{y}_d$.

10. IMPACTION EFFICIENCY

The impaction efficiency was defined as the ratio of the cross sectional area of the far upstream envelope of particles which eventually hit the target to the cross sectional area of the target itself. For a transverse cylindrical target this is simply given by the value of $\bar{y}_d$. For a spherical target it is given by $\bar{y}_d{}^2$.

11. COMPUTER PROGRAMS

Computer programs were written in Fortran IV for the DRES IBM 1130 computer to obtain solutions to the drag coefficient equations by the method described in Section 9 of this report. The complete programs, together with one set of results for each, are shown in Appendix A for the cylinder in free space and in Appendix B for the sphere in a tube or in free space (Appendix B). These programs are annotated so that the functions of their various parts can be understood without further description. The application of the method described in Section 9, which was used in the programs, is straight forward with the exception of the step by step integration procedure near the target sphere or cylinder. This procedure is therefore explained as follows.

The particle motion was calculated in time steps, ΔT, until the particle was just one time step away from the target. Then the time increment size was decreased by a factor of ten and step by step integration continued until the particle was one new time step away from the target. At this point the time increment size was decreased by another factor of ten and the integration allowed to proceed until the target was reached. This method ensured that the position of the target and particle coincided within a maximum error given by the distance travelled during one percent of the original time step size, while allowing the particle to reach the proximity of the target in an adequate but small number of steps.

12. RESULTS

Drag coefficients and impaction efficiencies were calculated by means of the computer programs (Appendix A and B) from two main points of view. First, they were calculated using the particle sizes, fluid velocities and target configurations used in experiments done at Suffield many years ago (Gillespie and Gunter, 1957; 1959). This was done so that a comparison of the results from the theoretical calculation of drag due to particles in air could be made with existing information obtained from experiments. The results were obtained for a narrow range of particle sizes and fluid velocities and, correspondingly, a narrow range of inertia parameters and free stream particle Reynolds numbers. Next, calculations of impaction efficiencies and drag coefficients were done for a wide range of inertia parameters. The impaction efficiencies were compared with the results of other workers (Friedlander, 1977) but no results were available for comparison to the calculated drag coefficients over this wide range of the inertia parameter. All the results previously mentioned are shown in Tables 1 and 2 and Fig. 4 - Fig. 9, along with plots of the calculated distribution of forces due to elastic reflection of particles from a cylinder (Fig. 10) and a sphere (Fig. 11). The latter two figures were plotted from the sample computer results shown with their corresponding programs in Appendix A and B. Further details explaining the significance of the tables and figures are described as follows.

The calculated results for the various particle sizes and fluid velocities used in the experiments done at Suffield (Gillespie and Gunter, 1957; 1959) are shown for a cylinder (Table 1) and for a sphere (Table 2). The sizes of these two targets also correspond to those in the experiments. The calculations were done for a 0.2 centimetre diameter cylinder in free space and a 0.8 centimetre sphere in a 2.54 centimetre tube. The tube which was used in all the experiments was included for the sphere since the velocity potential was readily available (Smythe, 1964). However, the effect of the presence of the tube on the potential flow and the final theoretical results was found to be negligible. Similarily, the effect of not including the tube in the potential flow for the cylinder is expected to be negligible.

A reason for tabulating the calculated results is to show clearly that the effect of particle size on the calculated drag coefficient for the sizes used in the experiments was negligibly small. This conclusion was also drawn from the experiments reported by Gillespie and Gunter (1957; 1959). The drag coefficients due to the middle size particles alone were plotted against target Reynolds number for the cylinder (Fig. 4) and for the sphere (Fig. 5). Also shown for the purpose of comparison of experiment to theory, described in the following section, are the drag coefficients due to air alone.

The impaction efficiencies for a wide range of the inertia parameter were plotted for cylinders (Fig. 6) and for spheres (Fig. 8). The cylinder results were plotted for $\phi = 1000$. The definition of ϕ

and the reason for its value choice are described as follows. The collection efficiency and also the drag coefficient are a function of two dimensionless groups, the inertial parameter, K, and the free stream particle Reynolds number, Re_o. A new dimensionless group

$$\phi = \frac{Re_o^2}{K} \qquad \text{(Eq. 44)}$$

independent of particle size was introduced. According to the rules of dimensionless analysis this is permissible, but the efficiency is still determined by two groups which for convenience were chosen to be K and ϕ. The collection efficiencies were calculated by means of a mechanical analog and plotted for values of ϕ by other workers (Brun et al., 1953). The results for the middle value of ϕ from this work are also shown (Fig. 6) for comparison. The sphere results were plotted (Fig. 8) for an Re_o value of 128. Again this choice was made for convenience of comparison to other work. The choice was the middle value of seven for which curves calculated from inviscid flow theory by Dorsch, Saper and Kadow are shown (Friedlander, 1977). The curve is also shown in Fig. 8.

The calculated drag coefficient due to particles alone were plotted for a wide range of inertia parameters for cylinders (Fig. 7) and for spheres (Fig. 9). In these two figures each curve was fitted by eye through several data points obtained by applying the computer programs (Appendix A and B) to various particle diameters for the cylinder case and various target diameters for the sphere case.

10. COMPARISON OF THEORETICAL RESULTS TO EXPERIMENTAL DATA

The experimental work previously done at Suffield (Gillespie and Gunter, 1957; 1959) indicated that the drag force of a sphere or cylinder is given by the following equation:

$$F = 1/2\rho^* U^2 C_{D_e} A \qquad \text{(Eq. 45)}$$

where

$$\rho^* = \rho + \rho_p \qquad \text{(Eq. 46)}$$

and generally ρ and ρ_p are unequal. For Eq. 45 to be true, the drag coefficients due to particles alone carried by air and due to air alone must be equal. This is shown as follows. The total drag force consisted of two parts expressed by the following equation:

$$F_p + F_a = 1/2\rho_p U^2 A C_{Dp} + 1/2\rho U^2 A C_D \qquad \text{(Eq. 47)}$$

Comparison of Eq. 45 and 46 to Eq. 47 shows that

$$(\rho + \rho_p)C_{De} = \rho_p C_{Dp} + \rho C_D \qquad \text{(Eq. 48)}$$

C_{Dp} and C_D must be equal for Eq. 48 to be true.

The results obtained herein (Tables 1 and 2) and illustrated in Fig. 4 and 5 show that the calculated drag coefficients due to particles carried by air are considerably greater than the established drag coefficients due to air alone (Schlicting, 1960). The discrepancies between the experimental and calculated drag coefficients were accounted for by showing that the velocity of the particles in the experiments were considerably lower than the mean air velocity.

The particles which were introduced by Gillespie and Gunter into the air stream with zero velocity in the direction of air flow were accelerated along a horizontal section of 2.54 cm tube, 17 cm long, and a vertical section, 78 cm long, giving a total tube length of 95 cm between the starting point and the test section. Calculations of velocity as a function of distance for particles accelerated from rest were done to simulate Gillespie's experiments (Mellsen, in draft). The results for a travelled distance of 95 cm for the various mean air velocities and number median diameter used in the experiments are shown in Table 3. Also, the curves of velocity versus distance for the three diameters and one air velocity are illustrated in Fig. 14.

The particle velocities were difficult to calculate because the behavior of the particles travelling in the air stream around the corner in the tube was very complex. The velocity of the air in a straight cross section of the tube was zero at the walls and increased to the centre to 1.22 to 1.25 times the mean velocity (Prandtl and Tietjens, 1957). Therefore, the particle velocities were affected by the location of the particles in the tube cross section. This could not be accounted for because the time history of the location of the particles was not known. As a simplifying approximation, the mean air velocity was assumed to act on the particles over their length of travel in the tube which was also assumed vertical over its entire length. The experimental results indicate that the variation in velocities was less than this method of calculation shows. A mean value of drag force due to the presence of all particle sizes was used to offset this.

The drag coefficients which were calculated for particles with a free stream velocity equal to the mean air velocity (Table 1 and 2) were then adjusted to simulate the special conditions of the experiments as follows. The drag coefficients in the experiments were based on the mean air velocities as measured by a rotameter. The drag coefficients due to the particles travelling at lower velocities were adjusted to the mean air velocity by the following relationship for flow over the cylinder.

$$C_{DpU} = C_{Dc}\left(\frac{v_{rms}}{U}\right)^2 \qquad \text{(Eq. 49)}$$

This method was justified because, for the purpose of comparison, the calculated drag coefficients over the range of velocities and particle sizes used were sufficiently near a constant value (Table 1). All three particle sizes were assumed present in equal quantities in the air flow. Then, since the drag forces vary as the square of the velocity, the root mean square particle velocity, v_{rms}, was calculated for each mean fluid velocity (Table 3) and used in Eq. 49 to obtain a value for the drag coefficient due to dust alone at each mean air velocity (Table 4). The overall drag coefficients due to air and particles were then calculated for the dust concentrations used in the experiments, namely 2 and 5 kilograms per cubic metre of space. This was done by solving Eq. 48 for C_{De}. That is

$$C_{De} = \frac{\rho_p C_{Dc} + \rho C_D}{\rho + \rho_p} \qquad \text{(Eq. 50)}$$

The results are shown for each mean air velocity (Table 4) and plotted along with the drag coefficient for air alone (Fig. 12) as was done for the experiments of Gillespie and Gunter (1957; 1959).

A similar method was used to obtain the overall drag coefficients for the sphere with the exception that, as for the experimental results, they were corrected for the presence of the tube walls by the Landenburg correction (Gillespie and Gunter, 1957). That is, the drag coefficients due to particles alone were divided by the factor 1 + 2.4 L/a to obtain the corrected drag coefficient, C_{DL}. For the sphere and tube radii used, the factor is 1.756. Then, the overall drag coefficient for the sphere was obtained from

$$C_{De} = \frac{\rho_p C_{DL} + \rho C_D}{\rho + \rho_p} \qquad \text{(Eq. 51)}$$

The results are shown for each mean air velocity in Table 4, and plotted along with the drag coefficient for air alone in Fig. 13.

14. DISCUSSION

The equations of motion (Eq. 1 and 2) have never been proven experimentally (Friedlander, 1977) although they have been used to calculate deposition efficiencies for cylinders (Glauert, 1940). The mathematical model described herein used these equations to calculate collection efficiencies for cylinders which compare favorably (Fig. 6) with the results obtained using a differential analyzer (Brun et al., 1953). Also, the impaction efficiencies for spheres (Fig. 8) agree with solutions to the equations obtained by other workers (Friedlander, 1977).

In the work reported herein, the mathematical model for calculating impaction efficiency was extended to obtain the impaction forces due to particles in air flowing over spheres and cylinders. These results agree favorably with experiments previously done at Suffield (Gillespie and Gunter, 1957) when adjusted to the special conditions of the experiment. The experiments do support the theory, but the theoretical results show that the results of the experiments are not applicable to the free field case of drag due to dusty air flowing over a cylinder or sphere.

The model could also be extended to blast loading problems. Since the effect of compressibility becomes important in these problems it would have to be accounted for. This could be done in the mathematical model by changing the air flow field equations to include it. The model could also be used to calculate the actual distributions of impaction velocities on cylinders (Fig. 10) and spheres (Fig. 11) in special cases which may have applications in chemical and nuclear defence.

15. CONCLUSIONS

A mathematical model was developed for calculating the impaction forces due to airborne particles on spheres and cylinders. The model can also be used for calculating impaction efficiences, velocity distributions, and points of impact for a stream of airborne particles flowing over a sphere or cylinder. Hence, the drag coefficient for the particles on the sphere or cylinder can be determined.

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TABLE 1

COEFFICIENT OF DRAG ON A 0.2 cm CYLINDER FOR VARIOUS PARTICLE SIZES

	PARTICLE SIZE			AIR VELOCITY	P
	470μm	155μm	55μm	cm sec^{-1}	Re_o^2/K
Coefficient of Drag due to Particles alone, C_{Dc}	2.367	2.362	2.333	700	3.99
	2.367	2.361	2.327	560	3.19
	2.367	2.360	2.318	435	2.48
	2.367	2.357	2.300	294	1.67

TABLE 2

COEFFICIENT OF DRAG ON A 0.8 cm SPHERE IN A 2.54 cm TUBE FOR VARIOUS PARTICLE SIZES USING POTENTIAL FLOW

	PARTICLE SIZE			AIR VELOCITY	Re_o
	470μm	155μm	55μm	cm sec^{-1}	For 155μm
Coefficient of Drag due to Particles alone, C_{Ds}	2.048	2.036	1.970	700	73.6
	2.047	2.033	1.955	560	58.9
	2.047	2.030	1.934	426	44.8
	2.046	2.023	1.887	272	28.6

TABLE 3

VELOCITY OF PARTICLES AT TARGET POSITION ACCELERATED VERTICALLY UPWARD IN A TUBE FROM REST

95 cm UPSTREAM

PARTICLE DIAMETER cm	MEAN AIR VELOCITY IN TUBE cm sec^{-1}	PARTICLE VELOCITY AT TARGET POSITION cm sec^{-1}	RMS PARTICLE VELOCITY AT TARGET POSITION cm sec^{-1}
0.0470	700	299	534
0.0155	700	572	
0.0055	700	664*	
0.0470	560	191	398
0.0155	560	445	
0.0055	560	527*	
0.0470	435	75*	302
0.0155	435	325*	
0.0055	435	405*	
0.0470	426	66	295
0.0155	426	317	
0.0055	426	396*	
0.0470	294	-	188
0.0155	294	187*	
0.0055	294	267*	
0.0470	272	-	171
0.0155	272	166*	
0.0055	272	245*	

* Upward terminal velocity reached

TABLE 4

DRAG COEFFICIENT OF A 0.2 cm CYLINDER IN A 2.54 cm TUBE DUE TO AIR CONTAINING EQUAL MASSES OF 0.0470, 0.0155 AND 0.0055 cm PARTICLES CORRECTED FOR PARTICLE VELOCITY DEFICIENCY

U cm sec^{-1}	C_{Dc}	C_D	U_{rms}	C_{DpU}	C_{De} AIR + DUST		Re
					2 kg m^{-3} Dust	5 kg m^{-3} Dust	
700	2.354	1.00	534	1.369	1.23	1.30	949
560	2.352	1.05	398	1.188	1.14	1.16	759
435	2.348	1.15	302	1.132	1.14	1.14	590
294	2.341	1.25	188	0.957	1.07	1.01	399

TABLE 5

DRAG COEFFICIENT OF A 0.8 cm SPHERE IN A 2.54 cm TUBE DUE TO AIR CONTAINING EQUAL MASSES OF 0.0470, 0.155 AND 0.055 cm PARTICLES CORRECTED FOR REAL FLUID VELOCITY AND PARTICLE VELOCITY DEFICIENCY

U cm sec^{-1}	C_{Ds}	C_D	U_{rms}	C_{DpU}	C_{DL}	C_{De} AIR + DUST		Re
						2 kg m^{-3} Dust	5 kg m^{-3} Dust	
700	2.018	0.391	534	1.174	0.669	0.564	0.615	3796
560	2.012	0.396	398	1.016	0.579	0.510	0.543	3076
426	2.004	0.406	295	0.961	0.547	0.494	0.519	2310
272	1.985	0.433	171	0.785	0.447	0.442	0.444	1475

APPENDIX A

COMPUTER PROGRAM FOR A CYLINDER

```
PAGE   1

// JOB T

LOG DRIVE   CART SPEC   CART AVAIL  PHY DRIVE
  0000        0205        0205        0000

V2 M11   ACTUAL 16K  CONFIG 16K

// EJECT
```

```
PAGE   2

// FOR
*ONE WORD INTEGERS
*LIST ALL
      SUBROUTINE SBM28(G,TAU,DTAW,XK,REZ,UX,UY,XCDRE)
C
C     THIS SUBROUTINE CALCULATES PARTICLE MOTION DURING THE FINAL
C     INCREMENT OF TAU
C
      DIMENSION G(4),DG(4)
      M=0
C
C     CALL ON RUNGE KUTTA SUBROUTINE
C
    8 CONTINUE
      M=M+1
      CALL SBM22(4,G,DG,TAU,DTAW,IRUNG,M)
      IF(IRUNG-1)10,9,10
    9 RE=REZ*((UY-G(2))**2+(UX-G(1))**2)**0.5
      XCDRE=CDRE(RE)
      DG(1)=(XCDRE/(24.0*XK))*(UX-G(1))
      DG(2)=(XCDRE/(24.0*XK))*(UY-G(2))
      DG(3)=G(1)
      DG(4)=G(2)
      GO TO 8
   10 CONTINUE
      M=0
C
C     CALCULATE FLUID VELOCITY AT PARTICLE POSITION
C
      RSQSQ=(G(3)**2+G(4)**2)**2
      UX=1.0-(G(3)**2-G(4)**2)/RSQSQ
      UY=-(G(3)*G(4))/RSQSQ
C
C     INTEGRATE ACROSS ANOTHER STEP IF REQUIRED
C
      IF(G(4) - 1.0)11,11,12
   11 DELX = SQRT(1.0 - G(4)**2)
      GO TO 13
   12 DELX = 0.0
   13 HITS=G(3)+G(1)*1.1*DTAW+DELX
      IF(HITS)8,8,18
   18 CONTINUE
      RETURN
      END
VARIABLE ALLOCATIONS
   DG(R )=0006-0000    RE(R )=0008      RSQSQ(R )=000A      DELX(R )=000C      HITS(R )=000E       M(I )=0012
```

```
PAGE   3

 IRUNG(I )=0013

STATEMENT ALLOCATIONS
 8     =0055  9     =C06A  10    =C0C0  11    =0104  12    =0115  13    =0119  18    =012E

FEATURES SUPPORTED
 ONE WORD INTEGERS

CALLED SUBPROGRAMS
 SBM22    CORE     FSQRT    FAXB     FADD     FADDX    FSUB     FSUBX    FMPY     FMPYX    FDIV     FLD      FLDX     FSTO     FSTOX
 FSBR     FDVR     FAXI     SNR      SUBIN

REAL CONSTANTS
  .500000E 00=0018    .240000E 02=001A    .100000E 01=001C    .000000E 00=001E    .110000E 01=0020

INTEGER CONSTANTS
     0=0022       1=0023       4=0024       2=0025

CORE REQUIREMENTS FOR SBM28
 COMMON       0  VARIABLES      24  PROGRAM     280

RELATIVE ENTRY POINT ADDRESS IS 0026 (HEX)

END OF COMPILATION

// DUP

*STORE       WS  UA  SBM28
CART ID 0205   DB ADDR  5F48   DB CNT    0015

// EJECT
```

```
// FOR
*ONE WORD INTEGERS
*LIST ALL
C
      FUNCTION CDRE(RE)
C
C     THIS FUNCTION COMPUTES THE PRODUCT OF DRAG COEFFICIENT
C     AND REYNOLDS NUMBER FOR A SPHERE AS A FUNCTION OF
C     REYNOLDS NUMBER
C
C     CONSTANT COEFFICIENTS
C
      A1=1./24.
      A2=-2.3363*1.E-04
      A3=2.0154*1.E-06
      A4=-6.9105*1.E-09
      B0=-1.29536
      B1=9.86*1.E-01
      B2=-4.6677*1.E-02
      B3=1.1235*1.E-03
C
C     CHOOSE THE APPROPRIATE POLYNOMIAL
C
      IF(RE-4.0)2.7.7
C
C     INITIAL ESTIMATE
C
    2 IF(RE - 0.001)3.4.4
    3 CDRE = 24.0
      GO TO 30
    4 X=24.*RE
C
C     BEGIN NEWTON METHOD ITERATION
C
      CONTINUE
      DO 6 ITER=1.20
      FX=A1*X+A2*X**2+A3*X**3+A4*X**4-RE
      FPX=A1+2.*A2*X+3.*A3*X**2+4.*A4*X**3
      DELX=FX/FPX
      X=X-DELX
C
C     CHECK FOR CONVERGENCE
C
      EPS=1.E-06
      IF(ABS(DELX/X)-EPS)5.5.6
    5 CDRE=X/RE
```

```
      GO TO 30
    6 CONTINUE
      GO TO 29
C
C     INITIAL ESTIMATE
C
    7 CD = 1.0
      FLOG = 0.434294481903252
      X=ALOG(CD*RE**2)*ELOG
C
C     BEGIN NEWTON METHOD ITERATION
C
      DO 24 ITER=1,20
      FX=B0+B1*X+B2*X**2+B3*X**3 - ALOG(RE)*ELOG
      FPX=B1+2.*B2*X+3.*B3*X**2
      DELX=FX/FPX
      X=X-DELX
C
C     CHECK FOR CONVERGENCE
C
      EPS=1.E-06
      IF(ABS(DELX/X)-EPS)22,22,24
   22 CDRE=10.**X/RE
      GO TO 30
   24 CONTINUE
   29 WRITE(3,202)
   30 RETURN
C
C     FORMATS FOR OUTPUT STATEMENTS
C
  202 FORMAT(16H0 NO CONVERGENCE)
C
      END
VARIABLE ALLOCATIONS
  CDRE(R )=0000         A1(R )=0002         A2(R )=0004         A3(R )=0006         A4(R )=0008         B0(R )=000A
    B1(R )=000C         B2(R )=000E         B3(R )=0010          X(R )=0012         FX(R )=0014        FPX(R )=0016
  DELX(R )=001B        EPS(R )=001A         CD(R )=001C       ELOG(R )=001E       ITER(I )=0028

STATEMENT ALLOCATIONS
 202  =0057  2    =00A6  3    =00AD  4    =00B3  5    =012B  6    =0133  7    =013E  22   =01B6  24   =01C1  29   =01CA
 30   =01CF

FEATURES SUPPORTED
 ONE WORD INTEGERS

CALLED SUBPROGRAMS
 FABS     FALOG    FAXB     FADD     FSUB     FMPY     FDIV     FLD      FSTO     FSBR     FAXI     SWRT     SCOMP    SNR      SUBIN
```

```
PAGE   6

REAL CONSTANTS
  .100000E 01=002A    .240000E 02=002C    .233630E 01=002E    .100000E-03=0030    .201540E 01=0032    .100000E-05=0034
  .691050E 01=0036    .100000E-08=0038    .129536E 01=003A    .986000E 01=003C    .100000E 00=003E    .466770E 01=0040
  .100000E-01=0042    .112350E 01=0044    .100000E-02=0046    .400000E 01=0048    .200000E 01=004A    .300000E 01=004C
  .434294E 00=004E    .100000E 02=0050

INTEGER CONSTANTS
      1=0052     20=0053       2=0054       3=0055       4=0056

CORE REQUIREMENTS FOR CDRE
 COMMON       0  VARIABLES      42  PROGRAM     424

RELATIVE ENTRY POINT ADDRESS IS 0061 (HEX)

END OF COMPILATION

// DUP

*STORE      WS  UA  CDRE
CART ID 0205   DB ADDR  5F60   DB CNT   0022

// EJECT
```

```
// FOR
*ONE WORD INTEGERS
*LIST ALL
      SUBROUTINE SRM22(N,Y,F,X,H,IRUNG,M)
C
C     FOURTH ORDER RUNGE KUTTA METHOD
C     FOR N FIRST ORDER O.D.E.
C
      DIMENSION PHI(50),SAVY(50),Y(50),F(50)
      GO TO (2,3,4,5,6),M
C
C     PASS 1
C
    2 IRUNG=1
      RETURN
C
C     PASS 2
C
    3 DO 22 J=1,N
      SAVY(J)=Y(J)
      PHI(J)=F(J)
   22 Y(J)=SAVY(J)+0.5*H*F(J)
      X=X+0.5*H
      IRUNG=1
      RETURN
C     PASS 3
    4 DO 33 J=1,N
      PHI(J)=PHI(J)+2.0*F(J)
   33 Y(J)=SAVY(J)+0.5*H*F(J)
      IRUNG=1
      RETURN
C
C     PASS 4
    5 DO 44 J=1,N
      PHI(J)=PHI(J)+2.0*F(J)
   44 Y(J)=SAVY(J)+H*F(J)
      X=X+0.5*H
      IRUNG=1
      RETURN
C     PASS 5
    6 DO 55 J=1,N
   55 Y(J) = SAVY(J) + (PHI(J) + F(J))*H/6.0
      IRUNG=2
      RETURN
C
      END
```

```
PAGE   8

VARIABLE ALLOCATIONS
   PHI(R )=0062-0000  SAVY(R )=00C6-0064    J(I )=00C8

STATEMENT ALLOCATIONS
 2    =0105  3    =0108  22   =011E  4    =0141  33   =0152  5    =016D  44   =017E  6    =019F  55   =01A3

FEATURES SUPPORTED
 ONE WORD INTEGERS

CALLED SUBPROGRAMS
 FADD     FADDX    FMPY     FMPYX    FDIV     FLD      FLDX     FSTO     FSTOX    SUBSC    SUBIN

REAL CONSTANTS
  .500000E 00=00CA    .200000E 01=00CC    .600000E 01=00CE

INTEGER CONSTANTS
     1=0000       2=0001

CORE REQUIREMENTS FOR SBM22
 COMMON        0  VARIABLES     202  PROGRAM     250

RELATIVE ENTRY POINT ADDRESS IS 0002 (HEX)

END OF COMPILATION

// DUP

*STORE      WS  UA  SBM22
CART ID 0205   DB ADDR  5F82   DB CNT    0012

// EJECT
```

```
PAGF   9

// FOR
*ONE WORD INTEGERS
*LIST ALL
      SUBROUTINE SBM29(G4LFT,G4RIT,SIGNL,DTAU,NIBP,NSBP,NX,XL,XK,REZ,
     1G4ZFR)
C
C     THIS SUBROUTINE CALCULATES THE
C     IMPACTION EFFICIENCY OF A CIRCULAR CYLINDER
C
      DIMENSION G(4),DG(4)
      WRITE(3,200)
      WRITE(3,201)G4LFT,G4RIT,SIGNL,DTAU,NIBP,NSBP,NX
C
C     HALF INTERVAL ITERATION FOR INITIAL G4 VALUE
C
      DO 47 ITER=1,NX
C
C     SET AND PRINT INITIAL CONDITIONS
C
      N=0
      NSTFP=0
      TAU=0.0
      G(3)=XL
      G4ZFR=(G4LFT+G4RIT)/2.0
      G(4)=G4ZER
      RSQSO=(G(3)**2+G(4)**2)**2
      UX=1.0-(G(3)**2-G(4)**2)/RSQSQ
      UY=-2.0*(G(3)*G(4))/RSQSQ
      G(1)=UX
      G(2)=UY
      RF=REZ*((UY-G(2))**2+(UX-G(1))**2)**0.5
      XCDRE=CDRE(RE)
      IP=ITER/NIBP*NIBP
      IF(IP-ITER)5,7,5
    5 CONTINUF
      IF(ITER-1)6,7,6
    6 CONTINUF
      IF(ITER-NX)8,7,8
    7 CONTINUE
      WRITE (3,205)
      WRITE(3,203)ITER,G4LFT,G4ZER,G4RIT,TAU,G(1),G(2),G(3),G(4),UX,UY,
     1XCDRE
C
C     CALL ON RUNGE KUTTA SUBROUTINE
C
    8 CONTINUE
```

```
      M=M+1
      CALL SRM22(4,G,DG,TAU,DTAU,IRUNG,M)
      IF(IRUNG-1)10,9,10
    9 RF=REZ*((UY-G(2))**2+(UX-G(1))**2)**0.5
      XCDRE=CDRE(RF)
      DG(1)=(XCDRE/(24.0*XK))*(UX-G(1))
      DG(2)=(XCDRE/(24.0*XK))*(UY-G(2))
      DG(3)=G(1)
      DG(4)=G(2)
      GO TO 8
   10 CONTINUE
      M=0
C
C     CALCULATE FLUID VELOCITY AT PARTICLE POSITION
C
      RSQSQ=(G(3)**2+G(4)**2)**2
      UX=1.0-(G(3)**2-G(4)**2)/RSQSQ
      UY=-2.0*(G(3)*G(4))/RSQSQ
C
C     PRINT SOLUTIONS
C
      IS = ITER/NIRP*NIRP
      IF(IS-ITER)11,13,11
   11 CONTINUE
      IF(ITER-1)12,13,12
   12 CONTINUE
      IF(ITER-NX)16,13,16
   13 CONTINUE
      NSTEP=NSTEP+1
      IF(NSTEP-NSRP)16,14,16
   14 CONTINUE
      NSTEP=0
      TAY = TAU+0.0001
      WRITE(3,204)TAW,G(1),G(2),G(3),G(4),UX,UY,XCDRE
C
C     INTEGRATE ACROSS ANOTHER STEP IF REQUIRED
C
   16 CONTINUE
      IF(G(4)-1.0)17,17,18
   17 DELX=SQRT(1.0-G(4)**2)
      GO TO 19
   18 DELX=0.0
   19 HITS=G(3)+G(1)*1.1*DTAU+DELX
      IF(HITS)9,8,20
   20 CONTINUE
```

```
C
C     CHANGE INCREMENT SIZE NEAR CYLINDER AND INTEGRATE FURTHER
C
      DTAW=DTAU/10.0
      CALL SBM28(G,TAU,DTAW,XK,REZ,UX,UY,XCDRE)
C
C     PRINT SOLUTIONS
C
      IF(IS-ITER)21,23,21
   21 CONTINUE
      IF(ITER-1)22,23,22
   22 CONTINUE
      IF(ITER-NX)24,23,24
   23 CONTINUE
      WRITE(3,204)TAU,G(1),G(2),G(3),G(4),UX,UY,XCDRE
   24 CONTINUE
      DTAW=DTAU/100.0
      CALL SBM28(G,TAU,DTAW,XK,REZ,UX,UY,XCDRE)
C
C     PRINT SOLUTIONS
C
      IF(IS-ITER)31,33,31
   31 CONTINUE
      IF(ITER-1)32,33,32
   32 CONTINUE
      IF(ITER-NX)34,33,34
   33 CONTINUE
      WRITE(3,204)TAU,G(1),G(2),G(3),G(4),UX,UY,XCDRE
   34 CONTINUE
C
C     CALCULATE ORDINATE AT TANGENT POINT OF TANGENT PATH
C
      ORD = G(1)/SQRT(G(1)**2 + G(2)**2)
C
C     FIND INTERVAL HALF WITH THE SIGN CHANGE
C
      IF((G(4)-ORD)*SIGNL-0.0)45,45,46
   45 G4RIT=G4ZER
      GO TO 47
   46 G4LFT=G4ZER
   47 CONTINUE
      EM = G4ZER
      WRITE(3,207) EM
C
      RETURN
C
```

```
C      FORMATS FOR OUTPUT STATEMENTS
C
  200 FORMAT( 1H0, 35X, 44H IMPACTION EFFICIENCY OF A CIRCULAR CYLINDER/
     1   1H0 )
  201 FORMAT(  10H0G4LEF =  ,F10.6/ 10H G4RIT =  ,F10.6/10H SIGNL =  ,
     1    F3.0/ 10H DTAU  =  ,F10.6/ 10H NISP  =  ,I3/ 10H NSBP  =  ,I3/
     2    10H NX    =  ,I3)
  203 FORMAT( 10H0ITER  =  ,I3/ 10H G4LEF =  ,F10.6/ 10H G4ZER =  ,
     1    F10.6/ 10H G4RIT =  ,F10.6/ 7H0   TAU, 11X, 4HG(1), 12X,
     2    4HG(2), 12X, 4HG(3), 12X, 4HG(4), 14X, 2HUX, 14X,  2HUY ,
     3    12X, 4HCDRF /
     4    1H0, F7.4, 4F16.6, 3F16.4 )
  204 FORMAT ( 1H , F7.4, 4F16.6, 3F16.4 )
  205 FORMAT ( 46H1THE MOTION OF A CRITICAL PARTICLE IS GIVEN BY )
  207 FORMAT( 30H0THE IMPACTION EFFICIENCY IS  ,E10.4)
      END
VARIABLE ALLOCATIONS
      G(R )=0006-0000     DG(R )=000E-0008   TAU(R )=0010     RSQSQ(R )=0012        UX(R )=0014        UY(R )=0016
     RE(R )=0018       XCDRE(R )=001A        TAW(R )=001C      DELX(R )=001E      HITS(R )=0020      DTAW(R )=0022
    ORD(R )=0024          EM(R )=0026       ITER(I )=002C         M(I )=002D      NSTEP(I )=002E        IP(I )=002F
  IRUNG(I )=0030          IS(I )=0031
STATEMENT ALLOCATIONS
 200  =0040  201  =006B  203  =00A3  204  =00EB  205  =00F3  207  =010C  5   =020A  6   =0210  7   =0216  8   =021E
 9    =0253  10   =02A9  11   =02FA  12   =0300  13   =0306  14   =0312  16  =0338  17  =0341  18  =0352  19  =0356
 20   =036B  21   =0381  22   =0387  23   =038D  24   =03A9  31   =03BF  32  =03C5  33  =03CB  34  =03E7  45  =040F
 46   =0415  47   =0419
FEATURES SUPPORTED
 ONE WORD INTEGERS
CALLED SUBPROGRAMS
 CORE    SRM22   FSQRT   SBM28   FAXB    FADD    FADDX   FSUB    FSUBX   FMPY    FMPYX   FDIV    FLD     FLDX    FSTO
 FSTOX   FSBR    FDVR    FDVRX   FAXI    SQRT    SCOMP   SIOFX   SIOF    SIOI    SNR     SUBIN
REAL CONSTANTS
  .000000E 00=0036    .200000E 01=0038    .100000E 01=003A    .500000E 00=003C    .240000E 02=003E    .100000E-03=0040
  .110000E 01=0042    .100000E 02=0044    .100000E 03=0046
INTEGER CONSTANTS
     3=0048       1=0049       0=004A       2=004B       4=004C
CORE REQUIREMENTS FOR SRM29
 COMMON       0  VARIABLES      54  PROGRAM    1016
```

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RELATIVE ENTRY POINT ADDRESS IS 011E (HEX)

END OF COMPILATION

// DUP

*STORE      WS  UA  SBM29
CART ID 0205   DB ADDR  5F94   DB CNT   0047

// EJECT
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```
// FOR
*ONE WORD INTEGERS
*LIST ALL
      SUBROUTINE SBM26(XL,G4ZER,DTAU,XK,REZ,PX,NZER,NJBP)
C
C     THIS SUBROUTINE CALCULATES THE MOTION OF PARTICLES
C     IN A FLUID STREAM MOVING TOWARD A CIRCULAR CYLINDER AND
C     CALCULATES THE FORCE OF PARTICLE IMPACT ON THE CYLINDER
C
      DIMENSION G(4),DG(4)
      DIMENSION YZER(500)
C
C     SET NUMBER OF INCREMENTS AT INITIAL POSITION
C
      WRITE(3,200)
      WRITE(3,201)NZER,NJBP
      WRITE(3,202)
      NCE=NZER+1
      DELY=G4ZER/FLOAT(NZER)
      FSUM=0.0
C
C     STEPWISE INTEGRATION FOLLOWING PARTICLE POSITION
C
      DO 40 ITER=1,NCE
C
C     SET AND PRINT INITIAL CONDITIONS
C
      M=0
      TAU=0.0
      G(3)=XL
      YZER(ITER)=FLOAT(ITER-1)*DELY
      G4ZER=YZER(ITER)
      G(4)=G4ZER
      RSQSQ=(G(3)**2+G(4)**2)**2
      UX=1.0-(G(3)**2-G(4)**2)/RSQSQ
      UY=-2.0*(G(3)*G(4))/RSQSQ
      UXZER=UX
      UY=-(G(3)*G(4))/RSQSQ
      G(1)=UX
      G(2)=UY
C
C     CALL ON RUNGE KUTTA SUBROUTINE
C
    8 CONTINUE
      M=M+1
      CALL SBM22(4,G,DG,TAU,DTAU,IRUNG,M)
```

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      IF(IRUNG-1)10,9,10
    9 RE=REZ*((UY-G(2))**2+(UX-G(1))**2)**0.5
      XCDRE=CDRE(RE)
      DG(1)=(XCDRE/(24.0*XK))*(UX-G(1))
      DG(2)=(XCDRE/(24.0*XK))*(UY-G(2))
      DG(3)=G(1)
      DG(4)=G(2)
      GO TO 8
   10 CONTINUE
      M=0
C
C     CALCULATE FLUID VELOCITY AT PARTICLE POSITION
C
      RSOSQ=(G(3)**2+G(4)**2)**2
      UX=1.0-(G(3)**2-G(4)**2)/RSOSQ
      UY=-2.0*(G(3)*G(4))/RSOSQ
C
C     INTEGRATE ACROSS ANOTHER STEP IF REQUIRED
C
      IF(G(4) - 1.0)11,11,12
   11 DELX = SQRT(1.0 - G(4)**2)
      GO TO 13
   12 DELX = 0.0
   13 HITS=G(3)+G(1)*1.1*DTAU+DELX
      IF(HITS)8,8,18
   18 CONTINUE
C
C     CHANGE INCREMENT SIZE NEAR CYLINDER AND INTEGRATE FURTHER
C
      DTAW=DTAU/10.0
      CALL SBM28(G,TAU,DTAW,XK,REZ,UX,UY,XCDRE)
      DTAW=DTAU/100.0
      CALL SBM28(G,TAU,DTAW,XK,REZ,UX,UY,XCDRE)
C
C     CO-ORDINATES, VELOCITY, AND PRESSURE DERIVATIVE AT CYLINDER
C
      VX=G(1)
      VY=G(2)
      X=G(3)
      Y=G(4)
      V=SQRT(VX**2+VY**2)
      PI = 3.14159265358979323846264
      GAMMA = PI - 2.0*ATAN(Y/SQRT(1.0-Y**2))- ATAN(VY/VX)
      FY= UXZER*(VX- V*COS(GAMMA))
      IF(ITER-1)24,25,24
   24 CONTINUE
```

```
      IF(ITER-NCE)26,25,26
   25 FSUM=FSUM+0.5*FY
      GO TO 29
   26 FSUM=FSUM+FY
   29 CONTINUE
C
C     PRINT SOLUTIONS
C
      IS = ITER/NJBP*NJBP
      IF(IS-ITER)31,33,31
   31 IF(ITER-1)32,33,32
   32 IF(ITER-NCE)34,33,34
   33 CONTINUE
      TAW = TAU+0.0001
      WRITE(3,203)TAW,VX,VY,X,Y,UX,UY,FY
   34 CONTINUE
   40 CONTINUE
C
C     DRAG COEFFICIENT OF CYLINDER DUE TO PARTICLE IMPACT
C
      PX=2.*FSUM*DELY
      WRITE(3,204)PX
      RETURN
C
C     FORMATS FOR OUTPUT STATEMENTS
C
  200 FORMAT( 65H1THE MOTION OF THE PARTICLES AT THE CYLINDER POSITION I
     1S GIVEN BY )
  201 FORMAT( 10HONZER   =   ,I3,20X, 10H NJBP   =   ,I3)
  202 FORMAT( 7H0    TAU, 11X, 4H VX , 12X, 4H VY , 11X, 4H  X , 12X,
     1    4H  Y , 14X, 2HUX, 14X, 2HUY , 14X, 4H FY /1H  )
  203 FORMAT ( F7.4, 4F16.6, 2F16.4, F16.5)
  204 FORMAT( 77H0THE DRAG COEFFICIENT OF THE CYLINDER DUE TO THE REFLEC
     1TION OF PARTICLES IS   , F9.4 )
      END
VARIABLE ALLOCATIONS
     G(R )=0006-0000      DG(R )=000E-0008  YZER(R )=03F6-0010  DELY(R )=03F8         FSUM(R )=03FA          TAU(R )=03FC
 RSQSQ(R )=03FE           UX(R )=0400         UY(R )=0402     UXZER(R )=0404           RE(R )=0406       XCDRE(R )=0408
  DFLX(R )=040A         HITS(R )=040C       DTAW(R )=040E        VX(R )=0410           VY(R )=0412           X(R )=0414
     Y(R )=0416            V(R )=0418         PI(R )=041A     GAMMA(R )=041C           FY(R )=041E         TAW(R )=0420
   NCE(I )=042A         ITER(I )=042B          M(I )=042C      IRUNG(I )=042D           IS(I )=042E

STATEMENT ALLOCATIONS
 200  =0440  201  =047C  202  =0480  203  =04A3  204  =04AA  8    =05A3  9    =05B8  10   =060E  11   =0658  12   =066
 13   =066D  18   =0682  24   =0707  25   =070D  26   =0717  29   =071D  31   =072D  32   =0733  33   =0739  34   =07
 40   =0753
```

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FEATURES SUPPORTED
 ONE WORD INTEGERS

CALLED SUBPROGRAMS
 SBM22    CORE     FSQRT    SBM28    FATAN    FCOS     FAXB     FADD     FADDX    FSUB     FSUBX    FMPY     FMPYX    FDIV     FLD
 FLDX     FSTO     FSTOX    FSBR     FDVR     FAXI     FLOAT    SWRT     SCOMP    SIOF     SIOI     SUBSC    SNR      SUBIN

REAL CONSTANTS
  .000000E 00=0434    .100000E 01=0436    .200000E 01=0438    .500000E 00=043A    .240000E 02=043C    .110000E 01=043E
  .100000E 02=0440    .100000E 03=0442    .314159E 01=0444    .100000E-03=0446

INTEGER CONSTANTS
     3=0448       1=0449       0=044A       2=044B       4=044C

CORE REQUIREMENTS FOR SBM26
 COMMON        0  VARIABLES    1076  PROGRAM     824

RELATIVE ENTRY POINT ADDRESS IS 04D4 (HEX)

END OF COMPILATION

// DUP

*STORE        WS  UA  SBM26
CART ID 0205   DB ADDR  5FDB   DB CNT    003B

// EJECT
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```
// FOR
*ONE WORD INTEGERS
*IOCS(CARD)
*IOCS(1132 PRINTER)
*LIST ALL
C
C     ESTABLISH PHYSICAL PROPERTIES FOR CALCULATING IMPACTION EFFICIENCY
C
C     ESTABLISH PARAMETERS FOR STEP BY STEP INTEGRATION AND HALF
C     INTERVAL METHOD
C     G4LFT AND G4RIT ARE LOWER AND UPPER BOUNDS RESPECTIVELY FOR
C     STARTING LOCATION OF PARTICLE
C     SIGNL IS SIGN CORRECTION FOR HALF INTERVAL METHOD
C     IN THIS PROBLEM IT IS -1.0
C     DTAU  IS THE TIME INCREMENT FOR STEP BY STEP INTEGRATION
C     NIBP IS INTERVAL OF WRITTEN PARTICLE PATHS
C     NSBP IS INTERVAL OF WRITTEN INTEGRATION STEPS
C     NX IS NUMBER OF ITERATIONS USED TO DETERMINE PARTICLE STARTING
C     POINT
    1 READ(2,100)G4LFT,G4RIT,SIGNL,DTAU,NIBP,NSBP,NX
C
C     ESTABLISH PHYSICAL PROPERTIES FOR CALCULATING IMPACTION EFFICIENCY
C
C     DC    IS CYLINDER DIAMETER, CM
C     DP    IS PARTICLE DIAMETER, CM
C     RHO   IS FLUID DENSITY, GM/CC
C     SIGMA IS PARTICLE DENSITY, GM/CC
C     XMU   IS ABSOLUTE VISCOSITY OF FLUID, POISE
C     UF    IS FREE STEAM VELOCITY, CM/SEC
C
      READ(2,101)DC,DP,RHO,SIGMA,XMU,UF
C
C     XL IS UPSTREAM STARTING POSITION FOR CALCULATING PARTICLE MOTION
C
      READ(2,102)XL
      REZ=RHO*DP*UF/XMU
      XK=SIGMA*DP**2*UF/(9.*XMU*DC)
      P = 9.*RHO**2*UF*DC/(XMU*SIGMA)
      WRITE(3,200)
      WRITE(3,201)
C
C     XK IS INERTIAL PARAMETER CALLED STOKES NUMBER
C     REZ IS FREE STREAM REYNOLDS NUMBER OF SPHERICAL PARTICLE
C     P IS DIMENSIONLESS GROUP INDEPENDENT OF DROP SIZE
C
      WRITE(3,202)XL,REZ,XK,P,DC,DP,RHO,SIGMA,XMU,UF
```

```
C
C     CALCULATE STARTING POSITION OF TANGENT PARTICLE
C
      CALL SBM29(G4LFT,G4RIT,SIGNL,DTAU,NIBP,NSBP,NX,XL,XK,REZ,G4ZER)
C
C     ESTABLISH NUMBER OF INCREMENTS FOR INTEGRATING PARTICLE FORCE
C
C     NZER IS NUMBER OF INCREMENTS
C     NJBP IS INTERVAL OF WRITTEN INCREMENTS
C
      READ(2,103)NZER,NJBP
C
C     CALCULATE FORCE PRODUCED BY PARTICLES
C
      CALL SBM26(XL,G4ZER,DTAU,XK,REZ,PX,NZER,NJBP)
C
C
C     CDAIR IS DRAG COEFFICIENT OF CYLINDER IN FREE AIR
C
      READ(2,104) CDAIR
      RECYL=RHO*DC*UF/XMU
      WRITE(3,204)RECYL
      WRITE(3,206)CDAIR
      READ(2,111)NSTOP
      IF(NSTOP)1,30,30
   30 CALL EXIT
C
C     FORMATS FOR INPUT AND OUTPUT STATEMENTS
C
  100 FORMAT(10X,F10.7,20X,F10.7,19X,F3.0/ 10X,F10.7,20X,I3,27X,I2/
     1   10X,I2)
  101 FORMAT(F10.5,F10.7,F10.6, F10.6,F10.7,F10.1 )
  102 FORMAT (F10.7)
  103 FORMAT(I5,I5)
  104 FORMAT(F10.6)
  111 FORMAT(I5)
  200 FORMAT( 1H1, 34X, 51HTHE DRAG ON CYLINDERS IN A STREAM OF DUST-LAD
     1EN AIR / 1H0 )
  201 FORMAT(  28H0THE PHYSICAL PARAMETERS ARE )
  202 FORMAT(  10H0XL     =  ,F10.7/10H REZ     =  ,E12.6/10H XK     =  ,
     1   F12.6/10H P      =    ,E12.6/
     2   10H0DC      =  , F10.5/10H DP     =  ,F10.7/10H RHO    =  ,F10.6/
     3   10H SIGMA =  , F10.6/10H XMU    =  ,F10.7/10H UF     =  ,F10.1)
  204 FORMAT(  32H0THE TARGET REYNOLDS NUMBER IS  ,E10.4)
  206 FORMAT( 61H0THE DRAG COEFFICIENT OF THE CYLINDER DUE TO THE AIR AL
     1ONE IS,F9.3)
```

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      END
VARIABLE ALLOCATIONS
  G4LFT(R )=0000      G4RIT(R )=0002      SIGNL(R )=0004      DTAU(R )=0006        DC(R )=0008        DP(R )=000A
    RHO(R )=000C      SIGMA(R )=000E        XMU(R )=0010        UF(R )=0012        XL(R )=0014       REZ(R )=0016
     XK(R )=0018          P(R )=001A      G4ZER(R )=001C        PX(R )=001E      CDAIR(R )=0020     RECYL(R )=0022
   NIBP(I )=0026       NSBP(I )=0027         NX(I )=0028      NZER(I )=0029       NJBP(I )=002A     NSTOP(I )=002B

STATEMENT ALLOCATIONS
  100  =0030  101  =0041  102  =0048  103  =004A  104  =004D  111  =004F  200  =0051  201  =0073  202  =0083  204  =00D3
  206  =00E6  1    =011F  30   =01D8

FEATURES SUPPORTED
 ONE WORD INTEGERS
 IOCS

CALLED SUBPROGRAMS
 SBM29   SBM26   FMPY    FDIV    FLD     FSTO    FDVR    FAXI    CARDZ   PRNTZ   SRED    SWRT    SCOMP   SFIO    SIOF
 SIOI

REAL CONSTANTS
  .900000E 01=002C

INTEGER CONSTANTS
     2=002E       3=002F

CORE REQUIREMENTS FOR
 COMMON       0  VARIABLES      44  PROGRAM     430

END OF COMPILATION

// XEQ
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```
                                   THE DRAG ON CYLINDERS IN A STREAM OF DUST-LADEN AIR

    THE PHYSICAL PARAMETERS ARE

    XL     =  -5.0000009
    REZ    =  0.735665E 02
    XK     =  0.339462E 03
    P      =  0.159429E 02

    DC     =     0.80000
    DP     =  0.0155000
    RHO    =    0.001213
    SIGMA  =    2.600000
    XMU    =  0.0001789
    UF     =       700.0

                                    IMPACTION EFFICIENCY OF A CIRCULAR CYLINDER

    G4LFF  =    0.000000
    G4RIT  =    1.000000
    SIGML  =  -1.
    DTAU   =    0.100000
    NIRP   =   10
    NSEP   =    2
    NX     =   20
```

```
THE MOTION OF A CRITICAL PARTICLE IS GIVEN BY

ITER  =    1
G4LEF =    0.000000
G4ZER =    0.500000
G4RIT =    1.000000

   TAU        G(1)        G(2)        G(3)        G(4)        UX        UY        CDRE

 0.0000     0.961180    0.007842   -5.000000    0.500000    0.9611    0.0078    24.0000
 0.2000     0.961179    0.007842   -4.807763    0.501568    0.9581    0.0088    24.3728
 0.4000     0.961177    0.007843   -4.615527    0.503136    0.9546    0.0099    25.2001
 0.6000     0.961172    0.007844   -4.423291    0.504705    0.9508    0.0113    26.1397
 0.8000     0.961164    0.007847   -4.231056    0.506274    0.9464    0.0129    27.1814
 1.0000     0.961153    0.007851   -4.038823    0.507844    0.9415    0.0149    28.2857
 1.2000     0.961138    0.007856   -3.846592    0.509415    0.9358    0.0172    29.3761
 1.4000     0.961118    0.007864   -3.654366    0.510987    0.9293    0.0201    30.3616
 1.6000     0.961093    0.007874   -3.462144    0.512560    0.9218    0.0236    31.2151
 1.8000     0.961060    0.007887   -3.269928    0.514136    0.9131    0.0280    32.1351
 2.0000     0.961019    0.007904   -3.077720    0.515715    0.9029    0.0334    34.0064
 2.2000     0.960966    0.007928   -2.885521    0.517298    0.8908    0.0404    35.7737
 2.4000     0.960898    0.007960   -2.693334    0.518887    0.8765    0.0493    37.7117
 2.6000     0.960813    0.008002   -2.501165    0.520483    0.8595    0.0611    39.8650
 2.8000     0.960734    0.008060   -2.309010    0.522089    0.8389    0.0767    42.2912
 3.0000     0.960565    0.008141   -2.116882    0.523709    0.8139    0.0980    45.0651
 3.2000     0.960386    0.008253   -1.924786    0.525348    0.7836    0.1276    48.2873
 3.4000     0.960153    0.008415   -1.732731    0.527014    0.7467    0.1697    52.0961
 3.6000     0.959848    0.008654   -1.540730    0.528720    0.7025    0.2314    56.6872
 3.8000     0.959444    0.009018   -1.348799    0.530485    0.6515    0.3242    62.3423
 4.0000     0.958911    0.009596   -1.156961    0.532343    0.5989    0.4682    69.4785
 4.2000     0.958216    0.010551   -0.965246    0.534351    0.5638    0.6962    78.7198
 4.3199     0.957729    0.011394   -0.850289    0.535669    0.5724    0.4465    73.4206
 4.3259     0.957707    0.011418   -0.844543    0.535737    0.5739    0.4522    73.8589
```

```
THE MOTION OF A CRITICAL PARTICLE IS GIVEN BY

ITER   =    10
G4LEF  =     0.919921
G4ZER  =     0.920898
G4RIT  =     0.921875

    TAU          G(1)         G(2)          G(3)          G(4)        UX        UY        CDRE

  0.0000      0.963851     0.013783     -5.000000      0.920898    0.9638    0.0137     24.0000
  0.2000      0.963850     0.013783     -4.807229      0.923655    0.9612    0.0154     24.3607
  0.4000      0.963848     0.013784     -4.614459      0.926411    0.9583    0.0174     25.1591
  0.6000      0.963844     0.013787     -4.421689      0.929169    0.9551    0.0197     26.0637
  0.8000      0.963837     0.013791     -4.228919      0.931926    0.9516    0.0224     27.0664
  1.0000      0.963828     0.013798     -4.036151      0.934685    0.9476    0.0256     28.1339
  1.2000      0.963816     0.013807     -3.843386      0.937446    0.9432    0.0294     29.1998
  1.4000      0.963800     0.013819     -3.650624      0.940208    0.9383    0.0339     30.1789
  1.6000      0.963780     0.013835     -3.457865      0.942973    0.9329    0.0395     31.0271
  1.8000      0.963755     0.013857     -3.265111      0.945743    0.9268    0.0462     31.6578
  2.0000      0.963723     0.013884     -3.072362      0.948517    0.9201    0.0545     33.3259
  2.2000      0.963684     0.013921     -2.879621      0.951297    0.9126    0.0647     35.1617
  2.4000      0.963637     0.013969     -2.686889      0.954086    0.9045    0.0775     36.9541
  2.6000      0.963579     0.014033     -2.494166      0.956886    0.8958    0.0937     38.9166
  2.8000      0.963509     0.014117     -2.301457      0.959700    0.8868    0.1142     41.0873
  3.0000      0.963426     0.014229     -2.108763      0.962535    0.8780    0.1405     43.5126
  3.2000      0.963328     0.014379     -1.916086      0.965395    0.8707    0.1745     46.2488
  3.4000      0.963216     0.014583     -1.723432      0.968290    0.8668    0.2185     49.3641
  3.6000      0.963094     0.014860     -1.530800      0.971233    0.8703    0.2752     52.9396
  3.8000      0.962971     0.015242     -1.338194      0.974242    0.8878    0.3473     57.0675
  4.0000      0.962872     0.015768     -1.145610      0.977341    0.9305    0.4354     61.8417
  4.2000      0.962849     0.016487     -0.953039      0.980564    1.0152    0.5346     67.3322
  4.4000      0.963000     0.017445     -0.760457      0.983954    1.1630    0.6257     73.5249
  4.6000      0.963486     0.018646     -0.567813      0.987560    1.3876    0.6659     80.2095
  4.8000      0.964518     0.019978     -0.375021      0.991422    1.6672    0.5890     86.8126
  5.0000      0.966240     0.021135     -0.181953      0.995538    1.9132    0.3453     92.2867
  5.1099      0.967489     0.021292     -0.075599      0.997873    1.9871    0.0752     94.9705
  5.1249      0.967668     0.021300     -0.061086      0.998192    1.9924    0.0609     95.3191
```

```
THE MOTION OF A CRITICAL PARTICLE IS GIVEN BY

ITER  =    20
G4LEF =     0.921587
G4ZER =     0.921588
G4RIT =     0.921589

    TAU        G(1)        G(2)        G(3)        G(4)         UX        UY        CDRE

  0.0000     0.963856    0.013792   -5.000000    0.921588     0.9638    0.0137    24.0000
  0.2000     0.963855    0.013792   -4.807229    0.924346     0.9612    0.0154    24.3607
  0.4000     0.963853    0.013793   -4.614457    0.927104     0.9583    0.0174    25.1590
  0.6000     0.963849    0.013796   -4.421685    0.929863     0.9551    0.0197    26.0635
  0.8000     0.963843    0.013800   -4.228915    0.932623     0.9516    0.0224    27.0662
  1.0000     0.963834    0.013807   -4.036146    0.935384     0.9476    0.0256    28.1336
  1.2000     0.963822    0.013816   -3.843379    0.938146     0.9432    0.0294    29.1994
  1.4000     0.963806    0.013828   -3.650616    0.940910     0.9384    0.0340    30.1785
  1.6000     0.963785    0.013844   -3.457856    0.943677     0.9329    0.0395    31.0267
  1.8000     0.963760    0.013866   -3.265101    0.946448     0.9268    0.0462    31.8572
  2.0000     0.963729    0.013893   -3.072351    0.949224     0.9201    0.0545    33.3243
  2.2000     0.963690    0.013930   -2.879609    0.952006     0.9127    0.0647    35.1604
  2.4000     0.963642    0.013978   -2.686875    0.954797     0.9045    0.0776    36.9526
  2.6000     0.963585    0.014042   -2.494152    0.957599     0.8958    0.0937    38.9147
  2.8000     0.963515    0.014126   -2.301441    0.960415     0.8869    0.1142    41.0849
  3.0000     0.963432    0.014238   -2.108746    0.963251     0.8781    0.1406    43.5096
  3.2000     0.963334    0.014388   -1.916069    0.966114     0.8708    0.1746    46.2449
  3.4000     0.963222    0.014592   -1.723412    0.969011     0.8670    0.2185    49.3590
  3.6000     0.963100    0.014869   -1.530780    0.971956     0.8706    0.2752    52.9328
  3.8000     0.962977    0.015251   -1.338172    0.974966     0.8881    0.3472    57.0582
  4.0000     0.962879    0.015776   -1.145586    0.978067     0.9308    0.4352    61.8289
  4.2000     0.962857    0.016495   -0.953014    0.981291     1.0156    0.5341    67.3143
  4.4000     0.963008    0.017452   -0.760430    0.984683     1.1633    0.6250    73.5000
  4.6000     0.963495    0.018651   -0.567785    0.988290     1.3877    0.6649    80.1751
  4.8000     0.964526    0.019980   -0.374991    0.992153     1.6666    0.5879    86.7667
  5.0000     0.966245    0.021134   -0.181922    0.996270     1.9120    0.3445    92.2295
  5.1399     0.967850    0.021306   -0.046538    0.999243     1.9950    0.0464    95.3180
  5.1869     0.968415    0.021308   -0.001035    1.000241     1.9995    0.0010    95.5894

THE IMPACTION EFFICIENCY IS  0.9215E 00
```

```
THE MOTION OF THE PARTICLES AT THE CYLINDER POSITION IS GIVEN BY

NZER  =  120                    NJBP  =   4

   TAU         VX          VY          X           Y          UX        UY        FY

 4.1690     0.955357    0.000000   -1.000526    0.000000    0.0010    0.0000    1.83428
 4.1690     0.955364    0.000336   -1.000514    0.023886    0.0027    0.0236    1.83324
 4.1710     0.955378    0.000785   -0.998553    0.055736    0.0064    0.0556    1.82856
 4.1730     0.955417    0.001232   -0.996544    0.087586    0.0160    0.0871    1.82018
 4.1760     0.955474    0.001677   -0.993535    0.119438    0.0298    0.1183    1.80807
 4.1810     0.955537    0.002121   -0.988577    0.151292    0.0459    0.1495    1.79221
 4.1860     0.955628    0.002561   -0.983570    0.183147    0.0679    0.1797    1.77265
 4.1920     0.955739    0.002995   -0.977564    0.215004    0.0939    0.2094    1.74935
 4.2000     0.955858    0.003428   -0.969602    0.246867    0.1226    0.2388    1.72231
 4.2090     0.955999    0.003855   -0.960640    0.278733    0.1557    0.2674    1.69153
 4.2190     0.956184    0.004489   -0.950676    0.310616    0.1931    0.2951    1.65693
 4.2300     0.956354    0.004919   -0.939714    0.342495    0.2347    0.3216    1.61864
 4.2420     0.956547    0.005340   -0.927748    0.374378    0.2807    0.3467    1.57661
 4.2550     0.956763    0.005751   -0.914779    0.406266    0.3307    0.3702    1.53084
 4.2700     0.956996    0.006154   -0.899855    0.438165    0.3843    0.3929    1.48131
 4.2870     0.957249    0.006548   -0.882964    0.470077    0.4420    0.4145    1.42801
 4.3050     0.957529    0.006930   -0.865068    0.501995    0.5039    0.4339    1.37096
 4.3240     0.957769    0.007698   -0.846172    0.533943    0.5700    0.4508    1.30967
 4.3450     0.958093    0.008059   -0.825305    0.565890    0.6400    0.4657    1.24565
 4.3680     0.958439    0.008408   -0.802470    0.597853    0.7142    0.4784    1.17663
 4.3930     0.958810    0.008745   -0.777668    0.629832    0.7925    0.4883    1.10441
 4.4210     0.959137    0.009586   -0.749939    0.661867    0.8757    0.4958    1.02771
 4.4510     0.959565    0.009888   -0.720228    0.693903    0.9627    0.4995    0.94782
 4.4830     0.960020    0.010179   -0.688544    0.725957    1.0528    0.4987    0.86410
 4.5190     0.960474    0.011027   -0.652960    0.758082    1.1480    0.4939    0.77584
 4.5590     0.961009    0.011281   -0.613464    0.790235    1.2477    0.4839    0.68431
 4.6040     0.961593    0.011538   -0.569094    0.822441    1.3523    0.4677    0.58882
 4.6540     0.962233    0.012317   -0.519841    0.854754    1.4595    0.4435    0.48878
 4.7130     0.962973    0.013062   -0.461844    0.887174    1.5733    0.4094    0.38473
 4.7830     0.963851    0.013253   -0.393145    0.919734    1.6906    0.3612    0.27699
 4.8730     0.964955    0.013895   -0.305038    0.952602    1.8135    0.2902    0.16438
 4.9010     0.965300    0.013940   -0.277681    0.960891    1.8454    0.2665    0.13555

THE DRAG COEFFICIENT OF THE CYLINDER DUE TO THE REFLECTION OF PARTICLES IS     2.3435

THE TARGET REYNOLDS NUMBER IS  0.3796E 04

THE DRAG COEFFICIENT OF THE CYLINDER DUE TO THE AIR ALONE IS     1.000
// PAUS
```

APPENDIX B

COMPUTER PROGRAM FOR A SPHERE IN A TUBE OR IN FREE SPACE

```
PAGE   1

// JOB T

LOG DRIVE   CART SPEC   CART AVAIL   PHY DRIVE
  0000        0305        0305        0001

V2 M11   ACTUAL 16K  CONFIG 16K

// EJECT
```

```
PAGE   2

// FOR
*ONE WORD INTEGERS
*LIST ALL
      SUBROUTINE SBM34(G.UX.UY.ITUBE)
C
C     THIS SUBROUTINE CALCULATES THE FLOW ABOUT
C     A SPHERE IN A CIRCULAR TUBE FOR A SPHERE TO TUBE DIAMETER RATIO
C     OF 0.8/2.54 OR FOR A SPHERE IN FREE SPACE.
C
      DIMENSION G(4)
      IF(ITUBE)1.1.2
    1 RSQ =  G(3)**2+G(4)**2
      UX = 1.0 - 1.0/RSQ**1.5 + 1.5*G(4)**2/RSQ**2.5
      UY =-1.5*G(3)*G(4)/RSQ**2.5
      GO TO 3
    2 R = SQRT(G(3)**2 + G(4)**2)
      COST =-G(3)/R
      SINT = G(4)/R
      DC = 0.8
      C = 1.0
      A = (2.54/DC)*C
      S2 = 7.5098907
      CZERO = 1.5401075
      PI = 3.14159265358979323846264
      AZERO=-C*(C/A)**2*S2*CZERO/(9.0*PI)
      FT1 = CZERO*C**3/(3.*R**3)
      FT2=2.0*AZERO/A
      UT=SINT*(1.0+FT1-FT2)
      FR1 = 2.0*CZERO*C**3/(3.*R**3)
      FR2 = FT2
      UR=-COST*(1.0-FR1-FR2)
      UX = -UR*COST + UT*SINT
      UY =  UR*SINT + UT*COST
    3 CONTINUE
      RETURN
      END
VARIABLE ALLOCATIONS
   RSQ(R )=0000        R(R )=0002     COST(R )=0004     SINT(R )=0006       DC(R )=0008       C(R )=000A
     A(R )=000C       S2(R )=000E    CZERO(R )=0010       PI(R )=0012    AZERO(R )=0014     FT1(R )=0016
   FT2(R )=0018       UT(R )=001A      FR1(R )=001C      FR2(R )=001E       UR(R )=0020

STATEMENT ALLOCATIONS
 1     =0059  2     =00A3  3     =016E

FEATURES SUPPORTED
 ONE WORD INTEGERS
```

```
PAGE   3

CALLED SUBPROGRAMS
 FSQRT   FAXB    FADD     FSUB     FMPY     FMPYX   FDIV     FLD      FLDX     FSTO     FSBR     FDVR     FAXI     SNR      SUBIN

REAL CONSTANTS
  .100000E 01=0028    .150000E 01=002A    .250000E 01=002C    .800000E 00=002E    .254000E 01=0030    .750989E 01=0032
  .154010E 01=0034    .314159E 01=0036    .900000E 01=0038    .300000E 01=003A    .200000E 01=003C

INTEGER CONSTANTS
     2=003E      3=003F

CORE REQUIREMENTS FOR SBM34
 COMMON      0  VARIABLES      40  PROGRAM     328

RELATIVE ENTRY POINT ADDRESS IS 0040 (HEX)

END OF COMPILATION

// DUP

*STORE      WS  UA  SBM34
CART ID 0305   DB ADDR  5A0A   DB CNT   0018

// EJECT
```

```
// FOR
*ONE WORD INTEGERS
*LIST ALL
C
      FUNCTION CDRE(RE)
C
C     THIS FUNCTION COMPUTES THE PRODUCT OF DRAG COEFFICIENT
C     AND REYNOLDS NUMBER FOR A SPHERE AS A FUNCTION
C     OF REYNOLDS NUMBER LESS THAN 10000.
C
      A1=1./24.
      A2=-2.3363*1.E-04
      A3=2.0154*1.E-06
      A4=-6.9105*1.E-09
      B0=-1.29536
      B1=9.86*1.E-01
      B2=-4.6677*1.E-02
      B3=1.1235*1.E-03
C
C     CHOOSE THE APPROPRIATE POLYNOMIAL
C
      IF(RE-4.0)2,7,7
C
C     INITIAL ESTIMATE
C
    2 IF(RE-0.00001)3,4,4
    3 CDRE = 24.0
      GO TO 30
    4 X=24.*RE
C
C     BEGIN NEWTON METHOD ITERATION
C
      CONTINUE
      DO 6 ITER=1,20
      FX=A1*X+A2*X**2+A3*X**3+A4*X**4-RE
      FPX=A1+2.*A2*X+3.*A3*X**2+4.*A4*X**3
      DELX=FX/FPX
      X=X-DELX
C
C     CHECK FOR CONVERGENCE
C
      EPS=1.E-06
      IF(ABS(DELX/X)-EPS)5,5,6
    5 CDRE=X/RE
      GO TO 30
    6 CONTINUE
```

```
PAGE   5

      GO TO 29
C
C     INITIAL ESTIMATE
C
    7 CD = 1.0
      ELOG = 0.434294481903252
      X=ALOG(CD*RE**2)*ELOG
C
C     BEGIN NEWTON METHOD ITERATION
C
      DO 24 ITER=1,20
      FX=B0+B1*X+B2*X**2+33*X**3 - ALOG(RE)*ELOG
      FPX=B1+2.*B2*X+3.*B3*X**2
      DELX=FX/FPX
      X=X-DELX
C
C     CHECK FOR CONVERGENCE
C
      EPS=1.E-06
      IF(ABS(DELX/X)-EPS)22,22,24
   22 CDRE=10.**X/RE
      GO TO 30
   24 CONTINUE
   29 WRITE(3,202)
   30 RETURN
C
C     FORMATS FOR OUTPUT STATEMENTS
C
  202 FORMAT(16H0 NO CONVERGENCE)
C
      END
VARIABLE ALLOCATIONS
  CDRE(R )=0000        A1(R )=0002        A2(R )=0004        A3(R )=0006        A4(R )=0008        B0(R )=000A
    B1(R )=000C        B2(R )=000E        B3(R )=0010         X(R )=0012        FX(R )=0014       FPX(R )=0016
  DELX(R )=0018       EPS(R )=001A        CD(R )=001C      ELOG(R )=001E      ITER(I )=0028

STATEMENT ALLOCATIONS
 202   =0059  2    =00AB  3    =00AF  4    =00B5  5    =012D  6    =0135  7    =0140  22   =0188  24   =01C3  29   =01CC
 30    =01D0

FEATURES SUPPORTED
 ONE WORD INTEGERS

CALLED SUBPROGRAMS
 FABS    FALOG    FAXB    FADD    FSUB    FMPY    FDIV    FLD    FSTO    FSBR    FAXI    SWRT    SCOMP    SNR    SUBIN

REAL CONSTANTS
  .100000E 01=002A    .240000E 02=002C    .233630E 01=002E    .100000E-03=0030    .201540E 01=0032    .100000E-05=0034
```

```
PAGE   6

  .691050E 01=0036    .100000E-08=0038    .129536E 01=003A    .986000E 01=003C    .100000E 00=003E    .466770E 01=0040
  .100000E-01=0042    .112350E 01=0044    .100000E-02=0046    .400000E 01=0048    .100000E-04=004A    .200000E 01=004C
  .300000E 01=004E    .434294E 00=0050    .100000E 02=0052

INTEGER CONSTANTS
    1=0054     20=0055      2=0056      3=0057      4=0058

CORE REQUIREMENTS FOR CDRE
 COMMON        0  VARIABLES      42  PROGRAM     426

RELATIVE ENTRY POINT ADDRESS IS 0063 (HEX)

END OF COMPILATION

// DUP

*STORE      WS  UA  CDRE
CART ID 0305   DB ADDR  5A25   DB CNT   0022

// EJECT
```

```
// FOR
*ONE WORD INTEGERS
*LIST ALL
      SUBROUTINE SBM22(N,Y,F,X,H,IRUNG,M)
C
C     FOURTH ORDER RUNGE KUTTA METHOD
C     FOR N FIRST ORDER O.D.E.
      DIMENSION PHI(50),SAVY(50),Y(50),F(50)
C
      GO TO (2,3,4,5,6),M
C
C     PASS 1
C
    2 IRUNG=1
      RETURN
C
C     PASS 2
C
    3 DO 22 J=1,N
      SAVY(J)=Y(J)
      PHI(J)=F(J)
   22 Y(J)=SAVY(J)+0.5*H*F(J)
      X=X+0.5*H
      IRUNG=1
      RETURN
C     PASS 3
    4 DO 33 J=1,N
      PHI(J)=PHI(J)+2.0*F(J)
   33 Y(J)=SAVY(J)+0.5*H*F(J)
      IRUNG=1
      RETURN
C
C     PASS 4
    5 DO 44 J=1,N
      PHI(J)=PHI(J)+2.0*F(J)
   44 Y(J)=SAVY(J)+H*F(J)
      X=X+0.5*H
      IRUNG=1
      RETURN
C     PASS 5
    6 DO 55 J=1,N
   55 Y(J) = SAVY(J) + (PHI(J) + F(J))*H/6.0
      IRUNG=2
      RETURN
C
      END
```

```
PAGE   8

VARIABLE ALLOCATIONS
   PHI(R )=0062-0000  SAVY(R )=00C6-0064    J(I )=00C8
STATEMENT ALLOCATIONS
 2     =0105  3    =010B  22   =011E  4    =0141  33   =0152  5    =016D  44   =017E  6    =019F  55   =01A3
FEATURES SUPPORTED
 ONE WORD INTEGERS
CALLED SUBPROGRAMS
 FADD     FADDX    FMPY     FMPYX    FDIV     FLD      FLDX     FSTO     FSTOX    SUBSC    SUBIN
REAL CONSTANTS
  .500000E 00=00CA    .200000E 01=00CC    .600000E 01=00CE
INTEGER CONSTANTS
     1=0000       2=0001
CORE REQUIREMENTS FOR SBM22
 COMMON      0  VARIABLES     202  PROGRAM     250
RELATIVE ENTRY POINT ADDRESS IS 00D2 (HEX)
END OF COMPILATION
// DUP
*STORE      WS  UA  SBM22
CART ID 0305   DB ADDR  5A47   DB CNT    0C12
// EJECT
```

```
PAGE   9

// FOR
*ONE WORD INTEGERS
*LIST ALL
      SUBROUTINE SBM32(G,TAU,DTAW,XK,REZ,UX,UY,XCDRE)
C
C     THIS SUBROUTINE CALCULATES PARTICLE MOTION DURING THE FINAL
C     INCREMENT OF TAU
C
      DIMENSION G(4),DG(4)
      M=0
C
C     CALL ON RUNGE KUTTA SUBROUTINE
C
    8 CONTINUE
      M=M+1
      CALL SBM22(4,G,DG,TAU,DTAW,IRUNG,M)
      IF(IRUNG-1)10,9,10
    9 RE=REZ*((UY-G(2))**2+(UX-G(1))**2)**0.5
      XCDRE=CDRE(RE)
      DG(1)=(XCDRE/(24.0*XK))*(UX-G(1))
      DG(2)=(XCDRE/(24.0*XK))*(UY-G(2))
      DG(3)=G(1)
      DG(4)=G(2)
      GO TO 8
   10 CONTINUE
      M=0
C
C     CALCULATE FLUID VELOCITY AT PARTICLE POSITION
C
      CALL SBM34 (G,UX,UY,ITUBE)
C
C     INTEGRATE ACROSS ANOTHER STEP IF REQUIRED
C
      IF(G(4) - 1.0)11,11,12
   11 DELX = SQRT(1.0 - G(4)**2)
      GO TO 13
   12 DELX = 0.0
   13 HITS=G(3)+G(1)*1.1*DTAW+DELX
      IF(HITS)8,8,18
   18 CONTINUE
      RETURN
      END
VARIABLE ALLOCATIONS
     DG(R )=0006-0000     RE(R )=0008        DELX(R )=000A        HITS(R )=000C        M(I )=0010        IRUNG(I )=0011
  ITUBE(I )=0012
```

```
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STATEMENT ALLOCATIONS
 8     =0050  9     =0065  10    =00BB  11    =00CE  12    =00DF  13    =00E3  18    =00F8
FEATURES SUPPORTED
 ONE WORD INTEGERS

CALLED SUBPROGRAMS
 SBM22   CORE    SBM34   FSQRT   FAXB    FADD    FADDX   FSUB    FSUBX   FMPY    FLD     FLDX    FSTO    FSTOX   FSBR
 FDVR    FAX     SUBIN

REAL CONSTANTS
  .500000E 00=0018    .240000E 02=001A    .100000E 01=001C    .000000E 00=001E    .110000E 01=0020

INTEGER CONSTANTS
     0=0022      1=0023      4=0024      2=0025

CORE REQUIREMENTS FOR SBM32
 COMMON      0  VARIABLES      24  PROGRAM     226

RELATIVE ENTRY POINT ADDRESS IS 0026 (HEX)

END OF COMPILATION

// DUP

*STORE      WS  UA  SBM32
CART ID 0305   DB ADDR  5A59   DB CNT   0011

// EJECT
```

```
// FOR
*ONE WORD INTEGERS
*LIST ALL
      SUBROUTINE SBM30(G4LFT,G4RIT,SIGNL,DTAU,NIBP,NSBP,NX,XL,XK,REZ,
     1G4ZER)
C
C     THIS SUBROUTINE CALCULATES THE
C     IMPACTION EFFICIENCY OF A SPHERE
C
      DIMENSION G(4),DG(4)
      WRITE(3,200)
      WRITE(3,201)G4LFT,G4RIT,SIGNL,DTAU,NIBP,NSBP,NX
C
C     HALF INTERVAL ITERATION FOR INITIAL G4 VALUE
C
      DO 31 ITER=1,NX
C
C     SET AND PRINT INITIAL CONDITIONS
C
      M=0
      NSTEP=0
      TAU=0.0
      G(3)=XL
      G4ZER=(G4LFT+G4RIT)/2.0
      G(4)=G4ZER
      CALL SBM34 (G,UX,UY,ITUBE)
      G(1) = UX
      G(2)=UY
      RE=REZ*((UY-G(2))**2+(UX-G(1))**2)**0.5
      XCDRE=CDRE(RE)
      IP=ITER/NIBP*NIBP
      IF(IP-ITER)5,7,5
    5 IF(ITER-1)6,7,6
    6 IF(ITER-NX)8,7,8
    7 CONTINUE
      WRITE (3,205)
      WRITE(3,203)ITER,G4LFT,G4ZER,G4RIT,TAU,G(1),G(2),G(3),G(4),UX,UY,
     1XCDRE
C
C     CALL ON RUNGE KUTTA SUBROUTINE
C
    8 CONTINUE
      M=M+1
      CALL SBM22(4,G,DG,TAU,DTAU,IRUNG,M)
      IF(IRUNG-1)10,9,10
    9 RE=REZ*((UY-G(2))**2+(UX-G(1))**2)**0.5
```

```
      XCDRE=CDRE(RE)
      DG(1)=(XCDRE/(24.0*XK))*(UX-G(1))
      DG(2)=(XCDRE/(24.0*XK))*(UY-G(2))
      DG(3)=G(1)
      DG(4)=G(2)
      GO TO 8
   10 CONTINUE
      M=0
C
C     CALCULATE FLUID VELOCITY AT PARTICLE POSITION
C
      CALL SBM34 (G,UX,UY,ITUBE)
C
C     PRINT SOLUTIONS
C
      IS = ITER/NIBP*NIBP
      IF(IS-ITER)11,13,11
   11 IF(ITER-1)12,13,12
   12 IF(ITER-NX)16,13,16
   13 NSTEP=NSTEP+1
      IF(NSTEP-NSBP)16,14,16
   14 CONTINUE
      NSTEP=0
      TAW = TAU+0.0001
      WRITE(3,204)TAW,G(1),G(2),G(3),G(4),UX,UY,XCDRE
C
C     INTEGRATE ACROSS ANOTHER STEP IF REQUIRED
C
   16 CONTINUE
      IF(G(4)-1.0)17,17,18
   17 DELX=SQRT(1.0-G(4)**2)
      GO TO 19
   18 DELX=0.0
   19 HITS=G(3)+G(1)*1.1*DTAU+DELX
      IF(HITS)8,8,20
   20 CONTINUE
C
C     CHANGE INCREMENT SIZE NEAR SPHERE AND INTEGRATE FURTHER
C
      DTAW=DTAU/10.0
      CALL SBM32(G,TAU,DTAW,XK,REZ,UX,UY,XCDRE)
C
C     PRINT SOLUTIONS
C
      IF(IS-ITER)21,23,21
   21 IF(ITER-1)22,23,22
```

```
   22 IF(ITER-NX)24,23,24
   23 CONTINUE
      WRITE(3,204)TAU,G(1),G(2),G(3),G(4),UX,UY,XCDRE
   24 CONTINUE
      DTAW=DTAU/100.0
      CALL SBMS2(G,TAU,DTAW,XK,REZ,UX,UY,XCDRE)
C
C     PRINT SOLUTIONS
C
      IF(IS-ITER)25,27,25
   25 IF(ITER-1)26,27,26
   26 IF(ITER-NX)28,27,28
   27 CONTINUE
      WRITE(3,204)TAU,G(1),G(2),G(3),G(4),UX,UY,XCDRE
   28 CONTINUE
C
C     CALCULATE ORDINATE AT TANGENT POINT OF TANGENT PATH
C
      ORD = G(1)/SQRT(G(1)**2 + G(2)**2)
C
C     FIND INTERVAL HALF WITH THE SIGN CHANGE
C
      IF((G(4)-ORD)*SIGNL-0.0)29,29,30
   29 G4RIT=G4ZER
      GO TO 31
   30 G4LFT=G4ZER
   31 CONTINUF
      EM = G4ZER**2
      WRITE(3,207) EM
C
      RETURN
C
C     FORMATS FOR OUTPUT STATEMENTS
C
  200 FORMAT( 1H0, 41X, 33HIMPACTION EFFICIENCY OF A SPHERE /
     1   1H0 )
  201 FORMAT(  10H0G4LEF =   ,F10.6/ 10H G4RIT =  ,F10.6/10H SIGNL =   ,
     1   F3.0/ 10H DTAU   =   ,F10.6/ 10H NIBP   =   ,I3/ 10H NSBP   =   ,I3/
     2   10H NX     =   ,I3)
  203 FORMAT( 10H0ITER   =   ,I3/ 10H G4LEF =   ,F10.6/ 10H G4ZER =   ,
     1   F10.6/ 10H G4RIT =   ,F10.6/ 7H0    TAU, 11X, 4HG(1), 12X,
     2   4HG(2), 12X, 4HG(3), 12X, 4HG(4), 14X, 2HUX, 14X,  2HUY ,
     3   12X, 4HCDRE /
     4   1H0, F7.4, 4F16.6, 3F16.4 )
  204 FORMAT ( 1H , F7.4, 4F16.6, 3F16.4 )
  205 FORMAT ( 46H1THE MOTION OF A CRITICAL PARTICLE IS GIVEN BY )
```

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PAGE  14

  207 FORMAT( 30HOTHE IMPACTION EFFICIENCY IS  ,E10.4)
      END
VARIABLE ALLOCATIONS
     G(R )=0006-0000    DG(R )=000E-0008   TAU(R )=0010        UX(R )=0012          UY(R )=0014           RE(R )=0016
 XCDRE(R )=0018       TAM(R )=001A      DELX(R )=001C      HITS(R )=001E      DTAM(R )=0020          ORD(R )=0022
    FM(R )=0024      ITER(I )=002A         M(I )=002B     NSTEP(I )=002C     ITUBE(I )=002D           IP(I )=002E
 IRUNG(I )=002F        IS(I )=0030

STATEMENT ALLOCATIONS
 200  =004D  201  =0066  203  =009E  204  =00E6  205  =002E  207  =0107  5    =01CE  6    =01D4  7    =01DA  8    =0202
 9    =0217  10   =026D  11   =0287  12   =028D  13   =0293  14   =029F  16   =02C5  17   =02CE  18   =02DF  19   =02E3
 20   =02F8  21   =030E  22   =0314  23   =031A  24   =0336  25   =034C  26   =0352  27   =0358  28   =0374  29   =039C
 30   =03A2  31   =03A6

FEATURES SUPPORTED
 ONE WORD INTEGERS

CALLED SUPPROGRAMS
 SBM34    CDRE     SBM22    FSQRT    SBM32    FAXB     FADD     FADDX    FSUB     FSUBX    FMPY     FDIV     FLD      FLDX     FSTO
 FSTOX    FSBR     FDVR     FDVRX    FAXI     SWRT     SCOMP    SIOFX    SIOF     SIOI     SUBIN

REAL CONSTANTS
  .000000F 00=0036     .200000E 01=0038     .500000E 00=003A     .240000E 02=003C     .100000E-03=003E     .100000E 01=0040
  .110000E 01=0042     .100000E 02=0044     .100000E 03=0046

INTEGER CONSTANTS
      3=0048       1=0049       0=004A       2=004B       4=004C

CORE REQUIREMENTS FOR SBM30
 COMMON        0  VARIABLES       54  PROGRAM     904

RELATIVE ENTRY POINT ADDRESS IS 0119 (HEX)

END OF COMPILATION

// DUP

*STORE       WS  UA  SBM30
CART ID 0305   DB ADDR  5A6A   DB CNT    003E

// EJECT
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// FOR
*ONE WORD INTEGERS
*LIST ALL
      SUBROUTINE SBM31(XL,G4ZER,DTAU,XK,REZ,PX,NZER,NJBP)
C
C     THIS SUBROUTINE CALCULATES THE MOTION OF PARTICLES
C     IN A FLUID STREAM MOVING TOWARD A SPHERE
C     AND CALCULATES THE FORCE OF PARTICLE IMPACT ON THE SPHERE
C
      DIMENSION G(4),DG(4)
      DIMENSION YZER(500)
C
C     SET NUMBER OF INCREMENTS AT INITIAL POSITION
C
      WRITE(3,200)
      WRITE(3,201)NZER,NJBP
      WRITE(3,202)
      NCE=NZER+1
      DELY=G4ZER/FLOAT(NZER)
      FSUM=0.0
C
C     STEPWISE INTEGRATION FOLLOWING PARTICLE POSITION
C
      DO 30 ITER=1,NCE
C
C     SET AND PRINT INITIAL CONDITIONS
C
      M=0
      TAU=0.0
      G(3)=XL
      YZER(ITER)=FLOAT(ITER-1)*DELY
      G4ZER=YZER(ITER)
      G(4)=G4ZER
      CALL SBM34 (G,UX,UY,ITUBE)
      UXZER=UX
      G(1)=UX
      G(2)=UY
C
C     CALL ON RUNGE KUTTA SUBROUTINE
C
    8 CONTINUE
      M=M+1
      CALL SBM22(4,G,DG,TAU,DTAU,IRUNG,M)
      IF(IRUNG-1)10,9,10
    9 RE=REZ*((UY-G(2))**2+(UX-G(1))**2)**0.5
      XCDRE=CDRE(RE)
```

```
      DG(1)=(XCDRE/(24.0*XK))*(UX-G(1))
      DG(2)=(XCDRE/(24.0*XK))*(UY-G(2))
      DG(3)=G(1)
      DG(4)=G(2)
      GO TO 8
   10 CONTINUE
      M=0
C
C     CALCULATE FLUID VELOCITY AT PARTICLE POSITION
C
      CALL SBM34 (G,UX,UY,ITUBE)
C
C     INTEGRATE ACROSS ANOTHER STEP IF REQUIRED
C
      IF(G(4) - 1.0)11,11,12
   11 DELX = SQRT(1.0 - G(4)**2)
      GO TO 13
   12 DELX = 0.0
   13 HITS=G(3)+G(1)*1.1*DTAU+DELX
      IF(HITS)8,8,18
   18 CONTINUE
C
C     CHANGE INCREMENT SIZE NEAR SPHERE AND INTEGRATE FURTHER
C
      DTAW=DTAU/10.0
      CALL SBM32(G,TAU,DTAW,XK,REZ,UX,UY,XCDRE)
      DTAW=DTAU/100.0
      CALL SBM32(G,TAU,DTAW,XK,REZ,UX,UY,XCDRE)
C
C     CO-ORDINATES, VELOCITY, AND PRESSURE DERIVATIVE AT SPHERE
C
      VX=G(1)
      VY=G(2)
      X=G(3)
      Y=G(4)
      V=SQRT(VX**2+VY**2)
      PI = 3.14159265358979323846264
      GAMMA = PI - 2.0*ATAN(Y/SQRT(1.0-Y**2))- ATAN(VY/VX)
      FY= UXZER*(VX- V*COS(GAMMA))
      FSUM = FSUM + FLOAT(2*ITER-1)*FY
      CONTINUE
C
C     PRINT SOLUTIONS
C
      IS = ITER/NJBP*NJBP
      IF(IS-ITER)21,23,21
```

```
   21 IF(ITER-1)22,23,22
   22 IF(ITER-NCE)24,23,24
   23 CONTINUE
      WRITE(3,203)TAU,VX,VY,X,Y,UX,UY,FY
   24 CONTINUE
   30 CONTINUE
C
C     DRAG COEFFICIENT OF SPHERE DUE TO PARTICLE IMPACT
C
      PX = 2.*FSUM*DELY**2
      WRITE(3,204)PX
      RETURN
C
C     FORMATS FOR OUTPUT STATEMENTS
C
  200 FORMAT( 63H1THE MOTION OF THE PARTICLES AT THE SPHERE POSITION IS
     1GIVEN BY )
  201 FORMAT( 10H0NZER   =   ,I3,20X, 10H NJBP   =   ,I3)
  202 FORMAT( 7H0   TAU, 11X, 4H VX , 12X, 4H VY , 11X, 4H  X , 12X,
     1    4H   Y , 14X, 2HUX, 14X, 2HUY , 14X, 4H FY /1H  )
  203 FORMAT ( F7.4, 4F16.6, 2F16.4, F16.5)
  204 FORMAT( 75H0THE DRAG COEFFICIENT OF THE SPHERE DUE TO THE REFLECTI
     1ON OF PARTICLES IS   , F9.4 )
      END
VARIABLE ALLOCATIONS
     G(R )=0006-0000        DG(R )=000E-0008  YZER(R )=03F6-0010  DELY(R )=03F8        FSUM(R )=03FA         TAU(R )=03FC
    UX(R )=03FE             UY(R )=0400        UXZER(R )=0402          RE(R )=0404      XCORE(R )=0406        DELX(R )=0408
  HITS(R )=040A           DTAW(R )=040C           VX(R )=040E          VY(R )=0410          X(R )=0412           Y(R )=0414
     V(R )=0416             PI(R )=0418        GAMMA(R )=041A          FY(R )=041C        NCE(I )=0426        ITER(I )=0427
     M(I )=0428          ITUBE(I )=0429        IRUNG(I )=042A          IS(I )=042B

STATEMFNT ALLOCATIONS
 200   =0447  201   =0469  202   =0479  203   =049C  204   =04A3  8     =0557  9     =056C  10    =05C2  11    =05D5  12    =05E6
 13    =05EA  18    =05FF  21    =06A0  22    =06A6  23    =06AC  24    =06C0  30    =06C0

FEATURFS SUPPORTED
 ONE WORD INTEGERS

CALLED SUPPROGRAMS
 SBM34    SBM22    CDRE     FSQRT    SBM32    FATAN    FCOS     FAXB     FADD     FADDX    FSUB     FSUBX    FMPY     FDIV     FLD
 FLDX     FSTO     FSTOX    FSBR     FDVR     FAXI     FLOAT    SWRT     SCOMP    SIOF     SIOI     SUBSC    SUBIN

REAL CONSTANTS
  .000000E 00=0430     .500000E 00=0432     .240000E 02=0434     .100000E 01=0436     .110000E 01=0438     .100000E 02=043A
  .100000E 03=043C     .314159E 01=043E     .200000E 01=0440
```

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INTEGER CONSTANTS
     3=0442      1=0443      0=0444      4=0445      2=0446

CORF REQUIREMENTS FOR SBM31
 COMMON       0  VARIABLES   1072  PROGRAM    688

RELATIVE ENTRY POINT ADDRESS IS 04CC (HEX)

END OF COMPILATION

// DUP

*STORE       WS  UA  SBM31
CART ID 0305   DB ADDR   5AAB   DB CNT    0030

// EJECT
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// FOR
*ONE WORD INTEGERS
*IOCS(CARD)
*IOCS(1132 PRINTER)
*LIST ALL
C
C     ESTABLISH PARAMETERS FOR STEP BY STEP INTEGRATION AND HALF
C     INTERVAL METHOD
C     G4LFT AND G4RIT ARE LOWER AND UPPER BOUNDS RESPECTIVELY FOR
C     STARTING LOCATION OF PARTICLE
C     SIGNL IS SIGN CORRECTION FOR HALF INTERVAL METHOD
C     IN THIS PROBLEM IT IS -1.0
C     DTAU  IS THE TIME INCREMENT FOR STEP BY STEP INTEGRATION
C     NIBP IS INTERVAL OF WRITTEN PARTICLE PATHS
C     NSBP IS INTERVAL OF WRITTEN INTEGRATION STEPS
C     NX IS NUMBER OF ITERATIONS USED TO DETERMINE PARTICLE STARTING
C     POINT
C
C     ESTABLISH WHETHER A TUBE IS PRESENT
C
C     IF ITUBE IS POSITIVE A TUBE IS PRESENT
C     IF ITUBE IS ZERO OR NEGATIVE SPHERE IS IN FREE SPACE
C
    1 READ(2,100)G4LFT,G4RIT,SIGNL,DTAU,NIBP,NSBP,NX,ITUBE
C
C     ESTABLISH PHYSICAL PROPERTIES FOR CALCULATING IMPACTION EFFICIENCY
C
C     DC    IS SPHERE DIAMETER, CM
C     DP    IS PARTICLE DIAMETER, CM
C     RHO   IS FLUID DENSITY, GM/CC
C     SIGMA IS PARTICLE DENSITY, GM/CC
C     XMU   IS ABSOLUTE VISCOSITY OF FLUID, POISE
C     UF    IS FREE STEAM VELOCITY, CM/SEC
C
      READ(2,101)DC,DP,RHO,SIGMA,XMU,UF
C
C     XL IS UPSTREAM STARTING POSITION FOR CALCULATING PARTICLE MOTION
C
      READ(2,102)XL
      REZ=RHO*DP*UF/XMU
      XK=SIGMA*DP**2*UF/(9.*XMU*DC)
      P = 9.*RHO**2*UF*DC/(XMU*SIGMA)
      WRITE(3,200)
      WRITE(3,201)
C
C     XK IS INERTIAL PARAMETER CALLED STOKES NUMBER
```

```
C      REZ IS FREE STREAM REYNOLDS NUMBER OF SPHERICAL PARTICLE
C      P IS DIMENSIONLESS GROUP INDEPENDENT OF DROP SIZE
C
       WRITE(3,202)XL,REZ,XK,P,DC,DP,RHO,SIGMA,XMU,UF,ITUBE
C
C      CALCULATE STARTING POSITION OF TANGENT PARTICLE
C
       CALL SBM30(G4LFT,G4RIT,SIGNL,DTAU,NIBP,NSBP,NX,XL,XK,REZ,G4ZER)
C
C      ESTABLISH NUMBER OF INCREMENTS FOR INTEGRATING PARTICLE FORCE
C
C      NZER IS NUMBER OF INCREMENTS
C      NJBP IS INTERVAL OF WRITTEN INCREMENTS
C
       READ(2,103)NZER,NJBP
C
C      CALCULATE FORCE PRODUCED BY PARTICLES
C
       CALL SBM31(XL,G4ZER,DTAU,XK,REZ,PX,NZER,NJBP)
       RECYL=RHO*DC*UF/XMU
       CDAIR = CDRE(RECYL)/RECYL
C
C      OUTPUT RESULTS
C
       WRITE(3,204)RECYL
       IF(RECYL - 10000)10,11,11
    10 CCNTINUE
       WRITE(3,206)CDAIR
    11 CONTINUE
       READ(2,111)NSTOP
       IF(NSTOP)1,30,30
    30 CALL EXIT
C
C      FORMATS FOR INPUT AND OUTPUT STATEMENTS
C
   100 FORMAT(10X,F10.7,20X,F10.7,19X,F3.0/ 10X,F10.7,20X,I3,27X,I2/
     1  10X,I2,10X,I2)
   101 FORMAT(F10.5,F10.7,F10.6, F10.6,F10.7,F10.1 )
   102 FORMAT (F10.7)
   103 FORMAT(I5,I5)
   111 FORMAT(I5)
   200 FORMAT( 1H1, 35X, 49HTHE DRAG ON SPHERES IN A STREAM OF DUST-LADEN
     1 AIR / 1HO )
   201 FORMAT(  28HOTHE PHYSICAL PARAMETERS ARE )
   202 FORMAT(  10HOXL      =  ,F10.7/10H REZ     =  ,E12.6/10H XK      =  ,
     1   E12.6/10H P       =   ,E12.6/
```

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      2   10HODC      =  , F10.5/10H DP     =  ,F10.7/10H RHO    =  ,F10.6/
      3   10H SIGMA =  , F10.6/10H XMU    =  ,F10.7/10H UF     =  ,F10.1/
      4   10H ITUBE =  ,I3)
  204 FORMAT(  32HOTHE TARGET REYNOLDS NUMBER IS  ,E10.4)
  206 FORMAT( 59HOTHE DRAG COEFFICIENT OF THE SPHERE DUE TO THE AIR ALON
     1E IS,F9.4)
      END
VARIABLE ALLOCATIONS
 G4LFT(R )=C000       G4RIT(R )=0002       SIGNL(R )=0004       DTAU(R )=0006          DC(R )=0008          DP(R )=000A
   RHO(R )=000C       SIGMA(R )=000E         XMU(R )=0010         UF(R )=0012          XL(R )=0014         REZ(R )=0016
    XK(R )=0018           P(R )=001A       G4ZER(R )=001C         PX(R )=001E       RECYL(R )=0020       CDAIR(R )=0022
  NIBP(I )=0026        NSBP(I )=0027          NX(I )=0028      ITUBE(I )=0029        NZER(I )=002A        NJBP(I )=002B
 NSTOP(I )=002C

STATEMENT ALLOCATIONS
 100   =0033  101   =0046  102   =004D  103   =004F  111   =0052  200   =0054  201   =0075  202   =0085  204   =00DD  206   =00F0
 1     =0178  10    =01E0  11    =01E6  30    =01EF

FEATURES SUPPORTED
 ONE WORD INTEGERS
 IOCS

CALLED SUBPROGRAMS
 SRM30    SRM31    CDRE     FMPY     FDIV     FLD      FSTO     FSBR     FDVR     FAXI     FLOAT    CARDZ    PRNTZ    SRED     SWRT
 SCOMP    SFIO     SIOF     SIOI

REAL CONSTANTS
  .900000E 01=002E

INTEGER CONSTANTS
     2=0030      3=0031  10000=0032

CORE REQUIREMENTS FOR
 COMMON       0  VARIABLES      46  PROGRAM     450

END OF COMPILATION

// XEQ
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```
                         THE DRAG ON SPHERES IN A STREAM OF DUST-LADEN AIR

 THE PHYSICAL PARAMETERS ARE

 XL     =  -5.0000009
 REZ    =  0.735665E 02
 XK     =  0.339462E 03
 P      =  0.159429E 02

 DC     =     0.80000
 DP     =  0.0155000
 RHO    =    0.001213
 SIGMA  =    2.600000
 XMU    =  0.0001789
 UF     =       700.0
 ITURE  =    1

                            IMPACTION EFFICIENCY OF A SPHERE

 G4LEF  =    0.000000
 G4RIT  =    1.000000
 SIGNL  =  -1.
 DTAU   =    0.100000
 NIBP   =  10
 NSHP   =   2
 NX     =  20
```

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THE MOTION OF A CRITICAL PARTICLE IS GIVEN BY

ITER  =     1
G4LEF =     0.000000
G4ZER =     0.500000
G4RIT =     1.000000

    TAU         G(1)         G(2)         G(3)         G(4)        UX        UY        CDRE

  0.0000     0.992235     0.001170    -5.000000     0.500000    0.9922    0.0011    24.0000
  0.2000     0.992235     0.001170    -4.801552     0.500234    0.9912    0.0013    24.1144
  0.4000     0.992234     0.001170    -4.603104     0.500468    0.9901    0.0016    24.3744
  0.6000     0.992232     0.001171    -4.404656     0.500702    0.9887    0.0019    24.6838
  0.8000     0.992230     0.001171    -4.206209     0.500936    0.9871    0.0023    25.0545
  1.0000     0.992226     0.001172    -4.007762     0.501170    0.9851    0.0028    25.5006
  1.2000     0.992221     0.001173    -3.809317     0.501405    0.9828    0.0034    26.0393
  1.4000     0.992215     0.001175    -3.610873     0.501639    0.9799    0.0042    26.6888
  1.6000     0.992206     0.001177    -3.412430     0.501874    0.9764    0.0052    27.4637
  1.8000     0.992194     0.001180    -3.213989     0.502110    0.9719    0.0066    28.3636
  2.0000     0.992179     0.001184    -3.015552     0.502346    0.9664    0.0085    29.3535
  2.2000     0.992158     0.001190    -2.817118     0.502584    0.9592    0.0110    30.3524
  2.4000     0.992131     0.001198    -2.618688     0.502822    0.9500    0.0146    31.3039
  2.6000     0.992096     0.001210    -2.420265     0.503063    0.9379    0.0197    32.5793
  2.8000     0.992046     0.001227    -2.221849     0.503307    0.9216    0.0273    35.1527
  3.0000     0.991977     0.001253    -2.023447     0.503555    0.8993    0.0387    37.8687
  3.2000     0.991878     0.001294    -1.825060     0.503809    0.8683    0.0566    41.2151
  3.4000     0.991733     0.001361    -1.626698     0.504074    0.8241    0.0858    45.4350
  3.6000     0.991511     0.001477    -1.428372     0.504357    0.7601    0.1354    50.8992
  3.8000     0.991162     0.001689    -1.230102     0.504672    0.6665    0.2240    58.1905
  4.0000     0.990588     0.002105    -1.031924     0.505048    0.5317    0.3905    68.2492
  4.1599     0.989818     0.002824    -0.873486     0.505434    0.4024    0.6495    87.4018
  4.1699     0.989753     0.002896    -0.863587     0.505462    0.3930    0.6701    89.2955
```

```
THE MOTION OF A CRITICAL PARTICLE IS GIVEN BY

ITER  =    10
G4LEF =     0.986328
G4ZER =     0.987304
G4RIT =     0.988281

   TAU        G(1)        G(2)        G(3)        G(4)        UX        UY        CDRE

  0.0000    0.992871    0.002153   -5.000000    0.987304    0.9928    0.0021    24.0000
  0.2000    0.992871    0.002153   -4.801424    0.987735    0.9920    0.0025    24.1050
  0.4000    0.992870    0.002153   -4.602849    0.988165    0.9910    0.0029    24.3421
  0.6000    0.992869    0.002154   -4.404275    0.988596    0.9899    0.0034    24.6219
  0.8000    0.992866    0.002155   -4.205701    0.989027    0.9885    0.0041    24.9539
  1.0000    0.992863    0.002156   -4.007127    0.989458    0.9870    0.0049    25.3495
  1.2000    0.992859    0.002158   -3.808554    0.989889    0.9851    0.0059    25.8227
  1.4000    0.992854    0.002161   -3.609982    0.990321    0.9829    0.0072    26.3884
  1.6000    0.992847    0.002164   -3.411412    0.990754    0.9802    0.0089    27.0605
  1.8000    0.992838    0.002169   -3.212843    0.991187    0.9771    0.0111    27.8458
  2.0000    0.992826    0.002176   -3.014276    0.991622    0.9732    0.0139    28.7317
  2.2000    0.992811    0.002185   -2.815712    0.992058    0.9686    0.0176    29.6714
  2.4000    0.992792    0.002198   -2.617151    0.992496    0.9629    0.0226    30.5895
  2.6000    0.992767    0.002214   -2.418595    0.992937    0.9561    0.0294    31.4721
  2.8000    0.992737    0.002238   -2.220044    0.993382    0.9478    0.0388    32.7664
  3.0000    0.992696    0.002272   -2.021500    0.993833    0.9380    0.0519    35.1330
  3.2000    0.992644    0.002320   -1.822965    0.994292    0.9267    0.0704    37.5175
  3.4000    0.992577    0.002392   -1.624443    0.994763    0.9145    0.0966    40.3086
  3.6000    0.992492    0.002500   -1.425935    0.995252    0.9032    0.1338    43.5836
  3.8000    0.992388    0.002664   -1.227447    0.995767    0.8975    0.1858    47.4096
  4.0000    0.992271    0.002912   -1.028981    0.996324    0.9066    0.2551    51.8058
  4.2000    0.992165    0.003283   -0.830537    0.996941    0.9475    0.3376    56.6614
  4.4000    0.992129    0.003809   -0.632109    0.997649    1.0426    0.4116    61.6017
  4.6000    0.992268    0.004474   -0.433672    0.998476    1.2030    0.4248    65.8702
  4.8000    0.992700    0.005142   -0.235180    0.999439    1.3892    0.3089    68.5349
  5.0000    0.993434    0.005561   -0.036569    1.000514    1.4962    0.0545    69.3931
  5.0299    0.993572    0.005570   -0.006764    1.000681    1.5377    0.0103    71.6809
  5.0359    0.993600    0.005571   -0.000803    1.000714    1.5378    0.0012    71.6874
```

```
THE MOTION OF A CRITICAL PARTICLE IS GIVEN BY

ITER  =   20
G4LEF =     0.986612
G4ZER =     0.986613
G4RIT =     0.986614

    TAU          G(1)          G(2)          G(3)          G(4)        UX        UY        CDRE

  0.0000      0.992870      0.002152     -5.000000      0.986613    0.9928    0.0021    24.0000
  0.2000      0.992869      0.002152     -4.801426      0.987043    0.9920    0.0025    24.1050
  0.4000      0.992869      0.002152     -4.602852      0.987474    0.9910    0.0029    24.3421
  0.6000      0.992867      0.002153     -4.404278      0.987904    0.9899    0.0034    24.6220
  0.8000      0.992865      0.002154     -4.205704      0.988335    0.9885    0.0041    24.9540
  1.0000      0.992862      0.002155     -4.007130      0.988765    0.9870    0.0049    25.3498
  1.2000      0.992858      0.002157     -3.808557      0.989197    0.9851    0.0059    25.8231
  1.4000      0.992853      0.002159     -3.609985      0.989628    0.9829    0.0072    26.3889
  1.6000      0.992845      0.002163     -3.411415      0.990061    0.9802    0.0089    27.0612
  1.8000      0.992836      0.002168     -3.212846      0.990494    0.9771    0.0111    27.8467
  2.0000      0.992825      0.002175     -3.014279      0.990928    0.9732    0.0139    28.7328
  2.2000      0.992810      0.002184     -2.815715      0.991364    0.9686    0.0176    29.6726
  2.4000      0.992790      0.002196     -2.617155      0.991802    0.9629    0.0226    30.5908
  2.6000      0.992766      0.002213     -2.418598      0.992243    0.9561    0.0294    31.4736
  2.8000      0.992735      0.002237     -2.220047      0.992687    0.9478    0.0388    32.7702
  3.0000      0.992695      0.002270     -2.021504      0.993138    0.9380    0.0519    35.1374
  3.2000      0.992643      0.002319     -1.822969      0.993597    0.9266    0.0704    37.5233
  3.4000      0.992575      0.002391     -1.624447      0.994067    0.9143    0.0966    40.3164
  3.6000      0.992490      0.002499     -1.425940      0.994556    0.9031    0.1339    43.5941
  3.8000      0.992386      0.002663     -1.227452      0.995071    0.8973    0.1859    47.4243
  4.0000      0.992268      0.002912     -1.028986      0.995627    0.9064    0.2553    51.8262
  4.2000      0.992162      0.003283     -0.830543      0.996245    0.9472    0.3381    56.6896
  4.4000      0.992125      0.003810     -0.632116      0.996952    1.0424    0.4123    61.6398
  4.6000      0.992264      0.004476     -0.433679      0.997780    1.2031    0.4258    65.9190
  4.8000      0.992697      0.005147     -0.235187      0.998744    1.3899    0.3098    68.5916
  5.0099      0.993539      0.005525     -0.026640      0.999873    1.5374    0.0409    71.7265
  5.0289      0.993629      0.005529     -0.007762      0.999977    1.5388    0.0119    71.7420

THE IMPACTION EFFICIENCY IS  0.9734E 00
```

```
THE MOTION OF THE PARTICLES AT THE SPHERE POSITION IS GIVEN BY

NZER  =  120                    NJBP  =    4

   TAU          VX          VY           X            Y           UX          UY          FY

 3.9319      1.014362    0.000000    -1.001004     0.000000     0.0019      0.0000      2.06391
 3.9329      1.014354    0.000144    -0.999986     0.024930     0.0006      0.0383      2.06261
 3.9339      1.014364    0.000338    -0.998960     0.058170     0.0060      0.0892      2.05690
 3.9359      1.014378    0.000531    -0.996900     0.091410     0.0149      0.1395      2.04662
 3.9399      1.014386    0.000726    -0.992805     0.124651     0.0245      0.1900      2.03174
 3.9449      1.014403    0.000922    -0.987683     0.157893     0.0378      0.2399      2.01229
 3.9499      1.014435    0.001117    -0.982546     0.191134     0.0577      0.2878      1.98830
 3.9569      1.014465    0.001314    -0.975374     0.224377     0.0786      0.3356      1.95971
 3.9649      1.014502    0.001512    -0.967173     0.257621     0.1033      0.3820      1.92655
 3.9749      1.014538    0.001714    -0.956930     0.290867     0.1295      0.4283      1.88880
 3.9849      1.014594    0.001912    -0.946678     0.324113     0.1620      0.4710      1.84651
 3.9969      1.014649    0.002114    -0.934380     0.357362     0.1963      0.5132      1.79962
 4.0109      1.014813    0.002230    -0.920041     0.390612     0.2329      0.5547      1.74840
 4.0239      1.014887    0.002425    -0.906702     0.423860     0.2770      0.5892      1.69238
 4.0399      1.014956    0.002631    -0.890306     0.457115     0.3218      0.6242      1.63175
 4.0579      1.015036    0.002838    -0.871872     0.490374     0.3695      0.6574      1.56653
 4.0769      1.015130    0.003041    -0.852407     0.523634     0.4214      0.6860      1.49676
 4.0979      1.015234    0.003245    -0.830898     0.556900     0.4766      0.7117      1.42239
 4.1209      1.015395    0.003351    -0.807344     0.590168     0.5352      0.7336      1.34361
 4.1449      1.015517    0.003554    -0.782760     0.623440     0.5974      0.7489      1.26008
 4.1719      1.015653    0.003759    -0.755115     0.656722     0.6628      0.7609      1.17195
 4.2019      1.015804    0.003964    -0.724405     0.690015     0.7318      0.7681      1.07922
 4.2349      1.015970    0.004086    -0.690633     0.723312     0.8045      0.7690      0.98198
 4.2699      1.016153    0.004285    -0.654810     0.756620     0.8796      0.7606      0.88008
 4.3109      1.016330    0.004425    -0.612877     0.789950     0.9601      0.7463      0.77356
 4.3549      1.016556    0.004625    -0.567873     0.823292     1.0424      0.7195      0.66241
 4.4049      1.016812    0.004815    -0.516742     0.856660     1.1279      0.6802      0.54663
 4.4639      1.017063    0.004970    -0.456439     0.890066     1.2180      0.6248      0.42618
 4.5349      1.017361    0.005144    -0.383902     0.923532     1.3118      0.5456      0.30099
 4.6269      1.017753    0.005334    -0.289958     0.957102     1.4093      0.4272      0.17098
 4.7789      1.018412    0.005533    -0.134866     0.990992     1.5107      0.2057      0.03570
 4.8979      1.018916    0.005627    -0.013558     0.999916     1.5386      0.0208      0.00019

THE DRAG COEFFICIENT OF THE SPHERE DUE TO THE REFLECTION OF PARTICLES IS      2.0356

THE TARGET REYNOLDS NUMBER IS  0.3796E 04

THE DRAG COEFFICIENT OF THE SPHERE DUE TO THE AIR ALONE IS   0.3909
// PAUS
```

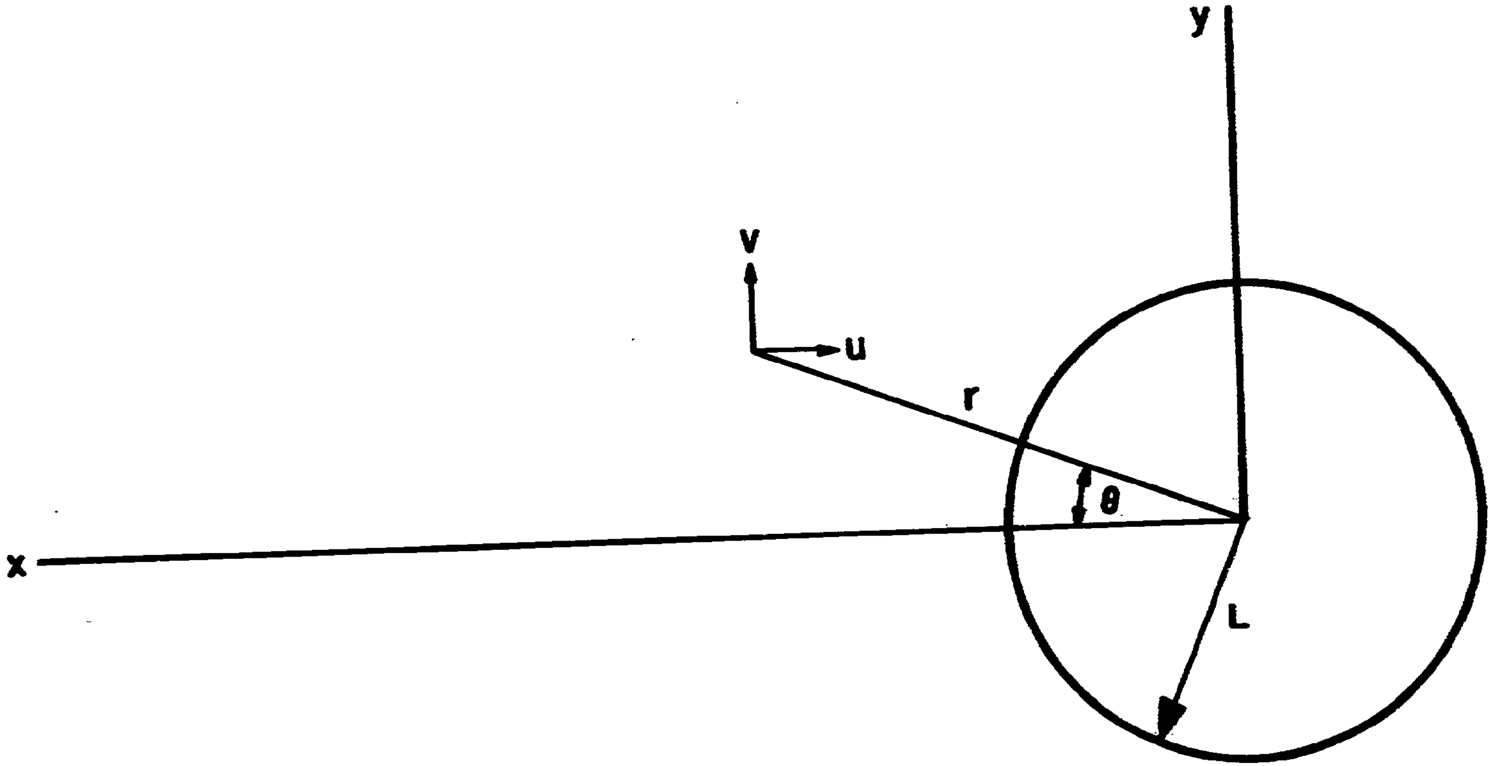

Fig. 1: Co-ordinate System for Transverse Flow with respect to a Circular Cylinder.

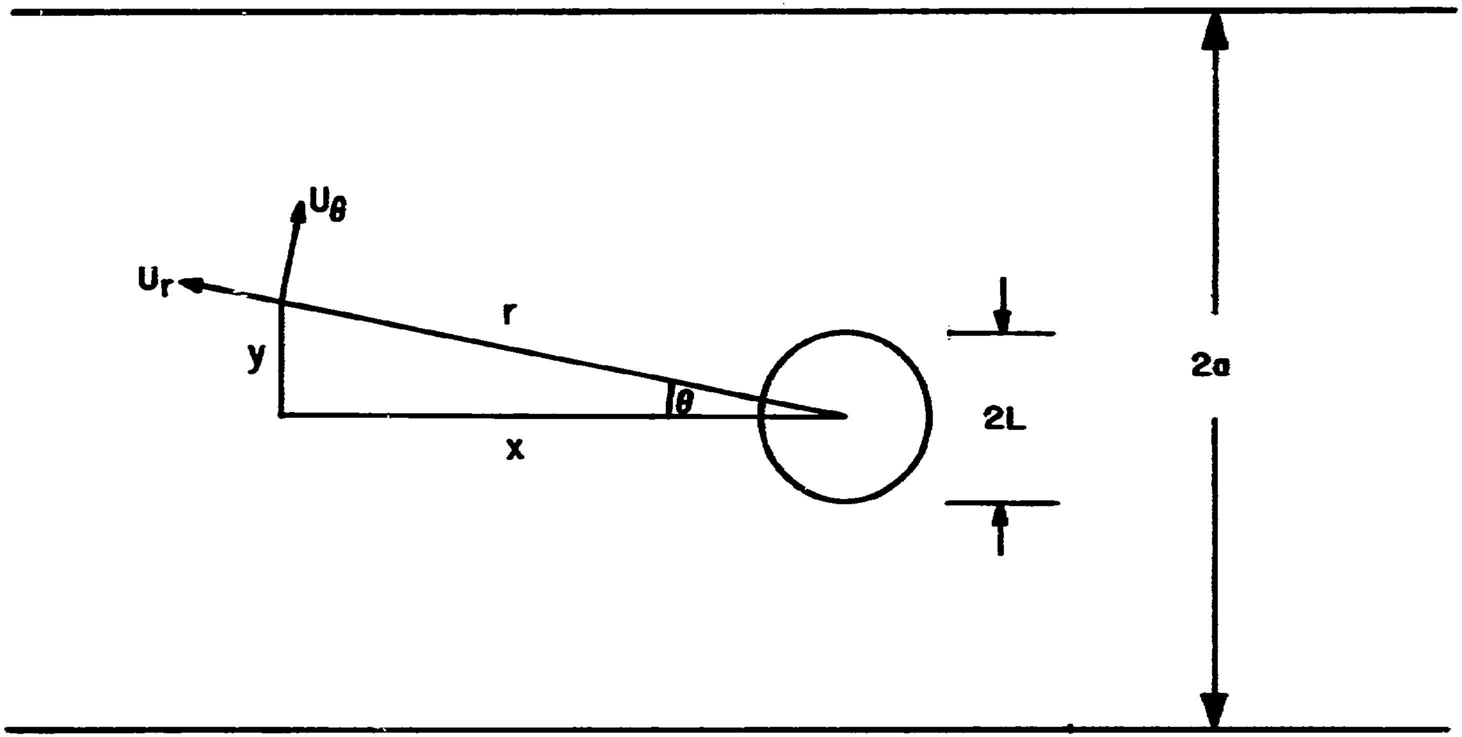

Fig. 2: Co-ordinate System for Flow around a Sphere in a Circular Cylinder.

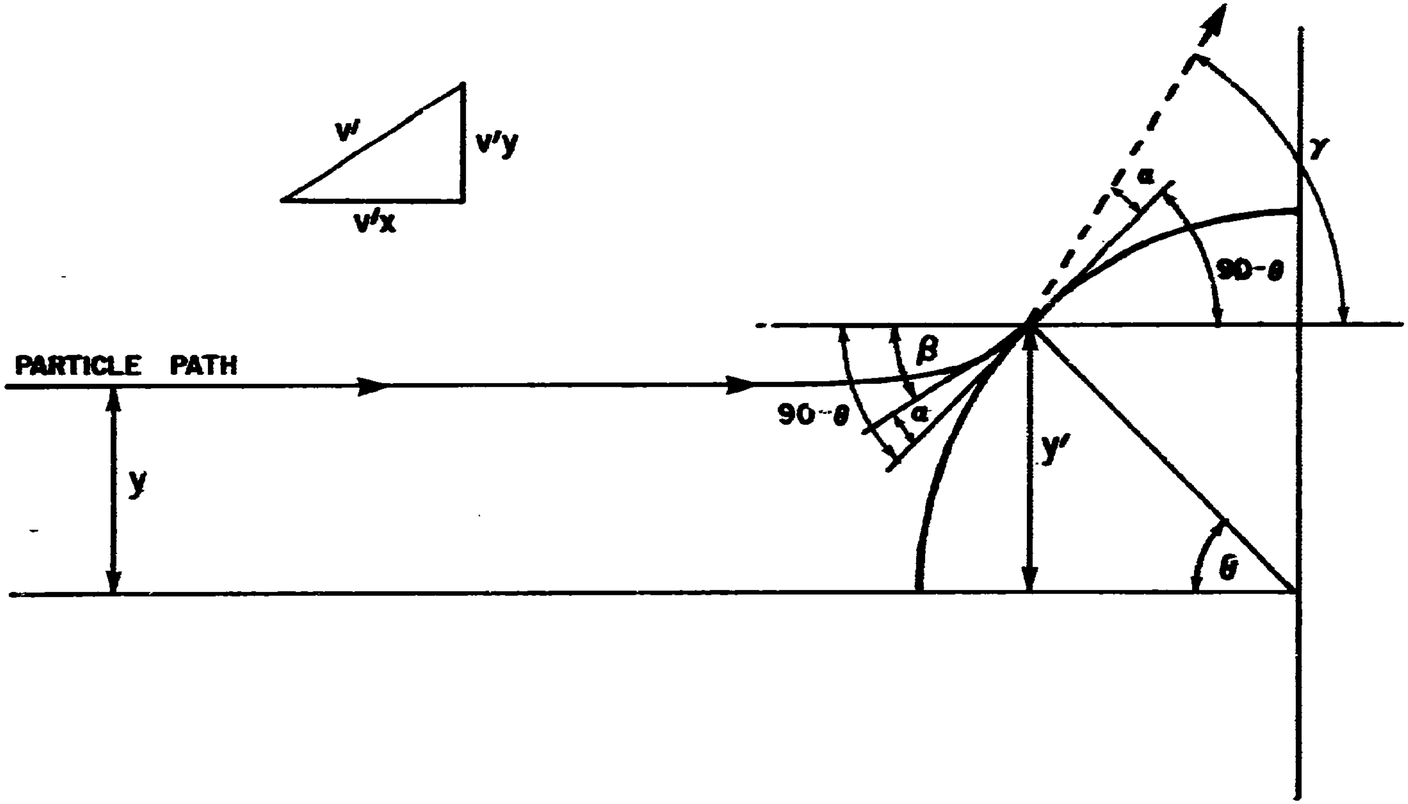

Fig. 3: Velocity Change of a Particle due to Interaction with a Sphere or Cylinder.

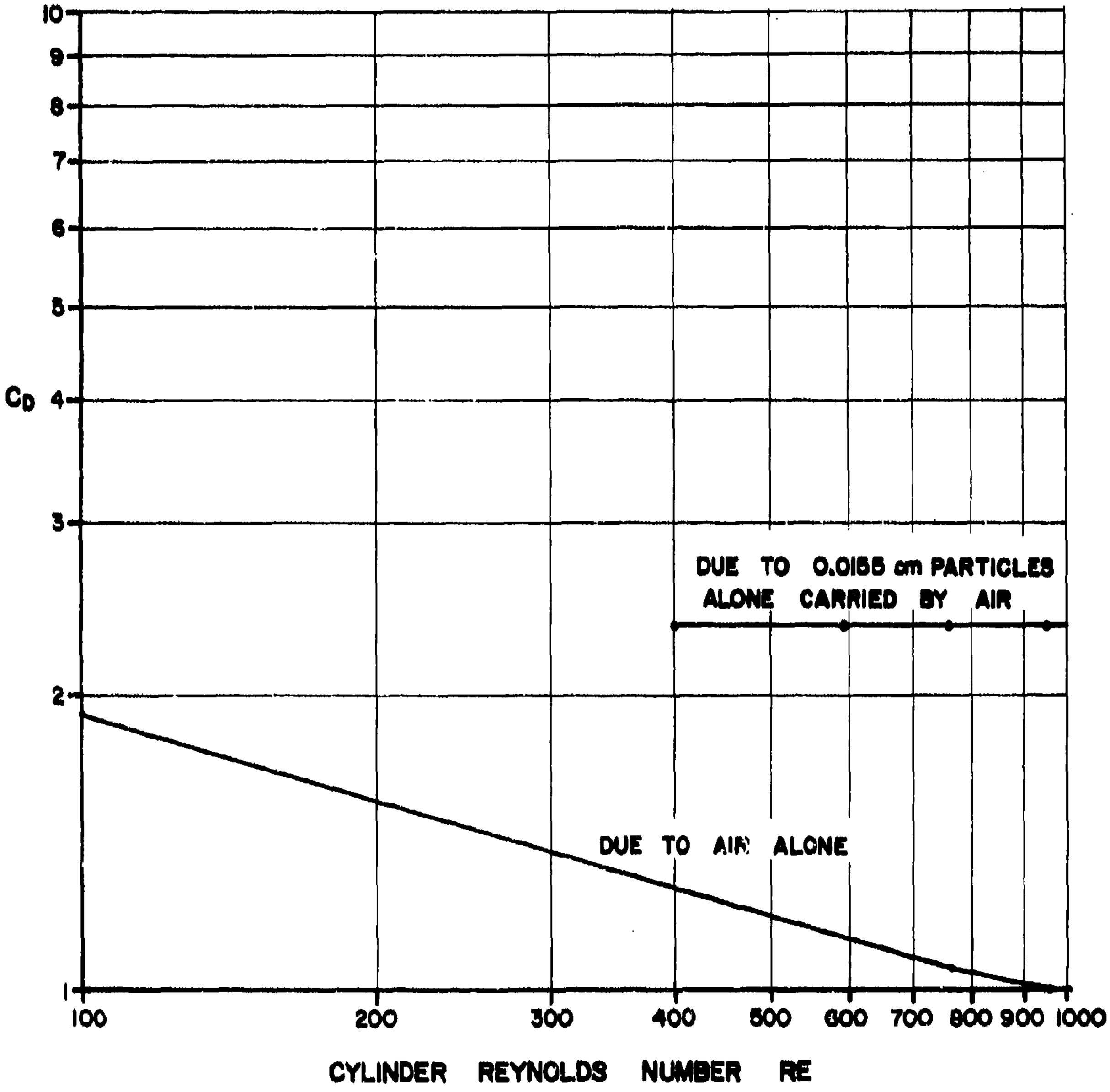

Fig. 4: Variation of Drag Coefficient for Cylinders.

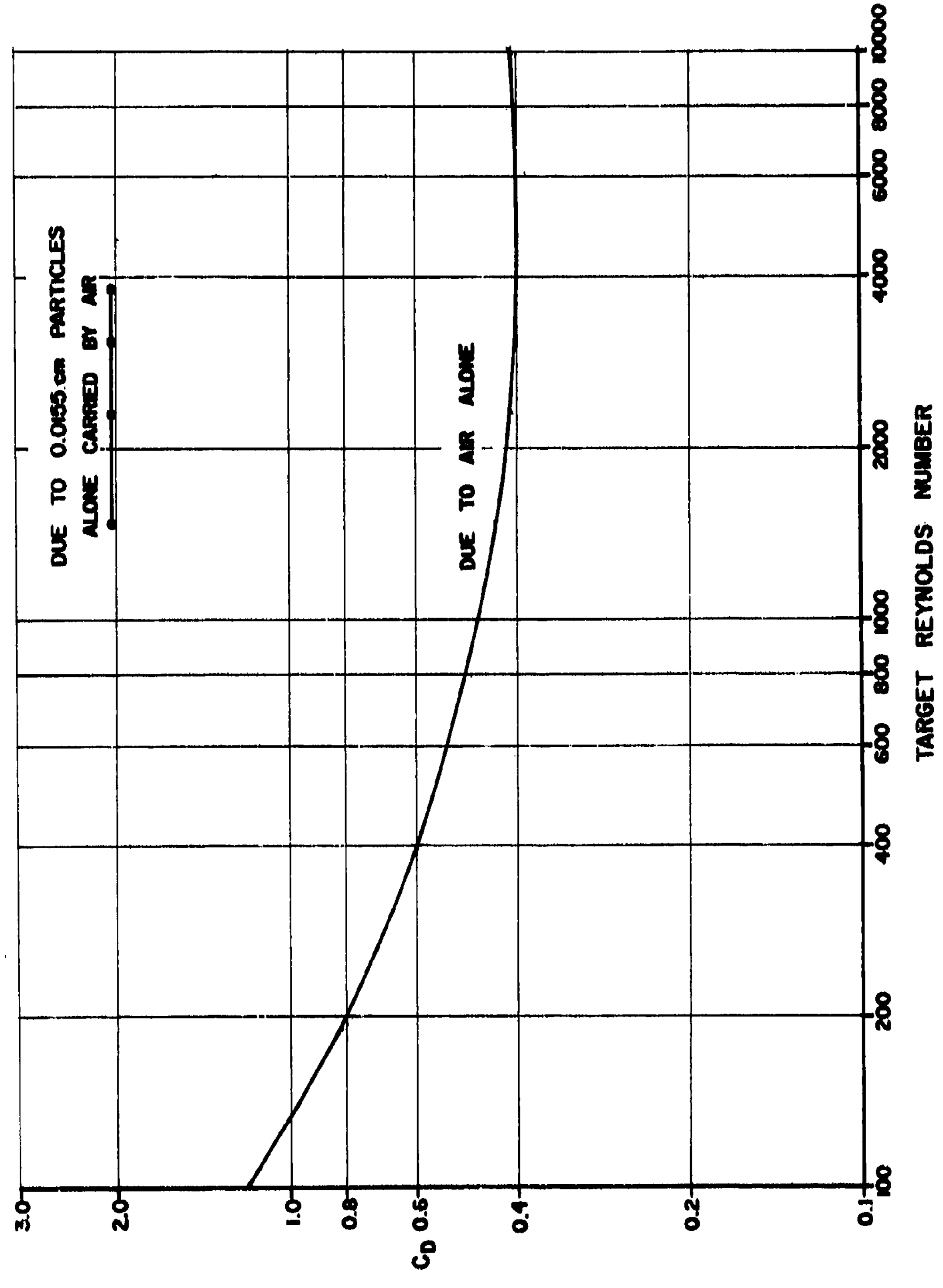

Fig. 5: Variation of Drag Coefficient for a Spherical Target.

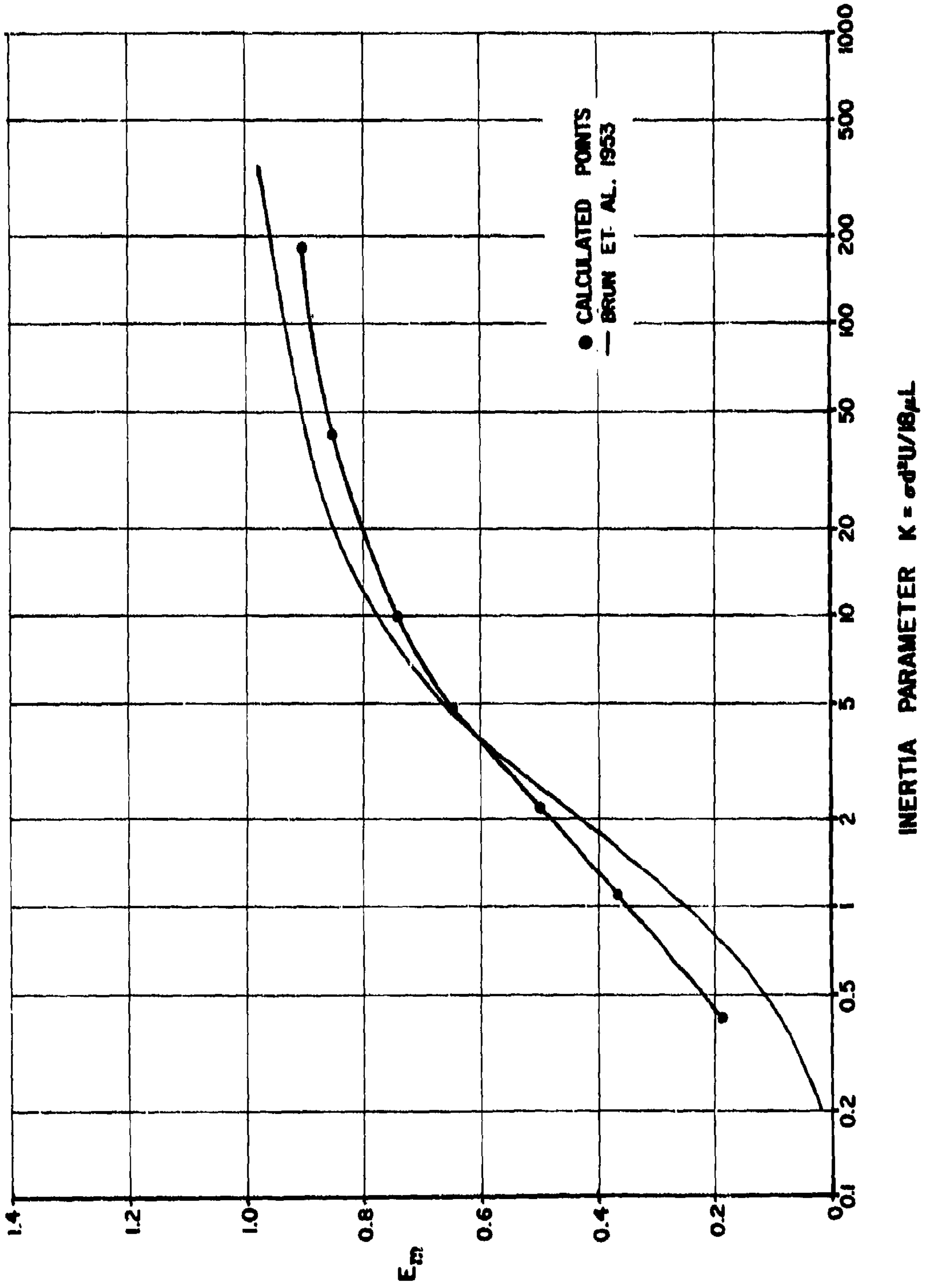

Fig. 6: Impaction Efficiency for Cylinders, φ = 1000.

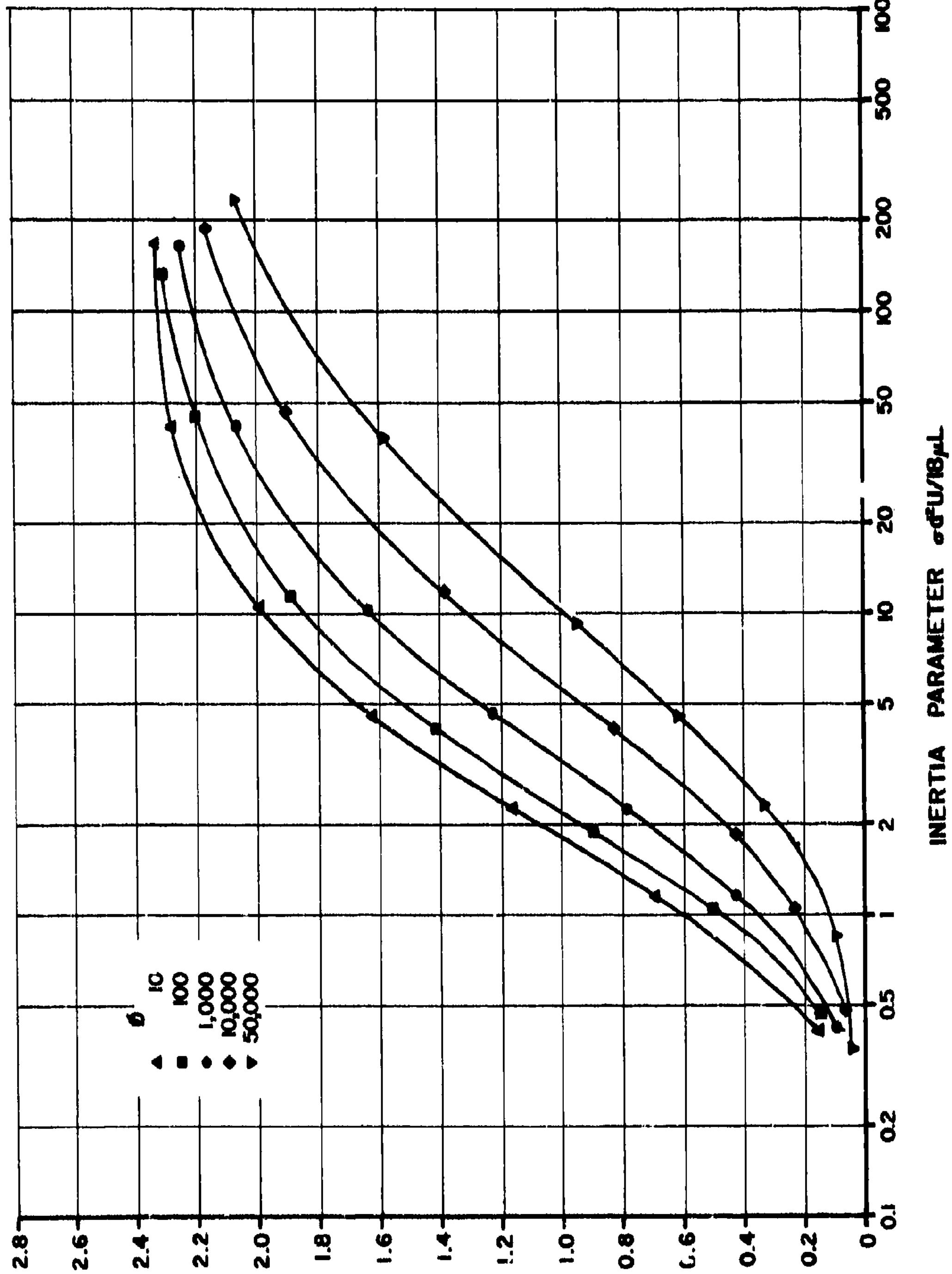

Fig. 7: Calculated Drag Coefficient for Cylinders due to Particles alone in an Inviscid Flow.

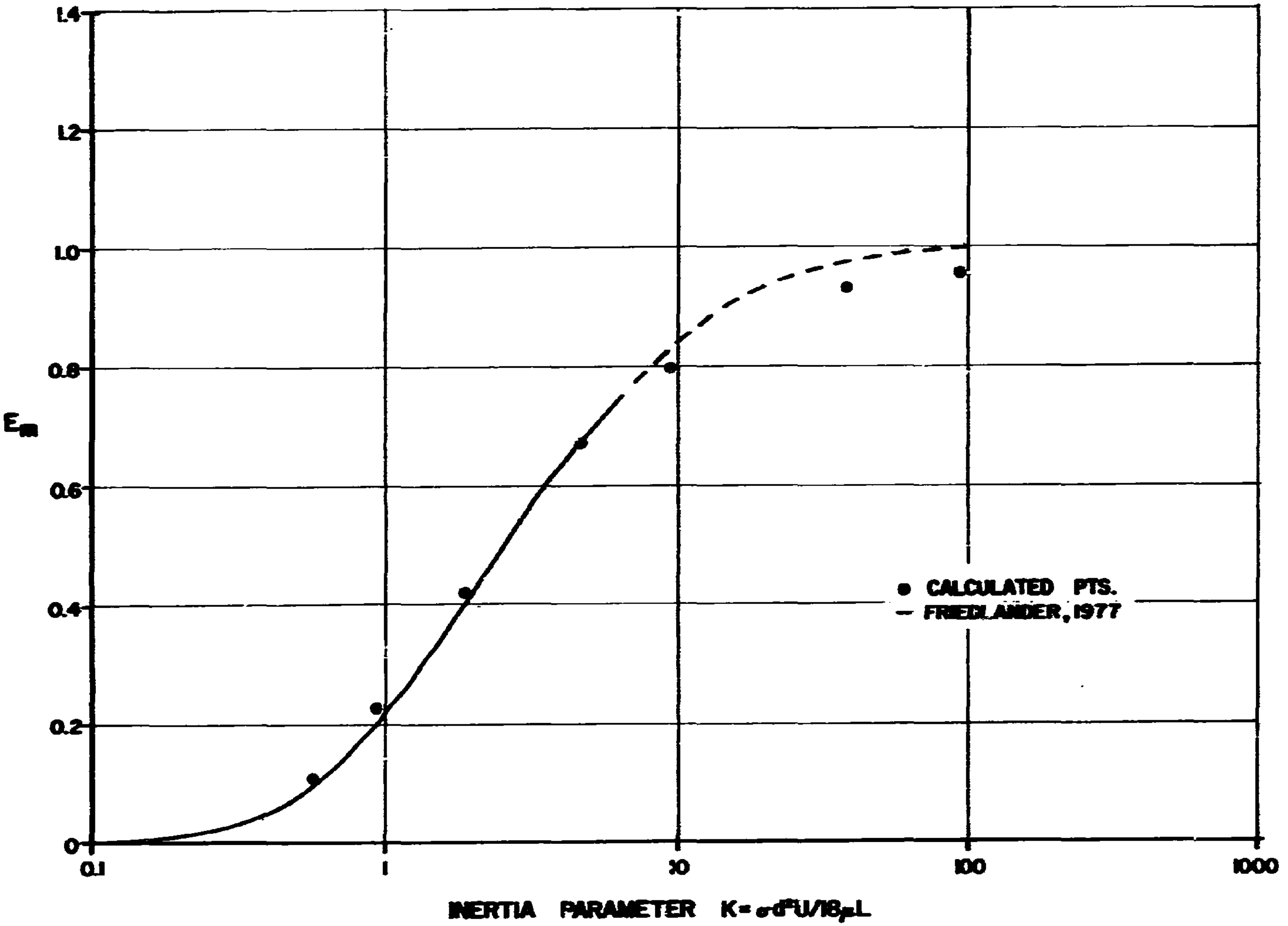

Fig. 8: Impaction Efficiency for Spheres for Free Stream Particle Reynolds Number of 128.

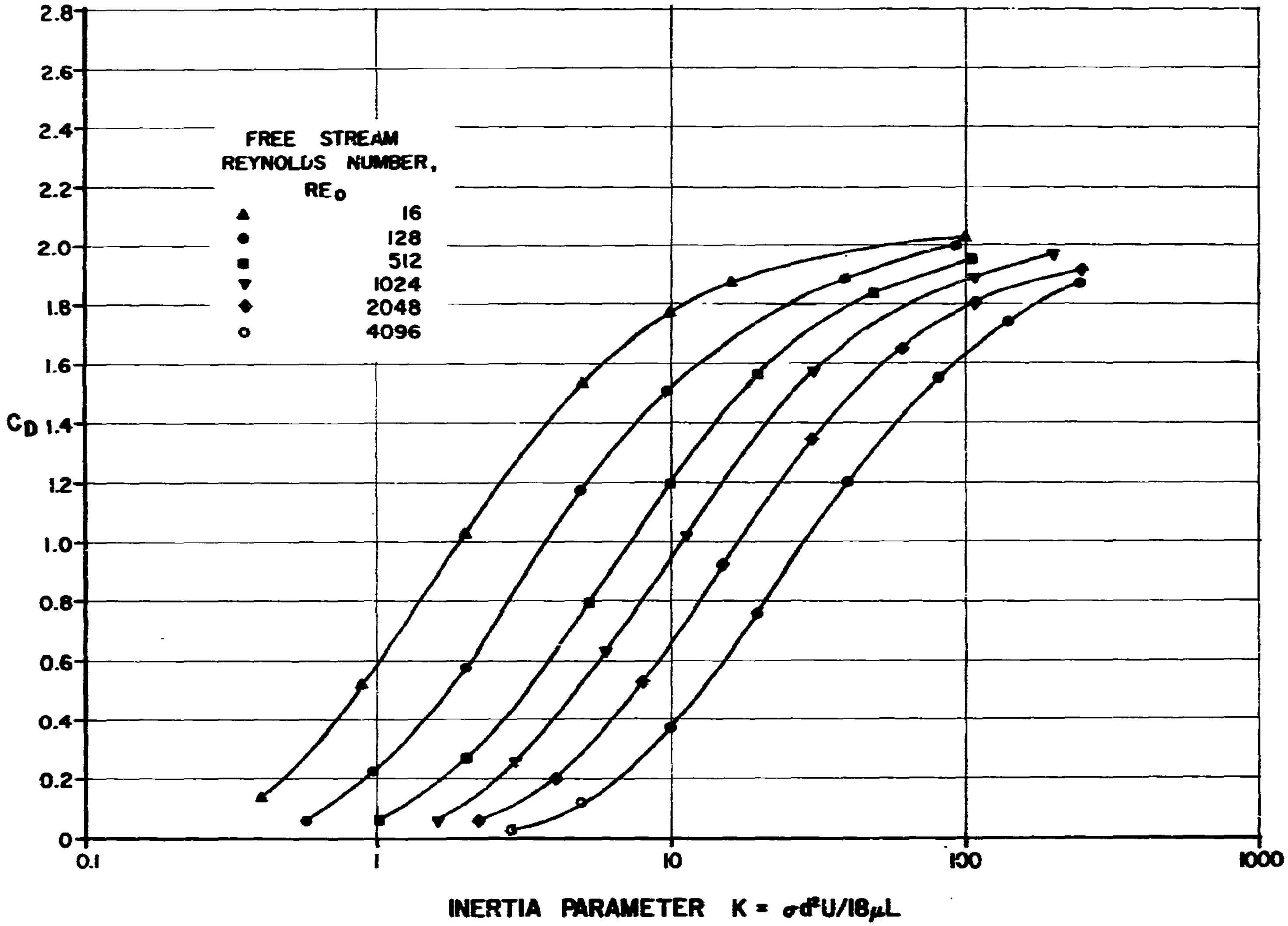

Fig. 9: Calculated Drag Coefficient for Spheres due to Particles alone in an Inviscid Flow.

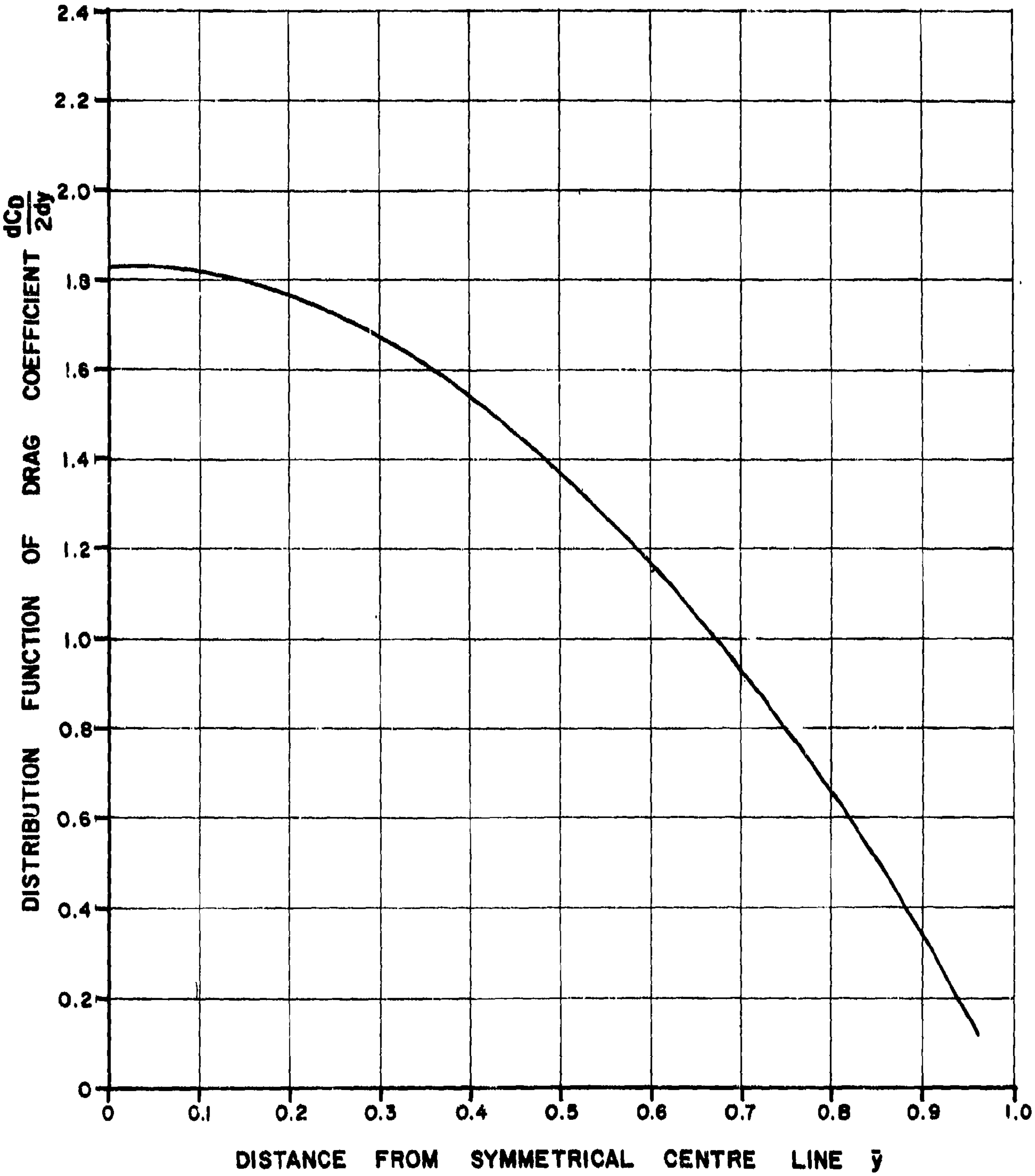

Fig. 10: Distribution of Forces due to Elastic Reflection of Particles from a Cylinder.

$Re_0 = 73.6$ and $K = 339$

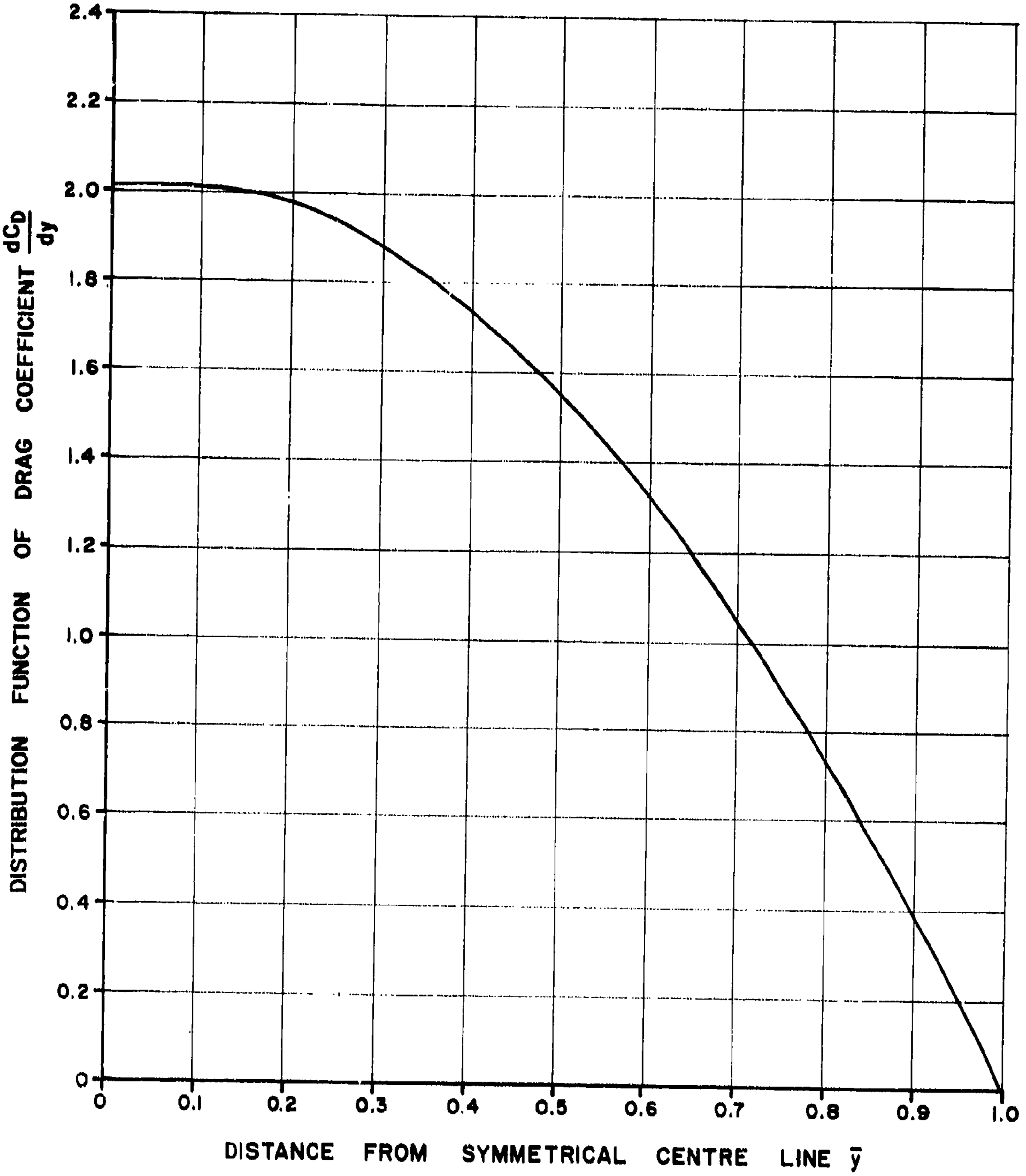

Fig. 11: Distribution of Forces due to Elastic Reflection of Particles from a Sphere in a Tube.

$Re_0 = 73.6$, $K = 339$ and $L = 0.3150\ a$

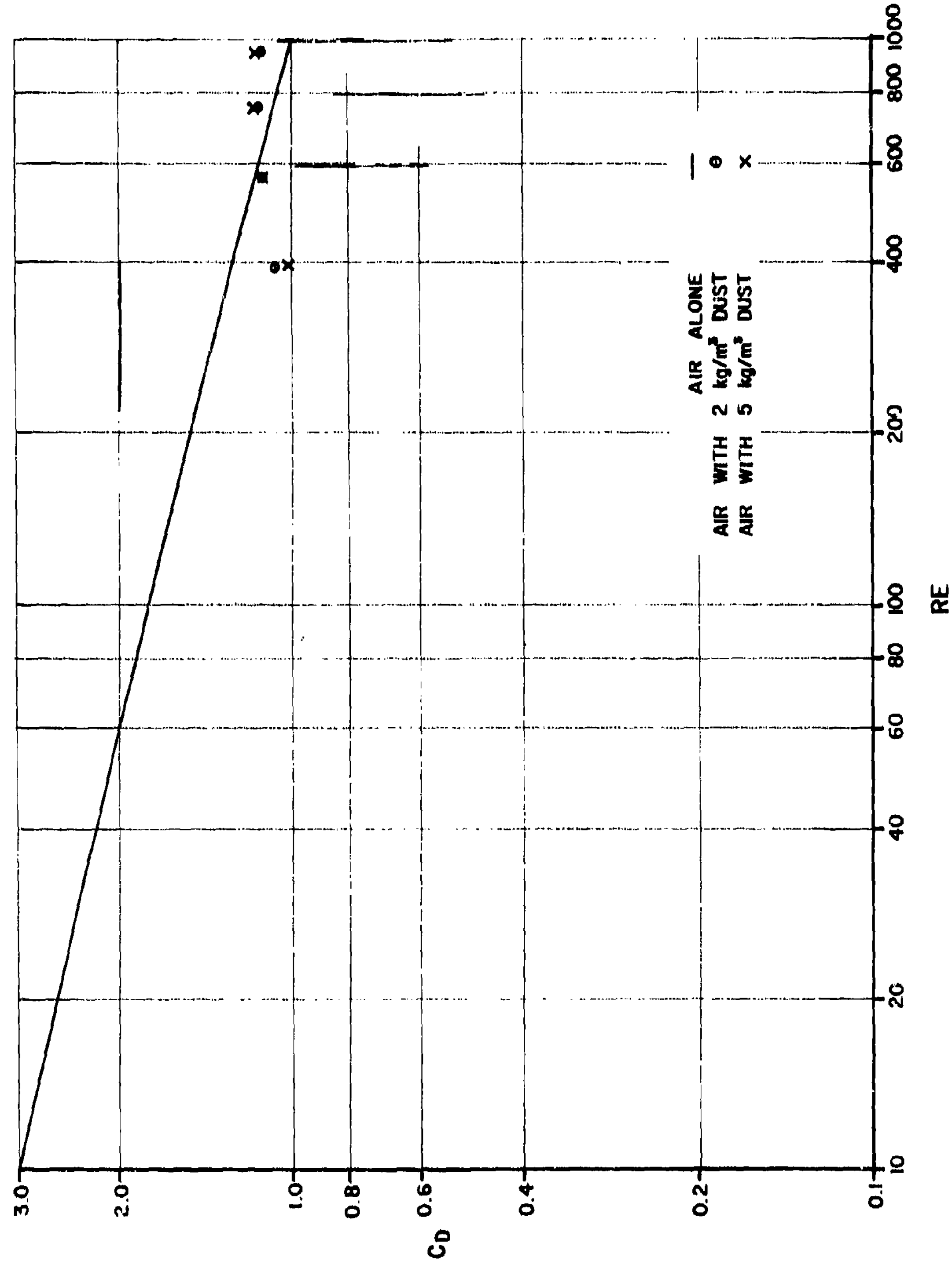

Fig. 12: Drag Coefficient of a 0.2 cm Cylinder in a 2.54 cm Tube due to Equal Masses of 0.0470, 0.0155 and 0.0055 cm Particles corrected for Particle Velocity Deficiency.

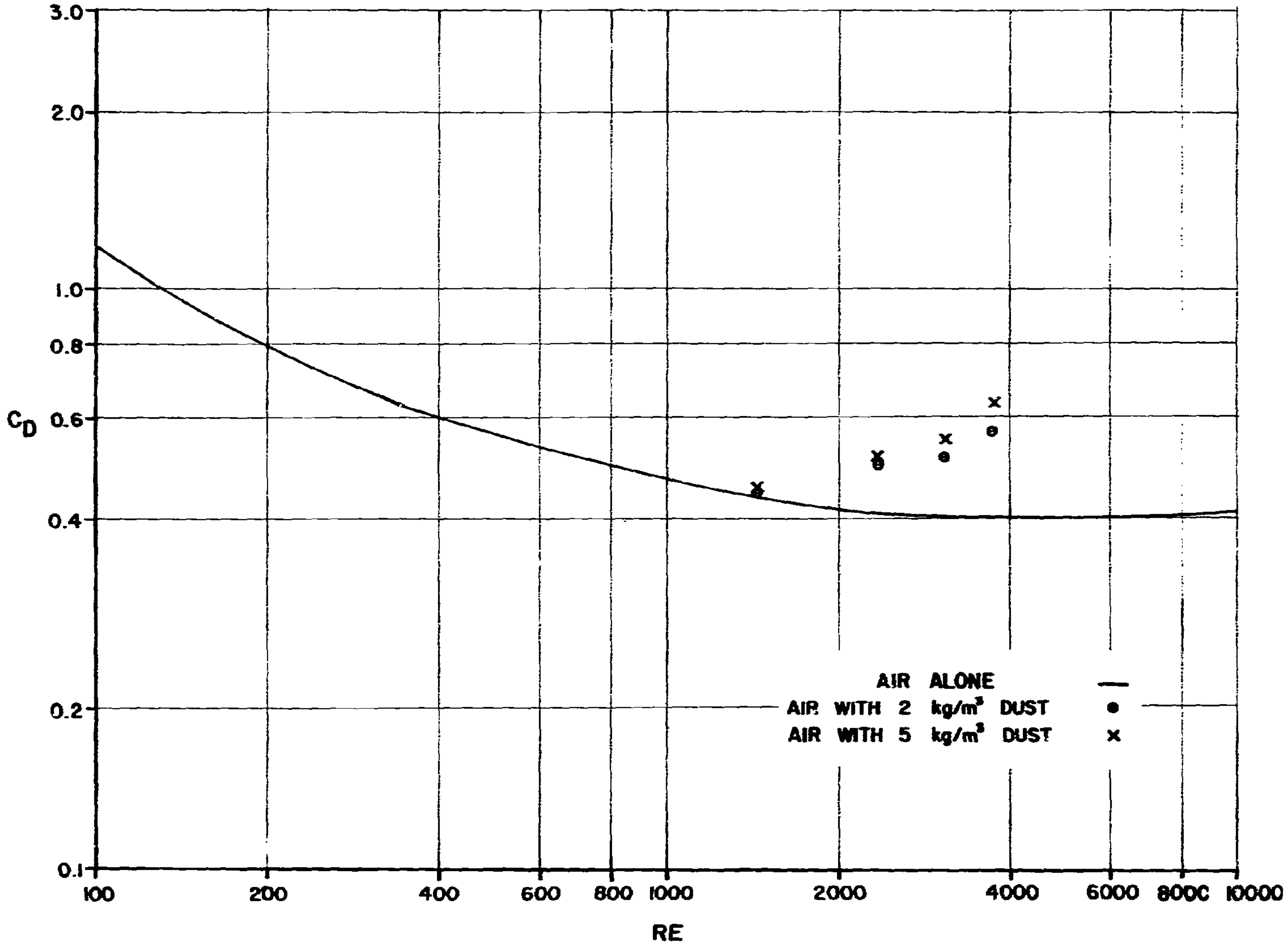

Fig. 13: Drag Coefficient of a 0.8 cm Sphere in a 2.54 cm Tube due to Equal Masses of 0.0470, 0.0155 and 0.0055 cm Particles corrected for real Fluid Velocity and Particle Velocity Deficiency.

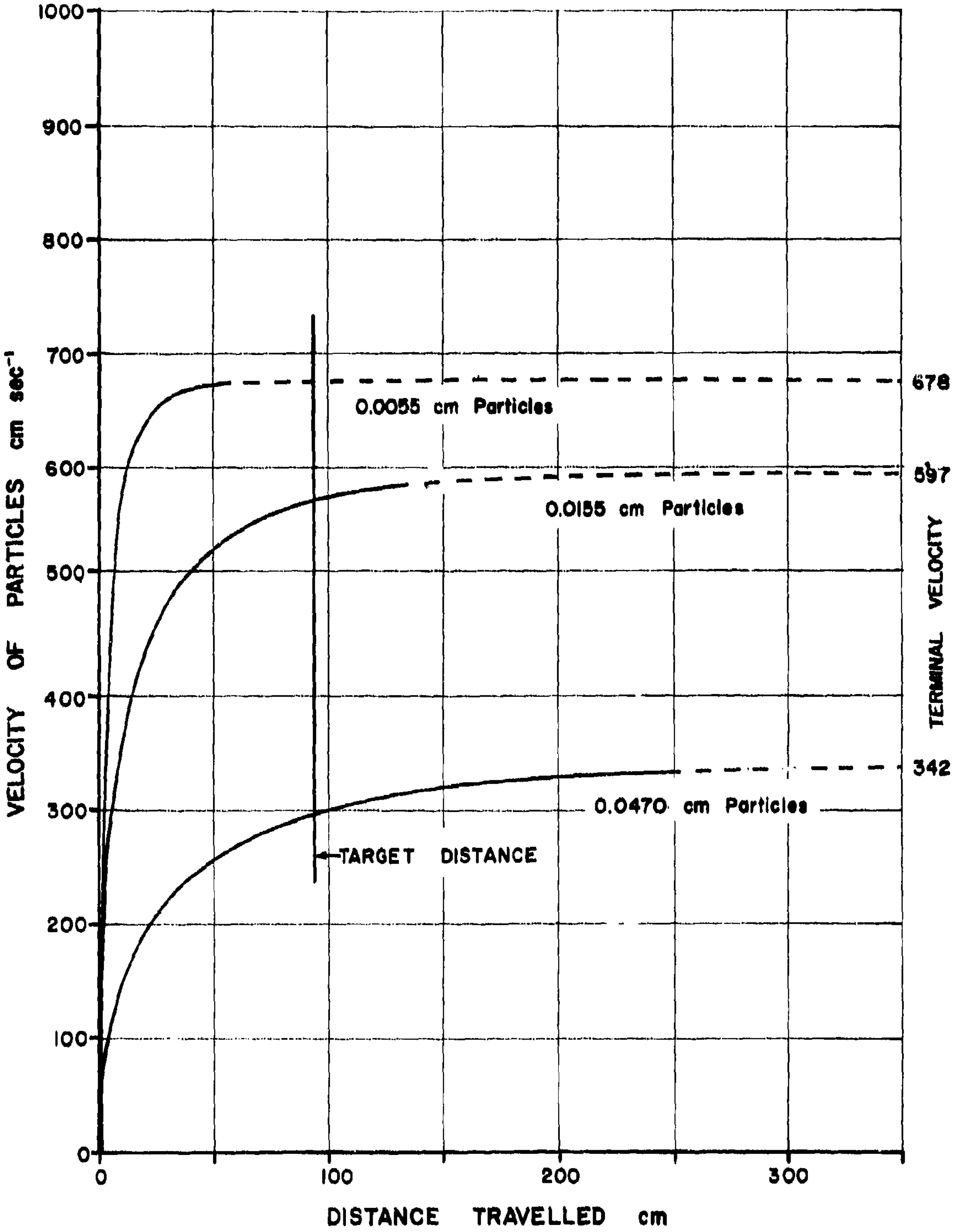

Fig. 14: Upward Velocity of Spherical Dust Particles accelerated from rest in a vertical Air Stream of 700 cm sec^{-1} Velocity

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The effect of dust on aerodynamic drag of spheres and cylinders was calculated by using a mathematical model developed for this purpose. The results were compared to experiments previously done by other workers. The calculated and experimental results agree favourably, showing that the mathematical model is satisfactory. Impaction efficiencies and drag coefficients due to dust alone were then obtained using the model for a wide range of the inertia parameter and the results are presented graphically. The model can also be used for calculating velocity distributions and points of impact for a stream of airborne particles flowing over a sphere or cylinder.

(U)

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Cylindrical Bodies
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Blast Loads
Dust Particles
Drops (Liquid)
Impact
Aerosols - Penetration
Aerosols - Sampling

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