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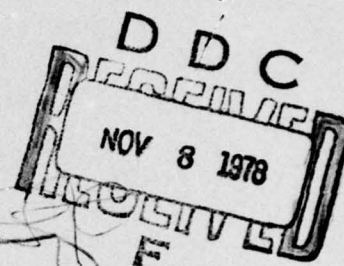
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(10) P.B. Walkley BSc, Grad IMA

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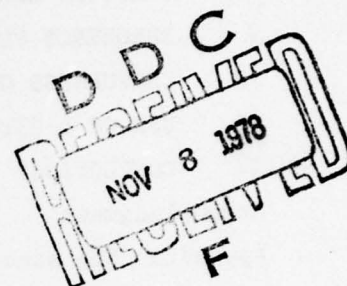
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IMPEDANCE-MATCHING NETWORKS OF THE L TYPE

by

P. B. Walkley, BSc, Grad IMA



SUMMARY

An L-network is an electric network comprising two reactive arms, one in series and one in shunt. Any two impedances can be matched at a single frequency by at least one such network. This Report explains relevant theory, sets out design equations, and examines the broadband matching performance of L-networks both algebraically and by means of computer analysis of examples. It is shown that these networks have occasional broadband uses.

Treatment is general but findings are expressed in terms of circuits which operate at frequencies in the low gigahertz region.

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1 INTRODUCTION

An L-network is a two-port network consisting of two reactive arms, one in series and one in shunt. Usually each arm comprises a single inductor or capacitor: the eight forms of two-element L-network are itemised in Fig 6.1.

The problem considered in this Report is how to design a network to put between a signal source and a load so as to convey to the load the whole of the power available from the source. Any two terminations can be matched at a particular frequency by a choice of one or more L-networks, and L-networks are the simplest class of transformerless networks with this property. An L-network used for matching has occasionally been called a "reactance transformer".

The investigation whose results are reported here had two main aims. The first was to produce a computer program which would design L-networks, since graphical methods of design^{1,3,5,7-10,14,15} had proved imprecise, time-consuming and error-prone. This involved establishing the design formulae. The second aim was to investigate how useful L-networks were for broadband matching. The literature found did not cover these areas.

First of all the design equations were derived, and the L-network design program was written and tested. Then an investigation was made of the range of terminations which each type of L-network can match, and the choice of types of L-network to match given terminations. Only matching at a single frequency was considered.

The second aim was achieved - so far as was practicable - by considering the variation of matching efficiency with frequency in very simple cases. The matching efficiency was investigated algebraically only for the case of matching two resistances; then examples were produced using the L-network design program and analysed using a computer program for frequency analysis. Some of the computer examples had both terminations resistive, and the rest had one resistive termination and one two-element termination.

This Report describes the main results of this work, including conclusions about the usefulness of L-networks and of the L-network design program. The program is described elsewhere¹⁶.

The L-network was found to give a very broad peak, but still it seems unsuitable for most high-performance broadband work, because the peak is far from flat. Nevertheless it seems to have two important broadband uses. Firstly, it can be used in conjunction with a matching network designed for resistive terminations, in order to transform each of the actual terminating impedances to a

resistance comparable to its own magnitude. Secondly, a multistage amplifier whose design will be optimised by computer could use L-networks if the optimisation of element values will remove irregularities from the response.

The next simplest class of matching networks comprises the T and Π networks. The L-network design program can make the design of such a network much easier.

2 LITERATURE

This survey is not comprehensive. Unless otherwise stated, matching at a single frequency only is considered.

Both terminations resistive

Elementary introductions are by the ARRL² and Ryder¹³, of which both give the design formulae and the latter gives their derivation. There is a set of design tables¹² for a certain class of broadband impedance-matching network of which certain L-networks are the simplest form.

One termination resistive, the other arbitrary

The usual practical problem of this kind is the matching of an arbitrary impedance to a long transmission line at a single frequency. Smith⁷ introduces the eight forms of two-element L-network and considers the feasibility of each for any given single-frequency matching task. He also gives design curves on a cartesian chart. In his later book³ he considers the same problem in terms of the Smith chart. In neither case does he give any formulae.

Martens⁸ covers the same material as Smith³ except for design curves. Phillips⁹ covers the same material as Smith³. Mathis¹⁰ considers the effect of lossy components, but gives no design curves. None of these authors gives any formulae.

Both terminations general

This problem occurs in the design of amplifier interstage networks, in which connection Hewlett-Packard¹ briefly treat the Smith chart method of finding the element values. Perna¹¹ gives an intuitive explanation. His account does not go into depth, and he seems to assume incorrectly that the selectivity equals the reactance/resistance ratio. Linvill and Gibbons¹⁵ consider a graphical method of L-network design using a cartesian plot. They then briefly consider how to design an L-network for band-pass matching. None of the above authors gives the formulae for the element values.

Theoretical treatments

Theoretical books⁴⁻⁶ tend to concentrate on more sophisticated filters, and therefore tend to give L-networks little attention and to discuss them using unnecessary concepts unfamiliar to most engineers. The most useful treatment found was by Everitt and Anner¹⁴, who treat only the case of resistive terminations. They briefly derive the design equations, discuss graphical design and give a formula for the frequency response of each of the two appropriate forms of L-network. The quantity $|I|/|I_d|$ which they plot is in fact the square root of the transducer gain.

The state of the literature

It seems very difficult to find a comprehensive treatment of the L-network, and possibly none exists. The treatments mentioned are all very restricted. None even gives design formulae for the case of general terminations. Furthermore, the references given in the papers mentioned have not led to any further useful information. This Report is intended to supply some of the important information which is not easily found.

3 IMPEDANCE-MATCHING NETWORKS

In order to clarify the task for which the L-network is used in the context of this Report, this section will outline the basic concepts of matching networks.

When a two-port network loaded by an impedance Z_L has an input impedance Z_{in} , the network can be said to *transform* Z_L to Z_{in} .

The *available power* of a signal source is the most power which any load can draw from it.

The *transducer gain* of a network joining a specified source to a specified load is the ratio

$$\frac{\text{power developed in load}}{\text{available power of source}} .$$

Clearly it can be used as a measure of the efficiency of a matching network.

It can be shown that, if a signal source has impedance Z_S , the only value of load impedance seen by the source which will draw the whole available power from it is Z_S^* . In other words *maximum power transfer* occurs if and only if there is a *conjugate match*.

The essential task of an impedance-matching network is to provide, between the terminations which it joins, a transducer gain which is fairly even and fairly close to unity over the wanted passband. It does this by loading the source with an impedance fairly close to the conjugate of the source impedance: this is a task of impedance transformation. Such a network uses passive elements, and usually its ohmic losses are minimised; theoretical designs are usually lossless.

Transformerless matching networks are common. These generally use inductors and capacitors only. They can only be used over a relatively narrow band, in contrast with the multi-decade capabilities of some transformers. Any impedance-matching network using lumped elements has a smooth curve of transducer gain versus frequency, so that if it gives a perfect match at one frequency it will also give a good match over a band of neighbouring frequencies. It is impossible, however, for a transformerless matching network to give a *perfect* match over any band of frequencies, however narrow.

For all transformerless matching network design tasks, there is a tradeoff between the number of elements needed in the network and its performance in terms of width of passband, minimum transducer gain over the passband, and the amplitude of variation of transducer gain over the passband.

In addition, if one termination contains a reactive element, there is a limit to the performance achievable by a matching network even if it can have any number of elements. The limit is set by the time constant of the termination.

In general, of course, the task of a matching network is to provide a transducer gain variation of a prescribed shape, which is not the same as providing a good match. It is intuitively clear that usually (a) the more reactive elements there are in the terminations, the more difficult it is to achieve a wanted transducer gain curve, and (b) the more elements are allowed in the matching network, the more closely such a curve can be approached.

The simplest class of transformerless matching network able to match any two impedances at one frequency is the class of two-element L-networks. A two-element L-network of a given form has its element values completely defined by the requirement of a match at the wanted frequency: this will be seen in the next part. The designer therefore has no freedom to choose the shape of the response.

4 THE BASIC IDEAS OF L-NETWORKS

Consider the problem of matching two resistive terminations R_1 and R_2 at a single frequency, with R_2 being the larger. The problem can be solved

like this. First, R_2 is shunted by a reactance such that the impedance of the combination has real part equal to R_1 . Then the reactance of the combination is balanced out by adding an opposite reactance in series. The impedance of the three-element combination now equals R_1 . That is the essential idea behind the L-network. Fig 4.1 shows the three elements, and will be used in the following derivation of expressions for the required reactances.

In general

$$Z_1 = \frac{(jX_2)R_2}{jX_2 + R_2} = \frac{X_2^2 R_2}{R_2^2 + X_2^2} + j \frac{X_2 R_2^2}{R_2^2 + X_2^2} \quad (4-1)$$

X_2 is to be chosen to make $\text{Re}(Z_1) = R_1$:

$$\frac{X_2^2 R_2}{R_2^2 + X_2^2} = R_1 \quad (4-2)$$

Therefore

$$X_2 = \pm \frac{R_2}{\sqrt{\frac{R_2}{R_1} - 1}} \quad (4-3)$$

Now X_1 must be chosen to balance out the reactance in Z_1 :

$$X_1 = - \frac{X_2 R_2^2}{R_2^2 + X_2^2} \quad (4-4)$$

Comparison with equation (4-2) gives

$$X_1 = - \frac{R_2 R_1}{X_2} \quad (4-5)$$

The two solutions are therefore:

(4-6)

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$$(i) \quad X_1 = R_1 \sqrt{\frac{R_2}{R_1} - 1} \quad \text{and} \quad X_2 = - \frac{R_2}{\sqrt{\frac{R_2}{R_1} - 1}} ;$$

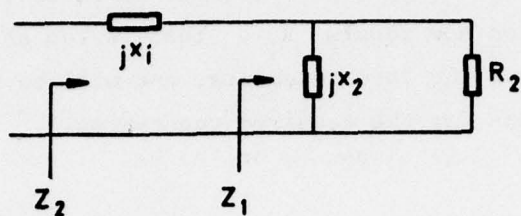


Fig 4.1

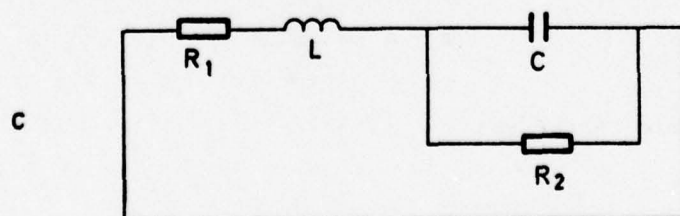
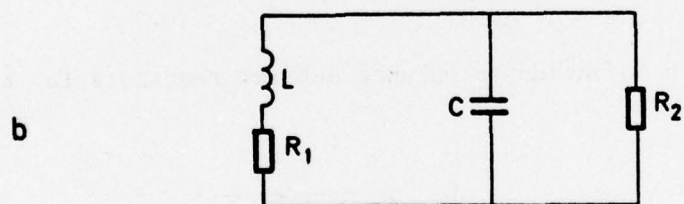
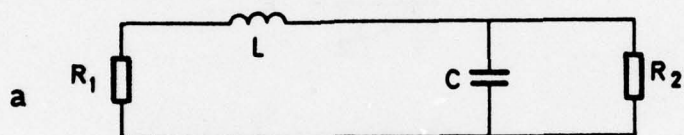


Fig 4.2 Three ways of drawing a circuit

$$(ii) \quad X_1 = -R_1 \sqrt{\frac{R_2}{R_1} - 1} \quad \text{and} \quad X_2 = \frac{R_2}{\sqrt{\frac{R_2}{R_1} - 1}}.$$

It can be seen that if R_2 is less than R_1 there is no solution. This means that the shunt reactance must always be on the side of the larger resistance.

The following points will be noted:-

(i) The reactances are opposite in sign.

$$(ii) \quad |X_2| > \sqrt{R_1 R_2} > |X_1| \quad (4-7)$$

$$(iii) \quad \frac{X_1}{R_1} = -\frac{R_2}{X_2}. \quad (4-8)$$

(iv) When $R_2 \gg R_1$, both reactances have magnitude approximately $\sqrt{R_1 R_2}$.

(v) When $R_1 = R_2$, the formulae give X_1 zero and X_2 infinite.

In other words, the matching network degenerates into a direct connection.

The obvious way to implement each reactance is with a single reactor. (Of course a reactance can be synthesised with any of an infinite number of combinations of reactors, for example series or parallel LC combinations.) So, for example, one possible matching L-network gives the circuit shown in Fig 4.2a, which resembles a low-pass filter. But that can also be redrawn as in Fig 4.2b, resembling a parallel tuned circuit, or as in Fig 4.2c, resembling a series tuned circuit. Not surprisingly, the frequency response of this circuit has a peak rather like that of a simple tuned circuit, as a later part of this Report will show. The behaviour at frequencies far from the peak resembles that of a low-pass filter.

5 L-NETWORK DESIGN EQUATIONS

This part will be less elementary than the last in its approach, and will use a few theorems relating to power flow in linear circuits, although the element reactances could have been derived in the same way as in the last part.

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To simplify the algebra, the termination next to the shunt arm of the L-network will be characterised by its admittance, the other by its impedance, and the L-network itself will be characterised by its series reactance and shunt susceptance.

The task of matching is here to set the transducer gain from source to load to the value of unity. However, the transducer gain between the two terminations is independent of which is considered to be the source and which the load, since the L-network is reciprocal. For this reason, the analysis below does not distinguish source and load, but refers to the terminations as A and B, with their impedances etc distinguished by suffixes a and b.

Since the L-network is lossless, the matching condition can be set at any plane between source and load, because all such conditions are equivalent.

Consider the circuit shown in Fig 5.1, where the marked quantities are element impedances at the frequency at which there is to be maximum power transfer. The Figure applies to one frequency only, and the representation of each termination does not relate to true resistive or reactive elements constituting the termination. There must be a conjugate match at the plane P, so that

$$R_a + jX_a + jX_1 = \left(\frac{1}{G_b + jB_b + jB_2} \right)^* \quad (5-1)$$

Equating the respective real and imaginary parts, one soon finds the two solutions (i) and (ii) to be

$$(i) \quad X_1 = -X_a + R_a \sqrt{\frac{1}{R_a G_b} - 1} \quad (5-2)$$

$$B_2 = -B_b + G_b \sqrt{\frac{1}{R_a G_b} - 1} \quad (5-3)$$

and

$$(ii) \quad X_1 = -X_a - R_a \sqrt{\frac{1}{R_a G_b} - 1} \quad (5-4)$$

$$B_2 = -B_b - G_b \sqrt{\frac{1}{R_a G_b} - 1} \quad (5-5)$$

Each exists if and only if $R_a \leq 1/G_b$, that is, if and only if the equivalent series resistance of termination A is not greater than the equivalent parallel resistance of termination B. In the case of equality, solutions (i) and (ii) are the same.

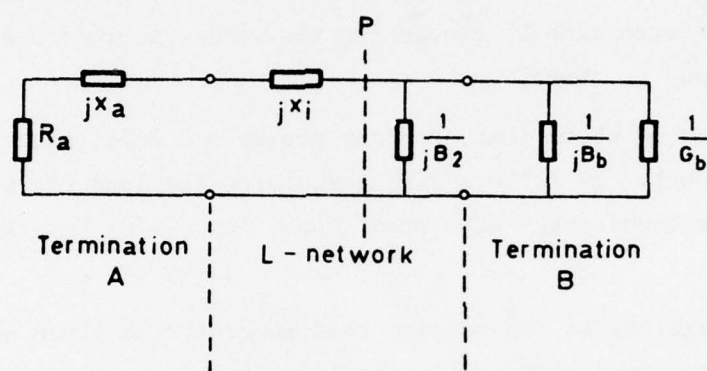


Fig 5.1

The significance of those results is this. Let the source impedance be $Z_S = R_S + jX_S$ and its admittance $Y_S = G_S + jB_S$; let the load impedance Z_L be described by similar symbols. If the series arm of the L is towards the load,

the series arm reactance of the L will be $-X_L \pm R_L \sqrt{\frac{1}{R_L G_S} - 1}$

and its parallel arm susceptance $-B_S \pm G_S \sqrt{\frac{1}{R_L G_S} - 1}$,

where both plus signs or both minus signs must be taken together. This kind of L-network (with the series arm on the load side) can be used if and only if $R_L \leq 1/G_S$.

A similar statement can be made about the use of an L-network with the series arm on the source side by exchanging the words 'source' and 'load' and the suffixes S and L above.

In fact it can be shown that, for any source and load, either $R_L \leq 1/G_S$ or $R_S \leq 1/G_L$ or both. It follows that any source and load can be matched at one frequency by an L-network. (The proof that ' $R_L \leq 1/G_S$ or $R_S \leq 1/G_L$ ' is not easy to see.)

From the foregoing, it can be seen that in general a given source and load can be matched at one frequency by a choice of either two or four L-networks. In certain special cases, these L-networks will not all be distinct. When $R_S = 1/G_L$ or $R_L = 1/G_S$ but not both, there will be three solutions. When both are true, it can be shown that the terminations are equal and resistive: the one L-network which will match is the trivial one with zero series reactance and zero shunt susceptance.

6 CONDITIONS FOR THE EXISTENCE OF A MATCHING L-NETWORK OF A GIVEN FORM

One can define eight forms of L-network according to the sign of each reactance and the side on which the shunt arm is placed. These forms are enumerated in Fig 6.1, in which a non-negative reactance is represented by an inductance symbol and a non-positive reactance by a capacitance symbol. In this Report, when the word 'form' is used to mean one of those forms, it will be written with a capital F. Nomenclature such as 'Form 1' is peculiar to this Report.

Two problems are of interest:- Firstly, what range of terminations can a given form of L-network match? Secondly, which forms of L-network can be used to match two given terminations? The two problems will be treated below under separate headings. During this work expressions have been derived constituting a

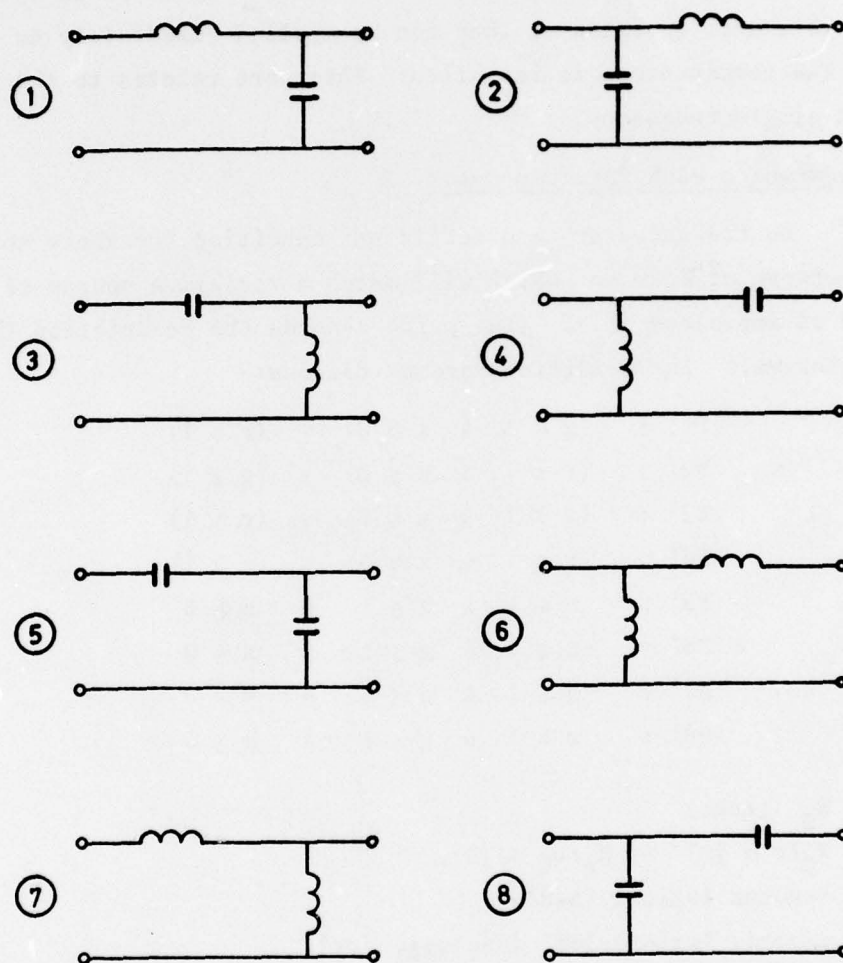


Fig 6.1 The eight forms of L-network

complete solution to the first problem, but they will not be given here because they are too long to be of much practical interest. However, from these expressions there have been derived the corresponding expressions relating to the simpler problem in which the source is a unit resistance. From those expressions, in turn, a graphical solution has been obtained to the problem of which Forms will match a given load to a resistive source.

Clearly, although the solutions given below are expressed in terms of a source which is a unit resistance, they can be applied immediately to any problem where one of the terminations is resistive. This part relates to the task of matching at a single frequency.

The loads which each Form can match

Let F'_n be the necessary and sufficient condition for there to exist at least one L-network of Form n which will match a resistive source of impedance Z_S to a load of impedance Z_L . (The prime denotes the restriction that the source is resistive.) The conditions are as follows:-

$$\begin{aligned} F1' : & (g \leq 1 \ \& \ x \geq 0) \vee (r \geq 1) \\ F2' : & (r \leq 1 \ \& \ x \leq 0) \vee (g \geq 1) \\ F3' : & (g \leq 1 \ \& \ x \leq 0) \vee (r \geq 1) \\ F4' : & (r \leq 1 \ \& \ x \geq 0) \vee (g \geq 1) \\ F5' : & r \leq 1 \ \& \ g \leq 1 \ \& \ x \geq 0 \\ F6' : & r \leq 1 \ \& \ g \leq 1 \ \& \ x \leq 0 \\ F7' : & r \leq 1 \ \& \ g \leq 1 \ \& \ x \leq 0 \\ F8' : & r \leq 1 \ \& \ g \leq 1 \ \& \ x \geq 0 \end{aligned}$$

where $Z_S = R_0$ (real)

and $Z_L = R_0(r + jx) = R_0(g + jb)$.

& denotes logical 'and'

\vee denotes logical 'or' (inclusive 'or').

Fig 6.2 shows the areas on the Smith chart³ corresponding to the inequalities in the conditions. From those it can be seen what area corresponds to the satisfaction of, for example, condition $F1'$: that area is shown shaded in the Figure.

The Forms which can match a given load

From the expressions given above it can be deduced which Forms can be used to match any given load. This deduction is easily made graphically, and a Smith chart giving the results appears as Fig 6.3. Consideration of the number of solutions (L-networks) in each area shows that, for a load impedance represented

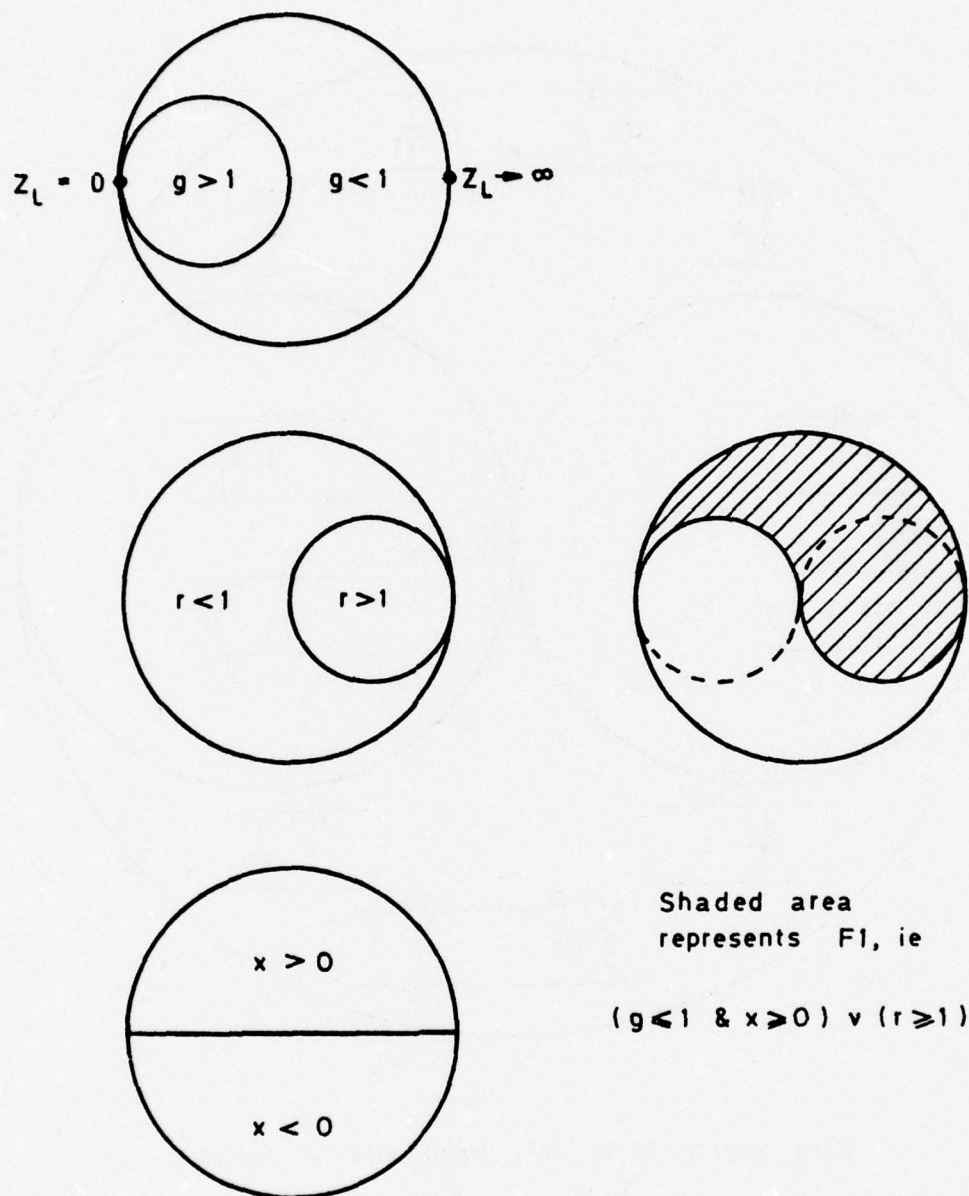
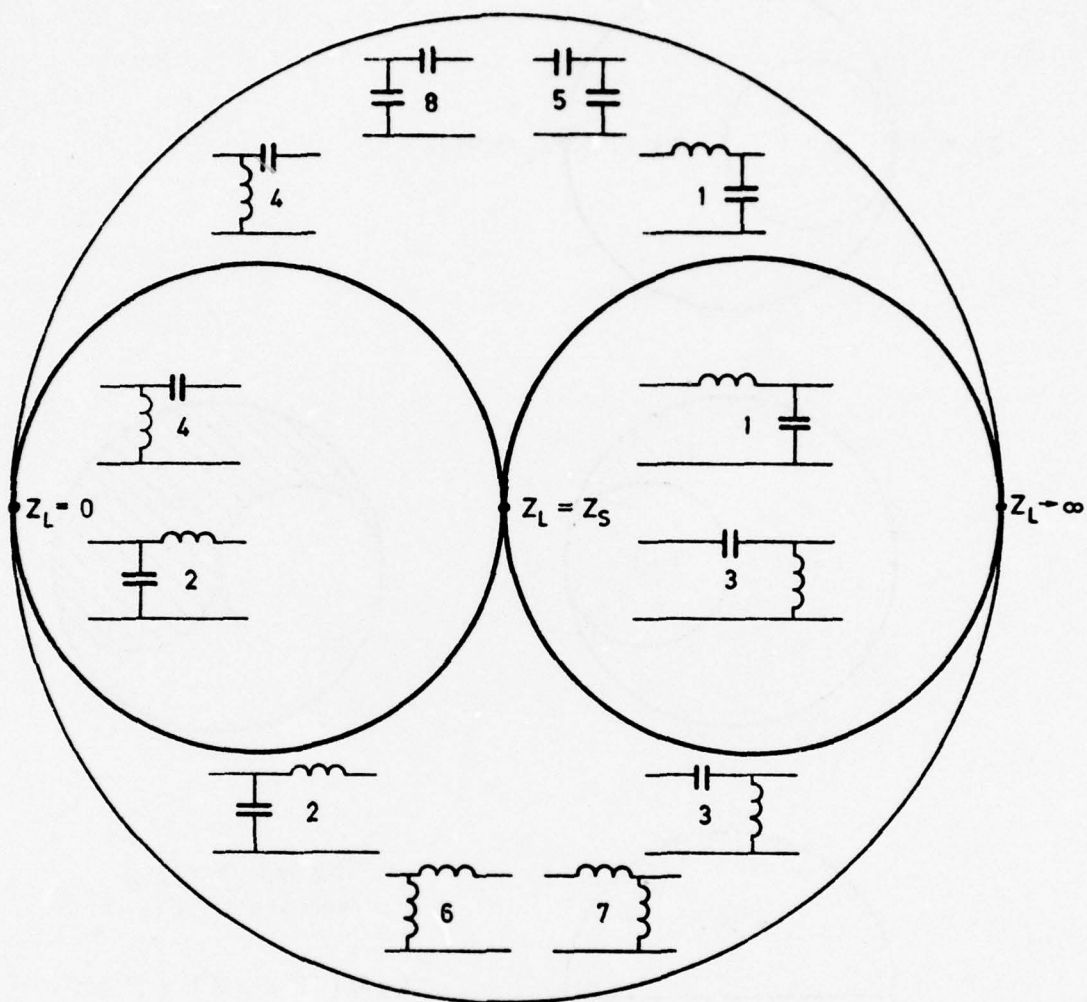


Fig 6.2 Areas corresponding to inequalities



Note: Source is on left hand side of network

Fig 6.3 The L-networks which can match a particular load to a resistive source

by a point in any of the four areas of the chart, matching may be effected by one L-network of each of the Forms shown in that area, and by no others.

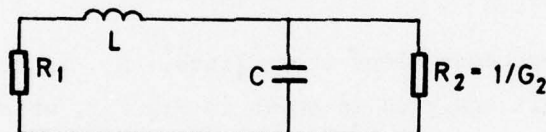
7 FREQUENCY ANALYSIS OF L-NETWORKS MATCHING RESISTIVE TERMINATIONS

In this section, the frequency behaviour of transducer gain will be discussed. Only the simplest case will be treated: two resistances matched by an L-network comprising one inductance and one capacitance. Even one extra reactive element makes the algebra very much more complicated.

The analysis does still apply, however, to a case where one of the reactive elements in the circuit analysed is a combination of two similar elements, one belonging to a termination and the other to the L-network.

In order to match two resistances, only Forms 1 to 4 can be used. Form 1 was analysed, and the results for the other three Forms were deduced using relationships between the Forms.

The circuit under discussion, shown in Fig 7.1, has the frequency response of a second-order low-pass filter. The properties of this kind of response are set out in the Appendix, and some were used in the analysis discussed below.



Note: $R_2 > R_1$

Fig 7.1

Apart from the formula giving transducer gain G_t as a function of frequency, the quantities Q , f_n and M_p implicit in this formula are also of interest. Q is the selectivity, and f_n is the natural frequency of the response. M_p is the ratio of the amplitude of the voltage gain at the peak to that at zero frequency in the low-pass case or infinite frequency in the high-pass case. Let f_0 be the frequency of perfect match.

It can be shown that in the low-pass case (Form 1 or 2):

$$G_t = \frac{1}{1 + \left\{ \frac{1}{2} \left(\sqrt{\frac{R_2}{R_1}} - \sqrt{\frac{R_1}{R_2}} \right) \left(\frac{f^2}{f_0^2} - 1 \right) \right\}^2}, \quad (7-1)$$

$$Q = \frac{1}{2} \sqrt{\frac{R_2}{R_1} + 1}, \quad (7-2)$$

$$M_p = \frac{1}{2} \left(\sqrt{\frac{R_2}{R_1}} + \sqrt{\frac{R_1}{R_2}} \right), \quad (7-3)$$

$$f_n = f_0 \sqrt{\frac{R_2 + R_1}{R_2 - R_1}}; \quad (7-4)$$

and that in the high-pass case (Form 3 or 4):

$$G_t = \frac{1}{1 + \left\{ \frac{1}{2} \left(\sqrt{\frac{R_2}{R_1}} - \sqrt{\frac{R_1}{R_2}} \right) \left(\frac{f_0^2}{f^2} - 1 \right) \right\}^2} \quad (7-5)$$

Q and M_p are as for the low-pass case, and (7-6)

$$f_n = f_0 \sqrt{\frac{R_2 - R_1}{R_2 + R_1}}. \quad (7-7)$$

The derivation proceeds along these lines, R_1 is arbitrarily taken to be the source. The circuit analysed is shown in Fig 7.2, which defines source voltage V_s and load voltage V_l , both being phasors whose magnitudes are rms quantities.

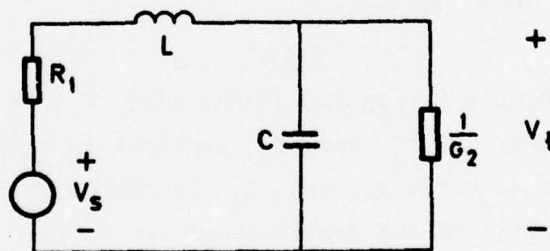


Fig 7.2

With those values for L and C which give the required reactances for a perfect match at f_0 , the complex-valued function V_l/V_s is obtained. From this, the values of Q and f_n are deduced, and M_p is inferred from Q .

Transducer gain has been defined as power delivered to load divided by power available from source.

Thus

$$G_t = \frac{|V_\ell|^2/R_2}{|V_s|^2/4R_1} . \quad (7-8)$$

The modulus squared of V_ℓ/V_s is therefore taken and G_t is obtained. Thus all the required formulae are obtained for Form 1.

The transducer gain function for Form 2 is identical to that of Form 1 (R_2 being the larger resistance in both cases) because Form 2 is Form 1 backwards. The functions G_t for Forms 3 and 4 are also identical by the same argument - this function is obtained from the low-pass function by substituting f_0/f for f/f_0 . That relationship, and the fact that Q and M_p are the same as for the low-pass case, arise because the Forms are related by a low-pass to high-pass transformation.

The expressions above for Q and M_p show that Q always exceeds 0.7 and M_p is at least unity. When $R_2 \gg R_1$, Q and M_p are both approximately equal to

$$\frac{1}{2}\sqrt{(R_2/R_1)} . \quad (7-9)$$

Curves for the response of a second-order low-pass filter are given in Ref 17, pp 226-228 and Ref 18, pp 158-159. These use the parameter ζ (zeta) which is $1/2Q$. The normalisation of those published curves sets the responses equal at zero frequency, as is the case for the voltage gain of the low-pass circuits discussed. However, the transducer gains are equal not at zero frequency but at the peak.

Q is not equal to any ratio of reactance to resistance or susceptance to conductance.

Another form¹⁴ of the formula for the transducer gain of a low-pass L-network matching resistances is

$$G_t = \frac{4n}{(n-1)^2 \left[(f/f_0)^4 - 2(f/f_0)^2 \right] + (n+1)^2} \quad (7-10)$$

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where $n = R_2/R_1$.

Examination of the formulae shows that the transducer gain curves for both low-pass and high-pass cases are smooth with a single peak. In the low-pass case the gain is non-zero at zero frequency, and falls asymptotically to zero at infinite frequency. In the high-pass case the gain is zero at zero frequency and falls asymptotically to a non-zero value at infinite frequency.

It can easily be shown that for Forms 1 and 2 at zero frequency, and for Forms 3 and 4 at infinite frequency,

$$G_t = \frac{4R_1 R_2}{(R_1 + R_2)^2} = 1 - \left(\frac{R_2 - R_1}{R_2 + R_1} \right)^2 \quad (7-11)$$

The term subtracted from 1 represents the proportion of available power which is reflected because of the mismatch between source and load resistances. This reflection is smaller than one might expect: this table gives some examples.

R_2/R_1	Transducer gain	
	Ratio	dB
1	1	0
2	0.89	-0.51
3	0.75	-1.25
20	0.18	-7.4
40	0.095	-10.2

For this reason, when two resistances matched by an L-network are comparable, the slope of the transducer gain from the peak towards the zero-frequency value (if low-pass) or asymptotic infinite-frequency value (if high-pass) is very gentle.

The formula (7-2) for Q shows that tuning is sharper for resistances of very different orders of magnitude. The Q values for practical cases are smaller than one might expect, eg $R_2 = 15R_1$ gives a Q of 2. With such low Q values, the relative bandwidth would be so large that the peak would be very asymmetrical.

Examples of the transducer gain curve will be given in the next part.

8 COMPUTER ANALYSIS OF EXAMPLES WITH RESISTIVE TERMINATIONS

Introduction

For a 50 ohm source and various resistive loads, matching L-networks were designed with the program LNETWORK¹⁶ and analysed with the program REDAP38.

LNETWORK designs two-element L-networks using equations (5-2) to (5-5). Given the frequency of match and the terminating impedances, it prints out the reactances and element values for all the two or four L-networks which can be used. REDAP38 is a fairly comprehensive program for frequency analysis of linear circuits. Both programs have been implemented for teletype use on the RAE's ICL 1906S remote-access computer, and at the time of writing they are on files in the machine.

The results, tables of transducer gain against frequency, are reproduced here only as graphs; the frequency axis has been chosen to be linear. The frequency of perfect match in each case was 2 GHz. The values of load resistance used are 12.5 ohms, 200 ohms and 2000 ohms. 200 ohms was chosen to make the ratio of resistances equal to the moderate value of 4. 12.5 ohms was used to allow comparison with 200 ohms, since it also gives a resistance ratio of 4. Finally 2 kilohms was used because it gives a high ratio of 40.

The matching networks

Fig 8.1 shows both circuits possible for each of the chosen loads. For each load there is one high-pass L-network which will match, and one low-pass, but no others. In every circuit the reactors are of opposite type and the shunt reactor is next to the larger resistance. For each load, the series reactances at 2 GHz of the two alternative L-networks were equal in magnitude and opposite in sign, and the same was true for the shunt reactances. The reactance values are not reproduced.

There is a close relationship between cases 200L, 200H, 12.5L and 12.5H in the Figure. For example, 200L and 12.5L are dual with respect to 50 ohms, as illustrated by the fact that

$$\frac{6.892\text{nH}}{(50\Omega)^2} \simeq 2.757 \text{ pF} \quad \text{to four significant figures.}$$

Again, 12.5L could be obtained from 200L by drawing the voltage generator on the other side and dividing all impedances by 4.

From either of those facts it could be deduced that the curves of transducer gain against frequency for 200L and 12.5L are the same, and indeed the printouts from the computer analyses of cases 200L and 12.5L were identical, as were those of 200H and 12.5H.

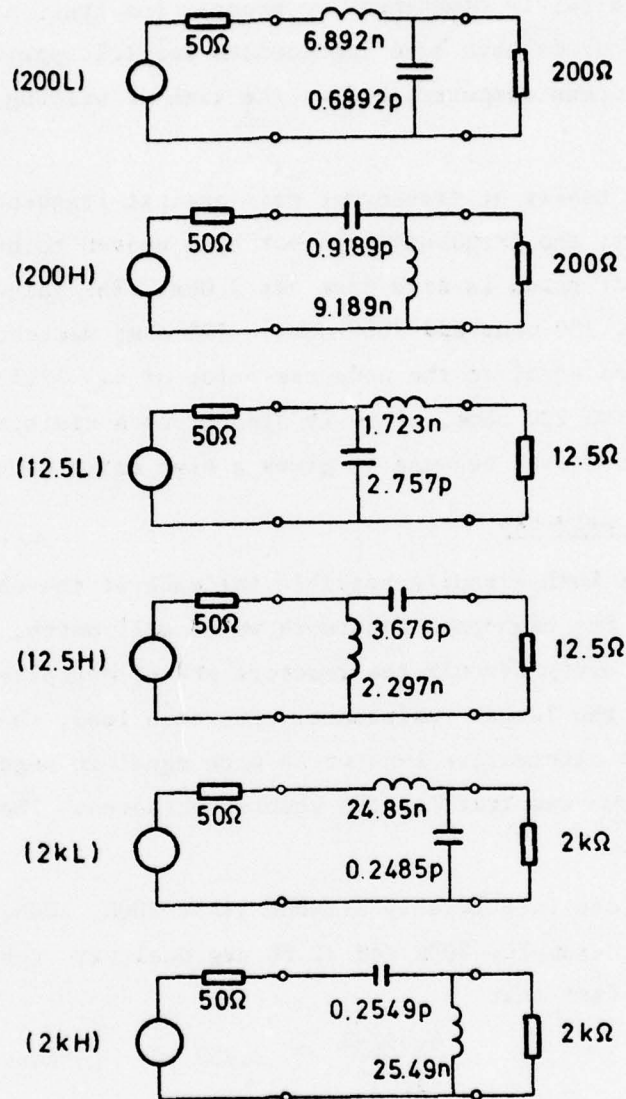


Fig 8.1 L-networks matching resistive terminations — element values

Curves for 0 to 4 GHz; 200 ohm load

Fig 8.2 shows the two L-networks which match 50 ohms to 200 ohms at 2 GHz, together with their transducer gain curves. The purposes of the Figure are to show the behaviour of the low-pass and high-pass cases over a wide range of frequency and to contrast them.

The nature of the curves is as expected in the last section. The curves would have been mirror-images about the 2 GHz line if a logarithmic frequency scale had been used. At zero frequency, the low-pass curve has a minimum of -1.9 dB. This is of course, the same as the value of transducer gain when the 50 ohm source is connected directly to the 200 ohm load. It has been stated that the gain of the low-pass circuit at high frequencies falls asymptotically to zero. The graph does not show this, but is consistent with it. The fact that the gain of the high-pass circuit falls to zero at zero frequency is shown fairly clearly.

The peaks of both circuits are, roughly speaking, equally broad. This reflects the fact that the Q values are the same. For the low-pass case, the -1 dB points are at 1.1 GHz and 2.6 GHz. This suggests that L-networks could often be used in broadband applications, despite their extreme simplicity.

Curves for 1 to 3 GHz: 12.5 ohm, 200 ohm and 2000 ohm loads

Fig 8.3 shows, for each load, the transducer gain curves of the low-pass and high-pass matching networks. One pair of curves applies to both 12.5 ohms and 200 ohms, as explained above. This Figure is drawn for frequencies between 1 and 3 GHz, in order to show the peak clearly.

The 200 ohm and 2000 ohm cases are seen to have roughly the same degree of asymmetry in the overall response. The peak of the response in the 2000 ohm case is sharper. The Q values, computed using equation (7-2), are 1.12 for 200 ohms (and 12.5 ohms) and 3.20 for 2000 ohms.

It is of interest to check the much used formula for half-power bandwidth to see how accurate it is here. The formula is

$$\text{half-power bandwidth} \simeq \frac{\text{centre frequency}}{Q} .$$

This arises from Theorem (A-9) in the Appendix and applies for $Q \gg \frac{1}{2}$, but little accuracy can be expected at the low Q values in these examples. If the centre frequency is taken to be 2 GHz the estimates from the formula are 1.70 GHz for

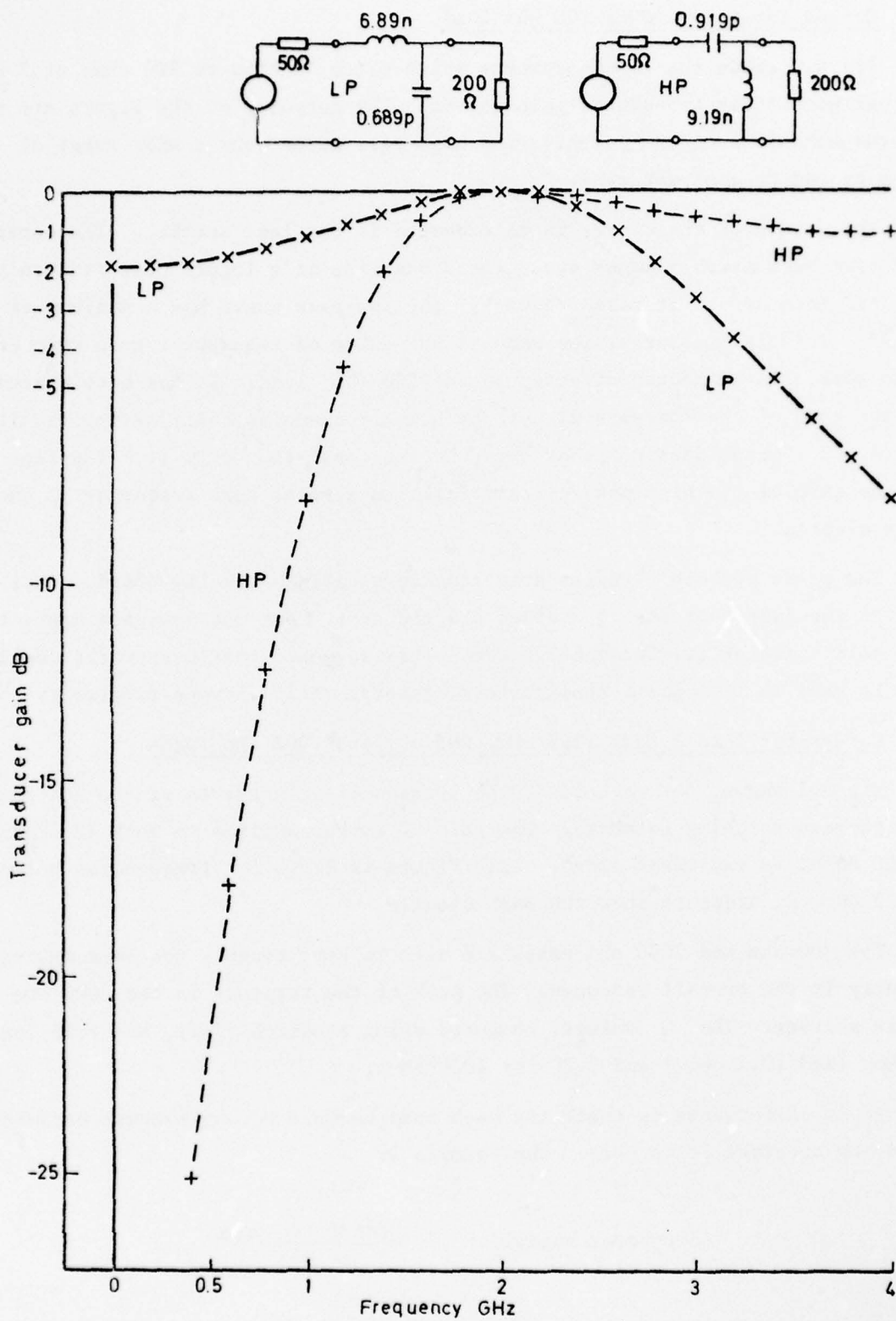


Fig 8.2

200L : 50Ω to 12.5Ω and 200Ω : low pass forms

200H : 50Ω to 12.5Ω and 200Ω : high pass forms

2KL : 50Ω to 2000Ω : low pass form

2KH : 50Ω to 2000Ω : high pass form

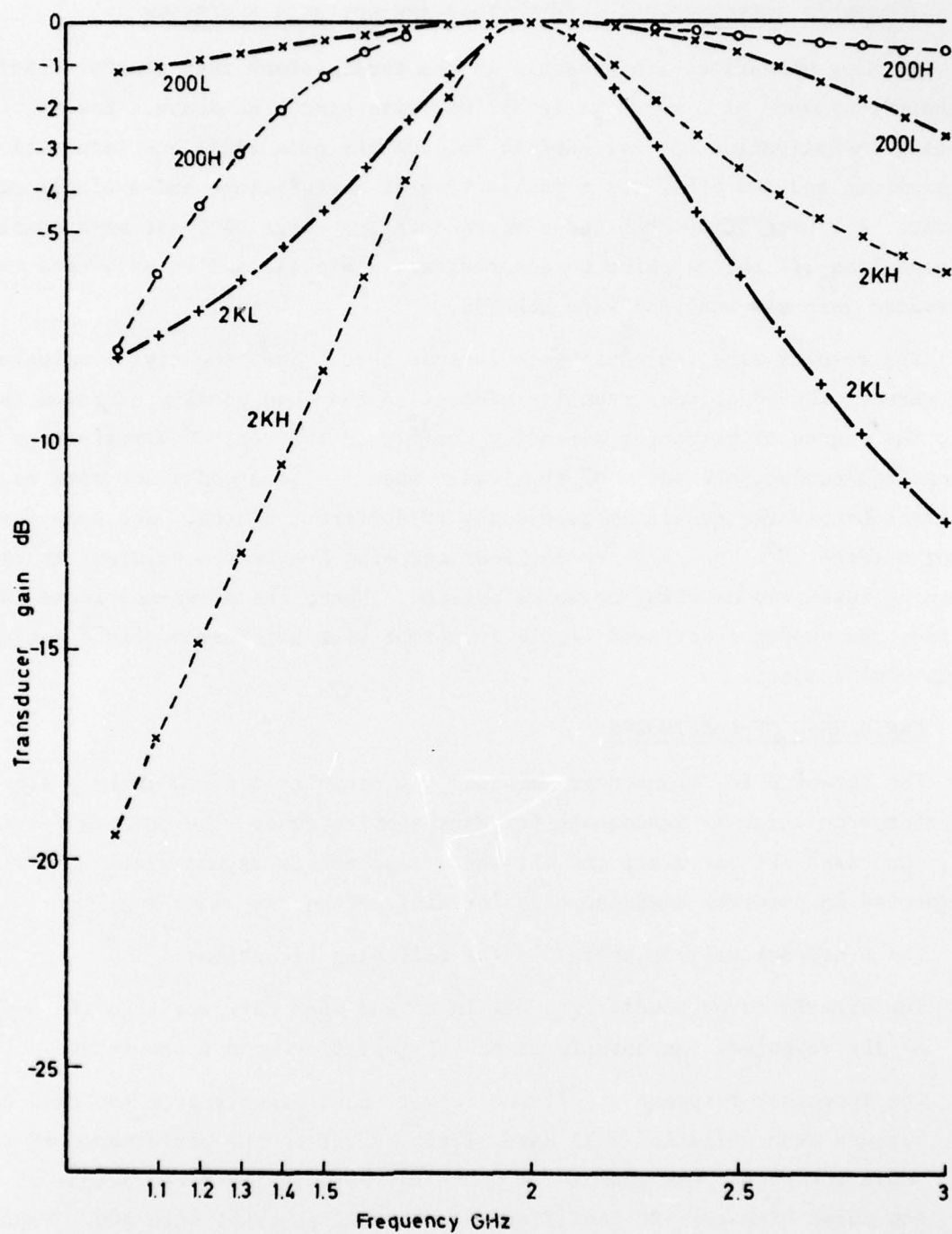


Fig 8.3

the 200 ohm load and 0.625 GHz for the 2000 ohm load. The graph shows that the former is a gross underestimate while the latter is reasonably close.

Around the top of the peak, the response for each load differs very little between the high-pass and low-pass cases.

9 FREQUENCY RESPONSE WHEN TERMINATIONS ARE NOT BOTH RESISTIVE

So many variations are possible in the terminations that general conclusions on the performance of L-networks in all circuits cannot be drawn. However, the computer investigation was extended to include the case where one termination was a resistance and the other was a combination of a resistance and a single reactive element. A source of 50 ohms and a representative range of loads were considered. For each load all the matching L-networks were designed, and in each case the transducer gain was analysed with REDAP38.

The results were too copious to include here. The essential conclusions were these. The additional reactive element in the load usually narrowed the peak, the degree of narrowing depending roughly on the reactance/resistance or susceptance/conductance ratio of the load. When the load contained some reactance, different L-networks generally gave peaks of different widths. For some loads having a large X/R or B/G ratio, four matching L-networks existed; in the remaining cases two matching networks existed. Where the above-mentioned ratio was low, the response differed little from that with both terminations resistive, as one would expect.

10 USEFULNESS OF L-NETWORKS

The formulae for transducer gain and the computer results imply a standard of performance which is inadequate for many applications. The peak is far from flat, the sides are not steep and the whole response is asymmetrical. This is to be expected in networks designed only for single-frequency matching.

The L-network will be useful in the following situations.

- (i) The signals to be handled are all in a band much narrower than the peak of the response. An example is an RF amplifier in a transmitter.
- (ii) The frequency response is allowed to vary considerably over the band used, because such variation will have little effect on the performance of the whole equipment. An example is where the network links the output of a low-noise high-gain RF amplifier to a tunable superhet with AGC. Variation of gain over the tuning band will not cause significant variation in either the noise figure or the gain of the overall receiver.

- (iii) The specification for the network's frequency response is undemanding because the equipment specification itself is undemanding.
- (iv) An L-network happens to give the frequency response wanted.
- (v) Irregularities in the response introduced by the L-network can be removed by adjusting the element values in another filter indirectly in cascade with it.
- (vi) The module being designed has many passive elements whose values can be chosen. The designer intends to make a rough initial design and optimise it (probably by computer), and the number of values to be chosen allows the optimisation enough scope for adjusting the frequency response to fulfil the specification.

Of those situations, (iii), (iv) and (v) are obviously not of great practical importance: (ii) would seem to arise only where there is gain to waste.

(i) is the classic situation where an L-network is used, and almost the only one discussed in the literature. Since the peak of the transducer gain of an L-network can be quite broad, such a network could sometimes cope with quite a large signal bandwidth. That was the justification for the derivation of the transducer gain formulae stated in an earlier part: to find out about the broadband capabilities of L-networks.

(vi) is a common situation nowadays, especially in the design of multistage microwave amplifiers, and the use of L-networks there seems quite promising.

Here is an example of situation (i). It is required to match two impedances over a fairly broad band. Neither is purely resistive, but the reactances are not large and do not change very much over the band. The designer has no program or tables to design a suitable matching network, and he wishes to make use of a set of tables for the element values of a broadband matching network which will match resistive terminations whose values are in discrete ratios (such as 2, 5, 10, 20 etc).

He can proceed as follows. He adds to one termination a series or parallel reactor which will transform it to a pure resistance near the centre frequency. Then he designs a matching network, using the tables, which will transform this resistance to a resistance fairly close to the impedance of the other termination. Finally he matches to that termination using an L-network. Although this is ugly from the theoretical point of view, one might sometimes need to do it. Trial and error might be needed to produce a satisfactory design, or of course the method might fail. Fig 10.1 shows an example of such a design.

is repeated for a range of values of the outer element, giving several designs for the T or Π network. Finally the whole circuit is analysed by program REDAP38 for each value of the outer element, and the design giving the best frequency response is identified. Further improvement in the shape of the response may be possible at the expense of the maximum gain.

12 CONCLUSIONS

If two terminations are to be matched at a single frequency, the matching task can be performed by a choice of two or four L-networks. The range of terminations which can be matched to a resistance by each of the eight forms of L-network has been stated in terms of formulae, corresponding to the graphical statements in several papers, and a graph has been presented showing for every possible terminating impedance which L-networks may be used to match it to a resistance.

Design formulae have been set out for the L-network which can cope with any pair of terminating impedances. These formulae have been embodied in a computer program.

When a two-element L-network is used to match two resistances, the peak is broader than one might expect, but its shape makes the characteristic unsuitable for most high-performance broadband work. The response is asymmetrical, the top is not flat and the slope of the skirts is not steep.

With terminations which are not both pure resistances, or an L-network with more than two elements, the frequency response is more complicated and few general comments can be made, but it is likely that the passband will be narrower and that the whole response will be irregular.

Nevertheless, the L-network is often worth considering for a broadband matching task. In particular, the use of L-networks should be considered when a multistage microwave amplifier is to be designed using a rough initial design and subsequent optimisation by computer.

The next simplest networks to the L are the Π and T, which allow the designer a measure of choice in the shape of the response. The L-network design program can be used to help in designing a T or Π network.

It is hoped that this Report has drawn attention to a neglected type of network, and provided information which will help designers to assess its suitability for tasks and to design networks of this type.

Acknowledgment

Some of the early work in this investigation was done by Mr J. Andrew Johnston, whose help is gratefully acknowledged.

Appendix

THE SECOND-ORDER LOW-PASS FILTER

This Appendix states formulae applying to transfer functions of the form

$$\frac{1}{a(j\omega)^2 + b(j\omega) + c} \quad (\text{A-1})$$

where a , b and c are real constants and a and c have the same sign. Many of these formulae are used in the derivation of the frequency response properties of the low-pass L-network matching resistive terminations.

The transfer functions examined here can be considered to be ratios of phasors, although from a more general point of view they are Fourier transfer functions, which are ratios of Fourier transforms.

Consider the circuit of Fig A.1.

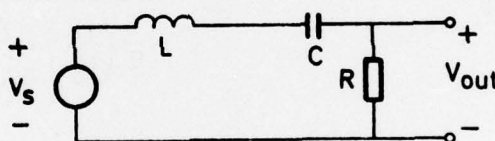


Fig A1 A commonly considered tuned circuit

It is very easily shown that the voltage transfer function is

$$\frac{V_{out}}{V_S} = \frac{RC(j\omega)}{1 + RC(j\omega) + LC(j\omega)^2} \quad (\text{A-2})$$

There is zero transmission at zero and infinite frequencies, and unity transmission at $\omega = 1/\sqrt{LC}$, which is the natural frequency of the circuit. The sharpness of the peak depends on the ratio L/CR^2 , whose square root is the selectivity Q . It can easily be shown that $Q = \omega_n L/R$, where ω_n is the natural frequency. This is typical of the sort of tuned circuit often considered in introductory textbooks, which usually proceed to give universal selectivity curves, showing for unit ω_n how the response varies with frequency for different values of Q .

When a narrow band about the top of the peak is considered, which is the case when Q is high, the response is little different when the input and/or output are at different points in the tuned circuit from those shown in Fig A.1 - for example, when the output voltage is taken across the capacitance instead of the resistance. That is because such a change of circuit changes only the numerator of the transfer function, which determines only the global slope of the response, whereas it is overwhelmingly the denominator which determines the position and width of the peak.

However, this Report relates to the response of the L-network at all frequencies, with a view to broadband use, and L-networks are usually used in such a way that the Q is very low. It is for this reason that the low-pass transfer function is being examined here. To facilitate comparison with the band-pass case discussed above, the symbols Q and ω_n , relating to the denominator of the transfer function, will again be used. Now that Q and ω_n have been discussed, the examination of the low-pass transfer function (A-1) can proceed.

Theorem (A-1):

$$\frac{1}{a(j\omega)^2 + b(j\omega) + c} \equiv \frac{1}{c} \frac{1}{1 + \frac{1}{Q} \left(\frac{j\omega}{\omega_n} \right) + \left(\frac{j\omega}{\omega_n} \right)^2}$$

where $Q = \sqrt{ac}/b$

and $\omega_n = \sqrt{c/a}$.

Thus the variables a , b and c can be replaced by ω_n , Q and a scaling factor. Accordingly, from here on the scaling factor will be ignored and this simpler form of transfer function will be considered:-

Let

$$T(j\omega) = \frac{1}{1 + \frac{1}{Q} \left(\frac{j\omega}{\omega_n} \right) + \left(\frac{j\omega}{\omega_n} \right)^2} \quad (A-3)$$

Note

$$T(0) = 1 + j0 \quad (A-4)$$

Note on ζ :- Control engineers characterise second-order systems in terms of ω_n as discussed above and a quantity ζ (zeta) called "damping ratio" or "damping factor".

$$\zeta = \frac{1}{2Q} . \quad (\text{A-5})$$

Theorem (A-2):

$$T(j\omega_n) = -jQ .$$

Proof Immediate, from equation (A-3).

Note This is the origin of the name "magnification" as applied to the Q-factor of a tuned circuit.

Now a further substitution will be made so that the response can be characterised in terms of the frequency of the peak, ω_0 , and the height of the voltage peak, M_p , instead of in terms of ω_n and Q as above. This applies only for $Q \geq 1/\sqrt{2}$, which is always true for the L-network when matching resistances.

Let the angular frequency maximising $|T|$ be ω_0 .

Let $|T(j\omega_0)|$ be M_p .

Theorem (A-3):

$$\omega_0 = \omega_n \sqrt{1 - \frac{1}{2Q^2}} \quad \text{when } Q \geq 1/\sqrt{2} ,$$

and

$$\omega_0 = 0 \quad \text{when } Q \leq 1/\sqrt{2} .$$

Proof From equation (A-3), differentiate $|T|^2$ with respect to ω^2 .

Note Clearly $\omega_0 \rightarrow \omega_n$ as $Q \rightarrow \infty$.

Note For a given non-zero ω_0 , $\omega_n \rightarrow \infty$ as $Q \rightarrow 1/\sqrt{2}$.

Theorem (A-4):

$$M_p = \frac{Q}{\sqrt{1 - \frac{1}{4Q^2}}} \quad \text{when } Q \geq 1/\sqrt{2} ,$$

and

$$M_p = 1 \quad \text{when } Q \leq 1/\sqrt{2} .$$

Proof Using theorem (A-3), find $M_p = |T(j\omega_0)|$.

Note $M_p > Q$.

Theorem (A-5): $M_p \rightarrow Q$ as $Q \rightarrow \infty$.

Proof Manipulation of theorem (A-4).

Note M_p deserves the name of "magnification" just as much as Q does, but the term is rarely if ever used to denote M_p .

Now that the new variables have been evaluated and some of their properties mentioned, the quantity $|T|^2$, which corresponds to power gain, will be given as a function of the new variables, and from this function further formulae will be deduced concerning the frequency response.

Theorem (A-6):- When $Q \geq 1/\sqrt{2}$:

$$|T|^2 = \frac{M_p^2}{1 + (M_p^2 - 1) \left(\omega^2 / \omega_0^2 - 1 \right)^2}.$$

Proof Lengthy manipulation using theorems (A-3) and (A-4).

Theorem (A-7):- If $\omega_1 > \omega_2 > 0$

and

$$|T(j\omega_1)| = |T(j\omega_2)|,$$

then

$$\omega_0^2 = \frac{1}{2} (\omega_1^2 + \omega_2^2).$$

Proof Manipulation from theorem (A-6).

Note This theorem can be used to find the peak frequency for given band edges.

Theorem (A-8):- For any M_p ,

$$|T(j\sqrt{2}\omega_0)| = 1.$$

Proof Theorem (A-6).

Definition The half-power bandwidth is the difference between the two positive values of ω for which $|T|^2 = \frac{1}{2}$.

Theorem (A-9):- The half-power bandwidth B approaches ω_0/Q as Q tends to infinity, in the sense that

$$\lim_{Q \rightarrow \infty} \frac{B}{\omega_0/Q} = 1.$$

Proof From theorem (A-6), fairly long manipulation.

Note Work reported in this Report suggests that ω_0/Q is fairly close to B when $Q > 4$.

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REPORT DOCUMENTATION PAGE

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17. Abstract An L-network is an electric network comprising two reactive arms, one in series and one in shunt. Any two impedances can be matched at a single frequency by at least one such network. This Report explains relevant theory, sets out design equations, and examines the broadband matching performance of L-networks both algebraically and by means of computer analysis of examples. It is shown that these networks have occasional broadband uses. Treatment is general but findings are expressed in terms of circuits which operate at frequencies in the low gigahertz region.					