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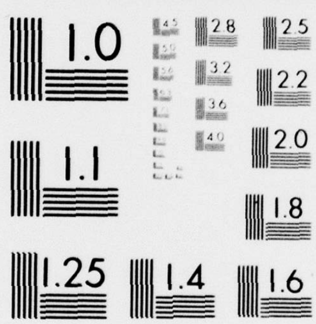
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MASSACHUSETTS INSTITUTE OF TECHNOLOGY

LINCOLN LABORATORY

THE PLATE OVERLAP TECHNIQUE

L. G. TAFF

Group 94

TECHNICAL NOTE 1978-30

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LEXINGTON MASSACHUSETTS

ABSTRACT

This report presents the development of the plate overlap technique originally suggested by H. Eichhorn. Included are discussions of standard coordinates, plate modeling, the plate overlap technique itself, and some computational details.

CONTENTS

I. INTRODUCTION

Over the past three years we have been systematically bringing together the tried and true techniques of classical photographic astrometry and the technology of today. Our aim has been the ultimate amalgam of the two. The result would be a completely automatic, real-time, accurate ($\approx 1''$) reduction of the position of any celestial object. The fulfillment of this endeavor involves the work of many people and the supplementary genesis of analytical tools, complicated software, and new hardware. Since this aim was clearly recognized long ago, wherever possible in earlier documents or software development flexibility was built in. Only the most germane parts of this material will be repeated here. For discussions of multi-color photometry and multi-color systems the reader should see 1, 2, and 3. The procedures one should use to update stellar positions are discussed in 4 with applications to the SAOC in 5. Non-stellar objects are discussed in 6. All of the pertinent aspects of astrometry, artificial satellite reductions, elementary plate modeling, and photographic astrometry are in 7 and 8.

The main purpose of this report is to present an even more powerful technique of data reduction which is known as the plate overlap technique. It was originally proposed by Eichhorn (9). His idea was to eliminate, as much as possible, systematic errors in the reduction of the positions, proper motions, and annual parallaxes of stars. While the conceptual idea is simple, its analytical formulation and computational implementation are not. We discuss this in §IV. First we establish our notation (§II) and review the properties of a gnomonic projection (§III).

II. NOTATION

We use the standard notation of the subject (unlike in 8).

$a^n_{ijk\ell}$, a^n_{ij} = plate constants in the xi standard coordinate

$b^n_{ijk\ell}$, b^n_{ij} = plate constants in the eta standard coordinate

c_m = the color of the m'th star

h^m = the number of epochs for which a catalogue position of the m'th star is available

H^m = the total number of catalogue positions for the m'th star

I,J = upper limits to indices i and j on the plate constants

m_m = the magnitude of the m'th star

M_n = the total number of different stars on the n'th plate

M = the total number of different stars

N_m = the total number of plates on which star number m appears

N = the total number of plates

P_{mn} see Eq. (10)

R_m see Eq. (11)

R_{plate}, R_{ref} = the residuals from the plate modeling and catalogue mismatch of the reference stars

$R = R_{plate} + R_{ref}$

t_{h^m} = the epoch numbered h^m

t_n = the epoch time of the n'th plate

x_{mn}, y_{mn} = the measured (not necessarily rectangular) coordinates of the m'th star on the n'th plate

α, δ = the equatorial coordinates of a celestial object

A, Δ = the equatorial coordinates of the tangential point

ξ,η = the standard coordinates related to α and δ

ξ_m, η_m = the standard coordinates of the m'th star at epoch t = 0

$\xi_{mn} = \xi_m + \mu_m^{\xi} t_n$

$\eta_{mn} = \eta_m + \mu_m^{\eta} t_n$

$\xi^c_{mh}m$, $\eta^c_{mh}m$ = the standard coordinates from a catalogue for the m'th star at epoch $t_h m$

$\mu_m^{\xi} = d\xi_m/dt|_{t=0}$

$\mu_m^{\eta} = d\mu_m/dt|_{t=0}$

III. THE GNOMONIC PROJECTION*

The imaging system (telescope plus camera) projects a portion of the celestial sphere onto the focal plane through a point on the optical axis of the imaging system. By analyzing the geometry of such a projection we can go from coordinates on the sky (right ascension α and declination δ) to coordinates on the focal plane (called standard coordinates and universally denoted by ξ,η). When the unit of length is the focal length of the telescope we can write

$$\cot\delta\sin(\alpha - A) = \xi\sec\Delta/(\eta + \tan\Delta), \tag{1a}$$

$$\cot\delta\cos(\alpha - A) = (1 - \eta\tan\Delta)/(\eta + \tan\Delta), \tag{1b}$$

$$\tan(\alpha - A) = \xi\sec\Delta/(1 - \eta\tan\Delta), \tag{1c}$$

$$\begin{aligned}\xi &= \cot\delta\sin(\alpha - A)/[\sin\Delta + \cot\delta\cos\Delta\cos(\alpha - A)], \\ &= \cos q\tan(\alpha - A)\sec(q - \Delta),\end{aligned} \tag{2a}$$

$$\begin{aligned}\eta &= [\cos\Delta - \cot\delta\sin\Delta\cos(\alpha - A)]/[\sin\Delta + \cot\delta\cos\Delta\cos(\alpha - A)], \\ &= \tan(q - \Delta),\end{aligned} \tag{2b}$$

$$\cot q = \cot\delta\cos(\alpha - A), \tag{2c}$$

where (A,Δ) are the equatorial coordinates of the extension of the optical axis of the imaging system on the celestial sphere. This is also known as the point of tangency. The angle q is a convenient auxiliary variable. Also, if $|\alpha - A|$ is small, one should use Eq. (1d) in place of Eqs. (1a, 1b);

$$\sin\delta = (\sin\Delta + \eta\cos\Delta)/(1 + \xi^2 + \eta^2)^{1/2}. \tag{1d}$$

The orientation of the ξ,η axes is such that the positive η axis points north and the positive ξ axis points east. Equations (1, 2) are completely

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*See §IVA of 7 for more detail.

rigorous, allow one to speak of an object's position in either coordinate system, and, just as the equatorial coordinates are measurable, so too are the standard coordinates.

IV. THE PLATE OVERLAP TECHNIQUE

A. Motivation

Suppose we have several plates* of the same area of the celestial sphere. On it will be the object of prime interest (the program object), stars with well known positions and proper motions (reference stars), many other stars (field stars), and images of various non-stellar objects (asteroids, planets, galaxies, etc.). Let us suppose that each plate is separately reduced by a model of the form

$$x_{mn} = \sum_{i=0}^{I_n} \sum_{j=0}^{J_n} \sum_{k=0}^{K_n} \sum_{\ell=0}^{L_n} a^n_{ijk\ell} \xi^i_{mn} \eta^j_{mn} m^k_m c^\ell_m, \tag{3a}$$

$$y_{mn} = \sum_{i=0}^{I'_n} \sum_{j=0}^{J'_n} \sum_{k=0}^{K'_n} \sum_{\ell=0}^{L'_n} b^n_{ijk\ell} \xi^i_{mn} \eta^j_{mn} m^k_m c^\ell_m, \tag{3b}$$

where the index m = 1, 2, 3,..., M numbers the stars, the index n = 1, 2, 3, ..., N numbers the plates (taken at epochs $\{t_n\}$), $\{x_{mn}, y_{mn}\}$ are the measured coordinates (not necessarily rectangular) of star number m on plate n, $\{\xi_{mn}, \eta_{mn}\}$ are the computed standard coordinates for star number m at epoch t_n, m_n is the magnitude of star number m, c_m is the color of star number m, and $\{a^n_{ijk\ell}, b^n_{ijk\ell}\}$ are the plate constants. Note that, except for the weak assumptions of a polynomial form for the model, Eqs. (3) are completely general.

Once we obtain, by an adjustment procedure, estimates for the plate constants, say $\{A^n_{ijk\ell}, B^n_{ijk\ell}\}$, we would take the measured values for the

*By "plate" we mean any completely formed image of a portion of the celestial sphere. Whether it be a photographic plate or television camera frame is immaterial.

program object, $\{x_n', y_n'\}$, invert the model and obtain estimates for its standard coordinates, $\{\xi_n' \; \eta_n'\}$. Finally, by inverting the gnomonic projection we obtain estimates for its equatorial coordinates, $\{\alpha_n' \; \delta_n'\}$. If only the reference stars were used in the adjustment for the plate constants (1) we would learn nothing more about these stars, (but we could compute the positions of all of the field stars and non-stellar objects), (2) we would more likely than not be extrapolating the magnitude and color terms rather than interpolating them, and (3) if the time scale over which the plates were taken was sufficiently small to render the effects of proper motion, foreshortening, and parallax factors totally insensible, we would find a total of N different values for the position of the program object. At most, one of these would be correct and the remainder would be systematically in error. The use of averaging, viz,

$$\alpha' = \sum_{n=1}^{N} \alpha_n'/N, \quad \delta' = \sum_{n=1}^{N} \delta_n'/N, \tag{4}$$

would reduce random errors by a factor of $1/\sqrt{N}$ but not the systematic ones.

As this point is one of the essential reasons for inventing the plate overlap technique, a few more words of explanation are necessary: Even if the geometry of the imaging system is known precisely and the errors of the measuring machine modeled exactly [so that Eqs. (3) contain both the necessary and sufficient number of parameters and an incorrect form of the model does not systematically bias the results], Eqs. (3) do not hold exactly. The unavoidable, random, errors of measurement imply that the left-hand sides of

Eqs. (3) should be written as

$$x_{mn} - X_{mn} = \qquad (5a)$$

$$y_{mn} - Y_{mn} = \qquad (5b)$$

where X_{mn} (Y_{mn}) is the noise associated with the measurement of x_{mn} (y_{mn}). Since a non-zero mean for X_{mn} (Y_{mn}) can be handled by a redefinition of a^n_{0000} (b^n_{0000}), we assume the noise to have a mean of zero and to be uncorrelated. Hence, our adjustment procedure would seek to minimize

$$R_{plate} = \sum_{n=1}^{N} \sum_{m=1}^{M} (X^2_{mn} + Y^2_{mn}), \qquad (6)$$

as a function of $\{a^n_{ijk\ell}, b^n_{ijk\ell}\}$.

The point is, because of the existence of $\{X_{mn}, Y_{mn}\}$, even if each plate images the same area of the celestial sphere, with the same tangential point, made on the same imaging system, measured with the same measuring machine, etc. it will not be true that

$$A^n_{ijk\ell} = A^{n'}_{ijk\ell}, \; B^n_{ijk\ell} = B^{n'}_{ijk\ell} \quad n \neq n', \qquad (7)$$

and these two effects will not cancel when computing $\{\xi_n, \eta_n\}$. Thus, while the program object clearly has one and only one position on the celestial sphere, we can not deduce it and each set of computed standard coordinates is systematically off from (ξ', η').

Now consider the case when we regard not only the plate constants to be unknown but also the positions and proper motions of <u>all</u> of the celestial objects imaged on the plate. Moreover, we constrain the equations of condition by insisting that the reference stars have their catalogue positions $\{\xi^c_{mh^m}, \eta^c_{mh^m}\}$ at the epochs $\{t_{h^m}\}$, $h^m = 1, 2, 3, \ldots, H^m$ for which the catalogues exist. Intuitively we feel that this "bonding" of the plates will eliminate all of the systematic effects. Moreover, the positions and proper motions of the reference stars are automatically computed and the color and magnitude terms are not extrapolated since all of the objects contribute to the solution. Obviously the number of unknowns has increased dramatically, the analytical and computational problem is exponentially more complex and time comsuming (and still can't be done rigorously!), and our ultimate gains may be small.

When the plates overlap in the center-edge pattern (Fig. 1a) or center-corner pattern (Fig. 1b) the power of the technique is more apparent. Now fewer reference stars are needed per square degree to obtain the same accuracy since, in effect, the total area covered by the overlapping plates is treated as one large plate.

B. Formulation

In order to proceed with a reasonable economy of notation, we drop the magnitude and color terms from Eqs. (3), set $I_n = I'_n = I$, $J_n = J'_n = J$ $\forall n \varepsilon [1,N]$, and ignore the possibility that, for some function f, $a^n_{ij} = f(\{b^{n'}_{i'j'}\})$. The first assumption is only to assist in the clarity of the presentation. The second is not at all restrictive if we regard $I = \max_n(I_n, I'_n)$, $J = \max_n(J_n, J'_n)$ and any plate constants artificially introduced by this to be identically zero and not subject to adjustment. The latter restriction is substantiative (consider the four constant plate model

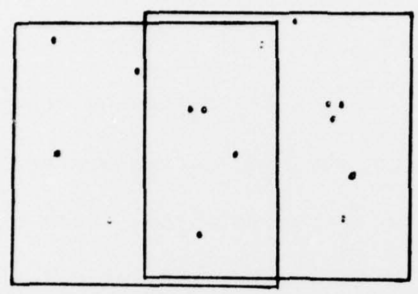

Fig. 1a. Illustration of the overlap pattern for the center-edge alignment (Displaced slightly for clarity)

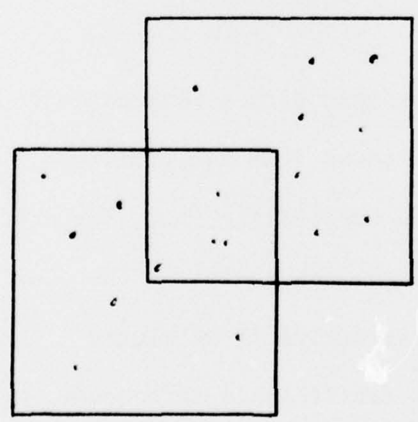

Fig. 1b. Illustration of the overlap pattern for the center-corner alignment

$$x = a\xi + b\eta + c, \tag{9a}$$

$$y = -b\xi + a\eta + d, \tag{9b}$$

which allows for an origin shift, a rotation, and a scale change) but necessary in order to make formal analytical progress. We shall also assume that the standard coordinates for each plate are referred to the same tangential point and let μ_m^ξ be the proper motion for the m'th star in the ξ direction and μ_m^η be the corresponding quantity in the η direction.

As before we will have a total of N different plates and M different stars. The plates are numbered by n, the stars by m, and the n'th plate was taken at epoch t_n relative to an arbitrary epoch of $t = 0$. On the n'th plate only M_n different stars appear (field stars plus reference stars: we ignore non-stellar objects in the sequel as they add nothing to the discussion). The total number of different plates the m'th star appears on is denoted by N_m. Finally, to allow unrestricted summations, we define P_{mn} and R_m by

$$P_{mn} = \begin{cases} 1 & \text{if star \#m appears on plate \#n} \\ 0 & \text{otherwise} \end{cases}, \tag{10}$$

$$R_m = \begin{cases} 1 & \text{if star \#m is a reference star} \\ 0 & \text{otherwise} \end{cases}. \tag{11}$$

The plate model is

$$\xi_{mn} = \xi_m + \mu^{\xi}_m t_n = \sum_{i=0}^{I} \sum_{j=0}^{J} a^n_{ij} x^i_{mn} y^j_{mn} + \Xi_{mn}, \tag{12a}$$

$$\eta_{mn} = \eta_m + \mu^{\eta}_m t_n = \sum_{i=0}^{I} \sum_{j=0}^{J} b^n_{ij} x^i_{mn} y^j_{mn} + H_{mn}, \tag{12b}$$

$$\forall\, m \in [1,M],\ n \in [1,N] \ni P_{mn} \neq 0.$$

The reversal with respect to Eqs. (3) and the precise meaning of the noise, $\{\Xi_{mn}, H_{mn}\}$ is discussed in §IVC below. That part of the residuals due to the plate modeling is

$$R_{\text{plate}} = \sum_{n=1}^{N} \sum_{m=1}^{M} P_{mn}[(\Xi_{mn})^2 + (H_{mn})^2]. \tag{13a}$$

The other part of the residuals are from the discrepancy between the catalogue position of a reference star number m, $(\xi^c_{mh^m}, \eta^c_{mh^m})$ at epoch $t = t_{h^m}$ for a total of H^m different catalogue positions, e.g.,

$$\xi_m + \mu^{\xi}_m t_{h^m} = \xi^c_{mh^m} + \Xi^c_{mh^m}, \tag{14a}$$

$$\eta_m + \mu^{\eta}_m t_{h^m} = \eta^c_{mh^m} + H^c_{mh^m}, \tag{14b}$$

$$h^m = 1, 2, 3, \ldots, H^m \ \forall m \in [1,M] \ni R_m \neq 0.$$

Hence, that part of the residuals due to the catalogue positions is

$$R_{ref} = \sum_{m=1}^{M} R_m \left[(\Xi^c_{mh^m})^2 + (H^c_{mh^m})^2 \right], \tag{13b}$$

$$R = R_{plate} + R_{ref}. \tag{13c}$$

If $R_m = 0$ then $h^m = H^m = 0$ and $\xi^c_{mh^m}$, $\eta^c_{mh^m}$ are undefined. Similarly, if $P_{mn} = 0$ x_{mn} and y_{mn} are undefined.

The normal equations are given by

$$\left.\begin{array}{l} \partial R/\partial a^n_{ij} = 0 \\ \partial R/\partial b^n_{ij} = 0 \end{array}\right\} \quad \begin{array}{l} n = 1, 2, 3, \ldots, N, \\ i = 0, 1, 2, \ldots, I,\ j = 0, 1, 2, \ldots, J; \end{array} \tag{15}$$

$$\left.\begin{array}{l} \partial R/\partial \xi_m = 0 \\ \partial R/\partial \eta_m = 0 \\ \partial R/\partial \mu^\xi_m = 0 \\ \partial R/\partial \mu^\eta_m = 0 \end{array}\right\} \quad m = 1, 2, 3, \ldots, M. \tag{16}$$

With the understanding that $\sum_{h^m=1}^{H^m} f(h^m) = 0$ if $R_m = 0$, these equations can be written as

$$\sum_{m=1}^{M} P_{mn} \left[\sum_{i=0}^{I} \sum_{j=0}^{J} a^n_{ij} x^{i+k}_{mn} y^{j+\ell}_{mn} \right] - \sum_{m=1}^{M} P_{mn} \xi_{mn} x^k_{mn} y^\ell_{mn} = 0, \tag{17a}$$

$$\sum_{m=1}^{M} P_{mn} \left[\sum_{i=0}^{I} \sum_{j=0}^{J} b^n_{ij} x^{i+k}_{mn} y^{j+\ell}_{mn} \right] - \sum_{m=1}^{M} P_{mn} \eta_{mn} x^k_{mn} y^\ell_{mn} = 0, \tag{18a}$$

$n = 1, 2, 3, \ldots, N$, $k = 0, 1, 2, \ldots, I$,

$\ell = 0, 1, 2, \ldots, J$;

$$- \sum_{n=1}^{N} P_{mn} \sum_{i=0}^{I} \sum_{j=0}^{J} a_{ij}^{n} x_{mn}^{i} y_{mn}^{j} + \xi_m (N_m + H^m) +$$

$$\mu_m^{\xi} \left(\sum_{n=1}^{N} P_{mn} t_n + \sum_{h^m=1}^{H^m} t_{h^m} \right) = \sum_{h^m=1}^{H^m} \xi_{mh^m}^{c}, \tag{17b}$$

$$- \sum_{n=1}^{N} P_{mn} \sum_{i=0}^{I} \sum_{j=0}^{J} b_{ij}^{n} x_{mn}^{i} y_{mn}^{j} + \eta_m (N_m + H^m) +$$

$$\mu_m^{\eta} \left(\sum_{n=1}^{N} P_{mn} t_n + \sum_{h^m=1}^{H^m} t_{h^m} \right) = \sum_{h^m=1}^{H^m} \eta_{mh^m}^{c}, \tag{18b}$$

$$- \sum_{n=1}^{N} P_{mn} t_n \sum_{i=0}^{I} \sum_{j=0}^{J} a_{ij}^{n} x_{mn}^{i} y_{mn}^{j} + \xi_m \left(\sum_{n=1}^{N} P_{mn} t_n + \sum_{h^m=1}^{H^m} t_{h^m} \right)$$

$$+ \mu_m^{\xi} \left(\sum_{n=1}^{N} P_{mn} t_n^2 + \sum_{h^m=1}^{H_m} t_{h^m}^2 \right) = \sum_{h^m=1}^{H^m} \xi_{mh^m}^{c} t_{h^m}, \tag{17c}$$

$$- \sum_{n=1}^{N} P_{mn} t_n \sum_{i=0}^{I} \sum_{j=0}^{J} b_{ij}^{n} x_{mn}^{i} y_{mn}^{j} + \eta_m \left(\sum_{n=1}^{N} P_{mn} t_n + \sum_{h^m=1}^{H^m} t_{h^m} \right)$$

$$+ \mu_m^{\eta} \left(\sum_{n=1}^{N} P_{mn} t_n^2 + \sum_{h^m=1}^{H} t_{h^m}^2 \right) = \sum_{h^m=1}^{H} \eta_{mh^m}^{c} t_{h^m}, \tag{18c}$$

$m = 1, 2, 3, \ldots, M.$

This completes the analytical specification of the problem.

C. On Statistical Rigor

Suppose two qantities, u and v, are linearly related, say

$$u = kv. \tag{19a}$$

Further suppose we have P sets of values of u and v, $\{u_p, v_p\}$ with random errors ε_p in u_p. Hence, the correct equations of condition are

$$u_p = kv_p + \varepsilon_p, \; p = 1, 2, 3, \ldots, P. \tag{20}$$

We determine an unbiased estimator for k (K) by minimizing

$$S = \sum_{p=1}^{P} \varepsilon_p^2 = \sum_{p=1}^{P} (u_p - kv_p)^2 \tag{21a}$$

with respect to k. The result is

$$K_P = \sum_{p=1}^{P} u_p v_p / \sum_{q=1}^{P} v_q^2 \tag{21b}$$

$$= k + \sum_{p=1}^{P} v_p \varepsilon_p / \sum_{q=1}^{P} v_q^2. \tag{22a}$$

When performed in this fashion the full set of results available from least squares theory is available to us and complete statistical rigor is satisfied. In particular, we know that

$$\lim_{P\to\infty} K_P = k. \tag{23a}$$

Had the relationship been written as

$$v = \kappa u \tag{19b}$$

with u still subject to random error (but v not), then blindly applying least squares procedures to

$$S = \sum_{p=1}^{P} (v_p - \kappa u_p)^2 \tag{21b}$$

results in

$$\begin{aligned} 1/K_p &= \sum_{p=1}^{P} v_p u_p / \sum_{q=1}^{P} u_q^2 \\ &= \frac{k \sum_{p=1}^{P} v_p^2 + \sum_{p=1}^{P} v_p \varepsilon_p}{k^2 \sum_{q=1}^{P} v_q^2 + 2k \sum_{q=1}^{P} v_q \varepsilon_q + \sum_{q=1}^{P} \varepsilon_q^2}, \end{aligned} \tag{22b}$$

and

$$\begin{aligned} \lim_{P \to \infty} 1/K_p &= k \langle v^2 \rangle / [k^2 \langle v^2 \rangle + \langle \varepsilon^2 \rangle] \\ &\neq 1/k. \end{aligned} \tag{23b}$$

Hence, the estimator for κ is biased.

Thus, Eqs. (3, 5) are rigorous and Eqs. (12) are not. Moreover, the exact interpretation of $\{\Xi_n, H_{mn}\}$ is unclear. Unfortunately the rigorous equations yield a non-linear estimation problem for which there is little theoretical basis. In addition, the resulting normal equations are horrendously complicated. We therefore have the choice of posing (correctly) an insoluble problem or posing (incorrectly) a soluble one. We have chosen the one which is computationally easier to handle (e.g., the latter).

D. On Solving the Normal Equations

It would be cavalier of us to remark that the normal equations are linear, and, therefore, standard techniques are applicable. Since their order is $2N(I + 1)(J + 1) + 4M$ one can easily have thousands of unknowns. When the pattern is of the center-edge or center-corner type the matrix of the equations is of the "banded-bordered" type and special techniques are available. If one is willing to iterate towards a solution then the problem is even simpler. In Eqs. (17b, 17c)* first set P_{mn} and N_m equal to zero. Solve these for the first approximation $\{\xi_m, \mu_m^\xi\}$ $\forall R_m \neq 0$. We now solve Eq. (17a) with $P_{mn} \neq 0$ iff $R_m \neq 0$. This provides the first approximation to $\{a_{ij}^n\}$. We can now compute $\{\xi_{mn}\}$ for the field stars and insert these values into our simplified versions of Eqs. (17b, 17c) for <u>all</u> of the stars. When this system is solved we have obtained our second approximation to $\{\xi_m, \mu_m^\xi\}$ for all of the stars. We then return to solving for the star constants using all of the stars, etc.

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*The assumption that $a_{ij}^n \neq f(\{b_{ij}^{n'}\})$ separates ξ from η and we only consider the ξ coordinate here. The η coordinate is treated in an analogous fashion.

V. ASTRONOMICAL APPLICATIONS

The astrophysical benefits of the plate overlap technique have been noteworthy. Gatewood and his collaborators at Allegheny Observatory have rapidly applied the technique to Allegheny's extensive plate collection. Among their major contributions have been (1) a complete rediscussion of the hypothetical planet around Barnard's star, (2) a test of the theory of general relativity and stellar structure theory by astrometrically inferring the gravitational redshift of the white dwarf van Maanen 2, and (3) the beginnings of a thorough rediscussion of the anomalous multiple systems used to define the zero-age main sequence mass-luminosity relationship (9 Pup, η Cor Bor, and 10 UMa). Perhaps similar gains in our knowledge of the geopotential, the selenopotential, solar radiation forces, etc. await us.

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This report presents the development of the plate overlap technique originally suggested by H. Eichhorn. Included are discussions of standard coordinates, plate modeling, the plate overlap technique itself, and some computational details.

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