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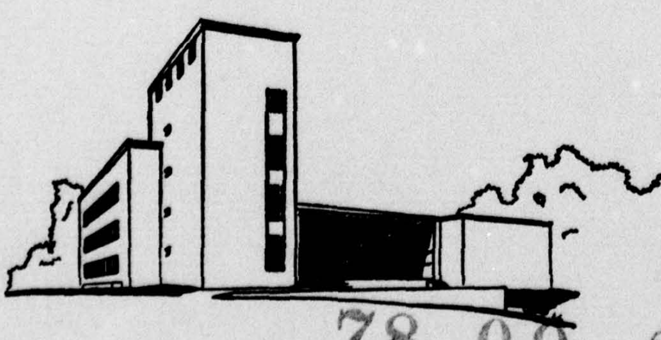

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(6) A NONLINEAR GOAL-ARC NETWORK EXTENSION OF THE MULTI-LEVEL COHERENCE MODEL FOR EEO.

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by

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KEY WORDS

Equal Employment Opportunity

Manpower Planning

Flexible Transfers

Outside Recruitment

Mathematical Programming

Goal Programming

Markoff Processes

Networks

Network Codes

Artifact Goals

Goal-Arcs

ABSTRACT

The model developed in this report is an extension and reformulation of a model called the Coherence model for guiding EEO (Equal Employment Opportunity) planning at the micro-level in the U.S. Navy's civilian workforce developed by Charnes, Cooper, Lewis and Niehaus. This model is called the Goal-Arc model.

Like its predecessors, the Goal-Arc model utilizes a goal programming approach with embedded Markoff processes. As in the Coherence model, piecewise linear goal functionals with "artifact goals" are used to approximate the transition relations of the Markoff process. The Goal-Arc model, however, carries this to another stage of development. Analytical as well as network formations and interpretations are provided in the following article. A numerical example with related interpretations for EEO planning is also provided.

Introduction

The Multi-Level Coherence Model for Equal Employment Opportunities (EEO) planning of Charnes, Cooper, Lewis and Niehaus (see [5]) was developed in a dyadic format. It was less general in its development, however, than might be required for some cases. For many applications, recourse is needed to large scale highly efficient network codes such as PNET[1/] (which the U.S. Navy Office of Civilian Personnel (OCP) has) which can readily handle multiple arcs between nodes as well as lower and upper bounds on arc flows. The new, more general, nonlinear goal-arc network model discussed in this paper was therefore developed to exploit these and other possibilities.

[1/] See [7] and [8]. Other codes, such as GNET, are discussed in [1] and [2]. See also [9].

In past research papers (Cass, Charnes, Cooper, Niehaus [3]) goal programming models of distribution (or assignment) type have been reformed into equivalent models of distribution type. The "MEEO" -- Multi-Level EEO -- model was the first to approximate Markoff transition constraints by "goal artifacts" which replaced the constraints by goals with convex goal functionals on certain dyadic cell elements. Here we use the analogous device for networks: the cells with nonlinear goal functions are replaced by arcs with goal functionals which we shall now call "goal-arcs." The network format with "transhipment" nodes which we shall now introduce allows us to simplify by dispensing with the transshipment elaboration and the extra rows and columns this required in the dyadic format. Finally, to fit the data format of the PNET code a "supersource" and "supersink" is introduced which connect to arcs whose bounds replace the influxes and effuxes in other model elaborations.

In summary, this is part of a continuing evolution involving an interplay between practically oriented implementations and research which started at the U.S. Navy, as in [6], with a combined goal-programming Markoff process model for joining EEO with civilian manpower planning in terms of targeted goal for each of them. This was followed by the Coherence (= MEEO) model which, as already noted, was the first to approximate Markoff transition constraints with "goal artifacts" in a suitable dyadic formulation. See [5]. Now we replace the latter with a network model with transshipment nodes that make it possible to obtain access to currently available ultra high speed computer codes. This, in turn, should make it possible to provide interactive computer capabilities, if desired, by means of which manpower-EEO planning can be directly integrated into management decision making instead of being confined for separate processing by customary personnel department specializations.

The way in which this network formulation is achieved and some of the uses to which it can (and will) be put are described in the sections that follow. First we shall describe some of the node-arc conventions we employ. Then we shall provide an analytic characterization. The network representation developed from this analytic model will then be depicted and the goal arc decompositions described to show how the convex functional elements are accommodated.

This will be followed by a numerical example which will be similarly developed and interpreted. The resulting solutions will be portrayed in the form of reports for possible managerial use that will help to point up some of these possibilities via the prototype (toy) example we shall be employing. This will be followed by a Summary and Conclusion section that will also suggest some possibilities for further research.

The Goal-Arc Model

We shall describe our transfers in terms of flows on several types of arcs between several types of nodes:

(1) To each job in each period we assign two nodes, an "antecedent" and a "consequent." We also designate as "job" nodes those corresponding to outside sources for recruitment (antecedents) and outside involuntary retirements (consequents). We also designate "job nodes" for normal organizational attrition (consequents). We designate the class of antecedent "job" nodes for period t as $J^-(t)$; the class of consequent "job" nodes by $J^+(t)$. $J_i^-(t)$ is the i^{th} job antecedent node; $J_j^+(t)$ is the j^{th} job consequent node.

(2) For each proper (real) job between two periods we designate a "valve" node to receive the goal arc flow from the consequent node of the immediate past period and to transmit an upper and lower bounded flow to the next period antecedent node. We let $V_i(t)$ denote the valve node for job i between periods t-1 and t.

(3) A supersource node, S_o, and a supersink node, S_{n+1}, are added for PNET code purposes. The supersink node is connected back to the supersource node. Thereby every node becomes a transhipment node.

The flow on every arc is unidirectional. The arcs may be "goal" arcs (with a nonlinear goal functional) involving multiple arcs between the same two nodes, or they may be simple arcs. Every simple arc (or individual arc of multiple arcs) may have an upper and a lower bound on its flow.

Let $x_{ij}^{k}(t)$ denote the flow from node $J_i^-(t)$ to node $J_j^+(t)$ on the k^{th} individual arc of a multiple "goal arc." The corresponding lower and upper bounds are $L_{ij}^{k}(t)$ and $U_{ij}^{k}(t)$.

Let x_{oi} denote the flow from the supersource to $J_i^-(1)$. Let $x_{i\ n+1}$ denote the flow from $J_i^+(n)$ to the supersink. Let $x_{n+1\ o}$ denote the flow from the supersink to the supersource.

Let $y_i^k(t)$ denote the flow on arc k of the goal-arc between $J_i^+(t-1)$ and $V_i(t)$. The corresponding upper and lower bounds are $L_i^k(t)$ and $U_i^k(t)$. Let $\bar{y}_i(t)$ denote the flow on the "valve" arc between $V_i(t)$ and $J_i^-(t)$.

The network node conditions may now be written explicitly:

(1) for supersource

$$x_{n+1\ o} - \sum_{i \varepsilon J^-(1)} x_{oi} = 0$$

(2) for $J^-(1)$

$$x_{oi} - \sum_{j \varepsilon J^+(1)} \sum_{k} x_{ij}^{k}(1) = 0$$

(3) for $J^+(1)$

$$\sum_{k} \sum_{i \varepsilon J^-(1)} x_{ij}^{k}(1) - \sum_{r} y_j^r = 0, \quad j \neq j_o$$

Where j_o is the "outside" node,

$$x_{oj_o} + \sum_k \sum_{i\epsilon J^-(1)} x^k_{ij_o}(1) - \sum_r y^r_{j_o}(1) = 0 .$$

Note that there is never flow from the "outside" node $J^-_{j_o}$ (1) to the natural attrition node $J_{i_o}{}^+(t)$. We also have

(4) for $V_i(t)$

$$\sum_k y^k_i(t) - \overline{y}_i(t) \qquad = 0$$

(5) for $J_i{}^-(t)$, t>1

$$\overline{y}_i(t) - \sum_k \sum_j x^k_{ij}(t) \qquad = 0$$

(6) for $J_j{}^+(t)$, t>1

$$\sum_k \sum_{i\epsilon J^-(t)} x^k_{ij} - \sum_r y^r_j(t) \qquad = 0$$

(7) for supersink S_{n+1}

$$\sum_t y_{i_o}(t) + \sum_{i\epsilon J^+(n)} x_{i\ n+1} - x_{n+1 o} = 0 .$$

We will now completely describe the Goal-Arc Model.

$$\text{Min} \left[\sum_{\substack{i,j,k,t \\ i \neq i_o}} c_{ij}^{k} \, x_{ij}^{k}(t) + \sum_{\substack{i,k,t \\ i \neq i_o, j_o}} d_i^k \, y_i^k(t) \right]$$

Subject to (1) - (7) above, and

$$L_{ij}^{k}(t) \leq x_{ij}^{k} \leq U_{ij}^{k}(t),$$

$$L_i^k(t) \leq y_i^k(t) \leq U_i^k(t),$$

where the $L_{ij}^{k}(t)$, $U_{ij}^{k}(t)$ and the $L_i^k(t)$, $U_i^k(t)$ are such that the

$x_{ij}^{k}(t)$, $y_i^k(t)$, $\bar{y}_i(t)$ are non-negative for all i, j, k and t.

An illustration of the Goal-Arc Model is given in Figure 1 for n time periods and m+2 job categories. S_o is the supersource node introduced on the left and S_{n+1} is the supersink node introduced on the right. In the diagram the antecedents and the consequents of the outside node are represented by $J_{m+1}^{-}(t) = J_{j_o}^{-}(t)$, $J_{m+1}^{+}(t) = J_{j_o}^{+}(t)$, and $J_{m+2}^{+}(t) = J_{i_o}^{+}(t)$.

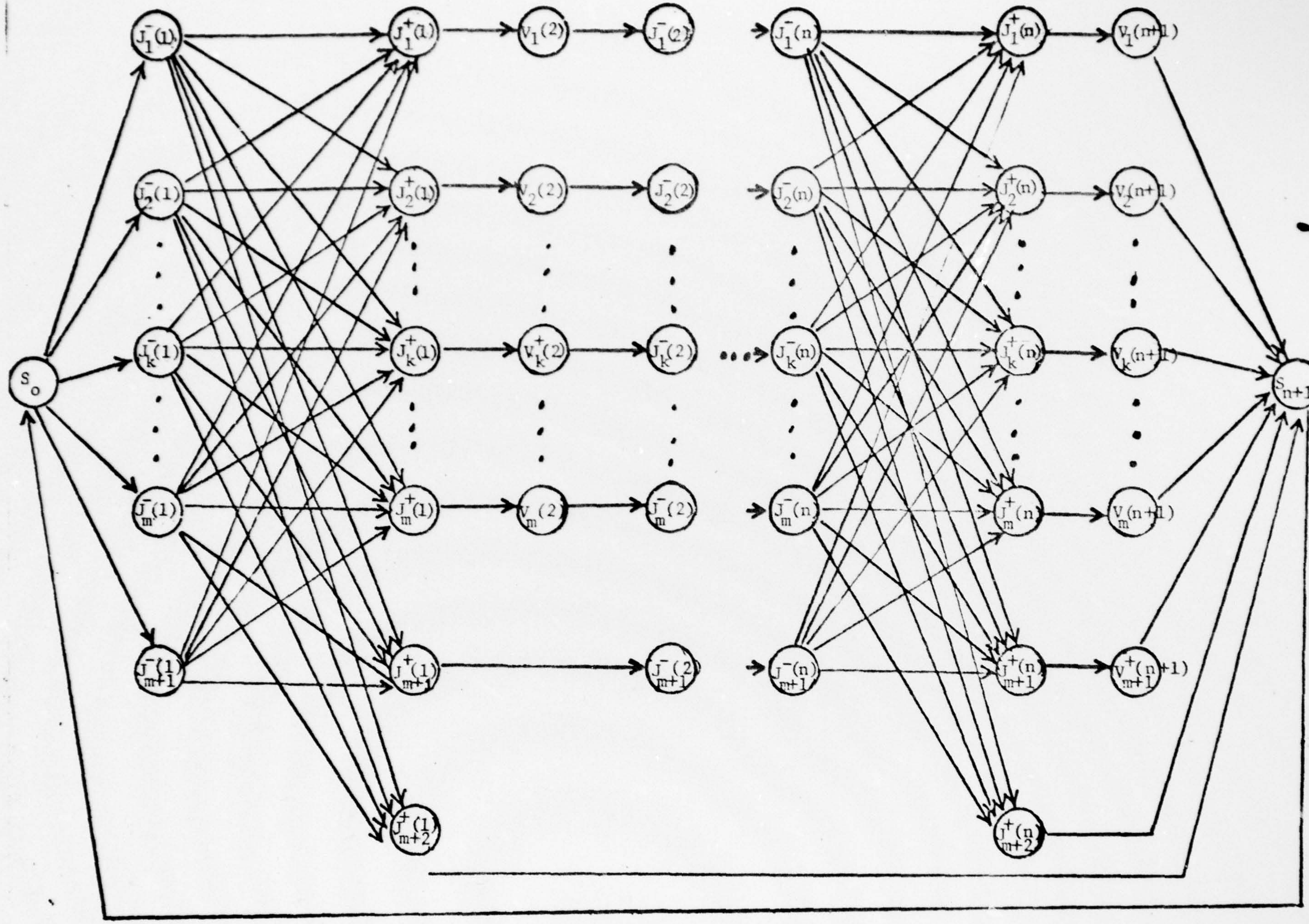


FIGURE 1: GOAL ARC NETWORK MODEL

Some of the arcs represent natural flows and some may be goal arcs. Recall that the purpose of each of the goal arcs is to represent a nonlinear goal functional element. To represent these piecewise linear (nonlinear) goal elements we can replace each goal arc by multiple capacitated arcs between the same two nodes.[1/]

An illustration is supplied in Figure 2. The arc G between nodes N_1 and N_2 is a goal arc. This is indicated by the symbol which we have omitted from these links in Figure 1 to avoid further cluttering of the diagram.

The lower portion of Figure 2 shows the decomposition. The flow z on G is broken up into flows z^k on G^k where $\sum_k z^k = z$. Each z^k is a bounded variable. Further we let c^k be the slope assigned for the flow z^k. Thus, the decomposition of the piecewise linear representation of the nonlinear functional on the goal arc is accomplished. The single arc with nonlinear functional between N_1 and N_2 is replaced by a finite number of arcs with linear functionals on each.

1/ For further detailed development of the underlying theory see Charnes and Cooper [4] Chapter XVII.

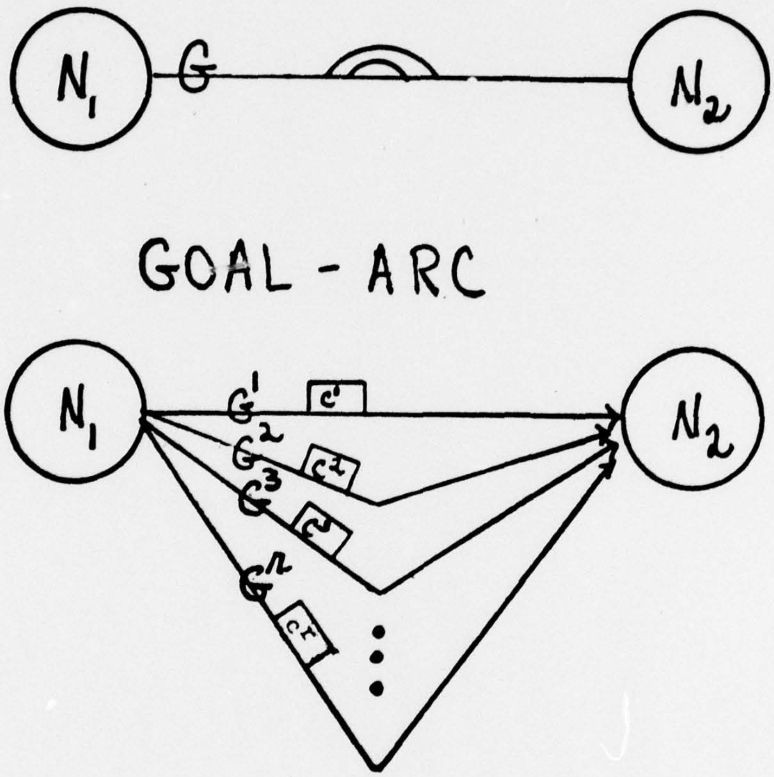


FIGURE 2

GOAL-ARC

AS

MULTIPLE ARCS

Numerical Illustration

In order to make the preceding development more concrete, we will now consider a numerical example. The problem that we will consider is the problem considered by Charnes, Cooper, Lewis and Niehaus.[1/]

Let there be two categories of personnel $\alpha = 1, 2$ (e.g., female and male) and three time periods, $t = 0, 1, 2$. For job categories we shall use the following:

i, j	Description	Abbreviation
0	Outside Source	O
1	Clerical	C
2	Technical	T
3	Administrative	A
4	Natural Attrition	N

Figure 3 provides targeted workforce goals $a_i(t)$ where $i = 1, 2, 3$ for the associated job category in each of the periods $t = 0, 1, 2$. Figure 4 provides a matrix of transition probabilities which is assumed to be applicable over these periods. Recall that N refers to natural attrition so that, e.g., there is 0.26 probability that clerical personnel will leave the organization in going from one period to another.

[1/] See [5].

i \ t	0	1	2
C	675	700	650
T	875	450	400
A	225	200	200

FIGURE 3

TARGETED WORKFORCE GOALS, $a_i(t)$.

FROM \ TO	N	C	T	A
C	.26	.7	.03	.01
T	.15	0	.8	.05
A	.13	0	.02	.85

FIGURE 4

EXAMPLE MARKOFF MATRIX

In Figure 5 the actual p_i^α proportions of personnel in each job category for the initial time period and the desired p_i^α proportion of personnel in each job category for future time periods are given. The actual proportions are obtained from the "on board" starting population. The desired proportions represent policy statements concerning the desired mix of personnel for the future.

Figure 6 provides the desired number of personnel of type $\alpha=1$ (female) for each job category in each period. These values are obtained from Figure 3 in the following manner. Let $b_i(t) = \left\langle p_i^1 a_i(t) \right\rangle$ where $\langle u \rangle$ is the smallest integer not less than u. Thus, e.g., in Figure 6 525 = .75 x 700 in the row for C where it intersects the column captioned "1" is obtained from the data of Figures 3 and 5.

		C	T	A
Actual Proportions	Female	.89	.20	.40
	Male	.11	.80	.60
Desired Proportions	Female	.75	.35	.45
	Male	.25	.65	.55

FIGURE 5

EXAMPLE OF PERSONNEL - JOB PROPORTIONS, p_i^α

i \ t	0	1	2
C	600	525	488
T	175	158	140
A	90	90	90
N		193	173

FIGURE 6

TARGETED FEMALE WORKFORCE GOALS, $b_i(t)$

In Figures 7 and 8 the "artifact goals" are given for each of the two periods as indicated in the titles of these Figures. The "artifact goals" are defined by $g_{ij}^{\alpha}(t) = \left\langle p_i^{\alpha} a_i(t-1) M_{ij} \right\rangle$ where M_{ij} is the i,j^{th} element of the Markoff matrix M. In this example we are confining our attention to $\alpha=1$ and so we can let $g_{ij}^{1}(t) = g_{ij}(t)$ without ambiguity.

Similarly let $x_{ij}(t)$ equal the number of females ($\alpha=1$) transferred from job category i to job category j in period t and let $y_j(t)$ represent the total number of females in job category j in period t. In this model the $y_j(t)$ and the $x_{ij}(t)$ are to conform "as close as possible" to the targeted workforce goals and the "artifact goals" respectively.

TO / FROM	N	C	T	A
C	156	420	18	6
T	25		140	9
A	12		2	76

FIGURE 7

ARTIFACT GOALS FOR THE FIRST PERIOD

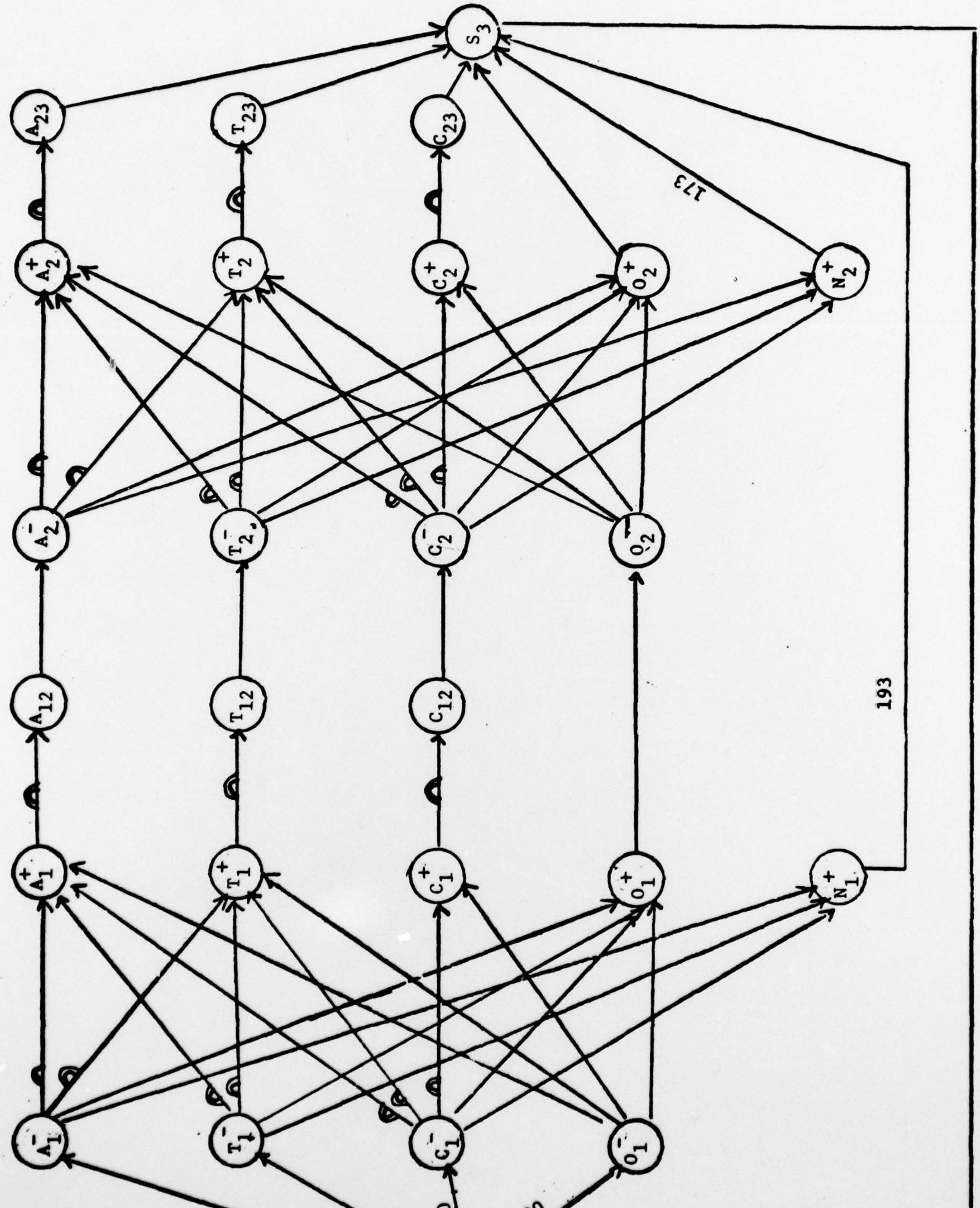
TO / FROM	N	C	T	A
C	135	368	15	5
T	24		126	8
A	12		2	76

FIGURE 8

ARTIFACT GOALS FOR THE SECOND PERIOD

<u>Reduction to Network Format</u>

We now formulate this as a network problem. This is shown graphically in Figure 9. In this example J_i^+, $J_i^-(t) = K_t^-$ and $V_i(t) = K_{(t-1)}$ where K is an abbreviation for "job category." Here, of course, K takes on the values A, C, T, N, O. As already noted, the symbol on an arc indicates that it is a "goal arc." Upper and lower bounds for the flow on the "valve" arcs are set, respectively, at the projected manpower requirements plus ten per cent of the requirements and minus ten per cent of the requirements.


90
175
600
500
193
173

In this example we will employ only two pieces in our piecewise linear goal functional, i.e., $k = 2$. Hence the decomposition on a "goal arc" is performed as described earlier with $k = 2$. We will now examine the decomposition of "goal arcs" in this example.

Consider any goal arc in Figure 9 between a K_t^- and a K_t^+. We replace this arc in Figure 10 with two arcs, say, $G_{ij}^k(t)$, where $k = 1$ or 2. Let $x_{ij}^k(t)$ denote the corresponding flows. These flows are bounded as follows: $0 \leq x_{ij}^1(t) \leq g_{ij}(t)$ and $0 \leq x_{ij}^2(t) < \infty$. Let c^k denote the functional coefficient on $G_{ij}^k(t)$. We assume that $c^1 < c^2$. In an optimal solution there will be no flow on $G_{ij}^2(t)$ until the flow on $G_{ij}^1(t)$ has reached $g_{ij}(t)$.

Now consider a goal arc between nodes K_t^+ and K_{t+1}. As above, we replace this arc with two arcs, $G_i^1(t)$ and $G_i^2(t)$. Let $y_i^k(t)$ denote the flow on $G_i^k(t)$. The flows on the two arcs are bounded as follows: $0 \leq y_i^1(t) \leq b_i(t)$ and $0 \leq y_i^2(t) < \infty$. Let d^k denote the functional coefficient for the flow on $G_i^k(t)$. We assume that $d^1 < d^2$.

Proceeding in this manner the problem is represented as a network with the "goal arcs" decomposed as in Figure 10.

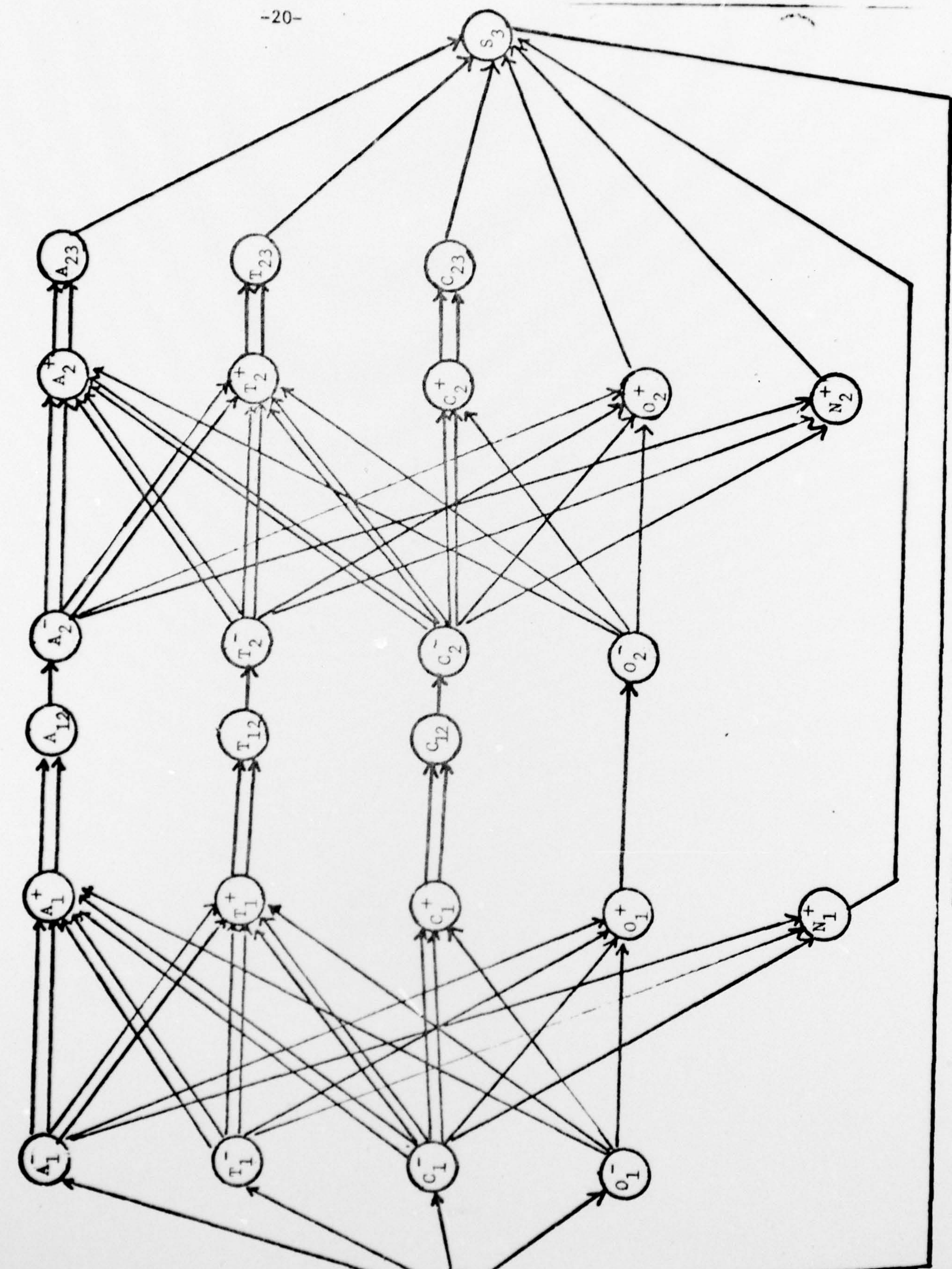


FIGURE 10: NETWORK WITH GOAL-ARCS DECOMPOSED

Since the objective function is to be minimized, a high positive value for the functional coefficient on an arc tends to make the resistance to flow on that arc high. In our penalty system the following priorities are established: Meeting the goal of a certain number of female personnel for each job category in each time period is given the higest priority. Firing is highly discouraged. Flexible movement has the second highest priority. The penalty on exceeding manpower requirements is greater than any other penalty except the penalty on firing. The penalty for hiring in the first period is greater than the penalty for hiring in the second period. The penalty on hiring is less than the penalty on exceeding manpower but greater than the penalty on flexible movement. The penalty on firing is set at an order of magnitude larger than the sum of all other weights.

The values for the functional coefficients on the arcs (with relevant interpretations) are given as follows:

H = hiring penalty = 5;

P = penalty on flexible movement = 2;

R = firing penalty = 1,000;

G = penalty on expected movement = -1;

Q = penalty on meeting manpower requirements = -6;

F = penalty on exceeding manpower requirements = 10.

The solution is summarized in four tables as follows: The projected personnel transfers for periods 1 and 2 are given in Tables 1 and 2, respectively. The 424 under "Normal ± Flexible" in row 1 of

Table 1 represents the planned retention of females in the clerical job category in the first time period. It is composed of 420 females via normal retention plus 4 more as a part of an optimum managerial plan to alter the present composition of the organization. The total of 525 females at the bottom of this column is to be obtained by recruiting an additional 101 females from outside the organization. Table 2 is similarly interpreted for the second time period.

PROJECTED PERSONNEL TRANSFERS
FEMALE

JOB CATEGORY	Number Aboard at Period (0)	CLERICAL		TECHNICAL		ADMINISTRATIVE		OUTSIDE LOSSES	
		Normal	Normal ± Flexible	Normal	Normal ± Flexible	Normal	Normal ± Flexible	Natural	Fires
Clerical	600	420	424	18	15	6	5	156	
Technical	175			140	141	9	9	25	
Administrative	90			2	2	76	76	12	
Hires			101						
Number Aboard at Period (1)			525		158		90		

TABLE 1

JOB CATEGORY	Number Aboard at Period (0)	CLERICAL		TECHNICAL		ADMINISTRATIVE		OUTSIDE LOSSES	
		Normal	Normal ± Flexible	Normal	Normal ± Flexible	Normal	Normal ± Flexible	Natural	Fires
Clerical	525	368	370	15	13	5	5	137	
Technical	158			126	126	8	8	24	
Administrative	90			2	1	76	77	12	
Hires			118						
Number Aboard at Period (2)			488		140		90		

TABLE 2

Table 3 compares workforce requirements and the optimal distribution from the model -- e.g., targeted workforce goals and optimal "aboards." The discrepancies between the two are given in the last column of Table 3. All discrepancies are at zero value which means that the optimum program achieves all of the indicated targets.

FEMALE

	On-Board	Hires	Fires	Workload Requirement	Discrepancy *
PERIOD 0:	Actual				
Clerical	600				
Technical	175				
Administrative					
PERIOD 1:	Projected				
Clerical	525	101		525	
Technical	158			158	
Administrative	90			90	
PERIOD 2:					
Clerical	488	118		488	
Technical	140			140	
Administrative	90			90	

*None: All goals attained.

WORKFORCE REQUIREMENTS AND DISTRIBUTIONS

TABLE 3

Table 4 is a summary of the personnel actions projected by the optimum plan. For example, 420 normal transfers plus 4 additional (flexible) transfers and 101 hires are projected for the clerical category in Period 1 and 368 normal transfers, 2 additional (flexible) transfers and 118 hires in period 2.

Summary and Conclusion

This concludes the present paper, but the above developments are a continuation of research in a series dealing with modeling for EEO planning. The first in this series of models was the FEEO model which provides for EEO planning at the macro-level. See [6]. The next in the series was the MEEO model. Also called the "Coherence Model," the MEEO model was developed to provide for EEO planning at the micro-level, e.g., at the activity level, which would be "coherent with" the FEEO model. For further discussion see [5].

The model developed above is an extension and reformulation of the MEEO model. As such we have a continuing evolution in a modeling strategy. The problem which was originally formulated as a capacitated distribution problem with "artifact goals" is now reformulated as a network problem with goal arcs. Thus we have alternate models for this same class of problems. The development portrayed in this paper was undertaken to take advantage of large capacity, fast and highly efficient network

	PERIOD 1						PERIOD 2				
CATEGORY	Number Aboard at Period (0)	Normal Trans.	Flexible Trans.	Hires	Fires	Aboard End Period (1)	Normal Trans.	Flexible Trans.	Hires	Fires	Aboard End Period (2)
Clerical	600	420	4	101		525	368	+2	118		488
Technical	175	160	-2			158	143	-3			140
Admin.	90	91	-1			90	89	+1			90

SUMMARY OF PROJECTED PERSONNEL ACTIONS

TABLE 4

codes such as PNET. See [7] and [8].[1/] Also in the model developed above the transhipment characteristics provide much greater convenience, simplicity and flexibility in representing desired personnel flows. In the MEEO model the dyadic character required special devices and redundant representation.

The Goal-Arc model of this paper currently handles the ethnosexual categories one at a time. This is done via the proportionate reduction devices described in the above paper. The next step in this ongoing research should develop a method for handling all of the ethnosexual categories simultaneously. This and other parts of this work in EEO modeling will be reported in subsequent papers of this series.

[1/] See also [1] and [2] and [9].

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