AD-A059 604 NAVAL POSTGRADUATE SCHOOL MONTEREY CALIF F/G 12/1

AN EVALUATION OF THREE RELIABILITY GROWTH MODELS.(U)

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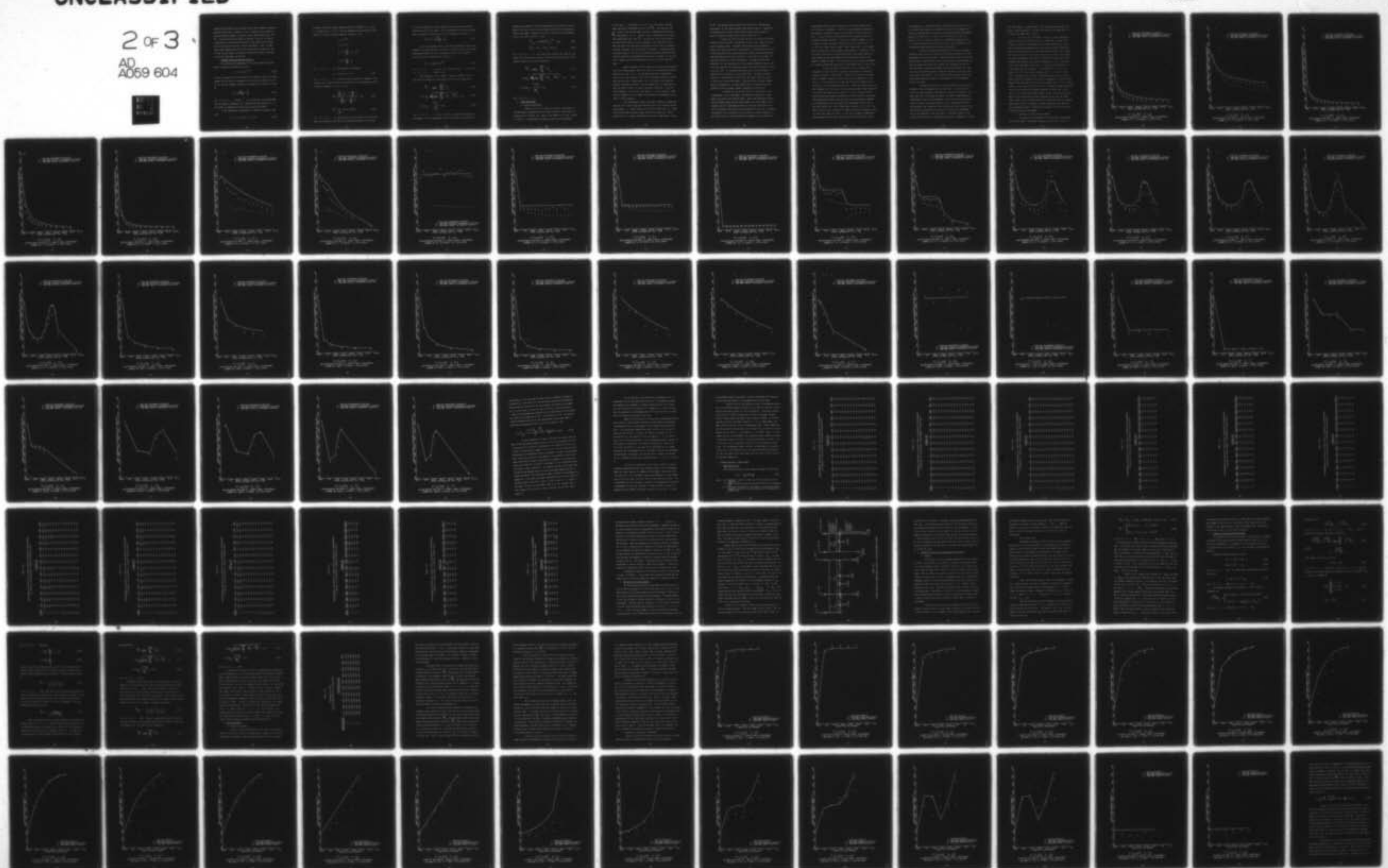

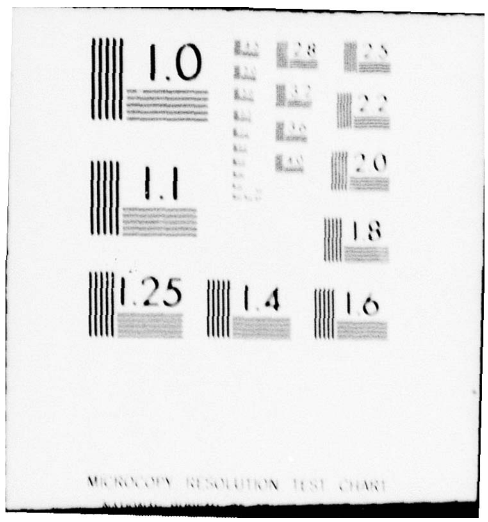
1.0
2.8
2.5
3.2
2.2
3.6
4.0
2.0
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1.8
1.25
1.4
1.6
MICROCOPY RESOLUTION TEST CHART

evaluating both models were collected from a single computer simulation program as described in chapter III.A.3. Features of the reliability testing procedure computer simulation such as phase planned test time PTT_i determination and assignment of one-half a failure in those instances when no failures occurred during a test phase were in affect for the evaluation of both the failure rate models. Also, the same twenty-eight lambda sets (failure rate sets) given in tables 3.1 and 3.2 and the same number of simulation replications (NSIMS = 100) were utilized for both model evaluations.

4. Computer Simulation Data Manipulation

The continuous instantaneous failure rate reliability growth model of equation 3.37 may be written as

$$\lambda_i = (1-a_i)b_i(TT_i)^{-a_i} . \tag{3.38}$$

As noted in equation 3.16 an estimate of the unknown, underlying instantaneous failure rate of a component for any given test phase of a reliability testing procedure corrected for planned test time bias is given by

$$\widehat{\lambda_i} = \frac{2NT_i}{1 + 2NT_i} \cdot \frac{F_i}{T_i} \tag{3.39}$$

for i = 1, 2, 3, ... , K where T_i is the total test time accumulated for the number of components NT_i tested during the phase and F_i is the number of component failures occurring during the phase.

If the logarithmic transformation is applied to equation 3.38, then

$$\ln \lambda_i = \ln (1-a_i)b_i - a_i \ln TT_i . \tag{3.40}$$

To obtain the ordinary least squares regression estimates for a_i and b_i the data pairs $(\ln \widehat{\lambda}_i, \ln TT_i)$ are employed from the results of the reliability testing procedure computer simulation. Let

$$Y_i = \ln \widehat{\lambda}_i ,$$

$$X_i = \ln TT_i ,$$

$$\bar{Y}_i = \frac{1}{i} \sum_{j=1}^{i} Y_j , \text{ and}$$

$$\bar{X}_i = \frac{1}{i} \sum_{j=1}^{i} X_j$$

for $i = 1, 2, 3, \ldots, K$. Then equation 3.40 becomes

$$Y_i = \ln b_i(1-a_i) - a_i X_i \qquad (3.41)$$

for $i = 1, 2, 3, \ldots, K$. By the derivation presented in Appendix A the ordinary least squares regression estimates for the instantaneous failure rate model parameters a_i and b_i are

$$\widehat{a}_i = \frac{\sum_{j=1}^{i} X_j Y_j - \bar{Y}_i \sum_{j=1}^{i} X_j}{\bar{X}_i \sum_{j=1}^{i} X_j - \sum_{j=1}^{i} X_j^2} \text{ and} \qquad (3.42)$$

$$\widehat{b}_i = \frac{1}{1-\widehat{a}_i} \exp(\bar{Y}_i + \widehat{a}_i \bar{X}_i) \qquad (3.43)$$

for $i = 2, 3, 4, \ldots, K$. Note that as with the cumulative failure rate model the regression requires at a minimum observations on the results of two test phases; thus, model parameter estimates are made for the second thru the K^{th} test phase. The instantaneous failure rate estimate given by equation 3.39 was used for the first phase of testing; i.e.,

$$\hat{\lambda}_{T_1} = \hat{\lambda}_1 = \frac{2NT_1}{1 + 2NT_1} \cdot \frac{F_1}{T_1} . \tag{3.44}$$

As the instantaneous failure rate model parameter estimates were obtained, they were applied to the model in equation 3.38 to obtain model determined estimates for the instantaneous failure rate reliability status at the end of each phase of testing; i.e.,

$$\hat{\lambda}_{T_i} = (1-\hat{a}_i)\hat{b}_i(TT_i)^{-\hat{a}_i} \tag{3.45}$$

for i = 2, 3, 4, ... , K. Again, the simulation index r is left implicit (r = 1, 2, 3, ... , NSIMS = 100).

Upon completion of all computer simulations (NSIMS = 100) for each specified lambda set, performance statistics were computed as

$$\overline{\hat{\lambda}}_{T_i} = \frac{1}{NSIMS} \sum_{r=1}^{NSIMS} \hat{\lambda}_{T_{i,r}} , \tag{3.46}$$

$$S.D._{\hat{\lambda}_{T_i}} = \sqrt{\frac{1}{NSIMS-1} \sum_{r=1}^{NSIMS} (\hat{\lambda}_{T_{i,r}} - \overline{\hat{\lambda}}_{T_i})^2} , \text{ and} \tag{3.47}$$

$$P.S.E._{\hat{\lambda}_{T_i}} = \frac{S.D._{\hat{\lambda}_{T_i}}}{\overline{\hat{\lambda}}_{T_i}} \times 100 \tag{3.48}$$

for i = 1, 2, 3, ... , K. Also, in order to evaluate the forecasting performance of the instantaneous failure rate model the same method of computing an estimate of the total accumulated test time for a one test phase in the future failure rate estimate utilized for the cumulative failure rate model in equation 3.31 was utilized for the instantaneous failure rate model forecasts. Hence,

$$\widehat{F\lambda}_{T_{i+1}} = (1-\hat{a}_i)\hat{b}_i(\widehat{TT}_{i+1})^{-\hat{a}_i} \quad \text{where} \tag{3.49}$$

$$\widehat{TT}_{i+1} = TT_i + (NT_{i+1} \times PTT_{i+1}) \tag{3.50}$$

for i = 2, 3, 4, ... , K. Note again that forecasts were made for test phases 3 thru K as was the case with the cumulative failure rate model. Forecast statistics were then computed as

$$\overline{\widehat{F\lambda}}_{T_i} = \frac{1}{NSIMS} \sum_{r=1}^{NSIMS} \widehat{F\lambda}_{T_{i,r}}, \tag{3.51}$$

$$S.D._{\widehat{F\lambda}_{T_i}} = \sqrt{\frac{1}{NSIMS-1} \sum_{r=1}^{NSIMS} (\widehat{F\lambda}_{T_{i,r}} - \overline{\widehat{F\lambda}}_{T_i})^2}, \text{ and} \tag{3.52}$$

$$P.S.E._{\widehat{F\lambda}_{T_i}} = \frac{S.D._{\widehat{F\lambda}_{T_i}}}{\overline{\widehat{F\lambda}}_{T_i}} \times 100 \tag{3.53}$$

for i = 3, 4, 5, ... , K.

5. Model Performance

a. Accuracy Performance

Figures 3.42 thru 3.77 depict the accuracy performance of the continuous instantaneous failure rate reliability growth model for selected tests per phase NT_i cases of the lambda sets listed in tables 3.1 and 3.2. In each graph the specified underlying instantaneous failure rate λ_i from table 3.1 or 3.2 (——, solid line), the mean model determined instantaneous failure rate $\overline{\widehat{\lambda}}_{T_i}$ from equation 3.46 (⊙ , circles), and the mean model forecast instantaneous failure rate $\overline{\widehat{F\lambda}}_{T_i}$ from equation 3.51 (X , crosses) are all plotted against the mean total accumulated test time $\overline{TT}_i$ from equation 3.29 for each phase of the specified reliability testing procedure of the acquisition cycle. While the specified underlying instantaneous failure rate is plotted as a smooth, continuous line for purposes of contrast, it should be noted from both figures 3.1 and 3.2 and the testing procedure design that the specified underlying instantaneous failure rate is actually a step function in shape with discontinuities occurring at the end of each test phase.

Examining figure 3.47 for example at a mean total accumulated test time of approximately 205.0 test time units the specified instantaneous failure rate is 0.30, the mean model determined instantaneous failure rate is approximately 0.24, and the mean model forecast instantaneous failure rate is approximately 0.19. As noted on the graph the accuracy performance plotted is for the lambda set 6, 16 test phase, 20 tests per phase reliability testing procedure simulation. Since the point examined is for the last test phase ($i = K = 16$), table 3.1 shows that for lambda set 6, test phase 16 the specified instantaneous failure rate is 0.30 as expected.

The instantaneous failure rate model frequently yielded mean forecast failure rates that were "off-the-scale" of the accuracy performance graphs; i.e., mean forecast failure rates greater than 1.0. These off-the-scale mean estimates ranged from failure rates only slightly greater than 1.0 to mean estimated failure rates of magnitudes in excess of 10^6. The extreme values occurred only for the first forecast and occasionally for the second forecast (test phases 3 and 4). Regardless of the magnitude of these off-the-scale forecast failure rates, all points of magnitude greater than 1.0 were plotted at 1.0.

Figures 3.42 thru 3.46 and 3.60 thru 3.64 display the instantaneous failure rate model's performance for the "nice" underlying failure rate progress paths. The model, while discerning the shape of these "nice" underlying failure rate patterns, is consistently optimistic in both determining the current failure rate status and forecasting the next test phase failure rate, especially in the case of the longer sixteen phase reliability testing procedures. The model performs more accurately for the more volatile six test phase procedures (figures 3.60 thru 3.64) than for the protracted sixteen test phase procedures (figures 3.42 thru 3.46). In fact mean determined failure rate performance is excellent for the shorter six test phase cases. Forecasting accuracy typically is off-the-scale for the first test phase improving rapidly by the third or fourth test phase. Once stabilized, the mean forecast failure rate generally is biased optimistically as expected from the method utilized to produce forecasts (equations 3.49 and 3.50).

In figures 3.47, 3.48, and 3.65 thru 3.67 accuracy of the instantaneous failure rate model is displayed for essentially linear underlying failure rate progress paths. Again, mean determined failure rate performance shows that the model detects the linear shape of the progress path with reasonable accuracy but provides consistently optimistic estimates of the underlying instantaneous failure rate. Forecasting accuracy, while consistent in the sixteen test phase procedures, is very poor being only around 50%-60% of the magnitude of the underlying instantaneous failure rate for the majority of the test phases until the final one or two phases. For the six phase procedures forecasting performance is extremely erratic as can be seen in figures 3.65 thru 3.67. In fact forecasting performance actually degrades as more test information becomes available in the case of lambda set MOD6; i.e., as the number of tests per phase increases from five to twenty ($NT_i = 5$ vs. $NT_i = 20$, figure 3.65 vs. 3.66). Also, figure 3.67 is for a twenty tests per phase ($NT_i = 20$) procedure; and therefore, in keeping with the convention established for presenting accuracy performance graphs this figure shows the best performance of the instantaneous failure rate model for lambda set MOD7. The five and ten tests per phase performance for lambda set MOD7 were worse than the performance shown in figure 3.67.

Figures 3.49 thru 3.52 and 3.69 thru 3.17 portray the instantaneous failure rate model's accuracy performance for permanently stagnated reliability status cases. The most vivid contrast for these cases is between situations in which failure rate stagnates at a high value ($\lambda_i = 0.70$; figures 3.49, 3.68, and 3.69) versus stagnation at a low rate ($\lambda_i = 0.05$; figures 3.52 and 3.71). When the underlying failure rate stagnates at a high value, the instantaneous model's mean determined failure rate estimates are very erratic ($\lambda_i = 0.70$; figures 3.49 and 3.68) and the mean forecast estimates are virtually useless from a magnitude consideration being, even once stabilized, approximately 40%-50% of the magnitude of the true underlying instantaneous failure rates. Note that figure 3.49 is for a "best case" situation of $NT_i = 20$. Forecasts at best give an indication of stagnation when the stagnation occurs at a high true underlying value. In the case of lambda set MOD8 determined estimated failure rate improves significantly when more testing is performed; i.e., figure 3.69 versus figure 3.68 (NT_i = 20 vs. NT_i = 5). On the other hand, when the true underlying failure rate stagnates at a low point, both the model's determined and forecast accuracy performance appear to be excellent (λ_i = 0.05, figures 3.52 and 3.71, NT_i = 5). Figures 3.50, 3.51, and 3.70 show for an intermediate stagnation level (λ_i = 0.30) how determined and forecast failure rate performance of the instantaneous model transitions from poor to excellent.

The contrast between the underlying failure rate permanent stagnation cases points out a general performance characteristic of the instantaneous failure rate model. Because the model produces consistent optimistically biased determined and forecast failure rate estimates, as the true underlying failure rate progress path approaches low failure rates the model's accuracy performance improves significantly by necessity since the estimates become "sandwiched" between the low true underlying failure rate and 0.0. This "closing of the brackets" effect may also produce favorable variability performance for the instantaneous failure rate model.

Figures 3.53, 3.54, 3.72, and 3.73 present the accuracy performance of the instantaneous failure rate model in the case of true reliability status progress interrupted by a period of stagnation. Only "best case" (NT_i = 20) graphs are shown. The model's mean determined instantaneous failure rate performance is fair for these cases and continues to exhibit consistent optimistic bias. Mean forecast failure rates generally are indicative of the underlying failure rate path trend only and are significantly off the mark in forecasting instantaneous failure rate magnitude. The "kick-down" in the mean forecast of the failure rate for the first phase after the period of stagnation in figure

3.53 (test phase 11 at approximately 130.0 total accumulated test time units) is more pronounced in the five and ten tests per phase (NT_i = 5, 10) cases for lambda sets 11 and 12.

For situations of temporary reliability status degradation figures 3.55 thru 3.59 and 3.74 thru 3.77 reveal that in these "worst situation" cases the instantaneous failure model performs surprisingly well in discerning the shape of the true underlying failure rate path, especially for the twenty tests per phase procedures (NT_i = 20; figures 3.57, 3.59, 3.75, and 3.77). During the period of failure rate degradation, the instantaneous model tends to produce determined failure rate estimates that overstate the degree of degradation. This is especially true in the "least data" cases (NT_i = 5, figures 3.55 and 3.58) of the sixteen phase testing procedures. This characteristic is not as pronounced for the six test phase procedures (NT_i = 5, figures 3.74 and 3.76). The model's mean forecast instantaneous failure rate estimates remain optimistic throughout the periods of degradation, but reflect the shape of the true underlying failure rate progress path with very credible fidelity. The figure series 3.55 thru 3.57 indicates how mean determined failure rate estimating performance improves with additional testing (NT_i = 5, 10, and 20) while mean forecast failure rate estimating performance is very stable after the third forecast for all test per phase sizes. Again, note in figures 3.59 and 3.77 that when the true instantaneous failure rate has decreased to a small magnitude (test phase 16), the instantaneous model's determined and forecast failure rate accuracy displays significant improvement.

b. Variability (Precision) Performance

The variability performance of the continuous instantaneous failure rate model is presented in tables 3.15 thru 3.26. Entries in

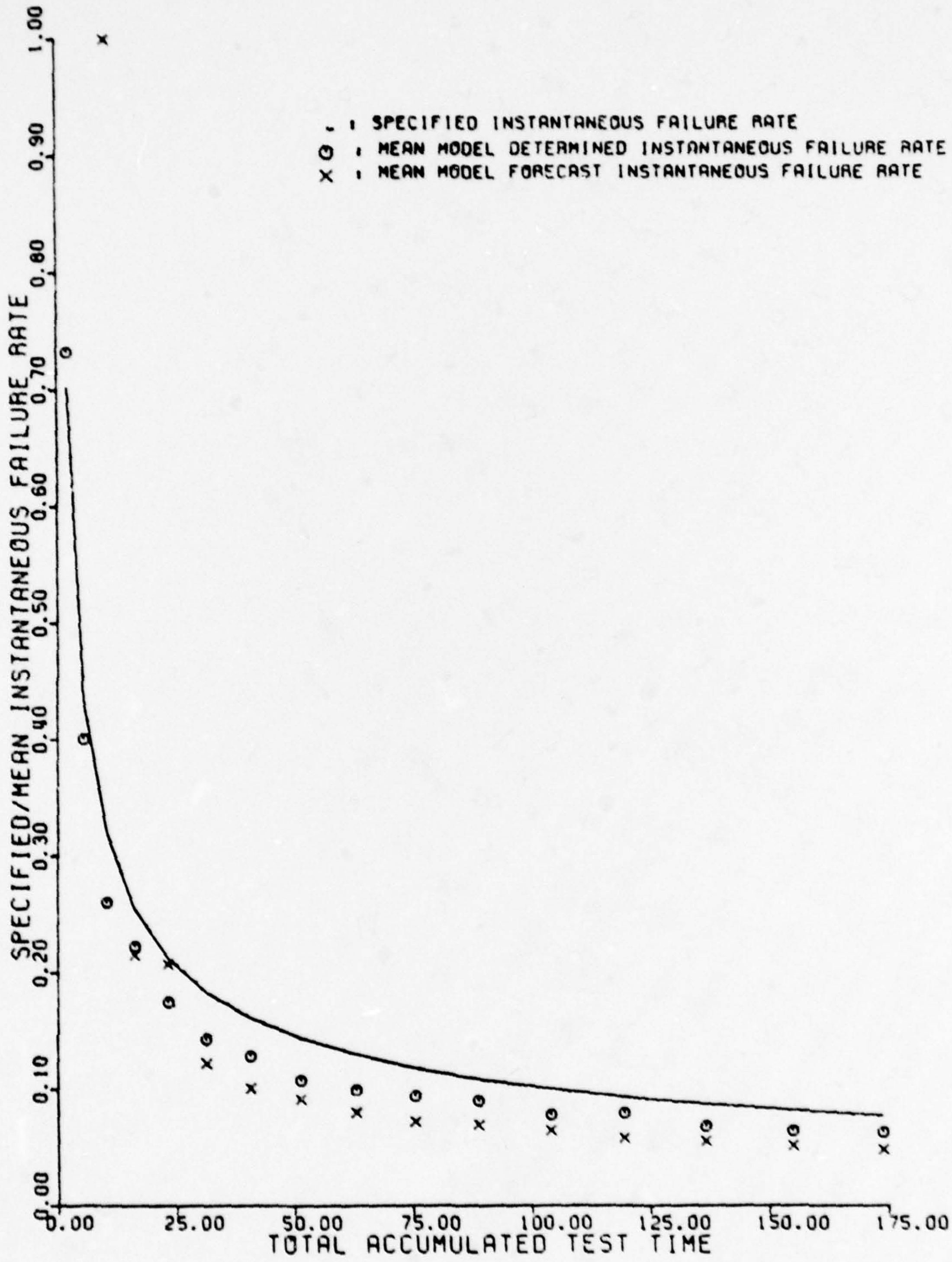

FIGURE 3.42

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE

LAMBDA SET 1: 16 PHASES, 5 TESTS/PHASE

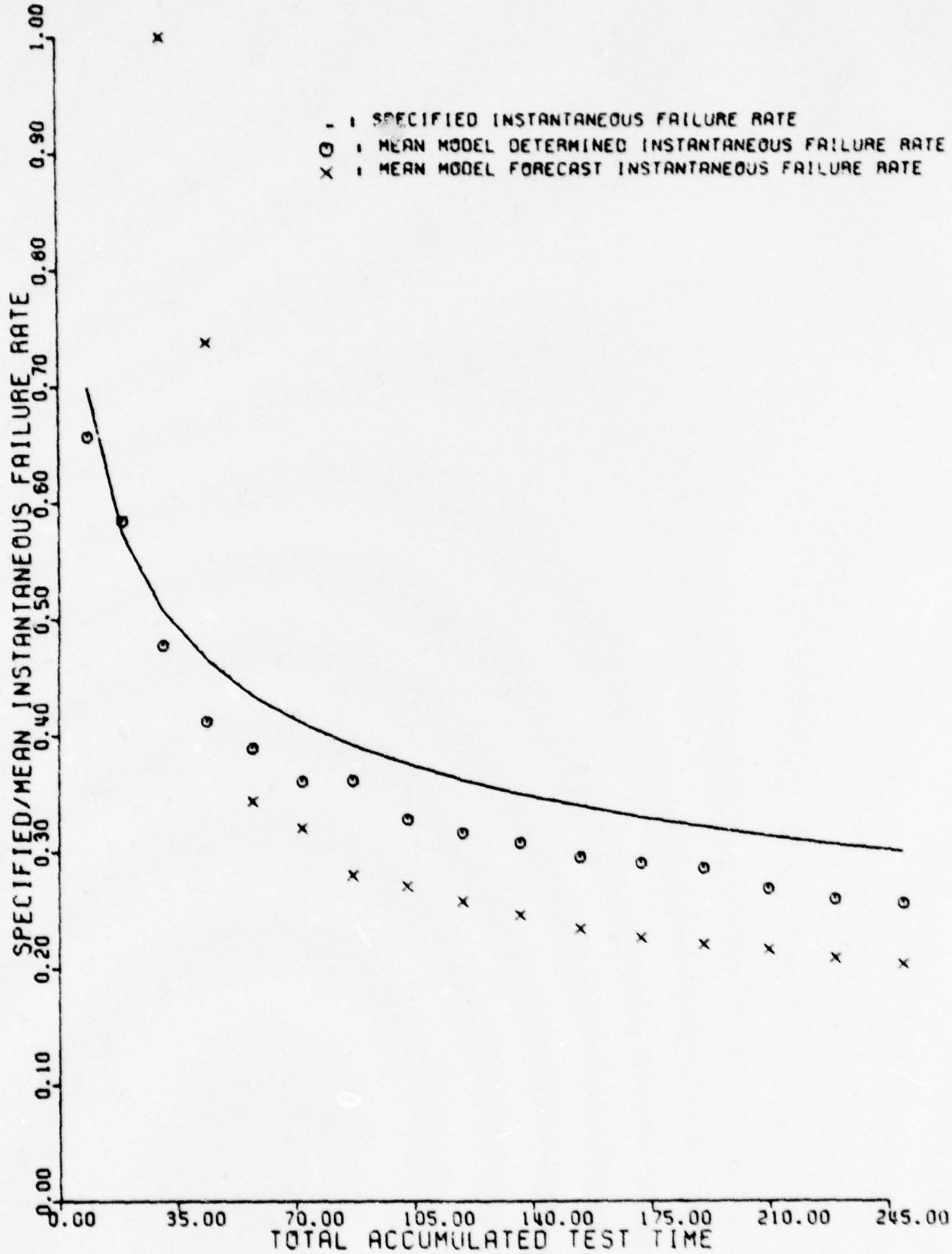

FIGURE 3.43

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE

LAMBDA SET 2: 16 PHASES, 20 TESTS/PHASE

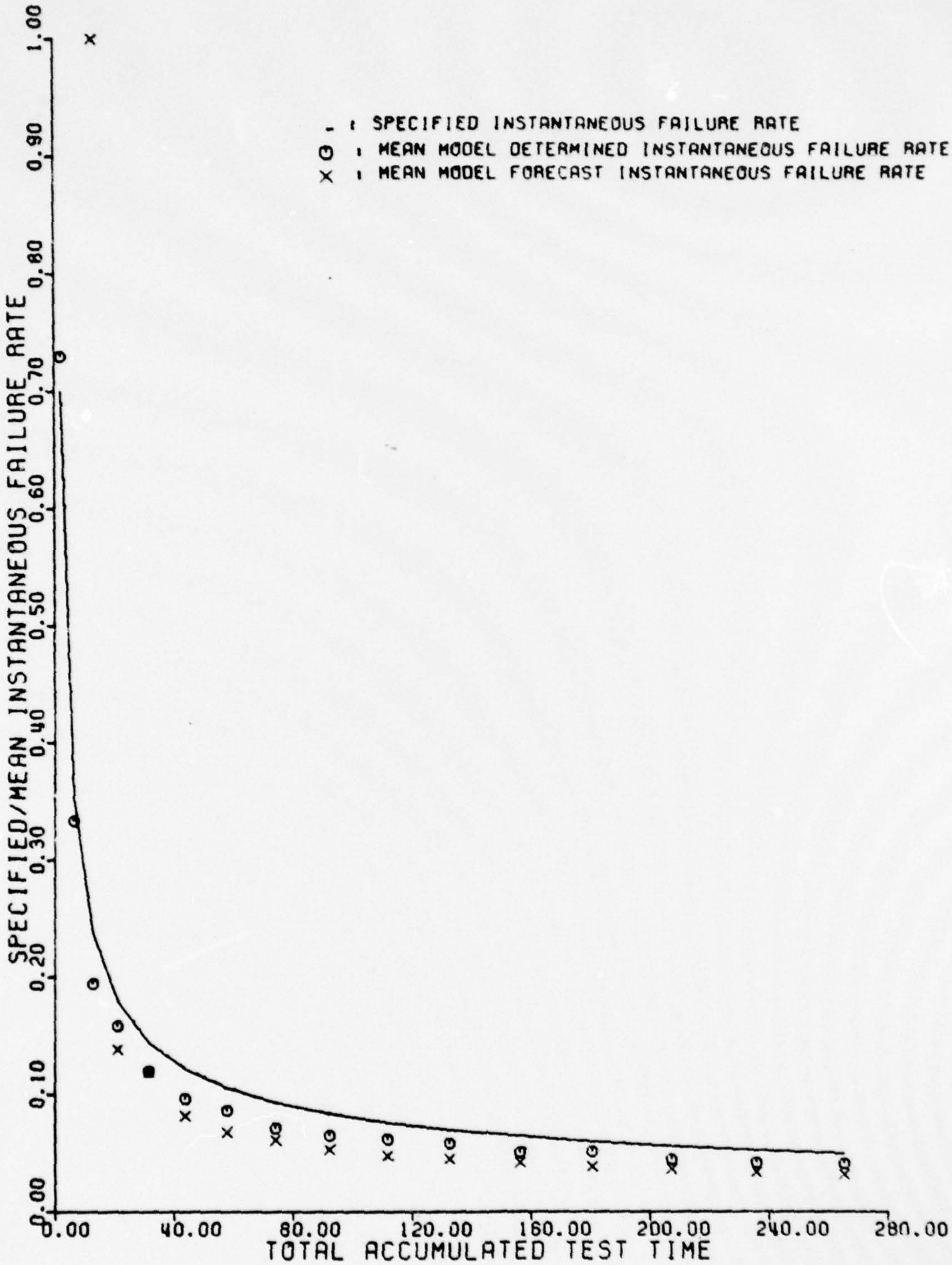

FIGURE 3.44

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 3: 16 PHASES, 5 TESTS/PHASE

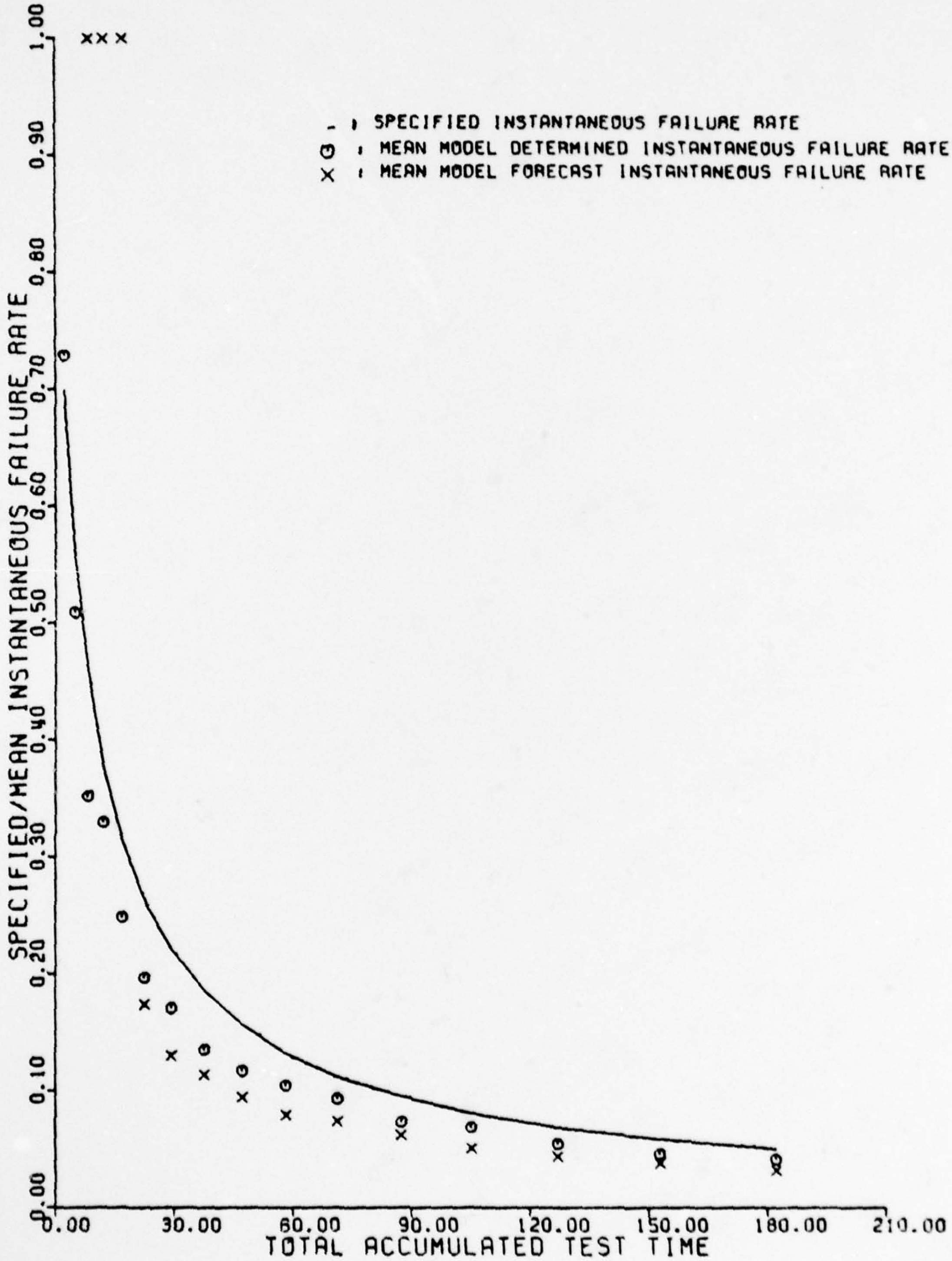

FIGURE 3.45

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 4: 16 PHASES, 5 TESTS/PHASE

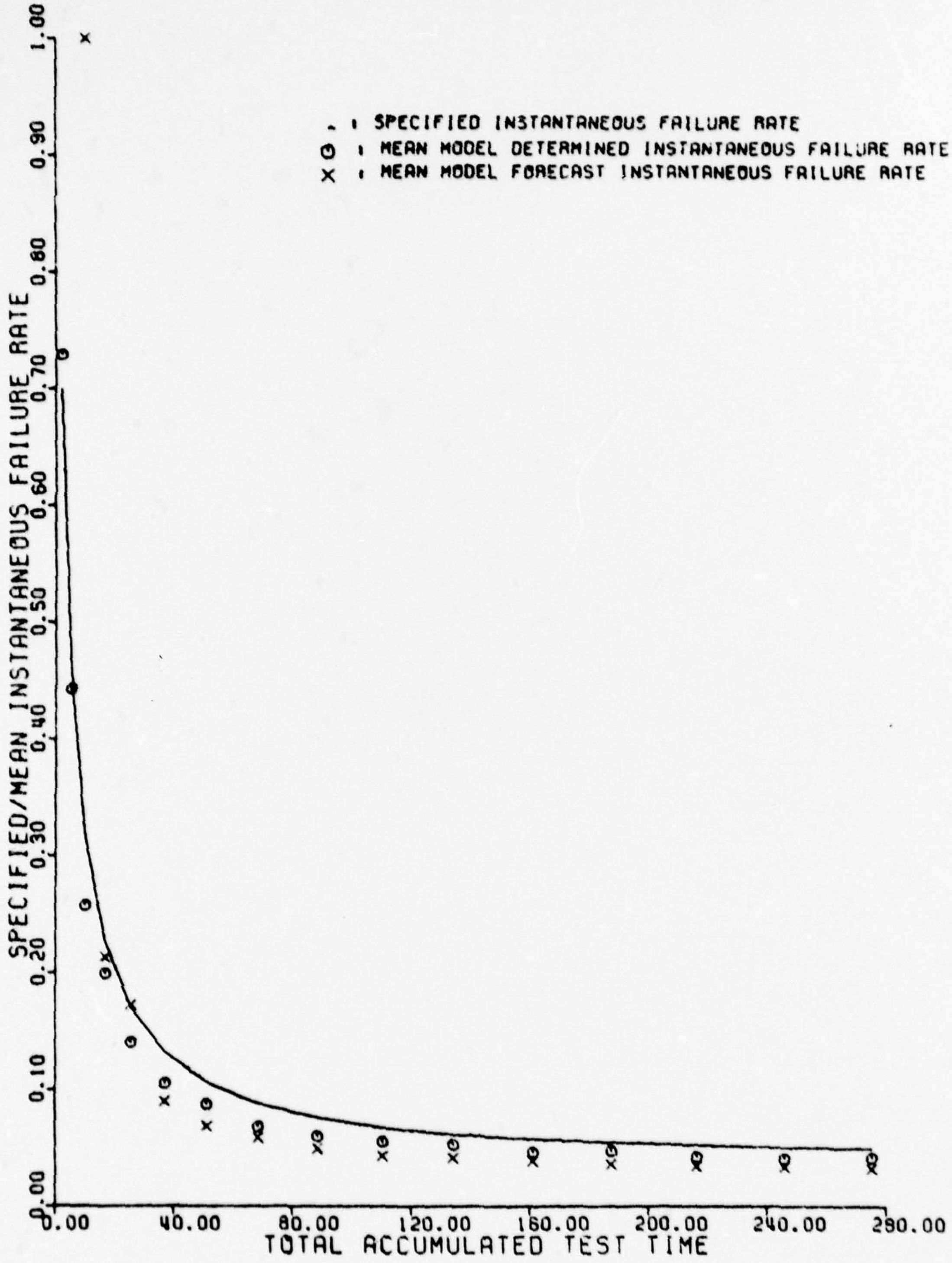

FIGURE 3.46

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 5: 16 PHASES, 5 TESTS/PHASE

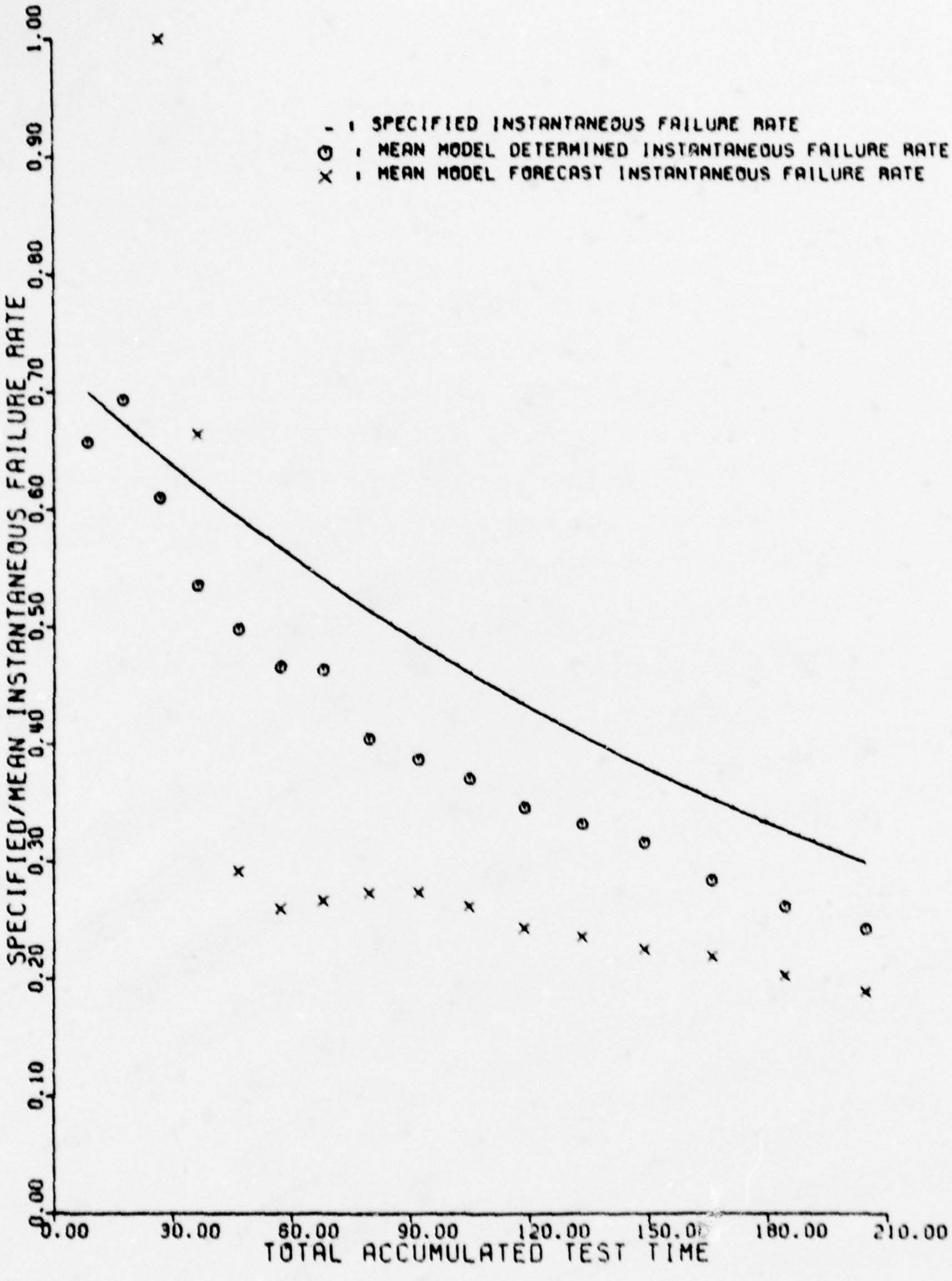

FIGURE 3.47

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 6: 16 PHASES, 20 TESTS/PHASE

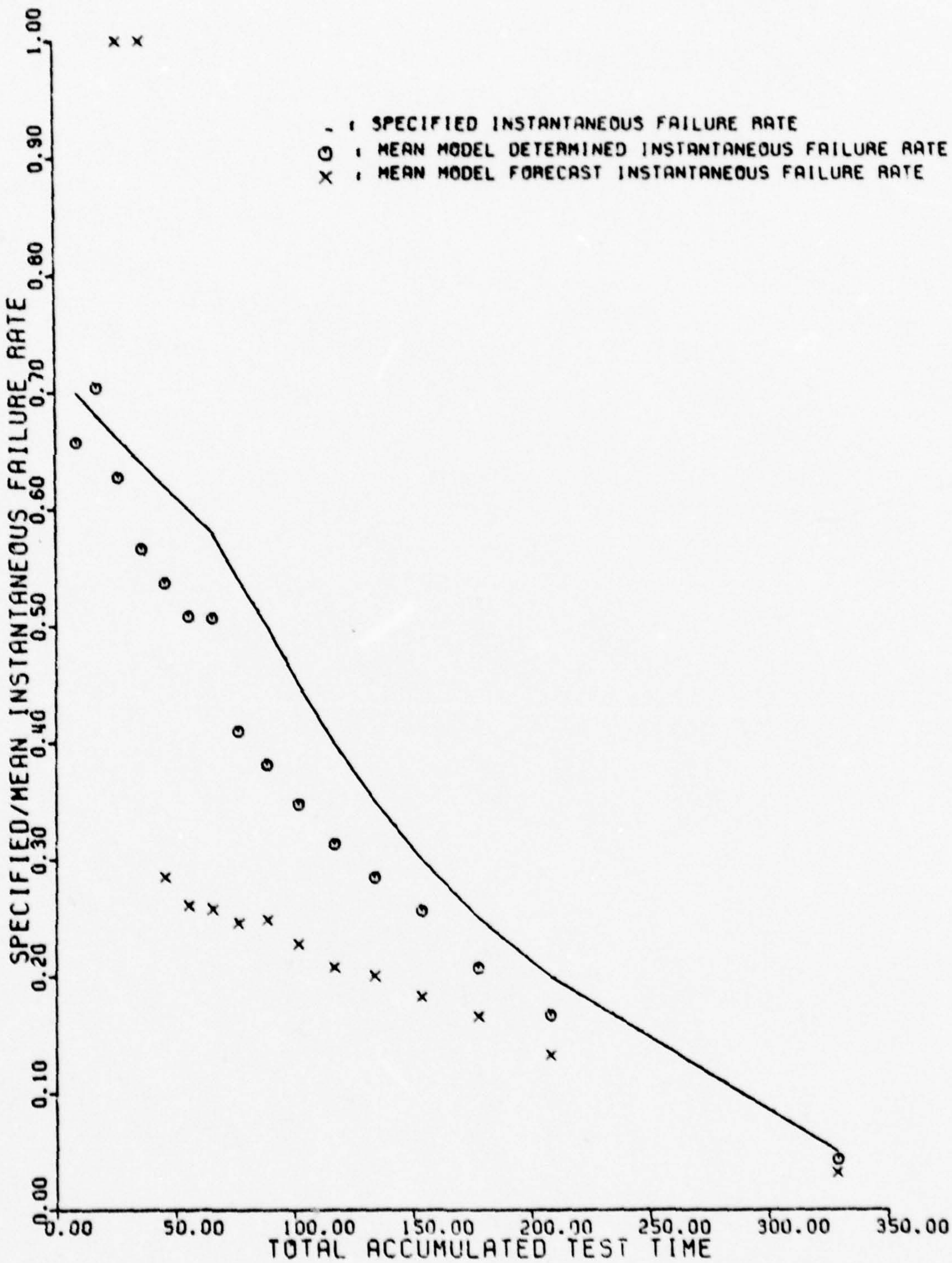

FIGURE 3.48

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 7: 16 PHASES, 10 TESTS/PHASE

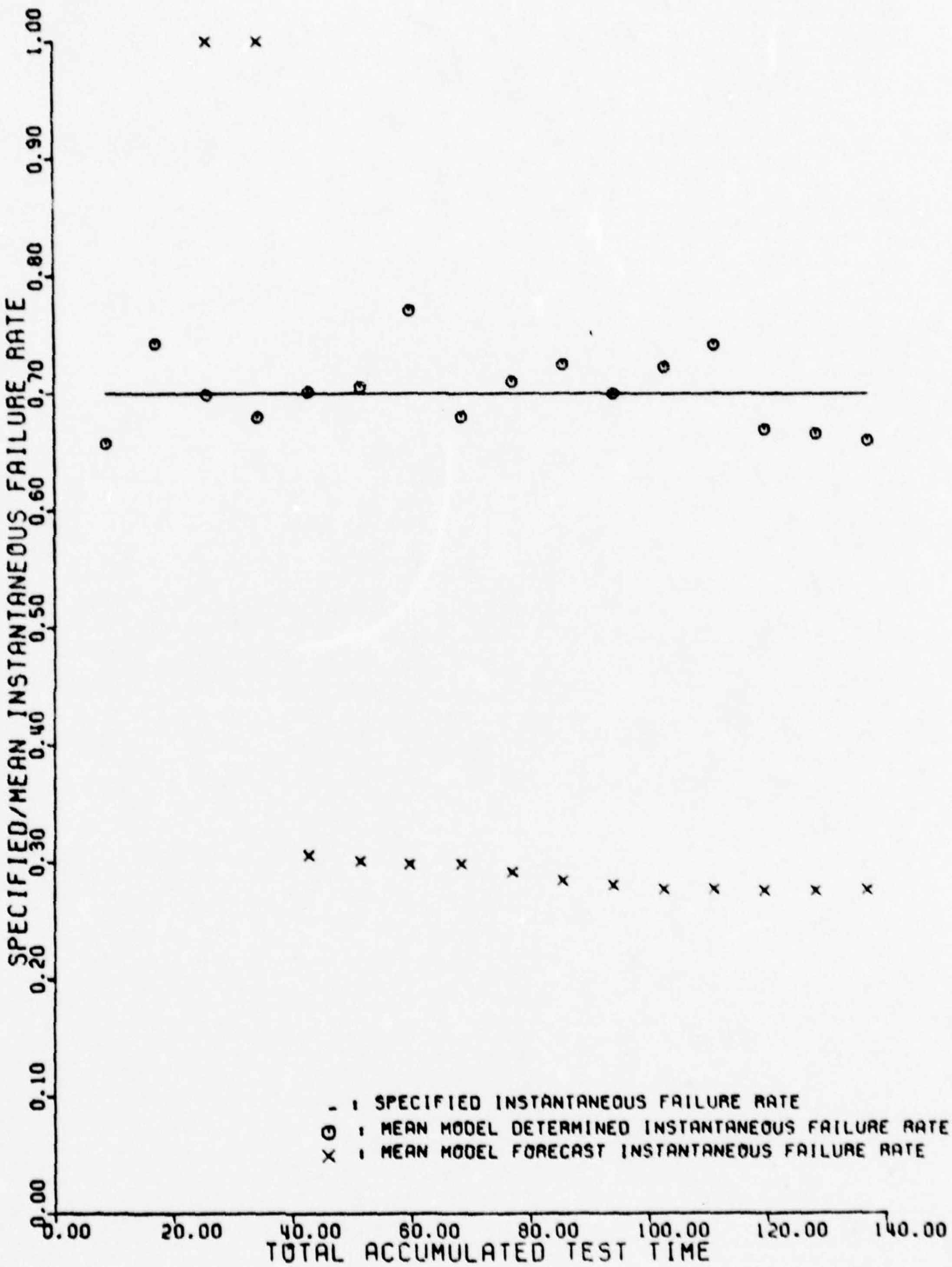

FIGURE 3.49

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 8: 16 PHASES, 20 TESTS/PHASE

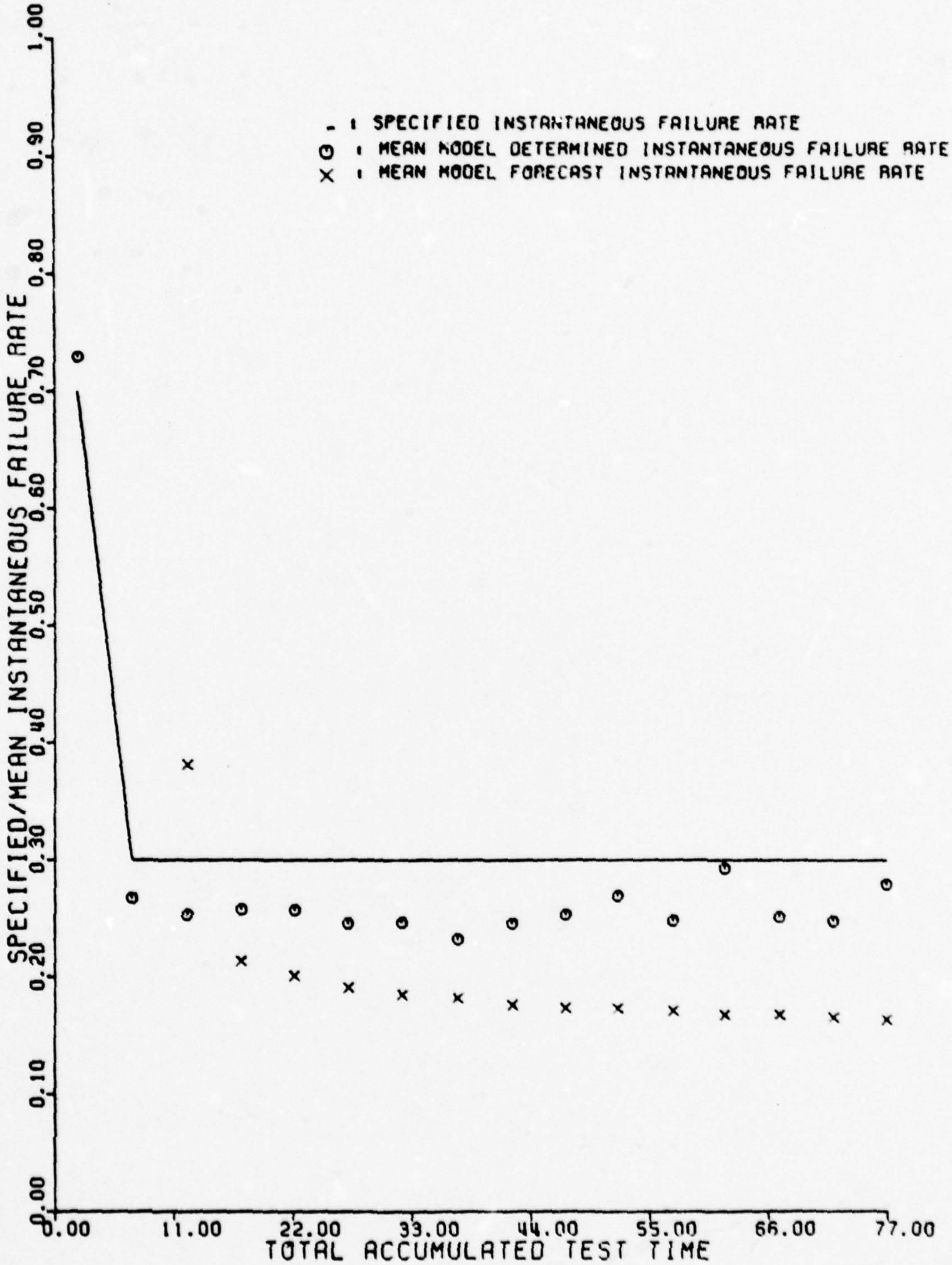

FIGURE 3.50

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 9: 16 PHASES, 5 TESTS/PHASE

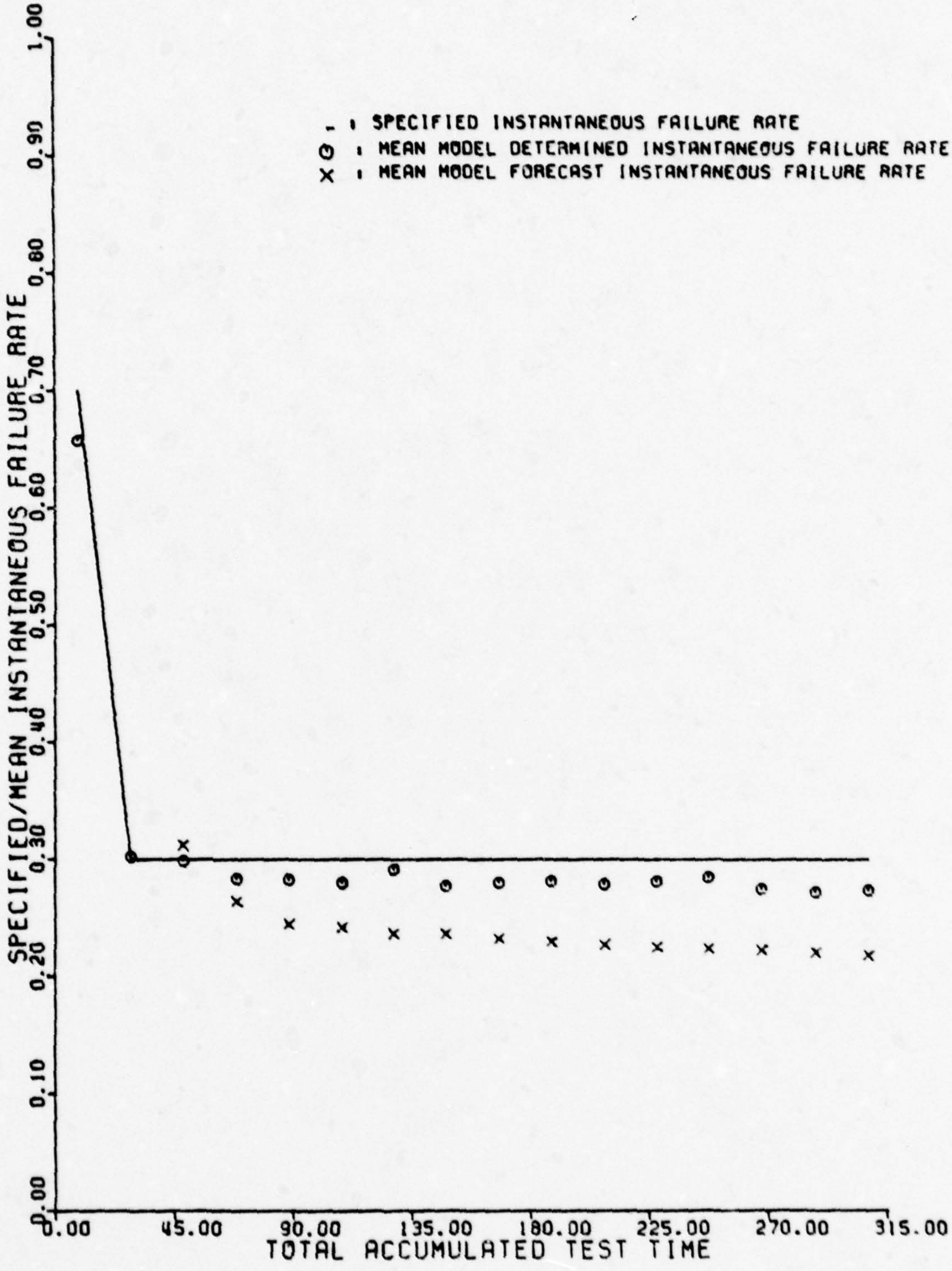

FIGURE 3.51

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 9: 16 PHASES, 20 TESTS/PHASE

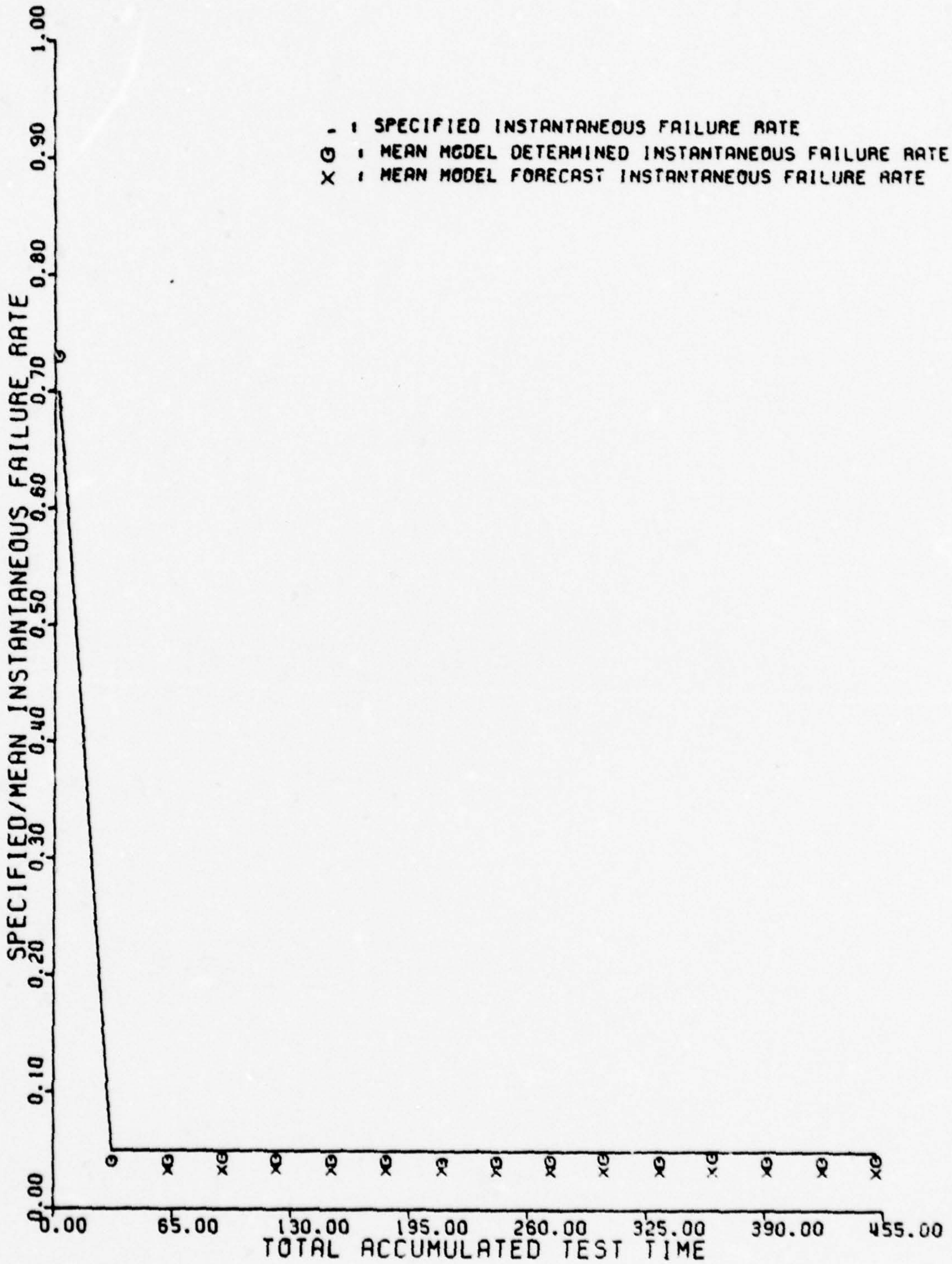

FIGURE 3.52

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 10: 16 PHASES, 5 TESTS/PHASE

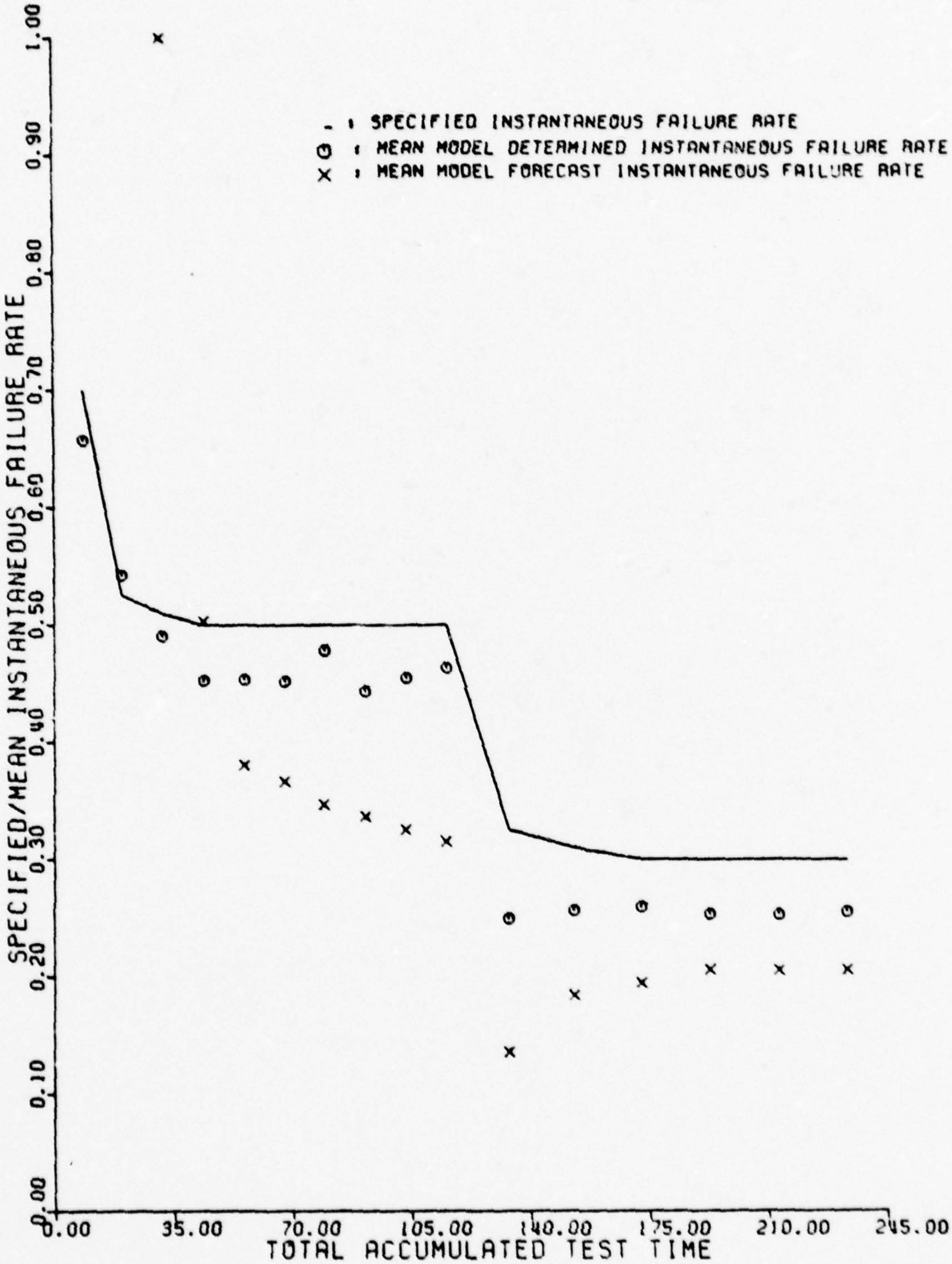

FIGURE 3.53

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE

LAMBDA SET 11: 16 PHASES, 20 TESTS/PHASE

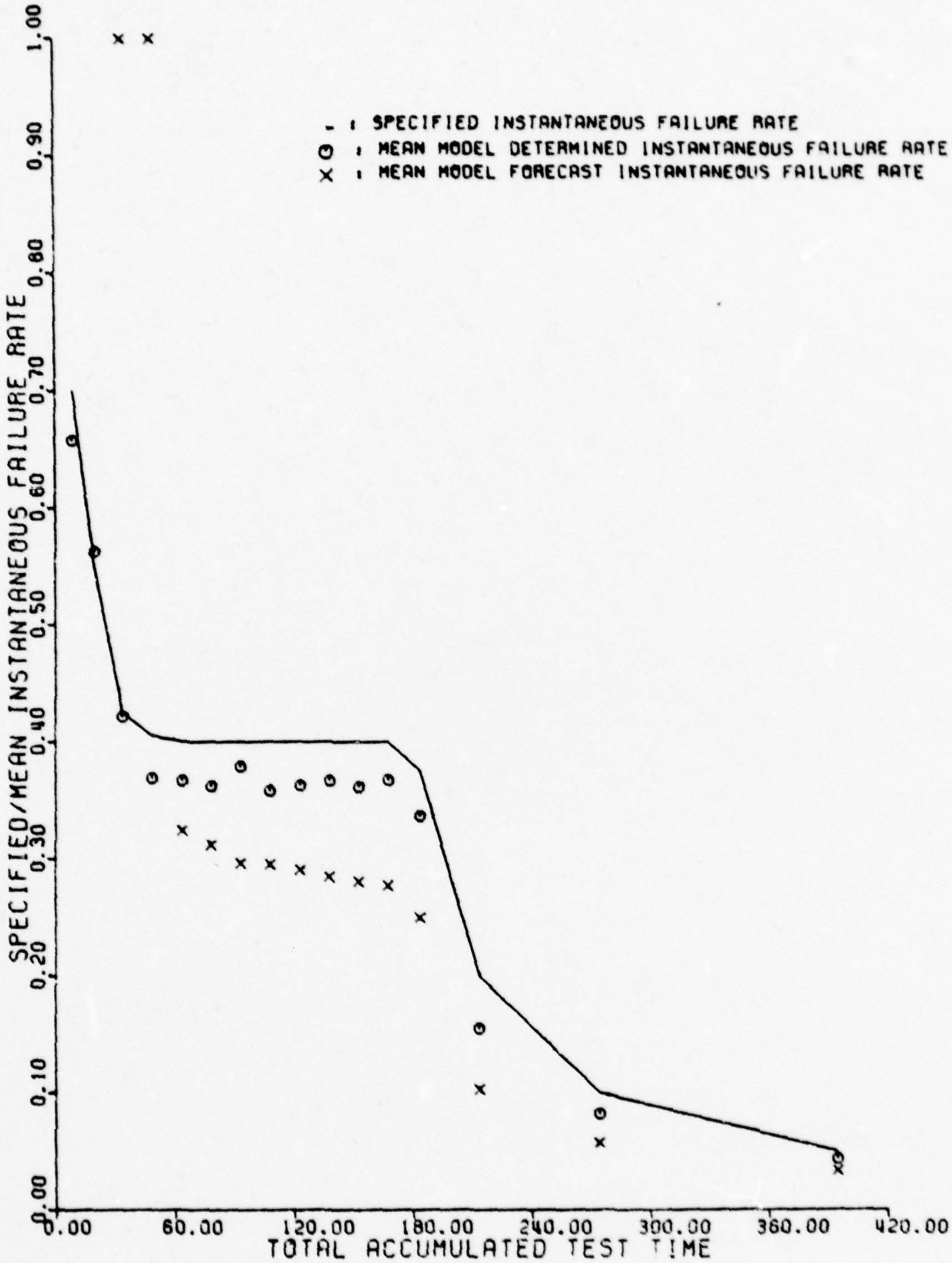

FIGURE 3.54

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 12: 16 PHASES, 20 TESTS/PHASE

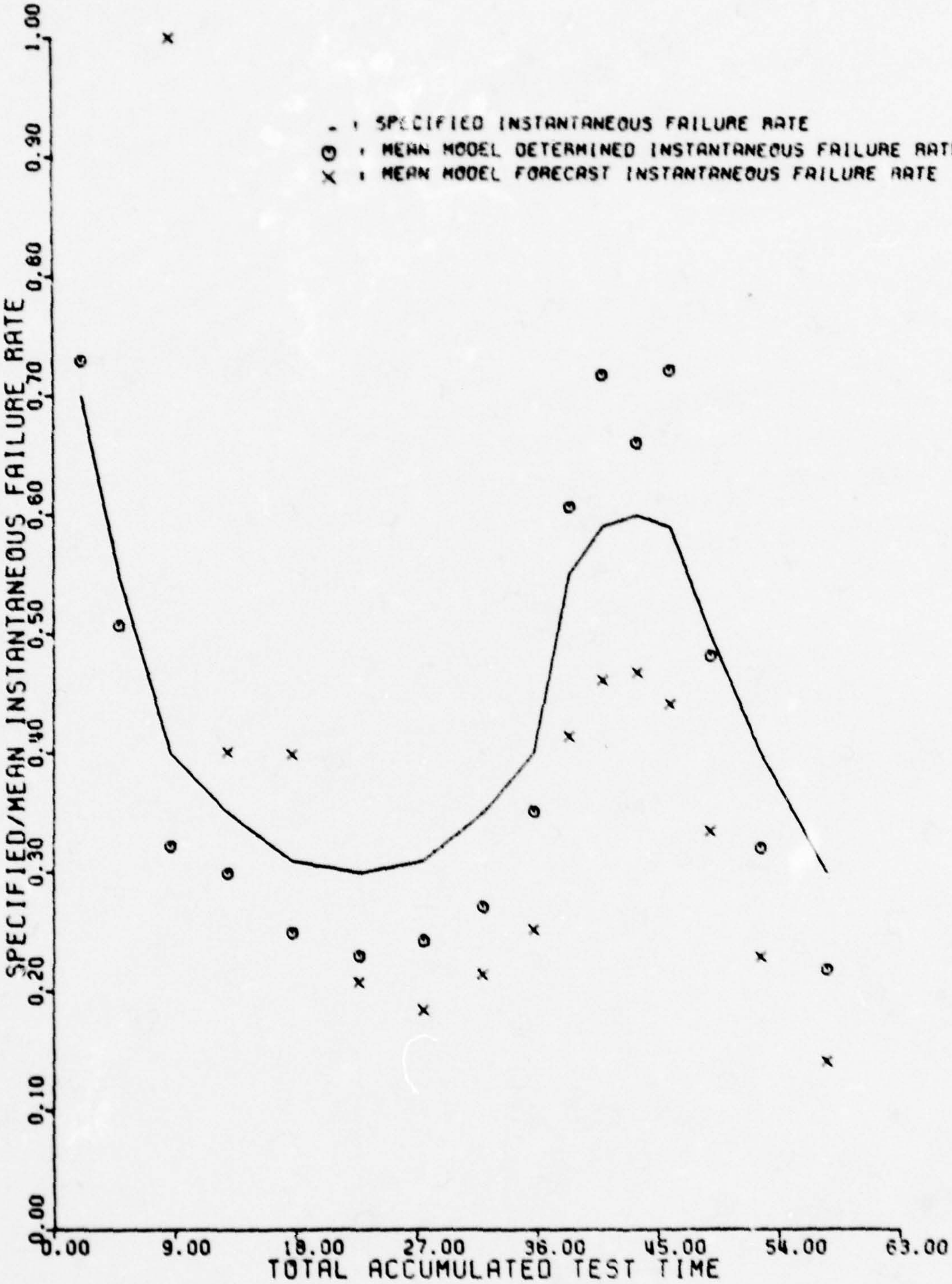

FIGURE 3.55

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 13: 16 PHASES, 5 TESTS/PHASE

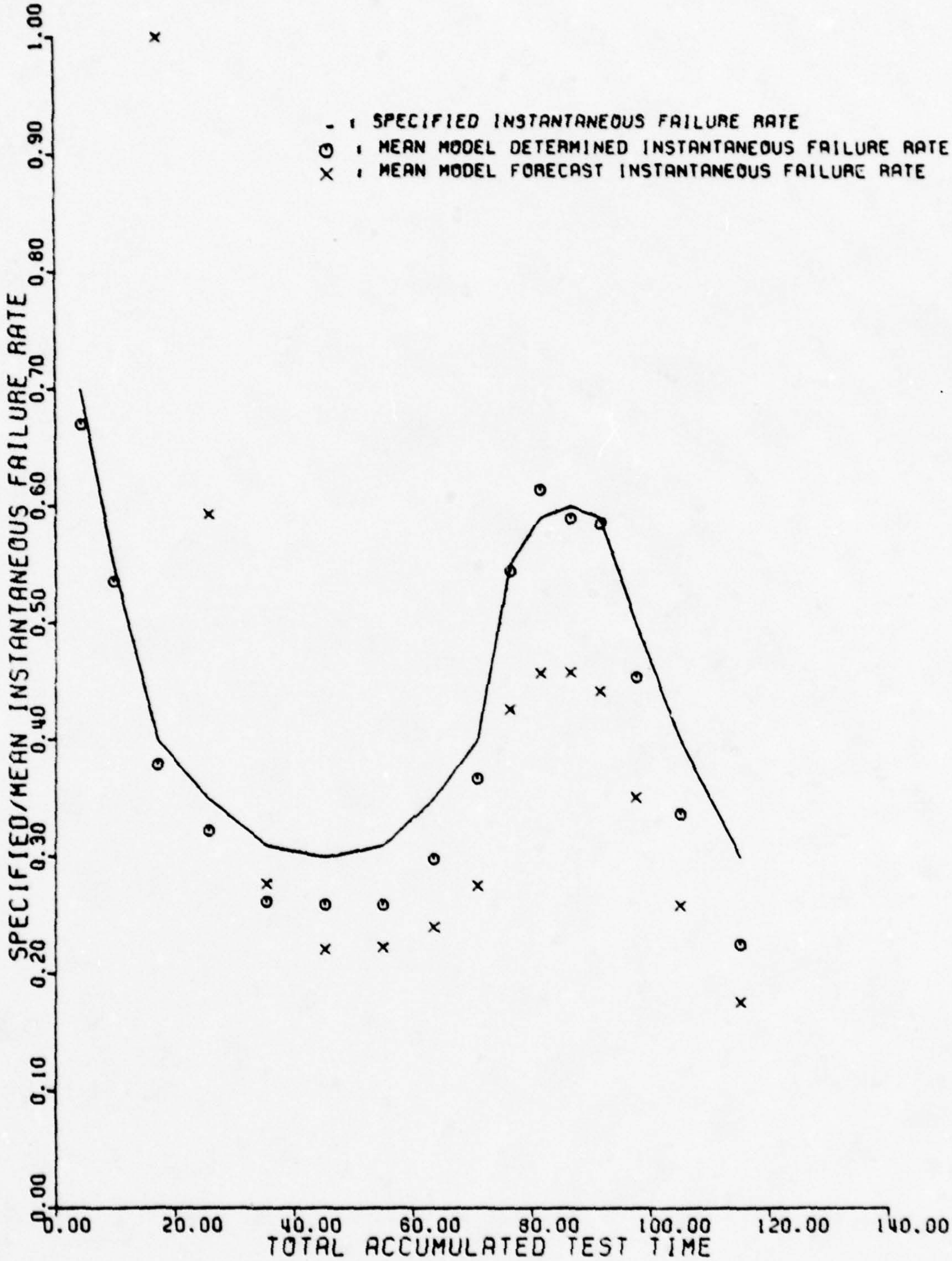

FIGURE 3.56

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 13: 16 PHASES, 10 TESTS/PHASE

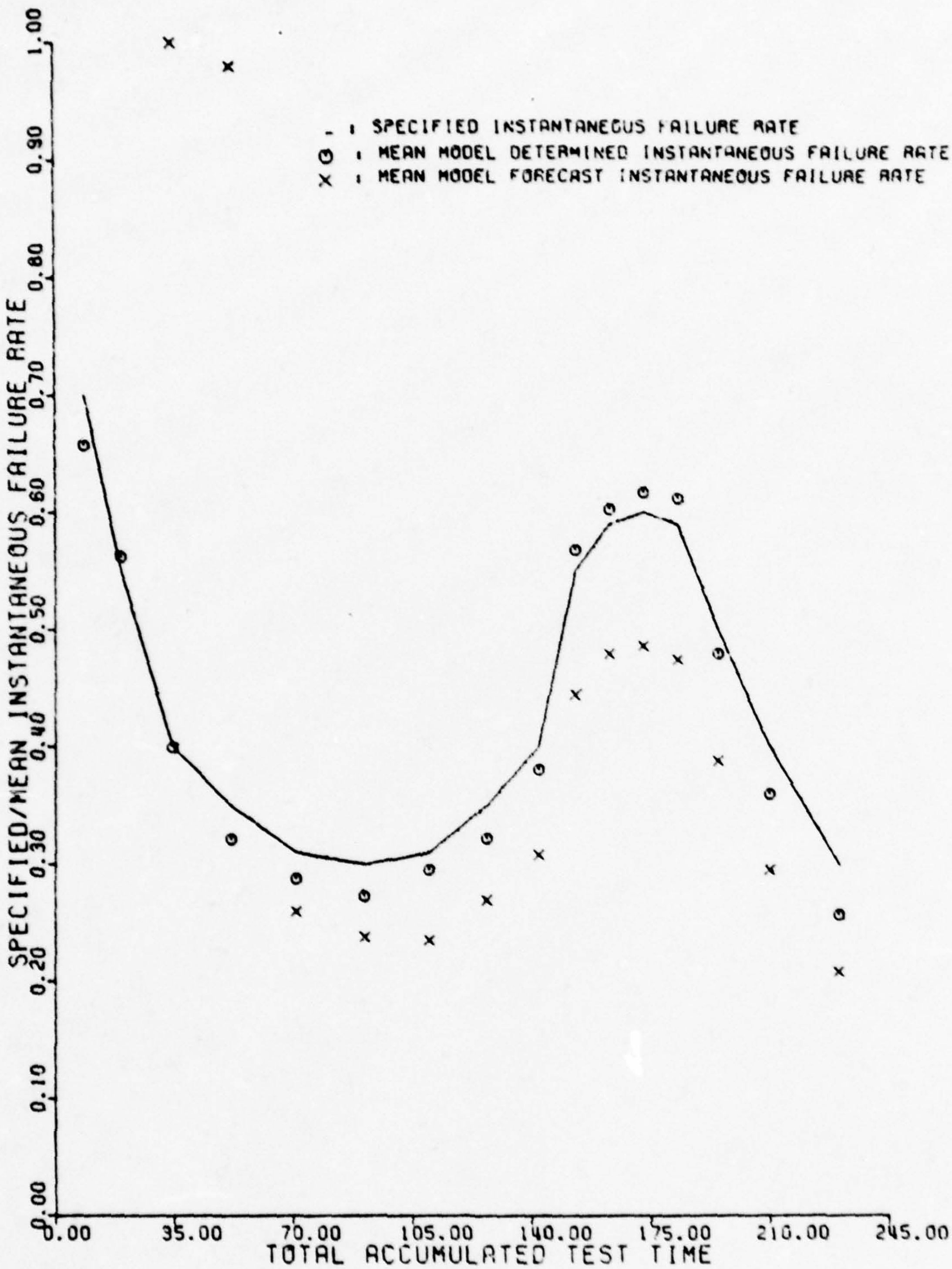

FIGURE 3.57

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 13: 16 PHASES, 20 TESTS/PHASE

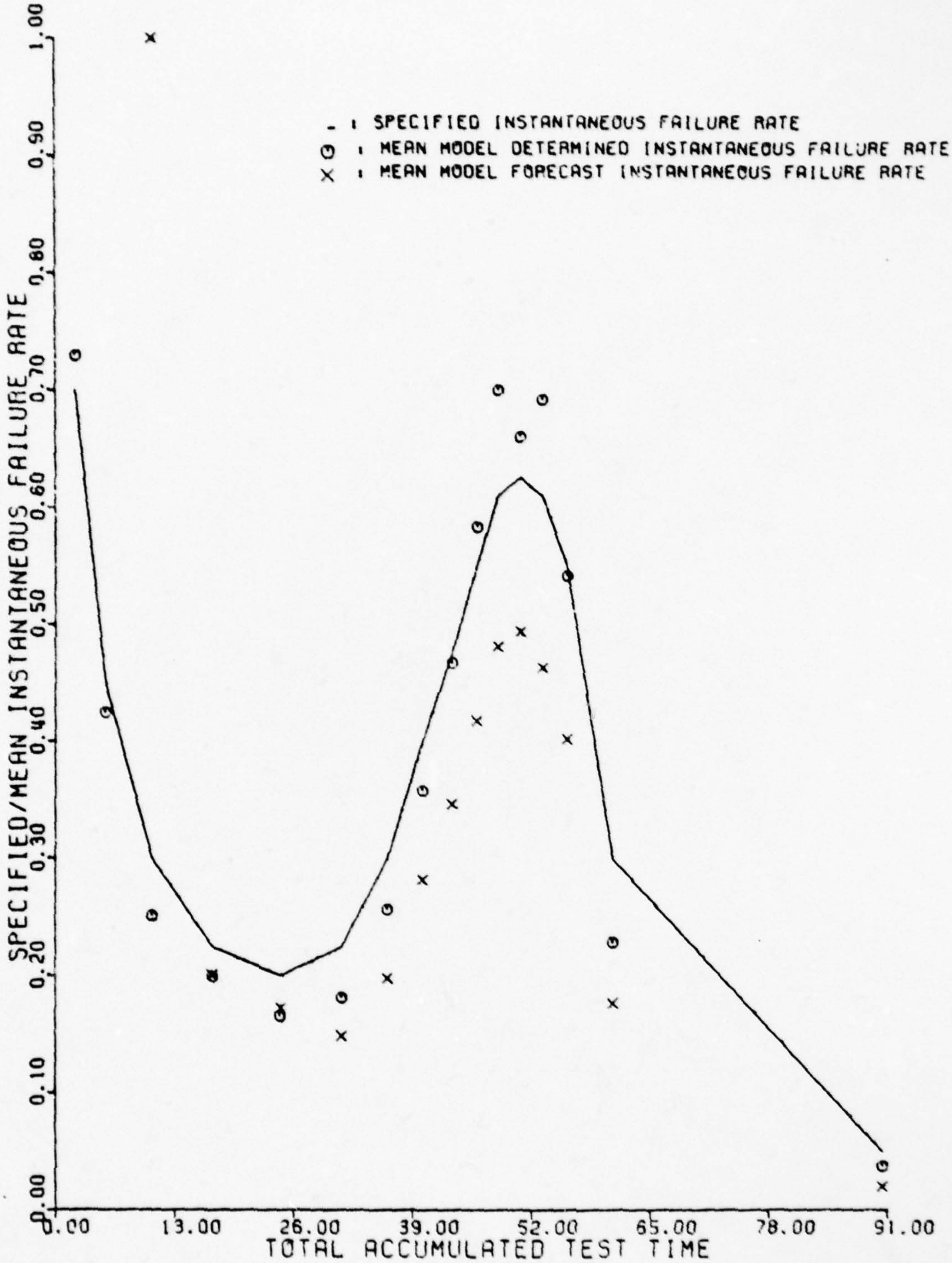

FIGURE 3.58

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 14: 16 PHASES, 5 TESTS/PHASE

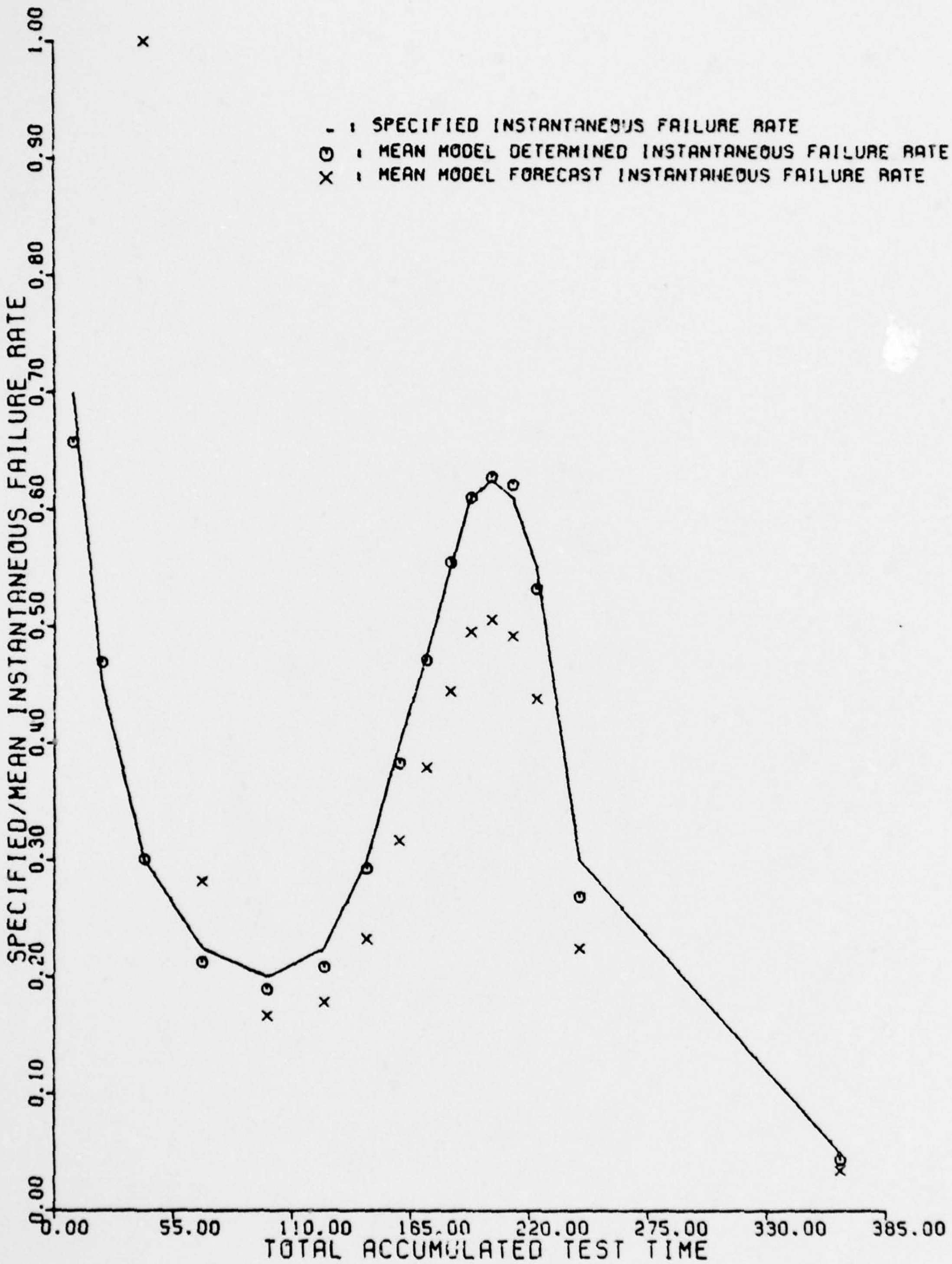

FIGURE 3.59

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 14: 16 PHASES, 20 TESTS/PHASE

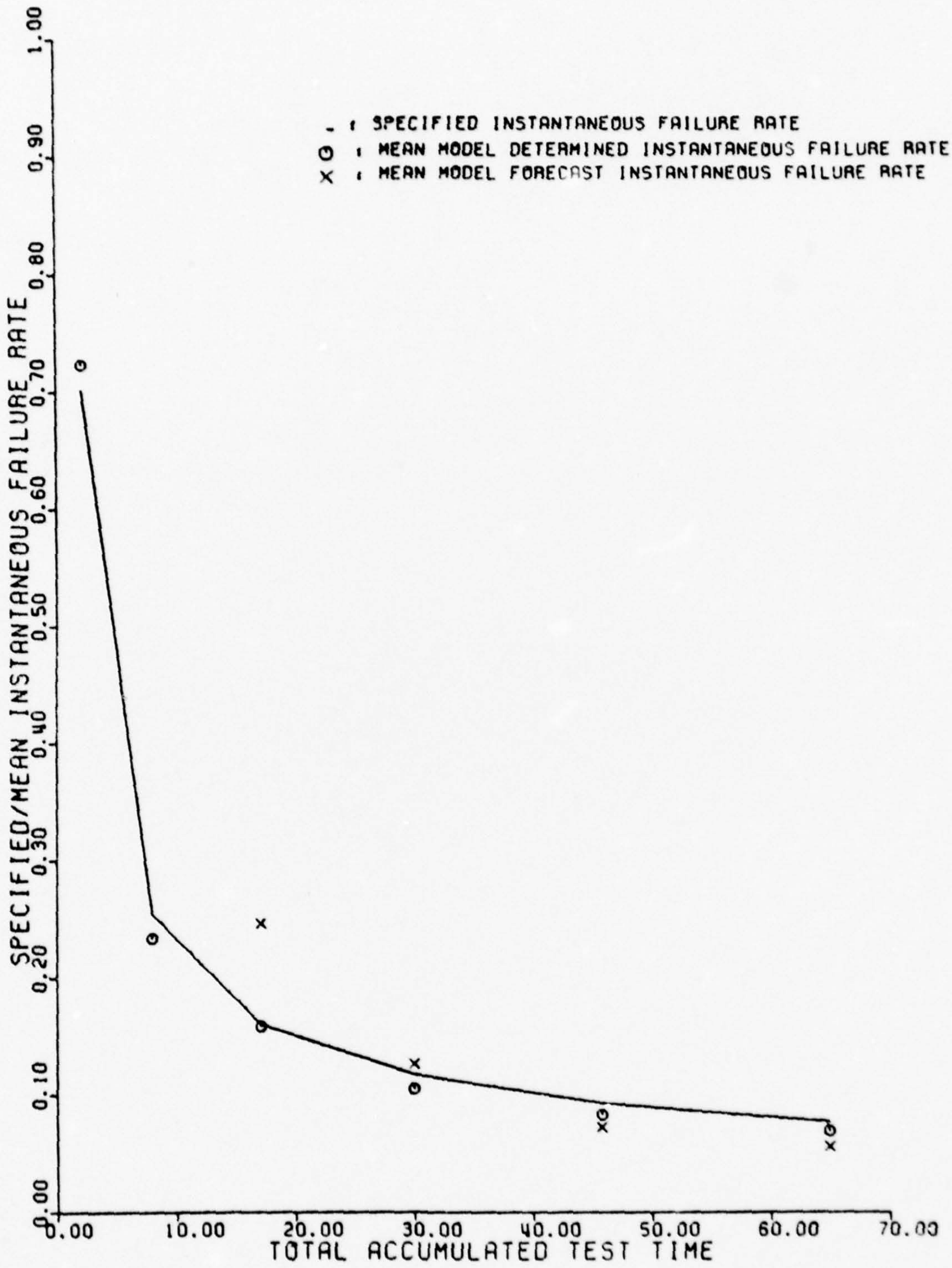

FIGURE 3.60

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE

LAMBDA SET MOD1: 6 PHASES, 5 TESTS/PHASE

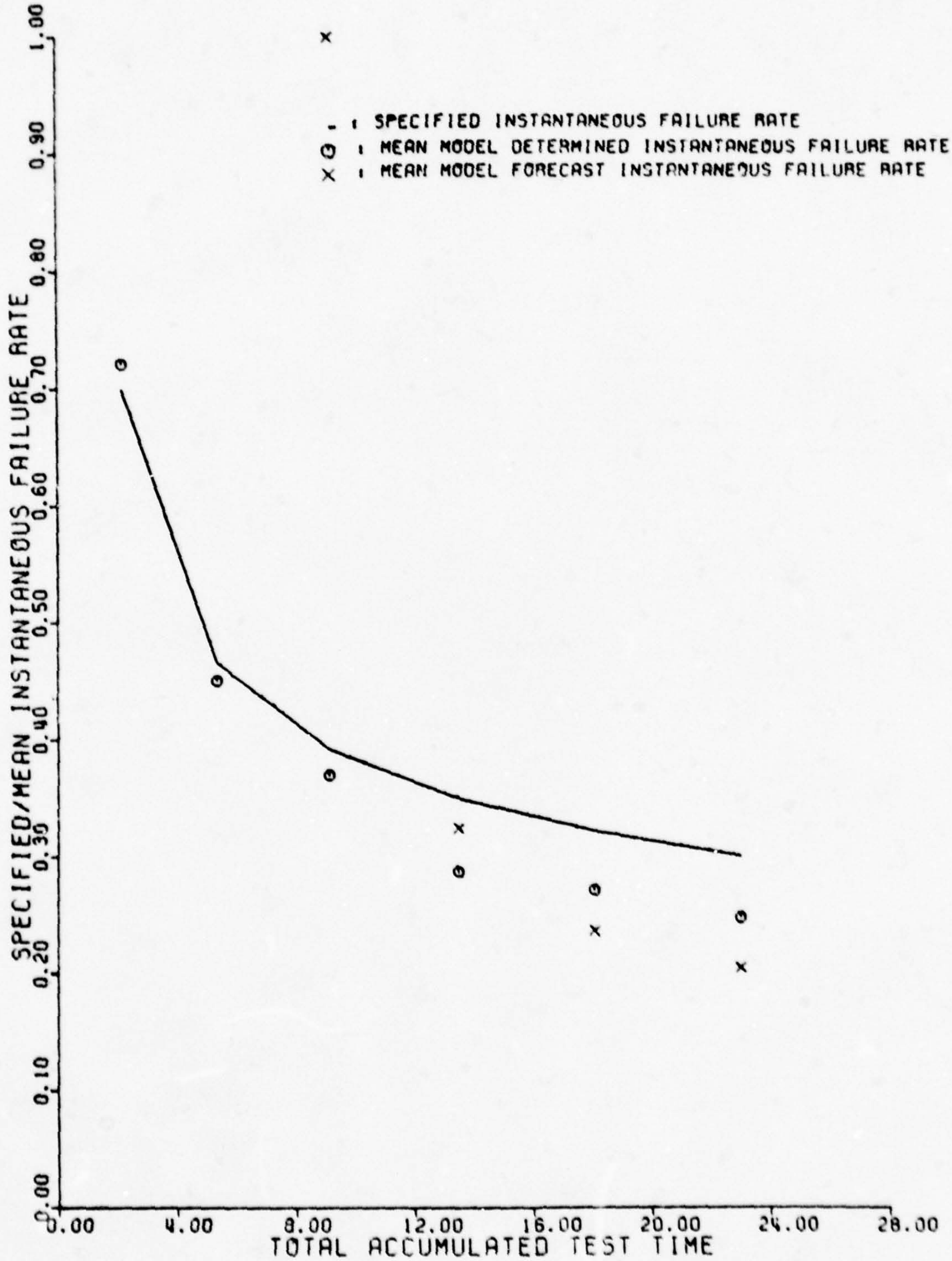

FIGURE 3.61

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE

LAMBDA SET MOD2: 6 PHASES, 5 TESTS/PHASE

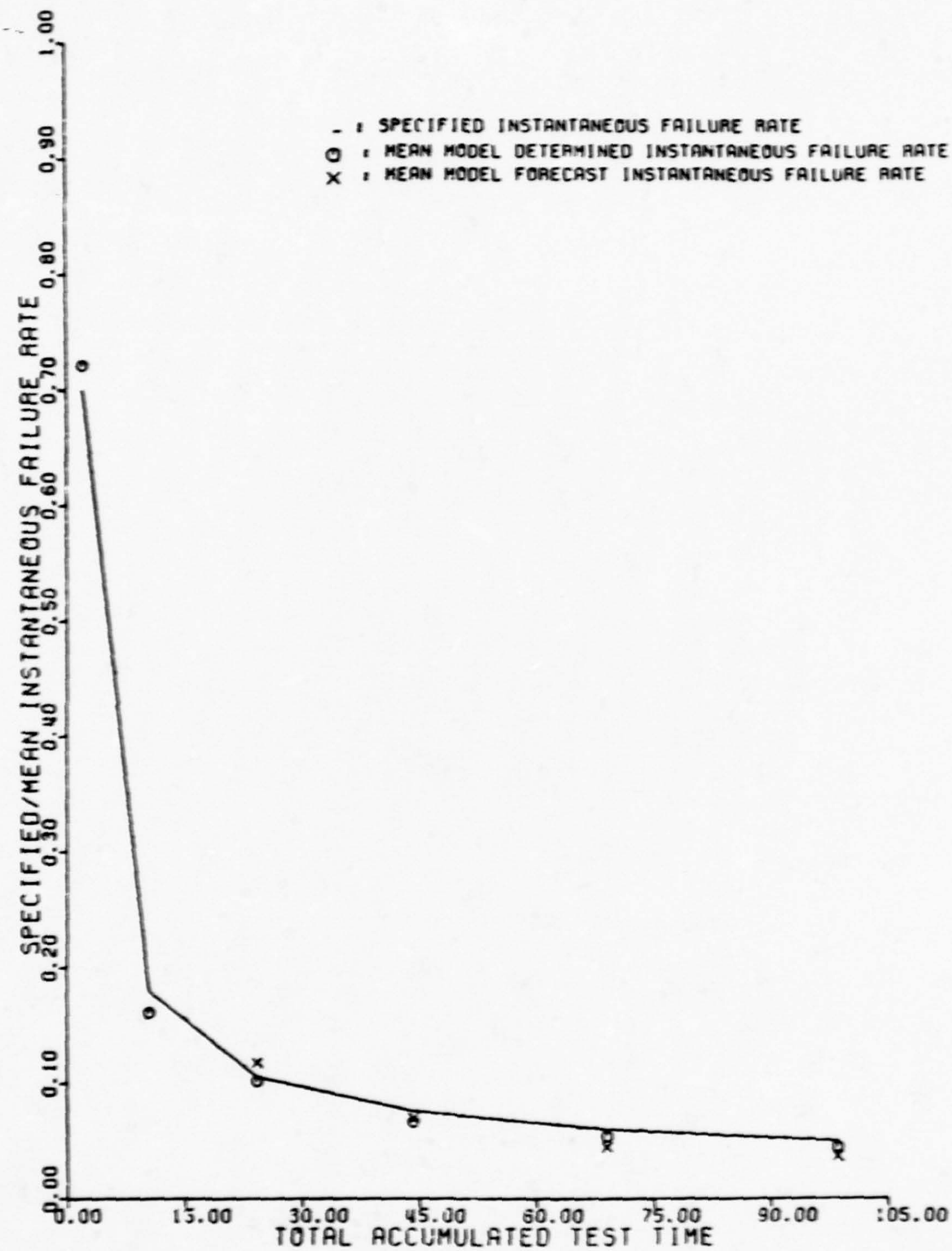

FIGURE 3.62

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE

LAMBDA SET MOD3: 6 PHASES, 5 TESTS/PHASE

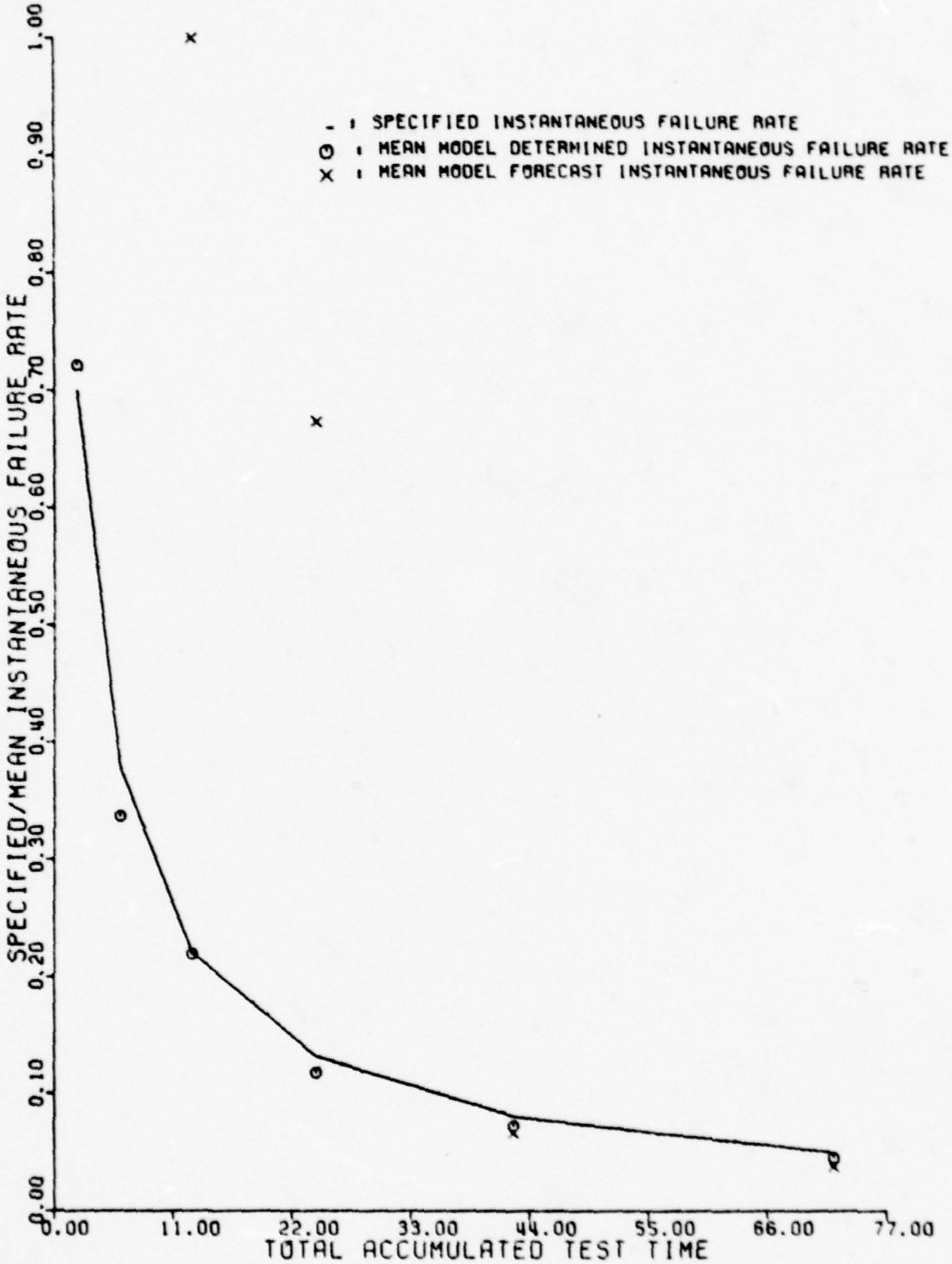

FIGURE 3.63

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD4: 6 PHASES, 5 TESTS/PHASE

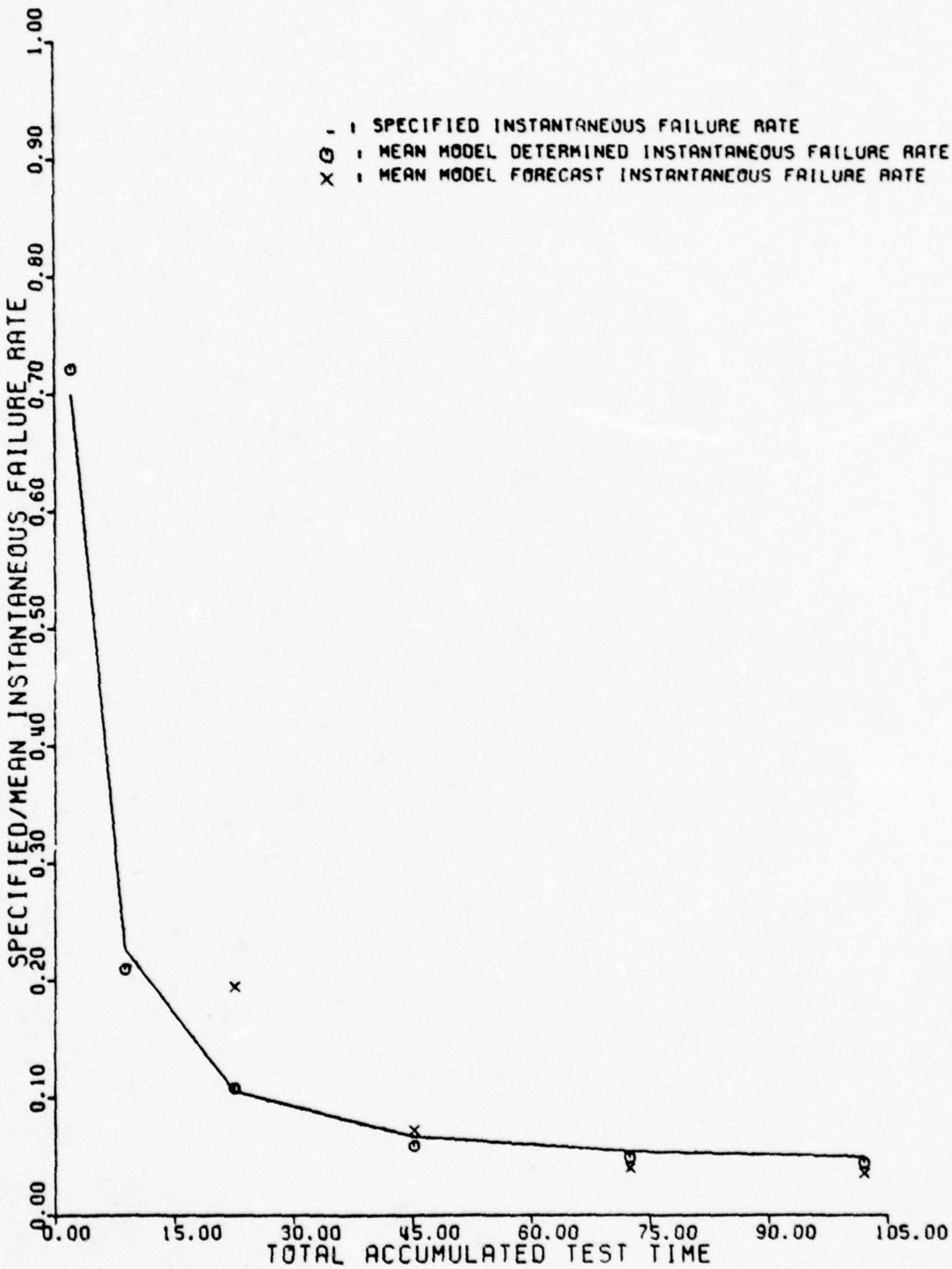

FIGURE 3.64

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MODS: 6 PHASES, 5 TESTS/PHASE

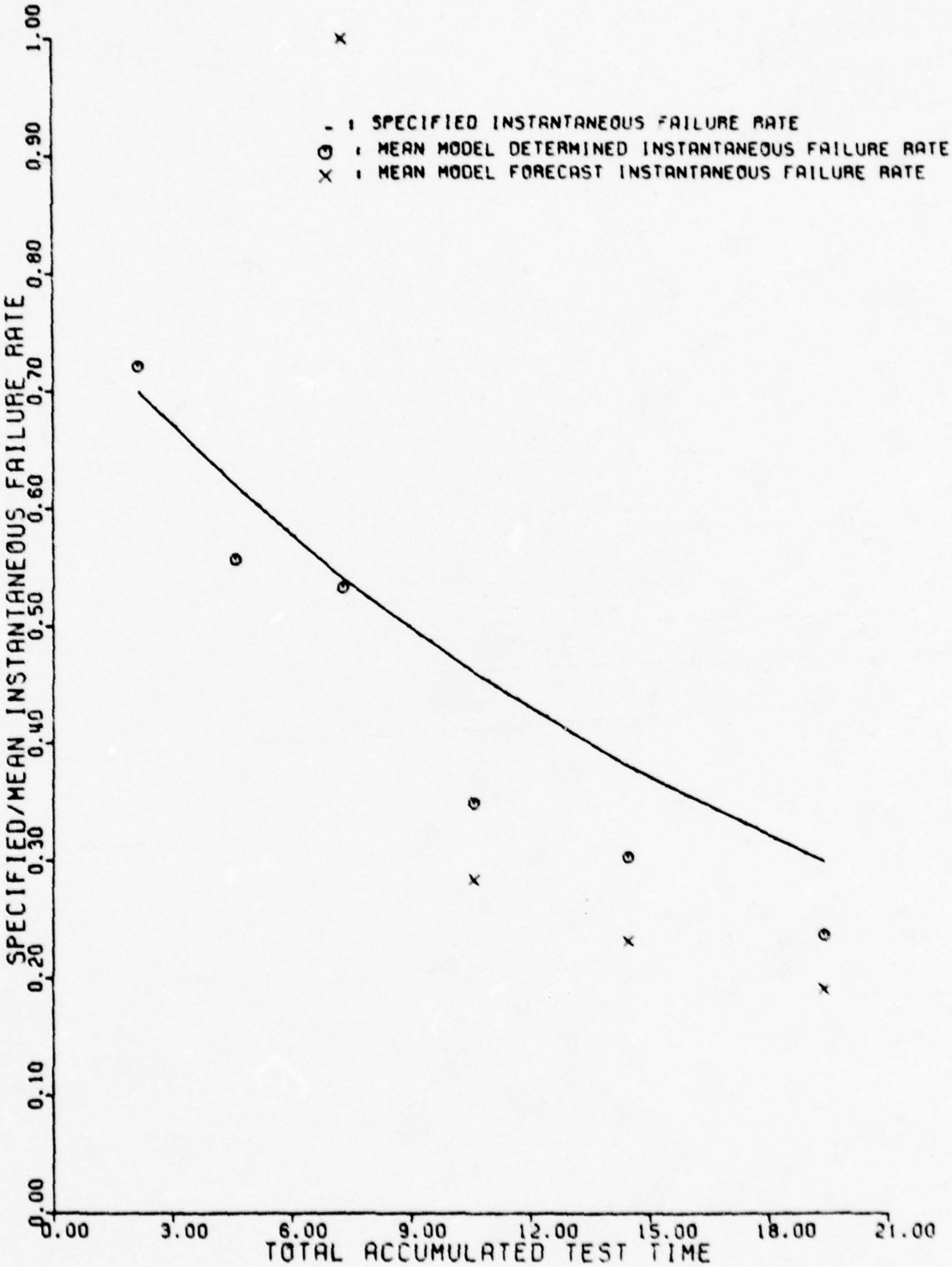

FIGURE 3.65

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD6: 6 PHASES, 5 TESTS/PHASE

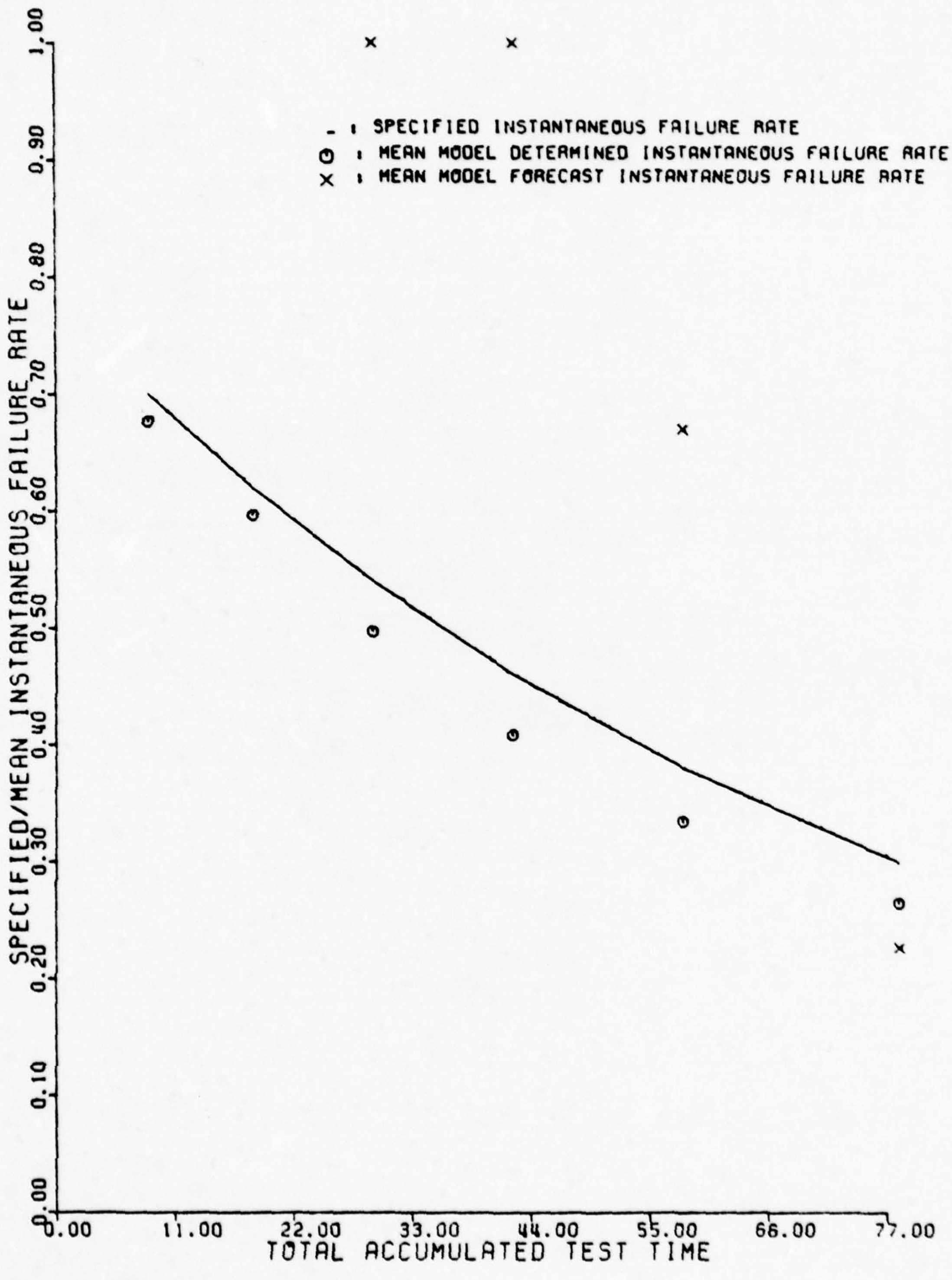

FIGURE 3.66

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD6: 6 PHASES, 20 TESTS/PHASE

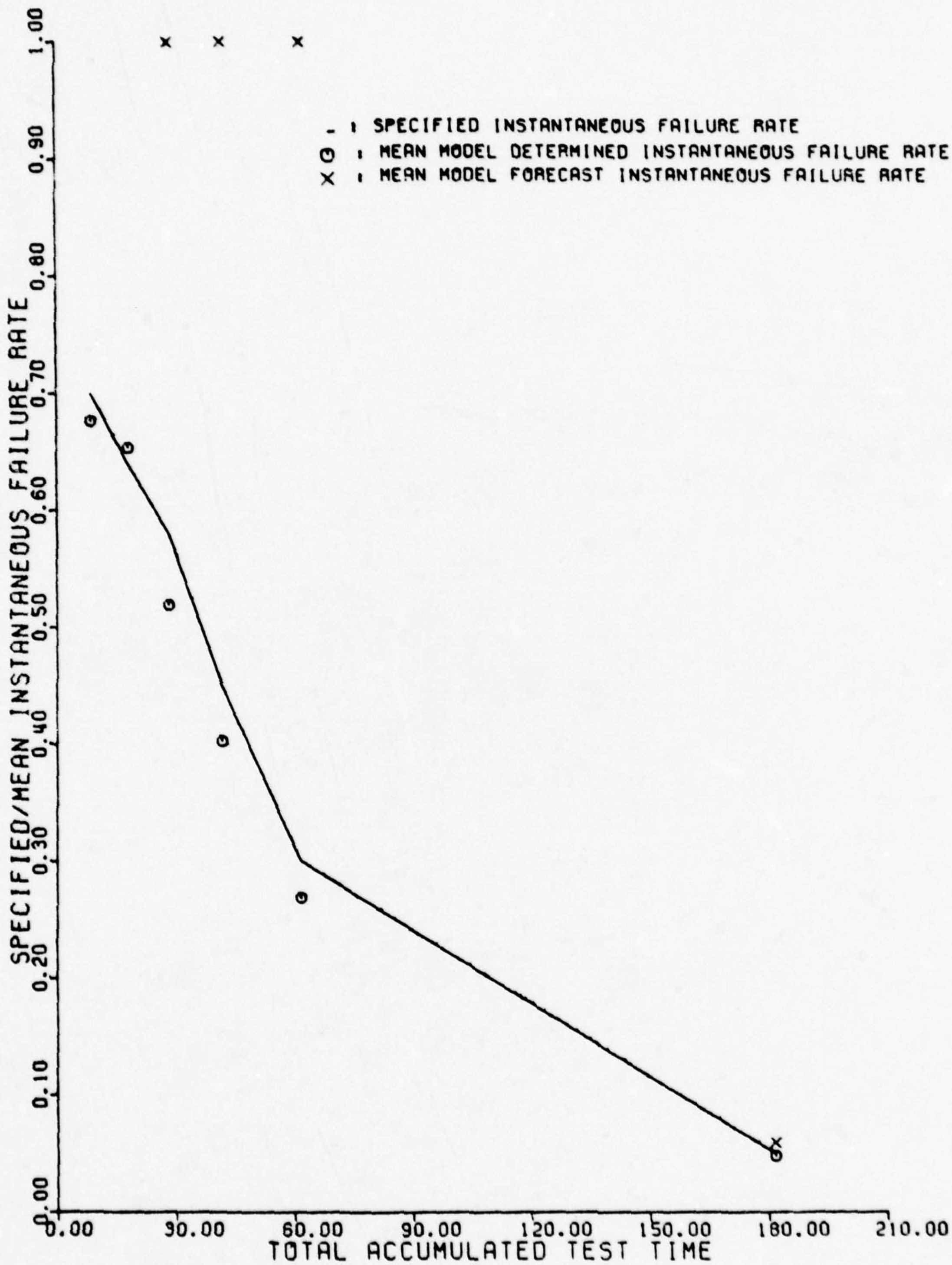

FIGURE 3.67

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD7: 6 PHASES, 20 TESTS/PHASE

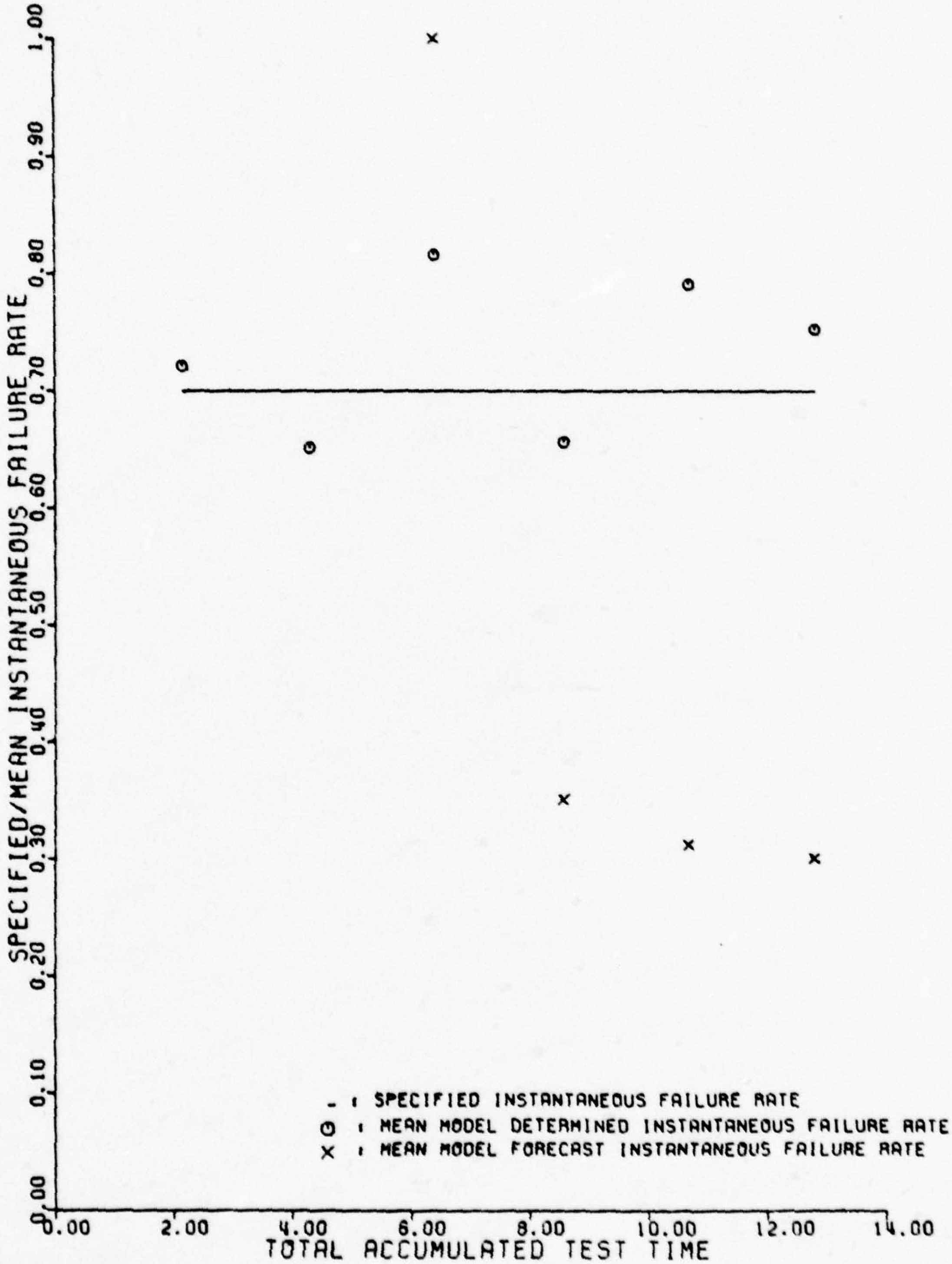

FIGURE 3.68

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD8: 6 PHASES, 5 TESTS/PHASE

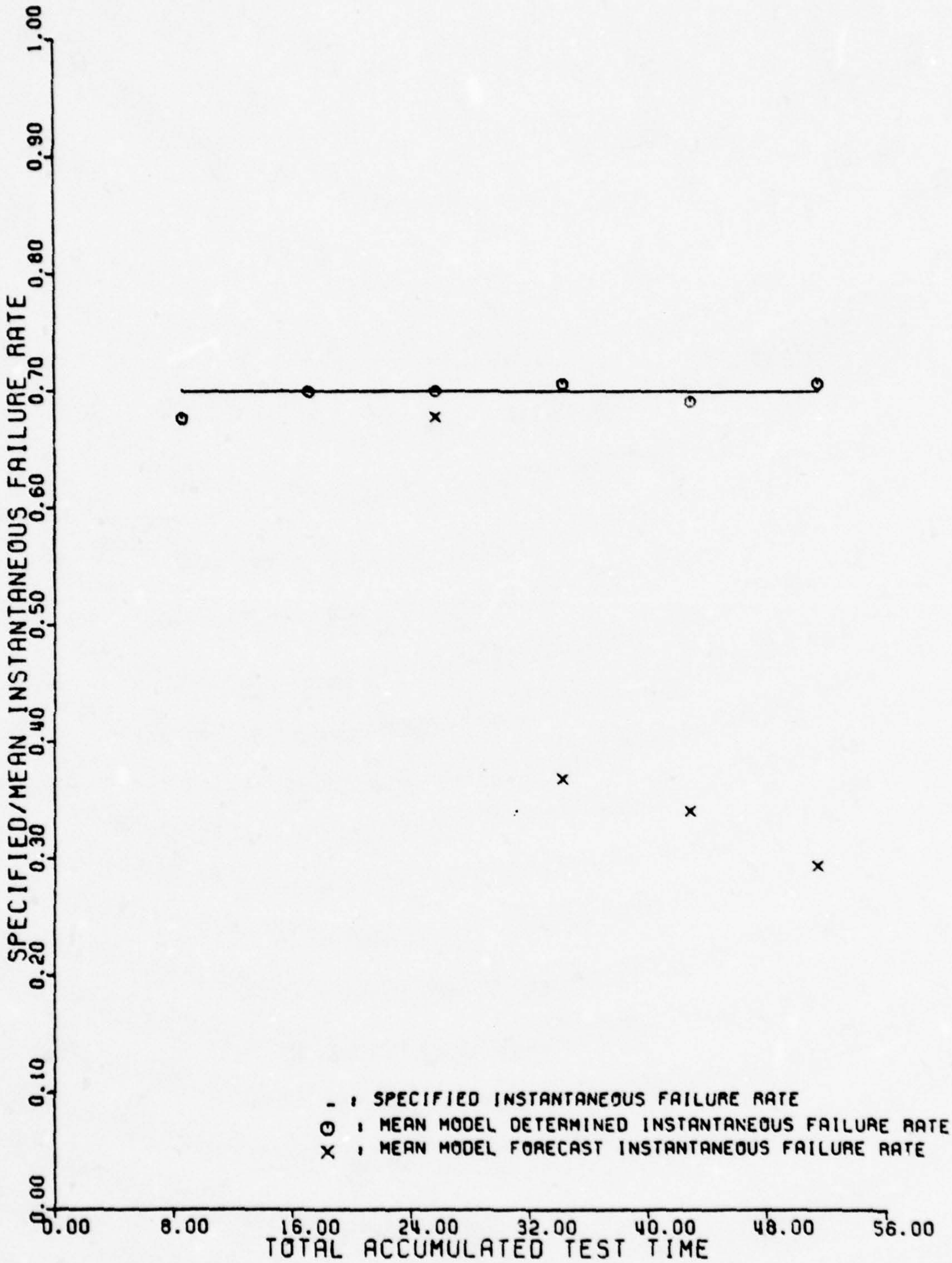

FIGURE 3.69

INSTANTANEOUS R[illegible]IABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET M[illegible]08: 6 PHASES, 20 TESTS/PHASE

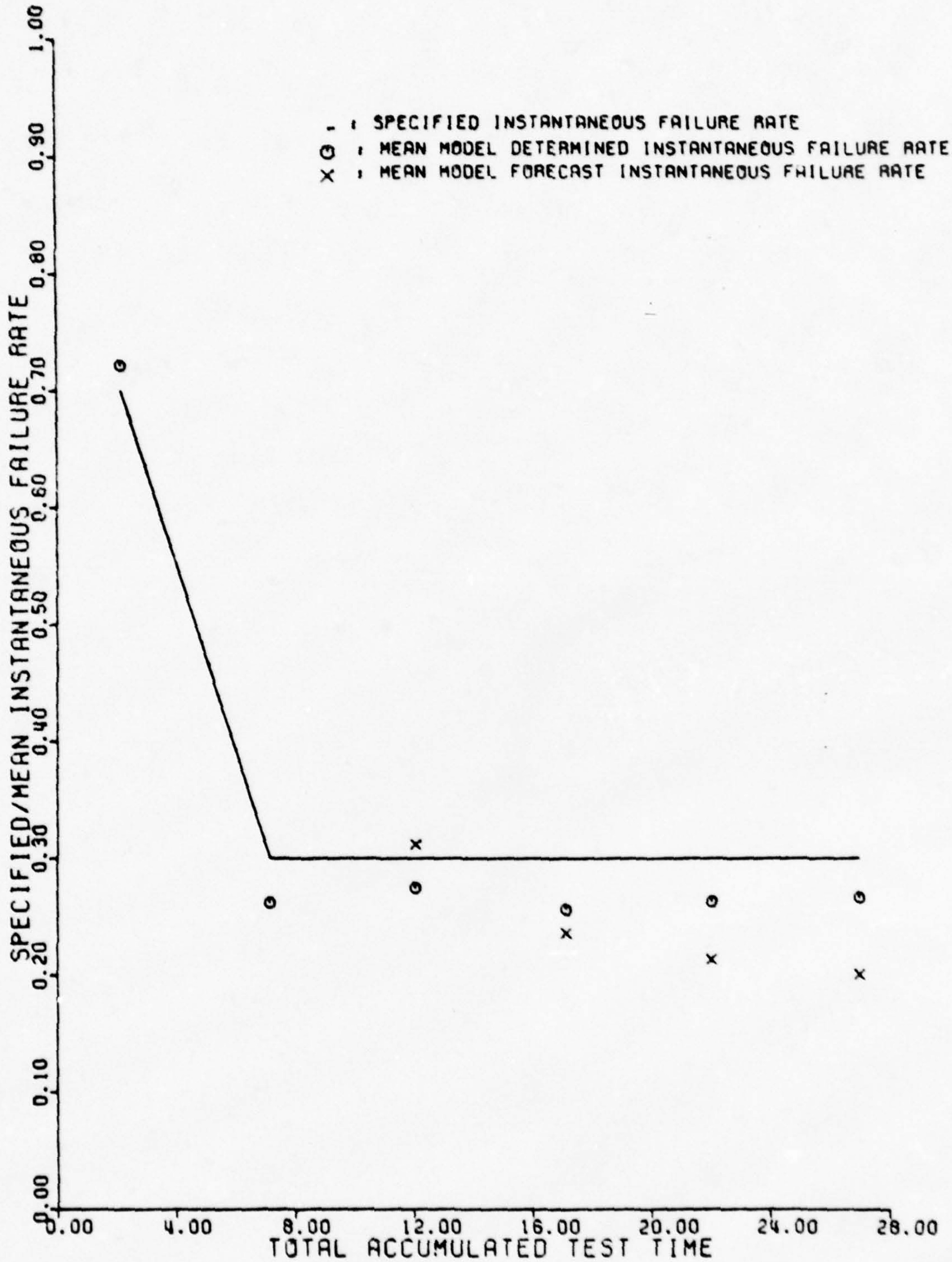

FIGURE 3.70

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD9: 6 PHASES, 5 TESTS/PHASE

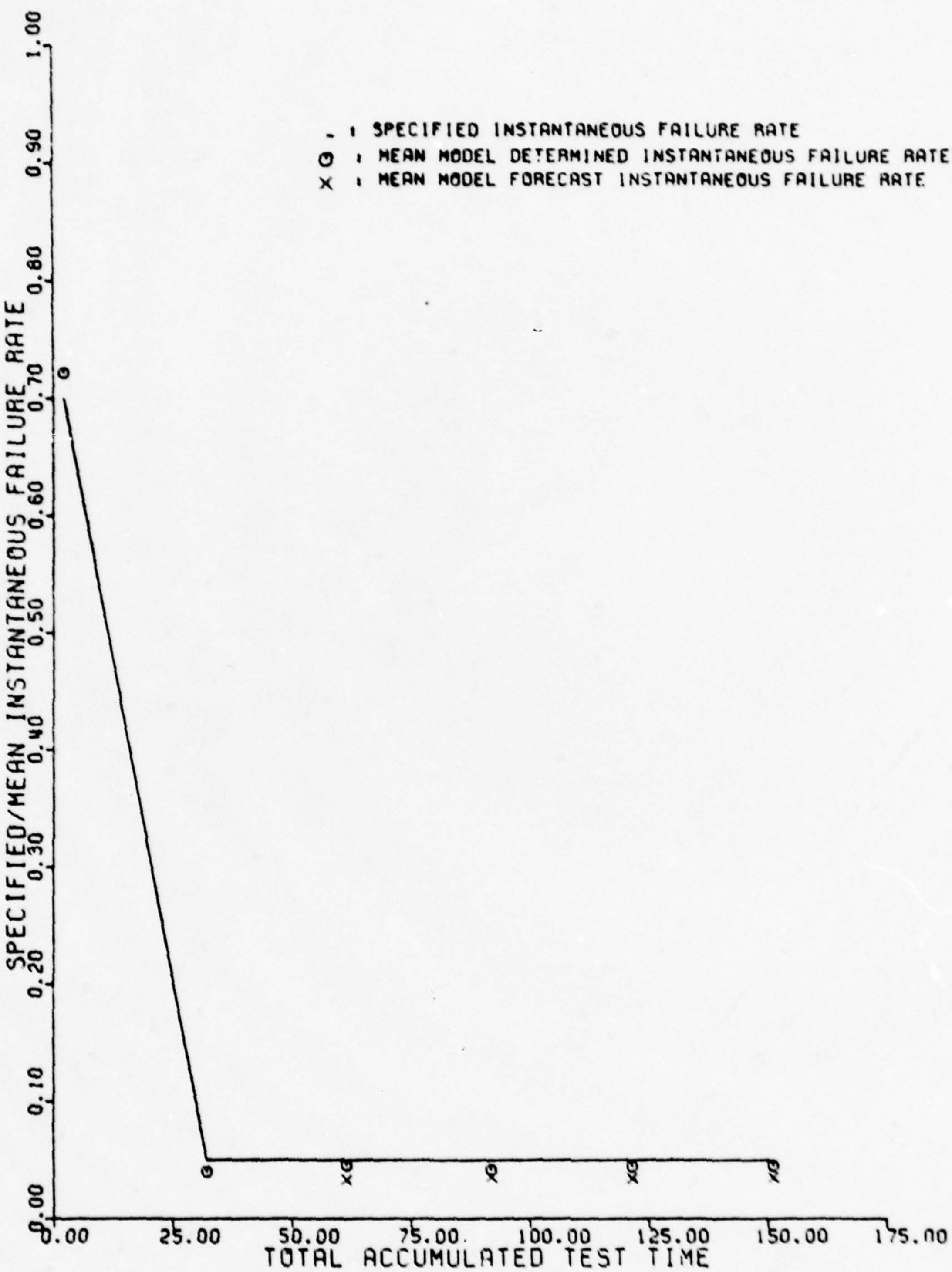

FIGURE 3.71

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD10: 6 PHASES, 5 TESTS/PHASE

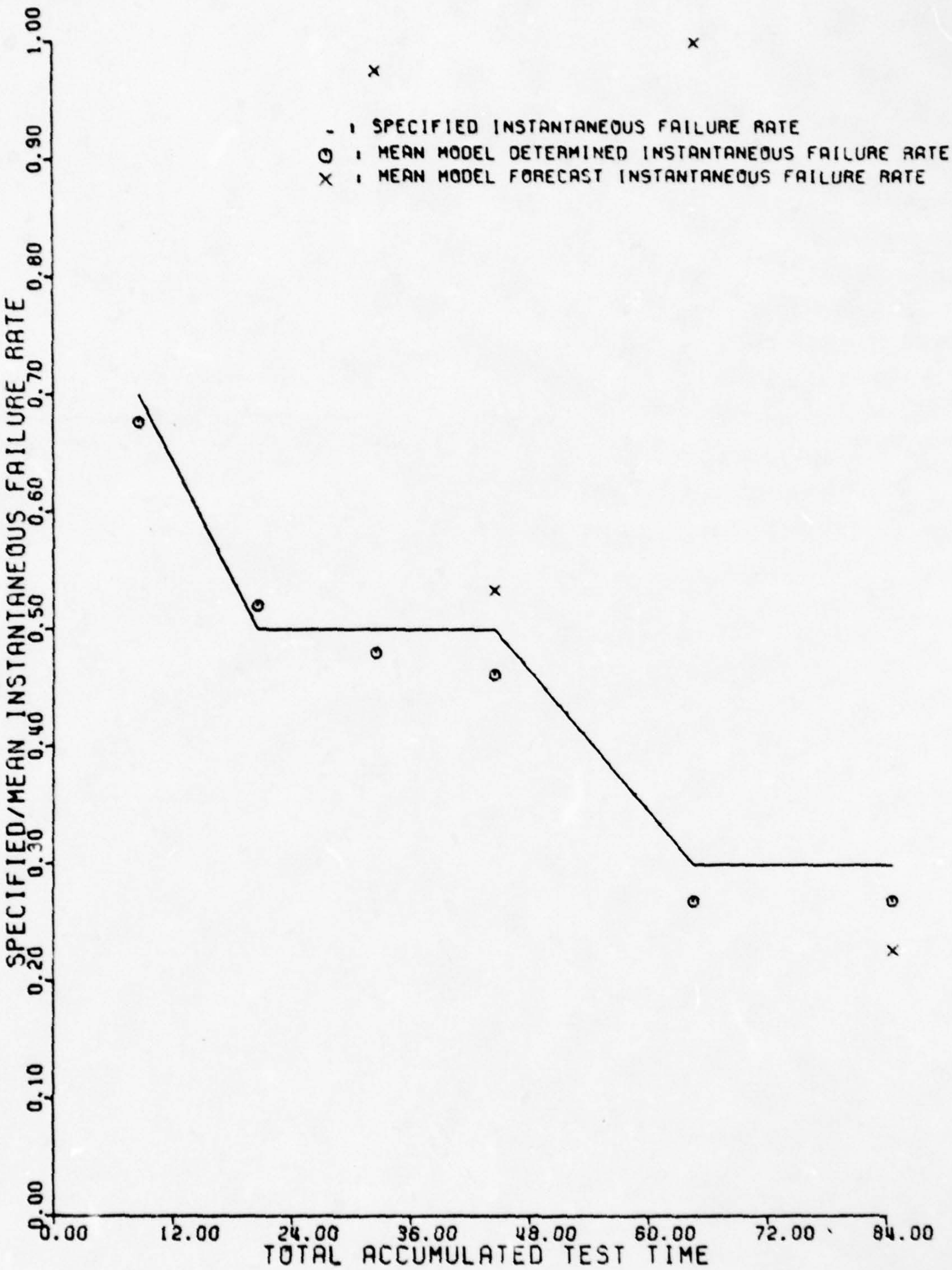

FIGURE 3.72

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD11: 6 PHASES, 20 TESTS/PHASE

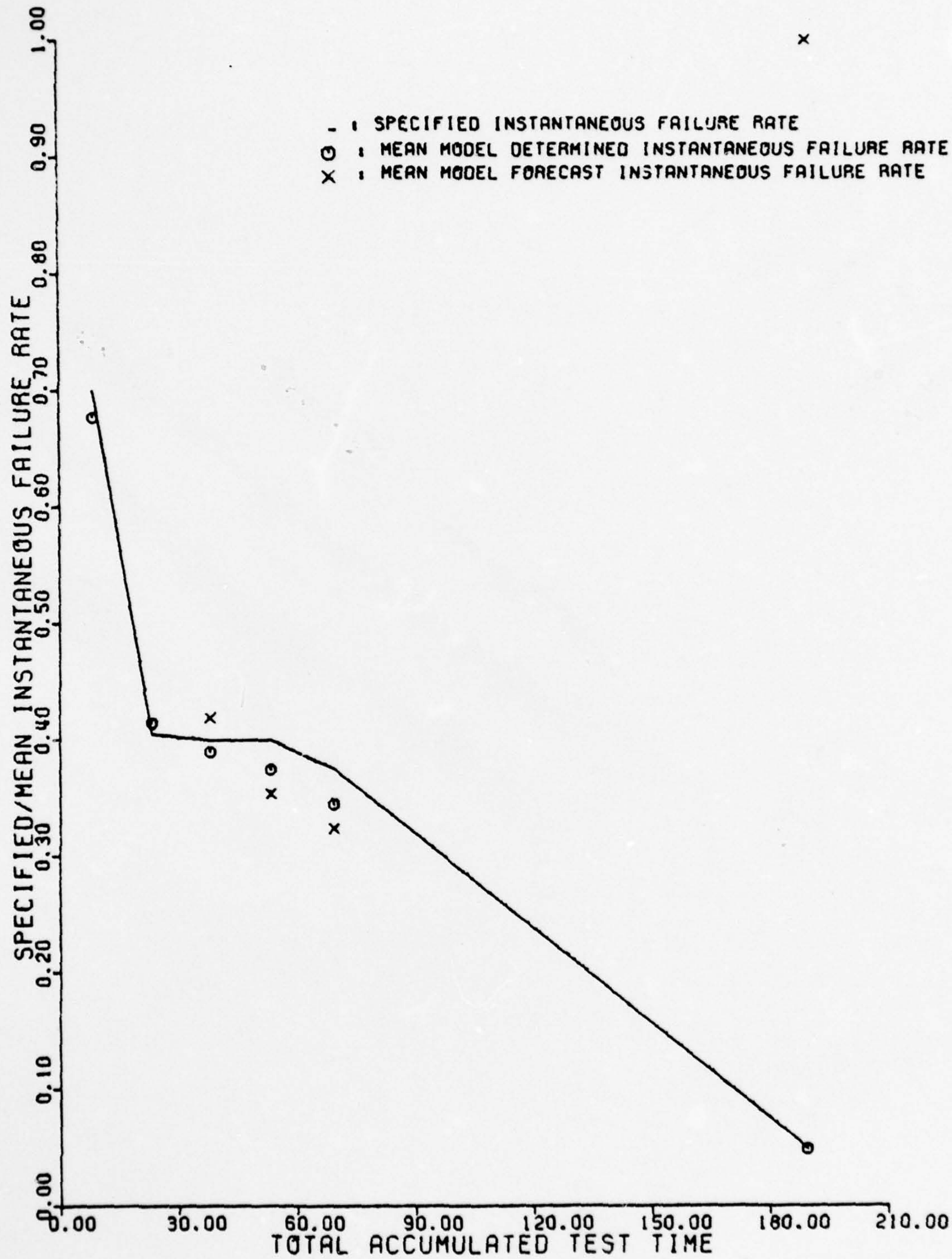

FIGURE 3.73

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD12: 6 PHASES, 20 TESTS/PHASE

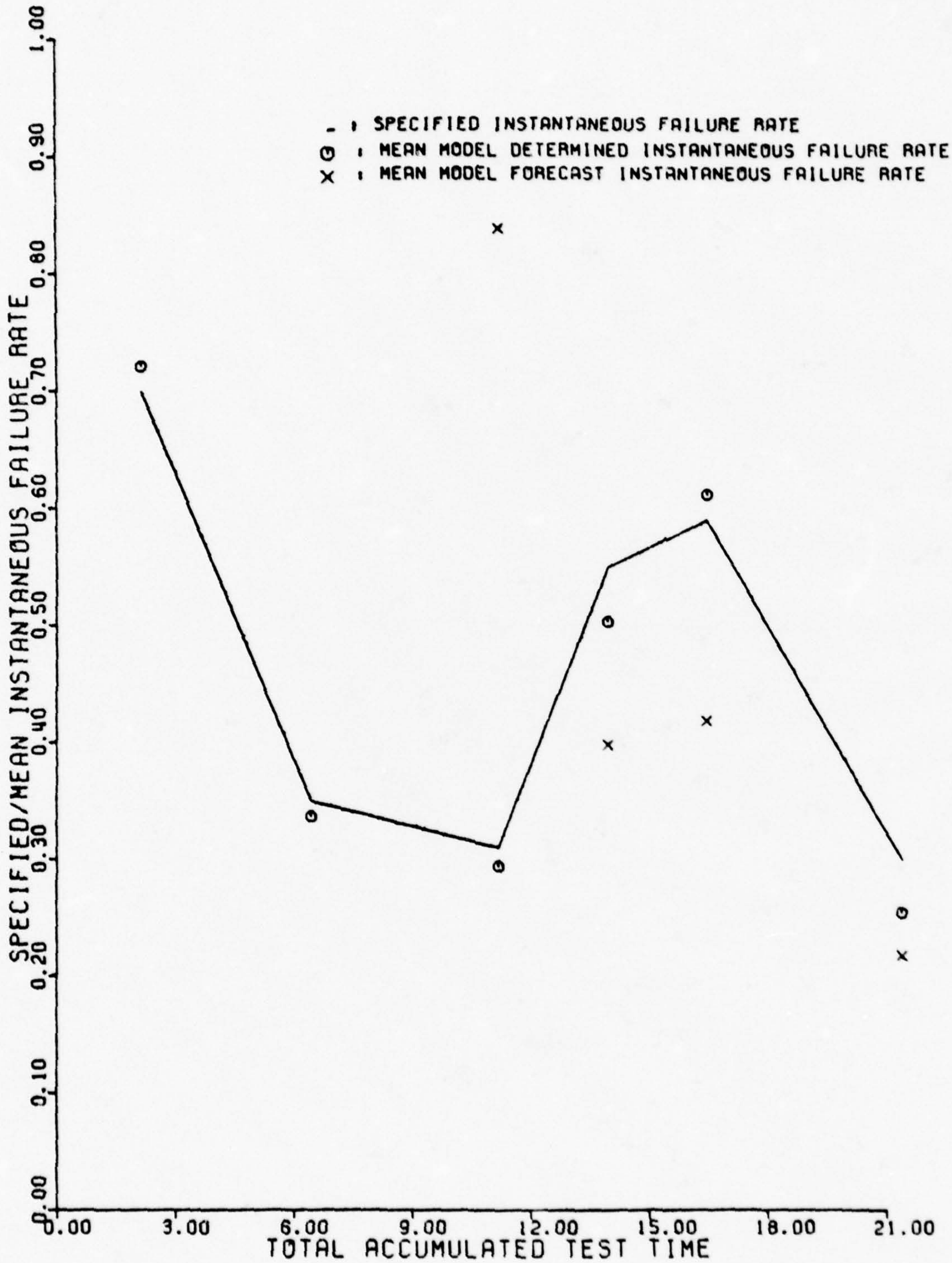

FIGURE 3.74

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD13: 6 PHASES, 5 TESTS/PHASE

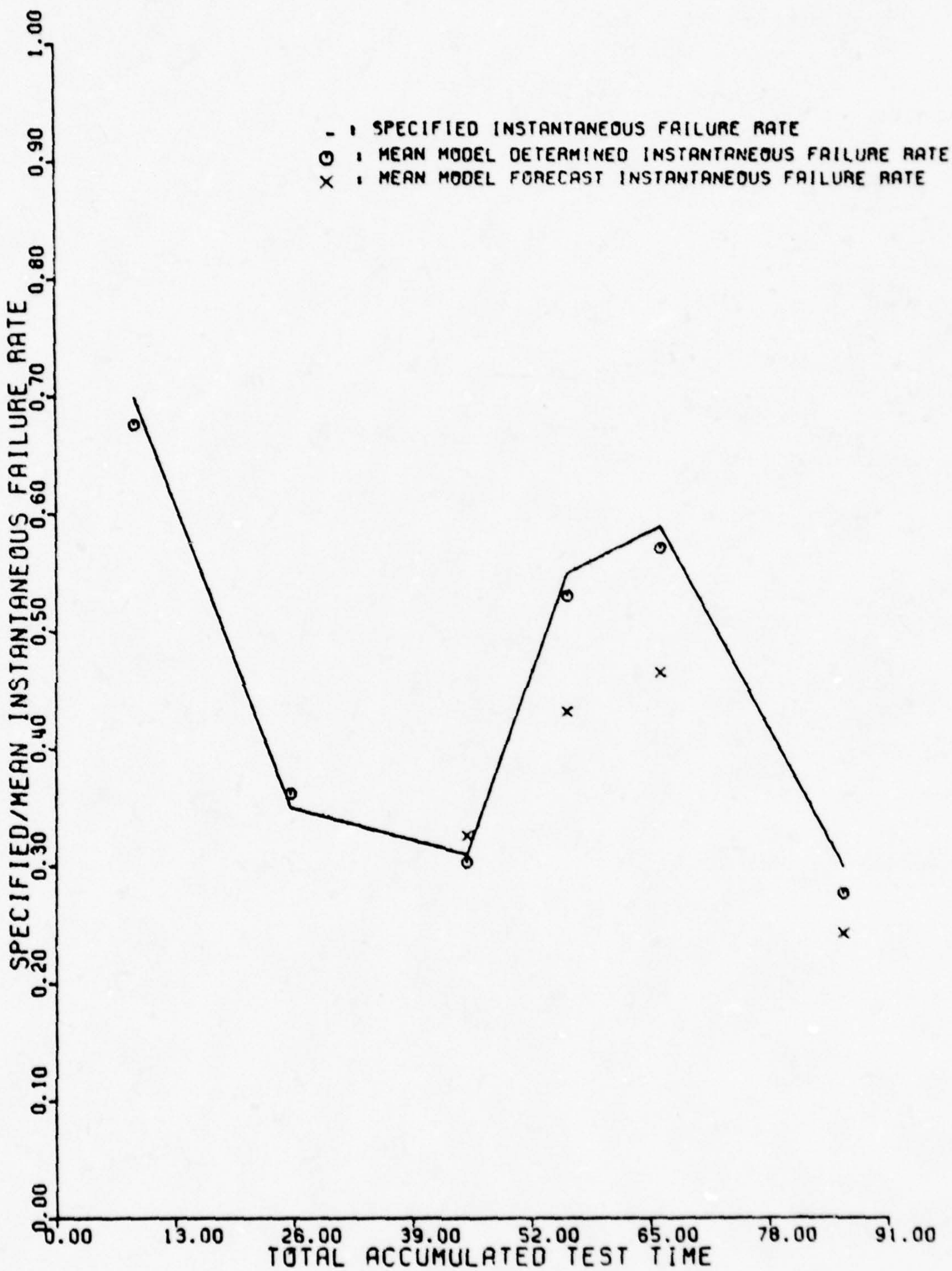

FIGURE 3.75

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE

LAMBDA SET MOD13: 6 PHASES, 20 TESTS/PHASE

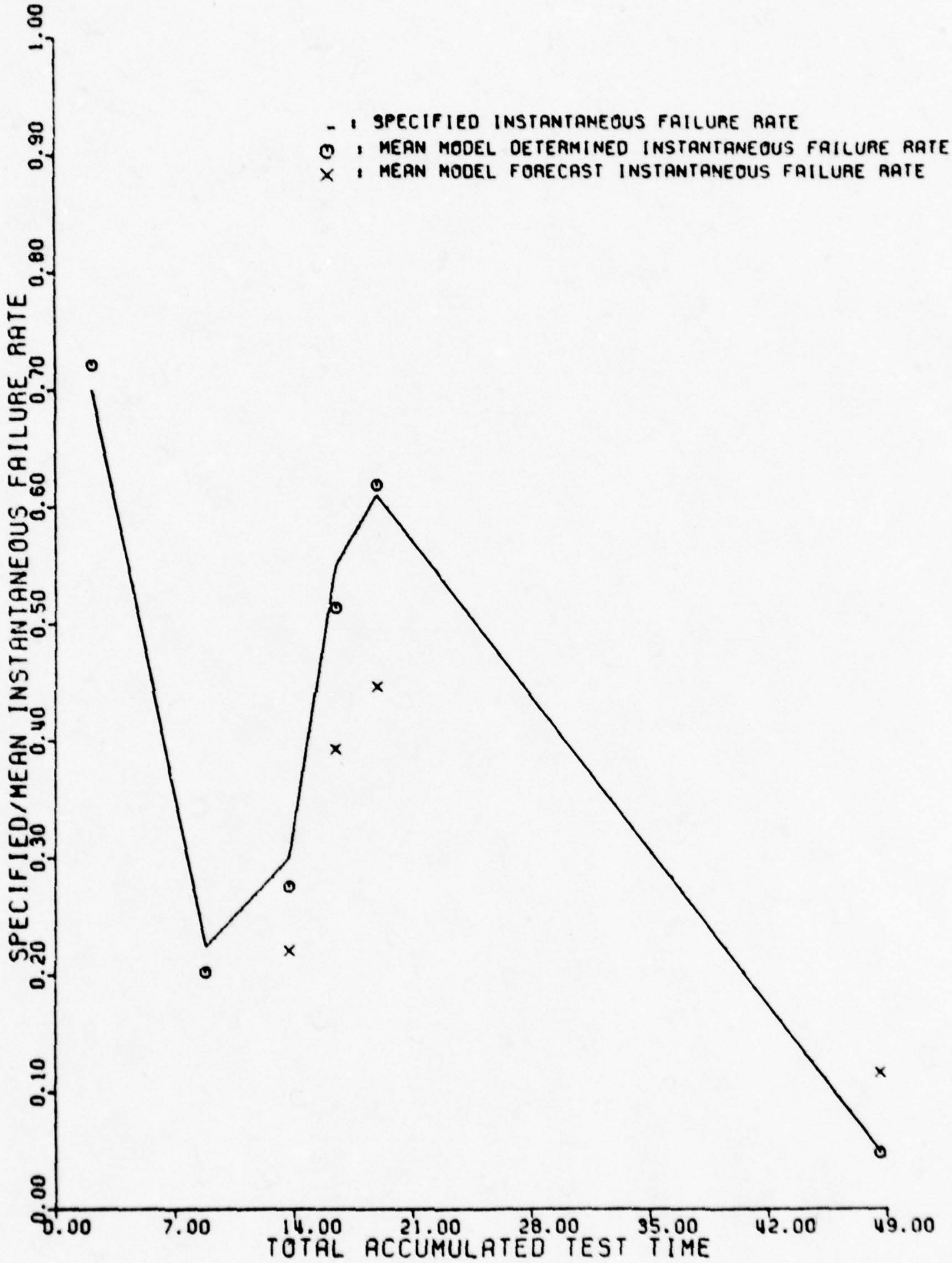

FIGURE 3.76

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD14: 6 PHASES, 5 TESTS/PHASE

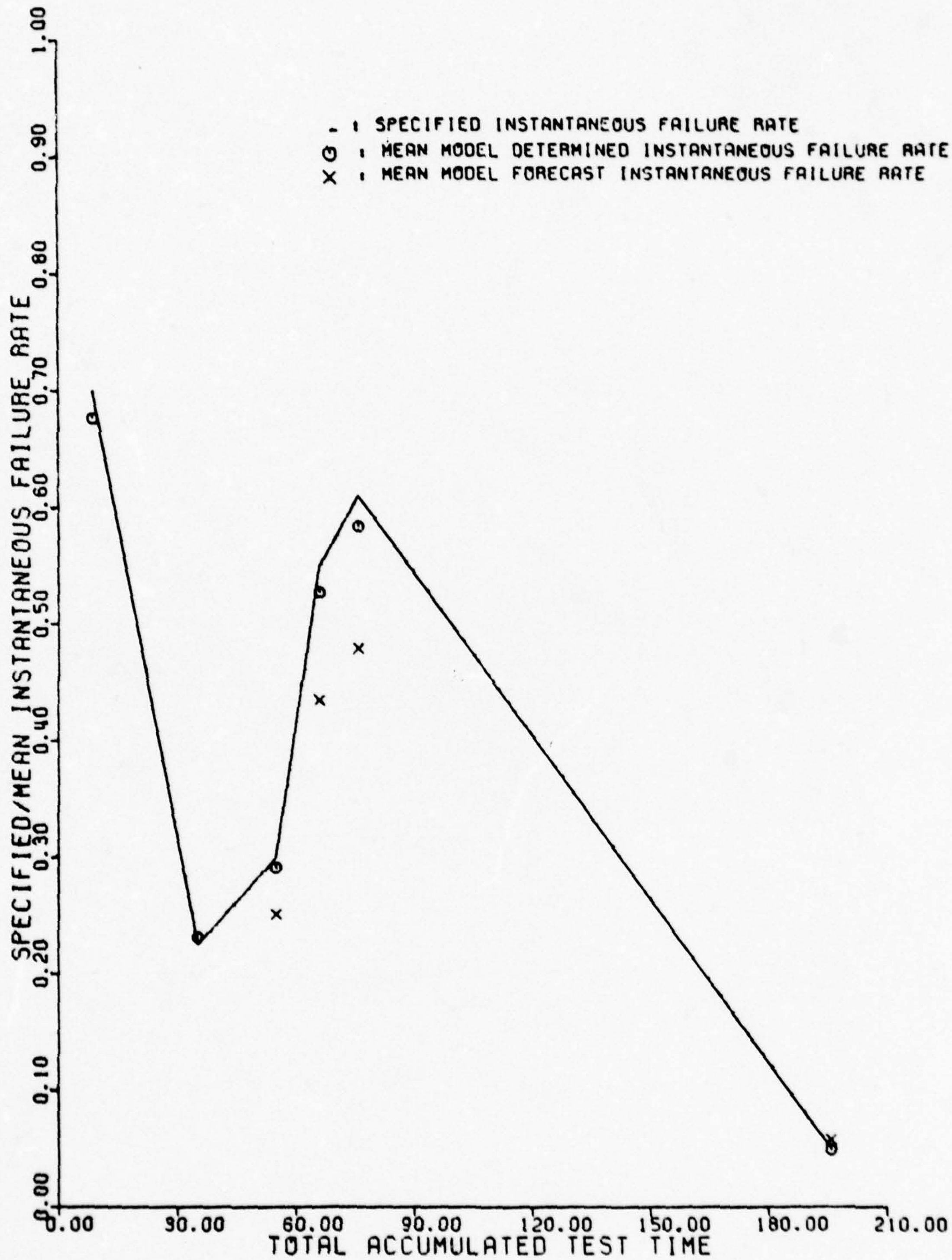

FIGURE 3.77

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD14: 6 PHASES, 20 TESTS/PHASE

these tables are the percentage standard errors as computed in equations 3.48 and 3.53. From figure 3.47 the mean model determined instantaneous failure rate was read as 0.24 for the sixteenth test phase of the lambda set 6, 16 test phase, 20 tests per phase reliability testing procedure. In table 3.17 the percentage standard error corresponding to the sixteenth test phase of the lambda set 6, $NT_i = 20$ simulations is 24%. By equation 3.48 the standard deviation of the instantaneous failure rate model's determined instantaneous failure rate for this phase is then

$$S.D.\,\widehat{\lambda}_{T_{16}} = \frac{P.S.E.\,\widehat{\lambda}_{T_{16}} \times \overline{\widehat{\lambda}}_{T_{16}}}{100} = \frac{24 \times 0.24}{100} = 0.0576\ . \qquad (3.55)$$

A quick examination of tables 3.15 thru 3.26 reveals that the goal of 30% percentage standard error of failure rate estimates utilized in evaluating the variability performance of the cumulative failure rate reliability growth model (chapter III.A.5.b.) will be too restrictive for evaluating the instantaneous failure rate model's variability performance. Also, unlike the variability performance of the cumulative failure rate model which almost uniformly improved as test phases of the testing procedure were completed, variability performance of the instantaneous model oscillates for many lambda sets. For example, keeping in mind that model variability performance begins with the percentage standard errors recorded for phase 2, the instantaneous model's variability performance for lambda set 1, five tests per phase, determined failure rate in table 3.15 improves steadily from 76% in phase 2 to 38% in phase 11. In phase 12 the model's variability performance jumps up to 41%, improves back to 32% by phase 14, and finally, climbs to 35% for the last test phase, phase 16.

This oscillation in the variability performance of the instantaneous failure rate model means that even if a reasonable percentage standard error benchmark is selected, determining the first phase for which variability performance on all lambda sets is within the goal does not guarantee the performance goal will be satisfied for all subsequent test phases of the acquisition cycle.

Tables 3.15, 3.16, and 3.17 display the instantaneous failure rate reliability growth model's variability performance for determined instantaneous failure rate estimates for the sixteen test phase procedures. If a percentage standard error goal of 40% is set, the goal is never achieved for $NT_i = 5$ or $NT_i = 10$ (tables 3.15 and 3.16), but is achieved for $NT_i = 20$ (table 3.17) on test phases 5, 7, 8, 14, and 16. Again, the oscillating character of the instantaneous model's variability performance is displayed by this method of examination. Tables 3.18, 3.19, and 3.20 show that in the case of the contracted six test phase procedures the instantaneous failure rate model's variability performance for determined instantaneous failure rate fails to attain the 40% goal for all simulated tests per phase specifications; i.e., $NT_i = 5$, 10, and 20.

In both the sixteen and six test phase reliability testing procedures determined failure rate estimates given by the instantaneous model were least accurate for lambda set 8 and MOD8. See figures 3.49, 3.68, and 3.69. Also, the model's accuracy performance for these lambda sets was the only case in which no definite bias trend was evident. The instantaneous failure rate model's variability performance for determined instantaneous failure rate estimates is almost uniformly the worst for lambda sets 8 and MOD8 as evidenced in tables 3.15 thru 3.20. This was to be expected based on the model's accuracy performance for these particular specified underlying instantaneous failure rate paths.

Tables 3.21 thru 3.26 contain variability performance results of the instantaneous model's forecast failure rates for both the sixteen and six test phase reliability testing procedures. Percentage standard error entries of 1000% indicate that the actual percentage standard errors in these cases were greater than or equal to 1000%. As with the cumulative failure rate model, lambda set 7, 12, 14, MOD7, MOD12, and MOD14 provide difficulty for the instantaneous model. These lambda sets are the ones which take the most rapid "plunge" to a low specified instantaneous failure rate of $\lambda_{16} = 0.0500$ or $\lambda_6 = 0.0500$. Again, if these lambda sets are not considered, then a 40% percentage standard error goal is achieved on phase 14 for $NT_i = 5$ and 10 and on phase 11 for $NT_i = 20$ in the case of the sixteen phase reliability testing procedure simulations. The instantaneous model's variability performance for forecasting during six test phase cycles is very poor achieving the 40% goal on the last test phase of the "most data" case ($NT_i = 20$) for only seven of the fourteen lambda sets.

C. DISCRETE RELIABILITY GROWTH MODEL

1. Model Description

The discrete reliability growth model evaluated is of the form

$$R_m = 1 - \frac{1}{\exp(\alpha + \beta m)} \qquad (3.56)$$

where R_m = component reliability after the m^{th} modification of the component

m = number of modifications that have been made to the component, and

α, β = constants that determine the change in the model estimated component reliability as modifications to the component are accomplished.

TABLE 3.15

INSTANTANEOUS RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE

16 PHASES, 5 TESTS/PHASE, DETERMINED FAILURE RATE

	LAMBDA SET													
PHASE	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	62	62	62	62	62	62	62	62	62	62	62	62	62	62
2	76	79	80	77	80	84	77	81	81	96	75	79	79	76
3	67	67	62	64	68	78	78	76	62	62	67	66	66	64
4	68	99	66	86	70	108	106	99	67	56	97	84	77	69
5	49	70	49	54	49	92	93	86	52	40	78	60	54	50
6	47	52	46	47	47	77	79	77	41	36	61	49	45	42
7	47	50	46	51	49	66	70	72	42	38	58	47	44	38
8	45	49	44	50	48	67	73	81	42	35	64	48	40	38
9	43	47	43	49	47	65	71	84	42	34	66	48	40	42
10	40	41	39	48	43	63	76	78	39	29	66	45	36	40
11	38	46	37	42	38	58	54	82	46	31	57	54	48	46
12	41	59	39	45	40	70	60	100	61	36	63	72	60	58
13	38	70	37	41	37	86	61	126	74	38	67	91	67	54
14	32	39	31	38	31	43	45	88	41	25	40	48	40	36
15	33	39	31	39	31	42	51	75	40	26	39	63	36	31
16	35	50	34	40	33	48	68	55	58	32	46	60	45	72

TABLE 3.16

INSTANTANEOUS RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE

16 PHASES, 10 TESTS/PHASE, DETERMINED FAILURE RATE

PHASE	LAMBDA SET 1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	58	58	58	58	58	58	58	58	58	58	58	58	58	58
2	66	73	66	67	68	70	70	69	70	61	63	67	67	64
3	49	51	49	52	51	55	58	53	44	46	46	50	49	51
4	45	46	44	48	46	58	61	60	39	39	43	41	43	44
5	46	46	45	47	47	67	69	68	38	36	49	42	44	43
6	39	40	40	41	43	54	58	59	34	32	43	36	36	33
7	33	34	32	38	37	54	61	65	28	26	41	30	29	26
8	31	33	30	36	33	41	45	54	27	24	38	29	26	29
9	31	32	30	36	33	38	41	52	26	24	36	29	26	30
10	36	37	34	42	37	46	48	70	31	26	49	35	37	36
11	35	40	34	41	36	49	50	79	34	26	54	39	37	35
12	33	34	32	39	33	41	46	58	28	24	45	31	29	29
13	27	29	26	31	27	34	37	65	26	21	35	30	30	28
14	27	29	26	31	26	34	38	65	26	21	33	44	25	25
15	25	27	24	28	23	32	36	62	25	19	28	44	22	21
16	27	30	26	31	25	36	52	62	29	21	31	50	30	48

TABLE 3.17

INSTANTANEOUS RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE

16 PHASES, 20 TESTS/PHASE, DETERMINED FAILURE RATE

	LAMBDA SET													
PHASE	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	42	42	42	42	42	42	42	42	42	42	42	42	42	42
2	45	42	39	45	41	39	42	40	39	42	41	40	40	41
3	36	35	36	37	38	46	44	43	30	29	33	38	40	38
4	36	34	36	38	38	43	46	47	26	26	29	33	36	38
5	31	28	31	32	33	31	33	37	23	22	24	26	30	30
6	29	29	29	32	31	38	44	48	22	22	27	25	27	24
7	25	24	25	27	26	32	36	40	20	20	24	22	22	20
8	26	26	25	28	27	31	33	40	20	19	24	22	20	19
9	23	23	22	26	24	28	30	43	17	16	23	20	17	22
10	22	23	21	25	22	27	29	43	19	17	26	21	26	26
11	22	21	21	25	22	25	28	44	16	15	33	18	21	22
12	20	20	20	22	20	24	26	42	18	16	28	20	22	22
13	22	22	21	25	21	27	30	43	18	16	27	22	24	24
14	19	20	19	22	18	25	28	37	16	15	23	34	16	17
15	19	20	19	22	18	25	28	45	17	15	21	37	16	17
16	18	20	18	21	17	24	34	38	17	15	20	33	20	34

TABLE 3.18

INSTANTANEOUS RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE

6 PHASES, 5 TESTS/PHASE, DETERMINED FAILURE RATE

	LAMBDA SET													
PHASE	MOD1	MOD2	MOD3	MOD4	MOD5	MOD6	MOD7	MOD8	MOD9	MOD10	MOD11	MOD12	MOD13	MOD14
1	65	65	65	65	65	65	65	65	65	65	65	65	65	65
2	80	74	76	73	79	81	83	76	80	90	67	74	71	78
3	64	67	67	70	70	89	91	81	57	62	76	64	57	56
4	56	58	58	64	61	71	70	69	50	54	59	53	53	50
5	52	53	53	59	52	59	61	83	50	46	68	54	60	58
6	52	53	52	62	52	57	78	88	50	44	55	81	55	77

TABLE 3.19

INSTANTANEOUS RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE

6 PHASES, 10 TESTS/PHASE, DETERMINED FAILURE RATE

LAMBDA SET

PHASE	MOD1	MOD2	MOD3	MOD4	MOD5	MOD6	MOD7	MOD8	MOD9	MOD10	MOD11	MOD12	MOD13	MOD14
1	46	46	46	46	46	46	46	46	46	46	46	46	46	46
2	56	60	65	64	63	64	63	64	68	54	62	56	61	66
3	54	53	52	55	53	52	57	58	44	50	47	44	49	40
4	46	46	46	51	49	51	53	63	41	41	45	42	40	40
5	39	36	37	46	39	42	50	62	31	30	49	35	35	35
6	42	40	40	49	39	51	57	64	34	32	45	57	42	55

TABLE 3.20

INSTANTANEOUS RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE

6 PHASES, 20 TESTS/PHASE, DETERMINED FAILURE RATE

	LAMBDA SET													
PHASE	MOD1	MOD2	MOD3	MOD4	MOD5	MOD6	MOD7	MOD8	MOD9	MOD10	MOD11	MOD12	MOD13	MOD14
1	37	37	37	37	37	51	37	37	37	37	37	37	37	37
2	45	45	44	45	45	37	51	48	45	43	49	44	47	44
3	38	39	39	40	39	38	37	43	33	35	33	33	35	30
4	34	34	33	38	35	35	40	43	27	27	28	27	26	27
5	31	32	30	35	31	36	37	44	25	24	37	28	27	27
6	24	25	23	27	22	28	34	45	19	19	25	36	25	37

TABLE 3.21

INSTANTANEOUS RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE

16 PHASES, 5 TESTS/PHASE, FORECAST FAILURE RATE

PHASE	LAMBDA SET 1	2	3	4	5	6	7	8	9	10	11	12	13	14
3	1000	1000	973	1000	978	997	1000	890	240	90	994	1000	1000	1000
4	133	168	94	612	161	99	99	76	64	54	764	244	281	133
5	213	140	117	673	207	162	168	48	52	44	69	292	339	135
6	82	61	65	153	75	131	170	39	43	36	41	368	167	52
7	54	55	49	67	54	51	44	33	36	30	34	49	45	29
8	45	45	42	59	49	59	53	30	32	28	32	41	31	22
9	44	41	41	58	48	52	47	26	30	26	30	35	22	21
10	45	39	42	62	46	41	47	22	28	25	27	31	19	21
11	44	36	40	71	45	42	51	21	25	23	65	27	18	21
12	36	28	34	45	35	30	38	19	21	20	46	22	16	18
13	33	27	31	41	31	29	43	18	20	19	34	23	15	17
14	30	27	28	36	28	32	47	17	19	18	33	51	15	15
15	29	27	27	37	26	36	55	17	18	17	30	83	19	22
16	28	26	26	36	24	41	144	16	17	16	30	98	28	112

TABLE 3.22

INSTANTANEOUS RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE

16 PHASES, 10 TESTS/PHASE, FORECAST FAILURE RATE

	LAMBDA SET													
PHASE	1	2	3	4	5	6	7	8	9	10	11	12	13	14
3	739	994	243	1000	997	998	1000	614	89	56	1000	1000	1000	978
4	93	806	76	240	129	615	114	550	52	42	192	111	329	118
5	70	644	58	179	76	78	75	58	43	35	157	76	84	58
6	60	59	54	105	85	64	64	52	40	31	57	46	53	41
7	49	59	46	63	54	57	56	48	37	27	51	40	39	25
8	43	51	39	61	46	49	54	41	30	22	45	34	26	22
9	36	49	33	49	38	53	56	40	27	21	37	32	19	23
10	37	53	34	48	37	56	59	40	27	20	37	33	20	23
11	40	66	35	53	37	80	84	35	26	20	91	33	20	22
12	38	61	34	48	34	70	73	33	24	19	64	33	19	21
13	38	67	33	46	32	78	73	30	24	18	49	40	17	19
14	29	33	27	34	26	39	47	29	23	18	37	93	16	17
15	26	30	25	31	24	38	47	28	22	17	34	108	19	20
16	25	27	23	29	21	36	85	24	19	15	29	63	26	73

TABLE 3.23

INSTANTANEOUS RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE

16 PHASES, 20 TESTS/PHASE, FORECAST FAILURE RATE

	LAMBDA SET													
PHASE	1	2	3	4	5	6	7	8	9	10	11	12	13	14
3	1000	949	820	874	1000	425	988	991	114	41	908	759	762	1000
4	233	334	111	999	340	437	1000	755	36	26	95	782	656	244
5	54	62	49	78	60	89	79	46	32	24	43	51	59	48
6	42	71	38	58	45	56	64	37	28	21	36	39	41	29
7	33	37	31	39	34	53	52	35	25	19	32	29	27	18
8	26	29	25	31	27	51	64	33	21	17	26	24	19	14
9	27	32	25	32	27	53	80	29	20	16	25	24	16	15
10	25	29	23	31	25	46	57	24	19	15	24	22	13	16
11	22	26	21	26	21	35	40	23	19	14	58	21	15	17
12	22	25	21	27	21	32	39	23	17	13	42	19	13	15
13	20	22	19	23	18	28	33	22	16	13	33	20	11	13
14	20	22	19	24	18	29	35	22	16	12	27	52	11	11
15	19	22	18	22	17	29	35	22	16	12	24	63	14	15
16	18	21	17	21	15	29	61	22	15	11	20	51	20	49

TABLE 3.24

INSTANTANEOUS RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE

6 PHASES, 5 TESTS/PHASE, FORECAST FAILURE RATE

LAMBDA SET

PHASE	MOD1	MOD2	MOD3	MOD4	MOD5	MOD6	MOD7	MOD8	MOD9	MOD10	MOD11	MOD12	MOD13	MOD14
3	212	626	138	897	223	962	1000	888	174	86	1000	934	442	74
4	172	116	108	745	146	132	999	81	69	57	325	116	37	43
5	62	87	60	81	61	123	133	53	51	47	475	77	34	32
6	55	83	51	71	46	147	273	38	42	36	208	857	77	403

TABLE 3.25

INSTANTANEOUS RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE

6 PHASES, 10 TESTS/PHASE, FORECAST FAILURE RATE

PHASE	LAMBDA SET													
	MOD1	MOD2	MOD3	MOD4	MOD5	MOD6	MOD7	MOD8	MOD9	MOD10	MOD11	MOD12	MOD13	MOD14
3	87	276	84	195	110	1000	996	1000	73	50	512	84	85	58
4	67	121	59	103	65	589	647	161	49	44	443	71	34	31
5	51	60	46	74	49	134	567	61	38	34	240	47	28	29
6	47	59	42	67	40	118	300	44	32	26	92	454	59	164

TABLE 3.26

INSTANTANEOUS RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE

6 PHASES, 20 TESTS/PHASE, FORECAST FAILURE RATE

LAMBDA SET

PHASE	MOD1	MOD2	MOD3	MOD4	MOD5	MOD6	MOD7	MOD8	MOD9	MOD10	MOD11	MOD12	MOD13	MOD14
3	76	268	63	245	95	916	658	410	51	39	198	67	69	36
4	50	87	45	77	50	831	999	117	36	31	132	47	23	23
5	46	85	40	67	41	473	971	77	33	25	991	67	19	20
6	38	62	34	51	31	70	180	37	29	22	42	1000	40	95

The modification number m takes on values 0, 1, 2, . . . with m = 0 designating the original version of the component. Component reliability R_m may be thought of as an instantaneous or modification reliability as opposed to a cumulative or average reliability since it is the intrinsic reliability of the m^{th} modification version of the item rather than a characteristic reliability for all m+1 versions of the item that have been produced. Under the assumption of reliability growth, the quantity exp $(\alpha + \beta m)$ in equation 3.56 is expected to increase as modifications to the component are made which hopefully improve its reliability. Hence, as modifications are accomplished the fraction $1/\exp(\alpha + \beta m)$ in equation 3.56 decreases and component reliability increases toward the maximum reliability of 1.0. The model is considered discrete since the single independent or control variable m, modification number, is discrete as opposed to a continuous variable such as total accumulated test time.

A mathematical analysis of the discrete reliability growth model is given in reference 1. Other Monte Carlo simulation evaluations and comparisons of the discrete model are contained in references 5 and 9.

2. Reliability Testing Procedure

For the discrete reliability growth model the appropriate reliability testing procedure of a system acquisition cycle is one in which modifications to an item are made as a specified number of failures of the item are observed during the testing procedure. The modifications of the item are made to improve reliability and hopefully result in such improvement. Modifications are indexed by m with a total of NTM modifications being considered during an acquisition cycle; i.e., m = 0, 1, 2, 3, ... , NTM. The total number of item failures that are observed during testing of the m^{th} modification version of an item before the item is modified again is denoted by NTF_m . The total number of item failures NTF_m is specified before reliability testing is conducted and may be varied from modification to modification. During testing of each modification version of an item, an underlying, inherent reliability that is unknown to the project managers/contractors is present in the item. The underlying modification reliability of the m^{th} modification version of the component is denoted as R_m .

Between each failure of the m^{th} modification version of an item a number of tests of the item are conducted which are denoted as $NT_{m,j}$; i.e., the number of tests run on the m^{th} modification version of the item between the $(j-1)^{th}$ failure and the j^{th} failure. $NT_{m,j}$ includes the j^{th} failure. So, $NT_{m,1}$ is the number of tests performed thru the first observed failure of the item when the underlying modification reliability is R_m . $NT_{m,2}$ is the number of tests performed from the first failure thru the second failure of the item while the underlying modification reliability remains R_m and so on. After NTF_m failures of the item under test are observed, a modification is accomplished and the underlying modification reliability of the component changes to R_{m+1} . Therefore, the failure number index j runs from 1 to NTF_m for each modification m ; i.e., $NT_{m,1}$, $NT_{m,2}$, ... , NT_{m,NTF_m} All $NT_{m,j}$ are random variables and cannot be specified beforehand. This is quite different from the testing procedure for the continuous reliability growth models evaluated where the number of tests per test phase were specified prior to any testing being performed.

Figure 3.78 is a schematic diagram of the discrete model reliability testing procedure. The total number of modifications NTM and the total number of failures permitted before modification NTF_m are

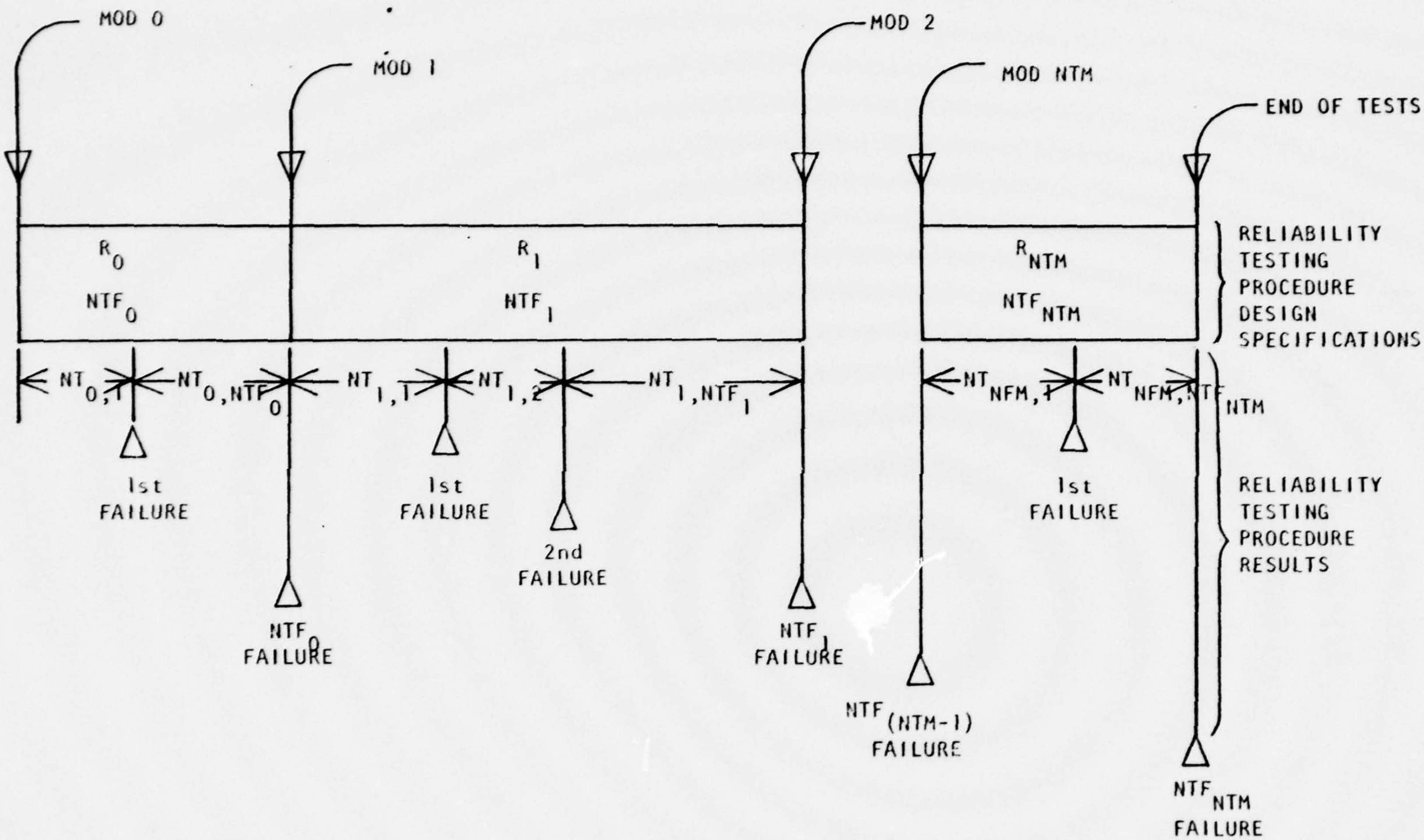

FIGURE 3.78

DISCRETE RELIABILITY GROWTH MODEL RELIABILITY TESTING PROCEDURE DIAGRAM specified prior to testing. The number of tests performed between failures $NT_{m,j}$ for each modification version of the item under test constitute the data collected from the reliability testing procedure. As with the continuous reliability growth models the most salient feature of the computer simulation of the discrete model reliability testing procedure is that the analyst also specifies the underlying modification reliability R_m that is effective for each modification version of the item. Thus an accurate assessment of the accuracy of the model can be made because the R_m values are known.

3. Reliability Testing Procedure Computer Simulation

a. Summary

The reliability testing procedure appropriate to the discrete reliability growth model was simulated on the Naval Postgraduate School W. R. Church Computer Center's IBM 360/67 System utilizing standard computer simulation techniques. Reliability testing procedure design parameters NTM and NTF_m were specified along with various underlying modification reliability sets. Tests of components were then simulated with component success or failure being determined for each test based on the appropriate underlying modification reliability R_m. Number of tests between failures $NT_{m,j}$ data were collected as each procedure simulation was accomplished. Simulations of reliability testing procedures for each specified modification reliability set were replicated one-hundred times in order to assess the discrete reliability growth model's variability performance.

After the reliability testing procedure simulations were completed, discrete reliability growth model estimates of the underlying modification reliability of the component were produced based on the simulated test data by computations utilizing ordinary least squares regression techniques to estimate the model parameters α and β. Mean and variability statistics of the model reliability estimates were then computed to permit performance evaluation of the discrete reliability growth model.

b. Detail Description

The computer simulation of the reliability testing procedure appropriate to the discrete reliability growth model was initiated by reading in the following reliability testing procedure design specifications: NTM, the total number of modifications to be made for the acquisition cycle under consideration; NTF_m, the total number of failures of the m^{th} modification version of the item to be observed until the $(m+1)^{th}$ modification is made; R_m, the underlying, intrinsic modification reliability of the m^{th} modification version of the item; and NSIMS, the number of times the specified reliability testing procedure was to be simulated. For this thesis NSIMS was always specified as one-hundred simulations.

After specification data were read into the computer, uniform (0,1) random variates were drawn in sequence corresponding to component tests starting for the modification 0 type component; i.e., uniform random variates were drawn in sequence and indexed as $U_{m,j,i}$ for m = 0, 1, 2, ..., NTM; j = 1, 2, 3, ... , NTF_m; and i = 1, 2, 3, ... starting with the $U_{0,1,1}$ test.

Each uniform variate was then compared with the appropriate underlying modification reliability R_m to determine if a success or a failure occurred during the test. The number of tests $NT_{m,j}$ and the number of failures accumulated NF_m data were recorded after each simulated test such that

$$NT_{m,j} = NT_{m,j} + 1 \text{ (}NT_{m,j}\text{ incremented by one test), and} \tag{3.57}$$

$$NF_m = \begin{cases} NF_m \text{ if } U_{m,j,i} < R_m & \text{(success)} \\ NF_m + 1 \text{ if } U_{m,j,i} \geq R_m & \text{(failure)} \end{cases} \tag{3.58}$$

for $m = 0, 1, 2, \ldots, NTM$; $j = 1, 2, 3, \ldots, NTF_m$; and $i = 1, 2, 3, \ldots$ If the test was a success, then the simulation proceeded to the next test. If the test was a failure, then the number of failures accumulated NF_m was compared to the total number of failures until modification NTF_m to determine if a modification was appropriate. If NF_m was less than NTF_m, testing continued for the given modification and number of tests were then counted until the next failure; i.e., $NT_{m,j+1}$. If NF_m was equal to NTF_m, then a modification of the item under test was simulated and testing continued for the next modification; i.e., NF_{m+1} and $NT_{m+1,1}$ being tallied next.

Testing in this sequence was simulated until NF_{NTM} was equal to NTF_{NTM} which signalled the termination of a given reliability testing procedure computer simulation. The simulation resulted in the number of tests until a failure $NT_{m,j}$ data being recorded for $m = 0, 1, 2, \ldots, NTM$ and $j = 1, 2, 3, \ldots, NTF_m$. These data constituted the total data gathered from a single computer simulation of the discrete model reliability testing procedure for a single specified underlying modification reliability progress path; i.e., a specified set of reliabilities which shall henceforth be referred to as a reliability set. The reliability testing procedure was then replicated one-hundred times ($r = 1, 2, 3, \ldots, NSIMS = 100$) such that reliability testing procedure results data were actually indexed as $NT_{m,j,r}$. Since ten reliability sets were utilized for evaluating the discrete reliability growth model with three different total number of failures until modification NTF_m specified for each reliability set, 3000 reliability testing procedures were simulated for the discrete reliability growth model evaluation.

4. Computer Simulation Data Manipulation

The discrete reliability growth model of equation 3.56 is proposed as a model of the unknown, underlying modification reliability R_m intrinsic to a component as it proceeds thru a system acquisition cycle. It is against this true reliability R_m that the model's performance was measured.

Equation 3.56 may be given in the form

$$R_m = 1 - \exp(-\alpha - \beta m) \text{ or} \tag{3.59}$$

$$\exp(-\alpha - \beta m) = 1 - R_m \tag{3.60}$$

for m = 0, 1, 2, ... , NTM. The logarithmic transformation of equation 3.60 yields

$$(\alpha + \beta m)_j = -\ln(1 - R_m)_j \tag{3.61}$$

for m = 0, 1, 2, ... , NTM and j = 1, 2, 3, ... , NTF_m .

An unbiased estimator of the quantity $\alpha + \beta m$ is given in reference 1 as

$$\widehat{(\alpha + \beta m)}_{m,j} = \begin{cases} 0.0 \text{ if } NT_{m,j} = 1 \text{ (first test was failure)} \\ 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{NT_{m,j} - 1} \text{ if } NT_{m,j} \geq 2 \end{cases} \tag{3.62}$$

for m = 0, 1, 2, ... , NTM and j = 1, 2, 3, ... , NTF_m .

By equation 3.61

$$\widehat{-\ln(1-R_m)}_{m,j} = \widehat{(\alpha + \beta m)}_{m,j} \qquad (3.63)$$

for $m = 0, 1, 2, \ldots, NTM$ and $j = 1, 2, 3, \ldots, NTF_m$. From this relation an average estimate may be computed as

$$\overline{\widehat{-\ln(1-R_m)}}_m = \overline{\widehat{(\alpha + \beta m)}}_m = \frac{1}{NTF_m} \sum_{j=1}^{NTF_m} \widehat{(\alpha + \beta m)}_{m,j} \, . \qquad (3.64)$$

Letting

$$Y_m = \overline{\widehat{-\ln(1-R_m)}}_m \, ,$$

then enuation 3.61 may be written as

$$Y_m = \widehat{\alpha_m} + \widehat{\beta_m} m \qquad (3.65)$$

for $m = 0, 1, 2, \ldots, NTM$ which is of the form $Y = \alpha + \beta X$. Applying ordinary least squares regression estimates as given in reference 6, the α_m and β_m estimates are

$$\widehat{\beta_m} = \frac{\sum_{i=0}^{m} (i - \bar{m})\, Y_i}{\sum_{i=0}^{m} (i - \bar{m})^2} \quad \text{and} \qquad (3.66)$$

$$\widehat{\alpha_m} = \bar{Y} - \widehat{\beta_m}\, \bar{m} \qquad (3.67)$$

for m = 1, 2, 3, ... , NTM where

$$\bar{Y} = \frac{1}{m+1} \sum_{i=0}^{m} Y_i \quad \text{and} \tag{3.68}$$

$$\bar{m} = \frac{1}{m+1} \sum_{i=0}^{m} i \ . \tag{3.69}$$

Finally, these $\widehat{\alpha}_m$ and $\widehat{\beta}_m$ estimates are utilized in the discrete reliability model, equation 3.56, to produce the model estimates of the unknown, underlying modification reliability. These estimates are given by

$$\widehat{R}_m = 1 - \frac{1}{\exp\left(\widehat{\alpha}_m + \widehat{\beta}_m m\right)} \tag{3.70}$$

for m = 1, 2, 3, ... , NTM. Note than since the regression procedure requires a minimum of two observations, model reliability estimates are produced from the first modification thru the last modification (NTM). A reliability estimate for original version of the component is derived from equation 3.56 and 3.64 as

$$\widehat{R}_0 = 1 - \frac{1}{\exp\widehat{(\alpha + \beta m)}_0} \ . \tag{3.71}$$

Again, since the reliability testing procedure was simulated one-hundred times (r = 1, 2, 3, ... , NSIMS = 100) for each reliability set, one-hundred estimates of the modification reliability were obtained for each modification version of the component under test. The mean value, standard deviation, and percentage standard error of these estimates were computed as

$$\overline{\widehat{R}}_m = \frac{1}{NSIMS} \sum_{r=1}^{NSIMS} \widehat{R}_{m,r} , \tag{3.72}$$

$$S.D._{\widehat{R}_m} = \sqrt{\frac{1}{NSIMS-1} \sum_{r=1}^{NSIMS} (\widehat{R}_{m,r} - \overline{\widehat{R}}_m)^2} , \text{ and} \tag{3.73}$$

$$P.S.E._{\widehat{R}_m} = \frac{S.D._{\widehat{R}_m}}{\overline{\widehat{R}}_m} \times 100 \tag{3.74}$$

for m = 0, 1, 2, ... , NTM.

To examine the discrete reliability growth model's forecasting capability a next modification reliability forecast was made for modifications m = 2, 3, 4, ... , NTM. Forecasting started for the second modification since model parameter estimates were not available until testing under the first modification was completed. No forecast was made past modification NTM because there would be no underlying modification reliability with which to compare. Forecasts were made as

$$\widehat{FR}_{m+1} = 1 - \frac{1}{\exp(\widehat{\alpha}_m + \widehat{\beta}_m (m+1))} \tag{3.75}$$

for m = 2, 3, 4, ... , NTM. Finally, forecast modification reliability statistics similar to the determined modification reliability statistics in equations 3.72, 3.73, and 3.74 were computed as

$$\overline{\widehat{FR}}_m = \frac{1}{NSIMS} \sum_{r=1}^{NSIMS} \widehat{FR}_{m,r} , \tag{3.76}$$

$$S.D._{\widehat{FR}_m} = \sqrt{\frac{1}{NSIMS-1} \sum_{r=1}^{NSIMS} (\widehat{FR}_{m,r} - \overline{\widehat{FR}}_m)^2} \text{ , and} \tag{3.77}$$

$$P.S.E._{\widehat{FR}_m} = \frac{S.D._{\widehat{FR}_m}}{\overline{\widehat{FR}}_m} \times 100 \tag{3.78}$$

for m = 2, 3, 4, ... , NTM.

To evaluate the discrete reliability growth model ten reliability sets were formed for use as the specified underlying reliability progress paths. These reliability sets are listed in table 3.27. All ten reliability progress paths start at a relatively low reliability of 0.200 for the original version of the component under scrutiny; i.e., $R_0 = 0.200$ for reliability sets 1 thru 10. The paths display reliability progress that ranges from extremely rapid reliability growth (reliability set 1; $R_0 = 0.200$, $R_1 = 0.925$) to linear growth (reliability set 6; $R_{m+1} = R_m + 0.150$) to permanently stagnated reliability progress (reliability set 10; R_0 thru $R_6 = 0.200$). Reliability progress was simulated only for components that were modified a total of five times during the acquisition cycle; i.e., NTM = 5 and m = 0, 1, 2, 3, 4, 5. Each reliability set progress path was simulated for three different total number of failures until modification specifications; i.e., NTF_m = 1, 3, and 5 failures which were held constant from modification to modification.

5. Model Performance

a. Accuracy Performance

Figures 3.79 thru 3.98 present all the cases (ten reliability sets) of the discrete reliability growth model's capability to determine and forecast the unknown, underlying modification reliability progress

TABLE 3.27

RELIABILITY SETS

DISCRETE RELIABILITY GROWTH MODEL

MODIFICATION	RELIABILITY SET 1	2	3	4	5	6	7	8	9	10
0	.200	.200	.200	.200	.200	.200	.200	.200	.200	.200
1	.925	.850	.700	.550	.450	.350	.225	.500	.650	.200
2	.950	.900	.825	.750	.650	.500	.275	.550	.650	.200
3	.950	.925	.900	.875	.775	.650	.350	.550	.350	.200
4	.950	.945	.925	.925	.875	.800	.475	.700	.600	.200
5	.950	.950	.950	.950	.950	.950	.950	.950	.950	.200

path during a reliability testing procedure for total number of failures until modification $NTF_m = 1$ and 5. These graphs display the "least data" and "most data" cases which can be contrasted for change and improvement as the testing sample is increased. For the $NTF_m = 3$ cases, the accuracy performance fell uniformly between the $NTF_m = 1$ and $NTF_m = 5$ accuracy performance.

The graphs depict the specified true underlying modification reliability R_m progress path (——, solid line), the mean model determined modification reliability $\overline{\widehat{R_m}}$ from equation 3.72 for each modification version of the component under test (⊙ , circles), and the mean model forecast modification reliability $\overline{\widehat{FR_m}}$ from equation 3.76 for the second thru the final modification version of the component (X , crosses) plotted versus the modification number m = 0, 1, 2, 3, 4, 5. Note that the point plotted for the mean model determined modification reliability of the original version (m = 0) of the component under test is not model determined; rather, it is the mean value of the estimate given by the estimator of equation 3.71. This point allows the accuracy of the reliability estimator utilized to be examined also.

To illustrate those quantities graphed in the accuracy performance figures, observe on figure 3.83 that for the third modification, the specified underlying reliability R_3 was 0.90, the mean model determined modification reliability $\overline{\widehat{R_3}}$ was approximately 0.86, and the mean model forecast modification reliability $\overline{\widehat{FR_3}}$ made from the second modification was approximately 0.77. Since this graph is for reliability set 3, referring to table 3.27, the specified modification reliability for the third modification of reliability set 3 is 0.900 as is graphed on figure 3.83. Finally, note on figure 3.83 for the unmodified version of the component tested ($m = 0$) that the reliability estimator of equation 3.71 produced a mean estimate $\overline{\widehat{R_0}}$ of approximately 0.175 for the true underlying reliability R_0 of 0.200.

With the exception of reliability set 9 (figures 3.95 and 3.96) the discrete model determines the shape of the underlying reliability progress path with very good accuracy. Forecasting accuracy, while fairly good for the "nice" reliability progress path sets 1 thru 4, displays difficulty with the anomalous reliability progress paths characterized in reliability sets 6 thru 10. The discrete model occasionally produced negative mean forecast estimates of reliability. The negative mean forecast reliability estimates, the magnitude of which never exceeded 0.20, are plotted as 0.0 on the accuracy performance graphs. For example see figure 3.97 for mean forecasts at modifications 2, 3, and 4. No negative mean forecasts of reliability were produced when the total number of failures until modification was specified as five ($NTF_m = 5$); i.e., the "most data" case.

The discrete model accuracy performance graphs display the marked improvement that takes place both in determining and forecasting reliability status as the specified total number of failures until modification NTF_m is increased from one to five which essentially increases the test data which the model can utilize. The improvement in accuracy is also displayed in the $NTF_m = 3$ accuracy performance graphs which are not presented. The reliability estimator of equation 3.71 displays the same improvement in accuracy when the testing sample size is increased as may be seen by examining the graph pairs for $NTF_m = 1$ and $NTF_m = 5$ at the modification $m = 0$ point.

Figures 3.79 thru 3.98 reveal that the discrete reliability model's accuracy performance for determining the underlying reliability is in general slightly pessimistic in the instances when mean estimates are not "on the money" as in figures 3.84 and 3.86. Except for the reliability sets 1 and 2, $NTF_m = 5$ cases shown in figures 3.80 and 3.82, mean model forecasting accuracy tends to run from slightly pessimistic as in figure 3.90; reliability set 6, $NTF_m = 5$ to grossly pessimistic as in figure 3.91; reliability set 7, $NTF_m = 1$ or figure 3.97; reliability set 10, $NTF_m = 1$. The general trend evidenced by the graphs is that the less test data available ($NTF_m = 1$) the more pessimistic the mean model estimates, determined or forecast. The same is true of the reliability estimator of equation 3.71.

Reliability set 9 which characterizes a reliability growth pattern interrupted by a period of reliability degradation is the only set with which the discrete model experiences a significant problem. While not detecting the full magnitude of the degradation, the model's mean determined reliability estimates do display the period of degradation credibly. See figures 3.95 and 3.96. However, the model's mean forecast modification reliability estimates are quite contrary to the true underlying reliability progress path. The significant problem being that the period of degradation is not forecast until the true underlying reliability status has recovered from the period of degradation and is in fact displaying significant reliability growth. This behavior is apparent even for the "most data" case $NTF_m = 5$ in figure 3.96. While model forecasting capability suffers in the "least data" case for reliability set 10, $NTF_m = 1$, the discrete model copes with permanent reliability stagnation satisfactorily as seen in figures 3.97 and 3.98.

b. Variability (Precision) Performance

Tables 3.28 thru 3.33 present the discrete reliability growth model's variability performance for determining and forecasting

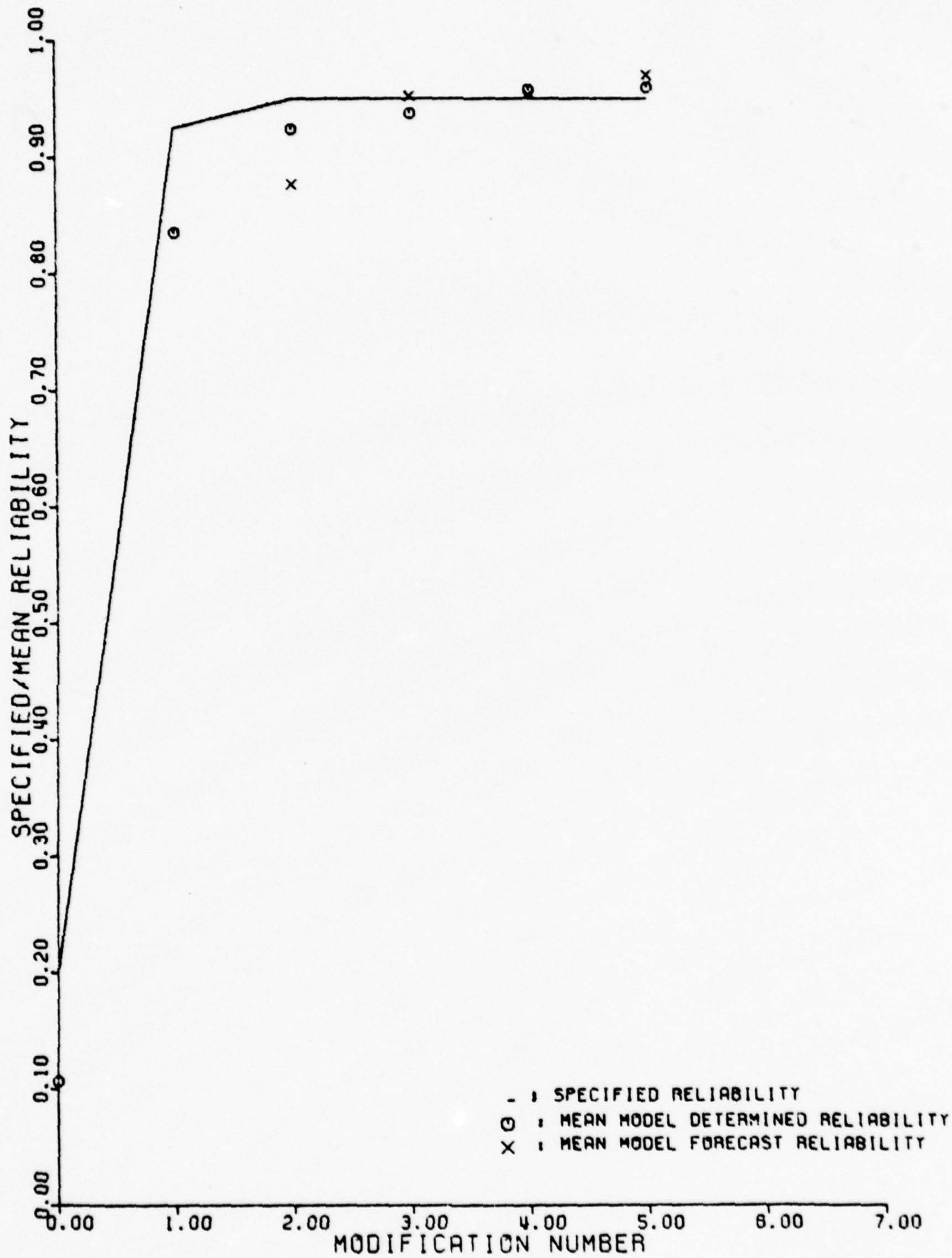

FIGURE 3.79

DISCRETE RELIABILITY GROWTH MODEL PERFORMANCE
RELIABILITY SET 1: 5 MODS, 1 FAILURE/MOD

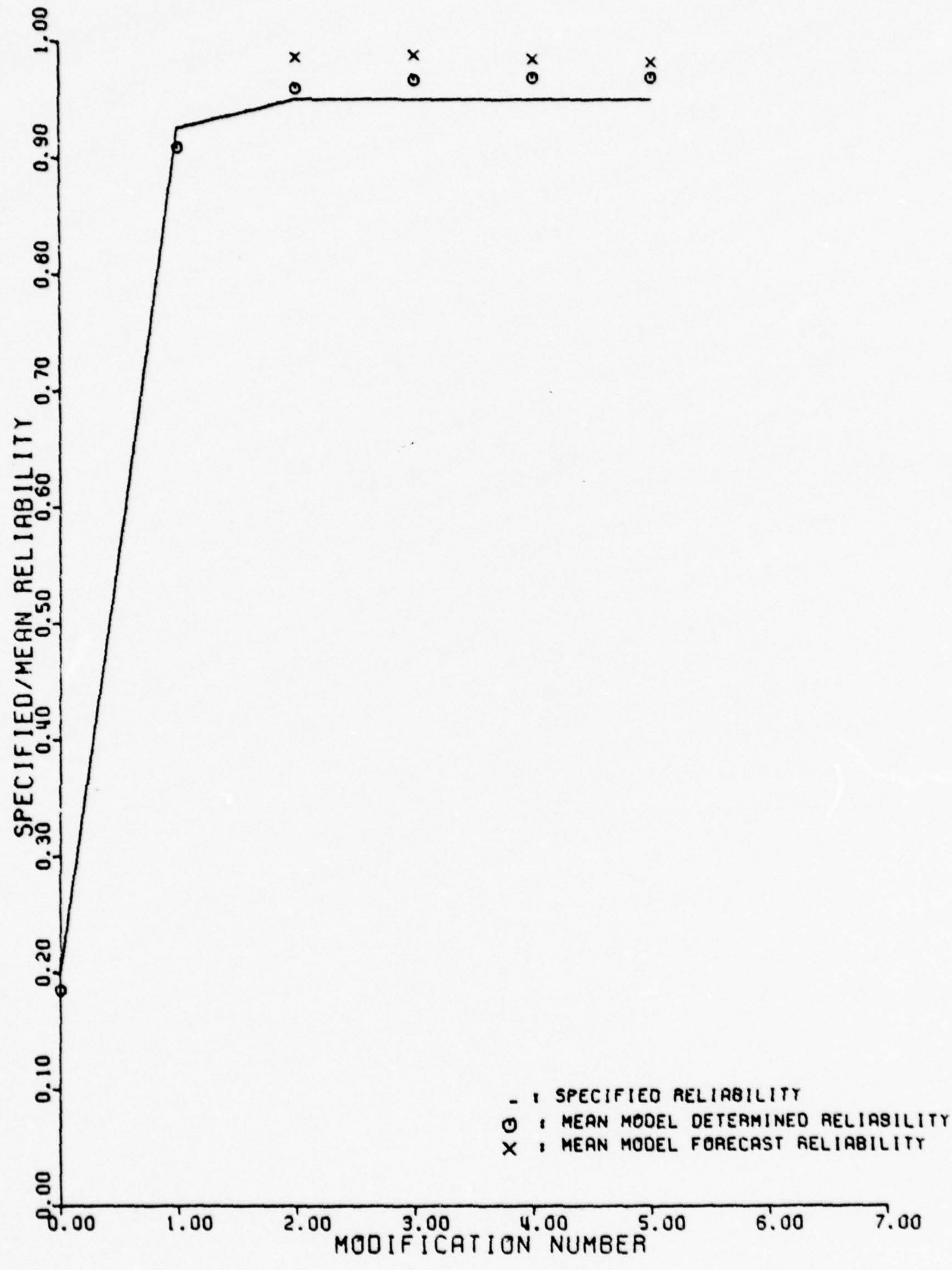

FIGURE 3.80

DISCRETE RELIABILITY GROWTH MODEL PERFORMANCE
RELIABILITY SET 1: 5 MODS, 5 FAILURES/MOD

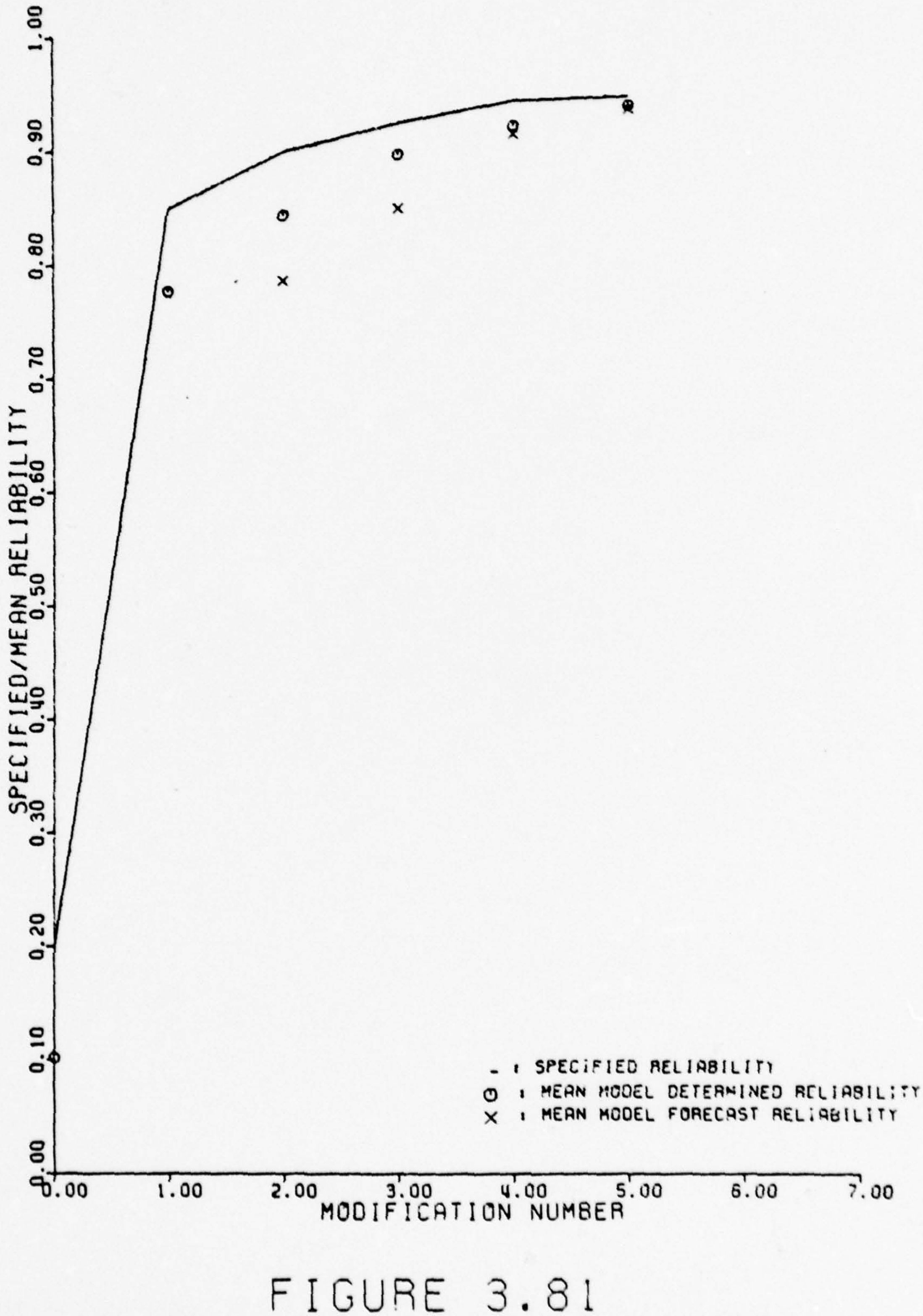

FIGURE 3.81

DISCRETE RELIABILITY GROWTH MODEL PERFORMANCE
RELIABILITY SET 2: 5 MODS, 1 FAILURE/MOD

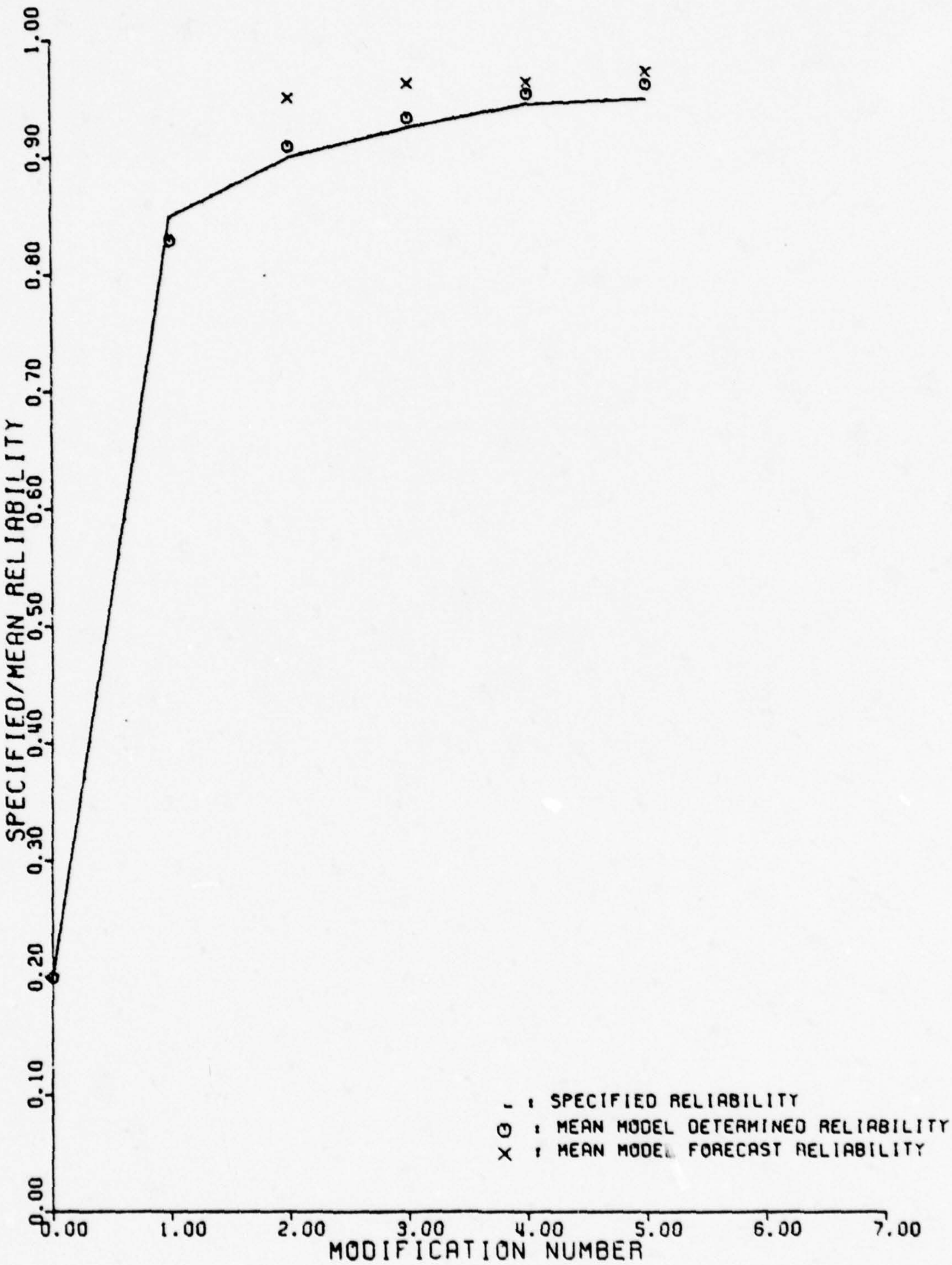

FIGURE 3.82
DISCRETE RELIABILITY GROWTH MODEL PERFORMANCE
RELIABILITY SET 2: 5 MODS, 5 FAILURES/MOD

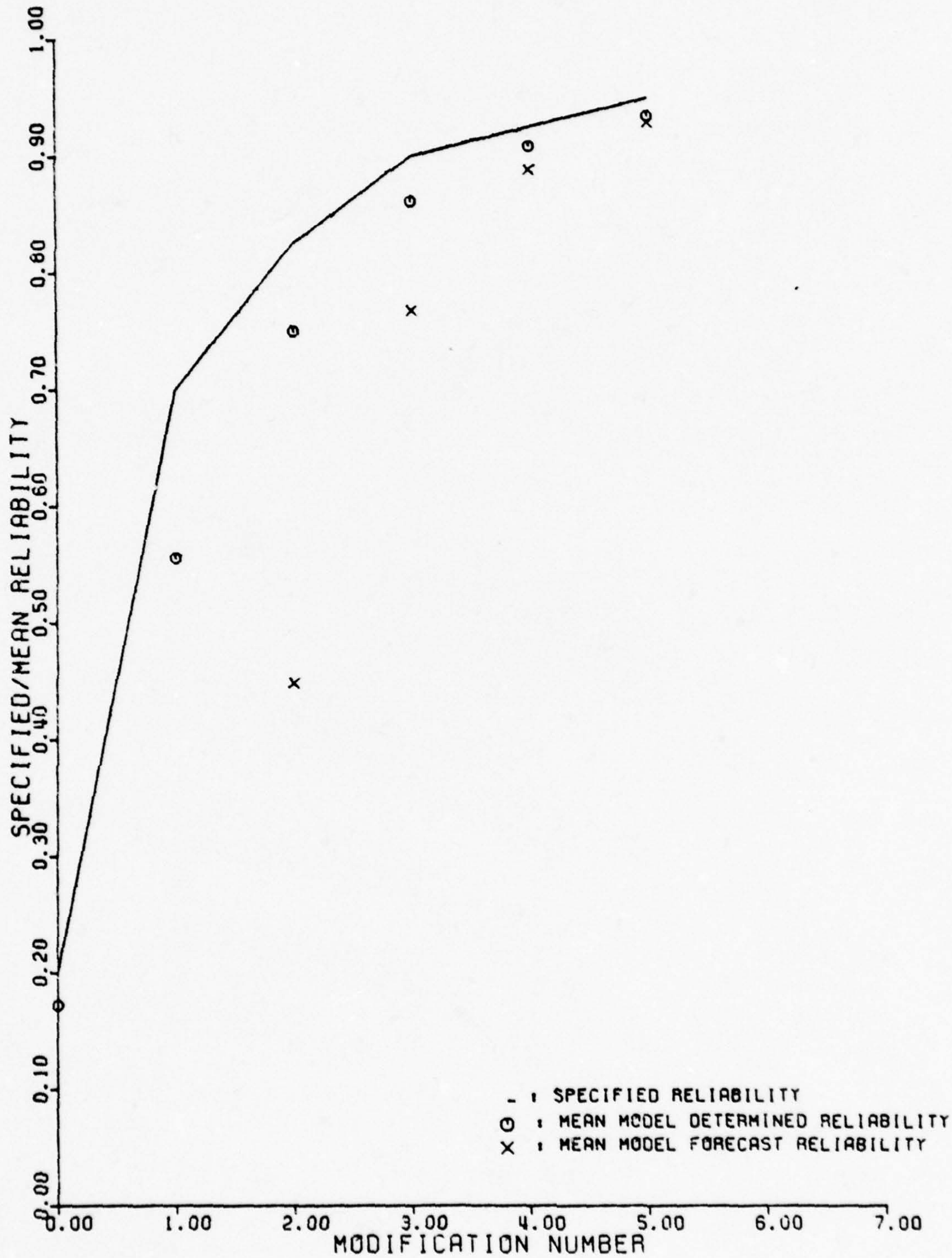

FIGURE 3.83

DISCRETE RELIABILITY GROWTH MODEL PERFORMANCE
RELIABILITY SET 3: 5 MODS, 1 FAILURE/MOD

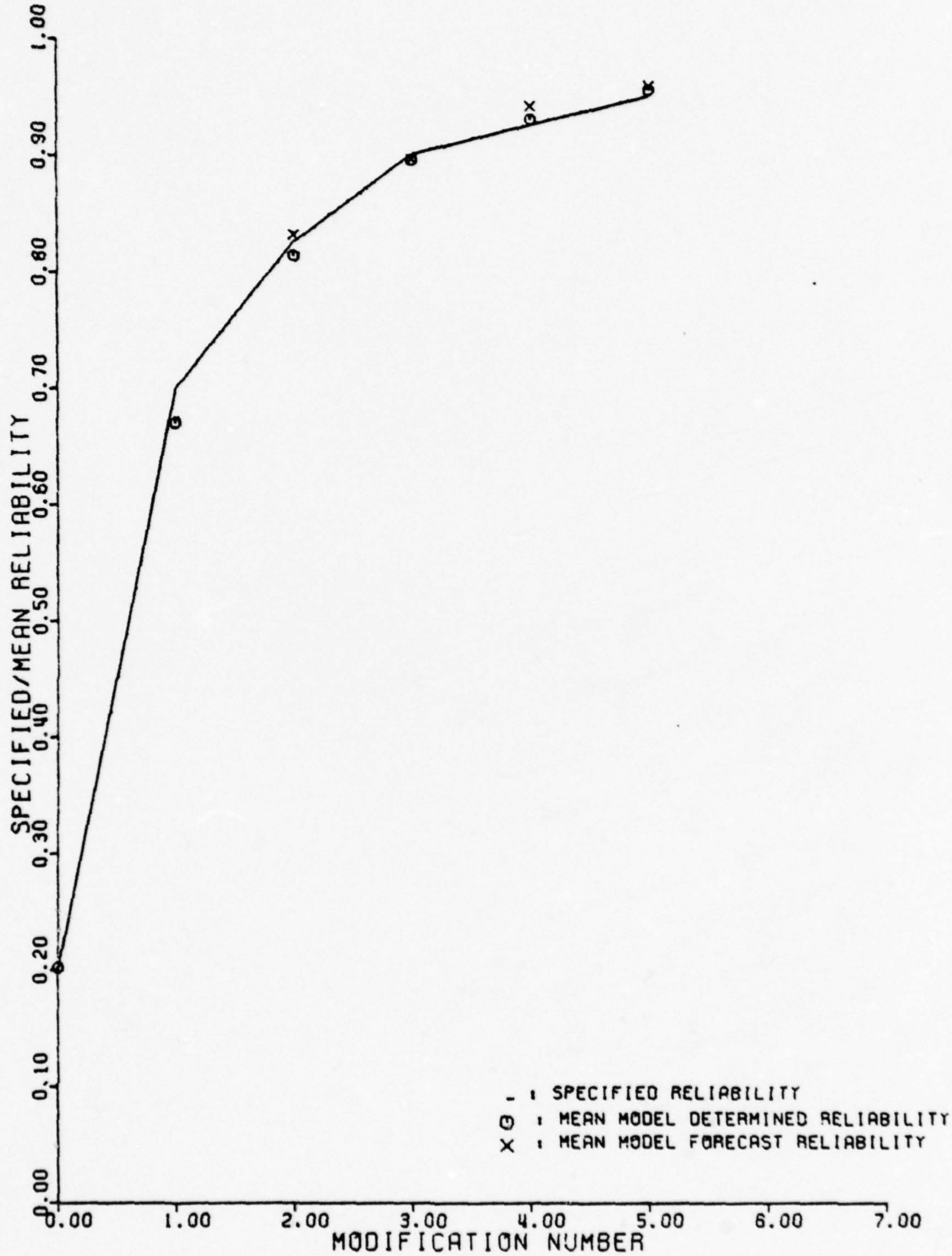

FIGURE 3.84

DISCRETE RELIABILITY GROWTH MODEL PERFORMANCE
RELIABILITY SET 3: 5 MODS, 5 FAILURES/MOD

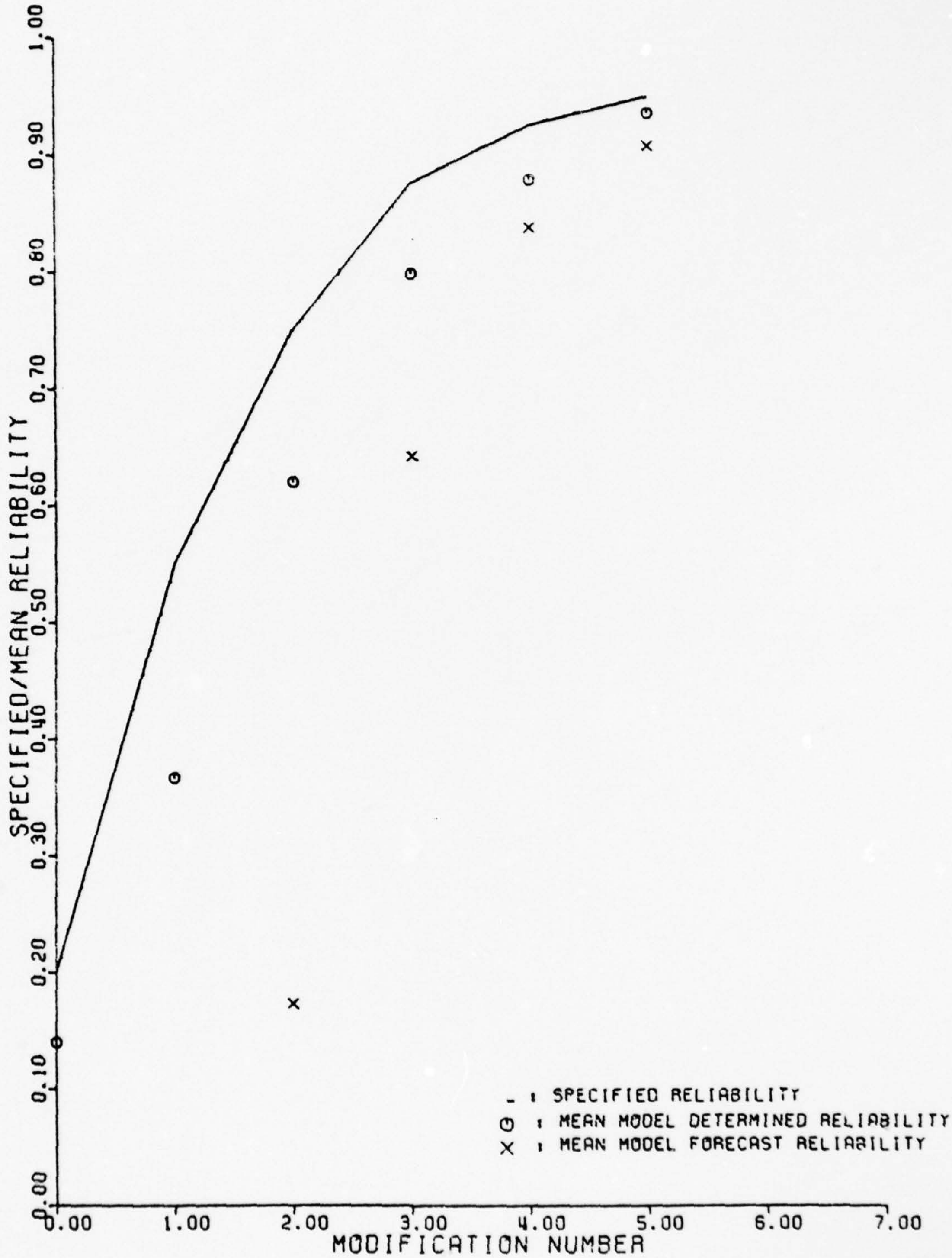

FIGURE 3.85

DISCRETE RELIABILITY GROWTH MODEL PERFORMANCE
RELIABILITY SET 4: 5 MODS, 1 FAILURE/MOD

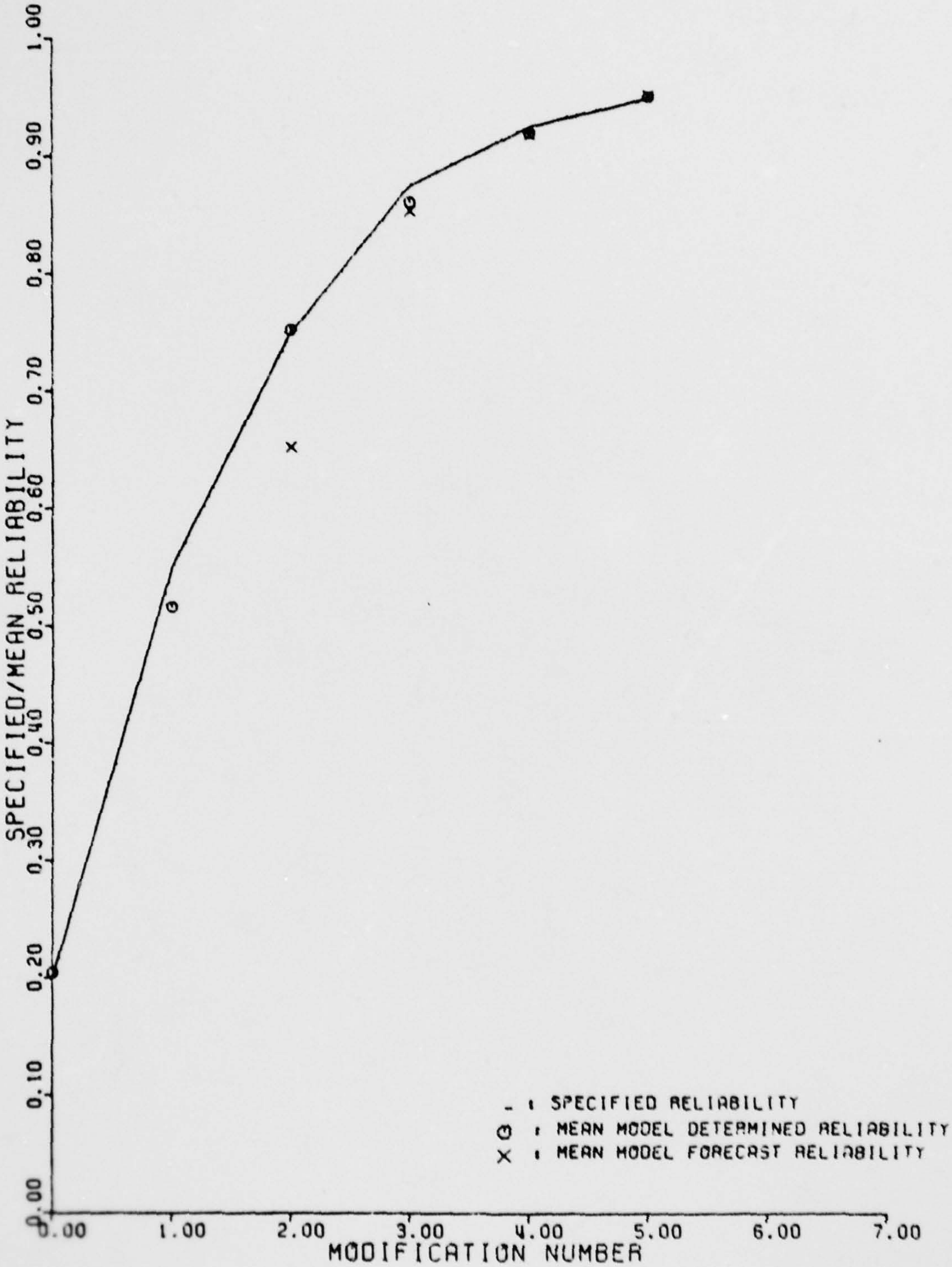

FIGURE 3.86

DISCRETE RELIABILITY GROWTH MODEL PERFORMANCE
RELIABILITY SET 4: 5 MODS, 5 FAILURES/MOD

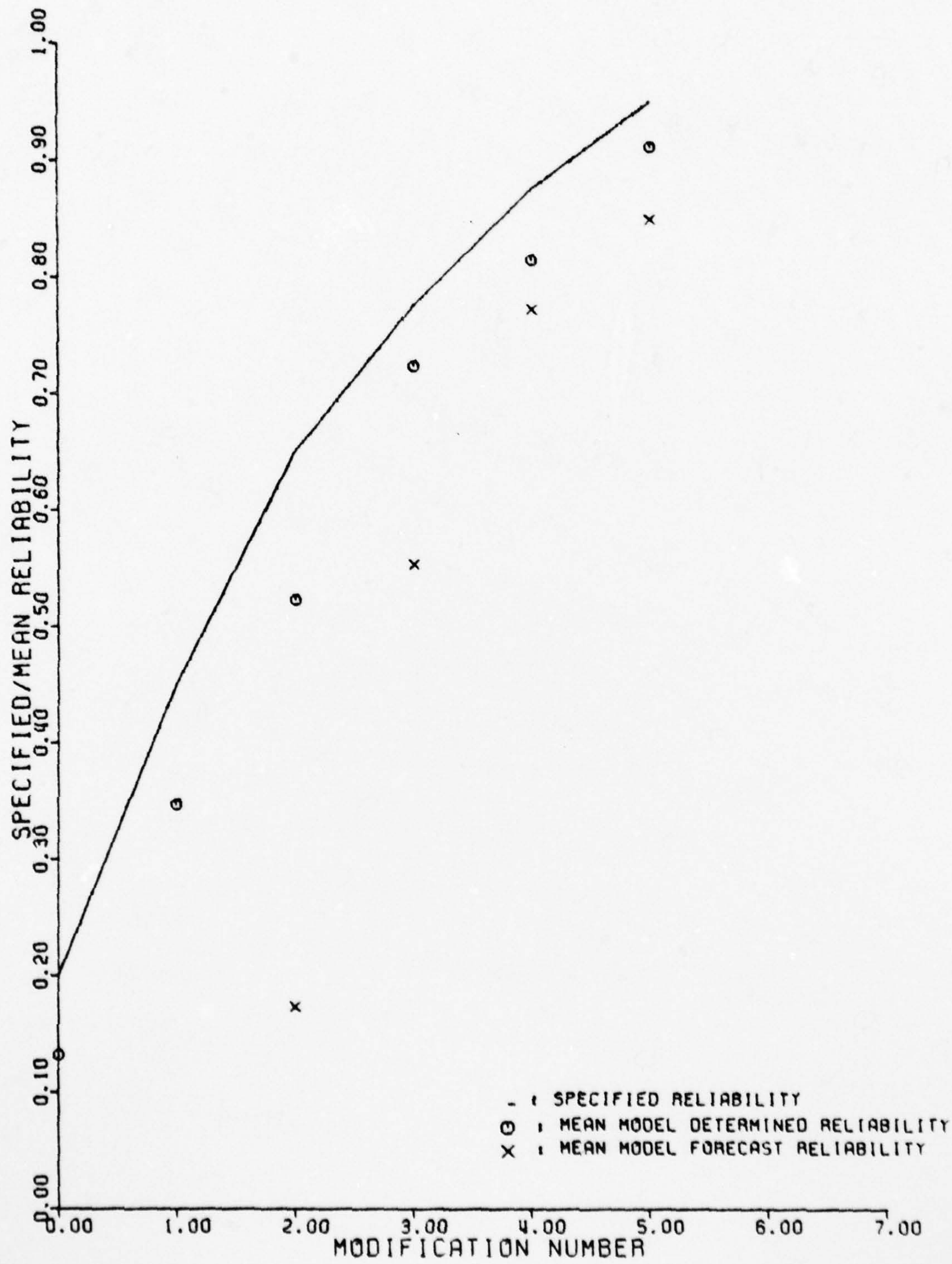

FIGURE 3.87
DISCRETE RELIABILITY GROWTH MODEL PERFORMANCE
RELIABILITY SET 5: 5 MODS, 1 FAILURE/MOD

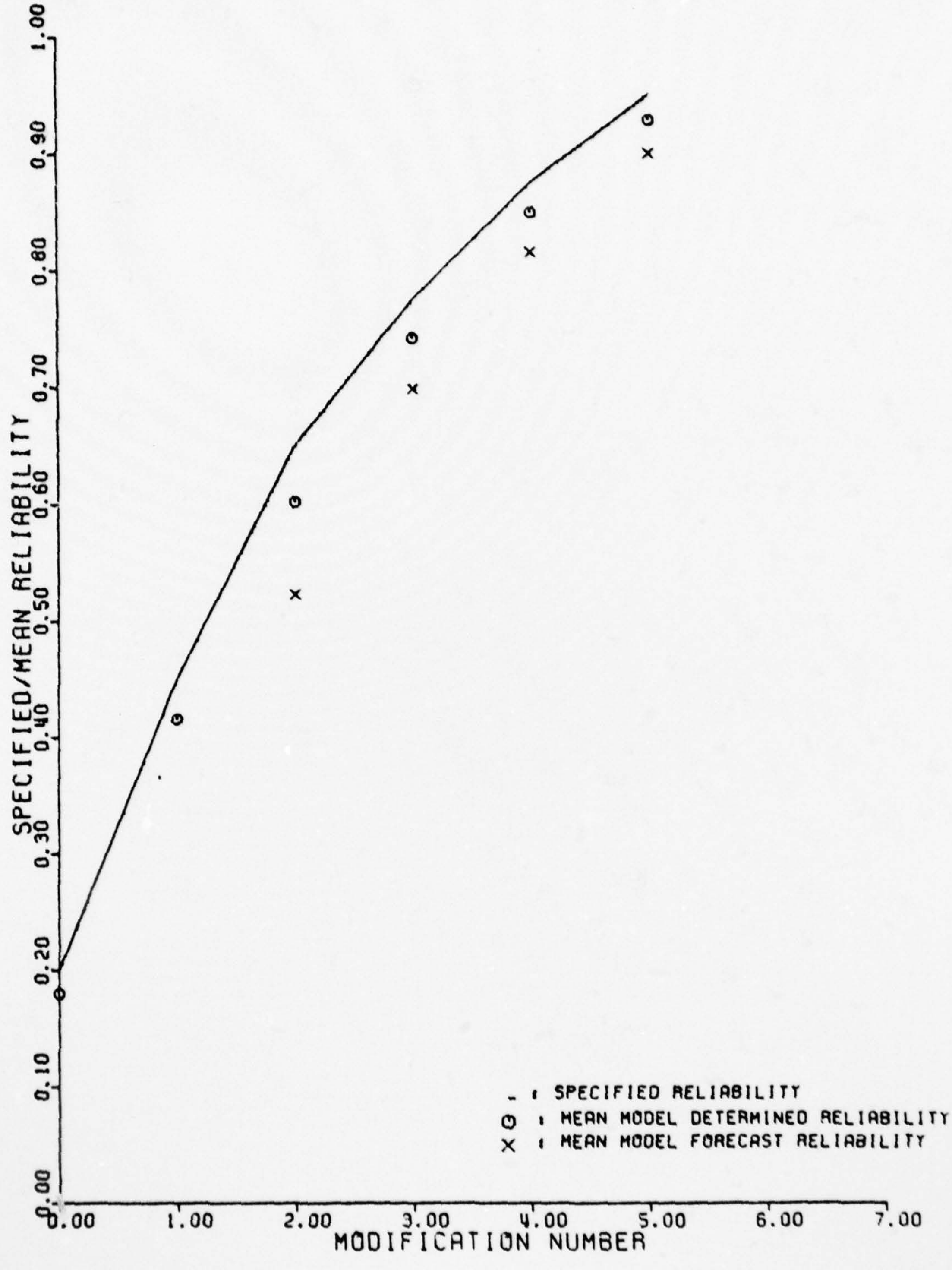

FIGURE 3.88

DISCRETE RELIABILITY GROWTH MODEL PERFORMANCE
RELIABILITY SET 5: 5 MODS, 5 FAILURES/MOD

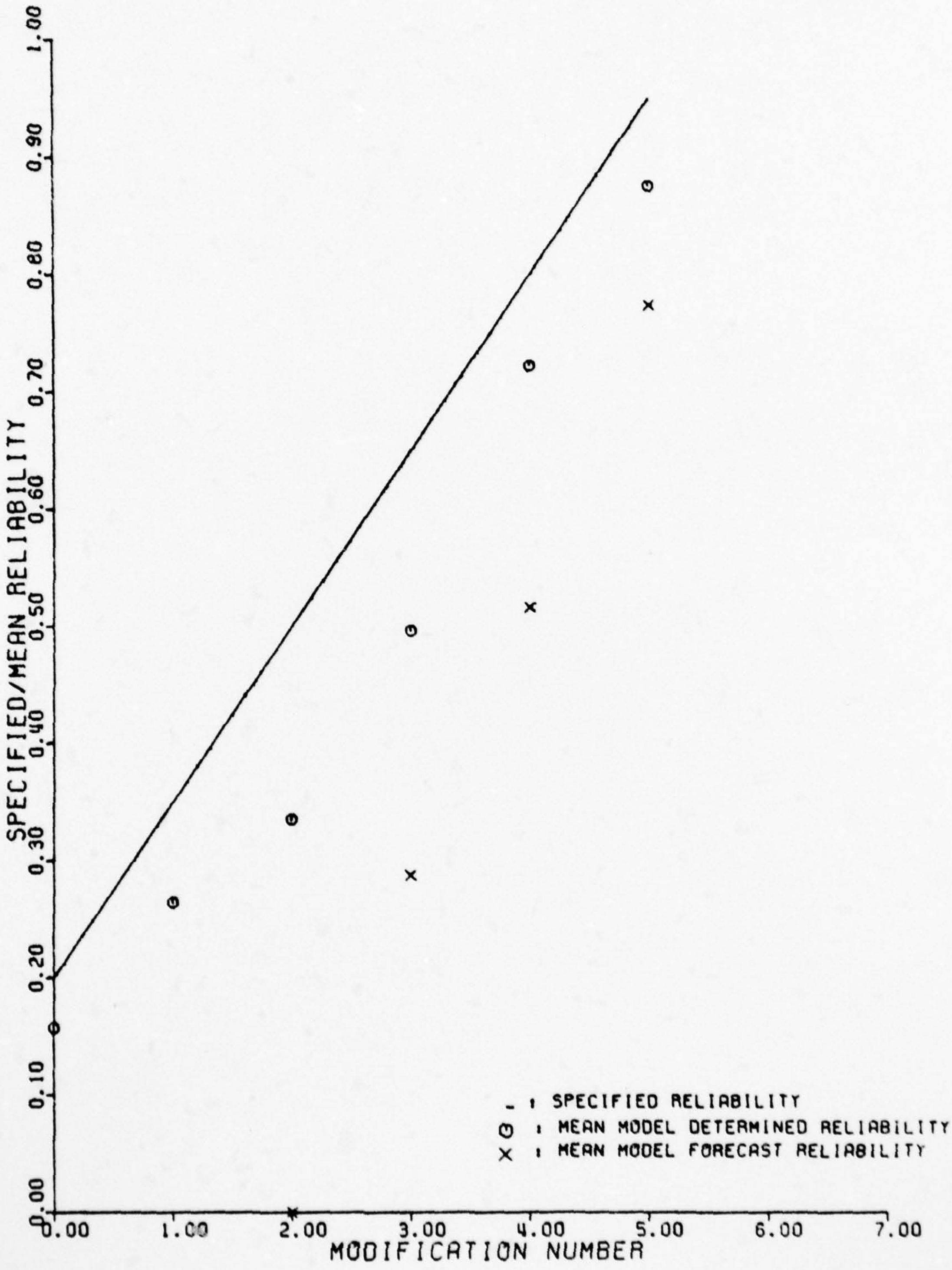

FIGURE 3.89

DISCRETE RELIABILITY GROWTH MODEL PERFORMANCE
RELIABILITY SET 6: 5 MODS, 1 FAILURE/MOD

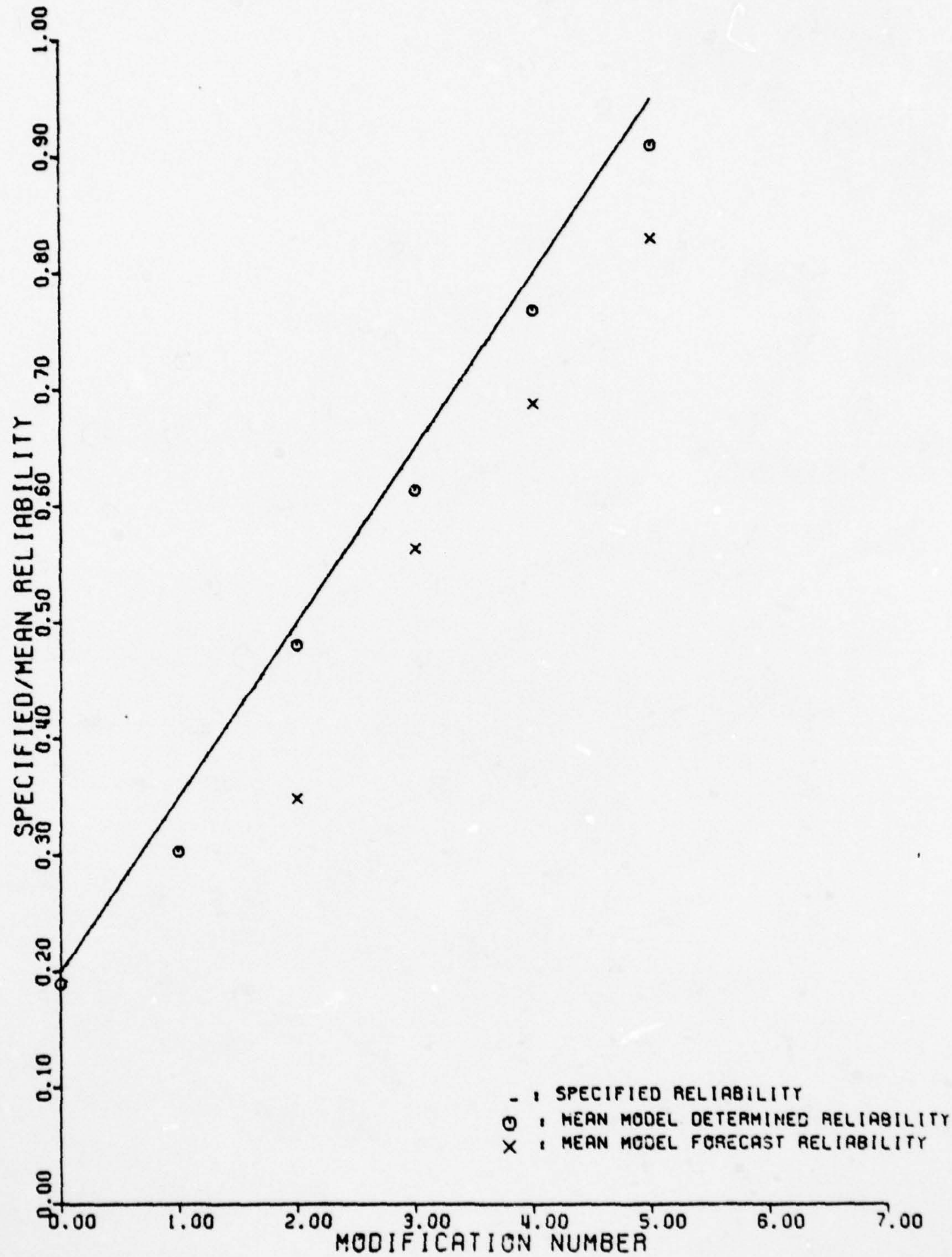

FIGURE 3.90

DISCRETE RELIABILITY GROWTH MODEL PERFORMANCE
RELIABILITY SET 6: 5 MODS. 5 FAILURES/MOD

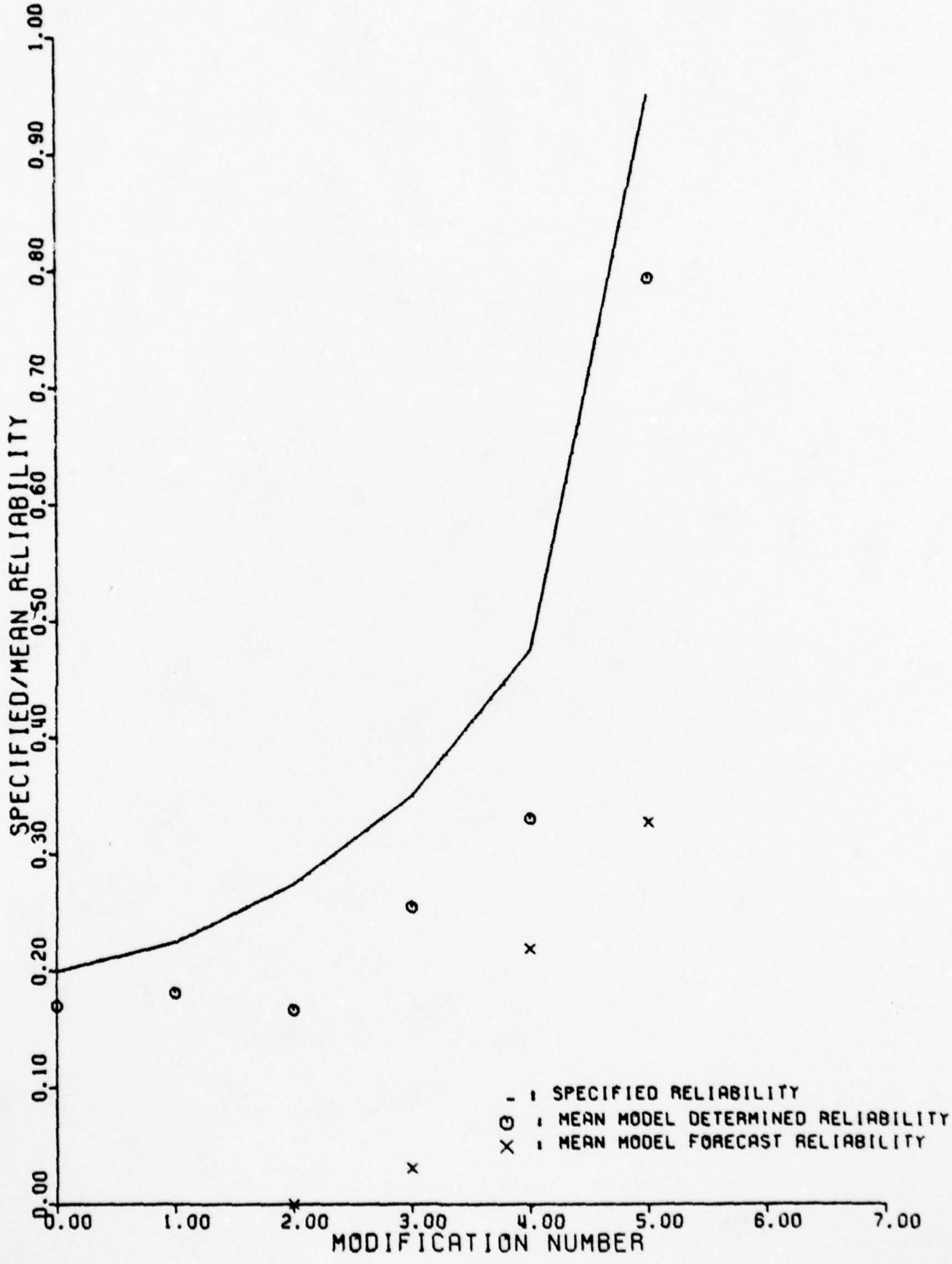

FIGURE 3.91

DISCRETE RELIABILITY GROWTH MODEL PERFORMANCE
RELIABILITY SET 7: 5 MODS, 1 FAILURE/MOD

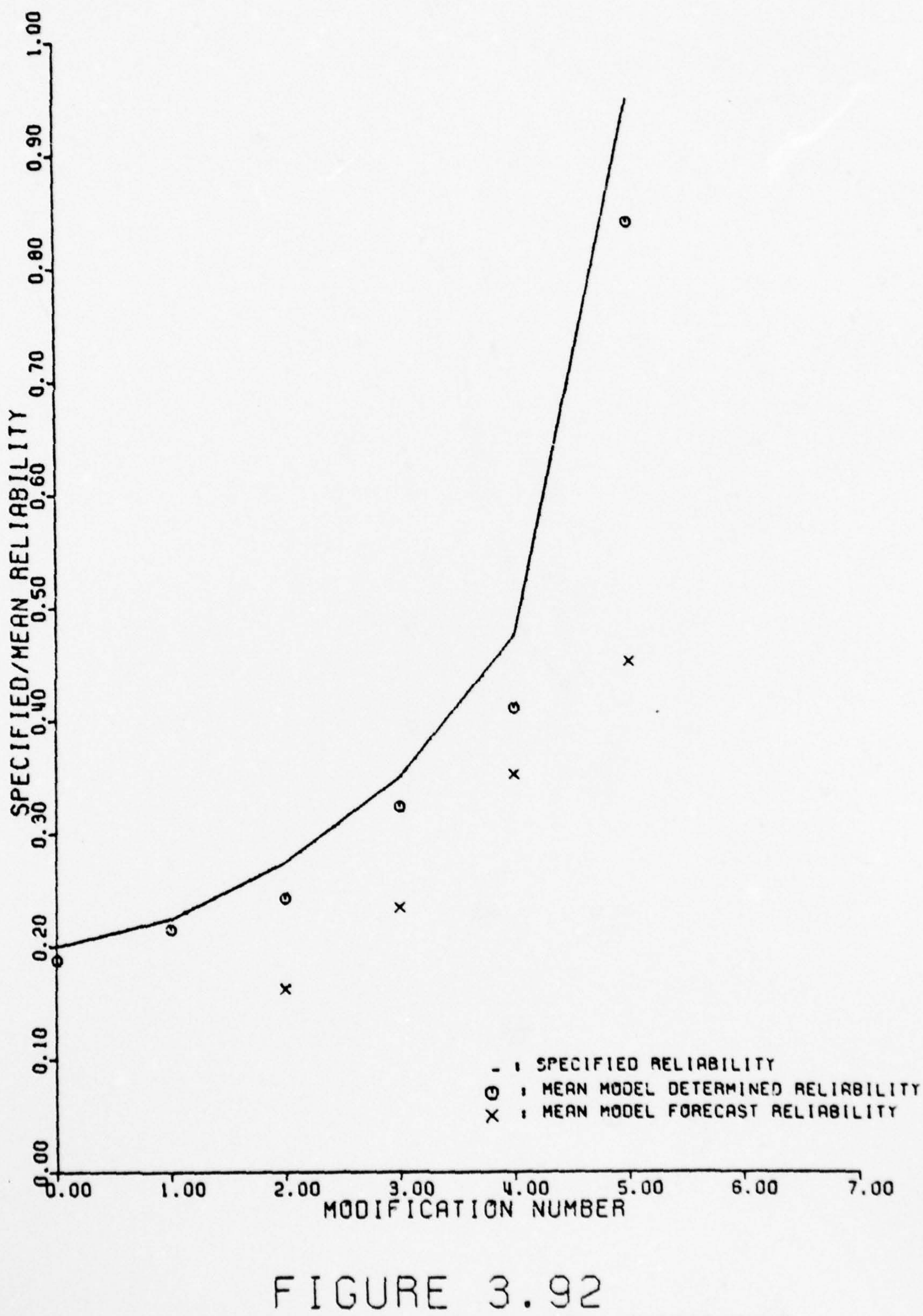

FIGURE 3.92

DISCRETE RELIABILITY GROWTH MODEL PERFORMANCE
RELIABILITY SET 7: 5 MODS. 5 FAILURES/MOD

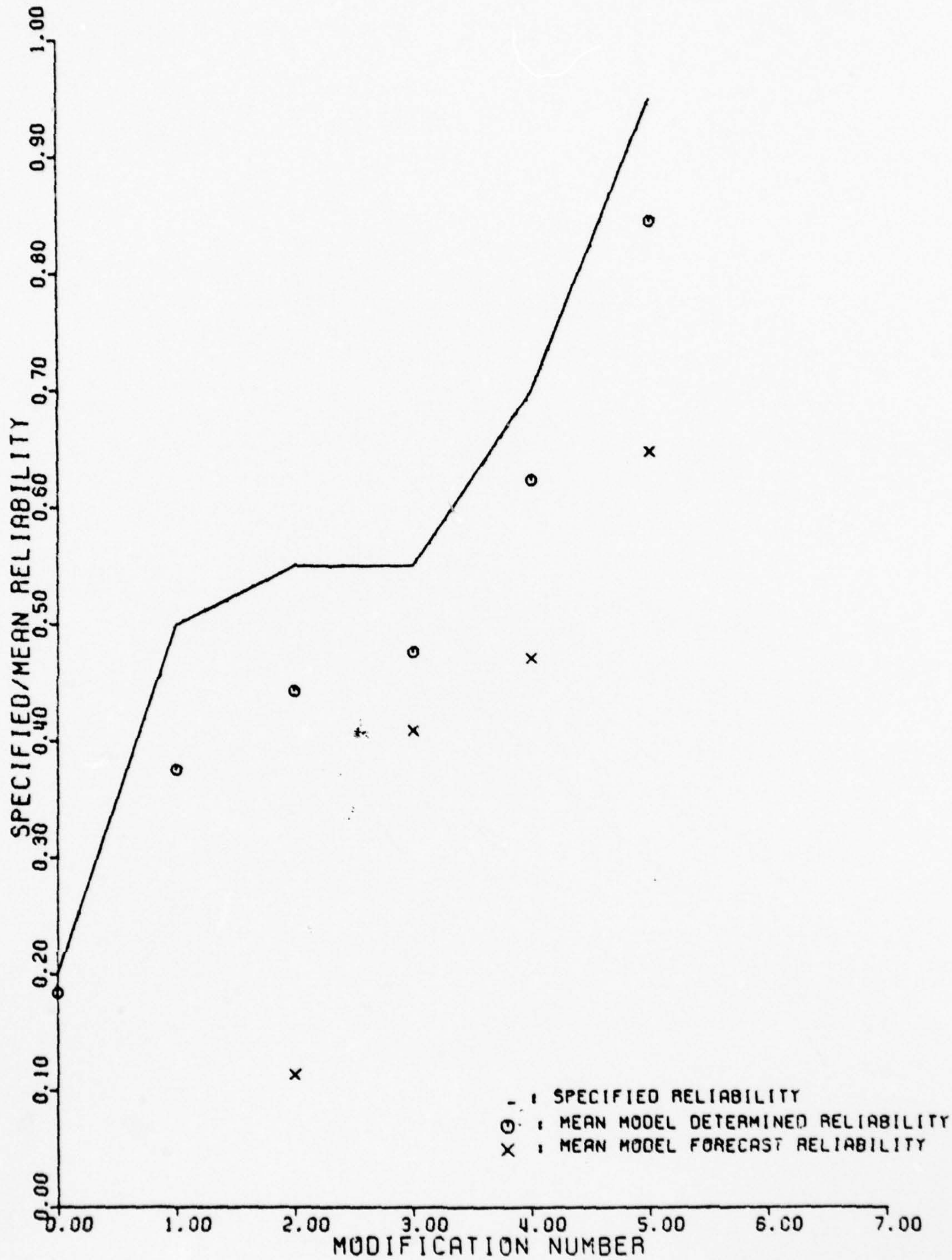

FIGURE 3.93

DISCRETE RELIABILITY GROWTH MODEL PERFORMANCE
RELIABILITY SET 8: 5 MODS. 1 FAILURE/MOD

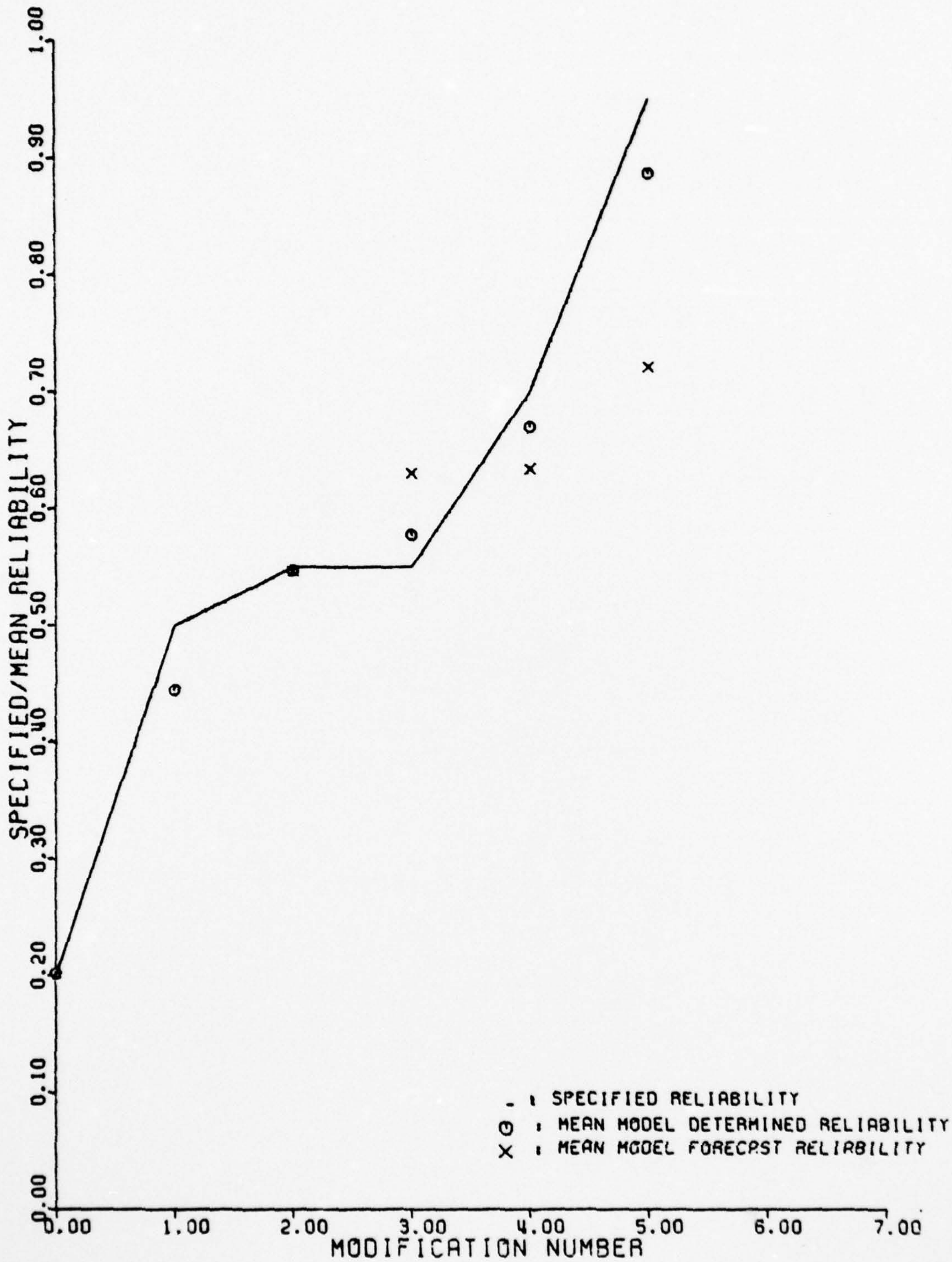

FIGURE 3.94

DISCRETE RELIABILITY GROWTH MODEL PERFORMANCE
RELIABILITY SET 8: 5 MODS, 5 FAILURES/MOD

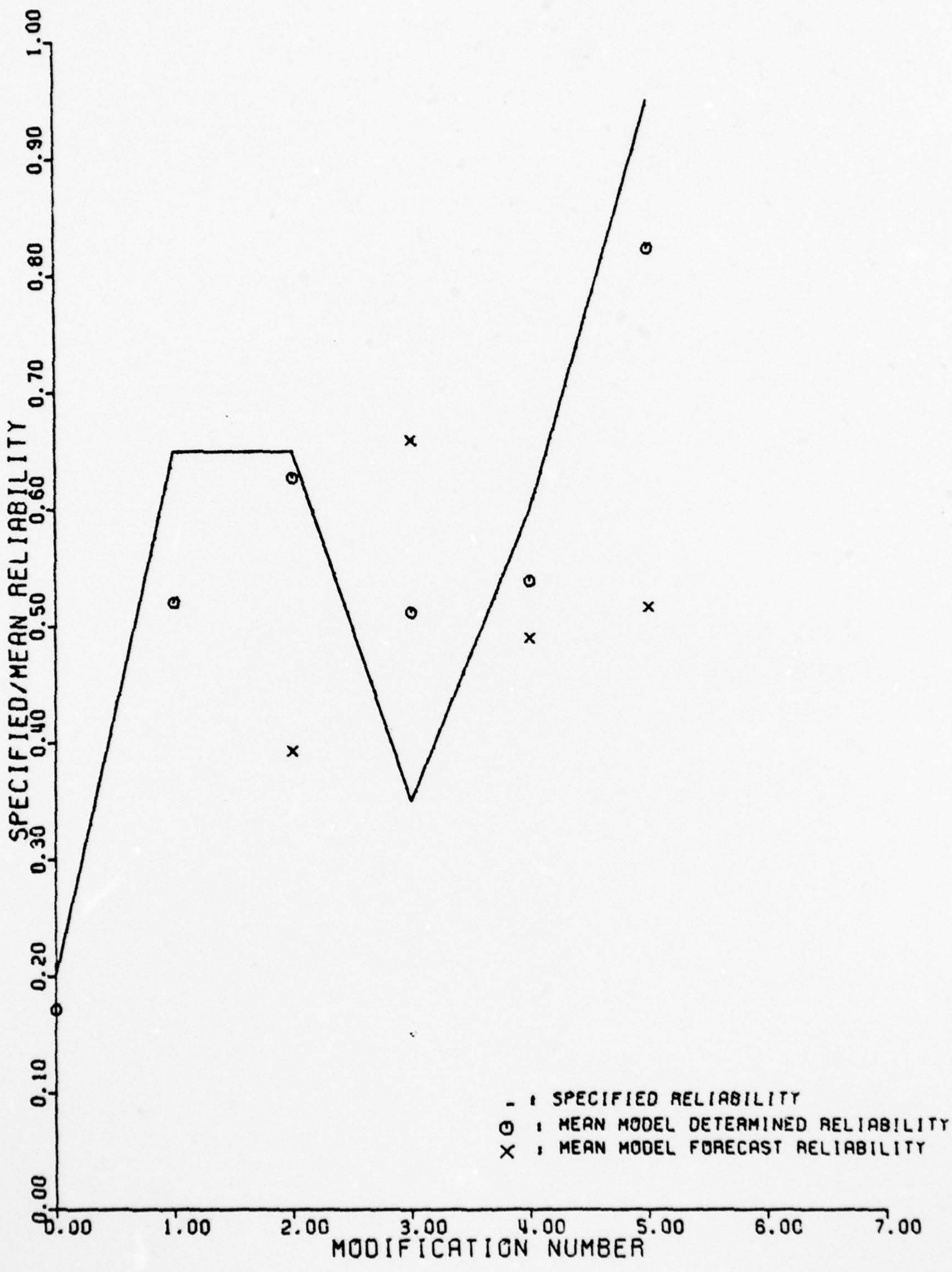

FIGURE 3.95

DISCRETE RELIABILITY GROWTH MODEL PERFORMANCE
RELIABILITY SET 9: 5 MODS, 1 FAILURE/MOD

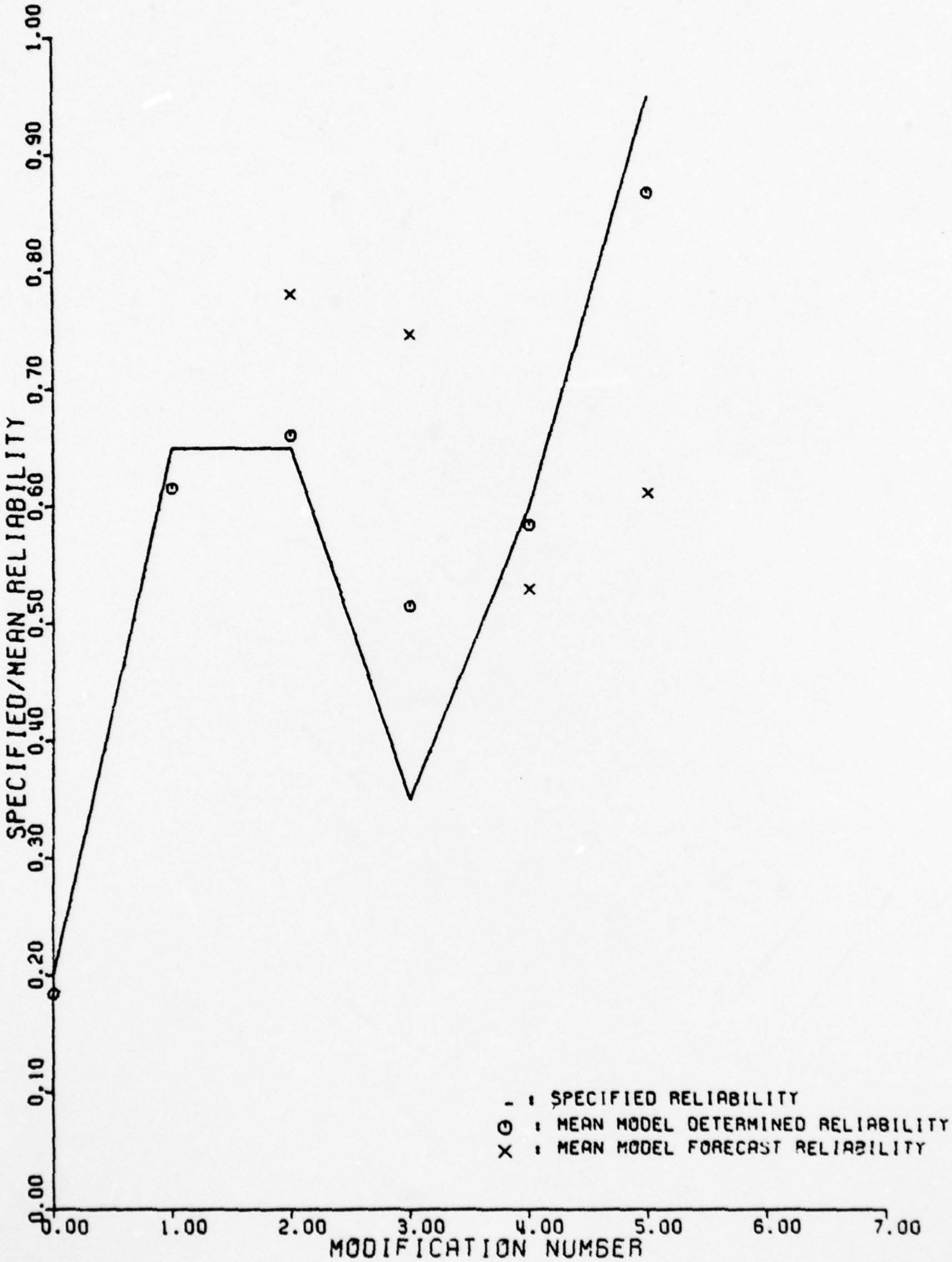

FIGURE 3.96

DISCRETE RELIABILITY GROWTH MODEL PERFORMANCE
RELIABILITY SET 9: 5 MODS, 5 FAILURES/MOD

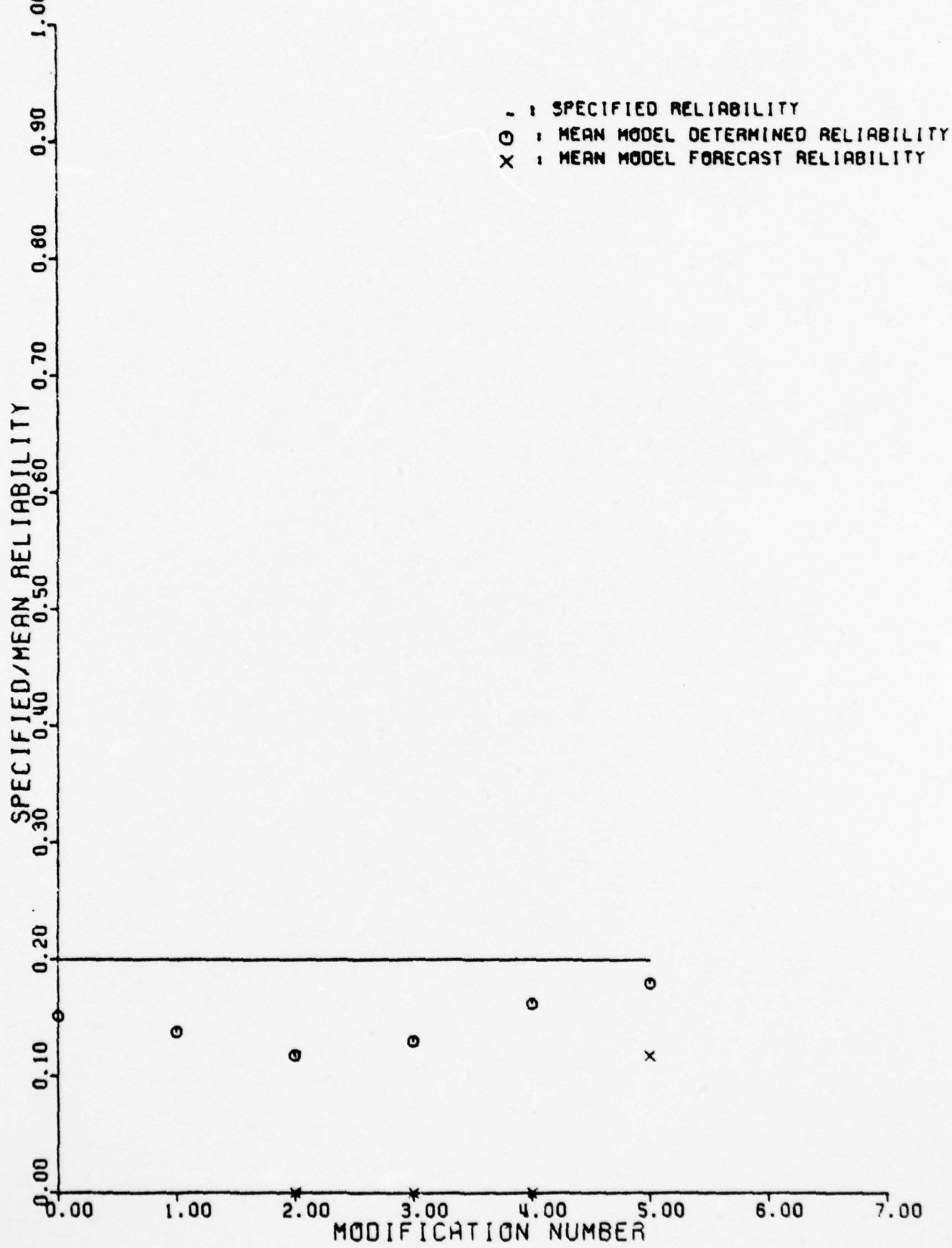

FIGURE 3.97

DISCRETE RELIABILITY GROWTH MODEL PERFORMANCE
RELIABILITY SET 10: 5 MODS, 1 FAILURE/MOD

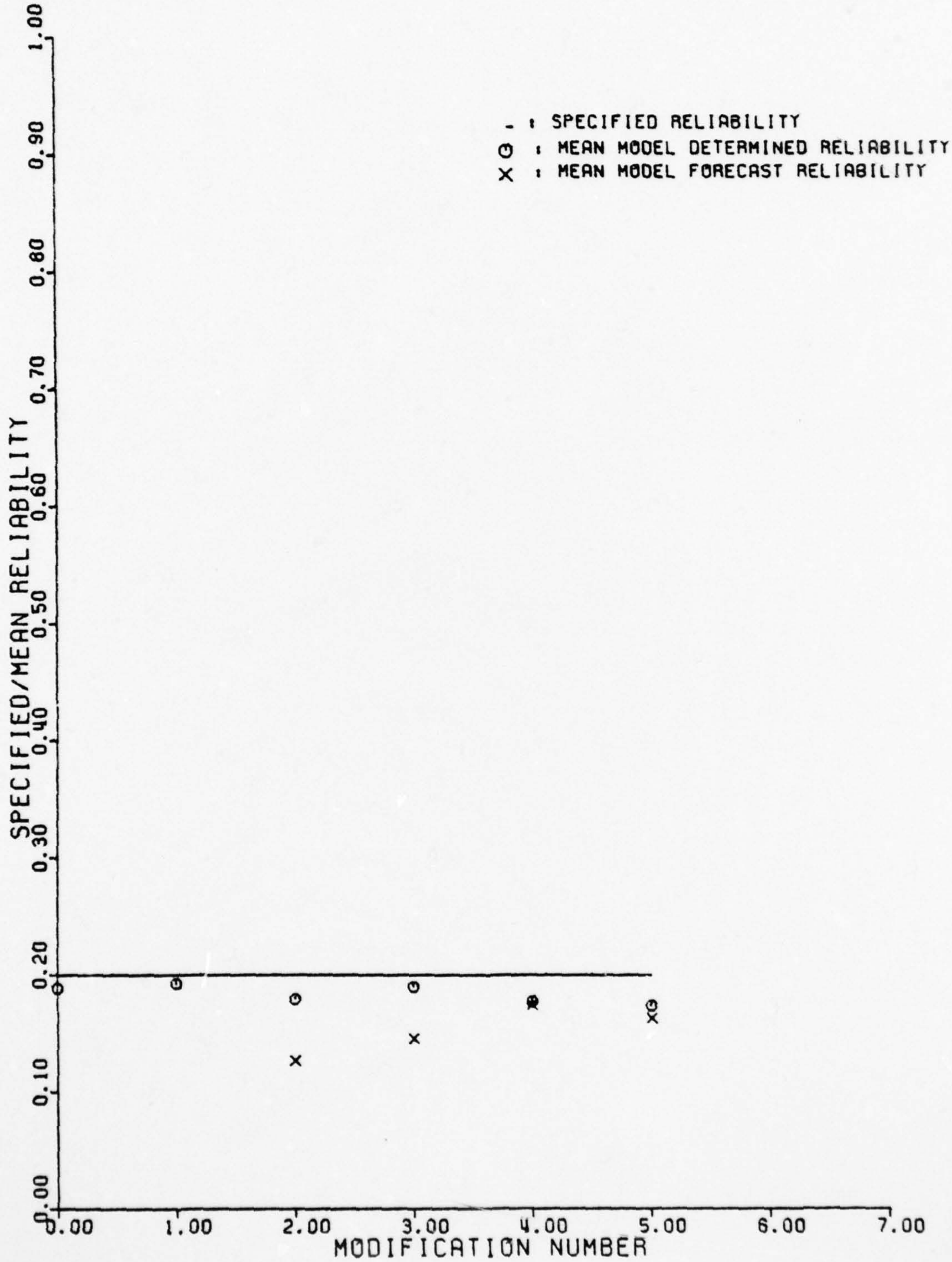

FIGURE 3.98

DISCRETE RELIABILITY GROWTH MODEL PERFORMANCE
RELIABILITY SET 10: 5 MODS, 5 FAILURES/MOD

the reliability status of a component as it undergoes modification in a system acquisition cycle. Entries in the tables are percentage standard errors as computed in equations 3.74 and 3.78. For example from figure 3.83 the mean model determined reliability for modification 3 from reliability set 3 was found to be $\overline{\hat{R}_3} = 0.86$ in the $NTF_3 = 1$ case. From table 3.28 the percentage standard error $P.S.E._{\hat{R}_3}$ corresponding to this mean model determined modification reliability is 20%. Therefore, by equation 3.74 the standard deviation for the observed mean model determined reliability is

$$S.D._{\hat{R}_3} = \overline{\hat{R}_3} \times \frac{P.S.E._{\hat{R}_3}}{100} = 0.86 \times \frac{20}{100} = 0.172 \ . \qquad (3.79)$$

In tables 3.28, 3.29, and 3.30 the discrete model's variability performance for determining modification reliability status is displayed for the specified total number of failures until modification cases of one, three, and five failures, respectively. Two salient characteristics displayed by the results are (1) the large variability of the estimates produced by the reliability estimator of equation 3.71 and (2) the large variability of the reliability estimates produced by the discrete model for reliability set 10, the permanently stagnated reliability case. The same difficulty is displayed to a lesser degree for reliability set 7.

If reliability set 10 is not considered, then a goal of 40% percentage standard error is achieved by the discrete reliability growth model on the fifth modification for the $NTF_m = 1$ and 3 cases and on the fourth modification for the $NTF_m = 5$ cases. If reliability set 7 is also dropped from consideration, the 40% goal is achieved on the fifth

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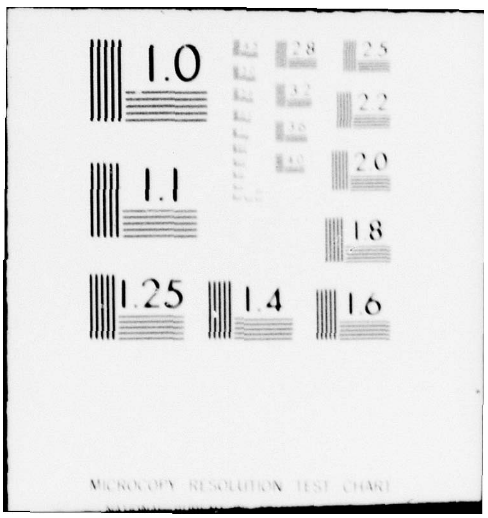
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MICROCOPY RESOLUTION TEST CHART

modification for $NTF_m = 1$, on the third modification for $NTF_m = 3$, and on the second modification for $NTF_m = 5$. Note that the reliability sets characterizing temporary reliability stagnation and degradation (sets 8 and 9) did not have to be taken out of consideration in order to find variability performance at a reasonable level.

Tables 3.31, 3.32, and 3.33 present the discrete model's variability performance in forecasting the reliability of the subsequent modification version of the component under test for the $NTF_m = 1$, 3, and 5 cases. Again, the worst variability performance occurs for reliability sets 7 and 10. Dropping reliability sets 7 and 10 from consideration, a 40% percentage standard error goal is achieved only in the case of $NTF_m = 5$ at the fourth modification. Hence, variability performance of the discrete reliability growth model for forecasting is not particularly good.

TABLE 3.28

DISCRETE RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE

5 MODIFICATIONS, 1 FAILURE/MODIFICATION, DETERMINED RELIABILITY

MODIFICATION	RELIABILITY SET 1	2	3	4	5	6	7	8	9	10
0	231	240	170	195	201	179	170	161	170	184
1	32	38	64	100	104	126	166	98	69	195
2	12	24	34	53	64	99	170	76	47	214
3	8	15	20	26	32	62	116	62	49	213
4	4	9	13	16	18	27	84	38	48	172
5	4	5	7	6	8	11	19	16	21	137

TABLE 3.29

DISCRETE RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE

5 MODIFICATIONS, 3 FAILURES/MODIFICATION, DETERMINED RELIABILITY

	RELIABILITY SET									
MODIFICATION	1	2	3	4	5	6	7	8	9	10
0	105	108	107	117	128	119	104	116	110	109
1	11	14	28	45	53	70	118	46	30	109
2	2	10	14	17	28	41	91	33	22	116
3	1	4	5	9	16	31	64	28	27	106
4	1	2	4	5	7	12	41	18	30	110
5	1	1	2	2	3	5	8	5	8	93

TABLE 3.30

DISCRETE RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE

5 MODIFICATIONS, 5 FAILURES/MODIFICATION, DETERMINED RELIABILITY

MODIFICATION	RELIABILITY SET 1	2	3	4	5	6	7	8	9	10
0	79	73	73	71	86	75	87	73	80	82
1	5	10	22	34	45	58	69	42	22	79
2	2	4	11	12	26	36	65	26	18	81
3	1	3	5	6	13	23	42	21	21	75
4	1	1	3	3	6	11	33	17	17	70
5	1	1	1	2	3	3	6	4	4	65

TABLE 3.31

DISCRETE RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE

5 MODIFICATIONS, 1 FAILURE/MODIFICATION, FORECAST RELIABILITY

	RELIABILITY SET									
MODIFICATION	1	2	3	4	5	6	7	8	9	10
2	42	80	217	606	554	4004	593	957	244	484
3	11	41	57	77	84	192	1869	134	60	4475
4	8	20	25	28	36	78	190	84	73	920
5	4	10	16	16	19	27	112	46	70	343

TABLE 3.32

DISCRETE RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE

5 MODIFICATIONS, 3 FAILURES/MODIFICATION, FORECAST RELIABILITY

MODIFICATION	RELIABILITY SET									
	1	2	3	4	5	6	7	8	9	10
2	7	11	31	53	87	150	923	56	31	767
3	1	9	14	16	29	53	166	38	24	440
4	1	3	4	7	15	37	82	35	41	205
5	1	2	3	4	6	12	46	19	41	196

TABLE 3.33

DISCRETE RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE

5 MODIFICATIONS, 5 FAILURES/MODIFICATION, FORECAST RELIABILITY

	RELIABILITY SET									
MODIFICATION	1	2	3	4	5	6	7	8	9	10
2	1	6	20	44	60	92	250	59	24	300
3	1	2	8	10	28	43	121	30	20	177
4	1	2	4	5	12	24	54	24	29	122
5	1	1	2	3	5	10	37	18	20	105

IV. CONTINUOUS FAILURE RATE RELIABILITY GROWTH MODELS COMPARISON

Codier in reference 2 gives a derivation in which an equation of the form of the continuous instantaneous failure rate reliability growth model (equation 3.37) is obtained by differentiating the equation of the continuous cumulative failure rate reliability growth model (equation 3.1) with respect to the total accumulated test time TT after substitution of the cumulative failure rate estimator of equation 3.13. The derivation essentially proceeds along the following line:

$$\lambda_{TT} \equiv \frac{TF}{TT} = \beta TT^{-\alpha} \text{ ; therefore,} \tag{4.1}$$

$$TF = \beta TT^{(1-\alpha)} \text{ .} \tag{4.2}$$

Now,

$$\frac{\partial TF}{\partial TT} = \frac{\partial}{\partial TT} (\beta TT^{(1-\alpha)}) = (1-\alpha)\beta TT^{-\alpha} \text{ , and} \tag{4.3}$$

$$\lambda_T \equiv \frac{\partial TF}{\partial TT} \text{ .} \tag{4.4}$$

Hence Codier derives an equation

$$\lambda_T = (1-\alpha)\beta TT^{-\alpha} \tag{4.5}$$

and claims that the equation is the continuous instantaneous failure rate model

$$\lambda_T = (1-a) \text{ b } TT^{-a} \text{ ; i.e.,} \tag{4.6}$$

equation 3.37. The implication being that the cumulative model parameters α and β are equivalent to the instantaneous model parameters a and b, respectively; and so, once either α and β or a and b have been estimated, it is unnecessary to estimate the other parameter pair.

The cumulative and instantaneous failure rate models were considered as two unrelated models which were proposed to model different aspects of reliability growth from the viewpoint of this thesis. The evaluation of the two models was conducted accordingly. In order to investigate the proposition that the cumulative and instantaneous models are related thru equivalence of model parameters a hybrid cumulative failure rate model was devised that utilized the parameter estimates generated for the instantaneous failure rate model. Also, a hybrid instantaneous failure rate model was devised that utilized the parameter estimates generated for the cumulative failure rate model. Specifically, the hybrid cumulative failure rate model is given by

$$\widehat{\lambda^{*}_{TT_{i,r}}} = \widehat{b}_{i,r} \, TT_{i,r}^{-\widehat{a}_{i,r}} \qquad (4.7)$$

where $\widehat{a}_{i,r}$ and $\widehat{b}_{i,r}$ were determined by the instantaneous failure rate model equations 3.42 and 3.43. The hybrid instantaneous failure rate model is given by

$$\widehat{\lambda_{T_{i,r}}} = (1 - \widehat{\alpha}_{i,r}) \, \widehat{\beta}_{i,r} \, TT_{i,r}^{-\widehat{\alpha}_{i,r}} \qquad (4.8)$$

where $\widehat{\alpha}_{i,r}$ and $\widehat{\beta}_{i,r}$ were determined by the cumulative failure rate model equations 3.21 and 3.22 after appropriate transformations given in chapter III.A.4.

As cumulative and instantaneous failure rate model parameter estimates were determined for each phase of each simulation for the different lambda sets they were applied to the hybrid models as well as to their appropriate models. Hybrid model estimates of the cumulative and instantaneous failure rates were recorded and the standard statistics of chapter III were compiled.

Table 4.1 contains the mean cumulative and instantaneous determined failure rate estimates produced by the hybrid models for the lambda set 3, ten tests per phase ($NT_i = 10$) case listed with the appropriate underlying mean test time weighted average cumulative failure rates $\overline{\lambda}_{TT_i}$ (equation 3.30) and the specified underlying instantaneous failure rates λ_i (table 3.1). Lambda set 3 is one of the "nice" failure rate sets for which both the cumulative and instantaneous failure rate models displayed some of their best accuracy and variability performance (see figures 3.5, 3.6, 3.7, and 3.44; and tables 3.4 and 3.16). Entries of 10.0000 for the hybrid cumulative model mean determined cumulative failure rate in table 4.1 indicate that the actual mean estimate was greater than or equal to 10.0000.

The performance of the hybrid models displayed in table 4.1 for a "nice" lambda set is typical of the behavior of the hybrid models for all the lambda sets, nice or anomalous. The hybrid cumulative model (instantaneous model's estimated parameters substituted into the cumulative model) produced failure rate estimates of magnitude greater than 10.0 for the first few test phases followed by very erratic estimates (including negative mean estimates) which displayed no relation to the mean test time weighted average cumulative failure rates supposedly being modelled. On the other hand the hybrid instantaneous model (cumulative

TABLE 4.1

HYBRID CUMULATIVE AND INSTANTANEOUS FAILURE RATE MODEL SIMULATION RESULTS
LAMBDA SET 3: 16 PHASES, 10 TESTS/PHASE

PHASE *****	MEAN TEST TIME WEIGHTED AVERAGE CUMULATIVE FAILURE RATE **********	HYBRID CUMULATIVE MODEL MEAN DETERMINED CUMULATIVE FAILURE RATE **********	SPECIFIED INSTANTANEOUS FAILURE RATE *************	HYBRID INSTANTANEOUS MODEL MEAN DETERMINED INSTANTANEOUS FAILURE RATE ************
1	0.7000	0.6702	0.7000	0.6702
2	0.4703	10.0000	0.3530	0.3605
3	0.3565	10.0000	0.2380	0.3048
4	0.2863	10.0000	0.1800	0.2669
5	0.2392	10.0000	0.1450	0.2362
6	0.2064	-0.1003	0.1220	0.2173
7	0.1818	1.9321	0.1060	0.2016
8	0.1625	-0.0171	0.0933	0.1889
9	0.1474	-0.0065	0.0837	0.1796
10	0.1346	0.0532	0.0760	0.1684
11	0.1241	-0.0206	0.0697	0.1626
12	0.1152	0.0237	0.0644	0.1548
13	0.1076	0.0070	0.0600	0.1497
14	0.1009	0.2491	0.0562	0.1442
15	0.0952	0.0518	0.0529	0.1402
16	0.0900	0.0060	0.0500	0.1357

model's estimated parameters substituted into the instantaneous model) produced what appeared to be a good failure rate estimate for the first one or two phases ($i = 2, 3$; models do not estimate for the first phase) but quickly transitioned to a dampened, sedate pattern of estimates which was very unresponseive to even the most radical variations in the underlying instantaneous failure rate progress path. Also, the hybrid instantaneous model's failure rate estimates after the second or third phase generally exhibited increasing magnitude error except for crossing situations in the cases of temporary stagnation or degradation of the specified underlying instantaneous failure rates.

Regarding the negative failure rate estimates produced by the hybrid cumulative model (instantaneous model parameters in cumulative model), the following observations on the parameter estimates $\widehat{a}_{i,r}$ and $\widehat{b}_{i,r}$ of the continuous instantaneous failure rate model were made during the reliability testing procedure computer simulations. During the simulations the magnitude of the parameter estimate $\widehat{a}_{i,r}$ was continually very close to 1.0 often being more or less than 1.0 only in the tenth, eleventh, or twelfth decimal place. From the instantaneous model equation 4.6 it can be seen that logically if the $\widehat{a}_{i,r}$ estimate is less than 1.0, the $\widehat{b}_{i,r}$ estimate should be positive; and, if the $\widehat{a}_{i,r}$ estimate is greater than 1.0, the $\widehat{b}_{i,r}$ estimate should be negative in order to produce positive estimates of the instantaneous failure rate. Exactly such behavior was observed during the computer simulations; as the magnitude of the $\widehat{a}_{i,r}$ estimate moved to either side of 1.0 the sign of the $\widehat{b}_{i,r}$ estimate would "flip-flop" accordingly. Of course if a negative $\widehat{b}_{i,r}$ estimate is plugged into the cumulative failure rate model for β in equation 4.1, a negative estimate of the cumulative failure rate necessarily results no matter what the sign or magnitude of the $\widehat{a}_{i,r}$ estimate which is substituted for α (save a value of $\widehat{a}_{i,r} = \infty$). Hence, it is very plausible for the hybrid cumulative model to have produced negative estimates of cumulative failure rate.

In contrast to the hybrid cumulative model's erratic failure rate estimates the hybrid instantaneous model (cumulative model parameters in instantaneous model) produced failure rate estimates which, after one or two test phases, displayed a "sluggish" growth pattern and generally increased in magnitude error from the underlying instantaneous failure rate λ_i being modelled. This situation is not surprising because the cumulative model parameters being employed in the hybrid instantaneous model are parameters utilized in modelling a "smoothed" quantity, the underlying cumulative (average) failure rate that is collectively characteristic of all the versions of an item tested thru a given point in the acquisition cycle. As the history of the item stretches farther and farther (total accumulated test time TT increases) the cumulative failure rate of the item becomes increasingly insensitive to the current instantaneous failure rate intrinsic to the item. Hence, the parameters of the cumulative model likewise become increasingly insensitive to the current instantaneous reliability status. When these relatively stable parameters are utilized in another model whatever its form, "smoothed" stable output is reasonably expected.

V. EVALUATION SUMMARY AND CONCLUSIONS

This thesis examined three reliability growth models for which accuracy, precision, and robustness characteristics were investigated over a wide variety of true underlying growth patterns, both "nice" and anomalous. The evaluation of a continuous cumulative failure rate model, an instantaneous failure rate model, and discrete reliability model was accomplished by computer simulation of reliability testing procedures that were appropriate to the reliability growth model types for a systems acquisition cycle. Performance of the models both in tracking and predicting true underlying reliability progress patterns was measured for mean accuracy and variability (precision) over progress patterns which included most situations that can be encountered, both good and bad. The models' performance characteristics are summarized in the following paragraphs.

A. CONTINUOUS CUMULATIVE FAILURE RATE RELIABILITY GROWTH MODEL

The continuous cumulative failure rate model examined generally displayed very good to excellent accuracy performance in tracking both "nice" and anomalous true underlying cumulative failure rate patterns. The cumulative model did exhibit difficulty in determining and forecasting true underlying patterns that characterize rapid reliability growth (failure rate decay) in the latter phases of the systems acquisition cycle. The difficulty was evidenced by mean model determined/forecast cumulative failure rate estimates that diverged from the true underlying values on the pessimistic side; thus, the cumulative model provided estimates that could be considered as upper bounds on failure rate in this situation. The other anomalous situation with which the cumulative model experienced difficulty in tracking and predicting accuracy was one of reliability growth interrupted by a period of reliability degradation. The modelled cumulative failure rate path both lagged the true underlying cumulative failure rate path and failed to reflect the full magnitude of the failure rate degradation. Thus dependence on the model in this situation could hamper response to the degradation by giving a "too little too late" signal of the problem. The cumulative model coped with true underlying cumulative failure rate patterns that were permanently stagnated quite adequately.

As the reliability testing procedures of the acquisition cycles completed more test phases, the cumulative model's variability (precision) performance improved uniformly for both "nice" and anomalous true underlying failure rate paths. Variability percentage standard error goals were satisfied earlier and earlier in the acquisition cycle as more testing data was made available to the model. This good variability performance provides a degree of confidence in employing the cumulative failure rate model in actual systems acquisition programs where the intrinsic cumulative failure rate status is truly unknown.

B. CONTINUOUS INSTANTANEOUS RAILURE RATE RELIABILITY GROWTH MODEL

The continuous instantaneous failure rate model examined, while exhibiting the capability to track and predict the shape of various unknown, underlying instantaneous failure rate path types with very good accuracy and robustness, consistently produced optimistically biased estimates of the instantaneous failure rate. If bias must be accepted in a reliability growth model, then pessimistic bias is preferred to optimistic bias. Consistent pessimistic bias permits managers to utilize the model produced estimates as lower bound type values and have confidence that reliability status is being observed from the "right side of the fence". On the other hand consistently optimistic model produced estimates impart a great deal of uncertainty as to how bad reliability status might be and what degree of corrective action is required.

The anomalous situations in which the instantaneous model experienced difficulty in tracking and predicting the shape of the true failure rate path were (1) the initial period of recovery in reliability growth after a period of temporary reliability stagnation, (2) permanently stagnated reliability growth at high failure rates, and (3) reliability growth interrupted by a temporary period of reliability degradation. In the case of temporary reliability degradation the instantaneous model charted a determined failure rate path that was an exaggeration of the underlying instantaneous failure rate path; i.e., the periods of growth were displayed optimistically and the period of degradation was portrayed to a magnitude greater than it was in truth. The instantaneous model's forecasts, while accurately capturing the shape of the underlying progress path, were consistently optimistic during the period of degradation.

The instantaneous model's accuracy performance was generally better for contracted acquisition cycles and suffered when the reliability testing procedure was extended to encompass a large number of test phases. Because of the model's consistent optimistic bias, accuracy performance improved as the true underlying failure rate decreased thereby "sandwiching" the model estimates between the true failure rate and 0.0. Also, the instantaneous model routinely furnished "off-the-scale" forecasts for the first one or two phases of an acquisition cycle.

The instantaneous model's demonstrated capability to determine and forecast the true underlying reliability status was overshadowed by its poor variability performance which (1) never achieved a really comfortable percentage standard error goal uniformly for all failure rate patterns simulated and, more damaging, (2) oscillated as test phases of the reliability testing procedure were accomplished. Because of this poor variability performance, the instantaneous failure rate model cannot be employed by itself with any degree of confidence.

C. DISCRETE RELIABILITY GROWTH MODEL

The discrete reliability growth model examined generally displayed very good determined reliability accuracy performance even for the most restricted test data cases. Slight pessimistic bias was demonstrated for model determined reliability status while a greater magnitude of pessimistic bias was present in model forecast reliability. Again, pessimistic bias is favored over optimistic bias in charting a reliability progress path. The degree of pessimism in determined and forecast reliability decreased markedly as more test data on each modification version of an item were gathered. In cases of limited test data the discrete model often forecast negative estimates of reliability for the early modification versions of a component under test.

For the anomalous situation of temporary reliability degradation the discrete model (1) failed to determine the full magnitude of the degradation and (2) provided reliability status predictions that lagged the true underlying reliability path outcomes significantly. In actual testing this performance characteristic would give a delayed signal of a situation that required corrective action. Also, gathering additional test data failed to remedy this deficiency in the discrete model's performance.

Variability performance of the discrete model, while generally good, revealed difficulty with the permanently stagnated reliability case and the case of rapidly increasing reliability during the latter modification versions of an item. Variability performance improved uniformly for all progress path characterizations as more modifications of the item being tested were accomplished. Also, variability goals were satisfied earlier in the acquisition cycle as more complete testing was accomplished on each modification version of the item. This nice behavior of the discrete model lends confidence to its utilization; and therefore, the discrete model is preferred to the continuous instantaneous failure rate model for obtaining a measure of the current or "instantaneous" reliability status of an item proceeding thru an acquisition cycle.

D. GENERAL OBSERVATIONS

For determining current reliability status it may be suggested that rather than employing the reliability growth models, why not utilize the point estimators of reliability status which were used to provide data to the models? Although the performance of the point estimators appropriate to each reliability growth model were only observed at one point for each model in the reliability testing procedure simulations (first phase of testing or mod 0 version of a component), at that point the estimators displayed very poor variability performance upon which the reliability growth models improved rapidly and significantly which tends to negate any confidence in utilizing the point estimators based on their good accuracy performance.

In retrospect the performance characteristics demonstrated by the reliability growth models examined suggest that as a model is employed the estimated reliability progress path it produces be compared with the appropriate model accuracy performance graphs in chapter III. If the estimated reliability progress path corresponds to a case for which the computer simulation performance results show the model's performance was good, then confidence can be placed in the estimated progress path. On the other hand if the estimated path corresponds to one of the anomalous cases where simulation results revealed a deficiency in the model's performance, then the simulation performance results at least give an indication of which direction the true underlying reliability progress path lies.

Finally, although use of the continuous instantaneous failure rate reliability growth model is questionable, since the test data required for the instantaneous model is identical to the data collected for the continuous cumulative failure rate model, application of the instantaneous model simultaneously with the cumulative model utilizing the method described in the preceding paragraph may provide some insight to the shape of the true underlying instantaneous failure progress path from the instantaneous model. But the comparison performed between the two continuous failure rate models definitely indicates that the cumulative failure rate and instantaneous failure rate models' parameters not be interchanged based on their hypothetical equivalence.

APPENDIX A

Derivation of Ordinary Least Squares Regression Estimates of the Continuous Instantaneous Failure Rate Reliability Growth Model Parameters

The continuous instantaneous failure rate model reliability growth model as given in equation 3.37 may be written as

$$\lambda_i = b(1-a)TT_i^{-a} . \tag{A.1}$$

Taking the logarithmic transformation of this equation yields

$$\ln \lambda_i = \ln b(1-a) - a \ln TT_i . \tag{A.2}$$

Let

$$\sum = \sum_{i=1}^{n} ,$$

$$Y_i = \ln \lambda_i ,$$

$$X_i = \ln TT_i ,$$

$$\bar{Y} = \frac{\sum Y_i}{n} , \text{ and}$$

$$\bar{X} = \frac{\sum X_i}{n} .$$

Therefore,

$$Y_i = \ln b(1-a) - aX_i \tag{A.3}$$

The residual error for regression is

$$\varepsilon_i = Y_i - \widehat{Y}_i = Y_i - [\ln b(1-a) - aX_i] \ . \qquad (A.4)$$

The goal of the regression is to minimize the sum of the residual errors squared; i.e.,

$$\min \sum \varepsilon_i^{\,2} = \sum (Y_i - \widehat{Y}_i)^2 = \sum [Y_i - \ln b(1-a) + aX_i]^2 .$$

$$(A.5)$$

Taking partial derivatives of equation A.5 with respect to b and a ; then equating the partials to 0 yields the following two equations in two unknowns:

(1) $$\frac{\partial \sum \varepsilon_i^{\,2}}{\partial b} = \sum [2\ (Y_i - \ln b(1-a) + aX_i)(-\frac{1}{b(1-a)}\ (1-a)] = 0$$

(2) $$\frac{\partial \sum \varepsilon_i^{\,2}}{\partial a} = \sum [2\ (Y_i - \ln b(1-a) + aX_i)(-\frac{1}{b(1-a)}(-b)+X_i)] = 0$$

Simplification of (1) and (2) yields in sequence:

(1') $$\sum [Y_i - \ln b(1-a) + aX_i] = 0 \ , \text{ and}$$

(2') $$\sum [(Y_i - \ln b(1-a) + aX_i)\ (\frac{1}{(1-a)} + X_i)] = 0 \ . \text{ Then,}$$

(1'') $$\sum Y_i = \sum \ln b(1-a) - \sum aX_i \ .$$

(2'') $$\sum \frac{1}{(1-a)} Y_i + \sum Y_i X_i = \sum \frac{1}{(1-a)} \ln b(1-a) + \sum X_i \ln b(1-a) - \sum \frac{aX_i}{(1-a)} - \sum aX_i^{\,2} \ . \text{ Finally,}$$

(1''') $\sum Y_i = n \ln b(1-a) - a \sum X_i$, and

(2''') $\frac{1}{(1-a)} \sum Y_i + \sum Y_i X_i = \frac{n}{(1-a)} \ln b(1-a) + (\ln b(1-a)) \sum X_i -$

$$\frac{a}{(1-a)} \sum X_i - a \sum X_i^2 .$$

Equation (1''') may be manipulated such that

(1'''') $\ln b(1-a) = \frac{1}{n} \sum Y_i + \frac{a}{n} \sum X_i = \bar{Y} + a\bar{X}$ or

(1''''') $b(1-a) = \exp (\bar{Y} + a\bar{X})$.

Finally, the estimate for b is

$$\hat{b} = \frac{1}{(1-\hat{a})} \exp (\bar{Y} + \hat{a} \bar{X}) . \qquad \text{(A.6)}$$

Substituting equation A.6 into (2''') yields

(2'''') $\frac{1}{(1-a)} \sum Y_i + \sum Y_i X_i = \frac{n}{(1-a)} \ln[\frac{\exp (\bar{Y} + a\bar{X})}{(1-a)} (1-a)] +$

$$\ln[\frac{\exp (\bar{Y} + a\bar{X})}{(1-a)} (1-a)] \sum X_i - \frac{a}{(1-a)} \sum X_i - a \sum X_i^2 .$$

Clearing the logarithms in (2'''') and transposing leaves

(2''''') $\frac{n}{(1-a)} (\bar{Y} + a\bar{X}) + (\bar{Y} + a\bar{X}) \sum X_i - \frac{a}{(1-a)} \sum X_i - a \sum X_i^2 -$

$$\frac{1}{(1-a)} \sum Y_i = \sum Y_i X_i .$$

Expanding (2''''') yields

(2*) $\frac{n}{(1-a)} \bar{Y} + \frac{an}{(1-a)} \bar{X} + \bar{Y} \sum X_i + a\bar{X} \sum X_i - \frac{a}{(1-a)} \sum X_i -$

$$a \sum X_i^2 - \frac{1}{(1-a)} \sum Y_i = \sum Y_i X_i .$$

Substituting and clearing terms in (2*) leaves

$$(2^{**})\quad \frac{1}{(1-a)} \sum Y_i + \frac{a}{(1-a)} \sum X_i + \bar{Y} \sum X_i + a\bar{X} \sum X_i - \frac{a}{(1-a)} \sum X_i -$$

$$a \sum X_i^2 - \frac{1}{(1-a)} \sum Y_i = \sum Y_i X_i .$$

Solving for a in (2**) gives

$$(2^{***})\quad a(\bar{X} \sum X_i - \sum X_i^2) = \sum Y_i X_i - \bar{Y} \sum X_i \quad \text{or}$$

$$\hat{a} = \frac{\sum Y_i X_i - \bar{Y} \sum X_i}{\bar{X} \sum X_i - \sum X_i^2} . \blacksquare \qquad (A.7)$$

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