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by

10 SOL R./BODNER

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FACULTY OF MECHANICAL ENGINEERING
TECHNION—ISRAEL INSTITUTE OF TECHNOLOGY
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studies on anelastic materials. Other topics in the program were investigations of ballistic penetration of single target plates under oblique impact and of multilayered targets subject to normal impact, impact strength of composites, and acoustic emission of composites. The accomplishments of the program are discussed and proposed directions of future research are indicated.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report reviews the program of work performed under Grant AFOSR-74-2607, sponsored by the U.S. Air Force Office of Scientific Research through the European Office of Aerospace Research and Development (EOARD) during the period 1 Dec 73 to 31 May 1978. The main subject of the research program was the further development of elastic-viscoplastic constitutive equations to represent a wide range of materials behavior and loading conditions and application of the equations to static and dynamic structural problems. Associated investigations included generalization of the constitutive equations to multiaxial stress states and

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FINAL SCIENTIFIC REPORT

DYNAMIC INELASTIC BEHAVIOR OF MATERIALS
1 December 1973 - 31 May 1978

by

Sol R. Bodner*
Principal Investigator

Abstract

This report reviews the program of work performed under Grant AFOSR-74-2607, sponsored by the United States Air Force Office of Scientific Research through the European Office of Aerospace Research and Development (EOARD) during the period 1 December 1973 to 31 May 1978. The main subject of the research program was the further development of elastic-viscoplastic constitutive equations to represent a wide range of material behavior and loading conditions and application of the equations to static and dynamic structural problems. Associated investigations included generalization of the constitutive equations to multiaxial stress states and studies on anelastic materials. Other topics in the program were investigations of ballistic penetration of single target plates under oblique impact and of multi-layered targets subject to normal impact, impact strength of composites, and acoustic emission of composites. The accomplishments of the program are discussed and proposed directions of future research are indicated.

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* Professor, Faculty of Mechanical Engineering
Technion - Israel Institute of Technology

OBJECTIVES

a. Constitutive Equations

The main objective of the research program was the continued development of equations that would adequately model material behavior in the time dependent inelastic range for arbitrary loading and temperature histories. Once the material constants in the equations are determined for a given material, the same equations should apply for all applicable loading circumstances. These equations would be numerically integrable to give material response results that should correspond to those actually obtained by a similarly controlled testing machine. A reasonable initial stage in the development of the equations would be the representation of the range of behavior under uniaxial stress of constant sign, i.e. inelastic response for constant and varying strain rates, unloading and reloading, creep, and stress relaxation. The equations should cover a range of temperatures so that they could handle situations in which both temperatures and loading are varied. The next level of generalization would be for conditions of uniaxial stress of varying sign with consideration of arbitrary combinations of monotonic and cyclic loading. Complete generalization of the material characterization to apply to multi-dimensional stress states over wide temperature ranges is the ultimate objective and is an extremely formidable problem.

In addition to the primary elastic-viscoplastic formulation, the material characterization should include anelasticity which would indicate energy losses for reversible deformations and other time dependent effects. Anelasticity could be a reasonably important factor for polymers at all temperatures and metals at high temperatures.

The end use of material characterizations is their inclusion in computer programs for structural analysis. This aspect should be considered at the initial stage of development of the constitutive equations to ensure their compatability with standard numerical procedures. In particular, these constitutive equations should be suitable for finite element and finite difference programs for the static and dynamic analyses of structures.

b. Ballistic Perforation

An engineering model of ballistic perforation of metallic target plates under perpendicular impact was developed in an earlier phase of the research program (by Awerbuch and Bodner). This analytical model seems capable of providing a reasonably accurate picture of the perforation process at standard ballistic velocities. Part of the research effort was devoted to generalizing the basic model to other cases. One was the oblique impact of single target plates, while another consideration was multilayered target plates under normal impact.

c. Composite Materials

Dynamic inelastic properties of composite materials were examined as part of the overall research program. Attention was given to the impact strength of angle ply laminates, and the application of acoustic emission techniques to examine time dependent failure processes.

Personnel

In addition to the Principal Investigator, Professor S. R. Bodner, the following persons were engaged for various times on the research program. They are current or former members or associates of the Faculty of Mechanical Engineering of the Technion.

Associate Professor (Adjunct) Yehuda Partom

Associate Professor Jacob Lifshitz

Dr. Malcolm Newman (presently Engineering Consultant, L.I.,N.Y.)

Dr. Jonathan Awerbuch (presently at Drexel University, Philadelphia, Pa.)

Dr. Assa Rotem (Senior Lecturer)

Dr. Anthony Merzer (Senior Lecturer)

Dr. Joseph Baruch (Research Engineer, Israel Metals Institute)

Dr. Dov Derman (formerly, Graduate Student)

Mr. Ascher Sperling (formerly, Graduate Student)

Mr. Issac Marom (formerly, Graduate Student)

Mr. Zvi Zaphir (Graduate Student)

LIST OF SCIENTIFIC REPORTS, 1973 - 78

Scientific Report No. 1: "Constitutive Equations for Elastic-Viscoplastic Strain Hardening Materials," by S.R. Bodner and Y. Partom, February, 1974; published in the Journal of Applied Mechanics, vol. 42, 1976, pp. 385-389.

Abstract

A set of constitutive equations has been formulated to represent elastic-viscoplastic strain-hardening material behavior for large deformations and arbitrary loading histories. An essential feature of the formulation is that the total deformation rate is considered to be separable into elastic and inelastic components which are functions of state variables at all stages of loading and unloading. The theory, therefore, is independent of a yield criterion or loading and unloading conditions. The deformation rate components are determinable from the current state which permits an incremental formulation of problems. Strain hardening is considered in the equations by introducing plastic work as the representative state variable. The problem of tensile straining has been examined for a number of histories that included straining at various rates, rapid changes of strain rate, unloading and reloading, and stress relaxation. The calculations were based on material constants chosen to represent commercially pure titanium. The results are in good agreement with corresponding experiments on titanium specimens.

Scientific Report No. 2: "Determining the Load-Time History of Fiber Composite Materials by Acoustic Emission," by A. Rotem and J. Baruch, March 1974; published in the Journal of Materials Science, vol. 9, 1975, pp. 1789-1798.

Abstract

Acoustic emission was monitored during the axial loading of unidirectional fiber composite tensile specimens. The material consisted of strong, brittle fibers (E glass) embedded in a viscoelastic matrix (epoxy). It was found that when the load was held constant the acoustic emission output continued, but at a decreasing rate with time at load. As the load level was increased, the acoustic emission output at load continued for a longer period. It is suggested that the acoustic emission under constant loading is a result of fiber fracture which continues after loading ceases because of the viscoelastic nature of the matrix which allows stress redistribution with time. The experimental results from acoustic emission are compared with computer calculations for fiber fracture based on theoretical considerations. Good agreement is noted between the theoretical and experimental results

Scientific Report No. 3: "A Representation of Elastic-Viscoplastic Strain-Hardening Behavior for Generalized Straining Histories," by S.R. Bodner and Y. Partom, April 1974; published in Proc. Conference on the Mechanical Properties of Materials at High Rates of Strain, Oxford, *Institute of Physics Conference* Series No. 21, London, 1975, pp. 102-110.

Abstract

A theory of elastic-viscoplastic, strain hardening material behaviour has been developed which is motivated in part from concepts of 'dislocation dynamics'. The resulting constitutive equations are intended to represent rate-dependent inelastic material behaviour for arbitrary loading and unloading histories. The basis of the formulation is the separation of the total deformation rate into elastic (geometrically reversible) and inelastic (geometrically irreversible) components at all stages of loading and unloading. The constitutive equations are relations between the deformation rate components and state variables which, in the present state of formulation, are the stress and the total plastic work which is introduced as the measure of strain hardening. The same equations hold at all stages of deformation which makes the formulation suitable for the computer solution of structural problems involving complicated loading histories. A number of examples are presented. The case of titanium tensile specimens subjected to a variety of different straining histories is described. The use of the equations in structural dynamic plasticity problems is indicated by a typical example. The description of the anelastic properties of materials by means of the general formulation is also discussed.

Scientific Report No. 4: "Constitutive Equations for Anelastic Materials at Finite Strains," by D. Derman, S.R. Bodner and Y. Partom, July, 1974; published in Proceedings IUTAM Symposium on the Mechanics of Visco-Elastic Media and Bodies, Springer-Verlag Publ., 1975, pp. 196-204.

Abstract

A representation of anelastic material behavior, i.e. materials that exhibit energy losses and rate dependent moduli for geometrically reversible deformations, is proposed for multiaxial stress states and large deformations (finite strains). The constitutive equations are relations between the deformation rate and the stress components. An "anelastic stress" is considered to be part of the total applied stress and to be responsible for the phenomena of energy losses, rate dependent moduli, and delayed elasticity (creep and recovery). In this incremental formulation, "memory" and "history" effects are incorporated into the current deformation state and the state variable, the elastic stress, which makes the method suitable for computer solution of structural problems involving arbitrary loading histories. The present analysis is limited to isothermal conditions but the procedure can be generalized to consider thermal effects.

Scientific Report No. 5: "Finite Element Analysis for Time-Dependent Inelastic Material Behavior," by M. Newman, Z. Zaphir and S.R. Bodner, February 1975; published in the Journal of Computers and Structures, vol. 6, 1976, pp. 157-162.

Abstract

A representation of inelastic, time-dependent material behavior, due to Bodner and Partom, in which both elastic and inelastic deformations are considered to be present at all stages of loading and unloading, is shown to be well-suited to structural analyses using finite element modeling techniques and high speed digital computation methods. The formulation considerably simplifies the computational logic for non-monotonic and cyclic loading problems since no special unloading criteria or yield conditions are required. Examples demonstrating strain-rate sensitivity, work-hardening, and reversed loading behavior are given for problems in the small strain range. Experimental results for a titanium tensile specimen subjected to changes in crosshead velocity are compared with predictions based on a plane stress finite element model. Numerical analyses using axisymmetric, solid of revolution finite element models are presented for unstiffened and ring-stiffened cylindrical shells subjected to time-dependent external pressure.

Scientific Report No. 6: "An Investigation of Oblique Perforation of Metallic Plates by Projectiles," by J. Awerbuch and S.R. Bodner, March 1975; presented at the SESA annual meeting 1976, and published in Experimental Mechanics, vol. 17, 1977, pp. 147-153.

Abstract

An experimental and analytical study was performed on the mechanics of oblique perforation of metallic plates by projectiles. The purpose was to determine the dependence of the velocity drop on the angle of impact for prescribed mechanical and physical properties of the projectile and the target plate. The ballistic experiments were carried out with 0.22-in.-caliber lead bullets on target plates of commercially pure aluminum and an aluminum alloy which ranged from 2.0 to 6.0 mm in thickness. Transient measurements were taken which included high-speed photographs of the perforation process. The theoretical model that had been developed previously by the authors for the case of normal perforation was modified to include the effects of the angle of impact. The experimental observations for the present test conditions indicate that the main modification to the analysis is the use of the total projectile path as the effective target-plate thickness. Reasonably good agreement between the experimental and theoretical results was obtained.

Scientific Report No. 7: "Impact Strength of Angle Ply Fiber Reinforced Materials," by J.M. Lifshitz, September 1975; published in the Journal of Composite Materials, vol. 10, 1976, pp. 92-101.

Abstract

Tensile strength of angle ply balanced laminates made of glass fibers and epoxy matrix has been investigated under dynamic loading using an instrumented drop weight apparatus. A comparison of theoretical and experimental stress-strain curves reveals that good agreement exists for a certain range of fiber orientation. Different failure criteria have to be used for each range. Failure stresses in the dynamic case are found to be considerably higher than the corresponding static values for the complete range of fiber orientation. Failure strains and initial effective moduli are the same for static and for impact loadings.

Scientific Report No. 8: "Numerical Analysis of Large Elastic-Plastic Deformation of Beams due to Dynamic Loading," by A. Sperling and Y. Partom, December 1976; published in the International Journal of Solids and Structures, vol. 13, 1977, pp. 865-876.

Abstract

Numerical calculations were performed for two examples of the response of elastic-plastic beams subjected to dynamic loads. These were a simply supported, axially restrained beam under suddenly applied uniform pressure, and an axially restrained, clamped beam with a central mass that is impacted by a projectile. Large elastic-plastic deflections were considered, and the method of finite differences was used. Two different constitutive equations were assumed: the elastic-perfectly plastic relation, and a special elastic-viscoplastic, strain hardening model. Analysis of the results included examining the interaction between the bending moment and the axial force, the variation of the axial force, bending moment and deflection with time, and the propagation velocities of the various phenomena during motion. Experiments were carried out in which a rifle projectile hit a central mass which had been fastened to a clamped beam. Comparison between the theoretical and experimental dynamic deflections shows good agreement for relatively short response times.

Scientific Report No. 9: "Constitutive Equations for Cyclic Loading of Rate Dependent Materials," by S.R. Bodner and Y. Partom, April 1976.

Abstract

The constitutive equations that have been developed by the authors to represent elastic-viscoplastic, strain-hardening materials have been modified to apply for changes in the direction of stressing. This modification involves altering the strain-hardening parameters with a change in the sign of the stress and also adding a linear viscosity term to the plastic deformation rate. These modifications are shown to have a physical basis and enable the constitutive equations to predict the general features of cyclic stress-strain behavior at various extensions rates and strain amplitudes including the tendency toward a limit cycle. An unusual feature of cyclic loading tests on some metals is a local concavity of the stress-strain curve which is predicable by these equations. Combined cyclic and unidirectional loading could also be treated by the equations. Cyclic stress-strain curves were calculated for titanium for various conditions and compared to corresponding experimental results.

Scientific Report No. 10: "A Hardness Law for Inelastic Deformation," by S.R. Bodner, November 1976; published in Letters in Applied and Engineering Sciences (Inter. Journal of Engineering Science), vol. 16, 1978, pp. 221-230.

Abstract

Certain formulations of elastic-viscoplastic constitutive equations lead to inelastic deformations being governed by a functional relation between the plastic deformation rate and the stress. A parameter that appears in certain forms of this equation can be interpreted as an internal variable measure of the overall resistance of the material to plastic flow. In this paper, it is taken to be a second order tensor and designated the "hardness stress". A procedure for calculating changes in the hardness stress parameter for general multiaxial loading histories is proposed in this paper. These evolutionary equations lead to induced plastic anisotropy. Stress-strain relations that would be predicted by this procedure appear to be consistent with experimental results for various multiaxial loading histories including proportional and non-proportional cyclic loading.

Scientific Report No. 11: "Nonlinear Anelastic Behavior of a Polymeric Rubber at Finite Strains," by D. Derman, Z. Zaphir and S.R. Bodner, March 1977; to be published in the Journal of Rheology, vol. 22, 1978.

Abstract

A set of constitutive equations has been developed to represent nonlinear anelastic behavior, i.e. energy losses and rate dependent moduli for geometrically large, reversible deformations. These equations are in incremental form and can be used to solve boundary value problems by numerical methods. Two problems were considered: thick walled spheres subjected to cycles of internal pressure, and a long rod subjected to imposed axial velocity (the simple tension test). Experiments were carried out for both conditions on specimens made of a synthetic rubber at three different temperatures (each under isothermal conditions) and over two decades of loading rates. Response curves were calculated based on the constitutive equations and chosen values of the material constants. Generally good agreement was obtained between the predicted and experimental results.

Scientific Report No. 12: "Viscoplastic Constitutive Equations for Copper with Strain Rate History and Temperature Effects," by S.R. Bodner and A. Merzer, January 1978; to be published in the Journal of Engineering Materials and Technology, Trans. ASME, vol. 100, 1978.

Abstract

Elastic-viscoplastic constitutive equations based on a single internal state variable which is a function of plastic work are used to calculate the response of copper to a six decade change of strain rate over a range of temperatures. Calculations were performed for the conditions of an experimental program on copper by Senseny, Duffy, and Hawley, namely, temperatures ranging from 77°K to 523°K and strain rate jumps from $2 \times 10^{-4} sec^{-1}$ to $3 \times 10^{2} sec^{-1}$ at three strain levels. The computed results are in good agreement with the experiments and show similar strain rate and strain rate history effects. Relations are obtained for the temperature dependence of certain parameters in the equations which indicate correspondence between plastic working and temperature and between strain rate sensitivity and temperature.

Scientific Report No. 13: "Projectile Perforation of Multi-Layered Beams," by I. Marom and S.R. Bodner, February 1978; submitted for publication in the International Journal of Mechanical Sciences.

Abstact

A combined analytical and experimental study has been performed on the ballistic resistance of layered targets, in particular, of flat, relatively thin beams with clamped ends in spaced and laminated (i.e., in contact without bonding) conditions. The theoretical analysis combines the effect of structural deformation with the mechanism of perforation. This requires a redefinition of the ballistic limit velocity as the initial impact velocity for which the post perforation velocity and the structural deformation velocity of the impact point would be equal at the same time. The ballistic tests were based on commercially pure and alloy (6061-T6) aluminum target specimens of various thicknesses and configurations, and a 0.22 (in.) caliber projectile which struck the targets perpendicularly at their center with a velocity of 375 m/s. The results for the velocity drop show fairly good agreement between experiments and predictions, and greater ballistic resistance of the laminated configuration.

Scientific Report No. 14: "Uniaxial Cyclic Loading of Elastic-Viscoplastic Materials," by S.R. Bodner, I. Partom and Y. Partom, June 1978.

Abstract:

Elastic-viscoplastic constitutive equations based on two internal state variables are utilized to determine material response for uniaxial cyclic loading conditions. These equations are capable of representing the principal features of cyclic loading behavior including softening upon stress reversal, cyclic hardening or softening, tendency towards a stable limit cycle, cyclic relaxation, and cyclic creep. Calculations were performed for various stress and strain controlled conditions using material constants intended to represent commercially pure titanium and aluminum and OFHC copper. Capabilities and limitations of the analytical formulations are discussed in relation to computed results and corresponding test data.

Publication of Scientific Reports issued under preceding contract during the reporting period:

1. Baruch, J. and Bodner, S.R., "Internal Friction During Repeated Discontinuous Yielding of Metals," Materials Science and Engineering, vol. 14, 1974, pp. 81-87.

2. Awerbuch, J. and Bodner, S.R., "Analysis of the Mechanics of Perforation of Projectiles in Metallic Plates," International Journal of Solids and Structures, vol. 10, 1974, pp. 671-684.

3. Awerbuch, J. and Bodner, S.R., "Experimental Investigation of Normal Perforation of Projectiles in Metallic Plates," International Journal of Solids and Structures, vol. 10, 1974, pp. 685-699.

Conference Presentations of Research Results:

1. "Review of Progress in the Development and Application of Elastic-Viscoplastic Constitutive Equations," by S.R. Bodner, Workshop on Applied Thermoviscoplasticity, sponsored by NSF, Northwestern University, October 1975.

2. "Representation of Generalized Elastic-Viscoplastic Material Behavior," by S.R. Bodner and Y. Partom, presented at the Second International Conference on Mechanical Behavior of Materials, Boston, Mass., August, 1976.

3. "A Review of Some Engineering Models of Ballistic Perforation," by S.R. Bodner, presented at ARO/BRL Workshop on Mechanics of Impact and Penetration," Aberdeen Proving Ground, Md., December, 1976.

ACCOMPLISHMENTS

a. Constitutive Equations

Consideration of strain hardening in the elastic-viscoplastic constitutive equations was achieved by introducing a history dependent internal state variable, Z, in the equation for plastic deformation rate. This parameter was taken to be a scalar function of plastic work, $Z = Z(W_p)$, for isotropic hardening conditions, e.g. uniaxial stress of constant sign. The resulting formulation was shown in Scientific Report (SR) 1 to lead to fairly accurate representation of the uniaxial response of titanium to constant and changing strain rates, unloading and reloading, and stress relaxation. An overall review of the work done on constitutive equations up to 1974 was presented in SR 3.

Inclusion of the elastic-viscoplastic constitutive equations in a quasi-static (i.e. without inertial forces) finite element code was demonstrated in SR 5 with various examples of structures, e.g. stiffened shells, subjected to time dependent loadings in the inelastic range. Consideration of structural inertial forces in dynamic structural analyses is sometimes best done by a finite difference code, and the constitutive equations were programmed in such a code similar to one developed at M.I.T., SR 8. The finite difference code was used for a number of examples of structures subjected to impulsive loading, and some of the calculated results were compared to corresponding experiments in SR 8.

The main relative advantages of the use of the constitutive equations developed in this research program in structural computer codes is that they do not require a yield criterion or loading and unloading conditions. Since the same basic equations are applicable under all circumstances, no auxiliary statements which depend on the loading situation are needed. As a consequence, it is possible to obtain the complete elastic-viscoplastic response of the structural elements through their deformation history.

The initial work on constitutive equations was confined to isothermal conditions; temperature effects were considered in SR 12. This involved identifying those viscoplastic material constants which were temperature dependent and determining the form of the temperature variation. A basis of this investigation was available experimental data on copper for steady and varying strain rates over a temperature range. Some of the conclusions of this exercise were that only two of the viscoplastic constants needed to be considered to be temperature dependent, and that the same work hardening function $Z = Z(W_p)$ held over a wide temperature range. Although the exercise did not include a hardness recovery (annealing) term, it is recognized that such a term in the form $\dot{Z} = - F_1(Z,J_2,T)$ would be important for high temperature behavior of metals. Secondary creep is the balanced condition when the increase of Z due to work hardening equals the decrease of Z due to annealing. Further work is in progress on the use of the constitutive equations for high temperature applications.

Under conditions of change of sign of the uniaxial stress, a second internal state variable is required in the constitutive equations to account for the induced directional character of resistance to plastic deformation. An initial study of the cyclic loading problem, reported in SR 9, utilized a discontinuous treatment of the $Z = Z(W_p)$ function instead of introducing a second inelastic variable. It showed a number of interesting features but is less capable of further generalization that the two variable approach used in SR 14. Results of that report, SR 14, showed the capability of the equations to represent a wide range of cyclic loading characteristics of titanium, copper, and aluminum including cyclic hardening or softening, limiting stable hysteresis loops, cyclic creep, and cyclic relaxation. Inclusion of this more general formulation into the structural computer programs requires a very simple modification of the instructions with no change in the actual computational procedure.

Complete generalization of the constitutive equations to multi-dimensional stress states and arbitrary loading paths requires development of evolutionary equations for the internal state variables that would represent the inelastic state of the material. These variables should probably be second order tensors to account for induced plastic anisotropy of the material. An initial proposal for treating this problem was reported in SR 10 which was subsequently revised for publication. To account for the directional as well as anisotropic nature of resistance to inelastic deformations, a pair of inelastic state variables, each being a second rank tensor, may be necessary. The rules governing the changes in the components of these variables with loading history should be identical or similar to those described in the published form of SR 10. Considerable effort has been given to this problem, and what seem to be workable techniques have been developed. However, considerably more work is required to obtain a general formulation of the constitutive equations that would realistically represent a wide range of loading situations for multi-dimensional stress states.

Inclusion of anelastic effects in the constitutive equations was achieved by adding an anelastic stress term which was a function of the deformation rate. Suitable expressions for this term were developed to account for energy losses associated with reversible deformations and for rate dependent effective moduli. The resulting equations were applied to the problem of the large deformation of a thick walled sphere of anelastic material subjected to cycles of internal pressure, SR 4 and SR 11. Comparisons were made to experiments on hollow spheres and strips of a synthetic rubber subjected to corresponding loadings with generally good agreement between calculated and test results, SR 11.

b. Ballistic Perforation

In SR 6, it is shown that the engineering model of ballistic perforation developed by Awerbuch and Bodner would apply to the oblique impact case by taking the effective target thickness to be the projectile path distance through the target. This assumption combined with some minor changes in the formulation leads to predictions of post perforation velocities that are in good agreement with experiments. These consisted of tests of 0.22 in. caliber lead bullets perforating aluminum target plates at various angles of incidence (SR 6).

An extensive investigation was performed on multi-layered target plates subjected to direct bullet impact, SR 13. Spaced, relatively thin strips, acting as beams, and strips in contact without bonding were used as the target specimens. In order to obtain a more appropriate reference analysis for this case, the previously developed engineering model of perforation was generalized to include the effect of overall structural bending. This was done by redefining the ballistic limit velocity as the initial impact velocity for which the exit velocity and the structural deformation velocity of the impact point would be equal at the same time. Calculated results for the revised analysis showed good agreement with corresponding experiments, SR 13.

c. Composite Materials

Acoustic emission techniques were employed in SR 2 to examine the time dependent failure of individual fibers in a stressed composite material. Time dependence of failure arises as a consequence of the viscoelasticity of the matrix which leads to readjustment of the load carrying characteristics upon fracture of some fibers. Detection and analysis of the sounds emitted by failure of individual fibers proves to be a useful method for examining the overall failure process.

Impact strength of composite materials is one of their important technological properties. An experimental investigation of the impact strength of balanced angle ply composites was performed in SR 7 using an instrumented drop tester. Results are generally predictable based on information of the maximum possible transverse strain of a single ply. The work reported in SR 7 served to initiate a separate research program on impact strength of composites that was partially supported by the U.S.-Israel Binational Science Fund.

PROPOSED DIRECTIONS OF FURTHER RESEARCH

Considerably more work is required to bring the constitutive equations to the state where they could be used as a standard method of material characterization. One aspect that requires more development is the use of the equations for high temperature applications. The loading conditions that should be within the representation capabilities of the equations include steady and interrupted creep, cyclic creep and relaxation, thermal cycling, and combinations of steady loading with thermal and stress cycling. The other major research effort on the constitutive equations is their generalization to realistically represent arbitrary loading histories involving multi-dimensional stress states. Progress on this subject is fundamental to the general application of the constitutive equations.

A field in which the constitutive equations would find direct, important application is the fracture mechanics of rate dependent materials. An open problem is the determination of appropriate rate dependent fracture criteria for such materials. A possible approach would be the determination of crack tip stress fields for elastic-viscoplastic materials based on the proposed constitutive equations. An auxiliary problem would be the identification of a suitable local failure criterion. Combining the local failure criterion with the calculated rate dependent stress distribution should provide essential information towards solving the problem.

There is an endless variety of ballistic perforation problems which require solution. A possible continuation of the work accomplished in this research program would be the basic determination of some of the empirical factors in the engineering model of ballistic perforation. Another topic worth investigating is the use of plates instead of beams as the multi-layered targets in a combined analytical and experimental study similar to that reported in SR 13.