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A FORMAL MODEL OF THE
ADAPTIVE AND DISCRETE CONTROL BEHAVIORS
OF HUMAN OPERATORS

Richard A. Miller
Industrial and Systems Engineering

For the Period
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) The focus of manual control research and the methods used for modelling and theorizing about manual control task performance are briefly reviewed. It is concluded that manual control research activities have been overwhelmingly concerned with constructing human operator transfer functions for simple tracking tasks. It is also concluded that the mathematical methods used in this type of research (typically difference or differential equations and their corresponding transfer function representations) constrain rather tightly the type of theories which can result. Further, generalization of these

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engineering theories beyond tracking is essentially impossible. It is argued, however, that control oriented concepts can be utilized to guide research on human adaptive, supervisory and co-ordinative control activities.

A meta-theoretic analysis of control problems is made to identify the types of objects and relations which must be addressed in any theory of manual adaptive control. A new formulation of adaptive control problems is then derived. From the problem description, the type of objects upon which any representation of the adaptor (the system which accomplishes adaptation) are determined.

Following in part work by Gaines, the adaptor is defined in terms of behaviors on a sequence of tasks. Tasks are formally characterized as well posed control problems consisting of a control context, goal structure, and set of allowable controllers. The adaptor processes information about past task performance to specify the proper controller for use on the next task.

Several suggestions about the construction of explicit models of the human adaptor are made. It is argued that the tasks are cognitive symbol manipulation and knowledge representation tasks. As such methods of representation used in cognitive psychology and artificial intelligence are probably the most applicable of the many structures currently available.

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1. Introduction

Manual control is typically defined to be a situation in which a person receives information about the system he controls through his various senses and provides directive or corrective information to the controlled system through one or more actuating devices. Practical situations in which humans perform manual control tasks are many (driving automobiles, flying aircraft, running machine tools) and much effort has been spent on constructing both engineering and scientific theories of manual control.

These theories do an excellent job of prediction within a limited domain. In particular, there exist both classical and optimal control theoretic methods for predicting performance on compensatory tracking tasks (see Sheridan and Ferrell (1974)). Unfortunately, human operators are included as components in control systems to provide control and information processing skills in addition to those required to successfully perform compensatory tracking.

To the control systems engineer the human is the ideal component to include in a system which must operate in a partially unknown environment. The human is adaptable. He can recognize and compensate for unusual circumstances and infrequent events. He can operate in nonconstant environments and provide context sensitive control unattainable with completely automatic systems. In other words, the human operator is the component which compensates for the engineer's necessarily incomplete knowledge of the world in which the system he is designing will be used. Human perceptual and cognitive abilities still far exceed those of any artificially intelligent system yet considered and the human

operator will certainly remain a key element in many control systems of the future.

Young (1969) and Sheridan and Ferrell (1974), among others, have recognized the above point and have argued that the primary role of the operator (or operators) in systems such as modern commercial aircraft, air traffic control systems, automated warehouses and machining centers, is control oriented supervision and management, and not tracking oriented control. The human is valued in these systems precisely for his information processing abilities and his ability to provide an adaptive decision-making capability in an otherwise automated system. In essence, the motor skill requirements necessary to control the system have been placed with the machines, and the routine information processing has also been mechanized, leaving the human with the poorly understood remainder.

The trend seems clear. The human operator in future man-machine systems will have to interact with and operate highly automated and computer controlled equipment. He will plan, sequence and co-ordinate rather than "control." It is equally clear that our understanding of these systems must improve if the design of these systems is to be placed on a solid theoretical and empirical foundation. More attention will have to be focused on the evolving role of the operator as supervisor, manager and adaptive controller. Engineering theories specific to these tasks will have to be developed. It is toward this end that this paper is directed.

The key hypothesis underlying the research reported here is that the concepts of control theory and engineering can enable progress in manual control research, but the typical association of control theory with the mathematics of linear systems, servomechanisms and optimal control is too restricting. For guiding research at least, the concepts should be separated from the usual methods. The use of a particular type of mathematics in control theory is motivated in part by the substance of control problems, but it is also motivated by the desire to get "results." Hence, the technical assumptions of a particular control theory are not necessarily desirable nor acceptable when the objective is a theory of some aspect of manual control. There seems to be a need to clearly delineate the domain of manual control theory, particularly the type of supervisory and adaptive manual control mentioned above. This paper is one attempt at such a delineation.

Specifically, the main objective of this paper is to provide a fairly formal characterization of systems in which the human operator fills the roles of adaptive controller, co-ordinator, and supervisor. The work presented is really at a meta-theoretic level in the sense that no specific theory is developed, but rather the types of objects and relations which must be addressed by specific theories are defined and put forward for examination. This hopefully will facilitate additional research and theory construction in this important area of manual control.

The paper is organized as follows. A brief review of the aims and methods of manual control research is presented in section 2, followed by a

discussion of the concept of a adaptation in section 3. The main body of the paper is section 4 in which a formalization of the generic control problem is developed and the objects and relations of an adaptive controller are derived. The implications for manual control modelling and research are discussed in section 5 and the results are summarized in section 6.

2. The Focus of Manual Control

The vast majority of manual control research, including that on adaptive manual control, consists of attempts to construct a transfer function representation of the human. Specifically, the human is represented by pairs of input and output signals, outputs having been obtained when the human subject performed a control task in which the input signal was presented to him. Through various curve fitting procedures a transfer function, or equivalently a differential equation, representation of an input to output mapping is obtained. The literature on simple tracking tasks is vast and the interested reader is referred to Sheridan and Ferrell (1974) for an excellent introduction to the area.

Study of adaptive manual control is much less comprehensive. The usual case consists of a compensatory tracking task in the face of a slowly changing controlled plant (controlled system) or in some cases a plant which undergoes step changes in gain. McRuer and Jex (1967); Miller and Elkind (1967); Phatak and Bekey (1969); Young (1969); Niemela and Krendel (1975) are representative of the cross section and provide an adequate introduction to this literature. There have also been some attempts at describing behavior during the learning phase when subjects are first exposed to the control task (see for example Preyss and Meiry (1968); Jagacinski et. al, (in press), Jagacinski and Miller (1978). Some switching theory and pattern recognition methods have been published including Angel and Bekey (1968); Gilstad and Fu (1971); and Witten and Corbin (1973).

All of these studies either describe the human's performance on a given task or compare human performance with some automatic system. In both cases, generalization is impossible until experiments are performed on a wide variety of tasks, and even then generalization is limited.

For the most part the above cites are from the engineering or engineering psychology literature. The objectives of these studies are essentially to describe human performance in sufficient detail and in a form suitable for predicting overall man-machine system performance. It is not surprising then that the methods used to describe the operator's behaviors are the same as those used to describe the machine's behaviors (e. g. differential equations or difference equations). In a sense the man is described in the same language used to describe his environment. This is a perfectly legitimate (probably even desirable) way to proceed, but it should be recognized that it puts many constraints on the types of behaviors that can be represented.

The strategy used to enrich the usual tracking theories with some features of adaptive control again follows the model of engineering practice. An additional control loop is added to the representation and this loop is intended to account for the process by which the operator modifies or regulates the parameters of the primary tracking loop in the face of a changing environment. Young (1969) and Pew (1974) provide particularly clear expositions of this strategy. This type of modelling process would be quite satisfactory for engineering purposes except that the mathematics of the control theoretic analysis essentially precludes all but linear, time-invariant representations. It is not clear that the human performs these tasks in a linear fashion using rules which do not change

over time. A mismatch between the technical means of description and the behaviors described again seems to pose some severe limits on this type of theory of manual adaptive control.

Also of interest is an emerging body of literature on supervisory control and monitoring. Sheridan and Johanssen (1976) is an excellent introduction to this literature and it contains substantial bibliographic information. This research emphasizes understanding human performance in monitoring and multitask decision-making. Monitoring generally refers to situations in which the operator systematically observes several information sources in order to classify operations as normal or abnormal and to diagnose the causes of abnormalities. Multitask decision-making refers to situations in which an operator has several tasks all of which must be accomplished with some degree of success if the system objectives are to be met.

The type of theories of supervisor control put forward to date are based on simple extensions of existing control models (Kleinman (1976); Kleinman and Curry (1977)), queuing theory (Rouse (1977); Carbonell (1966); Carbonell, Ward and Senders (1968) and utility based decision theory (Sheridan (1976))). On the surface at least it appears that these theories are highly specialized, highly structured and difficult to integrate. The results presented here will show that this is not necessarily the case. Each is simply the result of picking a particular type of mathematical representation as the vehicle for theory construction and modelling.

Another key trend which is found in the literature is the movement away from viewing the operator strictly as the pure "black box" responding invariantly to stimuli and toward viewing him as an adaptive processor of information. Moray (1976) discusses the importance of this view. In part this trend reflects the cognitive nature of the control tasks of interest. As cognition and perception become more important and skill at motor tasks becomes less important in automated systems, engineering theories of human performance will undoubtedly align themselves with some of those in cognitive or information processing psychology. As noted by Kantowitz in the preface of Kantowitz (1974), traditional control theories are used in psychology primarily for skill tasks; while theories of cognition tend more toward the types of theories used in computer science, e.g. automata theory. Further, some emerging theories of human motion and co-ordination provide some very fundamental insights into the ways in which humans accomplish these very complicated control tasks without excessive processing requirements. Turvey, Shaw and Mace (in press), Turvey (1977) and Greene (in press) are particularly useful in this regard.

In summary, the focus of manual control research seems to be moving away from theories about skill oriented tracking tasks toward a view of the operator as an adaptive information processor. Most of the engineering theories so far put forward are simply direct application of existing highly structured mathematical forms and it is not yet clear how these various schemes, ranging from optimal control theory to queuing theory, can be integrated to provide some cumulative knowledge applicable to a broad class of man-machine system

design problems. This paper puts forward a somewhat formal meta-theoretic analysis of the general adaptive control problem with the hope that it will contribute to a more comprehensive understanding of the problems, and therefore, speed integration of knowledge.

In the next section a brief discussion of the concept of adaptation is presented to put the formal analysis in some perspective.

3. The Concept of Adaptation

Berlinski writes "In biology and in the social sciences, one thinks of adaptive control as natural: the mechanisms by which social life is regulated, once one excludes certain artificial cases, inevitably involve adaptive processes, with the machinery of regulation employed in a continuous redefinition of the regulated process itself."¹ The statement applies as well to individual human behavior. Essentially, adaptation is a very natural mode of operation and the unusual event is its absence. It is therefore surprising that the concept is not better understood.

Gaines (1972) presented a very lucid discussion of many of the connotations of the terms which appear in the literature. In particular, he discusses some of the differences between structural and behavioral definitions of the term and he takes the point of view that adaptation can be formalized strictly on behavioral terms. In this regard his paper is a continuation of a long standing series of discussions on the philosophical foundations of the concepts of goal seeking and adaptation dating back at least to Wiener, Rosenblueth, and Bigelow (1943) and the foundations of cybernetics.

The debate about behavior versus structure seems to revolve around the question of purpose. Roughly speaking the behavioral view of adaptation argues that adaptive behaviors must be explained without recourse to speculation on the structure of the acting system. Adaptation in this view is therefore explained solely in terms of the responses, more correctly the inputs and outputs, of the acting systems. Purpose must be recognizable solely and completely

from the input-output pairs themselves without as Weiner states "any speculation on the structure and nature of the acting object."²

The opposite view posited by Taylor in response to the cybernetic view (see Taylor, 1950) and discussed by Berlinski (1976) is that purpose is an intentional concept and it is not behavioristically definable. Berlinski states "purpose is ordinarily taken as an intentional concept in the sense that some special relationship between a purposeful agent and the object of his actions is assumed in taking goal directed action..."³ In essence this school of thought argues that adaptive, goal-seeking behavior cannot be discussed without some recourse to the structure and organization of the system under study.

The point to be made here is that there is by no means agreement on the definition of adaptation nor is there agreement on the preferred methods for its study. It is the view of this author that the structural view is the more useful, particularly for the task of constructing engineering theories of manual control. Engineers are dealing with "artificial" systems (see Simon, 1968), systems designed to accomplish certain ends, and the human operator happens to be one component in these systems. The objects and relations needed for a model of human adaptive behavior should (must) incorporate this structural information as well as the behavior themselves. Even Gaines' "adaption automaton" (see Gaines, 1972) which presumably is to be considered an embodiment of the behavioral adaptation concept, is based on extra-behavioral assumption. Specifically the response of the controller performing a task must be evaluated via a performance measure and the performance classed as satisfactory or not

satisfactory. The automaton output behaviors are the sequences of performance classifications. A behavioral description has been given, but at a level of analysis one step removed from the behaviors of the controller itself and with the assumption that "adaptive behavior may be ascribed to an adaption automaton." The point is not to criticize Professor Gaines' important contribution, but to point out that it is exceedingly difficult to formally discuss adaptation without some reference to the overall system structure and some discussion of why it is structured that way.

With this material as background, a formalization of control problems, control systems and adaptive control will be developed.

4. Control Problems and Control Systems

Gaines (1972) lists the following as essential to the discussion of adaptive systems: 1) a controller, 2) an environment, 3) a measure of performance and 4) the notion of change over time. The controller, either artificial or natural, interacts with the environment to presumably further some purpose. The measure of performance is the means by which this interaction is evaluated. The notion of change over time is essential because the concept of adaptation is predicated on the change of behaviors over time.

The influence of control theory on the definition of these elements is obvious. The objective in this section is to present some general types of objects and relations which must be identified when control systems are described and control problems are posed. The relationship between control problems and control systems is examined and the concept of adaptation is introduced at this point. Discussion of the implications for manual control are deferred to the next section.

4.1 Control Problems and Controllers

When an engineer poses a control problem he has in mind the elements mentioned earlier, a controller, an environment usually thought of as a plant to be controlled and some set of performance specifications, if not a performance measure.

In its simplest form the problem can be addressed in terms of two structures, some representation of the controlled plant and a performance evaluation

structure. These are usually in the form of mathematical functions and relations (more precisely, they are usually statements about such mathematical structures). The controlled plant is typically characterized in terms of control inputs, disturbance inputs and output responses. A purely behavioral representation of the plant is then a relation S of the form

$$S \subseteq X \times W \times Y \quad (1)$$

where

$$X = \{x \mid x: T \rightarrow A_1\}$$

$$W = \{w \mid w: T \rightarrow A_2\}$$

$$Y = \{y \mid y: T \rightarrow A_3\}$$

The set T is linearly ordered by a binary relation denoted by $<$, has the structure of a monoid (see appendix) with binary operation Δ and identity t_0 , and represents time. The sets A_1, A_2, A_3 are the input, disturbance and output spaces respectively. The elements of these sets are the values which can be taken on by the input, disturbance and output time functions. In general these sets are multi-dimensional and are often given the structure of vector spaces.

The sets X, W and Y , which are used to define the relation S , contain as elements time functions which represent the system behaviors. Clearly, an element of S is a triple of these time functions, an input function x and disturbance function w matched with the corresponding response function y . Intuitively then, the relation S is nothing but a listing of all the possible behaviors of the system.

It is seldom that a control problem is posed using a system representation as general as the relation S . At a minimum some notion of time causality and state determinacy is invoked. Roughly speaking this corresponds to the idea that the plant cannot anticipate future inputs in advance but responds instead only to past inputs and further, the exhibited behaviors can be described to any desired degree in terms of a finite number of state or intervening variables. A formal characterization of these additional properties will be provided shortly, but some additional notation is needed first.

Let t_1, t_2 be any two elements in T and let $t_1 < t_2$ where $<$ denotes the relation which linearly orders T . Then, let

$$T_{t_1 t_2} = \{t \mid t \in T, t < t_2, t_1 < t\}$$

and let

$$\bar{T}_{t_1 t_2} = T_{t_1 t_2} \cup \{t_1\}.$$

The set $\bar{T}_{t_1 t_2}$ is therefore that subset of T which contains points "greater than or equal to" t_1 but "less than" t_2 . Now, consider any function $x \in X$. A new function $x_{t_1 t_2}$ can be constructed by restricting the domain of x to the set $\bar{T}_{t_1 t_2}$. Specifically, for the given function x ,

$$x_{t_1 t_2} = \{(t, x(t)) \mid t \in \bar{T}_{t_1 t_2}, (t, x(t)) \in x\}.$$

Then, the entire set of functions X can be so restricted,

$$X_{t_1 t_2} = \{x_{t_1 t_2} \mid t_1, t_2 \in T, t_1 < t_2, x \in X\}.$$

Similar notation will be used to reference the restrictions of the disturbance and output sets as necessary.

Now, state determinancy requires that the relation S be constructively specified in terms of two families of functions. First, let F be a set, the state space, and consider the two following functions:

$$\Phi_{t_1 t_2} : F \times X_{t_1 t_2} \times W_{t_1 t_2} \rightarrow F \quad (2)$$

$$\lambda_t : F \times A_1 \times A_2 \rightarrow A_3 \quad (3)$$

The first is a state transition function, the state at time t_1 and the inputs over the time interval from t_1 to t_2 combine to give the state at time t_2 . The second function establishes the output value at time t given the state and inputs at time the given point t . In the general case there is a different state transition function for each pair of time points $t_1, t_2 \in T$ and there is an output assignment function for each point $t \in T$. Most control problems are, however, posed with *at least time invariant systems* which provides some simplification. The interested reader is referred to Mesarovic and Takahara (1975) for a more complete discussion of these points.

The relationship between the system S given in equation (1) and the constructive specification of it (equations (2) and (3)) is straightforward. Essentially, all behaviors in S are constructed using (2) and (3) in the sense that an initial state and the inputs completely determine the response. For purposes of this discussion it is sufficient to say the following:

$(x, w, y) \in S$ if and only if

$\exists f \in F \exists \forall t \in T$

$$y(t) = \lambda_t \left(\Phi_{t_0 t} (f, x_{t_0 t}, w_{t_0 t}), x(t), w(t) \right)$$

In other words, the response observed at time t is determined by the starting state f , and the inputs applied over the interval from t_0 to t .

The above leads very naturally to a response function or initial state representation of the relation S . Specifically,

$$S_F: F \times X \times W \rightarrow Y \quad (4)$$

where

$$(f, x, w, y) \in S_F \Leftrightarrow$$

$$y = \{(t, y(t)) \mid t \in T,$$

$$y(t) = \lambda_t (\Phi_{t_0 t}(f, x_{t_0 t}, w_{t_0 t}), x(t), w(t))\}$$

Therefore, the response y is determined by the initial state, the control input function and the disturbance, i. e.,

$$y = S_F(f, x, w)$$

In almost all cases control problems start with some description of the state transition and output assignment apparatus (equations (2) and (3)) rather than the behavioral representation S or functional representation S_F . (This terminology is that of Zeigler, 1976.) That is, difference or differential equations are used to describe properties of the system behaviors, but the behaviors themselves are not typically given. The performance structure on the other hand is often expressed using the initial state representation (4).

The performance structure is easily characterized using a generalization of Mesarovic et. al. (1970). Given an initial state representation,

$$S_F: F \times X \times W \rightarrow Y$$

a performance measure is a function P ,

$$P : F \times X \times W \times Y \rightarrow V \quad (5)$$

where V is a linearly ordered set (a "value" set). P then associates a value with each appearance of the system (i. e. each element of S_F).

In addition to the measure of performance, in many problems a "tolerance" function and satisfaction relation is employed, particularly in cases where optimization is not the explicit concern. A tolerance function can be thought of as a map

$$TL : F \times W \rightarrow V. \quad (6)$$

Some value is assigned by the tolerance function to each initial state, disturbance function pair. This value can be thought of as the minimum acceptable level of performance, given the state and disturbance. The satisfaction relation is then a binary relation on V which provides the means of comparing the performance level with the tolerance level. In general,

$$SR \subseteq V \times V \quad (7)$$

but the specific properties must remain problem dependent. A specific appearance $(f, x, w, y) \in S_F$ is deemed satisfactory or acceptable if and only if the pair

$$(P(f, x, w, y), TL(f, w)) \in SR.$$

For example, if the set V is the real numbers, and SR is the relation less than or equal to, performance is satisfactory if and only if

$$P(f, x, w, y) \leq TL(f, w)$$

The general statement of the control problem can now be made. Given a representation of the plant, equation (4); a performance measurement structure, equations (5), (6), (7); some subset of the possible disturbance functions, say $W_d \subseteq W$; some set of initial states, $F_d \subseteq F$; and a set of allowable controls, $X_d \subseteq X$; find a set of controls X_s where,

$$X_s = \{ x \mid x \in X_d, \forall f \in F_d, \\ \forall w \in W_d, \\ (P(f, x, w, y), TL(f, w)) \in SR \}$$

The set X_s contains the controls which for any initial state in the design set F_d and for any disturbance in the design set W_d will produce satisfactory performance.

Almost always the sets F_d and W_d are highly constrained. For example only one initial state and a simple class of disturbances, perhaps step functions, are used. Also, the designer usually has no interest in enumerating the set X_s but rather he simply wants to find one of its elements assuming that there is none.

It is intuitively obvious that in almost every practical control problem the set X_s as presented is empty. That is, it is impossible to find one control function which achieves satisfactory performance with all initial states and disturbances. This is in part a function of the open-loop nature of the formulation, but only partly. Optimal control problems, for example, restrict the sets F_d and W_d so that this structure is formally, but not necessarily practically, meaningful.

A closed loop control design problem differs from the open loop problem in the information assumed. With the open loop problem the information available consists of the plant representation S_F (equation (4)), the performance structure, (equations (5), (6), (7)) and the design sets F_d , W_d , X_d . The closed loop problem is the same except for the specifications of X_d , the set of acceptable controls. In the closed loop case, control responses are presumed to be generated by state determined dynamic systems whose inputs are plant responses. Formally, consider the family of systems (controllers)

$$CN = \{C_E \mid C_E : E \times Y \rightarrow X\} \quad (8)$$

Y and X are as defined in the specification of S (see equation (1)). E is a controller state space. Each system in CN is assumed to be constructively specified with state transition functions and output assignment functions of the form

$$\Gamma_{t_1 t_2} : E \times Y_{t_1 t_2} \rightarrow E \quad (9)$$

$$\mu_t : E \times A_3 \rightarrow A_1 \quad (10)$$

Note well that each controller would in general have its own unique set of state transition and output assignment functions, including (possibly) a unique state space E .

The idea behind the closed loop controller is that the controlled plant and the controller are interconnected with the controller output input to the plant and information about the plant output input into the controller. This means that in operation the behaviors of both controller and plant are highly restricted and control actions are thereby responsive to the current situation. To formally

capture this for purposes of this discussion, a feedback composition S_C of the two systems S_F and C_E is used,

$$\begin{aligned} S_C &\subseteq F \times E \times W \times Y \\ &= \{ (f, e, w, y) \mid \exists x \in X \ni \\ &\quad (f, x, w, y) \in S_C \wedge (e, y, x) \in C_E \} \end{aligned}$$

Given that both S_F and C_E are assumed to be state determined, it easily follows that

$$S_C : F \times E \times W \rightarrow Y. \quad (11)$$

So, in the closed loop case, the plant response depends only on the disturbance, the controller initial state and the plant initial state.

Now, the controller design problem. To each controller in the set CN_d will correspond to a closed loop system of the form (11). A given controller would be classed acceptable if $\forall f \in F_d, \forall w \in W_d, \forall e \in E_d,$

$$\left(P(f, C_E(e, y), w, S_C(f, e, w)), TL(f, w) \right) \in SR.$$

That is, if performance is acceptable for all initial conditions and disturbances in the design sets. Note that $E_d \subseteq E$ is the set of controllers initial states considered. The control problem is to find some controller in CN_d which has these properties.

The successful solution of a control problem results in a functional representation of the controller. This provides a fairly complete description of the types of behaviors that the controller must exhibit when realized. Notice that in this form the controller D does not solve control problems, rather it is a control problem solution.

Further, it is extremely important to note that the control problem is solved in terms of some behavioral or functional representation of the plant, not the plant itself. If these structures, i. e. plant and controller representations, are in the same equivalence class as the behaviors of the plant and the controller which is ultimately realized, then actual performance will be acceptable as long as the design conditions are not violated (disturbances within the class designed for). Should the actual disturbance not be equivalent to an element of the design set, or should the plant representation used for design turn out not to be behaviorally equivalent to the plant, the controller realized from the control problem solution will then not necessarily produce acceptable performance.

The final observation to make here concerns this relationship between the control problem and the state determined controller behaviors which result. In essence, the final product of the control problem is a controller which presumably realizes the properties defined by equations (9) and (10), i. e. a system which processes information in a pre-specified way. Clearly, it is constructed in this way so that the control objectives will be achieved in at least those situations considered during design. But, this system has very little flexibility. Controller state changes follow from the organization of the "mechanism" and the information received from the environment, i. e. the plant response. The controller does not have the means to modify the rules of transition, i. e. to modify the functions Γ and μ listed in equations (9) and (10). Conceptually, but not practically, an all encompassing control problem might be envisioned

and the resulting solution, if it could be obtained, would result in a controller with a state space sufficiently "rich" that each context or each environmental situation would be properly handled. In terms of practical design this is impossible without further decomposition of the control problem. The usual notions of adaptive control provide this decomposition.

4.2 Adaptive Control, Some Preliminaries

A brief review of the previous section will show that a control problem and its closed loop solution involves three different types of structures,

$$EC = (S_F, F_d, W_d)$$

$$G = (P, TL, SR)$$

$$\overline{CN} = \{(C_E, E_d) \mid C_E \in CN_d, E_d \subseteq E\}$$

The first, EC , constitutes a particular view of the controller's environment expressed in terms of a plant representation S_F , a set of initial states F_d and a set of exogenous disturbance functions W_d . EC can be viewed as the "context" in which the controller is to operate. The second structure, G , is the performance evaluation structure or goal structure which is used to determine the acceptability of the possible controllers in the given context. The last, \overline{CN} , consists of the allowable controllers paired with the design initial states.

The design task can be summarized as follows: given a specific triple (EC, G, \overline{CN}) , find some element of \overline{CN} which is evaluated as acceptable by the goal structure G in the context EC . If CN^* denotes that subset of \overline{CN} which contains acceptable controllers, the task then is to find one of the elements in CN^* and implement it.

In the non-adaptive case this would be the end of it. A controller in CN^* , if one exists and is found, is implemented and things presumably work as intended. In the adaptive case, however, the context, or perhaps the goal structure, changes over time and the control system is expected to compensate for this change. Adaptive controllers can therefore be thought of as systems which are designed to solve, over time, control problems from some class of problems rather than simply realize one single, well specified, problem solution.

To formalize this, let \mathcal{E} denote a set of control contexts, \mathcal{G} a set of goal structures, and \mathcal{C} a set whose elements are sets of controllers and their design initial states.

The elements of \mathcal{E} are specific contexts $EC = (S_F, F_d, W_d)$; elements of \mathcal{G} are goal structures $G = (P, TL, SR)$; and elements of \mathcal{C} are sets of allowable controllers \overline{CN} . A control problem can then be viewed as a triple

$$(EC, G, \overline{CN}_d) \in \mathcal{E} \times \mathcal{G} \times \mathcal{C}.$$

Some consistency conditions must be imposed if the problems are to be well posed. Note that

$$(EC, G, \overline{CN}_d) = ((S_F, F_d, W_d), (P, TL, SR), \overline{CN}_d)$$

and recall that

$$S_F : F \times X \times W \rightarrow Y$$

$$P : F \times X \times W \times Y \rightarrow V$$

$$TL : F \times W \rightarrow V$$

$$SR \subseteq V \times V.$$

Also, note that elements C of CN_d are relations of the form, (C_E, E_d) ,

$$C_E: E \times Y \rightarrow X$$

A problem (EC, G, \overline{CN}) is said to be well posed if its components are all defined on the same objects F, X, W, Y and V and $F_d \subseteq F, W_d \subseteq W, E_d \subseteq E$. In other words, the plant representation, goal structure and controller representations must all be relations on the same objects.

A class of well posed control problems may then be thought of as a relation

$$\mathcal{J} \subseteq E \times \mathcal{G} \times C, \quad (12)$$

elements of which are well posed problems (EC, G, \overline{CN}) .

Similarly, the problem/acceptable solution relation $\mathcal{J}^* \subseteq \mathcal{J}$ can be defined as follows:

$$\begin{aligned} \mathcal{J}^* = \{ (EC, G, CN^*) \mid (EC, G, CN^*) \in \mathcal{J} \\ \text{and } (C, E_d) \in CN^* \Rightarrow \text{satisfactory} \\ \text{performance under } (EC, G) \} \end{aligned} \quad (13)$$

As a relation then, \mathcal{J}^* pairs the context and goal structure with those allowable controllers which produce satisfactory performance.

Using \mathcal{J} as a base, the adaptive control problem can be thought of in the following terms. At specified instants in time (perhaps continuously) the proper element of \mathcal{J} is identified, i. e. a control problem is defined. The system then finds a solution and implements it. The solution selected and implemented will not necessarily yield satisfactory performance, but over time, as experience

with the set of tasks is accrued, proper solutions are eventually achieved. In other words, as experience is gained, the problem base moves from \mathcal{J} to \mathcal{J}^* .

Gaines (1972) seems to have this type of operation in mind when he makes a strong case for axiomatizing adaptation in terms of tasks. He describes a task to be "... some specification of plant parameters, initial conditions and period of interaction, together with a tolerable performance level above which a control policy is considered satisfactory."⁴ His notion of a task then can be interpreted as a control problem, a means of measuring the level of performance of an implemented policy, and the means to classify this performance as acceptable or not acceptable. The basic objects and relations which characterize the adaptive controller will be shortly be described in these terms. But first, some relationships between the time sets used for control problem representation and the time set of the adaptive controller must be examined.

4.3 Implementation Time and Task Time

The notion that past experience is used to change, and hopefully improve, control strategy seems essential to any theory of adaptation. To get some picture of the implications of this, some of the technicalities of time sets must first be addressed.

Recall that the plant representation S_F is defined on sets of time functions and these time functions have as domain some linearly ordered set T (see equation (1) in the previous section). This set is presumably a representation of some of the properties of real time. In terms of the control problem, the identity element in T , namely t_0 , is usually interpreted as standing in

correspondence with that point in real time at which the controller starts to perform its assigned task, i. e. the point in time at which the solution is implemented. A more general view would consider t_0 as standing in correspondence with several real time points, those being task starting points in the sense of Gaines (1972). If the controller is turned on and left to run into the indefinite future, the first view is perhaps more appropriate. But even fixed structure control systems are considered to operate under some notion of invariance where activities over different intervals of time are construed as being essentially the same. In other words, the tasks are the same except they are performed at different points in time.

To provide a more precise characterization, let T_R serve as a time set representing the implementation time set (real time) and let T , as before, denote the time set used in posing the control problem. It is necessary to assume that there is a binary operation $+$ defined on T_R and a binary operation Δ defined on T such that the structures $\langle T_R, + \rangle$ and $\langle T, \Delta \rangle$ are both monoids (see appendix for a list of properties). Let $0 \in T_R$ denote the identity element of the first structure and t_0 the identity element of the second. Now, consider a map (homomorphism)

$$h : T \rightarrow T_R \tag{14}$$

with the properties that

$$\begin{aligned} \forall t_1, t_2 \in T \quad h(t_1 \Delta t_2) &= h(t_1) + h(t_2) \\ h(t_0) &= 0. \end{aligned}$$

Now, consider a map

$$m : T_R \times T \rightarrow T_R \quad (15)$$

defined such that $\forall \tau \in T_R, \forall t \in T$

$$m(\tau, t) = \tau + h(t)$$

The homomorphism h preserves interval information and m provides the means by which points in T can be interpreted in the set T_R .

Points in T , therefore, should be viewed as relative times with an element t denoting the time after start of operation on a task. The map m then provides the proper implementation time interpretation given the task starting time τ . The set T_R is then the natural set to use for description of the behaviors of the adaptive controller.

Finally, in terms of the notions described in the previous section, task starting times form some subset of the points in T_R , say $T_E \subset T_R$. The points in T_E then mark those points in times at which the controller structure can change. It is not necessarily the case that T_E is denumerable, but if it is not the implication is that the task definition is continuously changing over some interval of time. The denumerable assumption is made here.

4.4 A Description of an Adaptive Controller

Adaptive control will here be described in terms of a two-level system. The lower level at any instant in time is the realization of a controller of the type specified by equations (9) and (10). The upper level, the adaptor, is the part of the overall system responsible for adjusting the lower level. It operates on an information pattern consisting of past plant and controller responses and produces lower level controllers as outputs.

Roughly speaking, the evolution of an adaptive response can be thought of in terms of a sequence of activities including a control problem statement, determination of the solution (i. e., specification a controller), implementation of the solution, performance of the task by the controller, assessment of the performance by the adaptor, and then redefinition of the control problem for the next task. This scheme could operate in a number of different ways. Task duration could be fixed or variable. For example, performance evaluation could be continuous and the task terminated when performance falls below some specified level.

The respecification of the next control problem (next task) could involve activities as simple as modifying the design disturbance set W_d , or perhaps both W_d and F_d . It could also involve a redefinition of the plant model S_F . This might simply be the respecification of the function S_F on the same objects (i. e., the set X, W, Y, F) or it might involve some change in objects as well.

A less common, but legitimate type of adaptation, particularly for human, social and political systems, is some change in the goal structure G . The system might, for example, learn over time that certain performance levels are simply

not attainable with the resources available. The set of allowable controllers \overline{CN} might then be changed to allow achievement of the desired performance level.

In principle then there is no a priori reason to restrict the adaptor to act on one specific type of object, particularly since manual adaptive control is the ultimate aim. The adaptor is therefore assumed to be operating on the class of well posed control problems S . The information pattern is assumed to be past plant response and past controller behavior and the adaptor output behavior is a sequence of controllers which are intended to solve the control problems posed in the sequence of tasks.

The observable behavior of an operating adaptive system generally will not explicitly include a description of the tasks. (This it seems is a main point of those arguing against a strictly behavioral view of adaptation and purposive behavior.) But, before presenting a description of the objects upon which the adaptor can be defined, a brief characterization of the task sequence and the resulting controller sequence will be presented. This, hopefully, will provide a clearer view of the operation of an adaptive system than a pure input-output description.

Each task faced by the adaptor is assumed to be an element of the class of tasks \mathcal{J} (see equation (12)). The time set upon which the adaptor's behavior are defined is the set T_R , implementation time, and controller implementation times are the elements of the "event" time set $T_E \subset T_R$. The task sequence is then conveniently thought of in terms of a function

$$Y : T_E \rightarrow \mathcal{J} \quad (16)$$

A pair $(t, Y(t)) \in \mathcal{J}$ consists of the task starting time t and the specific task

$Y(t) \in \mathcal{Y}$ which is in effect at that time. Note that $Y(t)$ is a triple of structures,

$$Y(t) = [EC_t, G_t, (CN_d)_t]$$

where the subscript t is used to denote the event time dependence. The structure EC_t , for example, denotes the control context at time.

Parallel to the sequence defined by (16) is another sequence

$$Y^* : T_E \rightarrow \mathcal{Y}^* \quad (17)$$

where $[t, (CE_t, G_t, CN_t^*)] \in Y^*$ if and only if CN_t^* is the acceptable subset of $(CN_d)_t$ given CE_t and G_t , and $[t, (CE_t, G_t, (CN_d)_t)] \in Y$. Y^* , therefore, is the task sequence but with the set of allowable controllers replaced by the set of satisfactory controllers. From the set CN_t^* is selected the controller implemented at time t .

An output behavior of the adaptor is a function

$$C^* : T_E \rightarrow \overline{CN} \quad (18)$$

defined by the following conditions:

$$(t, C_t) \in C^* \Rightarrow C_t \in CN_t^*$$

$$[t, (CE_t, G_t, CN_t^*)] \in Y^* .$$

At best only C^* would be observable. The functions Y and Y^* would almost never be explicitly displayed, but conceptually they must underlie the observed sequence of controllers.

Before turning to the adaptor itself, a few additional observations should be made. First, the above functions (16), (17), and (18) all refer to single behaviors, i. e., one single response over time. The range of behaviors possible is determined by the set of possible task sequences and it is very broad. In general, the set T_E

is different for each possible behavior (unless task durations are fixed) and the occurrence of task start events is determined in part by the actual plant behaviors. Also, the task sequence which defines a given behavior presumably will reflect the learning (or lack thereof) which takes place. That is, the task definition at time t , and hence the controller implemented at t , will be a function of those controllers previously implemented and their behaviors on the previous tasks.

The point is that the task sequence and the sequence of controllers evolves over time. This evolution is restricted only by the alphabet of tasks \mathcal{J} and the rules under which the adaptor operates.

The inputs available to the adaptor, as stated above, are past plant and controller responses. These are functions defined on the time set T_R , but it is simpler and more clear to consider them as a special concatenation of functions defined on T and used in the representation of the plant.

It is here assumed that the set $T_E \subset T_R$ and the set T are related in the following way. Let t_a and t_b be two adjacent points in T_E , i. e., $t_a < t_b$ and \exists no element $t' \in T_E \ni (t_a < t' \text{ and } t' < t_b)$, then there exists a point $s \in T \ni t_b = m(t_a, s)$ where m is defined by equation (15). Equivalently, $t_b = t_a + h(s)$. In other words, task durations are assumed always to stand in correspondence with some element of T . Now, if t_a, t_b are successive task starting times, the corresponding interval in T is then $\bar{T}_{t_0 \Delta}$ where $t_b = m(t_a, s)$. Since all intervals in this representation start at the point $t_0 \in T$, for notational simplicity explicit dependence on t_0 will be dropped. Therefore

$$\begin{aligned}
\bar{T}_{t_0 s} &\triangleq \bar{T}_s \\
X_{t_0 s} &\triangleq X_s \\
Y_{t_0 s} &\triangleq Y_s
\end{aligned}
\tag{19}$$

With this notation, the information available to the adaptor upon completion of a task of duration s is a pair of functions $(x_s, y_s) \in X_s \times Y_s$. The first represents the controller response, the second the plant response. Note that the functions (x_s, y_s) are in some sense an internal representation used by the adaptor. That is, they represent an encoded version of the actual responses.

Now, if D denotes the state space of the adaptor, the behavior of the adaptor from task to task can be described in terms of a family of functions of the type

$$\Omega_{ts} : D \times X_s \times Y_s \rightarrow D \tag{20}$$

$$\mathcal{V}_t : D \rightarrow \overline{CN} \tag{21}$$

where $t \in T_E$ is the task starting time, $s \in T$ is the task duration.

In the usual fashion, equations (20) and (21) can be used to trace the response of the adaptor over time, and hence trace the evolution of the controller as well. Suppose the process starts at some time $t_1 \in T_R$ with the adaptor placed in the state $d_1 \in D$. The controller implemented at time t_1 is then determined by equation (21),

$$C_{t_1} = \mathcal{V}_{t_1}(d_1).$$

Presuming the first task to be of duration $s_1 \in T$, the starting time of the second task is

$$t_2 = m(t_1, s_1)$$

and after completion of the first task the plant and controller responses (x_{s_1}, y_{s_1}) will be available. The adaptor state at this time is

$$d_2 = \Omega_{t_1 s_1} (d_1, x_{s_1}, y_{s_1})$$

and the new controller is

$$C_{t_2} = V_{t_2} (d_2).$$

This process then continues indefinitely.

Clearly, the function C^* mentioned earlier (see equation (18)) can be constructed in this manner. Also, underlying the function C^* is a state trajectory, say Z^* ,

$$Z^* : T_E \rightarrow D \tag{22}$$

which has the property that

$$C_{t_i} = \Lambda_{t_i} (Z^* (t_i)).$$

Note, $Z^*(t_1) \in D$ is the adaptor state at time t_1 . The adaptor state trajectory therefore determines the controller trajectory.

Recall that it was argued earlier that the controller trajectory C^* followed from the sequence of tasks (i. e., control problems) and that this sequence reflected past experience. The state determined view provided by equations (20) and (21) provides a complementary, but somewhat more abstract interpretation of this process. The adaptor states, the elements of D , are an abstraction of the means by which tasks, and the past experiences of the adaptor with tasks, are encoded. Some representation of the task in effect at time t , $(EC_t, G_t, (CN_a)_t)$, presumably is encoded in the state at time t as is knowledge gained from past tasks and any diagnostic or strategic information used to guide the adaptor. The adaptor states therefore represent all of the factual information available to the adaptor.

Also, with this type of interpretation of the adaptor states, the output assignment function ∇_t (see equation (20)) can be viewed as a behavioral description of the control problem solution process. It converts the factual and procedural information encoded in the state into a controller which is expected to satisfactorily perform the task.

The state transition function (20) embodies a number of activities including the processes by which the control task definition is modified given the behavior observed on the previous task. The evaluation of the controller response to assess the acceptability is presumably accomplished here as well.

It is very important to note the level of description implied by equations (20) and (21). A family of state transition and output assignment functions so defined provides only sufficient structure to provide a causal description of the observed behaviors. The procedure by which these functions are computed or otherwise realized is not explained by the functions themselves. In most practical systems not involving a human operator some set of rules is used to specify how control problem (task) modifications are to be made. Also, some procedure for solving the control problem is specified and together these sets of rules define (20) and (21). The key point, however, is that the behavioral description of (20) and (21) in no way depends on the specific scheme of implementation.

Another point of interpretation should be made. Equation (20), the adaptor state transition function, was constructed under the assumption that the task duration was a constant s . There is absolutely no difficulty in treating s as a variable representing time so far spent on the current task. The function (20) then specifies

the adaptor state s units into a task which started at time t in state d and received information (x_s, y_s) . Tasks can be of indefinite length in this formulation, with the end of a task triggered whenever certain states are entered. These states presumably would be those for which the output assignment function (21) specifies a new controller.

In summary, the adaptor is described by two families of functions,

$$\Omega_{ts} : D \times X_s \times Y_s \rightarrow D \quad (20)$$

$$\nu_t : D \rightarrow \overline{CN} \quad (21)$$

where $t \in T_E \subset T_R$ and $s \in T$. These functions establish the sequence of controllers C^* in terms of a starting state and the observed controller and plant response on the previous tasks. The adaptor states are presumed to be some encoded representation of control tasks plus additional experiential and strategic information. The remaining objects X_s, Y_s are segments of the input and output sets used in the plant representation and here represent the possible information available to the adaptor.

4.5 The Performance of the Adaptor

Before moving to a discussion of the problems of modelling manual adaptors, a very brief discussion of the performance of the adaptor is presented. For purposes of discussion it is assumed that a sequence of k tasks starting at time $t_1 \in T_R$ is performed. The tasks are of durations $s_1, s_2, s_3, \dots, s_k$, respectively. The event times for this sequence are therefore

$$T_E = \{ t_1 \mid t_{i+1} = m(t_i, s_i), 1 \leq i \leq k-1, t_1 \text{ given} \} .$$

The adaptor is assumed to start in some state $d_1 \in D$, and the plant in some state $f_1 \in F$. The disturbance sequence which impacts the plant during the task sequence is denoted by $w_{s_1}, w_{s_2}, \dots, w_{s_k}$.

The first controller selected by the adaptor follows from equation (21) and the given information

$$C_{t_1} = \nabla_{t_1} (d_1).$$

Now, C_{t_1} is a controller specification and a set of initial states, say (C_{E_1}, E_1) .

Assume that $C_1 \in E_1$ is the actual initial state implemented.

The closed loop system will then generate behaviors x_{s_1}, y_{s_1} such that

$$y_{s_1} = S_F (f_1, x_{s_1}, w_{s_1})$$

$$x_{s_1} = C_{E_1} (C_1, y_{s_1})$$

At the end of the task the plant is in some state, say f_2 . The adaptor state at the end of the task is

$$d_2 = \Omega_{t_1 s_1} (d_1, x_{s_1}, y_{s_1})$$

and the next controller follows from (21)

$$C_{t_2} = \nabla_{t_2} (d_2).$$

The next pair of task behaviors x_{s_2}, y_{s_2} are then produced by the plant and controller

$$y_{s_2} = S_F (f_2, x_{s_2}, w_{s_2})$$

$$x_{s_2} = C_{E_2} (C_2, y_{s_2})$$

and so on.

Upon completion of all k tasks, the adaptor will have passed through a sequence of states d_1, d_2, \dots, d_k and behaviors $(x_{s_1}, y_{s_1}), (x_{s_2}, y_{s_2}), \dots, (x_{s_k}, y_{s_k})$

will have been observed. The question is, do these behaviors exhibit adaptation? Or in other words, did the adaptor perform acceptably?

Notice, the controller selected at any point $t_1 \in T_E$ is satisfactory in the sense that it is a satisfactory solution of the control problem posed at that point. Given the state of knowledge at t_1 , i. e., given the state of the adaptor, the selected controller is expected to perform adequately. But, it need not necessarily so perform for a number of reasons. For example, the actual plant may not sufficiently correspond to the representation used by the adaptor or the actual initial state or the actual disturbance might not be in the design set. So, the behaviors observed on any task may not be classed as satisfactory even though the controller used on the task was thought to be acceptable at the start.

In order to evaluate adaptor performance, a family of functions

$$EV_s : D \times X_s \times Y_s \rightarrow V_a, \quad (23)$$

where $s \in T$, must be considered. D , X_s , Y_s are as used above and V_a is the value set for performance evaluation. Performance on a given task is classed as acceptable if the observed performance value is an element of a specified set $V_a^* \subset V_a$. With the structure, performance on a task is determined by the adaptor state at the start of the task and the observed behaviors when the task is performed.

The evaluation functions EV defined above presumably incorporate knowledge of the goal structure G used in defining the task and they measure the degree to which the goals were achieved. Also, the adaptor state transition functions in general would incorporate some form of the evaluator functions in updating existing knowledge of plant and plant-controller interactions.

Whether or not the adaptor works successfully or not is a function of the evaluations over the entire sequence of tasks. Gaines (1972) discusses a number of modes of adaptation, all of which could be interpreted here. Suffice it to say that failure to perform successfully on any given task does not necessarily mean unacceptable adaptor behavior. The successful adaptor would, in general, be required to eventually achieve satisfactory performance on some proportion of the tasks.

5. Implications for Manual Control Research

The types of objects and relations needed to define and discuss adaptive control have now been presented. Roughly, these correspond to an adaptor and a family of control tasks. Each control task consists of a control context, a goal structure, and a set of allowable controllers. The control context is some plant representation together with sets of specified initial states and disturbances to be used for design. The goal structure consists of a performance function, tolerance function and satisfaction relation. The adaptor is an information processing system which essentially modifies task definitions based on past behaviors and produces as output the controller for the current task. It is defined in terms of family of state transition functions which establish how the current task performance affects the state of knowledge, and output assignment functions which produce controller specifications from the current knowledge state. A family of evaluator functions is required for an observer to assess the performance of the adaptor.

It was further argued that three time sets are useful in discussing adaptation. T_R , implementation time, is the most basic and may be thought of as real time. T_E is an event time base denoting task starting times. T is a relative time, or within task time, which is most convenient to use in describing plant and controller behaviors. The observable behaviors of the adaptor operate on T_E , in fact in general they define T_E .

It was argued in the introduction that humans are used in many sophisticated control systems for their ability to adapt and provide context sensitive

control. That is, the human is in the system to be the adaptor although he may well be the controller as well. The reason for his use rather than some artificially intelligent system is usually because the designer has incomplete knowledge of the set of tasks. That is, the class \mathcal{V} (equation (12)) cannot in general be sufficiently defined to allow an engineer to specify and realize an adaptor which will adequately function in complex environments. Since the human is very good at certain types of these tasks, he can provide acceptable adaptive performance where an artificial system cannot. In a sense then the human is the engineer's way of realizing the functions Ω and \mathcal{V} .

In terms of modelling or theorizing about adaptive control, this paper has explicitly used a two-level representation consisting of the adaptor and controller. Manual control modelling should follow the same scheme. Adaptation is accomplished through a system with the state transition structure and output assignment structure shown in equations (20) and (21). The human operator when performing adaptive control tasks realizes the system described by (20) and (21). So in behavioral terms, modelling the human adaptor is the process of constructing a structure with the properties required by equations (20) and (21).

Formally then, modelling the human adaptor requires determinations of a state space D , a task time set T , an event time set T_E or perhaps simply the set T_R , a family of input sets $(X_s \times Y_s)$ where $s \in T$, and a family of controllers \overline{CN} . In very few cases would all of these sets be obvious simply from observation of the human performing the tasks. Particularly, the state space D is not directly

observable. In all practical cases a great deal of structure is specified at the start of the modelling or theory construction. Specific representations would typically be assumed for each of the sets and structures mentioned above. Then, assumptions would be made about possible representations of the state transition functions and output assignment functions. Behaviors of the model would then be compared with data drawn from human task performance exercises to assess the adequacy of the model.

On the surface, none of this appears much different than the human transfer identification procedures discussed in section 2. There are some very real and very important differences however. First, transfer function (and optimal control models) use as basic objects sets of plant input and output behaviors. That is, they are at the level of describing controller behaviors. Any adaptation discussed is in terms of a very small number of parameters needed to specify the controller. The adaptor as described here is defined with controller behaviors as inputs, but controllers themselves, i. e., structures, as outputs.

Essentially the level of abstraction used in the analysis has been pushed up from controller behavior to controllers. The simple cases considered in adaptive manual tracking obviously fall in this class, but they are by no means general. The supervisory and discrete control tasks described in sections 1 and 2 do not fit in the tracking scheme, but they do fit the general structure proposed here. The general case allows any and all types of realizations of the functions Ω and \mathcal{V} and is clearly not restricted to difference or differential equation types of constructions.

The last point to be made here is really an extension of the point made above. It seems that manual control theory has for too long been tightly constrained by the type of mathematical structure used to model control behavior. The focus has been on technicalities of representation rather than on the properties and structure of the tasks involved. The framework presented here suggests that, abstractly at least, there are certain types of objects and certain types of relations which must be addressed, but the vehicle of expression need not be fixed. For example, information processing theories might naturally be used for constructively specifying the adaptor. Hunt and Poltrock (1974) and Anderson (1976) are examples of this type of structure. The stochastic automata based procedures currently being developed by this author under AFOSR Grant No. 77-3152 (see Miller, 1977) are also a possible means of modelling adaptation.

The direction seems clear. If manual control theories are ever to exhibit some of context sensitivity and adaptive behavior that humans actually display, new representations will have to be tried. Further, since the adaptation tasks are more cognitive in nature than tasks historically studied, the type of representations will probably have to move toward the information processing systems used and proposed in cognitive psychology and artificial intelligence.

Another very important point about the realization of the adaptor should be made. As pointed out earlier the functions Ω and \mathcal{V} are in essence functional descriptions. They do not in themselves explain or specify how they are to be computed or otherwise realized. It would probably be a mistake to consider

them as single level systems. Humans almost certainly accomplish these functions with a large number of small communicating systems, probably a hierarchy of systems. The ideas expressed by Turvey, Shaw and Macy (in press) might well help guide this modelling process.

In conclusion, a purely behavioral view of adaptation will almost certainly not lead to useful models of the human adaptor. Specific information processing structures will have to be postulated, including explicit statements about the goal structures and the control contexts involved, and model behaviors compared with human behaviors. The problems are abstract and complex and all of the representational issues are by no means clear.

6. Summary and Conclusions

The status of manual control was reviewed, including a brief description of the types of theories and representations commonly used. It was concluded that past research on manual control has been fairly tightly constrained by the types of mathematical forms commonly used to describe control systems in engineering terms. It was also stated, and several references were cited to support the statement, that the role of the human operator in control systems is evolving toward that of a supervisor and co-ordinator. This evolution means that new theories and models of manual control performance are required if engineering design of such systems is to proceed on sound theoretical and empirical principles.

With the objective of clarifying the type of objects and relations essential for representing the adaptive features of the human controller, a structural description of control tasks and adaptation was developed. This analysis resulted in a two-level description of the adaptive control system. The lower level, or controller, actually provides the interface with the environment (plant). The upper level, the adaptor, is an information processing system which maintains and updates the knowledge base about the environment and produces as output a specification of the lower level controller.

Following the work of Gaines (1972) the adaptor was defined in terms of tasks. Tasks were formally characterized as well posed control problems consisting of a control context, a goal structure, and a set of allowable controllers.

The adaptor uses information about past task performance to specify the next task and to produce the required controller for the task so defined.

Finally some suggestions about the construction of models of the human adaptor were made. It was argued that the most suitable models will probably be based on the symbol manipulation and knowledge representation methods currently used in cognitive psychology and artificial intelligence.

Appendix: Monoids

A monoid is simply a set, say A , and a binary operation, say $+$,

$$+ : A \times A \rightarrow A$$

(whose elements are typically written

$$(a_1, a_2, a_1 + a_2))$$

which satisfies the following properties:

1) \exists an element $0 \in A \ni \forall a \in A$

$$a + 0 = 0 + a = a$$

2) $\forall a, a', a'' \in A,$

$$a + (a' + a'') = (a + a') + a''.$$

The element 0 mentioned above is referred to as the identity.

Footnotes:

- 1) Berlinski (1976), p. 142.
- 2) As quoted in Berlinski (1976), p. 147.
- 3) Berlinski (1976), p. 146.
- 4) Gaines (1972), p. 182.

References

- Anderson, J. R. (1976). Language, Memory and Thought, Hillsdale, New Jersey: Erlbaum Associates.
- Angel, E. S. and Bekey, G. A. (1968). Adaptive finite-state models of manual control systems. IEEE Trans. Man-Machine Systems, MMS-9, 15-20.
- Berlinski, D. J. (1976). On Systems Analysis: An Essay Concerning the Limitations of Some Mathematical Methods in the Social, Political, and Biological Sciences. Cambridge, Mass: M. I. T. Press.
- Carbonell, J. R. (1966). A queueing model of many-instrument visual sampling. IEEE Trans. Human Factors in Electronics, HFE-7, 157-164.
- Carbonell, J. R., Ward, J. L., and Senders, J. W. (1968). A queueing model of visual sampling: experimental validation. IEEE Trans. Man-Machine Systems, MMS-9, 82-87.
- Gaines, B. R. (1972). Axioms for adaptive behavior. Int. J. Man-Machine Studies, 4, 169-199.
- Gilstad, D. W. and Fu, K. S. (1971). Two dimensional model of a human controller using pattern recognition techniques. IEEE Trans Systems, Man and Cybernetics, SMC-1, 261-266.
- Greene, P. H. (in press). Strategies for heterarchical control--an essay. To appear Int. J. Man-Machine Studies.
- Hunt, E. B. and Poltrock, S. E. (1974). The mechanics of thought. In Human Information Processing: Tutorials in Performance and Cognition, Hillsdale, N. J.: Erlbaum Associates.
- Jagacinski, R. J., Burke, M. W., and Miller, D. P. (in press). The use of schemata and acceleration information in stopping a pendulum-like system. J. of Experimental Psychology: Human Perception and Performance.
- Jagacinski, R. J. and Miller R. A. (1978). Describing the human operator's internal model of a dynamic system. Human Factors, in press.
- Kantowitz, B. H. (1974). Human Information Processing: Tutorials in Performance and Cognition. Hillsdale, N. J.: Erlbaum Associates.
- Kleinman, D. L. (1976). Solving the optimal attention allocation problem in manual control. IEEE Trans. Automatic Control, AC-21, 813-821.

- Kleinman, D. L. and Curry, R. E. (1977). Some new control theoretic models for human operator display monitoring. IEEE Trans. Systems, Man, and Cybernetics, SMC-7, 778-784.
- McRuer, D. T. and Jex, H. R. (1967). A review of quasi-linear pilot models. IEEE Trans. Human Factors in Electronics, HFE-8, 231-249.
- Mesarovic, M. D., Macko, D., and Takahara, Y. (1970). Theory of Hierarchical, Multilevel Systems. New York: Academic Press.
- Mesarovic, M. D. and Takahara, Y. (1975). General Systems Theory: Mathematical Foundations. New York: Academic Press.
- Miller, D. C. and Elkind, J. L. (1967). The adaptive response of the human controller to sudden changes in the controlled process dynamics. IEEE Trans. Human Factors in Electronics, HFE-8, 218-223.
- Miller, R. A. (1977). Identification of finite state models of human operators. Modelling and Simulation, 8, 923-926.
- Moray, N. (1976). Attention, Control and Sampling Behavior. In Monitoring Behavior and Supervisory Control, New York, London: Plenum Press.
- Niemela, R. J. and Krendel, E. S. (1975). Detection of a change in plant dynamics in a man-machine system. IEEE Trans. Systems, Man and Cybernetics, SMC-5, 615-617
- Pew, R. W. (1974). Human perceptual-motor performance. In Human Information Processing: Tutorials in Performance and Cognition, Hillsdale, N. J.: Erlbaum Associates.
- Phatak, A. V. and Bekey, G. A. (1969). Model of the adaptive behavior of the human operator in response to a sudden change in the control situation. IEEE Trans. Man-Machine Systems, MMS-10, 72-80.
- Preyss, A. E. and Meiry, J. L. (1968). Stochastic models of human learning behavior. IEEE Trans. Man-Machine Systems, MMS-9, 36-46.
- Rosenblueth, A., Wiener, N., and Bigelow, J. (1943). Behavior, Purpose, and Teleology. Philosophy of Science, 10, 18-24.
- Rouse, W. B. (1977). Human-computer interaction in multitask situations. IEEE Trans. Systems, Man and Cybernetics, SMC-7, 384-392.
- Sheridan, T. B. (1976). Toward a General Model of Supervisory Control. In Monitoring Behavior and Supervisory Control, New York, London: Plenum Press.

- Sheridan, T. B. and Ferrell, W. R. (1974). Man-Machine Systems: Information, Control and Decision Models of Human Performance. Cambridge, Mass.: MIT Press.
- Sheridan, T. B. and Johannsen, G. (eds.) (1976). Monitoring and Supervisory Control. New York, London: Plenum Press.
- Simon, H. A. (1969). The Sciences of the Artificial. Cambridge, Mass.: MIT Press.
- Taylor, R. (1950). Comments on a Mechanistic Conception of Purposefulness, Philosophy of Science, 17, 310-317.
- Turvey, M. T. (1977). Preliminaries to a theory of action with reference to vision. In R. Shaw and J. Bransford (eds.), Perceiving, Acting and Knowing: Toward an Ecological Psychology, Hillsdale, N. J.: Erlbaum Associates.
- Turvey, M. T., Shaw, R. E., and Mace, W. (in Press). Issues in the Theory of Action: Degrees of Freedom, Coordinative Structures and Coalitions. To appear in J. Requin (ed.) Attention and Performance VII, Hillsdale, N. J.: Erlbaum Associates.
- Witten, L. H. and Corbin, M. J. (1973). Human operators and automatic adaptive controllers: a comparative study on a particular control task. Int. J. Man-Machine Studies, 5, 75-104.
- Young, L. R. (1969). On adaptive Manual Control, Ergonomics, 12, 635-675.
- Zeigler, B. P. (1976). Theory of Modelling and Simulation. New York: John Wiley and Sons.