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TRANSIENT RESPONSE OF STRUCTURES BY USE OF COMPONENT MODES.(U)
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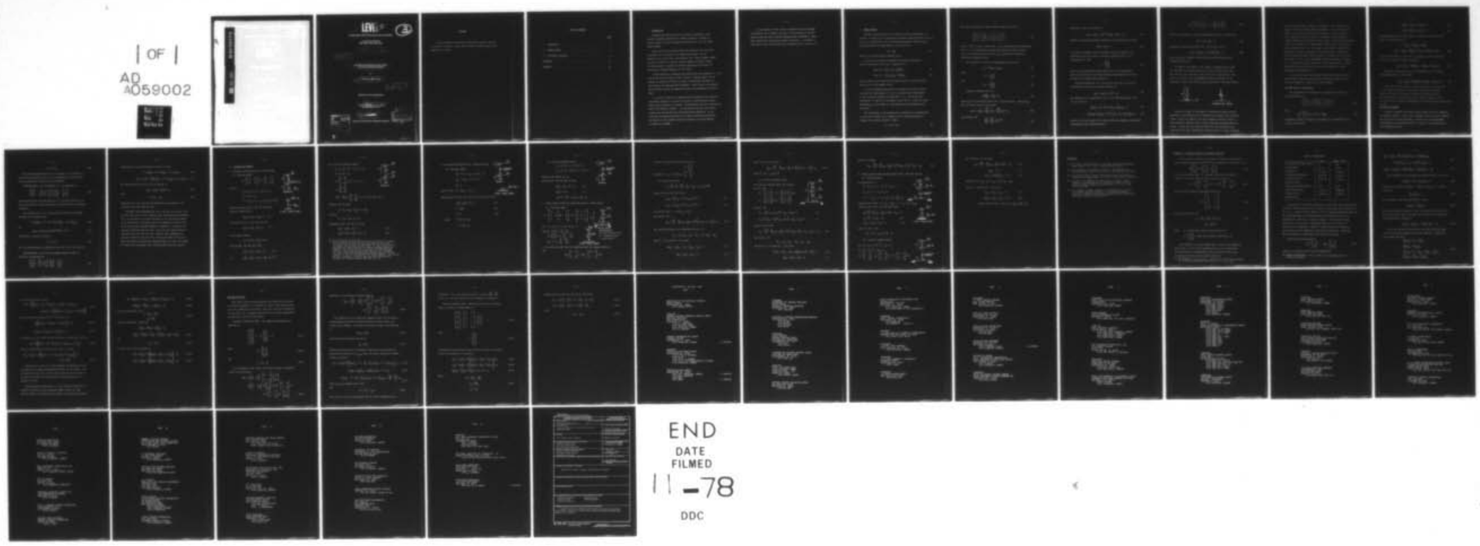

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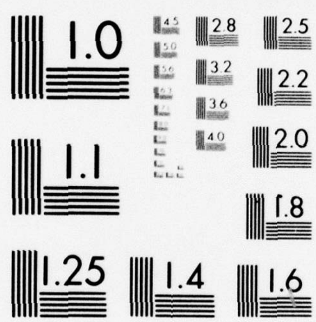

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TRANSIENT RESPONSE OF STRUCTURES BY USE OF COMPONENT MODES

by

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ABSTRACT

A general method is presented for obtaining the dynamic equations of an elastic structure to which elastic and/or nonlinear substructural elements are attached.

TABLE OF CONTENTS

I INTRODUCTION

Component mode methods had been successfully exploited to solve vibration problems long before the computer age made it feasible to analyze complex structural systems by discretization, e.g., the finite element method.

While analytical solutions abound in the literature, the structures considered, out of necessity, are comparatively simple, and are analyzed on an ad hoc basis. The extensive use of finite element methods has made it feasible to consider the most complex structural systems, and thereby made it necessary to develop comprehensive procedures for obtaining the dynamic equations for the system.

Various methods for component mode analysis have been suggested [1, 2, 3]. They differ from one another in their choice of component modes, but all express the equations of motion in terms of system modal stiffness and mass matrices and component mode coordinates. Their concern is essentially directed toward solving the eigenvalue problem and determining the system modes.

In a previous report [4] a general method was presented for obtaining the dynamic equations of an elastic structure to which arbitrary elastic substructural elements are attached. The methodology used was to express the kinetic and potential energies of the system as the sum of the contributions from the main structure and substructures, separately. It was shown that the equations of motion of the main structure were affected by the presence of the attached structure by means of the forces exerted at points of attachment.

It is the purpose of this report to formalize and fully automate the procedure, and to extend it so that it can be applied to the most complex structural elastic systems having nonlinear attachments and/or small substructural regions that respond nonlinearly. A treatment of some special cases involving nonlinear attachments may be found in [5].

II GENERAL METHOD

Consider an elastic structure S to which an elastic substructure σ is attached. We denote the in vacuo modal and diagonal generalized mass matrices of S by ϕ_S, μ_S, respectively, and the corresponding diagonal frequency matrix by ω_S^2, and express the displacement of the structure as

$$d_S = \phi_S q_S \tag{1}$$

where q_S is the generalized coordinate vector.

The kinetic and potential energies of the system, expressed as sums of contributions from S and σ separately, are

$$T(\dot{q}_S, \dot{d}_\sigma) = \frac{1}{2} \dot{q}_S^T \mu_S \dot{q}_S + \frac{1}{2} \dot{d}_\sigma^T M_\sigma \dot{d}_\sigma \tag{2}$$

$$V(q_S, d_\sigma) = \frac{1}{2} q_S^T \mu_S \omega_S^2 q_S + \frac{1}{2} d_\sigma^T K_\sigma d_\sigma \tag{3}$$

where M_σ and K_σ are the mass and stiffness matrices of the substructure, and d_σ is the σ-displacement vector.

Let the constrained base modes T_i^c be defined as the static elastic and rigid body displacements of σ due to successive unit displacements of the constrained degrees-of-freedom (dof) (i.e., those interface coordinates of σ which are constrained to move with S). Define the fixed-base modes $\phi_{f\sigma}$ as the modes of σ obtained when the constrained dof are held fixed.

The displacement of the substructure is expressed as a superposition of its static response due to movement of its constrained degrees of freedom and its motion relative to them:

$$d_\sigma = d_{c\sigma} + d_{f\sigma} \tag{4}$$

This can be expressed in a most convenient compact form [3] as

$$\begin{bmatrix} \bar{d}_\sigma \\ \hline \hat{d}_\sigma \end{bmatrix} = \left[\begin{array}{c|c} \bar{\phi}_S & 0 \\ \hline T^c\bar{\phi}_S & \hat{\phi}_{f\sigma} \end{array}\right] \begin{bmatrix} q_S \\ \hline q_{f\sigma} \end{bmatrix} \tag{5}$$

where $(\bar{\ })$ and $(\hat{\ })$ refer, respectively, to the constrained and unconstrained dof; T^c is the constrained base mode matrix whose successive columns are the constrained base mode vectors; $q_{f\sigma}$ represents the fixed-base generalized coordinate vector.

For convenience we express the displacement of Eq. (5) as:

$$d_\sigma = T^{c*}\bar{\phi}_S q_S + \phi_{f\sigma} q_{f\sigma} \tag{6}$$

where

$$T^{c*} = \begin{bmatrix} I \\ \hline T^c \end{bmatrix} \tag{7}$$

and

$$\phi_{f\sigma} = \begin{bmatrix} 0 \\ \hline \hat{\phi}_{f\sigma} \end{bmatrix} \tag{8}$$

Lagrange's equations for S are

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_S}\right) + \frac{\partial V}{\partial q_S} = Q_S \tag{9}$$

where Q_S are the generalized forces due to external forces. Substituting Eqs. (2) and (3) into (9) then yields

$$\mu_S \ddot{q}_S + \mu_S \omega_S^2 q_S + \frac{\partial \dot{d}_\sigma^T}{\partial \dot{q}_S} M_\sigma \ddot{d}_\sigma + \frac{\partial d_\sigma^T}{\partial q_S} K_\sigma d_\sigma = Q_S \tag{10}$$

But from Eqs. (6)

$$\frac{\partial d_\sigma^T}{\partial q_S} = \frac{\partial \dot{d}_\sigma^T}{\partial \dot{q}_S} = \bar{\phi}_S^T\ T^{c*T} \tag{11}$$

and thus Eq. (10) can be written as

$$\mu_S \ddot{q}_S + \mu_S \omega_S^2 q_S + \bar{\phi}_S^T T^{c*T} (M_\sigma \ddot{d}_\sigma + K_\sigma d_\sigma) = Q_S \tag{12}$$

Similarly by applying Lagrange's equations to the substructure we obtain

$$M_\sigma \ddot{d}_\sigma + K_\sigma d_\sigma = F \tag{13}$$

If we make the assumption that no external forces are applied to the substructure, the force vector F vanishes everywhere except at the constrained dof. Thus

$$F = \begin{Bmatrix} \bar{F} \\ -- \\ 0 \end{Bmatrix} \tag{14}$$

where $\bar{F}$ are the interaction forces exerted on σ at the constraints. Note that no difficultes arise when external forces applied to the substructure are included (see Appendix A).

Upon substituting Eqs. (7), (13) and (14) into (12), we can write the equations of motion for the main structure as

$$\mu_S \ddot{q}_S + \mu_S \omega_S^2 q_S + \bar{\phi}_S^T \bar{F} = Q_S \tag{15}$$

The equations for the substructure (13) can, by substituting Eqs. (6- 8, 14), be recast as

$$\bar{m}_\sigma \bar{\phi}_S \ddot{q}_S + (\bar{k}_\sigma + \bar{\hat{k}}_\sigma T^c)\, \bar{\phi}_S q_S + \bar{\hat{k}}_\sigma \hat{\phi}_{f\sigma} q_{f\sigma} = \bar{F} \tag{16}$$

$$\hat{m}_\sigma \hat{\phi}_{f\sigma} \ddot{q}_{f\sigma} + \hat{k}_\sigma \hat{\phi}_{f\sigma} q_{f\sigma} + \hat{m}_\sigma T^c\, \bar{\phi}_S \ddot{q}_S + (\hat{\bar{k}}_\sigma + \hat{k}_\sigma T^c) \bar{\phi}_S q_S = 0 \tag{17}$$

where m, k are the elements of the partitioned mass (assuming a lumped mass approximation) and stiffness matrices:

$$M_\sigma = \left[\begin{array}{c|c} \bar{m}_\sigma & 0 \\ \hline 0 & \hat{m}_\sigma \end{array}\right] \; ; \; K_\sigma = \left[\begin{array}{c|c} \bar{k}_\sigma & \bar{\hat{k}}_\sigma \\ \hline \hat{\bar{k}}_\sigma & \hat{k}_\sigma \end{array}\right] \tag{18}$$

By the very definition of the constrained base modes [3] it follows that

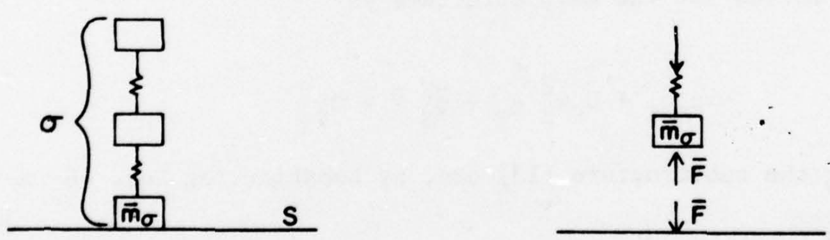
$$(\hat{\bar{k}}_\sigma + \hat{k}_\sigma T^c)\, \bar{\phi}_S q_S \equiv 0 \tag{19}$$

Consequently, upon pre-multiplying Eq. (17) by $\hat{\phi}^T_{f\sigma}$, we have

$$\mu_{f\sigma}\ddot{q}_{f\sigma} + \mu_{f\sigma}\omega^2_{f\sigma}q_{f\sigma} + \hat{\phi}^T_{f\sigma}(\hat{m}_\sigma T^c \bar{\phi}_S \ddot{q}_S) = 0 \tag{20}$$

where $\mu_{f\sigma}$ and $\omega_{f\sigma}$ represent, respectively, the generalized mass and frequency matrices of σ.

In summary, the dynamics of the system is expressed by Eqs. (15) and (20), where $\bar{F}$, the interaction force vector is given by Eq. (16). Note that the first term of Eq. (16) is that due to the inertia of the mass of σ located at the constrained dof, while the remainder of the expression is due to the stiffness forces exerted by σ (see figure below).



INTERACTION FORCE

There are a number of advantages in considering the main structure separately, and accounting for the substructure by means of the constraint forces. First, the computer core (central memory) required to solve a given problem is reduced, since no total system modal stiffness matrix is needed. Second, once the constraint modes have been determined, only those rows of K_σ (the unconstrained stiffness matrix of σ) that correspond to the constrained physical degrees of freedom of σ are required in the solution. This also reduces memory requirements. Third, the amount of computer time (central processor time) required to set up the matrices in the equation of motion is reduced, as compared to that required by a method employing a system modal stiffness matrix. Referring to the method of Ref. [3], for example, the system modal stiffness matrix is determined by the second of Eqs. (A-2). The computer time needed to evaluate this matrix may be excessive, especially if the problem under consideration requires many modes and/or physical degrees of freedom, and if K_σ is not well banded. In fact, the rearrangement of physical degrees of freedom of σ to partition K_σ usually results in a poorly banded matrix. Similar difficulties with the first of Eqs. (A-2) may be avoided by using a lumped mass approximation for σ.

In view of the above, the method developed here allows one to more readily handle complex systems with many substructures within the restrictions imposed by a given size of central memory.

FREE-FREE MODES OF SUBSTRUCTURE

The displacement of the substructure, expressed in terms of its free-free modes ϕ_σ, is:

$$\begin{bmatrix} \bar{d}_\sigma \\ \hline \hat{d}_\sigma \end{bmatrix} = \left[\begin{array}{c|c} \bar{\phi}_S & 0 \\ \hline 0 & \hat{\phi}_\sigma \end{array}\right] \begin{bmatrix} q_S \\ \hline q_\sigma \end{bmatrix} \tag{21}$$

since

$$\phi_\sigma = \begin{bmatrix} \bar{\phi}_\sigma \\ \hline \hat{\phi}_\sigma \end{bmatrix} \text{ and } \bar{d}_\sigma = \bar{\phi}_\sigma\, q_\sigma = \bar{\phi}_S q_S \tag{22}$$

Proceeding as before, we obtain the following set of equations in lieu of Eqs. (16) and (17).

$$\bar{m}_\sigma \bar{\phi}_\sigma \ddot{q}_\sigma + \bar{k}_\sigma \bar{\phi}_\sigma q_\sigma + \bar{\hat{k}}_\sigma \hat{\phi}_\sigma q_\sigma = \bar{F} \tag{23}$$

$$\hat{m}_\sigma \hat{\phi}_\sigma \ddot{q}_\sigma + \hat{\bar{k}}_\sigma \bar{\phi}_\sigma q_\sigma + \hat{k}_\sigma \hat{\phi}_\sigma q_\sigma = 0 \tag{24}$$

The generalized mass (μ_σ) and frequency (ω_σ) matrices for the free-free modes of the substructure are

$$\mu_\sigma = \phi_\sigma^T M_\sigma \phi_\sigma = \bar{\phi}_\sigma^T \bar{m}_\sigma \bar{\phi}_\sigma + \hat{\phi}_\sigma^T \hat{m}_\sigma \hat{\phi}_\sigma$$

$$\mu_\sigma \omega_\sigma^2 = \phi_\sigma^T K_\sigma \phi_\sigma = \bar{\phi}_\sigma^T (\bar{k}_\sigma \bar{\phi}_\sigma + \bar{\hat{k}}_\sigma \hat{\phi}_\sigma) + \hat{\phi}_\sigma^T (\hat{\bar{k}}_\sigma \bar{\phi}_\sigma + \hat{k}_\sigma \hat{\phi}_\sigma) \tag{25}$$

If we now pre-multiply Eq. (24) by ϕ_σ^T , and make use of the expressions in Eq. (25), then the following is obtained

$$\mu_\sigma \ddot{q}_\sigma + \mu_\sigma \omega_\sigma^2 q_\sigma - \bar{\phi}_\sigma^T [\bar{m}_\sigma \bar{\phi}_\sigma \ddot{q}_\sigma + (\bar{k}_\sigma \bar{\phi}_S q_S + \bar{\hat{k}}_\sigma \hat{\phi}_\sigma q_\sigma)] = 0 \tag{26}$$

The motion of the main structure is determined by Eq. (15), which when combined with Eq. (23) becomes

$$\mu_S \ddot{q}_S + \mu_S \omega_S^2 q_S + \bar{\phi}_S^T [\bar{m}_\sigma \bar{\phi}_\sigma \ddot{q}_\sigma + (\bar{k}_\sigma \bar{\phi}_S q_S + \bar{\hat{k}}_\sigma \hat{\phi}_\sigma q_\sigma)] = Q_S \tag{27}$$

Thus, when free-free modes of the substructure are used, Eqs. (26) and (27) express the motion of the system. Note that the bracketed terms in (26) and (27) represent the constraint interaction forces, and that the motion of the substructure, Eq. (26), is coupled to that of the main structure through the enforcement of the constrained dof relationship, Eq. (22).

NONLINEAR ATTACHMENTS

Consider a composite structural system in which each of the components is assumed to exhibit a linear elastic response, and in which the components are attached by nonlinear mountings. We denote the force-displacement relationships of the mountings by $\bar{F}(\delta, \dot{\delta})$, where δ is the relative interface displacement vector

$$\delta = \bar{d}_S - \bar{d}_\sigma \tag{28}$$

When component mode analysis is to be applied, it is necessary to isolate these attachments from all of the components, so that only the forces transmitted by them enter into the analysis.

FIXED-BASE MODES. The displacements of σ are expressed as

$$\begin{Bmatrix} \bar{d}_\sigma \\ \hat{d}_\sigma \end{Bmatrix} = \begin{bmatrix} \bar{\phi}_S & 0 \\ T^c \bar{\phi}_S & \hat{\phi}_{f\sigma} \end{bmatrix} \begin{Bmatrix} q_S \\ q_{f\sigma} \end{Bmatrix} - \begin{Bmatrix} \delta \\ T^c \delta \end{Bmatrix} \tag{29}$$

When determining the fixed-base modes of σ, it should be noted that the constrained dof must include those which are constrained by the nonlinear attachments.

Upon combining Eqs. (13), (14) and (29), we obtain the following equations for the substructure:

$$\bar{F} = \bar{m}_\sigma(\bar{\phi}_S \ddot{q}_S - \ddot{\delta}) + (\bar{k}_\sigma + \hat{\bar{k}}_\sigma T^c)(\bar{\phi}_S q_S - \delta) + \bar{\hat{k}}_\sigma \hat{\phi}_{f\sigma} q_{f\sigma} \tag{30}$$

and

$$\mu_{f\sigma}\ddot{q}_{f\sigma} + \mu_{f\sigma}\omega_{f\sigma}^2 q_{f\sigma} + \hat{\phi}_{f\sigma}^T \hat{m}_\sigma T^c (\bar{\phi}_S \ddot{q}_S - \ddot{\delta}) = 0 \tag{31}$$

The nonlinear constraint forces are

$$\bar{F} = \bar{F}(\delta, \dot{\delta}) \tag{32}$$

Thus the system dynamics is expressed by Eqs. (30), (31), (32) and (15).

FREE-FREE MODES. When free-free component modes are used, we express the displacements as

$$\begin{Bmatrix} \bar{d}_\sigma \\ \hat{d}_\sigma \end{Bmatrix} = \begin{bmatrix} \bar{\phi}_S & 0 \\ 0 & \hat{\phi}_\sigma \end{bmatrix} \begin{Bmatrix} q_S \\ q_J \end{Bmatrix} - \begin{Bmatrix} \delta \\ 0 \end{Bmatrix} \tag{33}$$

Utilizing Eq. (33) and proceeding as before, we obtain

$$\bar{F} = \bar{m}_\sigma(\bar{\phi}_S\ddot{q}_S - \ddot{\delta}) + \bar{k}_\sigma(\bar{\phi}_S q_S - \delta) + \hat{\bar{k}}_\sigma \hat{\phi}_\sigma q_\sigma \tag{34}$$

$$\mu_\sigma \ddot{q}_\sigma + \mu_\sigma \omega_\sigma^2 q_\sigma = \bar{\phi}_\sigma^T[\bar{m}_\sigma(\bar{\phi}_S\ddot{q}_S - \ddot{\delta}) + \bar{k}_\sigma(\bar{\phi}_S q_S - \delta) + \hat{\bar{k}}_\sigma \hat{\phi}_\sigma q_\sigma] \tag{35}$$

The substitution of Eq. (34) into (35) results in

$$\mu_\sigma \ddot{q}_\sigma + \mu_\sigma \omega_\sigma^2 q_\sigma = \bar{\phi}_\sigma^T \bar{F}(\delta, \dot{\delta}) \tag{36}$$

where

$$\delta = \bar{\phi}_S q_S - \bar{\phi}_\sigma q_\sigma \tag{37}$$

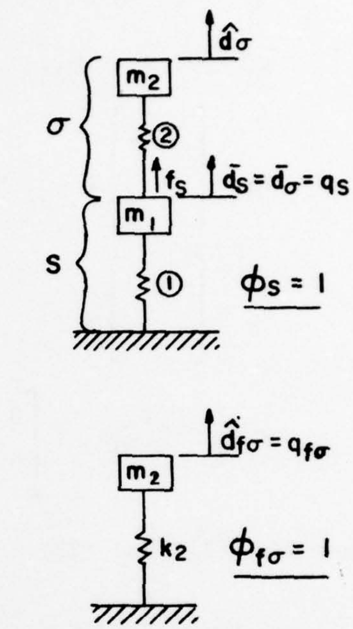
Equations (15), (32), (34) and (36) characterize the dynamics of the system when free-free modes are used.

One might, upon juxtaposing Eqs. (15), (30) and (31) and Eqs. (15), (34) and (36), arrive at the conclusion that, where nonlinear attachments are to be considered, it is more convenient to use the free-free modes of the substructure. Certainly, their use eliminates the need to store any of the mass and stiffness elements of component σ, as would be the case when fixed-base modes are used. Also, only two coupled systems of equations result when they are used. However, these advantages can readily be offset by the need to use a far greater number of free-free modes than would be required when fixed-base modes are used. For example, for rather stiff mountings, fewer fixed-base modes would be required.

III ILLUSTRATIVE PROBLEMS

A. Two-Degree-of-Freedom System - Linear Springs

(i) Fixed-Base Modes



FIXED - BASE MODEL

$$K_\sigma = \begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}; \; M_\sigma = \begin{bmatrix} 0 & 0 \\ 0 & m_2 \end{bmatrix}$$

and thus

$$\bar{k}_\sigma = k_2; \; \hat{\bar{k}}_\sigma = \bar{\hat{k}}_\sigma = -k_2; \; \hat{k}_\sigma = k_2$$

$$\bar{m}_\sigma = 0; \; \hat{m}_\sigma = m_2$$

$$T^c = 1; \; \hat{\phi}_{f\sigma} = 1; \; \mu_{f\sigma} = m_2; \; \omega_{f\sigma}^2 = k_2/m_2$$

Upon substituting into Eq. (20), we obtain the *equation of motion for* σ:

$$m_2\ddot{q}_{f\sigma} + k_2 q_{f\sigma} + m_2\ddot{q}_S = 0 \qquad \text{(i)}$$

But from Eq. (5): $\hat{d}_\sigma = q_S + q_{f\sigma}$, and thus

$$m_2\ddot{\hat{d}}_\sigma + k_2(\hat{d}_\sigma - \bar{d}_S) = 0 \qquad \text{(i}'\text{)}$$

For the main structure

$$\mu_S = m_1; \; \omega_S^2 = k_1/m_1; \; \bar{\phi}_S = 1$$

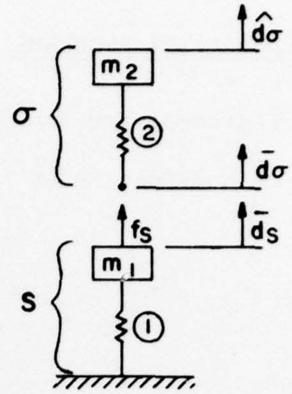
and thus Eqs. (15) and (16) yield:

$$m_1\ddot{q}_S + k_1 q_S - k_2 q_{f\sigma} = f_S \qquad \text{(ii)}$$

or

$$m_1\ddot{\bar{d}}_S + k_1\bar{d}_S - k_2(\hat{d}_\sigma - \bar{d}_S) = f_S \qquad \text{(ii}'\text{)}$$

(ii) Free-Free Component Modes*)



$$\bar{k}_\sigma = k_2;\ \hat{\bar{k}}_\sigma = \bar{\hat{k}}_\sigma = -k_2;\ \hat{k}_\sigma = k_2;$$

$$\bar{m}_\sigma = 0;\ \hat{m}_\sigma = m_2;$$

$$\phi_\sigma = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix};\ \omega_{\sigma 1} = 0;\ \omega_{\sigma 2} = \infty\ ;$$

$$\mu_\sigma = \begin{bmatrix} m_2 & 0 \\ 0 & 0 \end{bmatrix};$$

$$\mu_\sigma \omega_\sigma^2 = \phi_\sigma^T K_\sigma \phi_\sigma = \begin{bmatrix} 0 & 0 \\ 0 & k_2 \end{bmatrix},\ \mu_S = m_1;\ \omega_S^2 = k_1/m_1;\ \bar{\phi}_S = 1$$

From Eqs. (21) and (22),

$$\bar{d}_\sigma = \bar{d}_S = \bar{\phi}_\sigma q_\sigma = \bar{\phi}_S q_S;\ \hat{d}_\sigma = \hat{\phi}_\sigma q_\sigma$$

and thus,

$$\bar{d}_\sigma = q_{\sigma 1} + q_{\sigma 2};\ \hat{d}_\sigma = q_{\sigma 1}$$

Consequently, Eqs. (26) and (27) yield:

$$m_2 \ddot{\hat{d}}_\sigma - k_2(\bar{d}_\sigma - \hat{d}_\sigma) = 0 \qquad \text{(iii)}$$

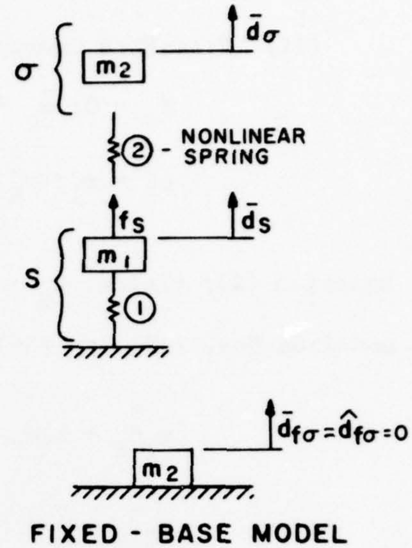
$$m_1 \ddot{\bar{d}}_\sigma + k_1 \bar{d}_\sigma + k_2(\bar{d}_\sigma - \hat{d}_\sigma) = f_S \qquad \text{(iv)}$$

---

*) When determining the free-free modes of the σ-structure, one might, at first glance, conclude that the constrained dof need not be considered, since no mass is placed there. But the σ-structure has two degrees of freedom and thus two free-free component modes are required. The first of these is, of course, the rigid body mode which has a zero frequency; the second, which is not so apparent, is a mode in which the constrained dof oscillates with an infinite frequency, so that the unconstrained dof remains immobile. Alternately, one can, by placing a fictitious mass m at the constrained dof, determine the free-free modes in the limit as $m \to 0$.

B. Two-Degree-of-Freedom System - Nonlinear Spring

(i) Fixed-Base Modes



$$K_\sigma = 0;\ \bar{m}_\sigma = \mu_{f\sigma} = m_2;\ \omega_{f\sigma}^2 = 0;$$

$$\hat{\phi}_{f\sigma} = 0;\ q_{f\sigma} = 0;\ T^c = 0;$$

$$\bar{\phi}_S = 1$$

From Eq. (29): $\bar{d}_\sigma = \bar{\phi}_S q_S - \delta;\ \hat{d}_\sigma = 0$

Substituting into Eqs. (15), (30), (32) and (31) we obtain

$$m_1 \ddot{\bar{d}}_S + k_1 \bar{d}_S + \bar{F} = f_S \qquad \text{(i)}$$

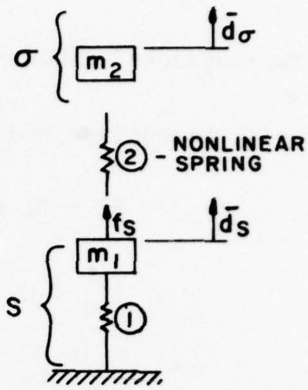
$$\bar{F} = m_2 \ddot{\bar{d}}_\sigma \qquad \text{(ii)}$$

$$0 = 0 \qquad \text{(iii)}$$

where $\bar{F} = \bar{F}(\delta, \dot{\delta})$ and

$$\delta = \bar{d}_S - \bar{d}_\sigma$$

(ii) Free-Free Component Modes



$$K_\sigma = 0;\ \bar{m}_\sigma = \mu_\sigma = m_2;\ \omega_\sigma^2 = 0;$$

$$\mu_S = m_1;\ \omega_S^2 = k_1/m_1;\ \bar{\phi}_\sigma = 1;\ \hat{\phi}_\sigma = 0$$

Equation (22) yields: $\bar{d}_\sigma = q_\sigma$

and from Eqs. (15) and (36), we have

$$m_1\ddot{\bar{d}}_S + k_1\bar{d}_S + \bar{F} = f_S \qquad \text{(iv)}$$

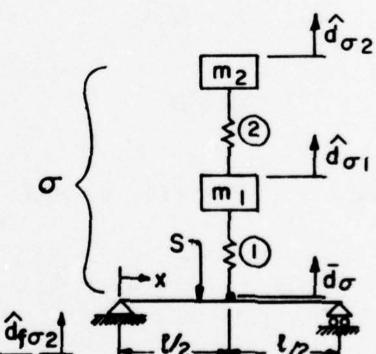
$$m_2\ddot{\bar{d}}_\sigma - \bar{F} = 0 \qquad \text{(v)}$$

where $\bar{F} = \bar{F}(\delta, \dot{\delta})$ and $\delta = \bar{d}_S - \bar{d}_\sigma$

C. Simply Supported Beam and Spring Mass System - Linear Springs

(i) Fixed-Base Modes

$$K_\sigma = \begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & (k_1+k_2) & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix},\ M_\sigma = \begin{bmatrix} 0 & 0 & 0 \\ 0 & m_1 & 0 \\ 0 & 0 & m_2 \end{bmatrix}$$

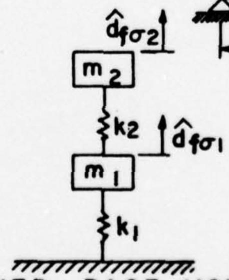


Let: $k_1 = 2k;\ k_2 = k;\ m_1 = 2m,\ m_2 = m$

Then: $\bar{k}_\sigma = 2k;\ \overset{\frown}{k}_\sigma = \begin{bmatrix} -2 & 0 \end{bmatrix} k$

$$\hat{k}_\sigma = \begin{Bmatrix} -2 \\ 0 \end{Bmatrix} k;\ \hat{k}_\sigma = \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix} k$$

$$\bar{m}_\sigma = 0;\ \hat{m}_\sigma = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} m$$



FIXED-BASE MODEL

BEAM CONSTANTS

A = cross-sectional area

EI = bending rigidity

ρ = mass density

It can readily be shown that the fixed-base modal and frequency matrices are:

$$\hat{\phi}_{f\sigma} = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix};\quad \omega_{f\sigma}^2 = \begin{bmatrix} 1/2 & 0 \\ 0 & 2 \end{bmatrix} k/m$$

and that the constrained base mode matrix is:

$$T^c = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

Consequently, $\mu_{f\sigma} = \hat{\phi}^T_{f\sigma}\, \hat{m}_\sigma \hat{\phi}_{f\sigma} = \begin{bmatrix} 6 & 0 \\ 0 & 3 \end{bmatrix} m$

For the main structure:

$$\mu_S = \frac{\rho A \ell}{2};\ \omega_S^2 = \frac{\pi^4 n^4}{\ell^4} \cdot \frac{EI}{\rho A}\ ;\ \phi_S q_S = \sum q_{Sn} \sin \frac{n\pi x}{\ell}$$

Substitution into Eq. (20) yields:

$$\ddot{q}_{f\sigma 1} + \frac{1}{2}\frac{k}{m} q_{f\sigma 1} + \frac{2}{3} \sum \ddot{q}_{Sn} \sin \frac{n\pi}{2} = 0 \qquad \text{(i)}$$

$$\ddot{q}_{f\sigma 2} + 2\frac{k}{m} q_{f\sigma 2} + \frac{1}{3} \sum \ddot{q}_{Sn} \sin \frac{n\pi}{2} = 0 \qquad \text{(ii)}$$

Now from Eq. (16): $F = -2k(q_{f\sigma 1} + q_{f\sigma 2})$

and from Eq. (15):

$$\ddot{q}_{sn} + \frac{\pi^4 n^4}{\ell^4} \cdot \frac{EI}{\rho A} q_{Sn} + \frac{2}{\rho A \ell} \sin \frac{n\pi}{2} \left[-2k(q_{f\sigma 1} + q_{f\sigma 2}) \right] = Q_{Sn} \qquad \text{(iii)}$$

The following identities are established from Eq. (5):

$$\hat{d}_{\sigma 1} - \bar{d}_\sigma = q_{f\sigma 1} + q_{f\sigma 2}\ ;\ \hat{d}_{\sigma 2} - \bar{d}_\sigma = 2q_{f\sigma 1} - q_{f\sigma 2}$$

Thus (i) + (ii) and 2(i) - (ii) yield

$$2m\ddot{\hat{d}}_{\sigma 1} + 2k(\hat{d}_{\sigma 1} - \bar{d}_\sigma) + k(\hat{d}_{\sigma 1} - \hat{d}_{\sigma 2}) = 0 \qquad \text{(i')}$$

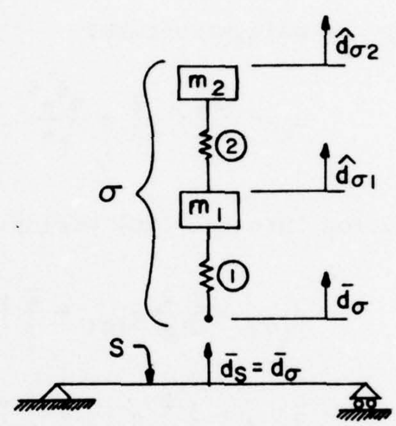
$$m\ddot{\hat{d}}_{\sigma 2} + k(\hat{d}_{\sigma 2} - \hat{d}_{\sigma 1}) = 0 \qquad \text{(ii')}$$

while (iii) can be written as

$$\ddot{q}_{Sn} + \frac{\pi^4 n^4}{\ell^4} \cdot \frac{EI}{\rho A} q_{Sn} + \frac{2}{\rho A \ell} \sin \frac{\pi n}{2} \left[-2k(\hat{d}_{\sigma 1} - \bar{d}_{\sigma}) \right] = Q_{Sn} \qquad \text{(iii')}$$

where $\bar{d}_{\sigma} = \bar{d}_{S} = \sum q_{Sn} \sin \frac{n\pi}{2}$

(ii) Free-Free Component Modes



The free-free component modal and frequency matrices are:

$$\phi_{\sigma} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & -2 & 0 \end{bmatrix} ; \; \omega_{\sigma}^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 3/2 & 0 \\ 0 & 0 & \infty \end{bmatrix} k/m ;$$

$$\mu_{\sigma} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 0 \end{bmatrix} ; \; \mu_{\sigma}\omega_{\sigma}^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 2 \end{bmatrix} k$$

From Eq. (26)

$$\ddot{q}_{\sigma 1} - \frac{3}{2}\frac{k}{m}\left[\sum q_{Sn} \sin \frac{n\pi}{2} - (q_{\sigma 1} + q_{\sigma 2}) \right] = 0 \qquad \text{(iv)}$$

$$\ddot{q}_{\sigma 2} + \frac{3}{2}\frac{k}{m} q_{\sigma 2} - \frac{1}{3}\frac{k}{m}\left[\sum q_{Sn} \sin \frac{n\pi}{2} - (q_{\sigma 1} + q_{\sigma 2}) \right] = 0 \qquad \text{(v)}$$

Equation (27) yields:

$$\ddot{q}_{Sn} + \frac{\pi^4 n^4}{\ell^4} \cdot \frac{EI}{\rho A} q_{Sn} + \frac{2}{\rho A \ell} \sin \frac{n\pi}{2} \left[\sum q_{Sn} \sin \frac{n\pi}{2} - (q_{\sigma 1} + q_{\sigma 2}) \right] = Q_{Sn} \qquad \text{(vi)}$$

But from Eq. (21)

$$\hat{d}_{\sigma 1} = q_{\sigma 1} + q_{\sigma 2} \quad ; \quad \hat{d}_{\sigma 2} = q_{\sigma 1} - 2q_{\sigma 2}$$

and thus (iv) + (v) and (iv) - 2(v) yield

$$2m \ddot{\hat{d}}_{\sigma 1} + k(\hat{d}_{\sigma 1} - \hat{d}_{\sigma 2}) - 2k\left[\sum q_{Sn} \sin \frac{n\pi}{2} - \hat{d}_{\sigma 1} \right] = 0 \qquad \text{(iv')}$$

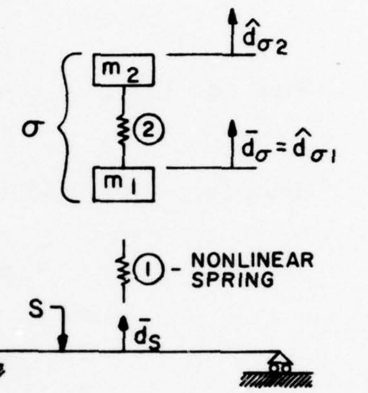
$$m\ddot{\hat{d}}_{\sigma 2} - k(\hat{d}_{\sigma 1} - \hat{d}_{\sigma 2}) = 0 \qquad \text{(v')}$$

while (vi) becomes

$$\ddot{q}_{Sn} + \frac{\pi^4 n^4}{\ell^4} \cdot \frac{EI}{\rho A} q_{Sn} + \frac{2}{\rho A \ell} \sin \frac{n\pi}{2} [\sum q_{Sn} \sin \frac{n\pi}{2} - \hat{d}_{\sigma 1}] = Q_{Sn} \qquad \text{(vi)}$$

D. Simply Supported Beam and Spring Mass System - Nonlinear Springs

(i) Fixed-Base Modes



For substructure σ :

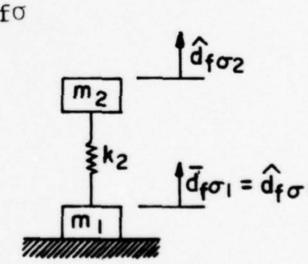
$\bar{k}_\sigma = \hat{k}_\sigma = k_2 = k$; $\bar{\hat{k}} = \hat{\bar{k}} = -k_2 = -k$

$\bar{m}_\sigma = m_1 = 2m$; $\hat{m}_\sigma = m_2 = m$

$T^c = 1$; $\hat{\phi}_{f\sigma} = 1$; $\mu_{f\sigma} = m_2 = m$; $\omega^2_{f\sigma} = \frac{k_2}{m_2} = \frac{k}{m}$

Equation (29) yields: $\bar{d}_\sigma = \hat{d}_{\sigma 1} = \bar{\phi}_S q_S - \delta$; $\hat{d}_{\sigma 2} = \bar{d}_\sigma + \hat{\phi}_{f\sigma} q_{f\sigma}$

Thus Eqs. (15), (30), (32) and (31) become:



FIXED - BASE MODEL

$$\ddot{q}_{Sn} + \frac{\pi^4 n^4}{\ell^4} \cdot \frac{EI}{\rho A} q_{Sn} + \frac{2}{\rho A \ell} \sin \frac{n\pi}{2} \bar{F} = Q_{Sn} \qquad \text{(i)}$$

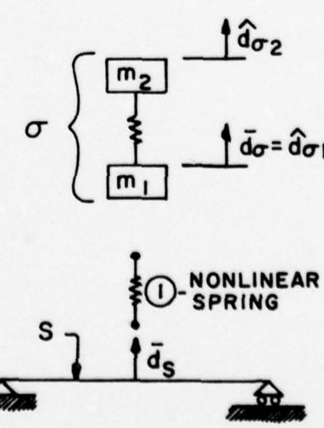
$$\bar{F} = 2m\ddot{\bar{d}}_\sigma - k(\hat{d}_{\sigma 2} - \hat{d}_{\sigma 1}) \qquad \text{(ii)}$$

$$m \ddot{\hat{d}}_{\sigma 2} + k(\hat{d}_{\sigma 2} - \hat{d}_{\sigma 1}) = 0 \qquad \text{(iii)}$$

where $\bar{F} = F(\delta, \delta)$ and

$$\delta = \bar{d}_S - \bar{d}_\sigma = \sum q_{Sn} \sin \frac{n\pi}{2} - \hat{d}_{\sigma 1}$$

(ii) Free-Free Component Modes



$\bar{k}_\sigma = \hat{k}_\sigma = k_2 = k$; $\bar{\hat{k}} = \hat{\bar{k}} = -k_2 = -k$;

$\bar{m}_\sigma = m_1 = 2m$; $\hat{m}_\sigma = m_2 = m$;

$$\phi_\sigma = \begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix} ; \quad \mu_\sigma = \begin{bmatrix} 3 & 0 \\ 0 & 6 \end{bmatrix} m ; \quad \omega^2_\sigma = \begin{bmatrix} 0 & 0 \\ 0 & 3/2 \end{bmatrix} k/m$$

Thus from Eqs. (15) and (36):

$$\ddot{q}_{Sn} + \frac{\pi^4 n^4}{\ell^4} \cdot \frac{EI}{\rho A} q_{Sn} + \frac{2}{\rho A \ell} \sin \frac{n\pi}{2} \bar{F} = Q_{Sn} \qquad \text{(iv)}$$

$$3m\ddot{q}_{\sigma 1} - \bar{F} = 0 \qquad \text{(v)}$$

$$6m\, \ddot{q}_{\sigma 2} + (6m)\frac{3}{2}\frac{k}{m} q_{\sigma 2} - \bar{F} = 0 \qquad \text{(vi)}$$

From Eq. (33): $\hat{d}_{\sigma 1} = q_{\sigma 1} + q_{\sigma 2}$; $\hat{d}_{\sigma 2} = q_{\sigma 1} - 2q_{\sigma 2}$

Thus (v) - (vi) and 2(v) + (vi) yield

$$m\ddot{\hat{d}}_{\sigma 2} - k(\hat{d}_{\sigma 1} - \hat{d}_{\sigma 2}) = 0 \qquad \text{(v)}$$

$$2m\ddot{\hat{d}}_{\sigma 1} + k(\hat{d}_{\sigma 1} - \hat{d}_{\sigma 2}) - \bar{F} = 0 \qquad \text{(vi)}$$

where $\bar{F} = \bar{F}(\delta, \dot{\delta})$ and $\delta = \bar{d}_S - \hat{d}_{\sigma 1} = \sum q_{Sn} \sin \frac{n\pi}{2} - \hat{d}_{\sigma 1}$

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[4] D. Ranlet, F.L. DiMaggio, H.H. Bleich and M.L. Baron, "Elastic Response of Submerged Shells with Internally Attached Structures to Shock Loading", Computers and Structures, Vol. 7, No. 3, June 1977, pp. 355-364.

[5] F.L. DiMaggio, "Dynamic Response of a Submerged Elastic Structure with Elastic Structures Attached to It by Inelastic Springs", Contract No. N00014-72-C-0119, Office of Naval Research, Weidlinger Associates, Technical Note, August 1977.

APPENDIX A - ALTERNATE DERIVATION OF DYNAMIC EQUATIONS

In this section we consider the equations of motion for the system as given by Benfield and Hruda [3]*), except for the addition of generalized forces:

$$M^c \begin{Bmatrix} \ddot{\xi}_a \\ \ddot{\xi}_{nb} \end{Bmatrix} + K^c \begin{Bmatrix} \xi_a \\ \xi_{nb} \end{Bmatrix} = \begin{Bmatrix} Q_a \\ Q_{nb} \end{Bmatrix} \qquad \text{(A-1)}$$

When using fixed-base modes for component b and free-free modes for component a, the system modal mass and stiffness matrices are

$$M^c = T_5^{cT} \begin{bmatrix} m_a & 0 \\ 0 & m_b \end{bmatrix} T_5^c \; ; \; K^c = T_5^{cT} \begin{bmatrix} k_a & 0 \\ 0 & k_b \end{bmatrix} T_5^c \qquad \text{(A-2)}$$

and the system coordinate transformation matrix is

$$T_5^c = \begin{bmatrix} \bar{\phi}_a & 0 \\ \hat{\phi}_a & 0 \\ \bar{\phi}_a & 0 \\ T_{cb}\bar{\phi}_a & \hat{\phi}_{nb} \end{bmatrix} \qquad \text{(A-3)}$$

The generalized forces are

$$Q_a = \hat{\phi}_a^T \hat{f}_a + \bar{\phi}_a^T \bar{f}_b + \bar{\phi}_a^T T_{cb}^T \hat{f}_b \qquad \text{(A-4)}$$

$$Q_{nb} = \hat{\phi}_{nb}^T \hat{f}_b$$

where $\hat{f}_a$ = known forces acting on main structure (S)

$$f_b = \begin{Bmatrix} \bar{f}_b \\ \hat{f}_b \end{Bmatrix} = \text{known forces acting on substructure } (\sigma)$$

For convenience to those who might wish to refer to the formulation given in Ref. [3], we use the nomenclature that is to be found there. The relationship of the nomenclature used in the body of this report to that used in this section is shown in Table A-1.

*) For simplicity, the superscript c which refers to the fixed-base modes in [3] has been replaced by the subscript n; e.g., $\xi_b^c \rightarrow \xi_{nb}$.

TABLE A-1 NOMENCLATURE

	WA	BH, Ref. [3] *)
Constrained Base Mode Matrix	T^c	T_{cb}
Displacements	d_S, d_σ	q_a, q_b
Frequencies	$\omega_S, \omega_{f\sigma}, \omega_\sigma$	$\omega_a, \omega_{nb}, \omega_b$
Generalized Coordinates	$q_s, q_{f\sigma}, q_\sigma$	ξ_a, ξ_{nb}, ξ_b
Generalized Forces	$Q_S, Q_{f\sigma}, Q_\sigma$	Q_a, Q_{nb}, Q_b
Main Structure	S	a
Mass and Stiffness Matrices	M_σ, K_σ	m_b, k_b
Modal Mass Matrices	$\mu_S, \mu_{f\sigma}, \mu_\sigma$	M_a, M_{nb}, M_b
Modal Matrices	$\phi_S, \hat{\phi}_{f\sigma}, \phi_\sigma$	$\phi_a, \hat{\phi}_{nb}, \phi_b$
Substructure	σ	b

The equations of the system as given by Eq. (A-1) have, with vanishing right-hand side, been successfully used to obtain the system modes and frequencies, in Ref. [3]. It should be noted, however, that, for complex systems having many dof, the system modal mass and stiffness matrices would require the use of an extensive part of the core of a digital computer. For transient or shock problems this would result in long running times, so that a direct use of Eqs. (A-1) is inadvisable. However, in what follows we will show that these equations can be simplified so that they do indeed yield expressions which are identical to those given in the body of this report.

Defining the following matrices

$$T^*_{cb} = \begin{Bmatrix} I \\ T_{cb} \end{Bmatrix} \quad ; \quad \phi_{nb} = \begin{Bmatrix} 0 \\ \hat{\phi}_{nb} \end{Bmatrix} \qquad \text{(A-5)}$$

allows us to express Eq. (A-1) in the following form

*) Symbols not appearing in [3] are chosen to be consistent with the notation found there.

$$M_a \ddot{\xi}_a + M_a \omega_a^2 \xi_a + \bar{\phi}_a^T (\overset{*T}{T}_{cb} m_b \overset{*}{T}_{cb}) \bar{\phi}_a \ddot{\xi}_a + \bar{\phi}_a{}^T \overset{*T}{T}_{cb} m_b \phi_{nb} \ddot{\xi}_{nb}$$

$$+ \bar{\phi}_a^T (\overset{*}{T}_{cb}{}^T k_b \overset{*}{T}_{cb}) \bar{\phi}_a \xi_a + \bar{\phi}_a^T \overset{*T}{T}_{cb} k_b \phi_{nb} \xi_{nb} = Q_a \qquad \text{(A-6)}$$

$$M_{nb} \ddot{\xi}_{nb} + M_{nb} \omega_{nb}^2 \xi_{nb} + \phi_{nb}^T m_b \overset{*}{T}_{cb} \bar{\phi}_a \ddot{\xi}_a + \phi_{nb}^T k_b \overset{*}{T}_{cb} \bar{\phi}_a \xi_a = Q_{nb} \qquad \text{(A-7)}$$

As a consequence of the constraint base mode definition in Ref. [3],

$$k_b \overset{*}{T}_{cb} \bar{\phi}_a \xi_a = (\hat{\bar{k}}_b + \hat{k}_b T_{cb}) \bar{\phi}_a \xi_a = 0 \qquad \text{(A-8)}$$

except at the interface coordinates. However, since $\phi_{nb} = 0$ at these coordinates,

$$\phi_{nb}^T k_b \overset{*}{T}_{cb} \phi_a \xi_a = 0 \qquad \text{(A-9)}$$

Also, assuming a lumped mass approximation, we have

$$\phi_{nb}^T m_b \overset{*}{T}_{cb} = \hat{\phi}_{nb}^T \hat{m}_b T_{cb} \qquad \text{(A-10)}$$

Upon substituting the identities of Eqs. (A-9) and (A-10) into Eq. (A-7), we obtain the final form of the equations of motion for the fixed-base (constraint) modes of component b

$$M_{nb} \ddot{\xi}_{nb} + M_{nb} \omega_{nb}^2 \xi_{nb} + \hat{\phi}_{nb}^T \hat{m}_b T_{cb} \bar{\phi}_a \ddot{\xi}_a = Q_{nb} \qquad \text{(A-11)}$$

Let us now consider all but the first two terms on the left-hand side of Eq. (A-6) and designate them by the symbol LHS. Appealing to the following identities

$$\overset{*T}{T}_{cb} m_b \overset{*}{T}_{cb} = \bar{m}_b + T_{cb}^T \hat{m}_b T_{cb}$$

$$\overset{*T}{T}_{cb} m_b \phi_{nb} = T_{cb}^T \hat{m}_b \hat{\phi}_{nb}$$

$$\overset{*T}{T}_{cb} k_b \overset{*}{T}_{cb} = \bar{k}_b + \hat{\bar{k}}_b T_{cb} + T_{cb}^T \hat{\bar{k}}_b + T_{cb}^T \hat{k}_b T_{cb} \qquad \text{(A-12)}$$

$$\overset{*T}{T}_{cb} k_b \phi_{nb} = \hat{\bar{k}}_b \hat{\phi}_{nb} + T_{cb}^T \hat{k}_b \hat{\phi}_{nb}$$

and rearranging terms, we write

$$LHS = \bar{\phi}_a^T \Big[\bar{m}_b \bar{\phi}_a \ddot{\xi}_a + (\bar{k}_b + \bar{\hat{k}}_b T_{cb}) \bar{\phi}_a \xi_a + T_{cb}^T (\hat{\bar{k}}_b + \hat{k}_b T_{cb}) \bar{\phi}_a \xi_a + \bar{\hat{k}}_b \phi_{nb} \xi_{nb} + T_{cb}^T \Big\{ \hat{m}_b \hat{\phi}_{nb} \ddot{\xi}_{nb} + \hat{k}_b \hat{\phi}_{nb} \xi_{nb} + \hat{m}_b T_{cb} \bar{\phi}_a \ddot{\xi}_a \Big\} \Big] \tag{A-13}$$

Now the equilibrium equations (A-11) can be written as

$$\hat{\phi}_{nb}^T \Big\{ \hat{m}_b \hat{\phi}_{nb} \ddot{\xi}_{nb} + \hat{k}_b \hat{\phi}_{nb} \xi_{nb} + \hat{m}_b T_{cb} \bar{\phi}_a \ddot{\xi}_a \Big\} = Q_{nb} \tag{A-14}$$

or

$$\hat{m}_b \hat{\phi}_{nb} \ddot{\xi}_{nb} + \hat{k}_b \hat{\phi}_{nb} \xi_{nb} + \hat{m}_b T_{cb} \bar{\phi}_a \ddot{\xi}_a = \hat{f}_b \tag{A-15}$$

The identities of Eqs. (A-8) and (A-15) allow us to rewrite Eq. (A-13) as:

$$LHS = \bar{\phi}_a^T \Big[\bar{m}_b \bar{\phi}_a \ddot{\xi}_a + (\bar{k}_b + \bar{\hat{k}}_b T_{cb}) \bar{\phi}_a \xi_a + \bar{\hat{k}}_b \hat{\phi}_{nb} \xi_{nb} + T_{cb}^T \hat{f}_b \Big] \tag{A-16}$$

Finally, substituting Eq. (A-16) into Eq. (A-6), we obtain

$$\begin{aligned} M_a \ddot{\xi}_a + M_a \omega_a^2 \xi_a + \bar{\phi}_a^T \Big[\bar{m}_b \bar{\phi}_a \ddot{\xi}_a + (\bar{k}_b + \bar{\hat{k}}_b T_{cb}) \bar{\phi}_a \xi_a + \bar{\hat{k}}_b \phi_{nb} \xi_{nb} \Big] &= Q'_a \\ &= \hat{\phi}_a^T \hat{f}_a + \bar{\phi}_a^T \bar{f}_b \end{aligned} \tag{A-17}$$

Equations (A-11) and (A-17), when combined with appropriate initial conditions, are a complete mathematical statement of the problem. They are identical to Eqs. (15), (16) and (20), except for the fact that in the body of the report the forces acting on the substructure were assumed to be zero.

Free-Free Modes of Substructure. It can readily be shown that the equations of motion for the system which appear in Ref. [3], when free-free modes of the substructure are used, can be written as follows:

$$(M_a + \bar{\phi}_a^T \bar{m}_b \bar{\phi}_a)\ddot{\xi}_a + M_a \omega_a^2 \xi_a + \bar{\phi}_a^T(\bar{k}_b \bar{\phi}_a \xi_a + \bar{\hat{k}}_b \hat{\phi}_b \xi_b) = Q_a' \quad \text{(A-18)}$$

$$\hat{\phi}_b^T \hat{m}_b \hat{\phi}_b \ddot{\xi}_b + \hat{\phi}_b^T \hat{\bar{k}}_b \bar{\phi}_a \xi_a + \hat{\phi}_b^T \hat{k}_b \hat{\phi}_b \xi_b = Q_b \quad \text{(A-19)}$$

where the generalized forces

$$Q_a' = \hat{\phi}_a^T \hat{f}_a + \bar{\phi}_a^T \bar{f}_b$$
$$Q_b = \hat{\phi}_b^T \hat{f}_b \quad \text{(A-20)}$$

have been introduced. Noting that

$$\phi_b^T m_b \phi_b = \bar{\phi}_b^T \bar{m}_b \bar{\phi}_b + \hat{\phi}_b^T \hat{m}_b \hat{\phi}_b = M_b$$

$$\phi_b^T k_b \phi_b = \bar{\phi}_b^T(\bar{k}_b \bar{\phi}_b + \bar{\hat{k}}_b \hat{\phi}_b) + \hat{\phi}_b^T(\hat{\bar{k}}_b \bar{\phi}_b + \hat{k}_b \hat{\phi}_b) = M_b \omega_b^2 \quad \text{(A-21)}$$

and

$$\bar{\phi}_a \xi_a = \bar{\phi}_b \xi_b \quad \text{(A-22)}$$

we rewrite Eqs. (A-18) and (A-19) as:

$$M_a \ddot{\xi}_a + M_a \omega_a^2 \xi_a + \bar{\phi}_a^T \left[\bar{m}_b \bar{\phi}_b \ddot{\xi}_b + \bar{k}_b \bar{\phi}_a \xi_a + \bar{\hat{k}}_b \hat{\phi}_b \xi_b \right] = Q_a' \quad \text{(A-23)}$$

$$M_b \ddot{\xi}_b + M_b \omega_b^2 \xi_b - \bar{\phi}_b^T \left[\bar{m}_b \bar{\phi}_b \ddot{\xi}_b + \bar{k}_b \bar{\phi}_b \xi_b + \bar{\hat{k}}_b \hat{\phi}_b \xi_b \right] = Q_b \quad \text{(A-24)}$$

NONLINEAR MOUNTINGS

When linear elastic system components are attached to one another by nonlinear mountings, it is necessary to isolate those mountings from all of the components of the system. Thus, only the forces transmitted by them, which are, in general, functions of the relative displacements and velocities, enter into the analysis.

Fixed-Base (Constraint) Modes. The component displacements are expressed as

$$\begin{Bmatrix} \bar{q}_a \\ \hat{q}_a \\ \bar{q}_b \\ \hat{q}_b \end{Bmatrix} = T_5^c \begin{Bmatrix} \xi_a \\ \xi_{nb} \end{Bmatrix} - \Delta^c \qquad \text{(A-25)}$$

where

$$\Delta^c = \begin{Bmatrix} 0 \\ 0 \\ \delta \\ T_{cb}\delta \end{Bmatrix} \qquad \text{(A-26)}$$

and

$$\delta = \bar{q}_a - \bar{q}_b \qquad \text{(A-27)}$$

As a consequence of Eq. (A-25), the potential energy of components a and b can be written as

$$PE_{ab} = \frac{1}{2}\begin{bmatrix} q_a^T & q_b^T \end{bmatrix} \begin{bmatrix} k_a & 0 \\ 0 & k_b \end{bmatrix} \begin{Bmatrix} q_a \\ q_b \end{Bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} \xi_a^T & \xi_{nb}^T \end{bmatrix} K^c \begin{Bmatrix} \xi_a \\ \xi_{nb} \end{Bmatrix} + \frac{1}{2}\Delta^{cT} \begin{bmatrix} k_a & 0 \\ 0 & k_b \end{bmatrix} \Delta^c$$

$$-\Delta^{cT} \begin{bmatrix} k_a & 0 \\ 0 & k_b \end{bmatrix} T_5^c \begin{Bmatrix} \xi_a \\ \xi_{nb} \end{Bmatrix} \qquad \text{(A-28)}$$

Similarly, we can express the kinetic energy as

$$KE_{ab} = \frac{1}{2}\begin{bmatrix}\dot{\xi}_a^T & \dot{\xi}_{nb}^T\end{bmatrix} M^c \begin{Bmatrix}\dot{\xi}_a \\ \dot{\xi}_{nb}\end{Bmatrix} + \frac{1}{2}\dot{\Delta}^{cT}\begin{bmatrix} m_a & 0 \\ 0 & m_b\end{bmatrix}\dot{\Delta}^c - \dot{\Delta}^{cT}\begin{bmatrix} m_a & 0 \\ 0 & m_b\end{bmatrix} T_5^c \begin{Bmatrix}\dot{\xi}_a \\ \dot{\xi}_{nb}\end{Bmatrix} \tag{A-29}$$

For simplicity, let us assume that components a and b are attached by mountings whose properties are characterized by nonlinear elastic and linear viscous elements. We express the potential energy of the mountings as

$$(PE)_M = V_M(\delta) \tag{A-30}$$

and the viscous dissipation function as

$$D_M = D_M(\dot{\delta}) \tag{A-31}$$

Using the generalized form of Lagrange's equations, and noting that the generalized coordinates are ξ_a, ξ_{nb}, and δ, we readily obtain the following system of equations

$$M_a\ddot{\xi}_a + M_a\omega_a^2\xi_a + \bar{\phi}_a^T\left[\bar{m}_b(\bar{\phi}_a\ddot{\xi}_a - \ddot{\delta}) + (\bar{k}_b + \hat{\bar{k}}_b T_{cb})(\bar{\phi}_a\xi_a - \delta) + \hat{\bar{k}}_b\hat{\phi}_{nb}\xi_{nb}\right] = Q_a \tag{A-32}$$

$$M_{nb}\ddot{\xi}_{nb} + M_{nb}\omega_b^2\xi_{nb} + \hat{\phi}_{nb}^T\hat{m}_b T_{cb}(\bar{\phi}_a\ddot{\xi}_a - \ddot{\delta}) = Q_{nb} \tag{A-33}$$

$$\bar{m}_b(\bar{\phi}_a\ddot{\xi}_a - \ddot{\delta}) + (\bar{k}_b + \hat{\bar{k}}_b T_{cb})(\bar{\phi}_a\xi_a - \delta) + \hat{\bar{k}}_b\hat{\phi}_{nb}\xi_{nb} = \frac{\partial V_M}{\partial\delta} + \frac{\partial D_M}{\partial\dot{\delta}} - Q_\delta \tag{A-34}$$

where Q_a, Q_{nb} are defined in Eq. (A-4)

and

$$Q_\delta = -\bar{f}_b - T_{cb}\hat{f}_b \tag{A-35}$$

Note that Eq. (A-34) is an expression for the forces transmitted by the attachments. For a more general material we replace $(\frac{\partial V_M}{\partial \delta} + \frac{\partial D_M}{\partial \dot{\delta}})$ by $F(\delta, \dot{\delta})$, the forces exerted by the attachments on component b.

Free-Free Component Modes. When using the free-free component modes, we express the displacements as

$$\begin{Bmatrix} \bar{q}_a \\ \hat{q}_a \\ \bar{q}_b \\ \hat{q}_b \end{Bmatrix} \begin{bmatrix} \bar{\phi}_a & 0 \\ \hat{\phi}_a & 0 \\ \bar{\phi}_a & 0 \\ 0 & \hat{\phi}_b \end{bmatrix} \begin{Bmatrix} \xi_a \\ \xi_b \end{Bmatrix} \quad -\Delta \tag{A-36}$$

where

$$\Delta = \begin{Bmatrix} 0 \\ 0 \\ \delta \\ 0 \end{Bmatrix} \tag{A-37}$$

Following the same procedure as was previously described, we readily obtain the following set of equations

$$M_a\ddot{\xi}_a + M_a\omega_a^2\xi_a + \bar{\phi}_a^T\left[\bar{\bar{m}}_b\bar{\phi}_b\ddot{\xi}_b + (\bar{k}_b\bar{\phi}_b + \bar{\hat{k}}_b\hat{\phi}_b)\xi_b\right] = Q_a' \tag{A-38}$$

$$M_b\ddot{\xi}_b + M_b\omega_b^2\xi_b - \bar{\phi}_b^T\left[\bar{\bar{m}}_b\bar{\phi}_b\ddot{\xi}_b + (\bar{k}_b\bar{\phi}_b + \hat{\bar{k}}_b\hat{\phi}_b)\xi_b\right] = Q_b \tag{A-39}$$

$$\bar{\bar{m}}_b\bar{\phi}_b\ddot{\xi}_b + \bar{k}_b\bar{\phi}_b\xi_b + \bar{\hat{k}}_b\hat{\phi}_b\xi_b = F(\delta, \dot{\delta}) - Q_\delta \tag{A-40}$$

where

$$Q_a' = \hat{\phi}_a^T\hat{f}_a + \bar{\phi}_a^T\bar{f}_b$$

$$Q_b = \hat{\phi}_b^T\hat{f}_b \tag{A-41}$$

$$Q_\delta = -\bar{f}_b$$

Substituting Eq. (A-40) into Eqs. (A-38, 39) yields:

$$M_a \ddot{\xi}_a + M_a \omega_a^2 \xi_a + \bar{\phi}_a^T F(\delta, \dot{\delta}) = \bar{\phi}_a^T Q_\sigma + Q_a' = \hat{\phi}_a^T \hat{f}_a \quad \text{(A-42)}$$

$$M_b \ddot{\xi}_b + M_b \omega_b^2 \xi_b - \bar{\phi}_b^T F(\delta, \dot{\delta}) = \bar{\phi}_b^T \bar{f}_b + \hat{\phi}_b^T \hat{f}_b \quad \text{(A-43)}$$

where

$$\delta = \bar{\phi}_a \xi_a - \bar{\phi}_b \xi_b \quad \text{(A-44)}$$

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A general method is presented for obtaining the dynamic equations of an elastic structure to which elastic and/or nonlinear substructural elements are attached.

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