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ON THE NEGATIVE BINOMIAL CONVERGENCE IN A CLASS OF M-DIMENSIONA--ETC(U)
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REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: AFOSR-TR- 78-1232
2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle): ON THE NEGATIVE BINOMIAL CONVERGENCE IN A CLASS OF M-DIMENSIONAL SIMPLE EPIDEMICS
5. TYPE OF REPORT & PERIOD COVERED: Interim
6. PERFORMING ORG. REPORT NUMBER: FSU No. M464
7. AUTHOR(s): H. Lacayo and Naftali A. Langberg
8. CONTRACT OR GRANT NUMBER(s): AFOSR 76-3109
9. PERFORMING ORGANIZATION NAME AND ADDRESS: The Florida State University, Department of Statistics, Tallahassee, Florida 32306
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 61102F 2304/A5
11. CONTROLLING OFFICE NAME AND ADDRESS: Air Force Office of Scientific Research/NM, Bolling AFB, Washington, DC 20332
12. REPORT DATE: July 1978
13. NUMBER OF PAGES: 10
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)
15. SECURITY CLASS. (of this report): UNCLASSIFIED
15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report): Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)
18. SUPPLEMENTARY NOTES
19. KEY WORDS (Continue on reverse side if necessary and identify by block number): Exponential distributions, convergence in law, stochastic processes, m-dimensional simple epidemics, negative binomial distributions.
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We consider a population which is exposed to m infections, and consist initially of N susceptibles. At each point in time at most one susceptible becomes infective, and only from one cause. This m-dimensional simple epidemic is a stochastic process, $(X_{N,1}(t), \ldots, S_{N,m}(t)$, with components counting the number of infectives from the respective causes at time t. We show that if the transition rates of cause 1 through m at time t are given by

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20. Abstract

$\alpha_i X_{N,i}(t)[1 - \frac{1}{N} \sum_{i=1}^{m} (X_{N,i}(t) - X_{N,i}(0))]$, $i = 1, \ldots, m$, $\alpha_1, \ldots, \alpha_m > 0$ and if $\lim_{N\to\infty} X_{N,i}(0) = b_i \in \{1, 2, \ldots\}$, then $(X_{N,1}(t), \ldots, X_{N,m}(t))$ converges as $N \to \infty$ to a random vector with independent negative binomial components.

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by (9) Interim technical rept.,

(10) H. Lacayo ~~and~~ Naftali A. Langberg

(11) Jul 78

FSU Statistics Report M464
AFOSR Technical Report No. 84
AFOSRN Technical Report No. 9

(12) 16p.

July, 1978
The Florida State University
Department of Statistics
Tallahassee, Florida 32306

(14) FSU-STATISTICS-M464, TR-84-AFOSR

Research sponsored by the Air Force Office of Scientific Research, AFSC, USAF, under Grants AFOSR 74-2581D and AFOSR 76-3109.

(15) ✓AFOSR-74-2581, ✓AFOSR-76-3109

(16) 2304

(17) A5

(18) AFOSR

(19) TR-78-1232

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On the Negative Binomial Convergence
in a Class of m-Dimensional Simple Epidemics

by

H. Lacayo and Naftali A. Langberg

ABSTRACT

We consider a population which is exposed to m infections, and consist initially of N susceptibles. At each point in time at most one susceptible becomes infective, and only from one cause. This m-dimensional simple epidemic is a stochastic process, $(X_{N,1}(t), \ldots, X_{N,m}(t))$, with components counting the number of infectives from the respective causes at time t. We show that if the transition rates of cause 1 through m at time t are given by $\alpha_i X_{N,i}(t)[1 - \frac{1}{N}\sum_{i=1}^{m}(X_{N,i}(t) - X_{N,i}(0))]$ $i = 1, \ldots, m$, $\alpha_1, \ldots, \alpha_m > 0$ and if $\lim_{N\to\infty} X_{N,i}(0) = b_i \in \{1, 2, \ldots\}$, then $(X_{N,1}(t), \ldots, X_{N,m}(t))$ converges as $N \to \infty$ to a random vector with independent negative binomial components.

Key Words: Exponential distributions, convergence in law, stochastic processes, m-dimensional simple epidemics, negative binomial distributions.

1. Introduction and Summary

Introduction.

We consider a population which is exposed to m infections and consists initially of N susceptibles and $b_{N,i}$ infectives from cause i, i = 1, ..., m. At each point in time at most one susceptible becomes an infective, and only from one cause. Once an individual enters the infective state he remains there and cannot be a carrier of any other infection. This m-dimensional simple epidemic is a stochastic process, $(X_{N,1}(t), \ldots, X_{N,m}(t))$, with components counting the number of infectives from the respective causes at time t. We show that if the transition rates of cause 1 through m at time t are given by:

$$\alpha_i X_{N,i}(t)[1 - \frac{1}{N}\sum_{i=1}^{m}(X_{N,i}(t) - b_{N,i})], \quad i = 1, \ldots, m, \quad \alpha_1, \ldots, \alpha_m > 0, \tag{1.1}$$

and if

$$\lim_{N\to\infty} b_{N,i} = b_i \in \{1, 2, \ldots\}, \quad i = 1, \ldots, m \tag{1.2}$$

then for every positive real numbers t and β, and m nonnegative integers $k_1, \ldots, k_m$,

$$\lim_{N\to\infty} P\{X_{N,i}(t) \geq k_i + b_i, \ i = 1, \ldots, m\} = \prod_{i=1}^{m} P\{X_i(t) \geq k_i + b_i\} \tag{1.3}$$

and

$$\lim_{N\to\infty} E(X_{N,i}(t))^{\beta} = E(X_i(t))^{\beta}, \quad i = 1, \ldots, m, \tag{1.4}$$

where $X_1(t), \ldots, X_m(t)$ are negative binomial random variables with respective parameters b_i and $e^{-\alpha_i t}$, $i = 1, \ldots, m$. The only previous work in the area of m-dimensional simple epidemics is the one by Billard, Lacayo and Langberg (1978), who proved (1.3) and (1.4) with the additional assumption that $\alpha_1 = \alpha_2 \cdots = \alpha_m$. In that case the interinfection random times are independent of the infection causes. Generally, as will be pointed out in Section 2 this property does not hold. The proof of (1.3) in the cited reference depends on the special structure of the interinfection times. To obtain (1.3) and (1.4) we have approached the problem from an alternate viewpoint.

Summary.

In Section 2 we present a rigorous definition of an m-dimensional simple epidemic, and describe the ones that are the subject of our analysis. Statements (1.3) and (1.4) are proven in Section 3. In this section we present an m-dimensional simple epidemic in a population consisting initially of infinitely many susceptibles. This theoretical simple epidemic is instrumental in the proofs of (1.3) and (1.4).

2. The m-dimensional Simple Epidemic

An m-dimensional simple epidemic in a population consisting initially of N susceptibles and $b_{N,i}$ infectives from cause i, i = 1, ..., m can be described by N bivariate random vectors $(T_{N,1}, \xi_{N,1}), \ldots, (T_{N,N}, \xi_{N,N})$. $T_{N,1}, \ldots, T_{N,N}$, are the random interinfection times, and $\xi_{N,1}, \ldots, \xi_{N,N}$ are discrete random variables with values in {1, ..., m} designating the respective infection causes. Let $S_{N,k}$ be equal to $\sum_{j=1}^{k} T_{N,j}$, k = 1, ..., N. The finite dimensional joint distributions of $(X_{N,1}(t), \ldots, X_{N,m}(t))$ are determined by $(T_{N,1}, \xi_{N,1}), \ldots, (T_{N,N}, \xi_{N,N})$, through the following set equalities:

$$X_{N,i}(t) = b_{N,i} + k) = \bigcup_{r=k}^{N} (S_{N,r} \leq t < S_{N,r+1}, \sum_{q=1}^{r} I(\xi_{N,q} = 1) = k), \; t > 0 \tag{2.1}$$

k = 0, ..., N, and i = 1, ..., m. $(S_{N,0} = 0,\ S_{N,N+1} = \infty)$.

We assume throughout that $(T_{N,1}, \xi_{N,1}), \ldots (T_{N,N}, \xi_{N,N})$, are determined by (2.2) and (2.3).

$$P\{\xi_{N,j} = \ell_j,\ j = 1, \ldots, N\} = \left[\prod_{i=1}^{m} \alpha_i^{r_i} \prod_{j=0}^{r_i - 1} (b_{N,i} + j)\right]\left[\prod_{k=1}^{N} \left(\sum_{i=1}^{m} \alpha_i (b_{N,i} + \sum_{q=1}^{k-1} I(\ell_q = i))\right)\right]^{-1}, \tag{2.2}$$

where $r_i = \sum_{q=1}^{N} I(\ell_q = i)$, i = 1, ..., m.

$$P\{T_{N,k} | \xi_{N,1}, \ldots, \xi_{N,k-1} > t_k,\ k = 1, \ldots, N\} = e^{-\sum_{j=1}^{N} \mu_j t_j} \tag{2.3}$$

where $t_1, \ldots, t_n$ are positive real numbers, and μ_j is equal to $\frac{N - j + 1}{N} \sum_{i=1}^{m} \alpha_i (b_i + \sum_{q=1}^{j-1} I(\xi_q = i))$, for j = 1, ..., N.

In the particular case discussed in Billard, Lacayo and Langberg (1978), condition (2.3) reduces to <u>independent</u> exponential interinfection times with rates equal to $\alpha\frac{N - k + 1}{N}(\sum_{i=1}^{m} b_{N,i} + k - 1)$, $k = 1, \ldots, N$, <u>independent</u> of $\xi_{N,1}, \ldots, \xi_{N,N}$. In addition equation (2.2) simplifies to $[\prod_{i=1}^{m} \prod_{j=0}^{r_i - 1} (b_{N,i} + j)][\prod_{k=1}^{N} (\sum_{i=1}^{m} b_{N,i} + k - 1)]^{-1}$. Finally we note that if (2.2) and (2.3) hold, then the m-dimensional simple epidemic satisfies (1.1). Conversly, if (1.1) is satisfied, then equations (2.2) and (2.3) follow by the inherited Markovian structure of the m-dimensional simple epidemic.

3. Main Results

Let $(X_1(t), \ldots, X_m(t))$ be an m-dimensional simple epidemic in a population consisting initially of b_i infectives from cause i, $i = 1, 2, \ldots, m$, and infinitely many susceptibles. This epidemic can be described by a sequence of bivariate random vectors, (T_1, ξ_1), (T_2, ξ_2). . . ., 2, ... $T_1, T_2, \ldots$, are the interinfection times, and $\xi_1, \xi_2, \ldots$, are random variables with values in $\{1, \ldots, m\}$, which designate the respective infection causes. Let S_k be equal to $\sum_{j=1}^{k} T_j$, $k = 1, 2, \ldots$. The finite dimensional joint distributions of $(X_1(t), X_m(t))$ are determined by (T_k, ξ_k), $k = 1, 2, \ldots$, through the following set equalities:

$$(X_i(t) = b_i + k) = \bigcup_{r=k}^{\infty} (S_r \le t < S_{r+1}, \sum_{q=1}^{r} I(\xi_q = i) = k) \tag{3.1}$$

for every positive real number t, positive integer k, and $i = 1, \ldots, m$. We assume throughout that the sequence (T_1, ξ_1), (T_2, ξ_2), ..., is determined by (3.2) and (3.3).

$$P\{\xi_k = \ell_k, k = 1, \ldots, n\} =$$

$$[\prod_{i=1}^{m} \prod_{j=0}^{r_i - 1} (b_i + j)][\prod_{j=1}^{n} (\sum_{i=1}^{m} \alpha_i (b_i + \sum_{q=1}^{j-1} I(\ell_q = 1))]^{-1} \tag{3.2}$$

where $r_i = \sum_{q=1}^{n} I(\ell_q = i)$, $i = 1, \ldots, m$, and $n = 1, 2, \ldots$.

$$P\{T_k | \xi_1, \ldots, \xi_{k-1} > t_k, k = 1, \ldots, n\} = e^{-\sum_{j=1}^{n} \mu_j t_j} \tag{3.3}$$

where $t_1, \ldots, t_n$ are positive real numbers, $\mu_j = \sum_{i=1}^{m} \alpha_i (b_i + \sum_{q=1}^{j-1} I(\xi_q = 1))$ $j = 1, \ldots, n$, and $n = 1, 2, \ldots$.

We will show that for every two positive real numbers t and β $(X_{N,1}(t), \ldots, X_{N,m}(t))$ converges in law to $(X_1(t), \ldots, X_m(t))$ as $N \to \infty$, and that $\lim_{N\to\infty} E(X_{N,i}(t))^{\beta} = E(X_i(t))^{\beta}$ for $i = 1, \ldots, m$. First we introduce two lemmas.

Lemma 3.1. Let $U_1, U_2, \ldots,$ be an i.i.d. sequence of exponential random variables with mean 1. Further let $(\tau_{b+r-1,1}, \ldots, \tau_{b+r-1,b+r-1})$ be the order statistic of a sample of size $b + r - 1$ from U_1. Then $\sum_{j=1}^{r} (b + j - 1)^{-1} U_j$ and $\tau_{b+r-1,r}$ are identically distributed.

Proof. It suffices to note that the spacings; $\tau_{r+b-1,r-j+1} - \tau_{r+j-1,r-j}$, $j = 1, \ldots, r$, $(\tau_{r+b-1,0} = 0)$ are independent exponential random variables with means equal respectively to $(b + j - 1)^{-1}$, $j = 1, \ldots, r$.

Lemma 3.2. Let t and β be positive real numbers; let $c = \sup_N \sum_{i=1}^{m} b_{N,i}$ and $\alpha = \max_{1\le i\le m} \alpha_i$. If (1.2) holds then; (i) $P(S_{N,r} \le t) \le (1 - e^{-\alpha ct})^r$ for $r = 1, \ldots, N$, and (ii) $\sup_N E(X_{N,i}(t))^{\beta} < \infty$, for $i = 1, \ldots, m$.

Proof. We note that (ii) is a corollary of (i). To prove (i) let $U_1, U_2, \ldots,$ be as in Lemma 3.1. Since $P\{S_{N,k} \le t\} \le P\{\sum_{j=1}^{r} j^{-1} U_j \le \alpha ct\}$, (i) follows from Lemma 3.1.

Theorem 3.3. Let β and t be positive real numbers. If (1.2) holds then; (i) $(X_{N,1}(t), \ldots, X_{N,m}(t))$ converges in law as $N \to \infty$ to $(X_1(t), \ldots, X_m(t))$, and (ii) $\lim_{N\to\infty} E(X_{N,i}(t))^{\beta} = E(X_i(t))^{\beta}$ for $i = 1, \ldots, m$.

Proof. We note that (ii) is a corollary of (i) and Lemma 3.2 (ii). To prove (i) let $k_1, \ldots, k_m$ be m nonnegative integers, A be the set $\{(r_1, \ldots, r_m) | r_i \geq k_i, i = 1, \ldots, m\}$ and let $R = \sum_{i=1}^{m} r_i$. Since for N sufficiently large, $P\{X_{N,i}(t) \geq k_i + b_i, i = 1, \ldots, m\} =$ $\sum_A P\{S_{N,R} \leq t < S_{N,R+1}, \sum_{q=1}^{R} I(\xi_{N,q} = i) = r_i, i = 1, \ldots, m) I(R \leq N)\}$, (i) follows from Lemma (3.2) (i), the dominated convergence theorem, and some simple calculations.

For reference purposes the following two well known results are presented.

Lemma 3.4. Let $U_1, U_2, \ldots,$ be as in Lemma 3.1, and let $\mu_1, \mu_2, \ldots,$ be a sequence of positive real numbers. Further we denote by f_k the density function of $\sum_{j=1}^{k} \mu_j^{-1} U_j$, k = 1, 2, ... Then, (i) $\mu_{r+1}^{-1} f_{r+1}(t) =$ $P\{\sum_{j=1}^{r} \mu_j^{-1} U_j \leq t < \sum_{j=1}^{r+1} \mu_j^{-1} U_j\}$, and (ii) $f_r(t) = (\prod_{j=1}^{r} \mu_j) \sum_{j=1}^{r} e^{-\mu_j t} \prod_{\substack{i=1 \\ i \neq j}}^{r} (\mu_i - \mu_j)^{-1}$, provided $\mu_1, \ldots, \mu_r$ are distinct.

We are ready to state and prove the main result of the paper.

Theorem 3.5. Let t and β be positive real numbers, and $k_1, \ldots, k_m$ be nonnegative integers. Further let A be the set $\{r_1, \ldots, r_m | r_i \geq k_i,$ $i = 1, \ldots, m.\}$. If conditions (1.1) and (1.2) are satisfied, then the following hold.

(i) $\lim_{N \to \infty} P\{X_{N,i}(t) \geq k_i + b_i, i = 1, \ldots, m\} = \sum_A \prod_{i=1}^{m} \binom{b_i + r_i - 1}{b_i - 1} e^{-\alpha_i b_i t} (1 - e^{-\alpha_i t})^{k_i}$.

(ii) $\lim_{N \to \infty} E(X_{N,i}(t))^{\beta}$ is the βth moment of a negative binomial random variable with parameters b_i and $e^{-\alpha_i t}$, i = 1, ..., m.

<u>Proof</u>. By Theorem 3.3 it suffices to show that $X_1(t), \ldots, X_m(t)$ are independent, and have the respective negative binomial marginal distributions. Let $k_1, \ldots, k_m$ be nonnegative integers, with sum equal to k. Further, let B denote the set

$$[(\ell_1, \ldots, \ell_k) | \ell_1, \ldots, \ell_k \in \{1, \ldots, m\}, \sum_{q=1}^{k} I(\ell_q = i) = k_i, i = 1, \ldots, m].$$

We note that $P\{X_i(t) = k_i + b_i, i = 1, \ldots, m\}$ equals

$$\sum_B P\{S_k \le t < S_{k+1} | \xi_q = \ell_q, q = 1, \ldots, k\} P\{\xi_q = \ell_q, q = 1, \ldots, k\}.$$

From Lemma 3.4 and equation (3.2) it follows that $P\{X_i(t) = k_i + b_i, i = 1, \ldots, m\}$ equals

$$\prod_{i=1}^{m} \binom{b_i + k_i - 1}{b_i - 1} e^{-\alpha_i b_i t} \alpha_i^{k_i} k_i! \sum_B \sum_{j=1}^{k+1} e^{-\theta_j t} \prod_{\substack{i=1 \\ i \ne j}}^{k+1} (\theta_i - \theta_j)^{-1}, \text{ where}$$

$$\theta_j = \sum_{q=1}^{j-1} \sum_{i=1}^{m} \alpha_i I(\ell_q = i), \; j = 1, \ldots, k+1.$$

By reusing Lemma 3.4 we obtain that $P\{X_i(t) = k_i + b_i, i = 1, \ldots, k\}$ equals

$$\prod_{i=1}^{m} \binom{b_i + k_i - 1}{b_i - 1} e^{-\alpha_i b_i t} \alpha_i^{k_i} k_i! \sum_B P\{\sum_{j=1}^{k+1} \theta_j^{-1} U_j \le t\} \prod_{j=2}^{k+1} \theta_j^{-1}.$$

To complete the proof it suffices to show that

$$\prod_{i=1}^{m} \alpha_i^{k_i} k_i! \sum_B P\{\sum_{j=2}^{k+1} \theta_j^{-1} U_j \le t\} \prod_{j=2}^{k+1} \theta_j^{-1} = \prod_{i=1}^{m} (1 - e^{-\alpha_i t})^{k_i}. \quad (3.4)$$

Let F denote the left side of equation (3.4). Further let

$$B_i = [\{i, \ell_2, \ldots, \ell_q\} | \ell_2, \ldots, \ell_k \in \{1, \ldots, m\}, \sum_{q=2}^{k} I(\ell_q = r) = k_r, r = 1, \ldots, m, r \ne i, \sum_{q=2}^{k} I(\ell_q = i) = k_i - 1].$$

We observe that F(t) equals $\prod_{i=1}^{m} \alpha_i^{k_i} k_i! \sum_{i=1}^{m} \sum_{B_i} P\{\sum_{j=2}^{k+1} \theta_j^{-1} U_j \le t\} \prod_{j=2}^{k+1} \theta_j^{-1}$.

By a simple calculation and Lemma 3.4, it follows that

$$\frac{dF}{dt} = \prod_{i=1}^{m} \alpha_i^{k_i} k_i! \sum_{i=1}^{m} e^{-\alpha_i t} \sum_{B_i} P\{\sum_{j=3}^{k+1} \underline{\theta}_{j,i}^{-1} U_j \le t\} \prod_{j=3}^{k+1} \underline{\theta}_{j,i}^{-1}, \text{ where } \underline{\theta}_{j,i} =$$

$\sum_{\substack{r=1 \\ r\neq i}}^{m} \alpha_r \sum_{q=2}^{j-1} I(\ell_q = r)$, i = 1, ..., m, j = 3, ..., k + 1. Consequently Equation (3.4) is established by an induction argument on k applied to each of the sums $\sum_{B_i} P\{\sum_{j=3}^{k+2} \underline{\theta}_{j,i} U_j \le t\} \prod_{j=3}^{k+1} \underline{\theta}_{j,i}^{-1}$, i = 1, ..., m.

Corollary 3.6. Let $X_N(t)$ the total number of infectives at time t. If (1.1) and (1.2) are satisfied, then the following hold.

(i) $X_N(t)$ converges in law as $N \to \infty$ to a sum of m independent negative binomial random variables with respective parameters b_i, $e^{-\alpha_i t}$, i = 1, ..., m, and

(ii) $\lim_{N\to\infty} E(X_N(t))^\beta$ equals to the βth moment of the limiting random variable given in (i).

In the particular case discussed in Billard, Lacayo and Langberg (1978), $X_N(t)$ converges in law to a negative binomial random variable with parameters $\sum_{i=1}^{m} b_i$ and $e^{-\alpha t}$. This fact played a major role in the proof of (1.3) for the symmetric case as presented in the cited reference. Finally we note that it is of interest to investigate the asymptotic behavior of m-dimensional simple epidemics, that do not have the Markovian structure.

REFERENCE

(1) Billard, L., Lacayo, H. and Langberg, Naftali A. (1978). The Symmetric m-dimensional Simple Epidemic. FSU Statistics Report M461.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1. REPORT NUMBER FSU No. M464 AFOSR No. 84 AFOSRN No. 9	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE On the Negative Binomial Convergence in a Class of M-Dimensional Simple Epidemics		5. TYPE OF REPORT & PERIOD COVERED Technical Report 6. PERFORMING ORG. REPORT NUMBER FSU Statistics Report M464
7. AUTHOR(s) H. Lacayo Naftali A. Langberg		8. CONTRACT OR GRANT NUMBER(s) AFOSR 74-2581D AFOSR 76-3109
9. PERFORMING ORGANIZATION NAME & ADDRESS The Florida State University Department of Statistics Tallahassee, Florida 32306		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME & ADDRESS The U.S. Air Force Air Force Office of Scientific Research Bolling Air Force Base, D.C. 20332		12. REPORT DATE July, 1978 13. NUMBER OF PAGES 10
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS (of this report) Unclassified 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from report)

18. SUPPLEMENTARY NOTES

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20. ABSTRACT

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$\alpha_i X_{N,i}(t)[1 - \frac{1}{N}\sum_{i=1}^{m}(X_{N,i}(t) - X_{N,i}(0))]$, $i = 1, \ldots, m$, $\alpha_1, \ldots, \alpha_m > 0$ and if $\lim_{N\to\infty} X_{N,i}(0) = b_i \in \{1, 2, \ldots\}$, then $(X_{N,1}(t), \ldots, X_{N,m}(t))$ converges as $N \to \infty$ to a random vector with independent negative binomial components.