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INTERACTION OF A LOW DENSITY RUNAWAY BEAM WITH CAVITY MODES. (U)
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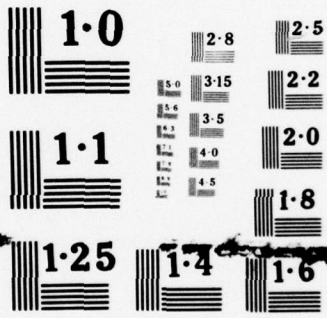


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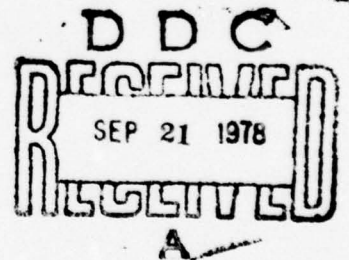
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INTERACTION OF A LOW DENSITY RUNAWAY BEAM
WITH CAVITY MODES

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June, 1978



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ABSTRACT

An idealized problem is investigated which illustrates the role of wave-particle interactions in the evolution of runaway beams. The model considers the interaction between a weak cold beam, driven by an external static electric field E , and waves quantized by the geometry. The waves may correspond to Gould-Trivelpiece modes fixed by the length of the experimental device, or to the finite Fourier modes encountered in computer simulations. The physics consists of the sweeping of the accelerated beam through the resonance provided by each cavity mode. This process is formulated in analogy with the O'Neil, Winfrey, Malmberg (OWM) problem but uses a spatially averaged description based on the exact energy and momentum conservation laws with the dynamics simplified through a WKB representation of the dispersion relation. This model shows that the beam can be clamped in velocity with the momentum push being transferred to the waves. The model has been extended to the relativistic and multi-mode cases. For $E = 0$ the nonrelativistic results are in good agreement with the work of (OWM), while in the relativistic case the model reproduces the nontrivial features of the computer study by Lampe and Sprangle.

I. INTRODUCTION

One of the more striking properties of hot tenuous plasmas is the virtual disappearance of collisional effects. A well known consequence of this behavior is the creation of runaway electrons¹ when a sufficiently strong external DC electric field is applied to a plasma. Although various aspects of this process have been extensively investigated, there has been a recent resurgence²⁻⁴ of interest in this topic. The reason for the renewed interest can be attributed to the successful operation of toroidal confinement devices capable of entering the hot plasma regime while providing sufficiently long confinement times so that a runaway population can be established.

The study of the properties of electron runaways is of interest because it provides an ideal arena for testing and developing various theoretical descriptions of fundamental wave-particle interactions which can have important consequences beyond the narrower realm of the runaway problem. On the more practical side, a better understanding of the behavior of runaways may be used as a diagnostic tool for uncovering internal phenomena (e.g., magnetic perturbations, transport, instabilities) not directly accessible to the standard diagnostic methods. Also, it may be possible that in certain parameter regimes the large streaming energy of the runaway electrons can be used to generate powerful microwave radiation, or even to achieve the direct heating of the background ions.

The motivation for the present study has been provided by the recent experimental observations⁵ made in the Microtor tokamak at the University of California, Los Angeles. By monitoring the time evolution and energy of the hard x-rays (>50 KeV) emitted from this device one deduces the maximum energy of the runaway population. At intermediate values of the plasma density ($n_e \sim 2 \times 10^{12} - 10^{13} \text{ cm}^{-3}$) it is found that the maximum energy of the runaways increases monotonically in time during the discharge, eventually

reaching levels as large as 2-3 MeV. However, in the low density regime ($n_e < 10^{12} \text{ cm}^{-3}$) it is found that the maximum runaway energy attained holds constant in time (5-10 msec) during a significant fraction of the discharge lifetime (20-30 msec). The peak energy achieved by these clamped runaways is of the order of 300-400 KeV.

Simultaneously with the clamping of the runaways it is observed that electromagnetic radiation in the neighborhood of the ion plasma frequency, ω_{pi} , is generated. Spontaneous emissions near the electron cyclotron frequency, Ω_e , and electron plasma frequency, ω_{pe} , are also detected⁶ and are present in both density regimes, i.e., the high frequency emission is triggered whether or not the runaways are clamped in energy. However, the lower frequency radiation is only present when the clamping process is at work, and the peak of this emission occurs at a frequency which scales proportionally to $\sqrt{n_e}$.

Evidently, such an interesting behavior in the complicated environment of a tokamak plasma can stimulate a wide variety of explanations. In particular, a simple physical picture suggested by these observations and whose interest goes far beyond the previously mentioned experiment is the following: a high energy electron is pushed directly by the external DC electric field, however, if the electron can resonate with a mode supported by the system then a significant part of the push may be transformed to the form of wave momentum, thus resulting in the clamping of the particle. This type of phenomenon is the subject of the present study.

Specifically, this study is concerned with the evolution of a low density beam which is continuously accelerated by an external DC electric field. The beam is envisioned to meet the criteria for the weak cold beam description^{7,8} used extensively in investigations of the beam-plasma interaction. The beam

is assumed to propagate along a strongly magnetized plasma capable of supporting modes whose wavenumbers are quantized by the geometry. The consideration of such modes is relevant to the low frequency ($\omega \ll \omega_{pe}$) oscillations in small toroidal machines in which an integral number of wavelengths fit around the torus. A similar situation is also encountered in computer simulations in which the periodicity length introduces the quantization of the Fourier modes.

The mathematical formulation used in the present study does not follow the detailed dynamics of the beam particles, which would require a computer simulation. The method used here relies on the exact conservation equations for the total energy and total momentum of the beam-wave system. To bypass the complexity associated with the simulation of beam dynamics, a WKB model is introduced. The foundations of this model can be traced to two previous unconnected investigations whose essence is briefly described in the following.

Several years ago⁹ O'Neil and Malmberg studied the topological transition in the linear dispersion relation for the cold and warm beam-plasma interaction. In analyzing the transition from a cold to a warm beam, it proved useful to consider a two parameter Lorentzian to represent the beam distribution function. One parameter describes the average velocity of the beam, while the other represents its thermal spread. The results obtained with this parametrization of the problem illustrated clearly the continuous transition from a hydrodynamic instability to a kinetic Landau-type of interaction.

The other investigation which has a bearing on the foundations of the WKB model used here concerns a laboratory experiment¹⁰ by Starke and Malmberg. This experiment has looked in detail at the connection between the well-known trapped particle sidebands excited by a launched large amplitude wave and the distortions in the time averaged velocity distribution function. The remarkable outcome of this experiment is that the behavior of the spontaneously excited

sidebands can be explained quantitatively by using the time averaged distribution to obtain a local linear dispersion relation.

The clues that have been extracted from the two previously described results to solve the present problem are: (1) the thermal spreading of the runaway beam which occurs as the beam encounters a cavity resonance can be reasonably described through a two parameter Lorentzian; the two parameters being allowed to change self-consistently with the conservation laws, (2) to bypass the calculation of the dynamics of the beam particles, one uses the slowly changing Lorentzian into the linear dispersion relation to obtain the time evolution of the wave energy and momentum.

To check the validity of the previously described formulation we have tested its predictions against the detailed particle simulation of the weak cold beam problem by O'Neil, et al.⁸ It is found that the spatially averaged technique¹¹ introduced in this work predicts the correct saturation time; the predicted saturation level does not exhibit the trapped particle oscillations (as expected) but rather it yields an average value which ranges between the peaks and the valleys of such oscillations.

When the effect of the external DC field is retained, one finds that for certain parameter values (e.g., the amplitude of the initial noise is small) the runaway beam can sweep through a cavity resonance and experiences only a slight drag and a correspondingly small thermal broadening. However, as the level of the initial noise is increased one finds that the beam momentum can be clamped. In the clamped state the beam momentum increases slowly while the amplitude of the cavity mode grows secularly in time. In this regime the external push by the DC field is converted into wave momentum (DC to AC conversion).

The spatially averaged formulation has been extended to include the many mode problem as well as the relativistic runaway case. The clamping behavior is also observed when these effects are present. The predictions of the relativistic formulation with the DC field set to zero have been compared with the computer calculations¹² of Lampe and Sprangle. It is found that one recovers the known but nontrivial decrease of the saturation level as the parameter s , first introduced¹³ by Thode and Sudan, is increased. In addition, one finds that the actual dependence on s is in excellent agreement with the results of Lampe and Sprangle.

The investigation of the restricted runaway beam interaction with two modes exhibits a wealth of structure that uncovers many features which have been previously observed in the laboratory^{10,14,15} in the study of trapped particle sidebands. Many of the features can be understood in terms of wave-particle-wave interactions in which the beam plays the role of an intermediate agent.

The manuscript is organized as follows. In Sec. II the problem is defined and the spatially averaged formulation is introduced. Section III discusses the application of the formulation to the weak cold beam problem. Section IV presents the nonrelativistic results and introduces the clamping behavior. Section V considers the relativistic formulation. Section VI presents a perturbation theory of the effects produced by cavity damping. Conclusions are presented in Sec. VII.

II. FORMULATION

The present idealized physical model consists of a low density beam (initially cold) which is pushed by an external DC electric field E_0 that points along a strong magnetic field B_0 . The plasma is taken to be a cylinder of radius a , surrounded by a perfectly conducting wall. For simplicity the plasma density n_0 is taken to be radially and axially uniform, and the plasma cylinder is envisioned to be axially periodic, with periodicity length L . The strong guide magnetic field points along the axis of the plasma cylinder (z axis).

In the strong magnetic field limit ($B_0 \rightarrow \infty$), as has been illustrated¹⁶ by Trivelpiece and Gould, the dispersion relation for the electromagnetic modes (E modes) can be solved analytically and is obtained from

$$\nabla_1^2 E_z + (k_0^2 - k^2) \epsilon_{zz} E_z = 0 \quad (1)$$

where ∇_1^2 refers to the perpendicular Laplacian operator, $k_0 = \omega/c$ corresponds to the vacuum wavenumber of a signal having frequency ω , and c is the speed of light. The axial component of the dielectric tensor is ϵ_{zz} .

The solution of Eq.(1) takes the form

$$E_z(z, r, \theta, t) = A J_n(k_1 r) \exp\{i(kz + n\theta - \omega t)\} \quad (2)$$

in which r, θ represent the radial and angular coordinates, t is the time variable, k is the axial wavenumber, A is the amplitude, J_n is the Bessel function of order n , and n is an integer.

The finite size constraints of the model require that

$$k = j \frac{2\pi}{L}, \quad k_1 a = P_{nv} \quad (3)$$

where j is an integer and P_{nv} refers to the zeroes of J_n . Utilizing these constraints one arrives at the following linear dispersion relation

$$\epsilon_{zz} = \frac{(P_{nv})^2}{(k_0 a)^2 - (ka)^2} \quad (4)$$

It is well known that in the absence of the beam and for a cold background plasma, Eq.(4) exhibits two branches: (1) the fast branch consists of waves whose phase velocities are larger than c and exists for frequencies $\omega > \omega_{pe}$ (the electron plasma frequency), (2) the slow branch has phase velocities less than c and exists in the frequency interval $\omega < \omega_{pe}$. It is this latter branch which is of interest to the present problem because it is possible for the velocity of the beam to coincide with the phase velocity of these modes, thus resulting in a strong resonant interaction.

The contribution of the low density beam to the finite size dispersion relation enters through the beam susceptibility χ_b , where

$$\epsilon_{zz} = \epsilon_{zz}^0 + \chi_b \quad (5)$$

and in which ϵ_{zz}^0 represents the contribution of the background plasma.

Defining the generalized finite size dielectric of the system ϵ by

$$\epsilon = \epsilon_{zz}^0 - \frac{(P_{nv})^2}{(k_0 a)^2 - (ka)^2} \quad (6)$$

results in the relation

$$\epsilon(k, \omega) = -\chi_b \quad (7)$$

which for a low density beam can be treated in analogy with the small cold beam problem, i.e., in zeroth order $\epsilon(k, \omega_0) \approx 0$, hence the perturbation produced by the beam can be described through $\omega = \omega_0 + \delta\omega$, $|\delta\omega/\omega_0| \ll 1$, and $\delta\omega$ is determined from

$$\left(\frac{\partial \epsilon}{\partial \omega}\right)_0 \delta \omega = -X_b \quad (8)$$

with the understanding that X_b depends functionally on $\delta \omega$. Once the dependence of X_b on $\delta \omega$ is established, Eq.(8) can be used to obtain $\delta \omega$, whose imaginary part determines the growth or damping of the cavity mode supported by the background plasma. In the present work Eq.(8) is used in its generalized WKB sense, i.e., one calculates the slow time evolution of $X_b = X_b(t, \delta \omega)$.

In general to find X_b one needs to follow the detailed dynamics of the beam electrons. To extend such an exercise beyond the linear regime one typically resorts to a digital computer in order to calculate the complicated trapped particle orbits. However, in the present study we wish to avoid such a procedure by relying on a spatially averaged description based on the exact conservation laws for the beam-wave system.

The derivation and meaning of the conservation laws are easier to understand if one represents the beam in terms of N discrete particles. Each particle is labeled by the index ℓ , $1 \leq \ell \leq N$, and in the nonrelativistic limit their motion is determined by

$$m \frac{d}{dt} (v_{z\ell}) = eE_0 - e \{ E(t) J_n(k_\perp r_\ell) \exp[i(kz_\ell + n\theta_\ell - \omega_0 t)] + c.c. \} \quad (9)$$

in which e , m refer to the electron charge and mass, $v_{z\ell}$ is the axial component of the velocity of particle ℓ with coordinates $(z_\ell, \theta_\ell, r_\ell)$, and

$$E(t) = E_z(0) \exp \left[-i \int_0^t dt' \delta \omega(t') \right] \quad (10)$$

with $\delta \omega$ complex.

After Fourier analyzing in time, Maxwell's equations take the form

$$\nabla \times \nabla \times \underline{E} = i \frac{4\pi\omega}{c} \underline{j} + \left(\frac{\omega}{c}\right)^2 \underline{E} \quad (11)$$

where \underline{j} refers to the oscillating plasma current, which in the strong B_0 limit points essentially in the z direction. Accordingly,

$$(\nabla \times \nabla \times \underline{E})_z = -\nabla_1^2 E_z + \partial_z (\nabla_1 \cdot \underline{E}_1) \quad (12)$$

and

$$\partial_z E_z + \nabla_1 \cdot \underline{E}_1 = 4\pi\rho \quad (13)$$

where ρ refers to the oscillating electron charge density. Using Eqs.(12) and (13) in Eq.(11) yields

$$\nabla_1^2 E_z + \partial_z^2 E_z + \left(\frac{\omega}{c}\right)^2 E_z = 4\pi\partial_z \rho - \frac{i4\pi\omega}{c} j_z \quad (14)$$

Operating on both sides of Eq.(14) with

$$\int_{-L/2}^{L/2} \frac{dz}{L} \int_0^{2\pi} \frac{d\theta}{2\pi} \exp[-i(kz + n\theta)] \quad (15)$$

and using the conservation of charge constraint together with the periodicity requirement along z results in

$$\nabla_1^2 \bar{E}_z + (k_0^2 - k^2) \bar{E}_z = -i \frac{4\pi}{k} (k_0^2 - k^2) \bar{\rho} \quad (16)$$

where the bar notation over E_z , ρ refers to the Fourier transformed quantities over z and θ .

The quantity $\bar{\rho}$ contains the contribution $\bar{\rho}_0$ arising from the background particles (which are assumed to behave linearly), as well as the contribution $\bar{\rho}_b$ due to the beam particles, i.e.,

$$\bar{\rho} = \frac{ik}{4\pi} \left(\frac{\omega}{c}\right)^2 \bar{E}_z + \bar{\rho}_b \quad (17)$$

Inserting Eq.(17) into Eq.(16) and integrating over the radial coordinate with the appropriate weight functions results in

$$2 \int_0^a \frac{rdr}{a^2} J_n(k_1 r) \left\{ \frac{\nabla_1 \bar{E}_z}{k_0^2 - k^2} + \epsilon_{zz}^0 \bar{E}_z \right\} \quad (18)$$

$$= i \frac{4\pi e}{k} \left(\frac{n_b}{N} \right) \sum_{\ell=1}^N J_n[k_1 r_\ell(t)] \exp\{-i[kz_\ell(t) + n\theta_\ell(t)]\}$$

where n_b represents the density of beam electrons.

For low density beams (i.e., $n_b/n_0 \ll 1$) the left-hand side of Eq.(18) vanishes to zero-order in $\delta\omega$, thus yielding the solutions described by Eqs.(2) and (4). To account for the interaction between the beam and the cavity mode one must retain the first order contributions in $\delta\omega$ which arise in the left-hand side of Eq.(18), namely

$$2 \int_0^a \frac{rdr}{a^2} J_n(k_1 r) \left[- \frac{(2k_0/c)}{(k_0^2 - k^2)^2} \nabla_1^2 \bar{E}_z + \frac{\partial \epsilon_{zz}^0}{\partial \omega} \bar{E}_z \right] \delta\omega \quad (19)$$

where it is understood that the terms inside the brackets are evaluated at $\omega = \omega_0$, i.e., the unperturbed frequency of the cavity mode. Since the zero order Eqs.(1) and (4) are satisfied identically, the expression in Eq.(19) takes the form

$$\left[\frac{2(k_0 a)^2}{\omega} \left(\frac{\epsilon_{zz}^0}{P_{nv}} \right)^2 + \left(\frac{\partial \epsilon_{zz}^0}{\partial \omega} \right) \right] [J_{n+1}^2(k_1 a)] \delta\omega(t) E(t) \quad (20)$$

Using the generalized dielectric ϵ defined in Eq.(6) one finally arrives at the equation governing the time evolution of $E(t)$

$$\left(\frac{\partial \epsilon}{\partial \omega} \right)_0 [J_{n+1}^2(k_1 a)] \frac{d}{dt} E = \frac{4\pi e n_b}{Nk} \sum_{\ell} J_n(k_1 r_\ell) \exp\{-i(kz_\ell + n\theta_\ell - \omega_0 t)\} \quad (21)$$

To obtain the time evolution of the total momentum of the system one returns to Eq.(9) and sums over ℓ , making use of Eq.(21), to yield

$$\frac{d}{dt} \left\{ n_b \sum_{\ell} \frac{mv_{z\ell}}{N} + k \left(\frac{\partial \epsilon}{\partial \omega} \right) [J_{n+1}^2(k_1 a)] \left(\frac{|E|^2}{4\pi} \right) \right\} = en_b E_0 \quad (22)$$

which states that the total momentum of the beam-wave system is increased by the presence of the external DC electric field.

In calculating the conservation of energy relationship it proves useful to introduce the relative coordinate $\xi_{\ell} = z_{\ell} - (\omega_0/k)t$ and the associated velocity $\dot{\xi}_{\ell}$. Multiplying both sides of Eq.(9) by $\dot{\xi}_{\ell}$ and summing over all beam particles yields

$$\frac{d}{dt} \left[\sum_{\ell} \frac{m(\dot{\xi}_{\ell})^2}{2} \right] = eE_0 \sum_{\ell} \dot{\xi}_{\ell} - e \sum_{\ell} \left\{ E(t) \dot{\xi}_{\ell} J_n(k_1 r_{\ell}) \exp[-i(k\xi_{\ell} + n\theta_{\ell})] + c.c. \right\} \quad (23)$$

but in the strongly magnetized limit

$$\sum_{\ell} \dot{\xi}_{\ell} \exp[i(k\xi_{\ell} + n\theta_{\ell})] J_n(k_1 r_{\ell}) = \frac{1}{ik} \frac{d}{dt} \left\{ \sum_{\ell} J_n(k_1 r_{\ell}) \exp[i(k\xi_{\ell} + n\theta_{\ell})] \right\} \quad (24)$$

using Eq.(21) this expression becomes

$$\frac{1}{ik} \frac{d}{dt} \left[\frac{Nk}{4\pi en_b} \left(\frac{\partial \epsilon}{\partial \omega} \right) J_{n+1}^2(k_1 a) \frac{d}{dt} E^* \right] \quad (25)$$

Inserting Eq.(25) in Eq.(23) and recognizing that

$$E \frac{d^2}{dt^2} E^* - E^* \frac{d^2}{dt^2} E = - \frac{d}{dt} [2\text{Re}(\delta\omega) |E|^2] \quad (26)$$

where Re refers to the real part, results in

$$\frac{d}{dt} \left[n_b \sum_{\ell} \frac{m(\dot{\xi}_{\ell})^2}{2N} + 2\text{Re}(\delta\omega) \left(\frac{\partial \epsilon}{\partial \omega} \right) J_{n+1}^2(k_1 a) \left(\frac{|E|^2}{4\pi} \right) \right] = en_b \sum_{\ell} \frac{\dot{\xi}_{\ell}}{N} E_0 \quad (27)$$

which states that the time rate of change of the total energy of the beam-wave system is equal to the power delivered by the DC electric field.

Equations (22) and (23) are the exact conservation laws for the runaway beam-cavity interaction used in the present work. Similar expressions have been found previously in the discussion of the weak cold beam problem, both in the nonrelativistic⁸ and relativistic¹⁷ cases, and also in the calculation of the nonlinear frequency shift¹⁸ due to particles trapped by an electron plasma wave. In the latter study it has been shown that to lowest order, the conservation of momentum regulates the damping or growth of the wave, while the conservation of energy requirement produces phase changes in the wave. In the present work both of these effects play a crucial role in determining the evolution of the runaway beam. For completeness, it should be mentioned that quasilinear theory retains only the effects associated with momentum changes and completely neglects the phase changes introduced by Eq.(27).

As can be seen from Eqs.(22) and (27) the conservation relations depend on two spatially averaged properties of the beam, i.e., the average velocity $\sum_{\ell} \dot{\xi}_{\ell}/N$, and the average of the squares of the velocity $\sum_{\ell} (\dot{\xi}_{\ell})^2/N$. If one finds the time evolution of these quantities then the behavior of $|E|^2$ and $\delta\omega$ can be determined. Such a procedure can be implemented if the orbits of the beam particles are known. In the case of a launched large amplitude wave this method of calculation has been implemented analytically,^{18,19} while in the small cold beam problem the orbits have been calculated numerically.⁸ In the present work we are interested in using the conservation equations in the reverse order. Essentially the idea is to determine $\delta\omega$ and $|E|^2$ first, and then use the conservation laws to extract the previously mentioned averages. Of course, such a procedure must be self-consistent thus implying that a method of closure must be implemented.

In order to calculate $\delta\omega(t)$ from Eq.(8) one needs to know $X_b(t, \delta\omega)$. Physically, one expects that as the runaway beam passes through a cavity resonance its average velocity is altered and simultaneously it acquires a

certain thermal spread. This implies that the initial cold beam is slowly transformed into a warm beam which suffers a recoil due to the fact that it excites a cavity mode. Thus one must deal with the transition from the hydrodynamic interaction (two-stream) to the kinetic interaction (Landau type). O'Neil and Malmberg have investigated the details of such a transition in the purely linear regime, and have found that it is extremely useful to parametrize the transition by means of an equivalent beam susceptibility

$$\chi_b = - \frac{\omega_{pb}^2}{(\omega - kv_b + ik\bar{v})^2} \quad (28)$$

where $\omega_{pb}^2 = \omega_{pe}^2 (n_b/n_o)$, v_b is the average velocity of the beam and \bar{v} represents the average thermal spread. In the limit $\bar{v} = 0$, Eq.(29) reduces to the cold beam result, while for $\bar{v} \neq 0$ this susceptibility can be identified with a beam having a Lorentzian distribution.

The closure procedure consists of using Eq.(28) in its WKB sense, i.e., $v_b = v_b(t)$ and $\bar{v} = \bar{v}(t)$, thus describing a slowly changing beam which undergoes a transition from cold to warm as it interacts with the cavity mode. To make the scheme self-consistent one relates v_b and \bar{v} to the spatially averaged quantities appearing in the conservation laws, namely

$$v_b(t) = \sum_{\ell} \frac{\dot{\xi}_{\ell}}{N} ; \quad \bar{v}(t) = \frac{1}{2} \left[\sum_{\ell} \frac{\dot{\xi}_{\ell}^2}{N} - \left(\sum_{\ell} \frac{\dot{\xi}_{\ell}}{N} \right)^2 \right]^{1/2} \quad (29)$$

With Eq.(28) the WKB (in time) dispersion relation takes the form

$$\left(\frac{\partial \epsilon}{\partial \omega} \right) \delta \omega(t) = \frac{\omega_{pe}^2 (n_b/n_o)}{[\delta \omega(t) - kv_b(t) + ik\bar{v}(t)]^2} \quad (30)$$

and with the understanding that v_b is measured relative to the wave frame.

Introducing the following scaling

$$\Omega \equiv \left[\omega_{pe}^2 \left(\frac{n_b}{n_o} \right) / \frac{\partial \epsilon}{\partial \omega} \right]^{1/3}$$

$$w = \delta\omega/\Omega, \quad \tau = \Omega t$$

$$p = kv_b/\Omega, \quad S = k\bar{v}/\Omega$$

$$u = \left[k^2 \left(\frac{\partial \epsilon}{\partial \omega} \right) J_{n+1}^2(k_1 a) |E|^2 \right] / (4\pi m n_b \Omega) \quad (31)$$

$$K = \frac{1}{2N} \sum_{\ell} \left(\frac{k \xi_{\ell}}{\Omega} \right)^2$$

$$\mathcal{E} = (ekE_o) / (m\Omega^2)$$

one obtains the scaled equations of the problem

$$\frac{d}{d\tau} (p + u) = \mathcal{E} \quad (32)$$

$$\frac{d}{d\tau} [K + 2\text{Re}(w)u] = \mathcal{E}p \quad (33)$$

$$w(w - p + iS)^2 = 1 \quad (34)$$

$$S = \frac{1}{2} (2K - p^2)^{1/2} \quad (35)$$

The simplification achieved by the spatially averaged description becomes evident by integrating Eqs. (32) and (33) directly

$$p(\tau) = \mathcal{E}\tau - u(\tau) - [p(0) + u(0)] \quad (36)$$

$$K(\tau) = 2\text{Re}[w(\tau)] + \int_0^{\tau} d\tau' p(\tau') - 2\text{Re}[w(0)]u(0) \quad (37)$$

with

$$u(\tau) = u(0) \exp\left\{ 2 \int_0^{\tau} d\tau' \text{Im}[w(\tau')] \right\} \quad (38)$$

and realizing that $w(\tau)$ can also be obtained in closed analytic form from Eq. (34) by means of the well-known²⁰ formula for the roots of a cubic

equation (not worth writing out in detail here). The latter simplification follows from the choice of Eq.(28), other methods of closure yield more complicated expressions. In practice the solution to the present problem reduces essentially to solving a set of coupled algebraic equations which can easily be done with very little computing effort.

It should be noted that the procedure used in generating Eqs.(32)-(35) can be implemented for many modes. In this more general case the resulting equations are slightly changed to the form

$$\frac{d}{d\tau} \left(p + \sum_k u_k \right) = \mathcal{E} \quad (39)$$

$$\frac{d}{d\tau} \left\{ K + \sum_k [\delta_k + 2\text{Re}(w_k)] \lambda_k u_k \right\} = \mathcal{E} p \quad (40)$$

$$w_k [w_k - (p-iS)/\lambda_k + \delta_k]^2 = D_k \quad (41)$$

$$u_k = u_k(0) \exp \left\{ 2 \int_0^\tau d\tau' \text{Im}[w_k(\tau')] \right\} \quad (42)$$

In scaling Eqs.(39)-(41) one selects a certain wavenumber k_1 , perhaps corresponding to the mode of principal interest to the problem, and defines

$$\Omega' \equiv \left[\omega_{pe}^2 \left(\frac{n_b}{n_0} \right) / \left(\frac{\partial \epsilon}{\partial \omega} \right)_1 \right]^{1/3}$$

$$w_k = \delta \omega_k / \Omega' , \quad \tau = \Omega' t$$

$$\lambda_k = (k_1/k) , \quad \delta_k = (\omega_k - \omega_1/\lambda_k) / \Omega'$$

(43)

$$p = k_1 v_b / \Omega' , \quad S = k_1 \bar{v} / \Omega'$$

$$D_k = \left(\frac{\partial \epsilon}{\partial \omega} \right)_k / \left(\frac{\partial \epsilon}{\partial \omega} \right)_1$$

$$u_k = \frac{kk_1 (\partial \epsilon / \partial \omega)_k J_{n+1}^2(k_1 a) |E_k|^2}{4\pi n_b m \Omega'}$$

where $(\partial\epsilon/\partial\omega)_k$ refers to $(\partial\epsilon/\partial\omega)$ evaluated at the unperturbed frequency of mode k , i.e., ω_k . It is clear that for $k = k_1$ the expressions in Eq.(43) reduce to those of Eq.(31).

III. THE WEAK COLD BEAM SATURATION

It is clear that the spatially averaged description neglects a variety of single particle dynamical effects which have been found to play an important role in previous investigations^{7,8} of the beam-plasma interaction. Outstanding among these is the trapping of the beam by the fastest growing linearly unstable mode. The trapping effect saturates the hydrodynamic instability and causes periodic oscillations of the wave amplitude. Physically, the trapping process causes the cold beam to acquire a velocity spread which in turn stops the growth and produces the temporary damping of the wave. However, due to the strong coherency of this system the velocity spread is only partially randomized. Accordingly, a significant fraction of the fake heating is reversible and manifests itself in the regrowth of the wave. This sequence of events repeats cyclically until the system phase mixes or other processes occur which make the beam heating practically irreversible.

The spatially averaged description can handle the effects associated with the increase in the velocity spread. However, the model intrinsically assumes that the velocity spread is an irreversible process. Consequently, the model can not reproduce the trapped particle amplitude oscillations, instead it gives an answer which is an average over these oscillations.

To check the predictive power of the model we consider the limit in which there is no external DC electric field present, and at first consider a single mode. In this limit the problem reduces to the weak cold beam-plasma interaction investigated by Shapiro, et al.⁷ and by O'Neil, et al.⁸

Figure 1 shows the comparison between the spatially averaged model (continuous curve) and the results of O'Neil, et al.⁸ (dashed curve). In the upper portion of Fig. 1 one plots the time evolution of the scaled wave amplitude \sqrt{u} . It is seen that the two results overlap in the linear

regime and reach saturation at the same time. As expected, after saturation the spatially averaged model yields a constant amplitude, while the computer simulation predicts amplitude oscillations about the saturation level of the averaged model. On the bottom of Fig. 1 one observes the time evolution of the scaled average momentum of the beam, and shows that the beam slows down as the instability saturates.

Although the beam momentum is converted into wave momentum, as illustrated in Fig. 1, this effect is not the primary reason for the saturation. To isolate the physics behind the saturation we have considered the evolution of the system when one legislates that $S = 0$ for all τ , i.e., one does not allow the beam to acquire a thermal spread as it recoils. The resulting behavior is shown in Fig. 2, where one observes that the instability does not saturate in this case. This result shows that it is crucial to retain the thermal spreading of the beam in order to correctly describe the saturation illustrated in Fig. 1.

The proper physical interpretation of the saturation is that as the beam acquires a thermal spread the hydrodynamic instability evolves into the kinetic instability. The consequence of this transition is illustrated in Fig. 3, where one observes the dependence of the growth rate Γ on p for a cold beam, $S = 0$, and for a slightly warm beam, $S = 0.5$. It is seen from Fig. 3 that as the beam acquires a thermal spread the region of unstable behavior shrinks and moves toward larger values of p . However, the conservation of momentum constraint requires that the momentum p must decrease as a function of time. Thus, as the beam recoils it eventually passes through the marginally stable point (i.e., $\Gamma = 0$) bringing about the saturation.

For completeness we exhibit in Fig. 4 the self-consistent evolution of the kinetic energy K , thermal spread S , and the frequency shift $\Omega = \text{Re}(w)$

corresponding to the solid curve of Fig. 1. It should be noted that the kinetic energy is measured relative to the initial wave frame, hence the beam recoil shows up as an increase in K .

Another topic which can be investigated by the formalism is the generation of sidebands due to wave-particle-wave interactions. This interaction has been recently identified as the fundamental process governing the excitation of the well-known trapped particle sidebands. The essence of this interaction is that a wave having a certain phase velocity can cause a non-local distortion in the spatially averaged velocity distribution function, thus modifying the damping or growth of other modes having different phase velocities. Such a process can be present in the beam-plasma interaction, as is illustrated in Fig. 5.

In Fig. 5 we exhibit the time evolution of the amplitude of two modes, $\sqrt{u_1}$, $\sqrt{u_2}$, obtained from the self-consistent numerical solution of Eqs. (39)-(42) for a fixed choice of $k_2/k_1 = 0.7$, and for different values of dispersion, i.e., $\delta_2 = 2.0, 1.5, 0.5$. The scaling wavenumber k_1 corresponds in this case to the fastest growing mode in the linear stage of the beam-plasma instability. It is seen in Fig. 5 that early in time both modes exhibit growth for all values of δ_2 . This is the linear stage during which the beam is not significantly perturbed. However, in the nonlinear regime one observes different responses for different values of δ_2 . For $\delta_2 = 2.0$ one has the more familiar situation arising due to the heating of the beam by the growing waves. As the beam acquires a thermal spread (mainly due to the growth of k_1) mode k_2 undergoes the transition from hydrodynamic to kinetic behavior, thus resulting in its damping. The final state consists of the saturated fastest growing mode k_1 .

It is seen in Fig. 5 that as the dispersion is decreased to $\delta_2 = 0.5$, the fastest growing mode undergoes a continuous transition that results in its eventual decay, i.e., a sideband decay whereby the energy in mode k_1 is transferred to the neighboring mode k_2 . This is the type of decay responsible for the excitation of the trapped particle sidebands¹⁰⁻¹⁴ and should be noted that it has nothing to do with the parametric couplings previously invoked²¹ to explain such a behavior.

To better understand the reason behind the nonlinear decay of the fastest growing mode for $\delta_2 = 0.5$, one should consider the self-consistent modification in both the real and imaginary parts of w_1 and w_2 . The relevant time evolution of $\Omega_j = \text{Re}(w_j)$ and $\Gamma_j = \text{Im}(w_j)$ is shown in Fig. 6 for $\delta_2 = 0.5$. Early in time Γ_1 is slightly larger than Γ_2 and $|\Omega_1| < |\Omega_2|$. As the beam acquires a thermal spread, Γ_1 decreases to zero as it would normally do just before saturation for the single mode problem. However, because mode k_2 is now present the beam continues to recoil and spread, thus causing mode k_1 to overshoot the marginally stable point ($\Gamma_1 = 0$) and eventually enter the region of damping. As mode k_1 damps, the recoil of the beam is slowed down thus preventing mode k_2 from crossing its marginally stable point (i.e., $\Gamma_2 = 0$). Also, from the scaling of Eq.(41) one observes that for a fixed thermal spread S , mode k_2 samples an effective thermalization which is 0.5 smaller than that seen by k_1 . As a consequence, Ω_1 suffers a much greater change from its linear value than Ω_2 , as is seen on the bottom of Fig. 6.

The present model can describe also the cascading of the spectrum to lower phase velocities (higher k). This effect is expected to be responsible for the long term slowing down of the beam and its final merger with the distribution function of the background plasma. To isolate this effect we

consider two modes of widely spaced phase velocities $\delta_2 = 3.0$, $k_2/k_1 = 0.7$, and legislate that the initial beam velocity lies between them, i.e., $p(0) = 2.0$. In this case k_1 corresponds to a mode which is initially stable while k_2 is slightly unstable, as predicted from Fig. 3 for $S = 0$. The corresponding evolution of the wave amplitudes is illustrated on the top part of Fig. 7, where it is seen that k_2 grows initially, saturates early in time and eventually decays. As mode k_2 decays one observes that mode k_1 is excited. The reason for the late excitation of mode k_1 is that the saturation of k_2 causes the beam to recoil, as indicated on the bottom portion of Fig. 7. As p is reduced, mode k_1 enters the region of growth (as in Fig. 3), thus bringing about the cascading to lower wavenumbers.

IV. CLAMPING BEHAVIOR

We proceed to consider the behavior of the beam when an external DC electric field is present. In addition to the strength of the DC field \mathcal{E} , one needs to legislate at $\tau = 0$ the momentum of the beam $p(0)$, its initial velocity spread $S(0)$, and the initial amplitude of the various modes, $u_k(0)$.

Figure 8 illustrates the time evolution of the wave amplitude and beam momentum for a case in which a cold beam interacts with a single mode having a relatively small initial amplitude, i.e., $u_1(0) = 1.0 \times 10^{-3}$, $p(0) = -5.0$, $S(0) = 0$, $\mathcal{E} = 1.0$. It is seen in Fig. 8 that for these parameters the beam is able to run through the phase velocity of the cavity resonance while experiencing only a slight recoil. This small recoil arises because for $p < 0$ the beam drives the cavity mode unstable (as in Fig. 3). However, since the beam is continuously accelerated by the external DC field its velocity eventually becomes larger than the phase velocity of the cavity mode, hence shutting-off the instability. For small initial noise levels and large DC fields the process consists of a runaway beam radiating energy into a cavity mode for a finite time and proceeding to increase its velocity until it encounters a new resonance with another mode, thus repeating the cycle. This behavior would show up experimentally in the form of a cascade in the spectrum of the observed radiation toward lower frequencies.

An interesting phenomenon occurs when either \mathcal{E} is decreased or $u_1(0)$ is increased. By decreasing \mathcal{E} one increases the time of interaction between the runaway beam and the cavity mode, while by increasing $u_1(0)$ the absolute amplitude of the wave can be increased. Both of these changes can make the rate of momentum transfer to the wave comparable to the magnitude of \mathcal{E} , thus preventing the beam from running away in velocity. Such a clamping behavior is illustrated in Fig. 9 for $p(0) = -5.0$, $S(0) = 0$, $\mathcal{E} = 1.0$, as in the case of Fig. 8 but now $u_1(0) = 5.0 \times 10^{-3}$. It is seen in Fig. 9 that early in

time the beam momentum exhibits the typical free-fall behavior associated with runaways. However, for $p \approx 0$ the beam recoils and its momentum proceeds to grow at a much slower rate while the wave amplitude (and wave momentum) continue to increase secularly. The clamping behavior exhibited in Fig. 9 is an example whereby an external push on a material particle can be transformed into field momentum. Alternatively, one can say that the beam acts as a converter of DC to AC energy.

The top of Fig. 10 illustrates the time evolution of the effective force acting on the beam. Early in time this force is equal to the DC push $\mathcal{E} = 1.0$, and later on it is seen that the effective force decreases, reverses sign for a short time, and finally it settles at the reduced value of 5×10^{-3} . One sees in the bottom of Fig. 10 the evolution of the kinetic energy K and the thermal spread S during clamping.

The functional dependence of the slow time evolution observed after the clamping stage sets in, illustrated in Figs. 9 and 10, can be seen to be a direct consequence of Eq. (39) in the regime of small growth rate β , but large amplitude so that $u(\tau) \approx \bar{u}(1 + 2\beta\tau)$ with \bar{u} defined by the condition that $dp/d\tau = 0$, i.e., $2\bar{\Gamma}\bar{u} \approx \mathcal{E}$. The slowly changing momentum and kinetic energy are of the form

$$p \approx \bar{p} + 2(\beta - \bar{\Gamma})\bar{u}\tau \tag{44}$$

$$K \approx \bar{K} + (\mathcal{E}\bar{p} + 2\bar{w}\bar{u})\tau + \frac{\mathcal{E}(\beta - \bar{\Gamma})}{2}\tau^2$$

where the bar notation denotes constants evaluated just after clamping is attained, and $\beta - \bar{\Gamma}$ is a small quantity.

In estimating the threshold value of the $u_1(0)$ required to obtain clamping for a given $p(0)$ and \mathcal{E} it is useful to consider the prediction made by the first Born approximation

$$u_1(0) > \frac{\mathcal{E}}{\Gamma_M} \exp \left[- (\Gamma_M + \Gamma_0) \frac{|p(0)|}{\mathcal{E}} \right] \quad (45)$$

in which Γ_M refers to the maximum linear growth rate and Γ_0 to the growth rate for $p(0)$ (as predicted from Fig. 3). By examining the clamping behavior for a wide range of initial conditions we have found that Eq.(45) gives good results for intermediate values of $p(0)$, i.e., $p(0) \approx -5.0$. As an illustration, for $p(0) = -5.0$, $\mathcal{E} = 1.0$, Eq.(45) predicts $u_1(0) > 1.6 \times 10^{-3}$ and by searching in parameter space one finds that $1.0 \times 10^{-3} < u_1(0) < 4.0 \times 10^{-3}$. However, Eq.(45) fails to predict the correct threshold for higher and lower values of p , i.e., $p(0) < -7.5$ and $-2.5 < p(0)$. The physics behind the failure is directly related to the thermal spreading of the beam and plays opposite roles for large and small values of $|p(0)|$.

For large $|p(0)|$ the beam acquires a finite velocity spread as it approaches the resonance, thus the effective growth rate is reduced, hence requiring a larger value to achieve clamping. For small $|p(0)|$ the beam penetrates deeper into the $p > 0$ region. However, for this situation the thermal spread elongates the region of growth toward larger p (as in Fig. 3) thus implying that the effective interaction time with the cavity mode is longer than estimated by the first Born approximation. As a consequence, the threshold $u_1(0)$ is lower than predicted by such a method. An illustration of the deeper penetration at smaller $p(0)$ is illustrated in Fig. 11 for $p(0) = -2.5$, $S(0) = 0$, $\mathcal{E} = 1.0$, $u_1(0) = 5.0 \times 10^{-3}$. It is seen in Fig. 11 that the beam penetrates up to $p = 0.8$ (to be compared with Fig. 9) and then recoils to the region $p < 0$ while the wave amplitude grows secularly.

The clamping process can also take place when more than one mode is considered. Figure 12 exhibits the behavior for two neighboring modes with $k_2/k_1 = 0.7$, $\delta_2 = 3.0$, $u_2(0) = u_1(0) = 1.0 \times 10^{-2}$, $p(0) = -5.0$ and $\mathcal{E} = 1.5$.

On the top part of Fig. 12 one sees the time evolution of the wave amplitudes, and it is observed that the slower mode k_1 saturates earlier and attains a lower amplitude level than mode k_2 . For large τ it is observed that both modes exhibit the characteristic secular growth associated with the clamping of the beam. By examining the evolution of the beam momentum shown on the bottom of Fig. 12, one finds that the beam actually runs through the phase velocity of mode k_1 until it encounters mode k_2 , which clamps the beam as in the single mode example of Fig. 9. By setting $u_2(0) = 0$ one verifies that mode k_2 is responsible for this effect, and in such a situation one finds that mode k_1 does not exhibit the late time secular growth; it simply saturates. The dominant multi-mode effect on the clamping process is that in addition to exciting the main mode responsible for the slowing down of the beam, neighboring modes are driven unstable due to the Landau-type of growth associated with a beam having a finite velocity spread.

The effect of finite velocity spread on the multi-mode interaction is illustrated in Fig. 13 for $S(0) = 1.0$, $u_2(0) = u_1(0) = 5.0 \times 10^{-2}$, with all other parameters as in Fig. 12. It is seen on the top part of Fig. 13 that both modes exhibit an early damping phase followed by growth and eventual saturation. This behavior is an entirely kinetic effect associated with the sampling of the negative slope of the velocity distribution function (i.e., $\partial f / \partial v < 0$) before the beam reaches the resonance. Following this stage there is a period of growth when the $\partial f / \partial v > 0$ part of the beam passes through the cavity modes in velocity space, as suggested on the bottom part of Fig. 13.

V. THE RELATIVISTIC PROBLEM

The relativistic runaway beam problem can also be handled through the appropriate conservation laws of the beam-wave system. These laws can be derived in analogy with the nonrelativistic case and for the single mode interaction take the form

$$\frac{d}{dt} \left[n_b \sum_{\ell} \frac{P_{\ell}}{N} + k \left(\frac{\partial \epsilon}{\partial \omega} \right) J_{n+1}^2(k_1 a) \left(\frac{|E|^2}{4\pi} \right) \right] = e n_b E_0 \quad (46)$$

$$\begin{aligned} \frac{d}{dt} \left[n_b \sum_{\ell} \frac{mc^2 \gamma_{\ell}}{N} - v_0 \sum_{\ell} P_{\ell} + 2 \operatorname{Re}(\delta \omega) \left(\frac{\partial \epsilon}{\partial \omega} \right) J_{n+1}^2(k_1 a) \left(\frac{|E|^2}{4\pi} \right) \right] \\ = e n_b E_0 \sum_{\ell} \frac{(v_{\ell} - v_0)}{N} \end{aligned} \quad (47)$$

where P_{ℓ} is the momentum of the ℓ beam particle and γ_{ℓ} is the corresponding relativistic factor. In Eq.(47) v_0 is the phase velocity of the cavity mode and $v_{\ell} = (P_{\ell}/m) [1 + (P_{\ell}/mc)^2]^{-1/2}$.

The dispersion relation for this problem is

$$\left(\frac{\partial \epsilon}{\partial \omega} \right) \delta \omega = \frac{\omega_{pb}^2}{\gamma^3 [\delta \omega - k(v - v_0) + ik\Delta / (m\gamma^3)]^2} \quad (48)$$

where v refers to the average velocity of the beam and Δ represents a small momentum spread, i.e., $\Delta/mv \ll 1$. The full frequency of the mode is $\omega = kv_0 + \delta \omega$.

The exact conservation laws shown in Eqs.(46) and (47) can be considerably simplified if one chooses to describe the evolution of the beam after it has acquired a large momentum P_0 relative to the lab, so that

$$\gamma_{\ell} = \gamma_0 (1 + \alpha \eta_{\ell}), \quad P = P_0 (1 + \alpha \gamma_0^2 q_{\ell}) \quad (49)$$

with the small parameter being

$$\alpha = \frac{1}{\gamma_0} \left[\frac{\omega_{pb}^2}{\left(\frac{\partial \epsilon}{\partial \omega} \right)} \right]^{1/3} \quad (50)$$

and γ_0 is obtained from P_0 through the relation $\gamma(P) = [1+(P/mc)^2]^{1/2}$. This relation also provides a connection between η_ℓ and q_ℓ for the limit $\alpha \ll 1$, namely

$$\eta_\ell = \left(\frac{v_0}{c}\right)^2 \frac{q_\ell}{\gamma_0^2} + \frac{1}{2} \left(\frac{P_0}{mc}\right)^2 \alpha q_\ell^2 \quad (51)$$

which implies that

$$k(v_\ell - v_0) = \gamma_0^3 \alpha q_\ell \quad (52)$$

Introducing the natural scaling of the problem $w = \delta\omega/\alpha$, $S = k\Delta/(m\alpha\gamma_0^3)$, transforms Eq.(48) into

$$w \left[w + i \left(\frac{\gamma_0}{\gamma}\right)^3 S - q \right]^2 = [1 + \alpha\eta]^{-3} \quad (53)$$

with $q = \sum q_\ell/N$, $\eta = \sum \eta_\ell/N$. Working to lowest order in q results in the dispersion relation

$$w[w + iS(1 + sq)^{-3} - q]^2 = (1 + sq)^{-3} \quad (54)$$

which is seen to depend explicitly on the value of the parameter $s \equiv (\alpha/kv_0)(v_0/c)^2\gamma_0^{-2}$ which was originally introduced¹³ by Thode and Sudan to study the beam-plasma interaction.

Using Eq.(49) in Eq.(47) and scaling $\tau = \alpha t$ yields

$$\frac{d}{d\tau} \left\{ \frac{1}{N_\ell} \left[\eta_\ell - \left(\frac{v_0}{c}\right)^2 \frac{q_\ell}{\gamma_0^2} \right] + 2 \frac{\text{Re}(w)\omega_0 (\partial\epsilon/\partial\omega) J_{n+1}^2(k_1 a) |E|^2}{4\pi n_b mc^2 \gamma_0} \right\} = \frac{eE_0 \omega_0}{kmc^2 \gamma_0 \alpha} q \quad (55)$$

generalizing the scaled variables to

$$u = \frac{k^2 (\partial\epsilon/\partial\omega) J_{n+1}^2(k_1 a) |E|^2}{4\pi n_b m \gamma_0^3 \alpha}$$

$$\mathcal{E} = \frac{ekE_0}{m\gamma_0^3(\alpha)^{2/3}} \quad (56)$$

$$K = \frac{1}{2} \sum_{\ell} \frac{q_{\ell}^2}{N}$$

results in

$$\frac{d}{d\tau} [q + u] = \mathcal{E} \quad (57)$$

$$\frac{d}{d\tau} [K + 2\text{Re}(w)u] = \mathcal{E}q \quad (58)$$

which are the analog of Eqs.(32) and (33) and in the limit $\gamma_0 = 1$ reduce identically to them.

Next we find the connection between the momentum spread Δ and K , where

$$\Delta^2 \equiv \sum_{\ell} \frac{(P_{\ell} - P)^2}{N} \quad (59)$$

or in scaled variables

$$\Delta^2 = \sum_{\ell} \frac{P_0^2 \alpha^2}{N \gamma_0^4} (q_{\ell} - q)^2 \quad (60)$$

resulting in

$$\Delta = \frac{P_0 \alpha \gamma_0^2}{\omega_0} (2K - q^2)^{1/2} \quad (61)$$

hence

$$S = \frac{1}{2} (2K - q^2)^{1/2} \quad (62)$$

To check the relativistic formulation we have compared its predictions with the computer study¹² of Lampe and Sprangle in which the full relativistic dynamics of the beam particles is followed for the case $E_0 = 0$, in analogy with the nonrelativistic work of O'Neil, et al.⁸ One finds again that the present formulation yields results which average over the relativistic trapped particle oscillations, as in Fig. 1. In addition, one uncovers a nontrivial

dependence of the saturation amplitude on the value of the parameter s . The existence of such a dependence was originally pointed out in a computer simulation¹³ by Thode and Sudan, and subsequently investigated¹² by Lampe and Sprangle for the single wave problem, and by Schamel, et al.¹⁷ in the context of parametric excitation of ion density fluctuations by the single wave.

The solid curve in Fig. 14 exhibits the predicted dependence of the scaled saturation level $(s/4)u_M$ on s . It is seen that for small s the curve increases linearly with s , as expected, and for $s > 1.0$ it follows a much slower dependence, as found earlier by Lampe and Sprangle whose results are indicated by the circles in Fig. 14. It is observed in Fig. 14 that the spatially averaged formulation yields an answer which agrees quantitatively with the results of the exact dynamical calculation, but averages over the small oscillations present in the more detailed description.

After establishing the predictive power of the formulation we have proceeded to investigate the existence of the clamping behavior in the relativistic regime. As is illustrated in Fig. 15 for $\mathcal{E} = 0.1$, $u(0) = 1.0 \times 10^{-6}$, $s = 0.5$, one finds that a runaway relativistic beam can experience clamping, but in general the time dependence of the phenomena exhibits more abrupt changes than in the nonrelativistic case, e.g., Fig. 9. The effects of multi-mode interactions and sideband generation have also been included in the formulation, and their behavior has been investigated. However, the results do not differ significantly from those discussed in Sec. IV for the nonrelativistic problem, consequently they need not be presented here.

VI. CAVITY DAMPING

The discussions presented in the previous sections did not include the effect of cavity damping. To illustrate some of the phenomena associated with such a damping, let us consider a simplified model in which the energy spread of the beam is neglected to lowest order. The spatially averaged equations which describe the runaway beam-cavity interaction for this case are

$$\frac{d}{d\tau} P = \mathcal{E} - 2\Gamma u \quad (63)$$

$$\frac{d}{d\tau} u = 2(\Gamma - D)u + \sigma \quad (64)$$

where Γ represents the scaled growth rate due to the beam plasma instability, D is the scaled effective damping of the cavity mode and σ is an unspecified source. In the absence of the beam the cavity mode has a scaled amplitude $u = \sigma/2D$, while in the absence of the cavity mode (i.e., $u = 0$) the beam runs away in time as $p = p(0) + \mathcal{E}\tau$. However, when both entities are present it is possible for the clamping behavior to set in, as can be seen from the steady state solution of Eqs.(63) and (64), namely

$$u_0 = \frac{\mathcal{E}}{2\Gamma_0} ; \Gamma_0 = D \left(\frac{\mathcal{E}}{\mathcal{E} + \sigma} \right) , \quad (65)$$

and $p(\tau) = p_0$, where p_0 must be obtained self-consistently from the dispersion relation.

Equation (65) describes the existence of a clamped state in which the momentum push of the external electric field is transferred to the cavity mode and subsequently this momentum is dissipated through the appropriate damping mechanisms acting on the mode (e.g., wall damping, Landau damping, radiation leakage). After finding the existence of such a dynamical equilibrium one must inquire into its stability. The stability properties are

determined from the evolution of small perturbations p' , u' , such that $p = p_0 + p'$, $u = u_0 + u'$, $p' \ll p_0$ and $u' \ll u_0$. The corresponding linearized system is

$$\frac{d}{d\tau} p' = -2\Gamma_0 u' - 2 \left(\frac{\partial \Gamma}{\partial p}\right)_0 u_0 p' \quad (66)$$

$$\frac{d}{d\tau} u' = 2(\Gamma_0 - D)u' + 2 \left(\frac{\partial \Gamma}{\partial p}\right)_0 u_0 p' \quad (67)$$

seeking a solution $p' = p'(0)\exp(\gamma\tau)$, $u' = u'(0)\exp(\gamma\tau)$ results in

$$\gamma = - \left[\left(\frac{\partial \Gamma}{\partial p}\right)_0 u_0 + \frac{\sigma}{\mathcal{E}} \Gamma_0 \right] \pm \left\{ \left[\left(\frac{\partial \Gamma}{\partial p}\right)_0 u_0 + \frac{\sigma}{\mathcal{E}} \Gamma_0 \right]^2 - 4D \left(\frac{\partial \Gamma}{\partial p}\right)_0 u_0 \right\}^{1/2} \quad (68)$$

From the dependence of Γ on p illustrated in Fig. 3 one observes that there are two possible values of p_0 such that $\Gamma_0 < D$, as required by the steady state solution. One root corresponds to $p_0 < 0$ having $(\partial\Gamma/\partial p)_0 > 0$, while the other has $p_0 > 0$ and $(\partial\Gamma/\partial p)_0 < 0$. The $p_0 < 0$ root is stable and can support damped oscillations about the stable point if

$$2 \left(\frac{\partial \Gamma}{\partial p}\right)_0 (\mathcal{E} + \sigma) > \left[\left(\frac{\partial \Gamma}{\partial p}\right)_0 \left(\frac{\mathcal{E} + \sigma}{2D}\right) + \frac{\sigma D}{\mathcal{E} + \sigma} \right]^2 \quad (69)$$

The $p_0 > 0$ root is stable provided that

$$D > (\mathcal{E} + \sigma) \left[\left| \left(\frac{\partial \Gamma}{\partial p}\right)_0 \right| / 2\sigma \right]^{1/2} \quad (70)$$

For large values of the external electric field \mathcal{E} the excitation of the cavity mode is not able to clamp the beam, as shown earlier in Fig. 8. In this case the beam momentum p increases monotonically in time, hence the condition $\Gamma > D$ is eventually attained and the amplitude of the cavity mode grows. The time evolution of this process can be described analytically by approximating $\Gamma(p) \approx \lambda |p|^{-1/2}$ where λ is a constant that can be determined

from the asymptotic behavior of the exact Γ shown in Fig. 3. In this regime $p(\tau) \approx -p(0) + \mathcal{E}\tau$ to lowest order, thus the time evolution of u is approximately determined by

$$\frac{d}{d\tau} u + 2 \left\{ D - \frac{\lambda}{[|p(0)| - \mathcal{E}\tau]^{\frac{1}{2}}} \right\} u = \sigma \quad (71)$$

in the region $p(\tau) < 0$.

The solution of Eq.(71) takes the form

$$u(\tau) = u(0) \left\{ \exp \left[-\int_0^\tau d\tau' g(\tau') \right] \right\} \left\{ 1 + 2D \int_0^\tau d\tau' \exp \left[\int_0^{\tau'} d\tau'' g(\tau'') \right] \right\} \quad (72)$$

with

$$g(\tau) = 2 \left\{ D - \frac{\lambda}{[|p(0)| - \mathcal{E}\tau]^{\frac{1}{2}}} \right\} \quad (73)$$

After performing the required integrations one obtains

$$u(\tau) = u(0) \exp \left\{ -2D\tau - (4\lambda/\mathcal{E}) \left[(|p(\tau)|)^{\frac{1}{2}} - (|p(0)|)^{\frac{1}{2}} \right] \right\} \otimes \quad (74)$$

$$\left\{ 1 + F(\tau) \exp \left\{ (2\lambda^2/\mathcal{E}D) + [2D|p(0)|/\mathcal{E}] - (4\lambda/\mathcal{E})(|p(0)|)^{\frac{1}{2}} \right\} \right\}$$

with

$$F(\tau) = \lambda(2\pi/\mathcal{E}D)^{\frac{1}{2}} \left\{ \text{erf}[y(\tau)] - \text{erf}[y(0)] \right\} \quad (75)$$

$$+ \exp[-y^2(\tau)] - \exp[-y^2(0)]$$

and

$$y(\tau) = (2D/\mathcal{E})^{\frac{1}{2}} (|p(\tau)|)^{\frac{1}{2}} - \lambda(2/D\mathcal{E})^{\frac{1}{2}} \quad (76)$$

where erf refers to the error function.

The behavior predicted by Eq.(74) is illustrated in Fig. 16 for $p(0) = -20.0$, $\mathcal{E} = 1.0$, $\lambda = 0.87$, $D = 0.25$. It is seen in Fig. 16 that

as the beam momentum approaches the threshold point (i.e., $D - \Gamma = 0$) the amplitude of the cavity mode increases linearly in time; beyond the threshold the amplitude grows exponentially, as expected.

VII. CONCLUSIONS

This study illustrates some of the basic processes associated with the interaction of a runaway electron beam with collective modes, thus broadening the present understanding of runaway behavior as well as of the beam-plasma interaction. The principal effect found is that a significant amount of the push provided by the external DC electric field can be transformed through the beam nonlinearities to the form of wave momentum, hence giving rise to the clamping of the beam. This phenomenon has two consequences: (1) it limits the maximum energy of runaways; (2) it provides a mechanism for converting DC to AC energy.

In solving the model problem a spatially averaged description has been introduced which permits the description of complicated events which would normally require the use of computer simulation. When the spatially averaged technique is applied to the pure beam-plasma interaction it is found that the results are in excellent agreement with previous computer studies, thus it provides additional insight into the basic physics behind those studies.

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FIGURE CAPTIONS

- FIG. 1 Time evolution of the scaled wave amplitude \sqrt{u} and scaled average momentum p for a single mode and $E_0 = 0$. Solid curves are the results of the present calculation, dashed curve is the result of O'Neil, et al. (Ref. 8).
- FIG. 2 Time evolution of the scaled wave amplitude \sqrt{u} and scaled average momentum p for a single mode and $E_0 = 0$, but setting $S = 0$ explicitly. To be compared with Fig. 1.
- FIG. 3 Dependence of the scaled growth rate Γ and scaled frequency shift Ω on p for a cold beam $S = 0$, and for $S = 0.5$.
- FIG. 4 Self-consistent time evolution of the scaled growth rate Γ , scaled frequency shift Ω , average kinetic energy K , and thermal spread S for the behavior shown in Fig. 1.
- FIG. 5 Time evolution of sideband decay for $E_0 = 0$, $k_2/k_1 = 0.7$, and for $\delta_2 = 2.0, 1.5$, and 0.5 .
- FIG. 6 Time evolution of the scaled growth rate Γ and frequency shift Ω during the sideband decay for $E_0 = 0$, $k_2/k_1 = 0.7$, $\delta_2 = 0.5$. To be compared with Fig. 5.
- FIG. 7 Cascading of the spectrum from a high phase velocity mode k_2 to a lower phase velocity mode k_1 and corresponding evolution of the average momentum p and thermal spread S . $u_1(0) = u_2(0) = 1.0 \times 10^{-2}$, $p(0) = 2.0$, $E_0 = 0$, $k_2/k_1 = 0.7$, $\delta_2 = 3.0$.
- FIG. 8 Time evolution of the wave energy u_1 and average momentum p for a runaway case $\mathcal{E} = 1.0$; single mode with $u_1(0) = 1.0 \times 10^{-3}$.

- FIG. 9 Example of clamping behavior. Time evolution of wave energy u_1 and average momentum p for $\mathcal{E} = 1.0$; single mode with $u_1(0) = 5.0 \times 10^{-3}$. To be compared with Fig. 8.
- FIG. 10 Time evolution of the effective force $dp/d\tau$ during clamping and self-consistent behavior of the kinetic energy K and thermal spread S for the case shown in Fig. 9.
- FIG. 11 Example of clamping behavior following an overshoot of the cavity mode. u_1 is the wave energy and p the average momentum. $\mathcal{E} = 1.0$, $p(0) = -2.5$, $S(0) = 0$, $u_1(0) = 5.0 \times 10^{-3}$.
- FIG. 12 Example of clamping behavior in the presence of two modes. $k_2/k_1 = 0.7$, $\delta_2 = 3.0$, $\mathcal{E} = 1.5$, $u_1(0) = u_2(0) = 1.0 \times 10^{-2}$, $S(0) = 0$.
- FIG. 13 Excitation of two cavity modes by a runaway warm beam. $\mathcal{E} = 1.5$, $u_1(0) = u_2(0) = 5.0 \times 10^{-2}$, $k_2/k_1 = 0.7$, $\delta_2 = 3.0$, $S(0) = 1.0$.
- FIG. 14 Dependence of scaled saturation energy $su_M/4$ on the parameter s for the relativistic single mode $E_0 = 0$ problem. Solid curve is the prediction of the present model, the circles are the computer simulation results of Lampe and Sprangle (Ref. 12).
- FIG. 15 Example of clamping behavior in the relativistic regime for a single mode case. $\mathcal{E} = 0.1$, $s = 0.5$.
- FIG. 16 Time evolution of the wave energy u and total growth rate $D-\Gamma$ for a damped cavity mode, as predicted by perturbation theory. $D = 0.25$, $\mathcal{E} = 1.0$, $p(0) = -20.0$, $\lambda = 0.87$.

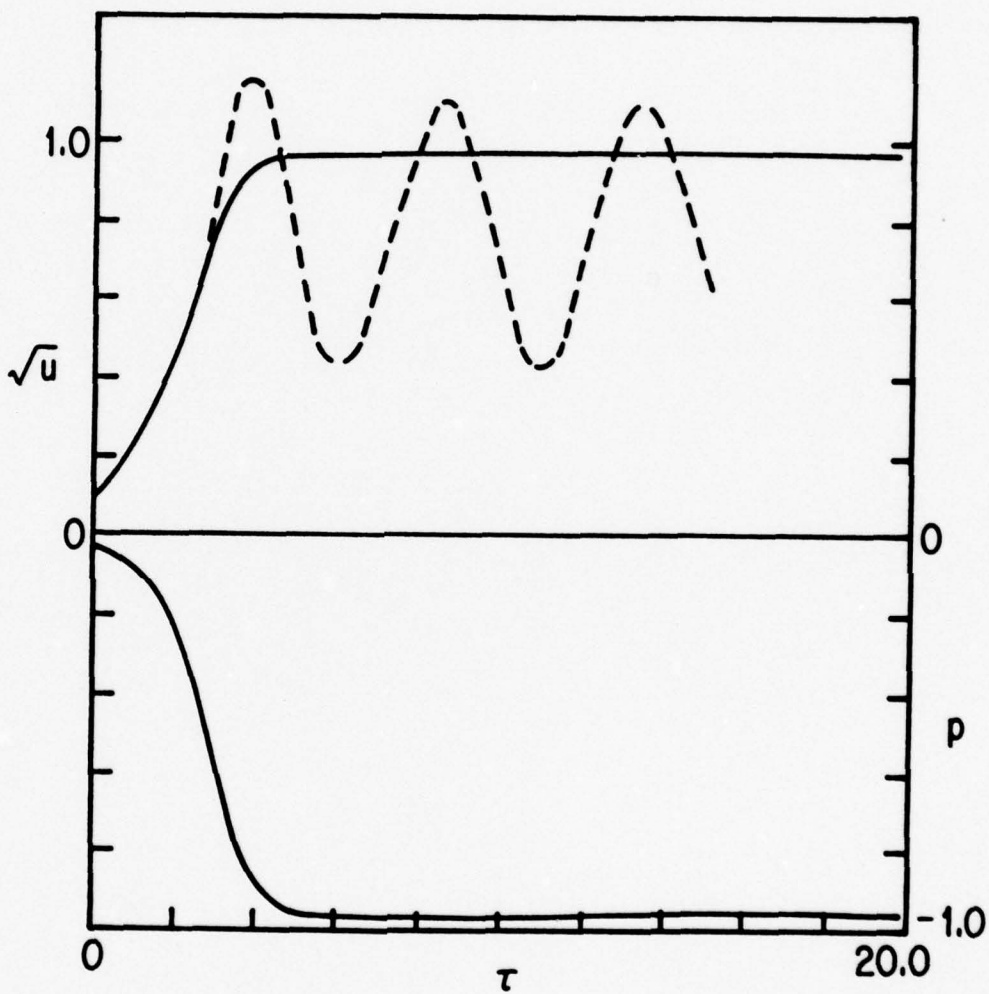


FIGURE 1

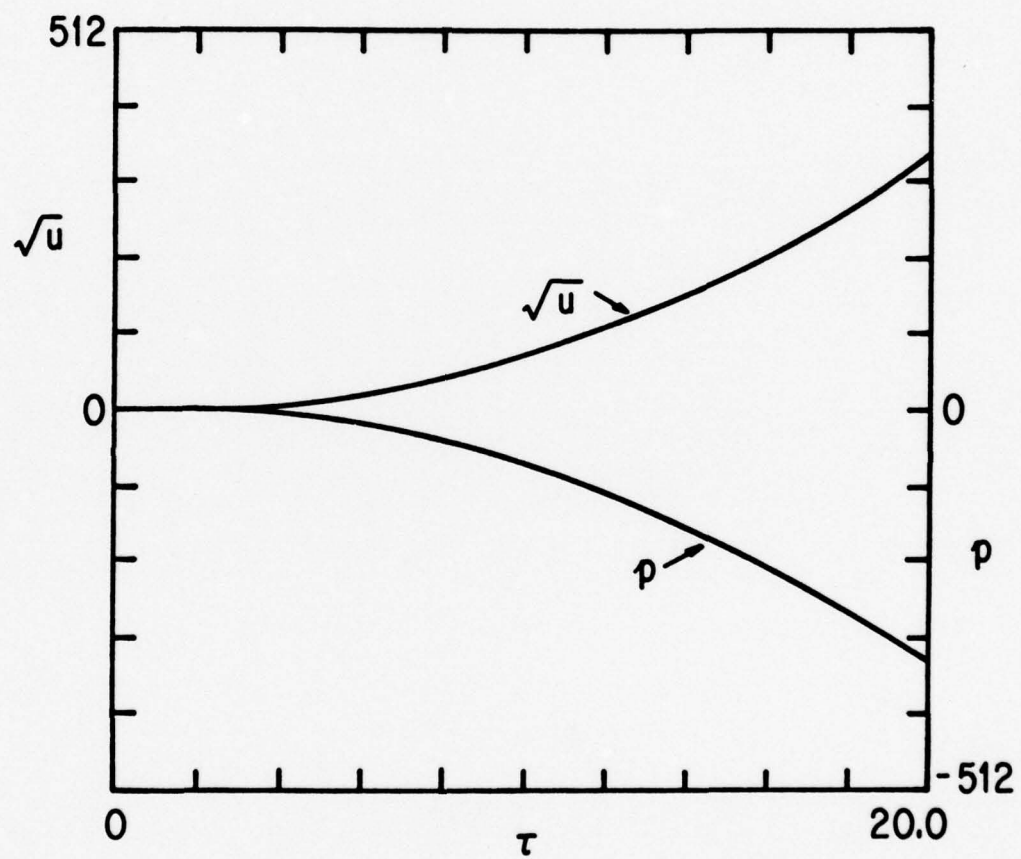


FIGURE 2

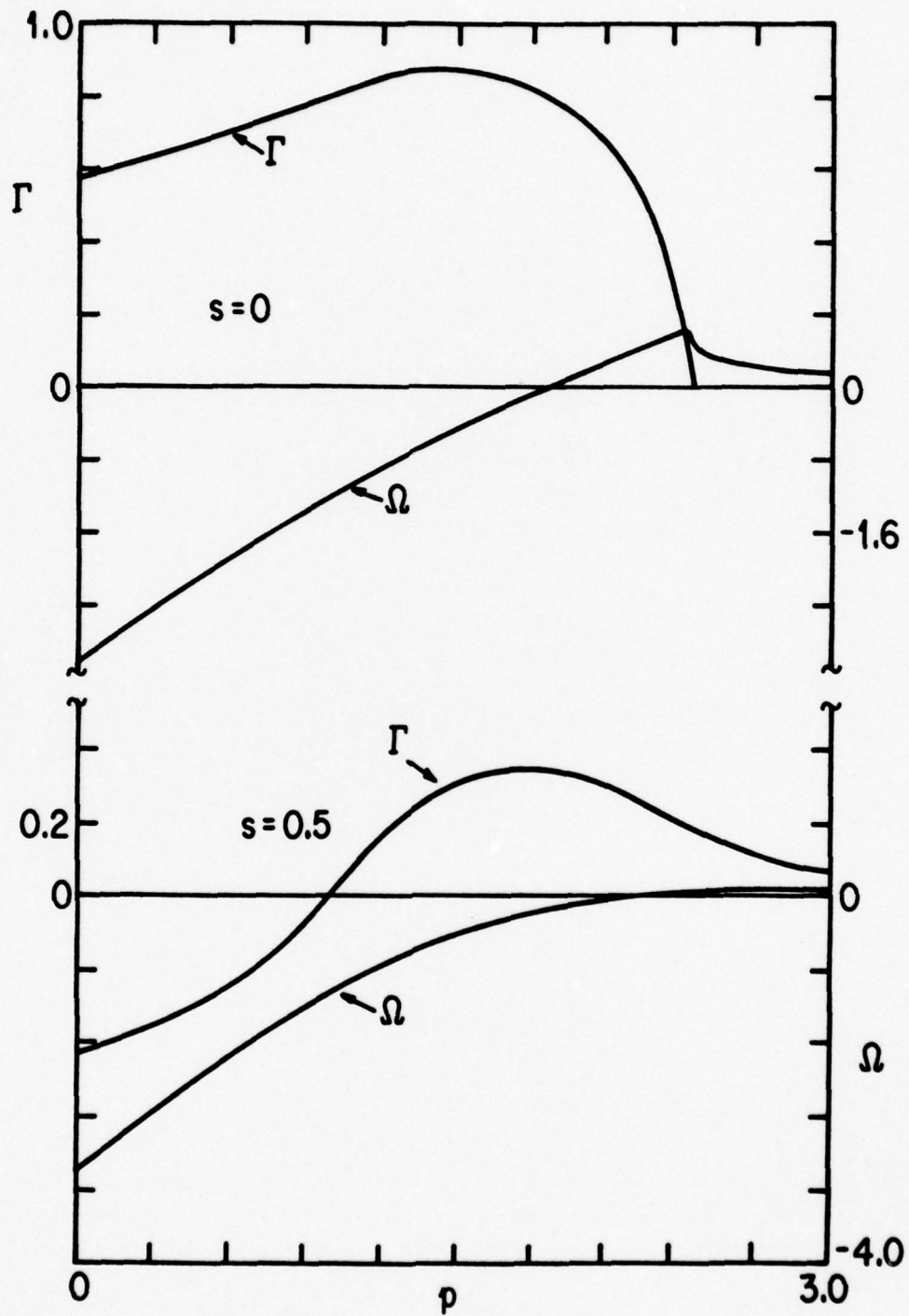


FIGURE 3

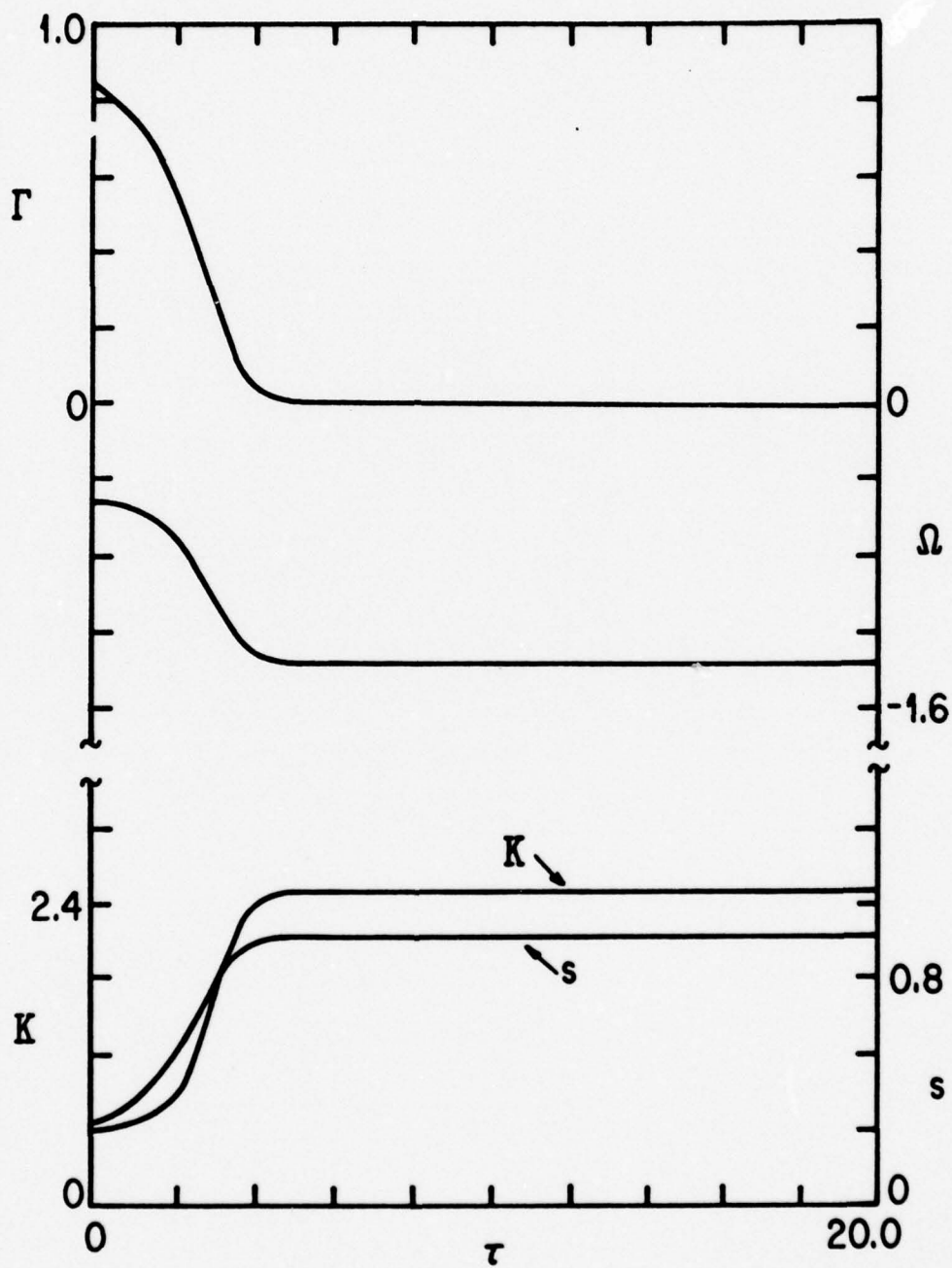


FIGURE 4

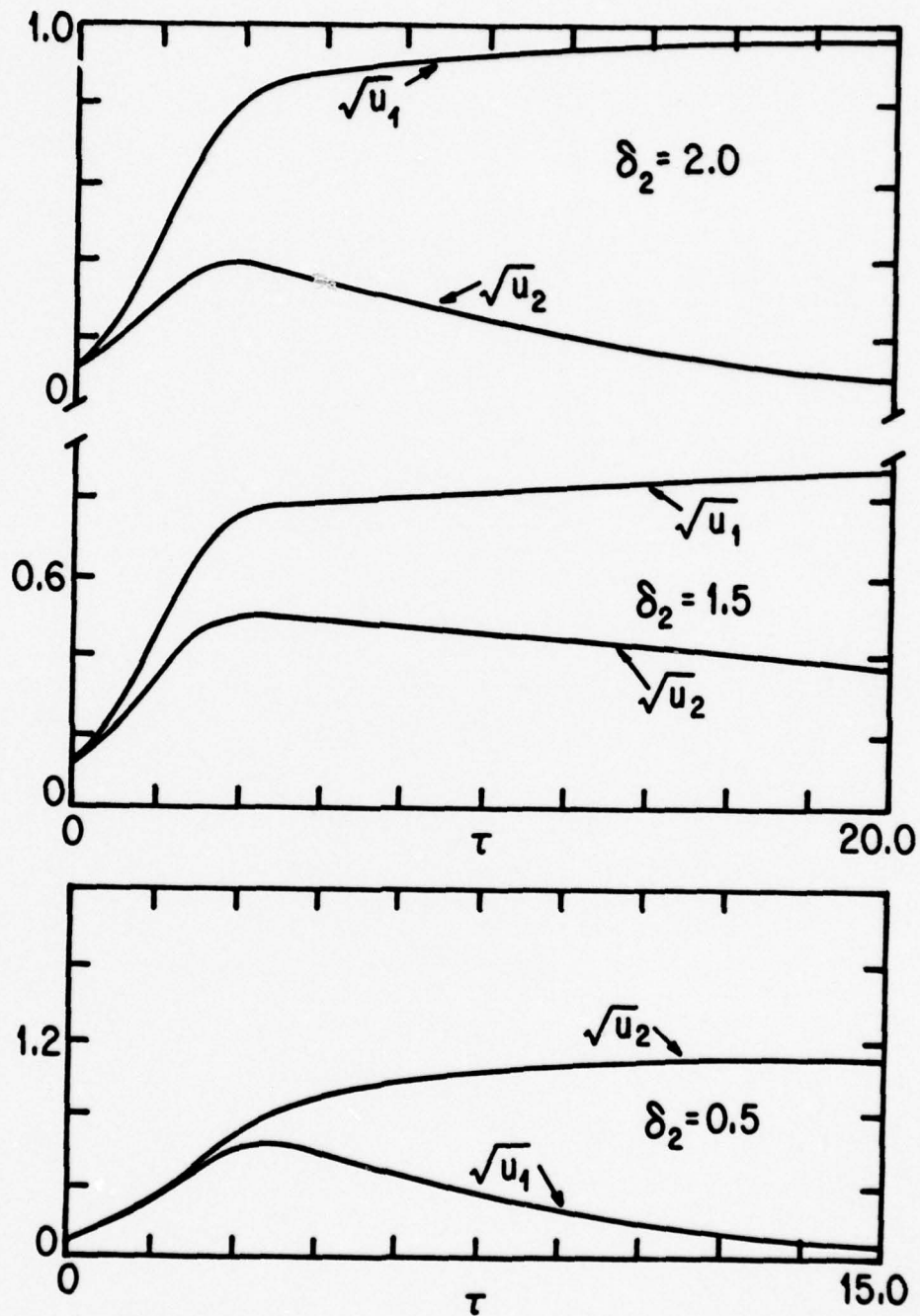


FIGURE 5

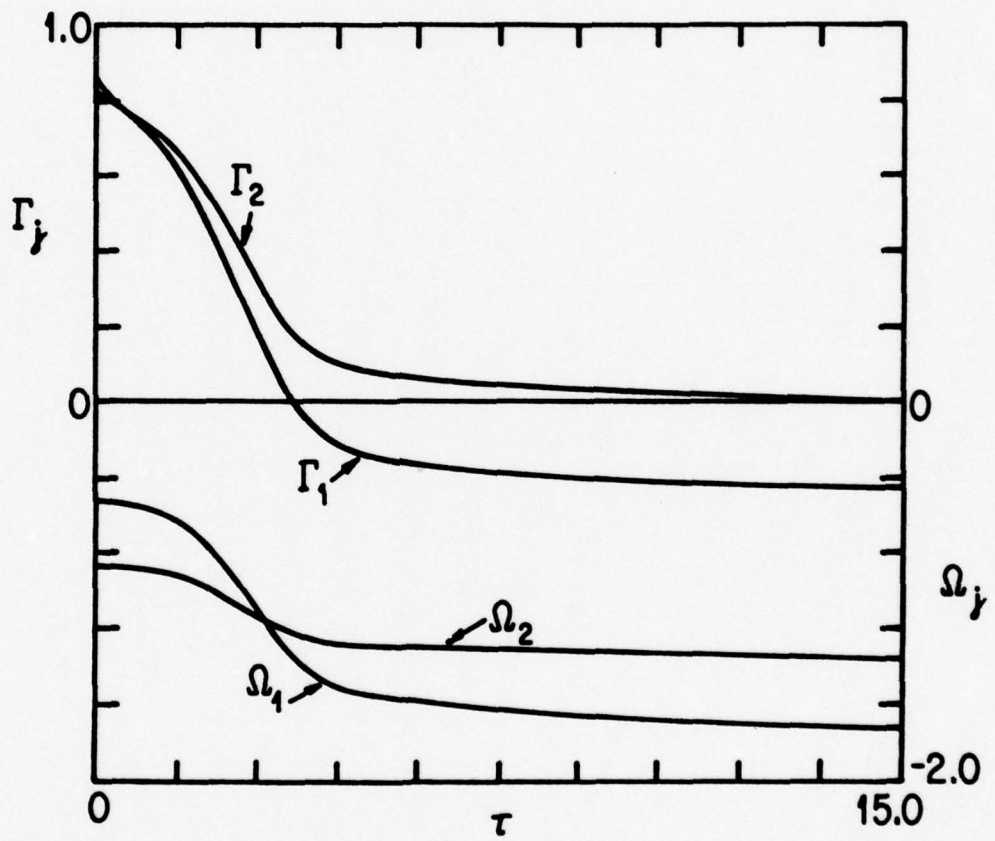


FIGURE 6

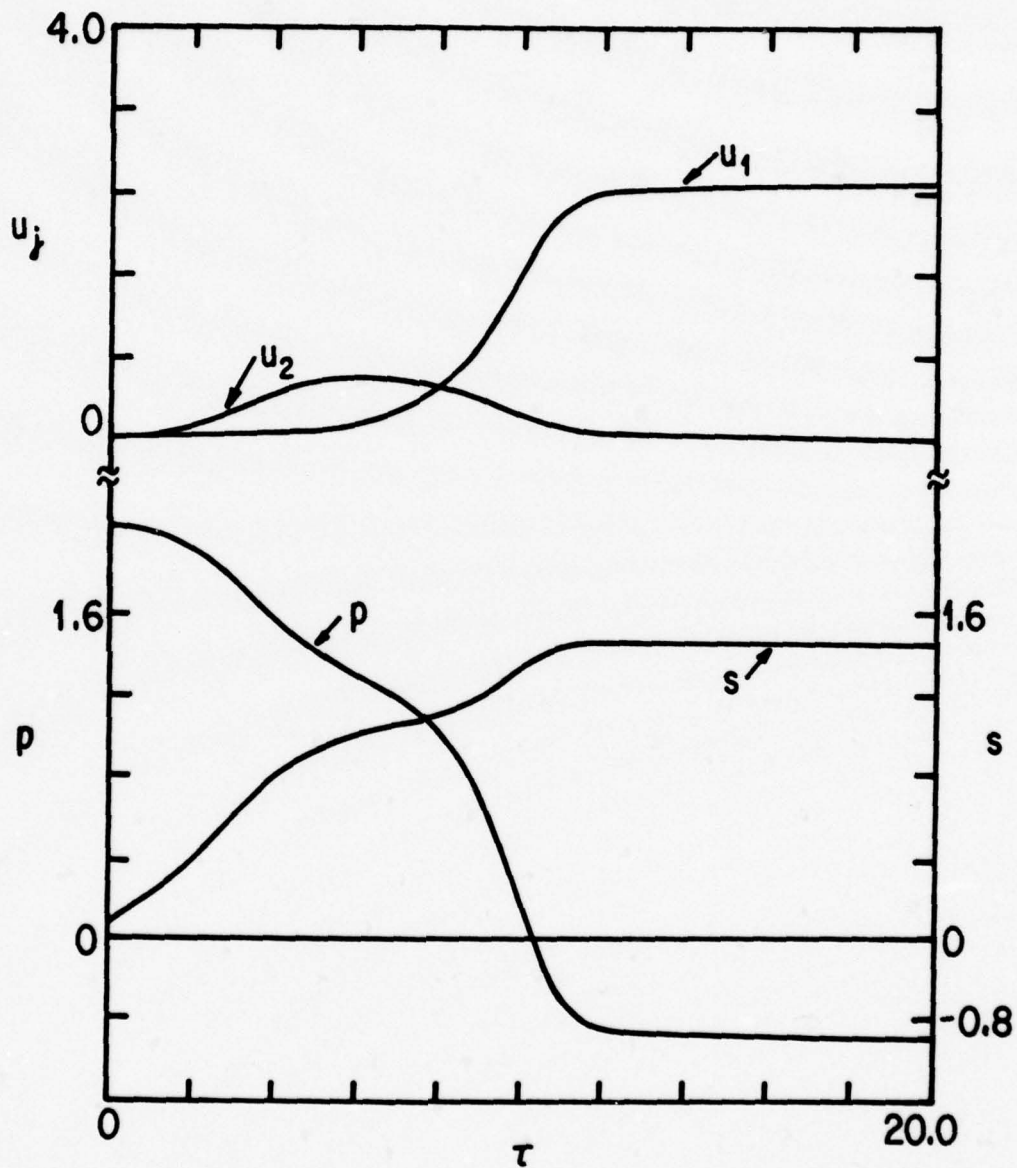


FIGURE 7

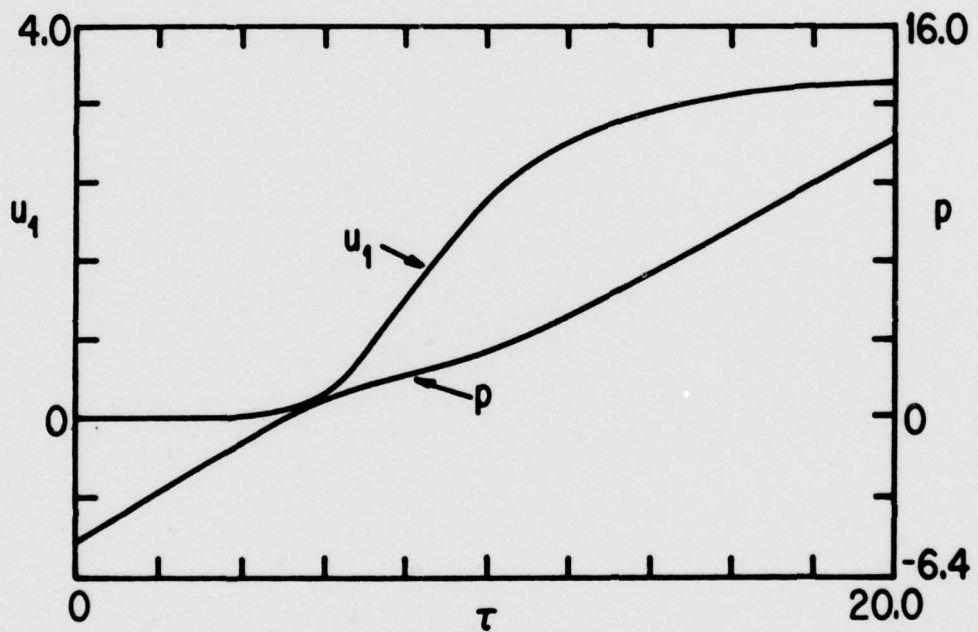


FIGURE 8

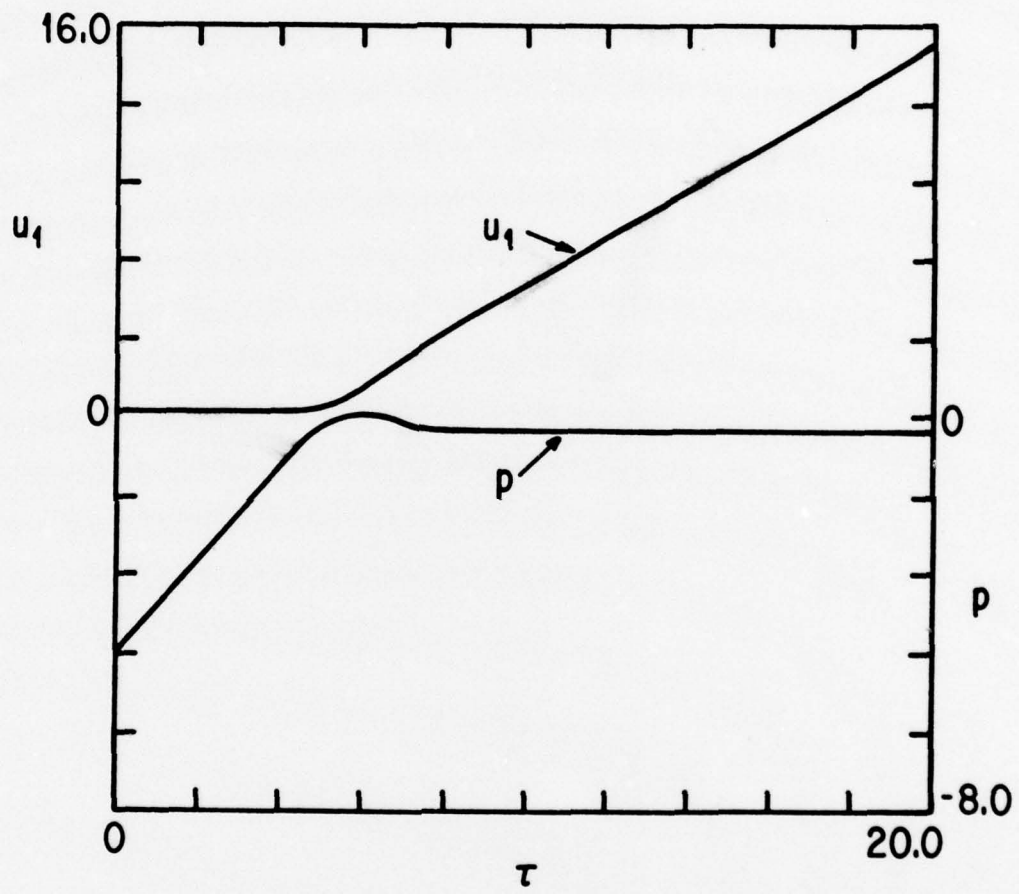


FIGURE 9

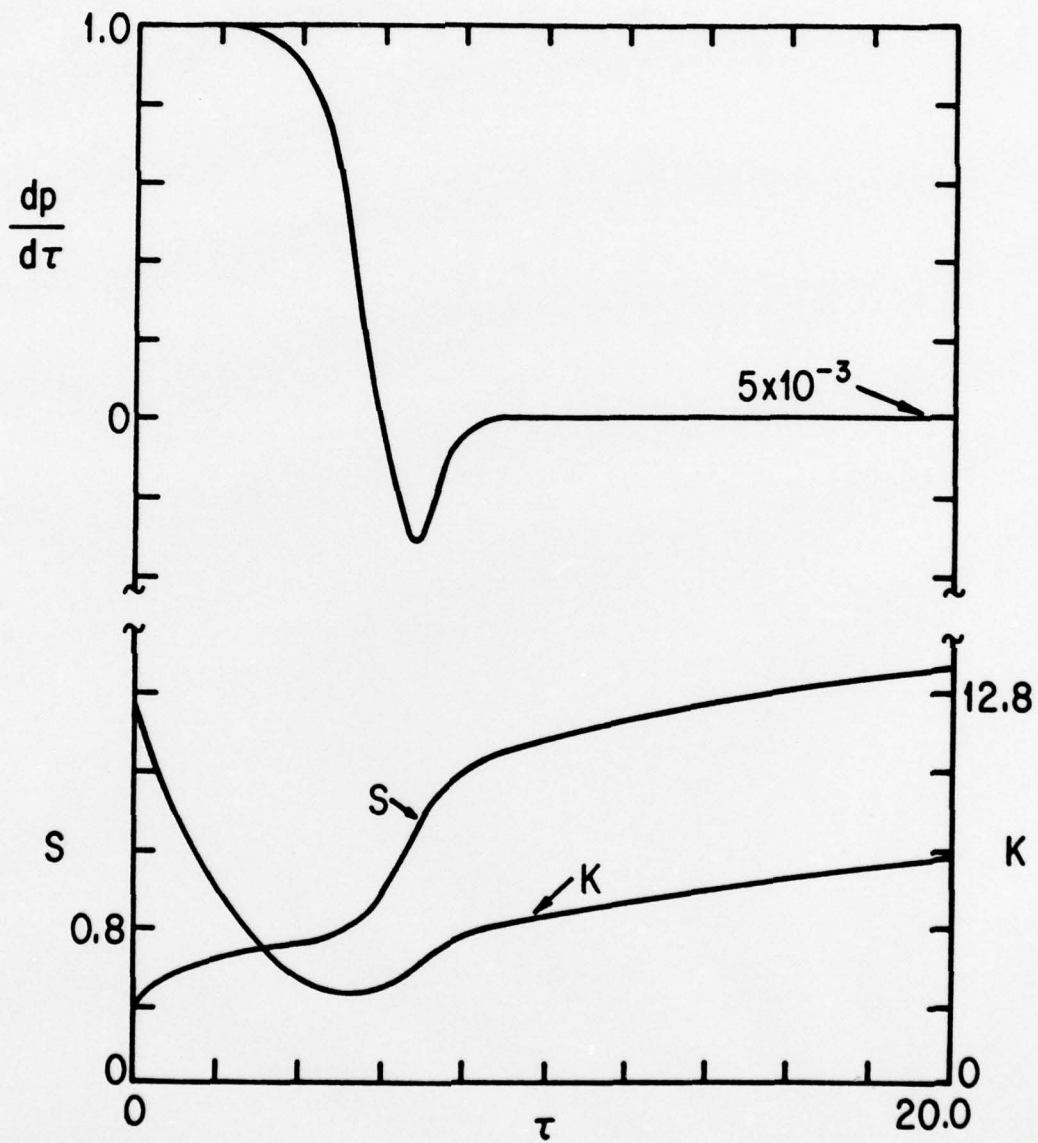


FIGURE 10

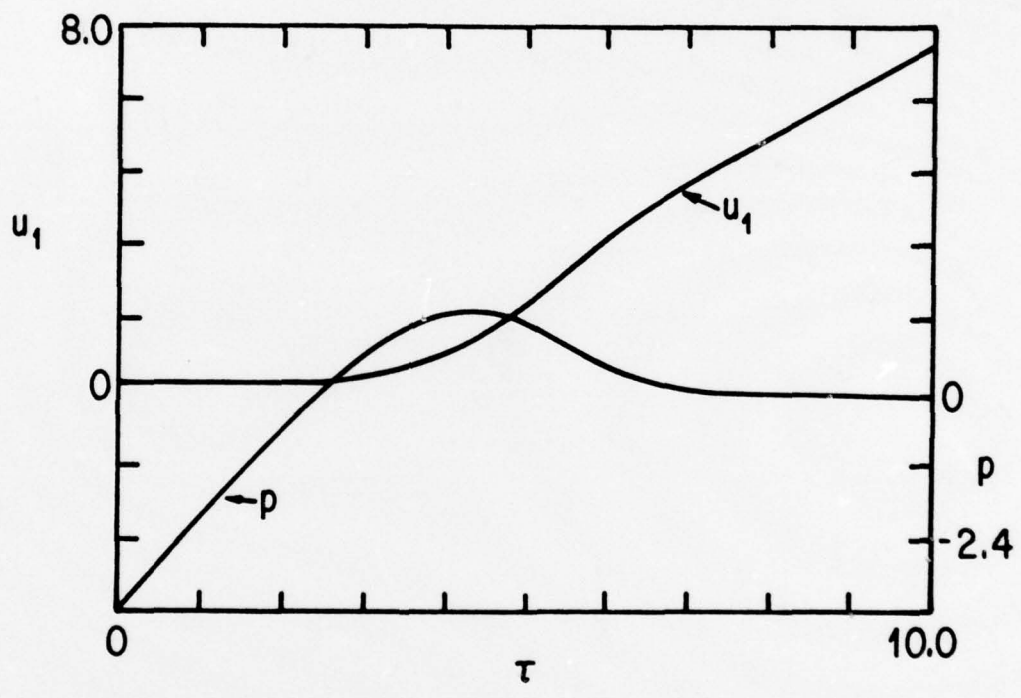


FIGURE 11

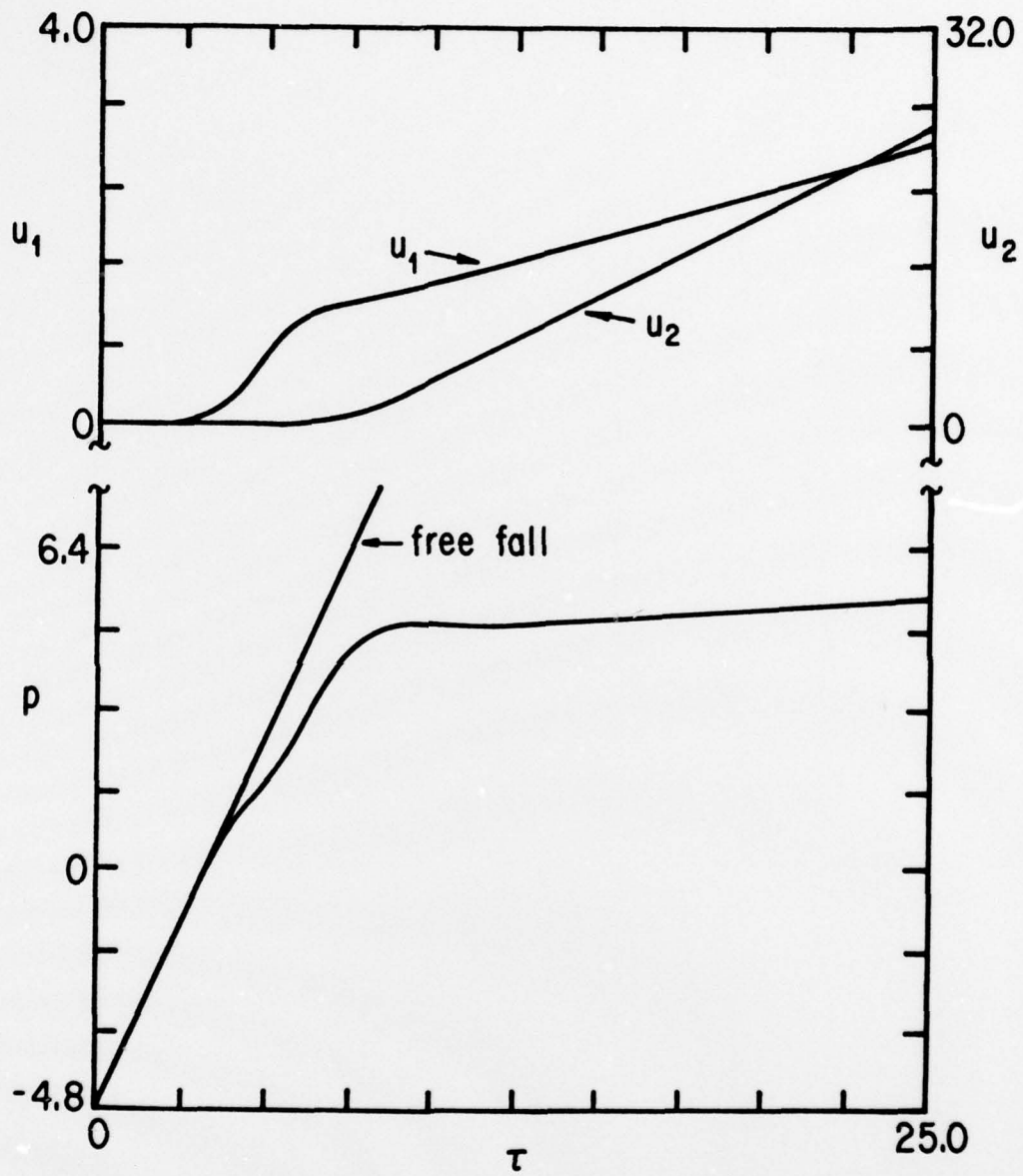


FIGURE 12

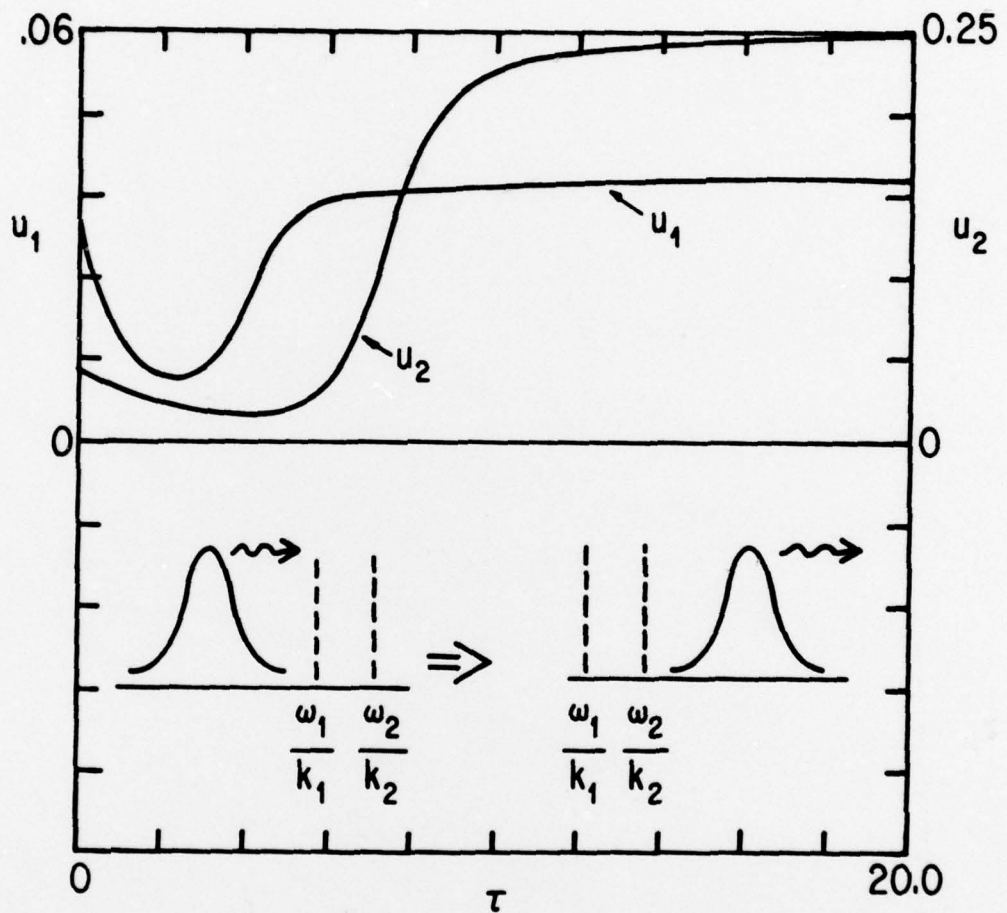


FIGURE 13

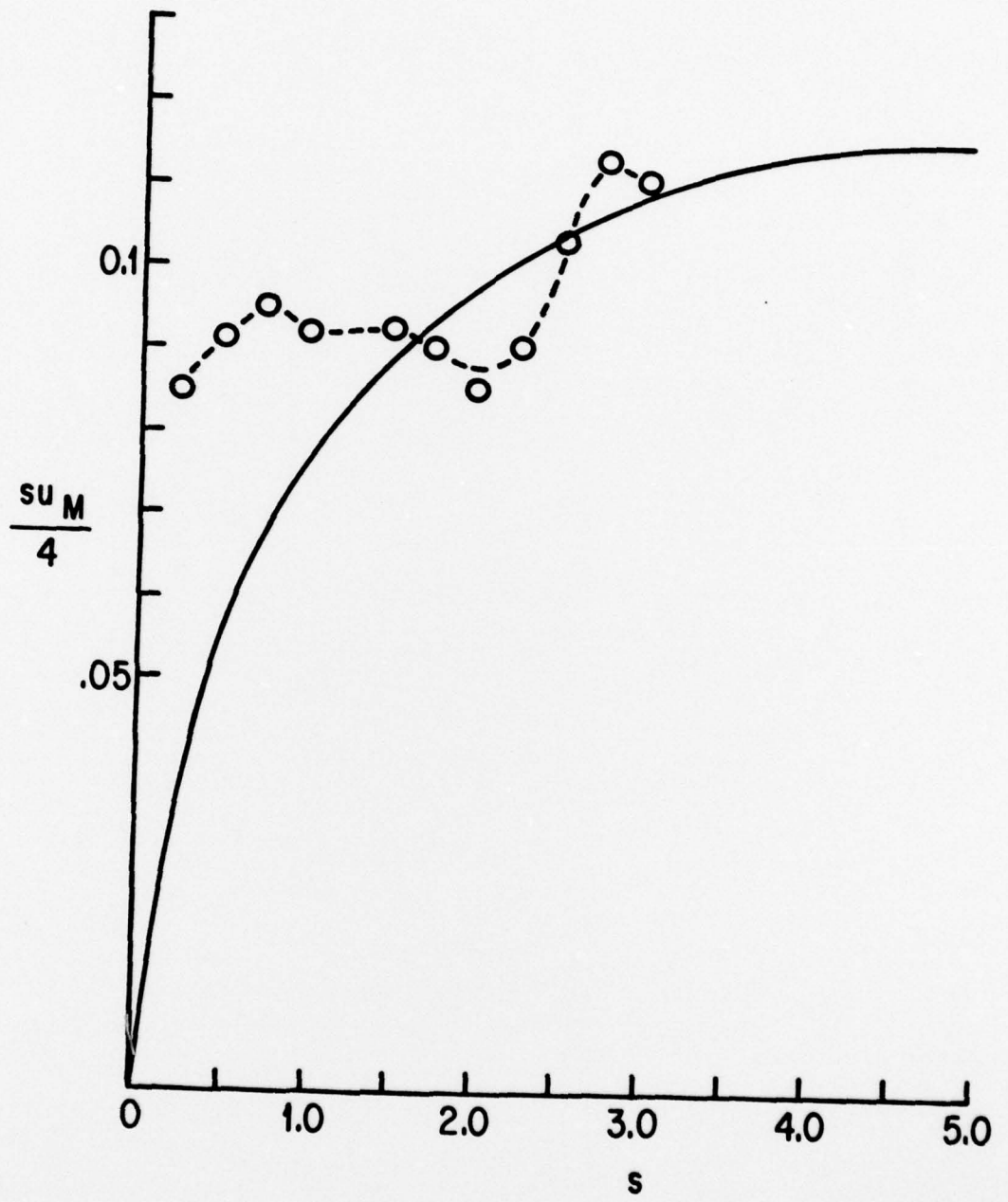


FIGURE 14

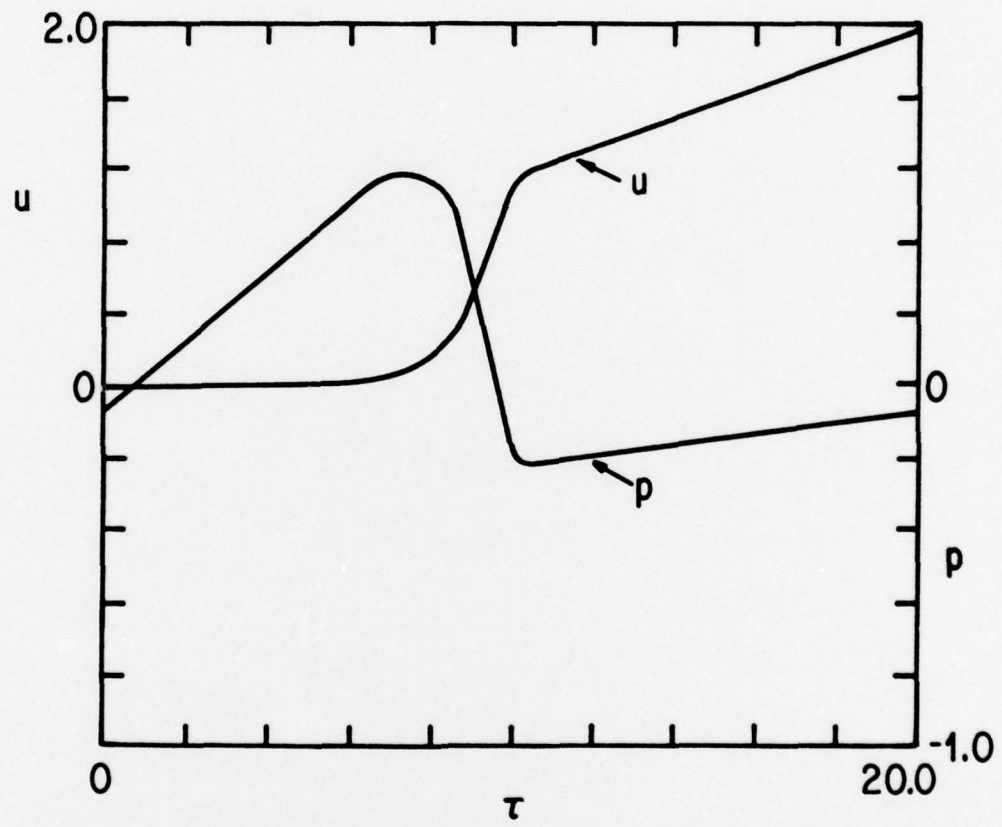


FIGURE 15

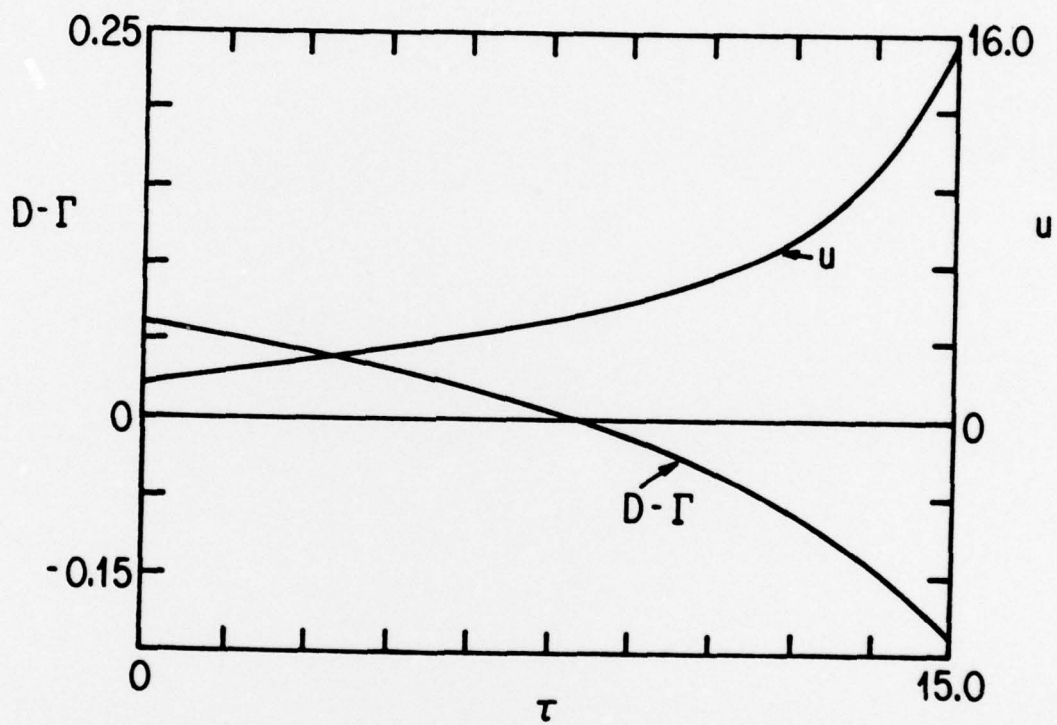


FIGURE 16

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