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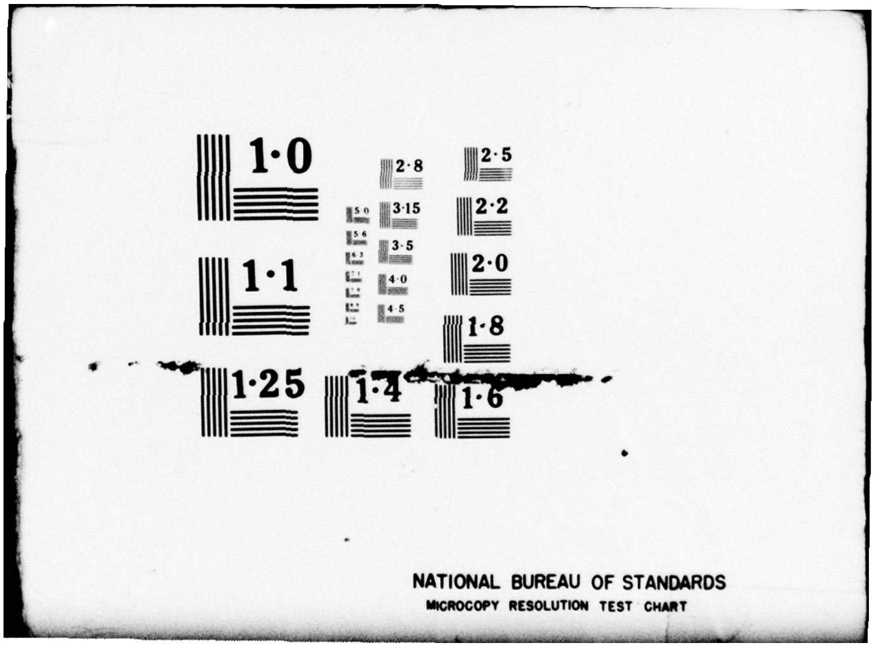

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RESONANT COLUMN TEST

by

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July 1978
Final Report

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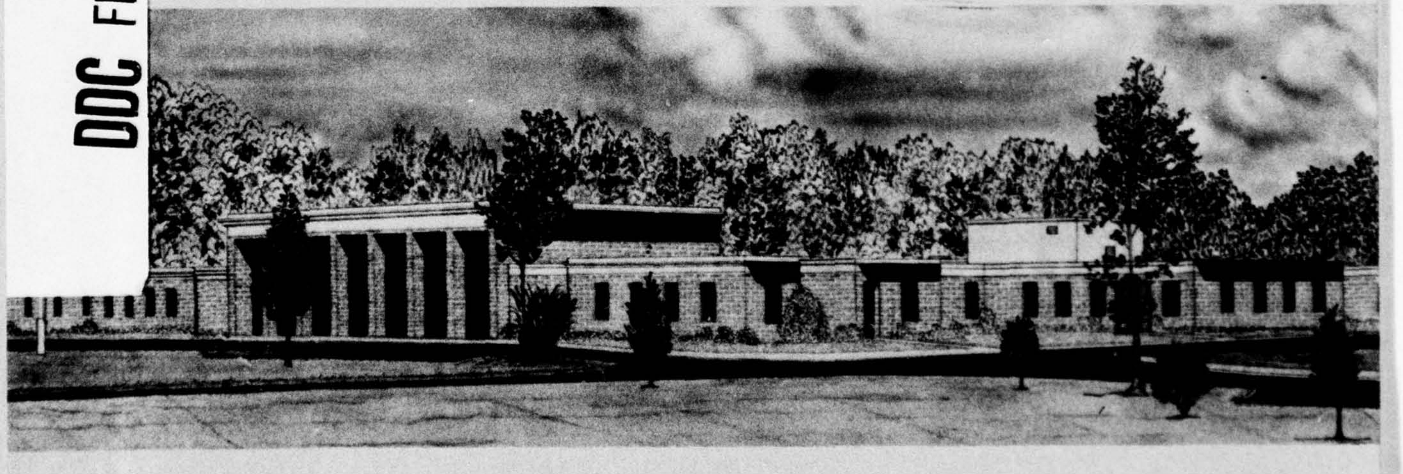

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The resonant column test is used to determine by vibration the shear modulus, shear damping, rod modulus (Young's modulus) and rod damping of cylindrical specimens of soil in the undisturbed and remolded conditions. The vibration apparatus, apparatus calibration, and calculations are described. The reduction of all resonant column test data is presented in a computer program.

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PREFACE

The study reported herein was conducted under Contract No. DACW39-77-M-1687. The work was administered under the direction of the Geotechnical Laboratory of the U. S. Army Engineer Waterways Experiment Station (WES), Vicksburg. Miss. Dr. Vincent P. Drnevich, Soil Dynamics Instruments, Inc., prepared the report.

Director of the WES during the preparation and publication of this report was COL John L. Cannon, CE. Technical Director was Mr. F. R. Brown.

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CONVERSION FACTORS, U. S. CUSTOMARY TO METRIC (SI) UNITS OF MEASUREMENT

U. S. customary units of measurement used in this report can be converted to metric (SI) units as follows:

Multiply	By	To Obtain
degrees	0.01745329	radians
feet	0.3048	metres
inches	25.4	millimetres

RESONANT COLUMN TEST

1. INTRODUCTION

These methods cover the determination of the shear modulus, shear damping, rod modulus (commonly referred to as Young's modulus), and rod damping of cylindrical specimens of soil in the undisturbed and remolded conditions by vibration using the resonant column. The vibration of the specimen may be superposed on a controlled ambient state of stress in the specimen. The vibration apparatus and specimen may be enclosed in a triaxial chamber and subjected to an all around pressure and axial load. In addition, the specimen may be subjected to other controlled environment conditions (for example temperature). These methods for modulus and damping determination are considered nondestructive when the strain amplitudes of vibration are less than 10↑-4 in./in. and many measurements may be made on the same specimen at a given ambient stress and at various other states of ambient stress. These methods cover the determination of the modulus and damping , the necessary vibration, and specimen preparation procedures related to the vibration, etc., and the application, measurement and control of the ambient stress. The application, measurement, and control of ambient stress is quite similar to that given in Appendix X: Triaxial Compression Tests.* The description of apparatus, apparatus calibration, and calculations (including the enclosed computer program) have been

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* Engineering Manual 1110-2-1906, "Laboratory Soils Testing," November 1970.

adapted from Drnevich et al.*

2. SIGNIFICANCE

The modulus and damping of a given soil, as measured by the resonant column, depend upon the strain amplitude of vibration, the ambient state of effective stress, and the void ratio of the soil as well as other, less significant factors. The applicability of the results to the field conditions will depend on the degree to which the application and control of the ambient stresses and the void ratio, as well as other parameters, duplicate field conditions.

3. DEFINITIONS

a. Resonant Column. - A cylindrical specimen or column of soil that has platens attached to each end as shown in Figure 1. A sinusoidal vibration excitation device is attached to the active end platen. The other end is the passive end platen. It may be rigidly fixed or its mass and rotational inertia must be known. The vibration excitation device may incorporate springs and dashpots of known characteristics connected to the active end platen. Vibration excitation may be longitudinal or torsional. A given apparatus may have the capability of one or the other, or both. The mass and rotational inertia of the active end platen and portions of the vibration excitation device moving with it

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* Drnevich, V.P., Hardin, B.O., and Shippy, D.J.,"Modulus and Damping of Soils by the Resonant Column Method," ASTM Symposium on Dynamic Soil and Rock Testing in the Field and Laboratory for Seismic Studies, Denver, Colorado, June, 1977.

must be known. Transducers are used to measure the vibration amplitudes for both types of motion at the active end and also at the passive end if it is not rigidly fixed. The frequency of excitation will be adjusted to produce resonance of the system, composed of the specimen and its attached platens and vibration excitation device.

b. System Resonant Frequency. - The definition of system resonance depends on both apparatus and specimen characteristics.

(1) Where the passive end platen is fixed, motion at the active end is used to establish resonance which is defined as the lowest frequency for which the sinusoidal excitation force (or moment) is in phase with the velocity of the active end platen.

(2) For the case where platen mass (or passive end platen rotational inertia) is greater than 100 times the corresponding value of the specimen and is not rigidly fixed, resonance is the lowest frequency for which the sinusoidal excitation force (or moment) is 180 degrees out of phase with the velocity of the active end platen.

(3) Otherwise, motion at the passive end is used to establish resonance which is the second lowest frequency for which the sinusoidal excitation force (or moment) is in phase with the velocity of the passive end platen. (The lowest frequency for this condition is not used because it does not produce significant strains in the specimen.)

(4) In general, the resonant frequency for torsional excitation will be different from the system resonant frequency for longitudinal excitation.

c. Ambient Stress. - Static confining stresses (whether isotropic or anisotropic) applied to the specimen during the test are termed ambient stresses. The ambient stress at the time of measurement of the system resonant frequency and system damping shall be measured and recorded in accordance with Section 7.

d. Moduli and Damping Capacities. - (1) Young's modulus (herein called rod modulus), E, is determined from longitudinal vibration and the shear modulus, G, is determined from torsional vibration. The rod and shear moduli shall be defined as the elastic moduli of a uniform, linearly viscoelastic specimen of the same mass density and dimensions as the soil specimen, necessary to produce a resonant column having the measured system resonant frequency. The stress-strain relation for a steady-state vibration in the resonant column is a hysteresis loop. These moduli will correspond to the slope of a line through the end points of the hysteresis loop. Section 8 provides for computation of shear and rod moduli from the measured system longitudinal and torsional resonant frequencies, respectively.

(2) The energy dissipated by the system is a measure of the damping capacity of the soil. Damping will be defined by the rod damping ratio, D_L, and the shear damping ratio, D_T, which are analogous to the critical viscous damping ratio, c/c_{cr}, for a single degree of freedom system. The damping ratios for rod compression shall be defined by

$$D_L = 0.5(\eta\omega/E)$$

where: η = viscous coefficient for rod compression

ω = circular resonant frequency

and by

$$D = 0.5(\mu\omega/G)$$

where: μ = viscous coefficient for shear.

Values for damping determined in this way will correspond to the area of the stress-strain hysterisis loop divided by 4π times the elastic strain energy stored in the specimen at maximum strain. Methods for determining damping ratio are prescribed in 7 f. In viscoelastic theory, it is common to use complex moduli to express both modulus and damping. The complex rod modulus is given by

$$E^* = E(1 + 2iD)$$

where: i = square root of -1,

and the complex shear modulus is given by

$$G^* = G(1 + 2iD)$$

e. Cyclic Strain. - (1) The sinusoidal vibration excitation causes sinusoidal vibratory motion having equal positive and negative magnitudes.

(2) For longitudinal excitation, the strain, ϵ, for each half cycle (positive half or negative half) is the average axial strain in the entire specimen.

(3) For torsional excitation, the strain, γ, for each half cycle is the average shear strain in the specimen. In the case of torsion, shear strain in each cross section varies from zero along the axis of rotation to a maximum at the perimeter of the specimen and the average shear strain for each cross section occurs at a radius equal to 2/3 the radius of the specimen.

f. Apparatus Model and Constants. - (1) The rigidity and mass distribution of the resonant column shall be as required in Section 5 in order for the resonant column system to be accurately represented by the model shown in Figure 1.

(2) The apparatus constants are: the mass of the passive end platen, M_P, including the mass of all attachments rigidly connected to it; the rotational inertia of the passive end platen, J_P, including the rotational inertia of all attachments rigidly connected to it; similar mass, M_A, and rotational inertia, J_A, for the active end platen and all attachments rigidly connected to it such as portions of the vibration excitation device; the spring and damping constants for both the longitudinal and torsional springs and dashpots (K_L, K_T, ADC_L, ADC_T); the apparatus resonant frequencies for longitudinal vibration, f_{oL}, and torsional vibration, f_{oT}; the force/current constant, FCF, relating applied vibratory force to the current applied to the longitudinal excitation device; the torque/current

constant, TCF, relating vibratory torque to the current applied to the torsional excitation device; and factors (LCF_A, RCF_A, LCF_P, RCF_P) relating the transducer outputs to active and passive end, longitudinal and rotational motion.

4. APPARATUS

a. General. - The complete test apparatus includes: the platens for holding the specimen in the pressure cell, the vibration excitation device, transducers for measuring the response, the control and readout instrumentation, and auxiliary equipment for specimen preparation.

b. Specimen Platens. - (1) Both the active end and passive end platens shall be constructed of noncorrosive material having a modulus at least ten times the modulus of the material to be tested. Each platen shall have a circular cross section and a plane surface of contact with the specimen, except that the plane surface of contact may be roughened to provide for more efficient coupling with the ends of the specimen. Porous stones or porous metallic discs of non-corrosive materials may be used for this purpose.

(2) Pore water drainage lines may be connected to one or both platens; however, it is important that the lines, when pressurized, do not affect apparatus resonant frequency and damping or that their effect be accounted for in calibration.

(3) Pore pressure transducers may be mounted directly in the platens or may be mounted in the pore water line(s). When drained tests are performed, the transducer(s) measure the

ambient back pressures if back pressures are used. In undrained tests where pore water drainage valves are closed, the transducer(s) measure the ambient pore pressure(s) at the specimen boundaries. The transducer(s) cannot accurately measure dynamic pore pressures unless special precautions are taken in the design of the pore pressure measuring system and the pore pressure equalization time for the specimen is less than 0.01 times the period of the applied vibration (inverse of the system resonant frequency). For details of accurate pore pressure measurement see Bishop and Henkel.*

(4) The diameter of the platens shall be equal to or greater than the diameter of the specimen. The active end platen may have a portion of the excitation device, springs, and dashpots connected to it. These must be rigidly connected and are to be considered part of the platen. The passive end platen may have a mass rigidly attached to it or it may be rigidly fixed. The construction of the platens shall be such that they are essentially rigid with respect to the specimen.

c. Vibration Excitation Device. - (1) This shall be an electromagnetic device capable of applying a sinusoidal longitudinal vibration and/or torsional vibration to the active end platen to which it is rigidly coupled.

(2) The frequency of excitation shall be adjustable and controlled to within 0.1 percent. The rigidity and mass distribution of the vibration excitation device when fastened to

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* Bishop, A.H., and Henkel, D.J.,"The Triaxial Test," Edward Arnold Publishers, London, 1962, 228 pages.

the active end platen shall be such that it can be accurately represented as a rigid mass that may be attached to weightless springs and dashpots as shown in Figure 1. If weightless springs are used, the excitation device and active end platen (without the specimen in place) form a two degree-of-freedom system (one-degree-of-freedom system for devices designed for only longitudinal or only torsional motion), having undamped natural frequencies for longitudinal motion, f_{oL}, and torsional motion, f_{oT}. The device shall be constructed such that these modes of vibration are uncoupled.

(3) The excitation device shall have a means of measuring the applied current that has at least a 5 percent accuracy. The voltage drop across a fixed, temperature and frequency stable, power resistor in series with the excitation device may be used for this purpose. The force/current and torque/current factors for the vibration excitation devices must be linear within 5 percent for the entire range of operating frequencies anticipated when testing soils.

d. Sine Wave Generator. - (1) An electric instrument capable of producing a sinusoidal current with a means of adjusting the frequency over the entire range of operating frequencies anticipated. This instrument shall provide sufficient power to produce the required vibration amplitude, or its output may be electronically amplified to provide sufficient power. The total distortion of the signal applied to the excitation device shall be less than 3 percent. The sensitivity of the frequency adjustment shall be less than 0.1 percent.

e. Vibration Measuring Devices and Readout Instruments. - (1) The vibration measuring devices shall be acceleration, velocity, or displacement transducers that can accurately measure the motion of the active and passive end platens. A transducer is not needed on the passive end platen if it rigidly fixed.

(2) On each platen, one transducer shall produce a calibrated electrical output that is proportional to the longitudinal acceleration, velocity, or displacement of that platen (not required for torsion only apparatus). The other transducer shall produce a calibrated electrical output that is proportional to the rotational acceleration, velocity or displacement (not required for longitudinal only apparatus). The readout instrument and transducers shall have a sensitivity such that a displacement of 10↑-5 in. (2.5 x 10↑-7 m) and a rotation of 10↑-5 radian can be measured with 10 percent accuracy for the entire range of frequency anticipated.

(3) It is also necessary to have an x-y-time oscilloscope available for observing signal waveforms and for establishing the system resonant frequency. This oscilloscope must have at least one amplifier (vertical or horizontal) with sufficient gain to observe the motion transducer output over the entire range of output voltages and frequencies anticipated.

(4) For measurement of damping by the free vibration method, and for calibration of the apparatus damping, the readout instrument shall be capable of recording the decay of free vibration. Either a strip chart recorder with appropriate response time and chart speed or an oscilloscope and camera may

be used for this purpose.

f. Support for Vibration Excitation Device. - For the special case where the vibration excitation device and active end platen are placed on top of the specimen, it may be necessary to support all or a portion of the weight of the platen and excitation device to prevent excessive axial stress or compressive failure of the specimen. This support may be provided by a spring, counter balance weights or pneumatic cylinder as long as the supporting system does not prevent axial movement of the active end platen and it does not alter the vibration characteristics of the excitation device.

g. Temporary Platen Support Device. - This may be any clamping device that can be used to support one or both end platens during attachment of vibration excitation device to prevent specimen disturbance during apparatus assembly. This device is to be removed prior to the application of vibration.

h. Device(s) for Specimen Dimension Measurement. - Vernier calipers, girth measuring tapes, or dial indicators may be used for measuring the physical dimensions of the specimen as long as they do not disturb the specimen and are of sufficient accuracy to give the specimen dimensions to within 1 % of the dimensions being measured.

i. Weighing Device. - The weighing device shall be suitable for weighing soil specimens. Specimens of less than 100 g shall be weighed to the nearest 0.01 g, whereas specimens of 100 g or larger shall be weighed to the nearest 0.1 g.

j. Specimen Preparation and Triaxial Equipment. - Apparatus described in Appendices X or XI may be used for specimen preparation and application of ambient stresses. Additional miscellaneous apparatus and tools may be used for these purposes as required.

5. APPARATUS CALIBRATION

a. Motion Transducers. - (1) Motion transducers shall be calibrated with each other and with an independent method to insure calibration accuracy within 5 percent.

(2) Linear motion transducers whose axes are located fixed distances from the axis of rotation may be used to measure rotational motion if the cross-axis sensitivities of the transducers are less than 5 percent. For this case the distance between the axis of rotation and the transducer axes shall be known to within 5 percent.

(3) The calibration factors for longitudinal motion shall be expressed in terms of peak meters/peak volt. The calibration factors for rotational motion shall be expressed in terms of peak radians/peak volt. This means that for velocity and acceleration transducers, the vibration frequency shall be included as a term in the calibration factor. For velocity transducers, the calibration factors are given by

$$\text{Displ. Cal. Fact.} = \text{Vel. Cal. Fact.}/(2\pi f)$$

and for acceleration transducers, the calibration factors are given by

$$\text{Displ. Cal. Fact.} = \text{Accel. Cal. Fact.}/(2\pi f)\uparrow 2$$

Thus, for velocity and acceleration transducers, the calibration factors will not be constants but will vary with measured frequency, f.

(4) Calibration factors for longitudinal motion are given by the symbol LCF with a subscript A or P denoting whether the transducer is associated with the active end platen or passive end platen. Likewise, the calibration factors for rotational motion will be given by the symbol RCF and will have subscripts A or P depending on their location.

b. Passive End Platen Mass and Rotational Inertia. - (1) The mass and rotational inertia of the passive end platen shall be determined with all transducers and other rigid attachments securely in place. The mass, M_P, is calculated from

$$M_P = W_P/g$$

where: W_P = the weight of the passive end platen and its attachments

g = the acceleration of gravity

(2) The rotational inertia of the concentric solid cylindrical components of the passive end platen and its attachments is given by

$$(J_P)_1 = \frac{1}{8g} \sum_{i=1}^{n} W_i \, d_i \uparrow 2$$

where: W_i = weight of the i-th solid cylindrical component

d_i = diameter of the i-th solid cylindrical component

n = number of solid cylindrical components

(3) Transducers and other masses attached to this platen can be accounted for by

$$(J_P)_2 = \frac{1}{g} \sum_{i=1}^{n} W_i \, r_i \uparrow 2$$

where: W_i = weight of the i-th component

r_i = distance from the platen axis to the center of mass for the i-th component

n = number of components attached to passive end platen and not covered in the determination of $(J_P)_1$

(4) The total rotational inertia for the passive end is given by

$$J_P = (J_P)_1 + (J_P)_2$$

c. Active End Platen Mass and Rotational Inertia. - (1) The mass, M_A, and rotational inertia, J_A, of the active end platen shall be determined with all transducers and rigid attachments, including attached portions of the vibration excitation device, securely in place.

(2) If all components included above have simple geometry, then the equations of 5 b. may be used to obtain the mass and rotational inertia.

(3) An alternate procedure involves the use of a metal calibration rod of known torsional and axial stiffness. One end of the rod shall be rigidly fixed and the other end shall be rigidly fastened to the active end platen. Since it may be very difficult to fasten the calibration rod to the platen without adding mass or rotational inertia, it is recommended that the calibration rod be permanently fastened by welding, etc., to an auxiliary platen identical in all respects to the one to be used in testing. The axial and torsional stiffness of the calibration rod should be chosen such that the system resonant frequency with the calibration rod in place is near the middle of the range of system resonant frequencies anticipated for soil testing. Several calibration rods may be necessary to account for different specimen sizes.

(4) With the calibration rod in place, determine the low amplitude system resonant frequencies for longitudinal vibration, $(f_{rod})_L$ and for torsional vibration, $(f_{rod})_T$. The mass of the active end platen system is calculated from

$$M_A = \frac{(K_{rod})_L}{(2\pi)^2 \left[(f_{rod})_L^2 - f_{oL}^2 \right]}$$

where: $(K_{rod})_L$ = longitudinal stiffness of calibration rod $[(AE)/L]$

A = calibration rod cross-sectional area

E = Young's modulus for calibration rod material

L = length of calibration rod

f_{oL} = apparatus longitudinal resonant frequency as described in 5 d.

The rotational inertia of the active end platen system is calculated from

$$J_A = \frac{(K_{rod})_T}{(2\pi)^2 [(f_{rod})_T^2 - f_{oT}^2]}$$

where: $(K_{rod})_T$ = torsional stiffness of calibration rod $[(I_p G)/L]$

I_p = polar moment of inertia of the calibration rod $[(\pi d^4)/32]$

d = calibration rod diameter

G = shear modulus for calibration rod material

f_{oT} = apparatus torsional resonant frequency as described in 5 d.

The above equations assume that the mass and rotational inertia of the calibration rods are much less than the corresponding values for the active end platen system.

(5) A second alternate procedure is to couple the metal calibration rod to the platens in place of the specimen and then use the procedures in Section 8 to backfigure the active end

inertias from the known moduli of the rod.

d. Apparatus Resonant Frequencies and Spring Constants. - (1) Apparatus resonant frequencies and spring constants are defined only for those apparatus that have springs attached to the active end platen system (See 7d.). To determine the resonant frequencies , set up the apparatus complete with active end platen and O-rings but no specimen. Vibrate at low amplitude and adjust the frequency of vibration until the input force is in phase with the velocity of the active end platen system. For longitudinal vibration, this apparatus resonant frequency is f_{oL} and for torsional vibration it is f_{oT} .

(2) The longitudinal and torsional apparatus spring constants (K_{SL}, K_{ST}), may be calculated from

$$K_{SL} = (2\pi f_{oL})\uparrow 2 \; M_A$$

$$K_{ST} = (2\pi f_{oT})\uparrow 2 \; J_A$$

where M_A and J_A are defined in 5 c.

e. Apparatus Damping Factors. - (1) To measure the damping constants for the apparatus, attach the same masses as used for the determination of apparatus resonant frequencies. For apparatus without springs attached to the active end platen, insert the calibration rod described in 5 c. With the apparatus vibrating at the resonant frequency, cut off the power to the excitation device and record the decay curve for the vibration of the apparatus. From the decay curve, compute the logarithmic decrement, δ, as follows

$$\delta = \frac{1}{n} \ln \frac{A_1}{A_{n+1}}$$

where: A_1 = the amplitude of vibration for the first cycle after the power is cut off

A_{n+1} = the amplitude for the (n+1)th cycle

(2) The apparatus damping coefficient, ADC_L, from longitudinal vibration shall be given by

$$ADC_L = 2f_L M_A \delta_L$$

where: f_L = the longitudinal motion resonant frequency measured during apparatus damping determination

M_A = active end platen mass from 5 c.

δ = logarithmic decrement for longitudinal motion

(3) For torsional motion, the apparatus damping coefficient, ADC_T, is given by

$$ADC_T = 2f_T J_A \delta_T$$

where: f_T = the torsional motion resonant frequency measured during apparatus damping determination

J_A = active end rotational inertia from 5 c.

δ_T = logarithmic decrement for torsional motion

f. Force/Current and Torque/Current. - (1) For apparatus without springs attached to the active end platen, insert the calibration rod as described in 5 c. Determine the resonant frequency of this single degree-of-freedom system consisting of the active end platen and apparatus spring or calibration rod by use of the same procedure as described in 7 d.

(2) Then set the frequency to 0.707 times the resonant frequency and apply sufficient current to the vibration excitation device so that the vibration transducer output to the readout device has a signal-to-noise ratio of at least 10. Read and record the output of both active end vibration transducer and current measuring instrument.

(3) Next set the frequency to 1.414 times the system resonant frequency and obtain the vibration transducer and current instrument readings in a similar fashion to those at 0.707 times the resonant frequency.

(4) Calculate C_1 and C_2 from

$$C_1 = \frac{(VTCF)_A\ (TO1)}{2(CR1)}$$

$$C_2 = \frac{(VTCF)_A\ (TO2)}{CR2}$$

where: $(VTCF)_A$ = Active end vibration transducer calibration factor (LCF_A or RCF_A) depending on whether vibration is longitudinal or torsional.

TO1 = Active end transducer output at 0.707 x resonant frequency

CR1 = Current instrument reading at 0.707 x resonant frequency

TO2 = Active end transducer output at 1.414 x resonant frequency

CR2 = Current instrument reading at 1.414 x resonant frequency

C_1 and C_2 should agree within 10 percent.

(5) By use of C_1 and C_2 from longitudinal vibration, the Force/Current calibration factor, FCF, is obtained from

$$FCF = 0.5(C_1 + C_2)K_{SL}$$

where: K_{SL} = apparatus spring constant (or for apparatus without springs, the calibrating rod spring constant) for longitudinal motion

(6) By use of C_1 and C_2 from torsional vibration, the Torque/Current calibration factor, TCF, is obtained from

$$TCF = 0.5(C_1 + C_2)K_{ST}$$

where: K_{ST} = apparatus spring constant (or for apparatus without springs, the calibrating rod spring constant) for torsional motion.

6. TEST SPECIMENS

a. General. - These methods cover only the special specimen preparation procedures related to the vibration and resonant column technique. Since the resonant column test may be conducted in conjunction with controlled ambient stresses, the provisions for preparation of specimens in Appendix X or Appendix XI may be applicable or may be used as a guide in connection with other methods of application and control of ambient stresses.

b. Specimen Size. - (1) Specimens shall be of uniform circular cross section with ends perpendicular to the axis of the specimen.

(2) Specimens shall have a minimum diameter of 33 mm (1.3 in.) and the largest particle contained within the test specimen shall be smaller than one-tenth of the specimen diameter except that for specimens having a diameter of 70 mm (2.8 in.) or larger, the largest particle size shall be smaller than one-sixth of the specimen diameter. If, after completion of a test, it is found that larger particles than permitted are present, indicate this information in the report of test data under "Remarks."

(3) The length-to-diameter ratio shall be not less than 2 (this may be changed to 1 for torsional vibration only) nor more than 7, except that when an ambient axial stress greater than the ambient lateral stress is applied to the specimen, this ratio shall be between 2 and 3.

(4) Measure the length and diameter of the specimen using a Vernier caliper or other suitable device. Determine the weight of the test specimen. For determination of moisture content

(Appendix I), secure a representative sample of the cuttings from undisturbed specimens, or of the extra soil for remolded specimens, placing the sample immediately in a covered container.

c. End Coupling for Torsion. - (1) For torsional motion, complete coupling of the ends of the specimen to the specimen cap and base must be assured. Complete coupling for torsion may be assumed if the mobilized coefficient of friction between the end platens and the specimen is less than 0.2 for all shear strain amplitudes. (2) The coefficient of friction is approximately given by

$$\text{Coeff. of Frict.} = \gamma G/\sigma_a'$$

where: γ = shear strain amplitude (see Section 8m.)

G = shear modulus (see Section 8l.)

σ_a' = effective axial stress

When this criterion is not met, other provisions such as the use of razor blade vanes, adhesives, etc. must be made in order to assure complete coupling.

(3) The effectiveness of the coupling provisions shall be evaluated by testing two specimens of the same material but of different lengths. The lengths of these specimens shall differ by at least a factor of 1.5. The provisions for end coupling may be considered satisfactory if the values of the shear modulus for

these two specimens of different length do not differ by more than 10 percent.

d. Specimen Stiffness. - (1) The upper limit of specimen stiffness will depends on the specific apparatus being used. Refer to the recommendations of the manufacturer or to Drnevich* for methods of estimating limits of specimen stiffness.

e. Use of Side Drains. - (1) For tests on saturated or nearly saturated soils where the specimen will be consolidated, filter paper strips (Whatman No. 54 or similar) may be used around the perimeter of the specimen to speed up the consolidation and pore pressure equalization processes. Except for very soft specimens tested at low effective confining stresses, these strips should have negligible effects on the test results.

(2) Since the effect of side drains is inversely related to specimen diameter, the testing of larger diameter specimens can be used to check on the effects of side drains.

(3) For side drains to be effective, they should extend over the porous stones or discs at each end of the specimen.

7. PROCEDURE

a. General. - (1) Resonant column testing is quite similar to triaxial testing except that vibrations at the system resonant frequency are applied to the specimen instead of the deviator stress that causes failure. Consequently, the procedure for

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* Drnevich, V.P.,"Resonant Column Testing - Problems and Solutions," Proc. of the ASTM Symposium on Dynamic Soil and Rock Testing in the Field and Laboratory for Seismic Studies, STP 654, ASTM, June, 1977.

preparation of specimens in Section 4. of Appendix X. is applicable and the data sheet in Plate X-1 can be used to record the specimen data. Because the application of vibrations is relatively nondestructive, each sample can be tested at two or three confining stresses (staged testing), and thus triplicate samples are not usually needed.

(2) Once the specimen is placed in the apparatus, a confining stress is applied. Depending on the insitu conditions being simulated, the specimen may be dry, partially saturated, or saturated by means of a back pressure. Prior to the application of large amplitude vibration, the specimen may be undrained or consolidated. (Low amplitude vibrations where strain amplitudes are on the order of 0.001 percent or less are completely nondestructive and can be applied at any time.)

(3) After the confining stress has been applied (including completion of saturation and consolidation* if they are chosen), small amplitude vibrations at the system resonant frequency are applied and data are taken. Next, the amplitude of vibration is increased by 30 percent to 50 percent, the frequency is adjusted

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* It has been shown by D. G. Anderson and R. D. Woods ("Comparison of Field and Laboratory Shear Moduli," Proceedings of the Geotechnical Engineering Specialty Conference on Insitu Measurement of Soil Properties, ASCE, Raleigh, N.C., Vol. I, June 1975, pp 69-92) that for fine grained soils, the amount of time in secondary compression can significantly affect test results. To obtain a measure of this effect, it is recommended that low amplitude vibration be applied, the frequency be adjusted to resonance, and that sets of data be taken at convenient intervals of time during the consolidation process and to at least one log cycle of time into secondary compression. After this, testing at larger strain amplitudes can commence.

to resonance and another set of data are taken. This process is repeated until the desired amplitude is achieved or the vibration capacity of the apparatus is reached. Then, the amplitude of vibration is reduced to the level used at the beginning of the test, the frequency is adjusted to resonance, and another set of data is taken.

(4) The amount of disturbance to the specimen caused by the high amplitude vibration can be established by comparing the low amplitude data obtained before and after high amplitude vibration. For fine grained soils, even when tested with drainage allowed, excess pore pressures may be created by the high amplitude vibration. In this situation, it is adviseable to allow the excess pore pressures to dissipate (specimen consolidates or swells). Additional sets of data at low amplitude vibration may be taken during this process. If it is apparent from the data that only a small amount of disturbance was caused by the high amplitude vibration, another confining stress may be applied to the same specimen and the process outlined above can be repeated for the new confining stress.

b. Procedure for Placement of Specimen in Apparatus. - (1) The exact procedure to be followed during test setup will depend on the apparatus and electronic equipment used and on the methods used for application, measurement, and control of the ambient stresses. However, the specimen shall be placed in the apparatus by procedures that will minimize the disturbance of the specimen.

(2) Particular care must be exercised when attaching the end platens to the specimen, when applying filter paper strips,

membranes, and O-rings, and when attaching the vibration excitation device to the platens. A temporary support device for the platens as discussed in Section 4f may be needed.

(3) For the sake of good coupling between the specimen and the platens, the use of filter paper discs at the ends of the specimen is not recommended. The filter paper strips on the perimeter of the specimen, if used, should extend beyond the ends of the specimen and over the porous discs attached to the platens.

(4) When it is desired to saturate cohesive soil specimens, the saturation process is greatly facilited if a very small diameter flexible tube is inserted between the top platen and the membrane before the top O-rings are placed. A seal is formed by use of a liberal amount of vacuum grease. A vacuum is then introduced to the space between the specimen and membrane by means of this tube. Finally, the tube is gently withdrawn and the top O-rings applied.

(5) For cases where ambient isotropic stresses are to be applied to the membrane enclosed specimen, liquid or air confining media may be used for dry or partially saturated specimens. For tests where complete saturation is important, a liquid confining media such as deaired water or mineral oil should be used. Where the vibration excitation device is located within the pressure chamber, an air-liquid interface in the chamber just below the device is acceptable as long as the liquid covers the entire membrane that encloses the specimen.

(6) The system for providing confining stress and back

pressures to the specimen is identical to that given in Fig. 16 of Appendix X. except that a pressure line must be connected between valve C on the chamber pressure reservoir and valve B on the apparatus chamber lid. During the process of filling the chamber to the desired level with confining liquid, valve A is open. After this, valve A is closed and valves B and C remain open.

(7) If undisturbed specimens are being tested and saturation by means of back pressure is desired, it is recommended that consolidaton be done before back pressuring. This will provide a less disturbing effective stress history to the specimen.

c. Electric Equipment Connections. - (1) Connect the vibration excitation device to the sine wave generator (with amplifier, if required). The power supplied to the vibration excitation device should be very low in order not to exceed the amplitude of vibration prescribed in 7a.

(2) Connect the vibration transducers to the readout instruments for the type of motion (longitudinal or torsional) to be applied. Adjust the readout instruments according to the instruction manuals for these instruments.

d. Measurement of Resonant Frequency. - (1) The procedure for measuring system resonant frequency is the same for both longitudinal and torsional vibration except that the longitudinal motion transducer is used for longitudinal motion and the rotational motion transducer is used for torsional motion.

(2) If the passive end is fixed or if P>100 (See Section 8e for definition of P), motion of the active end platen is used to establish resonance. Otherwise, motion of the passive end platen is used.

(3) With the power as low as practical, increase the frequency of excitation from a very low value (e.g. 10 Hz.) until the resonant frequency is obtained. The phase relationship describing resonance can be established by observing the Lissajous figure formed on an x-y oscilloscope with the voltage proportional to the driving current applied to the horizontal amplifier and the output from the transducer applied to the vertical amplifier. If a velocity transducer is used for vibration measurement, the system resonant frequency occurs when the figure formed is a straight, sloping line. If a displacement or acceleration transducer is used, the frequency should be adjusted to produce an ellipse with axes vertical and horizontal.

(4) For the case where the motion of the passive end platen is being used to establish resonance, the resonant frequency is the second lowest frequency for which the Lissajous figure either forms a straight sloping line (velocity transducer being used) or an ellipse with axes vertical and horizontal (accelerometer or displacement transducer being used). When a velocity transducer is used for this case, the slope of the line has the opposite sign for the second lowest frequency as it does for the lowest frequency.

(5) It is recommended that the frequency be measured with a digital electronic frequency meter and be recorded to at least

three significant figures. The system resonant frequency for longitudinal motion shall be designated f_L and that for torsional motion shall be designated f_T.

e. Measurement of Strain Amplitude. - (1) The strain amplitude measurements shall only be made at the system resonant frequencies. Thus, for a given current applied to the excitation device, the vibratory motion transducer outputs recorded at the system resonant frequency give sufficient information to calculate strain amplitude. To increase or decrease strain amplitude, the current to the vibration excitation device must be increased or decreased. After making a change in current applied to the vibration excitation device, the procedure of 7 d must be followed to establish the corresponding system resonant frequency before the transducer outputs can be used to establish the new strain amplitude value.

f. Measurement of System Damping. - (1) Associated with each strain amplitude and system resonant frequency is a value of damping. Two methods are available for measuring system damping: the steady-state vibration method and the amplitude decay method. The procedures for both methods are independent of whether longitudinal or torsional motion is under consideration.

(2) For the steady state method, the active end or the passive end vibration transducer output (depending on which end is used to establish resonance) and the current applied to the vibration excitation device must be measured at each resonant frequency. The calculations are outlined in 8 k.

(3) For the free vibration method, with the system

vibrating at the system resonant frequency, cut off the power to the vibration excitation device and record the transducer output of the transducer used in establishing resonance as a function of time. This gives the decay curve for free vibration.

g. Explanation of Data Sheet. - (1) Data acquired during the performance of a test is recorded directly on the data sheet given in Plate 1. Columns (2) through (7) of this sheet can be used to record consolidation and/or saturation data. Low amplitude, nondestructive vibration data may also be recorded during the consolidation and saturation processes.

(2) Data in column (2) is the elapsed time from the beginning of the test or from the initiation of consolidation.

(3) Data in columns (3) and (4) are the readings from the chamber pressure gage and the pore pressure transducer, respectively (See Fig. 16, Appendix X). They may be in engineering units or they may be readings to which calibration factors must be applied. For the latter case, the calibration factors for each should be noted in the "Remarks" section at the bottom of the sheet. If a differential pressure transducer (one side measuring cell pressure and the other pore pressure) is used, record the cell pressure from the pressure gage in column (3) and the differential pressure in column (4). Relabel column (4) to note the use of the differential pressure transducer.

(4) Column (5) is used to record the static axial load that is applied in addition to the ambient confining stress.

(5) Columns (6) and (7) are for specimen length change measurement readings and pore volume change measurement readings.

(6) Place an "L" or a "T" in column (8) depending on whether longitudinal or torsional excitation is being applied to the specimen.

(7) Data in columns (9) through (12) are associated with the vibration of the specimen. Column (9) is used for the reading associated with the amount of force or torque applied to the excitation device as discussed in Sections 4c, 5e, and 7f. The outputs of the active end and passive end vibration transducers are recorded in columns (10) and (11), respectively. Finally, the system resonant frequency (See paragraph d. above) is recorded in column (12).

(8) If the free vibration method is also used to measure damping, each record of amplitude decay with time must be marked to associate it with its corresponding line in the data sheet.

8. CALCULATIONS

a. General. - (1) Calculations require the apparatus calibration factors (Section 5.) and the physical dimensions and weight of the specimen. In addition, for each ambient stress, one data set is required for each vibration strain amplitude. A data set consists of the information on one line of the data sheet given in Plate 1. Additionally, it may include the free vibration amplitude decay curve.

(2) The data reduction process requires several steps, the first of which is to determine the specimen volume, length, diameter, and weight for each data set (line on the data sheet). Record the proper values of these specimen characteristics in

columns (1) through (5) of the Intermediate Calculations Sheet given in Plate 2.

(3) Items in columns (6) through (11) of this sheet are calculated according to paragraphs b. through g. below. Items in columns (8) through (11) are input parameters for the computer program given in Section 11.

(4) If the computer program is used, values in columns (12) and (13) are calculated by the program and the information in column (14) is not needed. Alternatively, items in columns (12) through (14) may be determined for most cases from equations and graphs discussed in paragraphs h. through j.

(5) Information for the Final Calculations Sheet (Plate 3), columns (2) through (5) is available from the Data Sheet (Plate 1). Column (2) is the mean effective confining stress (average of the three principal effective stresses). Column (3) is the ratio of axial effective stress to lateral effective stress. Column (4) is simply carried over from column (5) of the Data Sheet. Columns (5) through (9) are determined according to procedures given in paragraphs k. through o. below.

b. Soil Mass Density. - (1) The soil mass density, ρ, is given by

$$\rho = \frac{W}{Vg}$$

where: W = total weight of the specimen (Col. (5), Intermediate Calculations Sheet)

V = volume of the specimen (Col. (2), Intermediate

Calculations Sheet)

g = acceleration of gravity

Record the soil mass density in column (6) of the Intermediate Calculations Sheet.

c. Specimen Rotational Inertia. - (1) The specimen rotational inertia about the axis of rotation is given by

$$J = \frac{Wd^{2}}{8g}$$

where: d = diameter of the specimen (Col. (4), Intermediate Calculations Sheet)

Record the value of specimen rotational inertia in column (7) of the Intermediate Calculations Sheet.

d. Active End Inertia Factors. - (1) The active end inertia factor for longitudinal motion, T_L, is calculated from

$$T_L = \frac{M_A g}{W} \left[1 - \left(\frac{f_{oL}}{f_L} \right)^{2} \right]$$

where: M_A = mass of the active end platen system as calculated in 5c.

f_{oL} = apparatus resonant frequency for longitudinal motion from 5d. (For apparatus without springs attached to the active end platen, this term is zero.)

f_L = system resonant frequency for longitudinal motion (Col.

(12), Data Sheet)

(2) The active end inertia factor for torsional motion, T_T, is given by

$$T_T = \frac{J_A}{J}\left[1 - \left(\frac{f_{oT}}{f_T}\right)^2\right]$$

where: J_A = rotational inertia of the active end platen system as calculated in 5c.

J = specimen rotational inertia as calculated in 8c.

f_{oT} = apparatus resonant frequency for torsional motion from 5d. (For apparatus without springs attached to the active end platen, this factor is zero.)

f_T = system resonant frequency for torsional motion (Col. (12), Data Sheet)

Record the values of T_L or T_T in column (8) of the Intermediate Calculations Sheet.

e. Passive End Inertia Ratios. - (1) For longitudinal motion, the passive end inertia ratio, P_L, is given by

$$P_L = \frac{M_P g}{W}$$

where: M_P = mass of the passive end platen system as described in 5b.

(2) For torsional motion, the passive end inertia ratio, P_T, is given by

$$P_T = \frac{J_P}{J}$$

where: J_P = rotational inertia of passive end platen system as calculated in 5b.

For the special case where the passive end of the specimen is rigidly fixed, P_L and P_T are equal to infinity. For most situations, the passive end may be assumed to be rigidly fixed if the value of P is greater than 10,000 . Record passive end inertia ratio in column (9) of Intermediate Calculations Sheet.

f. Apparatus Damping Factors. - (1) For longitudinal motion, the apparatus damping factor, ADF_L, is calculated from

$$ADF_L = ADC_L/[2\pi f_L (W/g)]$$

where: ADC_L= apparatus damping coefficient for longitudinal motion as described in 5e.

(2) For torsional motion, the apparatus damping factor, ADF_T, is calculated from

$$ADF_T = ADC_T/[2\pi f_T J]$$

where: ADC_T= apparatus damping coefficient for torsional motion as described in 5e.

The apparatus damping factors are recorded in column (10) of the Intermediate Calculations Sheet.

g. Magnification Factors. - (1) These factors are used in calculating damping. For longitudinal motion, the magnification factor is calculated from

$$MMF_L = [(LCF)(LTO)/(FCF)(CR_L)](W/g)(2\pi f_L)\uparrow 2$$

where: LCF = longitudinal motion transducer calibration factor for the transducer used in establishing resonance

LTO = longitudinal motion transducer output (Col. (10) or (11), Data Sheet depending on which end is used to establish resonance.)

FCF = force/current factor from 5f.

CR_L = current reading to longitudinal excitation system (Col. (9), Data Sheet)

(2) For torsional motion, the magnification factor is calculated from

$$MMF_T = [(RCF)(RTO)/(TCF)(CR_T)]J(2\pi f_T)\uparrow 2$$

where: RCF = rotational transducer calibration factor for the transducer used in establishing resonance

RTO = rotational transducer output (Col. (10) or (11), Data Sheet depending on which end is used to establish resonance.)

TCF = torque/current factor from 5f.

CR_T = current reading to torsional excitation system (Col. (9), Data Sheet)

Record magnification factors in column (11) of Intermediate Calulations Sheet.

h. Dimensionless Frequency Factor. - (1) The dimensionless frequency factor, F, is used in calculating modulus. It is a function of system factors T, P, and ADF and of specimen damping ratio, D. Values of F are provided by the computer program in Section 11.

(2) For cases where ADF is zero and specimen damping ratio is less than 10%, values of F can be obtained from Figure 2. Figure 2b is similar to Figure 2a except that the range of T is different. This figure is independant of which end of the specimen is used to determine resonance.

(3) For the cases where ADF and specimen damping ratio are small and where T and P are both greater than 10, dimensionless frequency factor may be approximated by

$$F = \sqrt{1/T + 1/P}$$

This simple equation is especially useful for checking results obtained from Fig. 2 or from the computer program. Record dimensionless frequency factors in column (12) of the

Intermediate Calculations Sheet.

i. Strain Factor. - (1) The strain factor, SF, is calculated by the program in Section 11. or, for cases of ADF = 0 and specimen damping equal to 10 percent, it may be obtained from Fig. 3. For other values of specimen damping ratio and ADF, values of from Fig. 3. are only approximately correct. (Note that Fig. 3a is for the case where resonance is established by phase mesasurement between input force and motion at the active end and Fig. 3b is for the case where resonance is established by phase measurement between input force and motion at the passive end.) Record strain factors in column (13) of the Intermediate Calculations Sheet.

j. Amplification Coefficient. - (1) When manually reducing data, the amplification coefficient, A, is needed to calculate damping. This coefficient may be obtained from Fig. 4 for cases where apparatus damping factor, ADF (Col. (10), Intermediate Calculations Sheet), is zero. For other values of ADF, the amplification coefficient is only approximate.

(2) The following equation may also be used to approximate the amplification coefficient for the case where resonance is determined by phase measurement between input force or torque and motion at the passive end. For this equation, the values of both T and P must be greater than 10.

$$A = 2(T + P)$$

Amplificaton coefficient is recorded in column (14) of the Intermediate Calculations Sheet.

k. Number of Vibration Cycles. - (1) The approximate number of vibration cycles, N, applied for each data set may be determined by

$$N = 60(t_i - t_{i-1})f$$

where: t_i = elapsed time in minutes for data set "i"

t_{i-1} = elapsed time in minutes for data set "i-1"

f = system resonant frequency (Col. (12), Data Sheet)

Both t_i and t_{i-1} are from column (2) of the Data Sheet. Number of vibration cycles is recorded in column (5) of the Final Calculations Sheet.

1. Moduli. - (1) The rod modulus is calculated from

$$E = \rho(2\pi L)\uparrow 2\ (f_L/F_L)\uparrow 2$$

where: ρ = specimen mass density (Col. (6), Intermediate Calculations Sheet)

f_L = system resonant frequency for longitudinal motion (Col. (12), Data Sheet)

F_L = dimensionless frequency factor (Col. (12), Intermediate Calculations Sheet)

(2) The shear modulus is calculated from

$$G = \rho(2\pi L)\uparrow 2\ (f_T/F_T)\uparrow 2$$

where: f_T = system resonant frequency for torsional motion (Col. (12), Data Sheet)

F_T = dimensionless frequency factor (Col. (12), Intermediate Calculations Sheet)

Record moduli in column (6) of Final Calculations Sheet.

m. Strain Amplitude. - (1) The average rod strain amplitude, ϵ, for longitudinal motion shall be calculated from

$$\epsilon = (LCF)(LTO)(SF/L)\ 100\%$$

where: LCF = longitudinal motion transducer calibration factor for the transducer used in establishing resonance

LTO = longitudinal transducer output (Col.(10) or (11), Data Sheet depending on transducer used in establishing resonance)

SF = strain factor (Col. (13), Intermediate Calculations Sheet)

L = specimen length (Col. (3), Intermediate Calculations Sheet)

(2) For torsional motion, the average shear strain amplitude, γ, shall be calculated from

$$\gamma = (RCF)(SF)[d/(3L)]\ 100\%$$

where: RCF = rotational motion transducer calibration factor for the transducer used in establishing resonance

RTO = rotational transducer output (Col. (10) or (11), Data Sheet depending on transducer used in establishing resonance)

SF = strain factor (Col. (13), Intermediate Calculations Sheet)

d = specimen diameter (Col. (4), Intermediate Calculations Sheet)

Strain amplitudes are recorded in column (7) of Final Calculations Sheet.

n. Damping Ratio from Steady State Vibration. - (1) If the computer program in Section 11 is used, the damping ratio of the specimen is established as part of the output.

(2) Manual calculation of damping ratio may be done for cases where the apparatus damping factor, ADF, is zero or may be assumed to be zero (See 8f. for definition of ADF).

The damping ratio, D, is calculated from

$$D = 100\%/[\ A\ (MMF)\ ]$$

where: A = Amplification Coefficient (Col. (14), Intermediate Calculations Sheet)

MMF = magnification factor (Col. (11), Intermediate Calculations Sheet)

o. Damping Ratio from Free Vibration. - (1) This procedure is theoretically exact for apparatus where the passive end can be assumed to be rigidly fixed. For cases where the passive end is not rigidly fixed, irrespective of which end of the specimen is used in establishing resonance, this method is approximate. The same transducer that is used to determine resonance must be used to obtain the amplitude decay curve.

(2) For the case where resonance is established by use of the passive end transducer, values of T and P should both be greater than ten when amplitude decay is used.

(3) For apparatus where the active end platen is restrained by a spring, a system energy ratio must be calculated. For other apparatus, this factor is zero.

(4) For longitudinal motion, the system energy ratio is calculated from

$$S_L = (M_A g/W)(f_{oL} F_L/f_L) \uparrow 2$$

where: F_L = dimensionless frequency factor for longitudinal motion (Col. (12), Intermediate Calculations Sheet)

and for torsional motion from

$$S_T = (J_A/J)(f_{oT} F_T/f_T) \uparrow 2$$

where: F_T = dimensionless frequency factor for torsional motion (Col. (12), Intermediate Calculations Sheet)

Record the system energy ratios in column (7) of the Amplitude Decay Data Sheet.

(5) Compute the system logarithmic decrement from the free vibration (obtained in 7f.) from

$$\delta_s = (1/n) \log (A_1/A_{n+1})$$

where: A_1 = amplitude of vibration for first cycle after power is cut off (Col. (3), Amplitude Decay Data Sheet)

A_{n+1} = amplitude of vibration for (n+1)th cycle of free vibration (Col. (4), Amplitude Decay Data Sheet)

n = number of free vibration cycles which must be 10 or less. (Col. (5), Amplitude Decay Data Sheet)

(6) Finally, calculate the damping ratio from

$$D = [\ \delta_s(1+S) - S\delta]100\%/(2\pi)$$

where: D = D_L or D_T, in percent, depending on whether vibration is longitudinal or torsional

δ_s = δ_{sL} or δ_{sT} depending on whether vibration is longitudinal or torsional

S = S_L or S_T depending on whether vibration is longitudinal or torsional

δ = δ_L or δ_T, the apparatus logarithmic decrement from 5e.

Record values of damping ratio in both column (8) of the Amplitude Decay Data Sheet and in column (9) of the Final Calculations Sheet.

p. Accuracy of Calculations. - (1) The items listed on the Final Calculations Sheet are those which will be used in analysis or design and consequently, it is important that they be accurate.

(2) In the foregoing sections, no units are specified. Any system of units, whether English, metric, or SI, will work as long as units are used in a consistent fashion.

(3) All numerical items in the Data Sheet (Plate 1) should be recorded to at least three significant figures. In cases where apparatus have springs attached to the active end platen, and where the system resonant frequencies are within 20 percent of the apparatus resonant frequency, the system resonant frequency should be recorded to at least four significant figures. A digital frequency meter is generally required for this accuracy and when the system resonant frequencies are less than 100 Hz, it may be easier and more accurate to measure the resonant period and invert it to get the resonant frequency.

(4) Items in columns (2) through (7) in the Intermediate Calculations Sheet (Plate 2) should be calculated and recorded with three significant figures. Any constants or units conversions for these items should also be to at least three significant figures. Items in columns (8) through (12) are all non-dimensional. It is imperatative that the parameters used in calculating these items be in consistent units. Special

precaution should be taken when determining the active end inertia ratio, T, in column (8) and subsequent parameters which are functions of this inertia ratio, when the system resonant frequency is within 20 percent of the apparatus frequency. For this case, it is also adviseable to use the computer program to determine the dimensionless frequency factor, F, the strain factor, SF, and the damping ratio, D.

(5) If all of the above guidelines are observed, the data on the Final Calculations Sheet (Plate 3) should be reported to three significant figures. The absolute accuracy of these data will then be directly related to the absolute accuracy of the calibrations.

9. REPORT

a. General. - The report shall include characteristics of the apparatus, specimen, ambient test conditions, and the results for each data set.

b. Apparatus Characteristics. - The following apparatus characteristics shall be included: apparatus name, model number and serial number; active end and passive end masses and rotational inertias (M_A, M_P, J_A, J_P); longitudinal and torsional apparatus resonant frequencies (f_{oL}, f_{oT}); longitudinal and torsional apparatus logarithmic decrements (δ_L, δ_T); the force/current and torque/current constants (FCF, TCF): and the applicable motion transducer calibration factors (LCF_A, LCF_P, RCF_A, RCF_P). (Note that if the passive end is fixed, inertias and transducers are not needed for the passive end. Likewise, if only one type of motion, longitudinal or torsional, is used, then only factors and inertias for that type need be given.)

c. Specimen Characteristics. - A visual description and origin of the soil shall be given including: name, group symbol, and whether undisturbed or remolded. This may be reported on the Specimen Data Sheet (Plate X-1).

d. Ambient Stress History. - This is to include a description of the sequence of loading, consolidating, and saturating the specimen and the application of axial loads. For staged testing where behavior is measured at more than one effective confining stress, the details of this staging should be described. Plots of specimen volume change and/or specimen length change versus log time or square-root-of-time during consolidation should also be given.

e. Vibration Test Results. - This is to include all of the applicable information on the data and calculation sheets (Plates 1 through -4) for each data set including those associated with consolidation and saturation. For each stage of testing, plots of modulus and damping (arithmetic scales) versus vibratory strain amplitude (either arithmetic or logarithmic scale) should also be given.

10. POSSIBLE ERRORS

a. Specimen Preparation. - (1) Since specimen preparation is identical to that for triaxial testing, the same sources of error discussed in Appendix X., especially in Section 9 b. are applicable here.

b. Apparatus Assembly. - (1) The most common source of error associated with apparatus assembly is disturbing the specimen when placing the end platens and when attaching the vibration excitation device to the platen. A temporary platen support device as described in Section 4 g. and a temporary support for the vibration excitation device as described in Section 4 f. should be used to avoid disturbance.

(2) Complete contact between the ends fo the specimen is essential for coupling in torsion and for accurate rod modulus and rod strain values. Careful use of the mitre box in trimming the specimen ends will circumvent the problem for cohesive soils and careful smoothing and leveling of the specimen top will circumvent this problem for cohesionless soils.

(3) When stiff to very stiff soils are tested and when razor blade vanes are protruding from the end platens, there is a possibility that the blades will not fully penetrate the specimen. This can be averted in most cases by shortening the distance the blades protrude from the porous discs and by keeping the blades razor sharp. For extremely stiff soils, the blades should not be used. Capping adhesives serve to provide the proper contact for these soils.

(4) Most apparatus rely on a coil-permanent magnet system for vibration excitation. For cases where the active end platen does not have springs attached to it, the coils may touch the magnets, especially as the specimen swells or consolidates. Usually there are provisions for adjusting the clearance between the coil(s) and magnet(s) without having to remove the confining

stresses or otherwise disturb the specimen. Adjustments should be made as needed.

(5) Associated with the vibration excitation device, the transducers, and the pore water control are various cables and tubes. It is important that these be arranged such that they do not affect the vibratory characteristics of the system.

c. Test Procedure. - (1) The definition of resonance is different for different apparatus boundary conditions. Particularly where the motion at the passive end is used to measure resonance, the wrong resonant frequency may be recorded as the system resonant frequency. Also, the apparatus may have some minor spurious resonances near the system resonant frequency. These should be known beforehand by use of the calibrating rod and should not be mistaken for the system resonant frequency.

(2) Another source of error in the apparatus where the passive end platen is not rigidly fixed, is the use of the wrong transducer to establish resonance. This problem can be averted by clearly labeling all leads from the apparatus.

(3) At all amplitudes of vibration, but particularly at higher strain amplitudes, the system resonant frequency changes with time due to changes occuring within the specimen from vibration and/or consolidation. If the voltage readings from the transducers are not taken while the system is at resonance, significant errors in strain amplitude and damping ratio will occur.

d. Data Reduction. - (1) Other than the common errors of calculation and graph interpretation, errors due to units conversion and use of inconsistent units are relatively frequent. It is preferred that one system of units be used throughout.

(2) Finally, it is important that all results be checked for reasonableness. Approximate values for test results may be obtained by use of the equations and curves by Hardin and Drnevich.* Gross deviations from expected values or inconsistent results for a given test should be cause for rechecking data and calculations and possibly redoing the test.

11. COMPUTER PROGRAM FOR DATA REDUCTION

a. General. - (1) All resonant column data associated with describing the system, the amplitude of vibration, and resonant frequency can be represented by five nondimensional terms that are independent of apparatus type and whether the vibratory motion is longitudinal or torsional. Four of the terms (T, P, ADF, and MMF) are given in columns (8) through (11) of the Intermediate Calculations Sheet. The remaining term, JPA, describes which end (active or passive) is used in establishing resonance.

(2) The FORTRAN program adapted from Drnevich et al.** calculates three nondimensional parameters. The first is the dimensionless frequency factor, F, which is used along the system

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* Hardin, B.O., and Drnevich, V.P.,"Shear Modulus and Damping in Soils: Design Equations and Curves," Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 98, No. SM7, July, 1972, pp. 667-692.

**Drenvich, Hardin, and Shippy, op. cit.

resonant frequency and specimen characteristics to calculate modulus. The second is the strain factor, SF, which is used along with vibration amplitude and specimen dimensions to calculate the average vibratory strain amplitude. The last parameter is the damping ratio of the specimen, in percent, which is calculated directly by the program.

(3) The program utilizes a Gaussian elimination procedure* to solve four simultaneous equations. A double iteration procedure (one for dimensionless frequency factor and one for damping ratio) is achieved by using two Mueller's iteration subroutines**. Each iteration subroutine requires an error criterion (EPSD is used for damping ratio and EPSF is used for dimensionless frequency factor). Both may be prescribed by the user or default values built into the program may be used.

(4) In addition to the output described above, the program outputs a calculated phase angle and a calculated magnification factor that may be used for the purpose of checking the accuracy of the iterative processes. The phase angle should be equal to zero and the calculated magnification factor should equal the input magnification factor, MMF, which is based on measurements and calibration factors. If the differences between the measured and calculated values of MMF are more than two percent a decrease in EPSD should be made until the difference is within two percent. Similarly, if the phase is greater than 0.001 radian, a

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* System/360 Scientific Subroutine Package, Version III, Fifth Edition, IBM, 1970.
** ibid.

decrease in EPSF should be made. Very low values of EPSD and EPSF should be avoided because they cause excessive iteration cycles and computer time.

b. Input Data Format. - (1) The first card sets the dimensionless frequency factor error criterion, EPSF, the maximum number of iterations for dimensionless frequency factor, ITERF, the damping error criterion, EPSD, and the maximum number of iterations for damping ratio, ITERD. The format for these data are 2(F10.0,7X,I3). The default values of EPSF = 0.01, ITERF = 40, EPSD = 0.0001, and ITERD = 40. To use the default values, put a blank card in for the first card.

(2) Next, there is one card for each data set (line on Intermediate Calculations Sheet). The data required: T, P, ADF, MMF, and JPA comes from columns (8) through (11) of the Intermediate Calculations Sheet for the first four and the value of JPA equals 0 when the active end is used for establishing resonance and equals 1 when the passive end is used. The format for these data sets is 4F10.0,9X,I1. Any number of data sets may be used as long as the last card in the data deck is a blank card.

Sample Input Data Set for Program

```
         1         2         3         4         5
12345678901234567890123456789012345678901234567890   Data Card Col. Nos.

0.01             050  0.0001         050             First Card
98.75     4.69      .1053     .6729              1
97.98     4.69      .1070     .5466              1   Data Set Cards
96.30     4.69      .1108     .3985              1
94.35     4.69      .1150     .3009              1
                                                     Last Card is Blank
```

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C.  COMPUTER PROGRAM FOR RESONANT COLUMN DATA REDUCTION                 RCP 0001
C                                                                       RCP 0002
C                                                                       RCP 0003
C-------------DEFINITIONS OF INPUT-OUTPUT VARIABLES---------------------RCP 0004
C                                                                       RCP 0005
C  ADF       APPARATUS DAMPING FACTOR (ADF > 0.)                        RCP 0006
C  D         SPECIMEN DAMPING RATIO (0.01% < D < 35%)                   RCP 0007
C  EPSD      ERROR CRITERION FOR D (DEFAULT VALUE:  0.0001)             RCP 0008
C  EPSF      ERROR CRITERION FOR F (DEFAULT VALUE:  0.01)               RCP 0009
C  F         FREQUENCY FACTOR                                           RCP 0010
C  ITERD     MAXIMUM NUMBER OF ITERATIONS ALLOWED FOR D (DEFAULT        RCP 0011
C              VALUE:  40)                                              RCP 0012
C  ITERF     MAXIMUM NUMBER OF ITERATIONS ALLOWED FOR F (DEFAULT        RCP 0013
C              VALUE:  40)                                              RCP 0014
C  JPA       INDICATOR OF END WHERE MEASUREMENTS WERE TAKEN:            RCP 0015
C                JPA = 0 FOR MEASUREMENTS AT THE ACTIVE END;            RCP 0016
C                JPA = 1 FOR MEASUREMENTS AT THE PASSIVE END.           RCP 0017
C  MMF       MODIFIED MAGNIFICATION FACTOR (MMF > 0.)                   RCP 0018
C  P         PASSIVE-END INERTIA RATIO (P > 0.;  IF JPA = 0, THEN       RCP 0019
C              P > 100. AND P > T) .                                    RCP 0020
C  SF        STRAIN FACTOR                                              RCP 0021
C  T         ACTIVE-END INERTIA FACTOR (T > -10.)                       RCP 0022
C                                                                       RCP 0023
C-----------------------------------------------------------------------RCP 0024
C                                                                       RCP 0025
C        GIVEN VALUES OF T, P, ADF, MMF, AND JPA, THIS PROGRAM CALCU-   RCP 0026
C  LATES VALUES OF F, D, AND SF AND WILL PRINT VALUES OF ALL THESE      RCP 0027
C  PARAMETERS.                                                          RCP 0028
C                                                                       RCP 0029
C----------------DATA INPUT INSTRUCTIONS---------------------------------RCP 0030
C                                                                       RCP 0031
C        VALUES OF EPSF, ITERF, EPSD, AND ITERD MAY BE SPECIFIED ON THE RCP 0032
C  FIRST DATA CARD ACCORDING TO THE FORMAT 2(F10.0,7X,I3).  THESE       RCP 0033
C  PARAMETERS ARE REQUIRED IN THE ITERATIVE SOLUTION PROCEDURE TO CON-  RCP 0034
C  TROL ACCURACY AND LIMIT THE NUMBER OF ITERATIONS.  THIS DATA CARD    RCP 0035
C  MAY BE LEFT BLANK IF THE USER CHOOSES TO USE THE DEFAULT VALUES      RCP 0036
C  SPECIFIED BY THE PROGRAM.  (SEE THE LIST OF DEFINITIONS OF INPUT-    RCP 0037
C  OUTPUT VARIABLES.)  THE DEFAULT VALUES WILL GIVE GOOD RESULTS IN MOSTRCP 0038
C  CASES INVOLVING SMALL APPARATUS DAMPING.  FOR LARGE APPARATUS DAMPINGRCP 0039
C  OR TO CHECK RESULTS OBTAINED WITH THE DEFAULT VALUES, SMALLER VALUES RCP 0040
C  OF ITERF AND ITERD SHOULD BE USED.  (HOWEVER, IN SOME CASES OF LARGE RCP 0041
C  APPARATUS DAMPING, ACCURATE CALCULATION OF D IS IMPOSSIBLE.)         RCP 0042
C                                                                       RCP 0043
C        EACH SUBSEQUENT DATA CARD SHOULD CONTAIN A VALUE OF EACH OF THE RCP 0044
C  PARAMETERS P, T, ADF, MMF, AND JPA ACCORDING TO THE FORMAT 4F10.0,   RCP 0045
C  9X,I1.  THUS, EACH OF THESE CARDS CORRESPONDS TO ONE SET OF EXPERI-  RCP 0046
C  MENTAL TEST DATA.  SEE THE LIST OF DEFINITIONS OF INPUT-OUTPUT       RCP 0047
C  VARIABLES FOR THE RANGES OF THESE VARIABLES ALLOWED BY THE PROGRAM.  RCP 0048
C  THE DATA CARDS SHOULD BE TERMINATED WITH A BLANK CARD.               RCP 0049
C                                                                       RCP 0050
C-----------------------------------------------------------------------RCP 0051
C                                                                       RCP 0052
C                                                                       RCP 0053
      DIMENSION C(4),UP(11)                                             RCP 0054
      REAL MMF,MMFCAL                                                   RCP 0055
```

```
      INTEGER OD
      COMMON /VALS/P,T,ADF,MMF,JPA,AMP,D,F /DLIM/DL,DR
      COMMON /ALARM/KALARM /EPSIT/EPSF,ITERF,EPSD,ITERD
      COMMON /PAR/PL,QL,C
      COMMON /CRIT/MMFCAL,PHASE
      EXTERNAL DELAMP
 1000 FORMAT(4F10.0,9X,I1)
 1005 FORMAT(2(F10.0,7X,I3))
 1110 FORMAT(49H * * * * *   W A R N I N G   * * * * *   POSSIBLY,
     1   23H NOT ENOUGH ITERATIONS )
 1115 FORMAT(50H TO OBTAIN SPECIFIED ACCURACY FOR D WITH THE ABOVE,
     1    18H PARAMETER VALUES.)
 1117 FORMAT(30H  TRY A LARGER VALUE OF ITERD.)
 1120 FORMAT(46H * * * * *   E R R O R   * * * * *   THE VALUE,
     1    19H OF D FOR THE ABOVE)
 1125 FORMAT(46H PARAMETERS LIES OUTSIDE THE ALLOWABLE RANGE, ,
     1    14H0.001 TO 0.35.)
 1140 FORMAT(31H   THE ERROR CRITERION FOR F IS,E12.2/)
 1150 FORMAT(52H   THE MAXIMUM NUMBER OF ITERATIONS ALLOWED FOR F IS,
     1    I4/)
 1160 FORMAT(31H   THE ERROR CRITERION FOR D IS,E12.2/)
 1170 FORMAT(52H   THE MAXIMUM NUMBER OF ITERATIONS ALLOWED FOR D IS,
     1    I4///)
 1210 FORMAT(5X,1HT,9X,1HP,8X,3HADF,6X,3HMMF,4X,3HJPA,3X,1HF,8X,2HD%,
     1 8X,2HSF,6X,9HMMF(CALC),3X,5HPHASE/)
 1220 FORMAT(3(1X,F9.4),1X,F9.5,1X,I1,2(1X,F9.6),1X,1PE9.3,
     1   2(1X,0PF9.5))
 1230 FORMAT(53H * * * * *   E R R O R   * * * * *   P MUST BE .GE.0.)
 1240 FORMAT(50H * * * * *   E R R O R   * * * * *   IF(JPA.EQ.0) ,
     1    19H P MUST BE .GE.100.)
 1250 FORMAT(50H * * * * *   E R R O R   * * * * *   IF(JPA.EQ.0) ,
     1    17H P MUST BE .GE. T)
 1260 FORMAT(55H * * * * *   E R R O R   * * * * *   T MUST BE .GE.-10.)
 1270 FORMAT(55H * * * * *   E R R O R   * * * * *   ADF MUST BE .GE.0.)
 1280 FORMAT(54H * * * * *   E R R O R   * * * * *   MF MUST BE .GT.0.)
 1310 FORMAT(49H * * * * *   W A R N I N G   * * * * *   POSSIBLY,
     1   23H NOT ENOUGH ITERATIONS )
 1315 FORMAT(50H TO OBTAIN SPECIFIED ACCURACY FOR F WITH THE ABOVE,
     1    12H PARAMETERS.)
 1317 FORMAT(30H  TRY A LARGER VALUE OF ITERF.)
 1320 FORMAT(48H * * * * *   E R R O R   * * * * *   THERE IS NO,
     1    10H RESONANCE)
 1330 FORMAT(51H  (DISPLACEMENT ONE-QUARTER CYCLE OUT OF PHASE WITH,
     1    19H FORCING FUNCTION) )
 1340 FORMAT(25H FOR THE ABOVE PARAMETERS)
 1350 FORMAT(51H * * * * *   W A R N I N G   * * * * *   BECAUSE OF,
     1    16H LARGE APPARATUS)
 1360 FORMAT(49H  DAMPING, THE CALCULATED AMPLITUDE IS RELATIVELY,
     1    15H INSENSITIVE TO)
 1365 FORMAT(49H  SPECIMEN DAMPING.  CONSEQUENTLY, THE CALCULATED,
     1 17H SPECIMEN DAMPING)
 1367 FORMAT(37H  RATIO ABOVE MAY BE VERY INACCURATE.)
C  SET INPUT AND OUTPUT DEVICE CODES
      ID = 5
      OD = 6
```

```
      READ(ID,1005) EPSF,ITERF,EPSD,ITERD                   RCP 0111
      IF(EPSD.EQ.0.)  EPSD = 1.E-4                          RCP 0112
      IF(ITERD.EQ.0)  ITERD = 40                            RCP 0113
      IF(EPSF.EQ.0.)  EPSF = 1.E-2                          RCP 0114
      IF(ITERF.EQ.0)  ITERF = 40                            RCP 0115
      WRITE(OD,1140) EPSF                                   RCP 0116
      WRITE(OD,1150) ITERF                                  RCP 0117
      WRITE(OD,1160) EPSD                                   RCP 0118
      WRITE(OD,1170) ITERD                                  RCP 0119
      WRITE(OD,1210)                                        RCP 0120
   10 READ(ID,1000) T,P,ADF,MMF,JPA                         RCP 0121
      IF(MMF.EQ.0.)  GO TO 190                              RCP 0122
      MAL = 0                                               RCP 0123
      IF(P.GE.0.)  GO TO 20                                 RCP 0124
      MAL = MAL + 1                                         RCP 0125
      IF(MAL.EQ.1)  WRITE(OD,1220) P,T,ADF,MMF,JPA          RCP 0126
      WRITE(OD,1230)                                        RCP 0127
   20 IF(JPA.EQ.1.OR.P.GE.100.)  GO TO 30                   RCP 0128
      MAL = MAL + 1                                         RCP 0129
      IF(MAL.EQ.1)  WRITE(OD,1220) P,T,ADF,MMF,JPA          RCP 0130
      WRITE(OD,1240)                                        RCP 0131
   30 IF(JPA.EQ.1.OR.T.LE.P)  GO TO 40                      RCP 0132
      MAL = MAL + 1                                         RCP 0133
      IF(MAL.EQ.1)  WRITE(OD,1220) P,T,ADF,MMF,JPA          RCP 0134
      WRITE(OD,1250)                                        RCP 0135
   40 IF(T.GE.-10.)  GO TO 50                               RCP 0136
      MAL = MAL + 1                                         RCP 0137
      IF(MAL.EQ.1)  WRITE(OD,1220) P,T,ADF,MMF,JPA          RCP 0138
      WRITE(OD,1260)                                        RCP 0139
   50 IF(ADF.GE.0.)  GO TO 60                               RCP 0140
      MAL = MAL + 1                                         RCP 0141
      IF(MAL.EQ.1)  WRITE(OD,1220) P,T,ADF,MMF,JPA          RCP 0142
      WRITE(OD,1270)                                        RCP 0143
   60 IF(MMF.GT.0.)  GO TO 70                               RCP 0144
      MAL = MAL + 1                                         RCP 0145
      IF(MAL.EQ.1)  WRITE(OD,1220) P,T,ADF,MMF,JPA          RCP 0146
      WRITE(OD,1280)                                        RCP 0147
   70 IF(MAL.NE.0)  GO TO 10                                RCP 0148
      KALARM = 0                                            RCP 0149
      DL = 0.0001                                           RCP 0150
      DR = .35                                              RCP 0151
      CALL RTMI2(D,AMPDEL,DELAMP,DL,DR,EPSD,ITERD,IER)      RCP 0152
      IF(KALARM.NE.1)  GO TO 80                             RCP 0153
      MAL = MAL + 1                                         RCP 0154
      IF(MAL.EQ.1)  WRITE(OD,1220) P,T,ADF,MMF,JPA          RCP 0155
      WRITE(OD,1320)                                        RCP 0156
      WRITE(OD,1330)                                        RCP 0157
      WRITE(OD,1340)                                        RCP 0158
      GO TO 10                                              RCP 0159
   80 IF(IER.NE.2)  GO TO 90                                RCP 0160
      MAL = MAL + 1                                         RCP 0161
      IF(MAL.EQ.1)  WRITE(OD,1220) P,T,ADF,MMF,JPA          RCP 0162
      WRITE(OD,1120)                                        RCP 0163
      WRITE(OD,1125)                                        RCP 0164
      GO TO 10                                              RCP 0165
```

```
   90 CONTINUE
      DO 180 I=1,11
      XR = (I-1)/10.
      QLX = QL*XR
      PLX = PL*XR
      CC = COSH(QLX)*COS(PLX)
      SC = SINH(QLX)*COS(PLX)
      SS = SINH(QLX)*SIN(PLX)
      CS = COSH(QLX)*SIN(PLX)
      UP(I) = C(1)*(-PL*CS+QL*SC) + C(2)*(-PL*SS+QL*CC) +
     1   C(3)*(PL*SC+QL*CS) + C(4)*(PL*CC+QL*SS)
  180 CONTINUE
      GAM = (UP(1)+UP(11)+4.*(UP(2)+UP(4)+UP(6)+UP(8)+UP(10))
     1    +2.*(UP(3)+UP(5)+UP(7)+UP(9)))/30.
      SF = ABS(GAM*F**2/MMF)
       D = D*100.
      WRITE(OD,1220) T,P,ADF,MMF,JPA,F,D,SF,MMFCAL,PHASE
      IF(IER.NE.1)  GO TO 110
      WRITE(OD,1110)
      WRITE(OD,1115)
      WRITE(OD,1117)
  110 IF(KALARM.NE.2)  GO TO 120
      WRITE(OD,1350)
      WRITE(OD,1360)
      WRITE(OD,1365)
      WRITE(OD,1367)
  120 IF(KALARM.NE.4)  GO TO 140
      WRITE(OD,1310)
      WRITE(OD,1315)
      WRITE(OD,1317)
  140 CONTINUE
      GO TO 10
  190 STOP
      END
C***********************
      FUNCTION DELAMP(D)
C***********************
      COMMON /VALS/P,T,ADF,MMF,JPA,AMP,DD,F /DLIM/DL,DR
      COMMON /ALARM/KALARM /EPSIT/EPSF,ITERF,EPSD,ITERD
      EXTERNAL VFCN
      INTEGER OD
      REAL MMF
      OD = 6
      DD = D
      IF(T.LT.0.)  GO TO 30
      AA = -48*T*P - 7*(T+P) - 1
      BB = 48*T*P + 20*(T+P) + 5
      CC = -4*(T+P) - 4
      BD2A = BB/(2*AA)
      F = SQRT(12*(-BD2A-SQRT(BD2A**2-CC/AA)))
      FL = .8*F
      FR = 1.2*F
      GO TO 60
   30 IF(T.LT.-.1)  GO TO 40
      FL = 1.4
```

```
      FR = 1.5*FL
      GO TO 60
   40 FL = 1.75
      FR = 1.5*FL
   60 CONTINUE
      CALL RTMI(F,V,VFCN,FL,FR,EPSF,ITERF,IER)
      IF(IER.NE.2)  GO TO 70
      FL = .99*FR
      FR = 1.5*FR
      IF(FL.LE.8.)  GO TO 60
      KALARM = 1
      GO TO 120
   70 IF(IER.EQ.1)  KALARM = 4
   90 CONTINUE
      V = VFCN(F)
      A    = AMP
      AM     = MMF/F**2
      DELAMP = A - AM
      IF(D.EQ.DL)  DELL = DELAMP
      IF(D.EQ.DL)  AML = A
      IF(D.NE.DR)  GO TO 120
      DELR = DELAMP
      AMR = A
      DIF = AML - AMR
      RELDIF = DIF/AML
      IF(RELDIF.LE.0.20)  KALARM = 2
  120 CONTINUE
      RETURN
      END
C***********************
      FUNCTION VFCN(F)
C**********************
      DIMENSION A(4,4),C(4)
      REAL MMFCAL
      COMMON /VALS/P,T,ADF,MMF,JPA,AMP,D,FF
      COMMON /PAR/PL,QL,C
      COMMON /CRIT/MMFCAL,PHASE
      BETA = SQRT(1.+(2.*D)**2)
      PL = F*SQRT((BETA+1.)/2.)/BETA
      QL = F*SQRT((BETA-1.)/2.)/BETA
      SNHQ = SINH(QL)
      CSHQ = COSH(QL)
      SNP = SIN(PL)
      CSP = COS(PL)
      CS = CSHQ*SNP
      SC = SNHQ*CSP
      SS = SNHQ*SNP
      CC = CSHQ*CSP
      CSSC = -PL*CS + QL*SC
      SSCC = -PL*SS + QL*CC
      SCCS = PL*SC + QL*CS
      CCSS = PL*CC + QL*SS
      PF2 = P*F**2
      A(1,1) = T*F**2
      A(1,2) = QL - 2*D*PL
```

```
      A(1,3) = - ADF*F**2
      A(1,4) = PL + 2*D*QL
      A(2,1) = -A(1,3)
      A(2,2) = -A(1,4)
      A(2,3) = A(1,1)
      A(2,4) = A(1,2)
      A(3,1) = CSSC - 2.*D*SCCS - PF2*CC
      A(3,2) = SSCC - 2.*D*CCSS - PF2*SC
      A(3,3) = SCCS + 2.*D*CSSC - PF2*SS
      A(3,4) = CCSS + 2.*D*SSCC - PF2*CS
      A(4,1) = -A(3,3)
      A(4,2) = -A(3,4)
      A(4,3) = A(3,1)
      A(4,4) = A(3,2)
      C(1) = 0
      C(2) = -1.
      C(3) = 0
      C(4) = 0
      EPS = 1.E-5
      NN = 4
      CALL  GELG(C,A,NN,1,EPS,IER,NN*NN,NN)
      U0 = C(1)
      V0 = C(3)
      AMPL0 = SQRT(U0**2 + V0**2)
      PHASE0 = ATAN2(V0,U0)
      IF(PHASE0.LT.0.)  PHASE0 = PHASE0 + 6.283185
      UL = C(1)*CC + C(2)*SC + C(3)*SS + C(4)*CS
      VL = C(3)*CC + C(4)*SC - C(1)*SS - C(2)*CS
      AMPLL = SQRT(UL**2 + VL**2)
      PHASEL = ATAN2(VL,UL)
      IF(JPA.EQ.0)  VFCN = PHASE0 - 3.141593
      IF(JPA.NE.0)  VFCN = PHASEL
      IF(JPA.EQ.0)  AMP = AMPL0
      IF(JPA.NE.0)  AMP = AMPLL
      MMFCAL = AMP*F**2
      PHASE = VFCN
      RETURN
      END
C*****************************************************
      SUBROUTINE RTMI2(X,F,FCT,XLI,XRI,EPS,IEND,IER)
C*****************************************************
      COMMON /ALARM/KALARM
      IER=0
      XL=XLI
      XR=XRI
      X=XL
      TOL=X
      F=FCT(TOL)
      IF(KALARM.EQ.1)  RETURN
      IF(F)1,16,1
    1 FL=F
      X=XR
      TOL=X
      F=FCT(TOL)
      IF(KALARM.EQ.1)  RETURN
```

```
      IF(F)2,16,2
    2 FR=F
      IF(SIGN(1.,FL)+SIGN(1.,FR))25,3,25
    3 I=0
      TOLF=100.*EPS
    4 I=I+1
      DO 13 K=1,IEND
      X=.5*(XL+XR)
      TOL=X
      F=FCT(TOL)
      IF(KALARM.EQ.1)  RETURN
      IF(F)5,16,5
    5 IF(SIGN(1.,F)+SIGN(1.,FR))7,6,7
    6 TOL=XL
      XL=XR
      XR=TOL
      TOL=FL
      FL=FR
      FR=TOL
    7 TOL=F-FL
      A=F*TOL
      A=A+A
      IF(A-FR*(FR-FL))8,9,9
    8 IF(I-IEND)17,17,9
    9 XR=X
      FR=F
      TOL=EPS
      A=ABS(XR)
      IF(A-1.)11,11,10
   10 TOL=TOL*A
   11 IF(ABS(XR-XL)-TOL)12,12,13
   12 IF(ABS(FR-FL)-TOLF)14,14,13
   13 CONTINUE
      IER=1
   14 IF(ABS(FR)-ABS(FL))16,16,15
   15 X=XL
      F=FL
   16 RETURN
   17 A=FR-F
      DX=(X-XL)*FL*(1.+F*(A-TOL)/(A*(FR-FL)))/TOL
      XM=X
      FM=F
      X=XL-DX
      TOL=X
      F=FCT(TOL)
      IF(KALARM.EQ.1)  RETURN
      IF(F)18,16,18
   18 TOL=EPS
      A=ABS(X)
      IF(A-1.)20,20,19
   19 TOL=TOL*A
   20 IF(ABS(DX)-TOL)21,21,22
   21 IF(ABS(F)-TOLF)16,16,22
   22 IF(SIGN(1.,F)+SIGN(1.,FL))24,23,24
   23 XR=X
```

```
      FR=F                                                              RCP 0386
      GO TO 4                                                           RCP 0387
   24 XL=X                                                              RCP 0388
      FL=F                                                              RCP 0389
      XR=XM                                                             RCP 0390
      FR=FM                                                             RCP 0391
      GO TO 4                                                           RCP 0392
   25 IER=2                                                             RCP 0393
      RETURN                                                            RCP 0394
      END                                                               RCP 0395
```

```
C************************************************                       RCP 0396
      SUBROUTINE GELG (R,A,M,N,EPS,IER,MM,MN)                           RCP 0397
C************************************************                       RCP 0398
      DIMENSION A(MM), R(MN)                                            RCP 0399
      IF(M)23,23,1                                                      RCP 0400
    1 IER=0                                                             RCP 0401
      PIV=0.                                                            RCP 0402
      MM=M*M                                                            RCP 0403
      NM=N*M                                                            RCP 0404
      DO 3 L=1,MM                                                       RCP 0405
      TB=ABS(A(L))                                                      RCP 0406
      IF(TB-PIV)3,3,2                                                   RCP 0407
    2 PIV=TB                                                            RCP 0408
      I=L                                                               RCP 0409
    3 CONTINUE                                                          RCP 0410
      TOL=EPS*PIV                                                       RCP 0411
      LST=1                                                             RCP 0412
      DO 17 K=1,M                                                       RCP 0413
      IF(PIV)23,23,4                                                    RCP 0414
    4 IF(IER)7,5,7                                                      RCP 0415
    5 IF(PIV-TOL)6,6,7                                                  RCP 0416
    6 IER=K-1                                                           RCP 0417
    7 PIVI=1./A(I)                                                      RCP 0418
      J=(I-1)/M                                                         RCP 0419
      I=I-J*M-K                                                         RCP 0420
      J=J+1-K                                                           RCP 0421
      DO 8 L=K,NM,M                                                     RCP 0422
      LL=L+I                                                            RCP 0423
      TB=PIVI*R(LL)                                                     RCP 0424
      R(LL)=R(L)                                                        RCP 0425
    8 R(L)=TB                                                           RCP 0426
      IF(K-M)9,18,18                                                    RCP 0427
    9 LEND=LST+M-K                                                      RCP 0428
      IF(J)12,12,10                                                     RCP 0429
   10 II=J*M                                                            RCP 0430
      DO 11 L=LST,LEND                                                  RCP 0431
      TB=A(L)                                                           RCP 0432
      LL=L+II                                                           RCP 0433
      A(L)=A(LL)                                                        RCP 0434
   11 A(LL)=TB                                                          RCP 0435
   12 DO 13 L=LST,MM,M                                                  RCP 0436
      LL=L+I                                                            RCP 0437
      TB=PIVI*A(LL)                                                     RCP 0438
      A(LL)=A(L)                                                        RCP 0439
   13 A(L)=TB                                                           RCP 0440
      A(LST)=J                                                          RCP 0441
      PIV=0.                                                            RCP 0442
      LST=LST+1                                                         RCP 0443
      J=0                                                               RCP 0444
      DO 16 II=LST,LEND                                                 RCP 0445
      PIVI=-A(II)                                                       RCP 0446
      IST=II+M                                                          RCP 0447
      J=J+1                                                             RCP 0448
      DO 15 L=IST,MM,M                                                  RCP 0449
      LL=L-J                                                            RCP 0450
```

```
      A(L)=A(L)+PIVI*A(LL)
      TB=ABS(A(L))
      IF(TB-PIV)15,15,14
   14 PIV=TB
      I=L
   15 CONTINUE
      DO 16 L=K,NM,M
      LL=L+J
   16 R(LL)=R(LL)+PIVI*R(L)
   17 LST=LST+M
   18 IF(M-1)23,22,19
   19 IST=MM+M
      LST=M+1
      DO 21 I=2,M
      II=LST-I
      IST=IST-LST
      L=IST-M
      L=A(L)+.5
      DO 21 J=II,NM,M
      TB=R(J)
      LL=J
      DO 20 K=IST,MM,M
      LL=LL+1
   20 TB=TB-A(K)*R(LL)
      K=J+L
      R(J)=R(K)
   21 R(K)=TB
   22 RETURN
   23 IER=-1
      RETURN
      END
C*****************>**********************************
      SUBROUTINE RTMI(X,F,FCT,XLI,XRI,EPS,IEND,IER)
C****************************************************
      IER=0
      XL=XLI
      XR=XRI
      X=XL
      TOL=X
      F=FCT(TOL)
      IF(F)1,16,1
    1 FL=F
      X=XR
      TOL=X
      F=FCT(TOL)
      IF(F)2,16,2
    2 FR=F
      IF(SIGN(1.,FL)+SIGN(1.,FR))25,3,25
    3 I=0
      TOLF=100.*EPS
    4 I=I+1
      DO 13 K=1,IEND
      X=.5*(XL+XR)
      TOL=X
      F=FCT(TOL)
```

```
      IF(F)5,16,5                                                       RCP 0506
    5 IF(SIGN(1.,F)+SIGN(1.,FR))7,6,7                                   RCP 0507
    6 TOL=XL                                                            RCP 0508
      XL=XR                                                             RCP 0509
      XR=TOL                                                            RCP 0510
      TOL=FL                                                            RCP 0511
      FL=FR                                                             RCP 0512
      FR=TOL                                                            RCP 0513
    7 TOL=F-FL                                                          RCP 0514
      A=F*TOL                                                           RCP 0515
      A=A+A                                                             RCP 0516
      IF(A-FR*(FR-FL))8,9,9                                             RCP 0517
    8 IF(I-IEND)17,17,9                                                 RCP 0518
    9 XR=X                                                              RCP 0519
      FR=F                                                              RCP 0520
      TOL=EPS                                                           RCP 0521
      A=ABS(XR)                                                         RCP 0522
      IF(A-1.)11,11,10                                                  RCP 0523
   10 TOL=TOL*A                                                         RCP 0524
   11 IF(ABS(XR-XL)-TOL)12,12,13                                        RCP 0525
   12 IF(ABS(FR-FL)-TOLF)14,14,13                                       RCP 0526
   13 CONTINUE                                                          RCP 0527
      IER=1                                                             RCP 0528
   14 IF(ABS(FR)-ABS(FL))16,16,15                                       RCP 0529
   15 X=XL                                                              RCP 0530
      F=FL                                                              RCP 0531
   16 RETURN                                                            RCP 0532
   17 A=FR-F                                                            RCP 0533
      DX=(X-XL)*FL*(1.+F*(A-TOL)/(A*(FR-FL)))/TOL                       RCP 0534
      XM=X                                                              RCP 0535
      FM=F                                                              RCP 0536
      X=XL-DX                                                           RCP 0537
      TOL=X                                                             RCP 0538
      F=FCT(TOL)                                                        RCP 0539
      IF(F)18,16,18                                                     RCP 0540
   18 TOL=EPS                                                           RCP 0541
      A=ABS(X)                                                          RCP 0542
      IF(A-1.)20,20,19                                                  RCP 0543
   19 TOL=TOL*A                                                         RCP 0544
   20 IF(ABS(DX)-TOL)21,21,22                                           RCP 0545
   21 IF(ABS(F)-TOLF)16,16,22                                           RCP 0546
   22 IF(SIGN(1.,F)+SIGN(1.,FL))24,23,24                                RCP 0547
   23 XR=X                                                              RCP 0548
      FR=F                                                              RCP 0549
      GO TO 4                                                           RCP 0550
   24 XL=X                                                              RCP 0551
      FL=F                                                              RCP 0552
      XR=XM                                                             RCP 0553
      FR=FM                                                             RCP 0554
      GO TO 4                                                           RCP 0555
   25 IER=2                                                             RCP 0556
      RETURN                                                            RCP 0557
      END                                                               RCP 0558
```

D. SAMPLE RUN OF DATA REDUCTION PROGRAM

```
THE ERROR CRITERION FOR F IS    0.10E-01

THE MAXIMUM NUMBER OF ITERATIONS ALLOWED FOR F IS  50

THE ERROR CRITERION FOR D IS    0.10E-03

THE MAXIMUM NUMBER OF ITERATIONS ALLOWED FOR D IS  50


    T          P         ADF       MMF    JPA    F        D%          SF       MMF(CALC)   PHASE

 98.7500    4.6900     0.1053    0.67290  1  0.457952  0.733725 1.052E 00    0.67283  -0.00001
 97.9800    4.6900     0.1070    0.54660  1  0.458071  0.910360 1.053E 00    0.54660  -0.00000
 96.3000    4.6900     0.1108    0.39850  1  0.458348  1.270199 1.054E 00    0.39850  -0.00005
 94.3500    4.6900     0.1150    0.30090  1  0.458716  1.715767 1.055E 00    0.30090  -0.00013


CORE USAGE        OBJECT CODE=   20472 BYTES,ARRAY AREA=      220 BYTES,TOTAL AREA AVAILABLE=  161792  BYTES
DIAGNOSTICS         NUMBER OF ERRORS=        0, NUMBER OF WARNINGS=         0, NUMBER OF EXTENSIONS=        0
COMPILE TIME=     0.89 SEC,EXECUTION TIME=      2.36 SEC,      20.14.08     TUESDAY       20 SEP 77     WATFIV
```

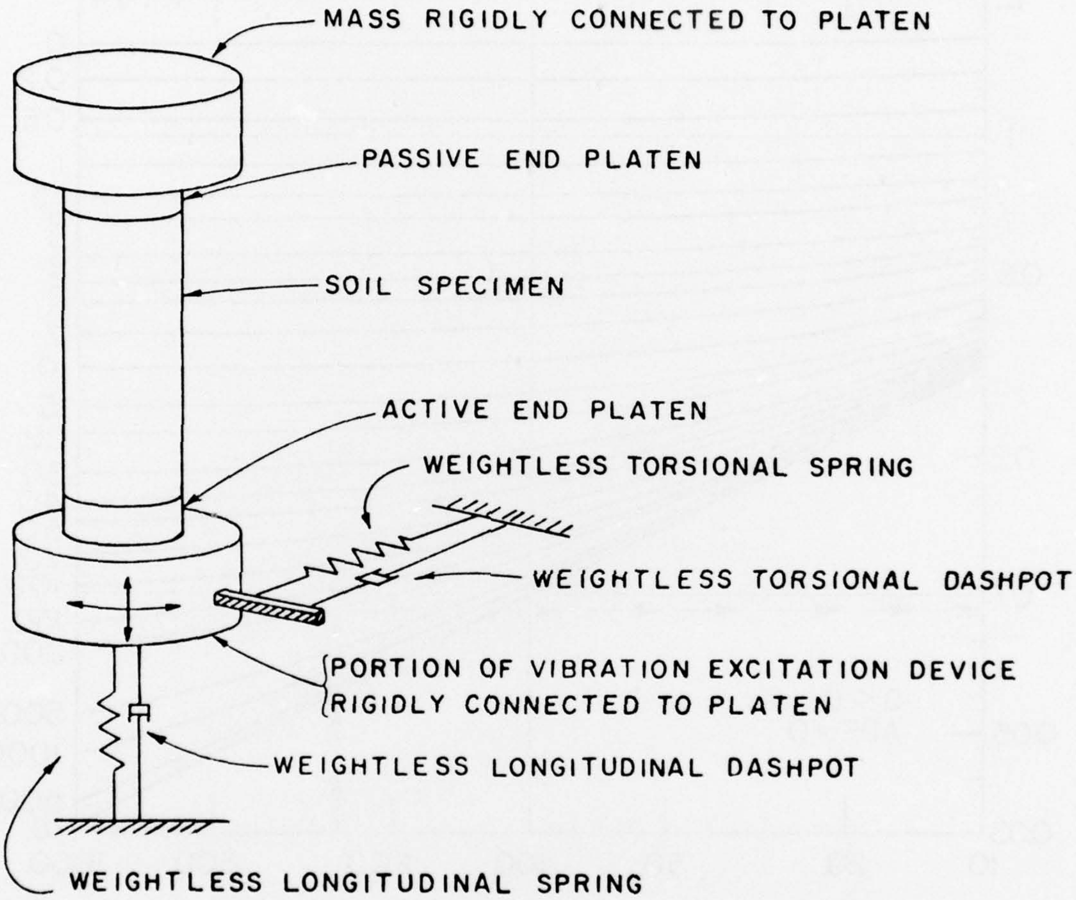

FIG. I RESONANT COLUMN SCHEMATIC

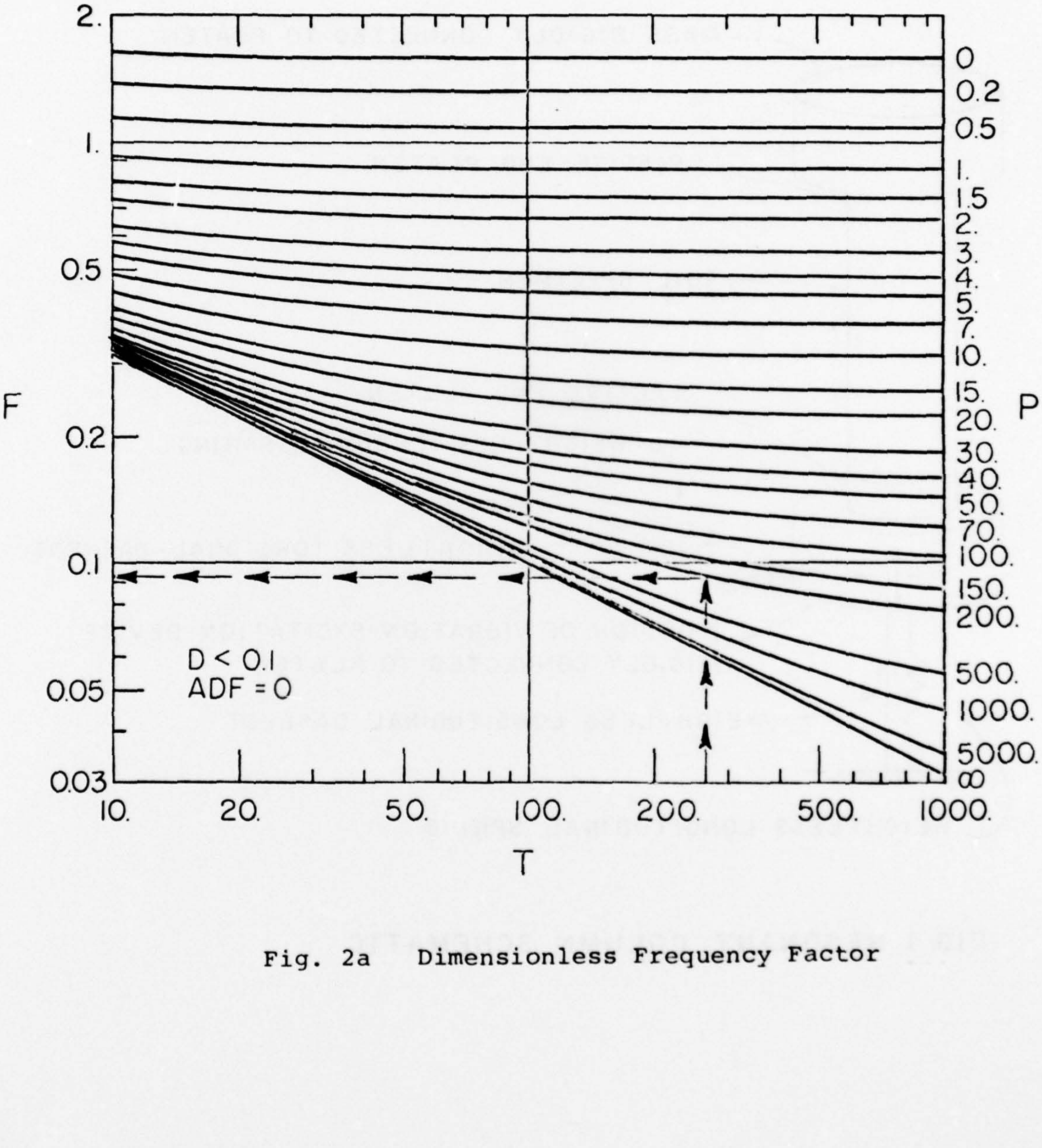

Fig. 2a Dimensionless Frequency Factor

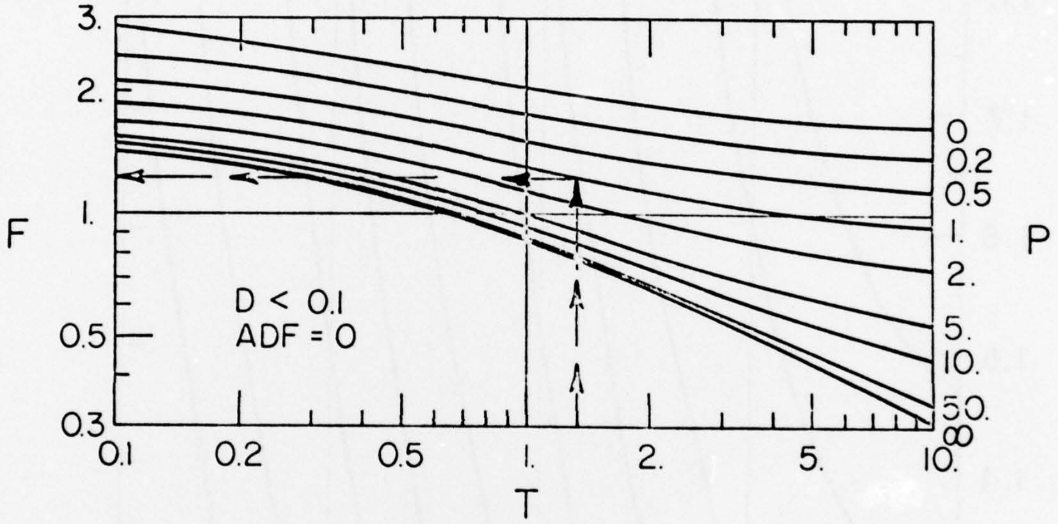

Fig. 2b   Dimensionless Frequency Factor

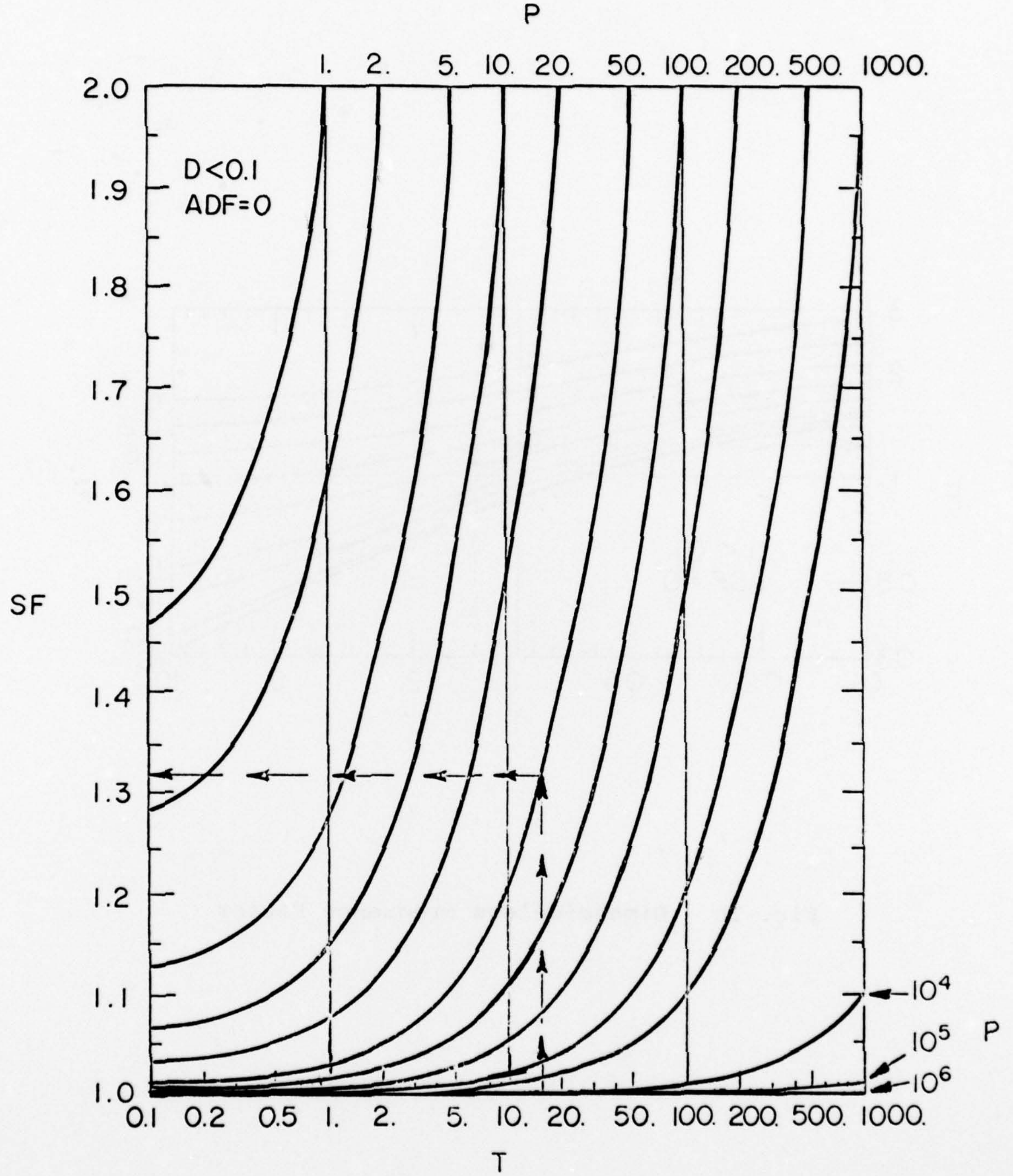

Fig. 3a Strain Factor for Resonance Determined by Phase Measurement Between Input Force and Motion at the Active End

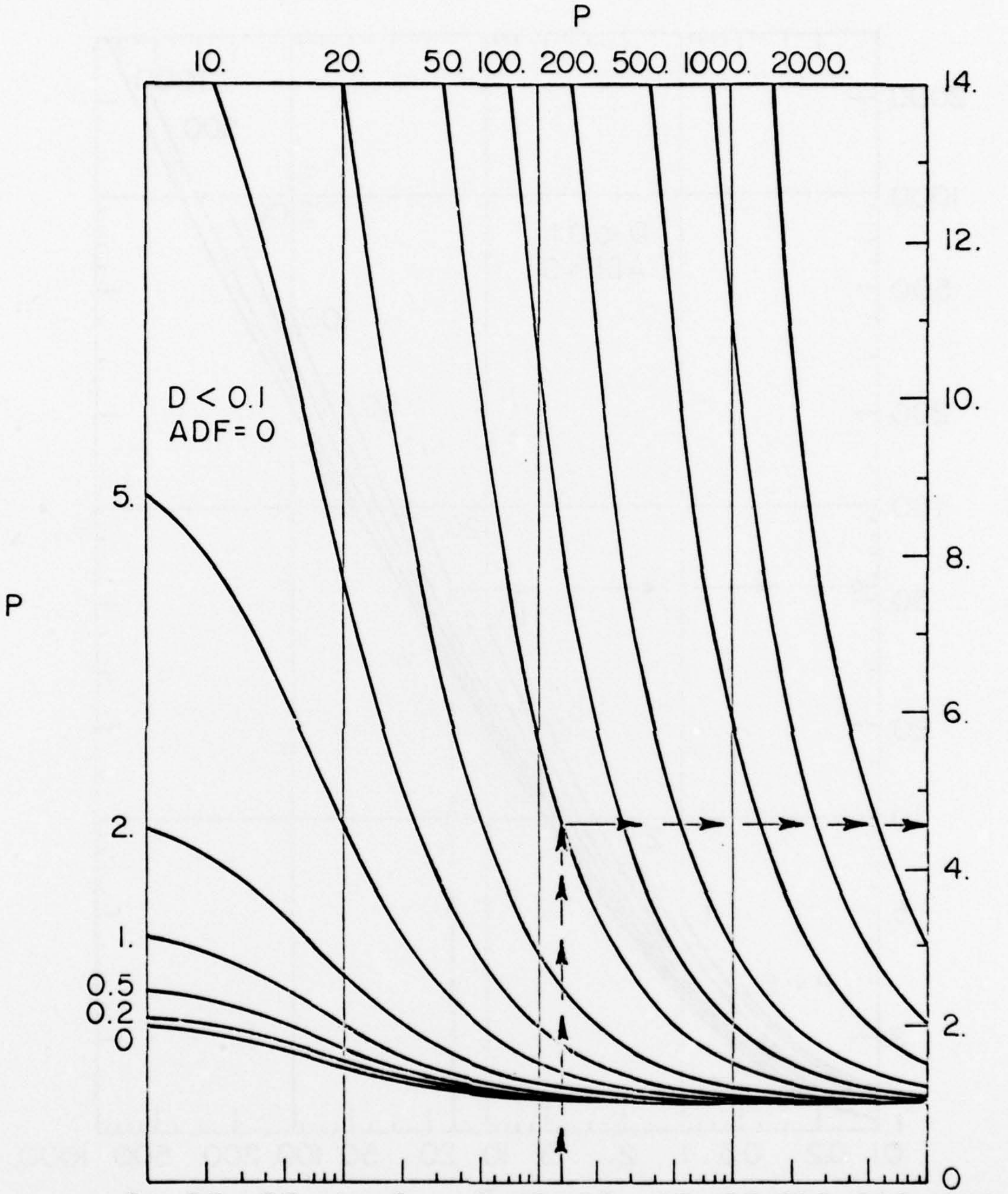

Fig. 3b Strain Factor for Resonance Determined by Phase Measurement Between Input Force and Motion at the Passive End

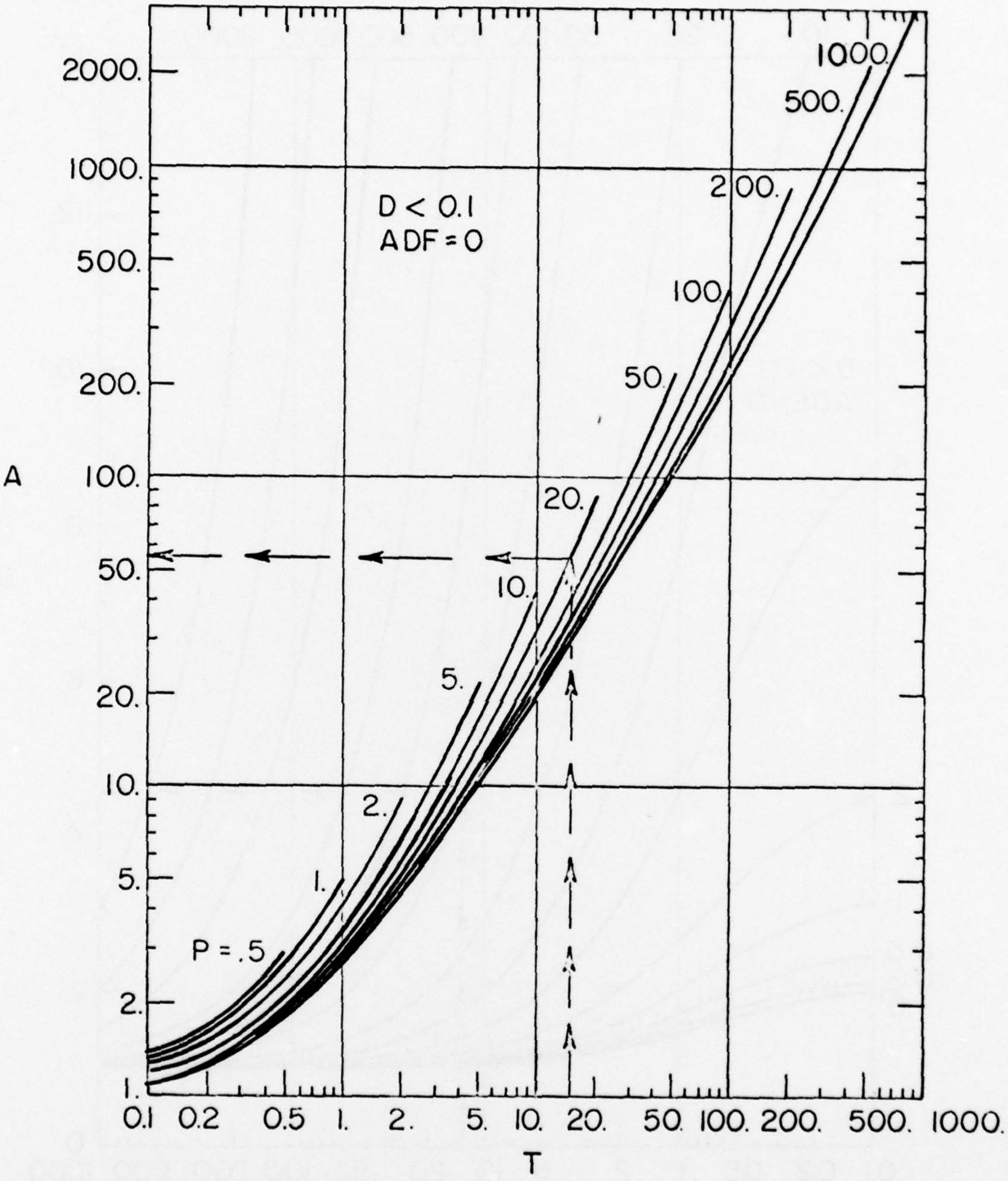

Fig. 4a Coefficient for Manual Calculation of Damping Ratio for Resonance Determined by Phase Measurement Between Input Force and Motion at the Active End

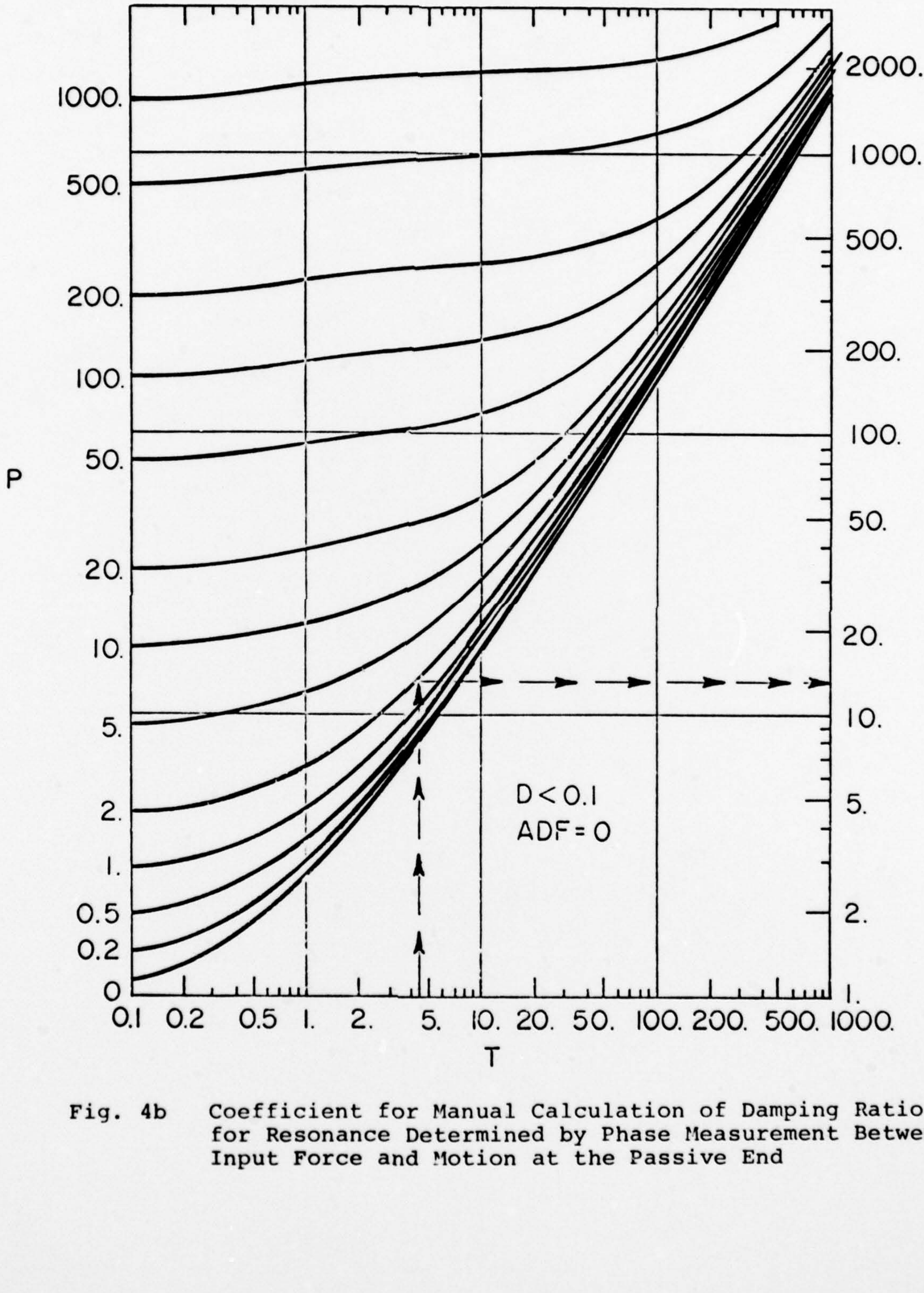

Fig. 4b Coefficient for Manual Calculation of Damping Ratio for Resonance Determined by Phase Measurement Between Input Force and Motion at the Passive End

RESONANT COLUMN TEST
(Data Sheet)

DATE ---------------

PROJECT ---------------------------------------

BORING NO. ------------------- SAMPLE NO. ------------------- APPARATUS -------------------

(1) Line No.	(2) Elapsed Time [min.]	(3) Cell Press. [ ]	(4) Pore Press. [ ]	(5) Axial Load [ ]	(6) Buret Rds. [ ]	(7) Length Rds. [ ]	(8) Long. or Tors.	(9) Force or Torque Rds. [mv(rms)]	(10) Act. End Trans.Rds. [mv(rms)]	(11) Pass. End Trans.Rds. [mv(rms)]	(12) Res. Freq. [Hz.]
1											
2											
3											
4											
5											
6											
7											
8											
9											
10											
11											
12											
13											
14											
15											

Technician ------------------- Computed By ------------------- Checked By -------------------

Remarks ---

PLATE 1

PLATE 2

RESONANT COLUMN TEST
(Intermediate Calculations Sheet)

DATE ______________

PROJECT ____

BORING NO. __________________ SAMPLE NO. __________________ APPARATUS ___________________

(1) Line No.	(2) Vol. ( )	(3) Length ( )	(4) Dia. ( )	(5) Wt. ( )	(6) ( )	(7) J ( )	(8) T	(9) P	(10) ADF	(11) MMF	(12) F	(13) SF	(14) A
1													
2													
3													
4													
5													
6													
7													
8													
9													
10													
11													
12													
13													
14													
15													

Technician __________________ Computed By _________________ Checked By __________________

Remarks ____

RESONANT COLUMN TEST
(Final Calculations Sheet)

DATE ______________

PROJECT ______________________________________

BORING NO. ________________ SAMPLE NO. ________________ APPARATUS ________________

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Line No.	Eff. Conf. Stress ( )	Princ. Stress Ratio	Long. or Tors.	No. of Vib. Cycles	Modulus E or G ( )	Strain Amplitude (%)	Damping Ratio (%)	
							Steady State	Ampl. Decay
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								

Technician ________________ Computed By ________________ Checked By ________________

Remarks ____

PLATE 3

PLATE 4

RESONANT COLUMN TEST
(Amplitude Decay Data Sheet)

DATE ----------------

PROJECT --

BORING NO. -------------------- SAMPLE NO. -------------------- APPARATUS --------------------

(1) Line No.	(2) Pict. or Record No.	(3) A	(4) A	(5) n	(6) System Log Dec.	(7) System Energy Ratio	(8) Damping Ratio (%)
1							
2							
3							
4							
5							
6							
7							
8							
9							
10							
11							
12							
13							
14							
15							

Technician -------------------- Computed By -------------------- Checked By --------------------

Remarks --

In accordance with letter from DAEN-RDC, DAEN-ASI dated 22 July 1977, Subject: Facsimile Catalog Cards for Laboratory Technical Publications, a facsimile catalog card in Library of Congress MARC format is reproduced below.

Drnevich, Vincent Paul
Resonant column test / by Vincent P. Drnevich, Soil Dynamics Instruments, Inc., Lexington, Kentucky. Vicksburg, Miss. : U. S. Waterways Experiment Station ; Springfield, Va. : available from National Technical Information Service, 1978.
63,[7] p., 4 leaves of plates : ill. ; 27 cm. (Miscellaneous paper - U. S. Army Engineer Waterways Experiment Station ; S-78-6)
Prepared for Office, Chief of Engineers, U. S. Army, Washington, D. C., under Contract No. DACW39-77-M-1687.
Includes bibliographical references.

1. Computer programs. 2. Damping. 3. Resonant column tests. 4. Shear properties. 5. Soil test specimens. 6. Vibrations. I. Soil Dynamics Instruments, Inc. II. United States. Army. Corps of Engineers. III. Series: United States. Waterways Experiment Station, Vicksburg, Miss. Miscellaneous paper ; S-78-6.
TA7.W34m no.S-78-6