AD-A058 874 BROWN UNIV PROVIDENCE R I DIV OF APPLIED MATHEMATICS F/G 12/1

OPTIMAL CONTROL OF MARKOV DIFFUSION PROCESSES. (U)

1978 W H FLEMING AFOSR-76-3063

UNCLASSIFIED AFOSR-TR-78-1360 NL

1 OF 1

AD A058874

END

DATE FILMED

11-78

DDC

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: AFOSR-TR- 78-1360	2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle): OPTIMAL CONTROL OF MARKOV DIFFUSION PROCESSES	5. TYPE OF REPORT & PERIOD COVERED: Interim rept.
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): Wendell H. Fleming	8. CONTRACT OR GRANT NUMBER(s): AFOSR 76-3063, NSF-MCS76-07261
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Brown University, Division of Applied Mathematics, Providence, RI 02912	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 61102F 2304/A1
11. CONTROLLING OFFICE NAME AND ADDRESS: Air Force Office of Scientific Research/NM, Bolling AFB, Washington, DC 20332	12. REPORT DATE: 1978
	13. NUMBER OF PAGES: 5
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report): UNCLASSIFIED
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

LEVEL

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

(12) 11 p.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Some results from optimal stochastic theory are surveyed in this paper, with particular emphasis on control of diffusion processes. Methods for obtaining necessary & sufficient conditions for an optimum are obtained, as well as some techniques for approximate solution. A new application of stochastic control methods is made to obtain Ventcel-Freidlin type estimates for the probability that the states of a diffusion process remain in a given region during a given time period.

DD FORM 1 JAN 73 1473

UNCLASSIFIED

065 300

DISCLAIMER NOTICE

AFOSR-TR- 78-1360

OPTIMAL CONTROL OF MARKOV DIFFUSION PROCESSES

Wendell H. Fleming

ACCOMPANYING STATEMENT

This is a concise summary of recent results about optimal control for Markov diffusion processes, and guide to recent literature. The paper is to be presented at the Joint Automatic Control Conference in October 1978. The emphasis is on completely observed diffusions, with briefer discussion of results for the case of partial observations. Various methods to deduce the basic dynamic programming principle are discussed; and some methods for approximate solution are indicated.

OPTIMAL CONTROL OF MARKOV DIFFUSION PROCESSES

Wendell H. Fleming*
Professor
Department of Mathematics
Brown University
Providence, Rhode Island 02912

ABSTRACT

Some results from optimal stochastic theory are surveyed in this paper, with particular emphasis on control of diffusion processes. Methods for obtaining necessary and sufficient conditions for an optimum are obtained, as well as some techniques for approximate solution. A new application of stochastic control methods is made to obtain Ventcel-Freidlin type estimates for the probability that the states of a diffusion process remain in a given region during a given time period.

INTRODUCTION

This paper is intended as a concise survey of recent results in the theory of optimal control for Markov diffusions. We mention results which establish rigorously conditions for an optimum, in case of complete or partial observations, as well as some techniques of approximate solution.

THE MODEL

Consider a control system with state space finite dimensional R^n, subject to random disturbances which are modelled as white noise. The state at time t is denoted by $\xi(t)$ and the control by $u(t)$. The state process obeys a stochastic differential equation (Ito sense)

$$d\xi = f[\xi(t),u(t)]dt + \sigma[\xi(t)]dw , \qquad (1)$$

with $w(t)$ a brounian motion process of some dimension m, and with $u(t) \in U$ where U is a given "control space". The controller may have complete or partial information about past system states. Various kinds of performance criteria have been considered. For instance, one may consider (1) on a finite time interval $0 \leq t \leq T$, and seek a control minimizing an expectation

$$J = E\{\int_0^T L[\xi(t),u(t)]dt + \Psi[\xi(T)]\} \qquad (2).$$

See [10, Chap. VI]. Another possible criterion, discussed below, is the probability of exit from a given region D.

The white noise idealization in (1) implies that the state process $\xi(t)$ is a Markov diffusion if the control enters in feedback form as $u(t) = \underline{u}(t,\xi(t))$. In [4] it is shown that certain stability and other properties of stochastic control systems continue to hold for wide band (approximately white) noise. If the noise coefficient σ is not constant, care must be used in passing from wide band to white noise. This is related to the matter of Ito vs. Stratonovich sense interpretation of (1) [10, pp 126-7].

*Supported in part by NSF Grant MCS76-07261 and U.S.A.F. Grant AFOSR76-3063B

We shall not review here the considerable recent literature on jump Markov processes. See for instance [5] [21] [23].

COMPLETELY OBSERVED SYSTEM STATES

Consider the problem of minimizing a criterion J of type (2), in which the controller can observe the state $\xi(t)$. The theory is in a rather mature state for this problem. To a considerable extent it is based on dynamic programming methods. Let $x = \xi(0)$, and let $V = V(x,T)$ denote the minimum of J. Earlier rigorous treatments of the dynamic programming method relied on the fact that V is a smooth solution of the Bellman equation, which is a uniformly parabolic second order partial differential equation if the problem is nondegenerate. By nondegenerate is meant that the symmetric matrices $\sigma(x)\sigma'(x)$ have eigenvalues bounded below by some $c > 0$. See [10, Chap. VI], and the more complete development in the new book [14]. More recently, other techniques have been developed to justify the dynamic programming principle without appealing to the theory of parabolic partial differential equations. A purely probabilistic method, relying heavily on the Girsanov transformation for measures and martingale representation theorems was used in [6]. Related ideas were developed further in [2] [7], and for the average cost per unit time problem in [16]. An elegant semigroup approach was used in [19]. For simplicity let $L = 0$ in (2), and write $V(x,T) = S_T\Psi(x)$. Then $\{S_T\}$ is a nonlinear semigroup, acting on functions Ψ. The method in [19] is to construct this semigroup directly by a suitable monotone sequence of approximations which are piecewise constant in time.

PARTIALLY OBSERVED SYSTEM STATES

Suppose that the controller can observe $\eta(t)$, which satisfies

$$d\eta = g[\xi(t)]dt + \sigma_1 dw_1 \tag{3}$$

with w_1 a brownian motion independent of w and $\eta(0) = 0$. From a practical viewpoint, the most important result is the classical separation principle in case of linear state and observation equations (1), (3). See [10] [15]. There remains a technical issue in connection with the separation principle, concerning the class of controls admitted [15] [22]. For nonlinear systems, general necessary and sufficient conditions for optimality have been given [6] [7] [13] [20]. However, it seems difficult to get practically useful information about the solution from these conditions.

Another point of view is the following. Let π_t denote the conditional distribution of $\xi(t)$ given $\eta(s)$ for $0 \leq s \leq t$. Even in the nonlinear case a kind of "separated" control problem can be introduced, in which the state is π_t (regarded as completely observed). If $\xi(t)$ is a finite state controlled Markov chain, rather than a solution to (1), and the observation process obeys (3), then the separated problem is itself a finite dimensional diffusion [3] [23]. When $\xi(t)$ obeys (1) and $\eta(t)$ obeys (3), the conditional distribution π_t is a measure-valued process obeying the nonlinear filter equation [17, Chap. 8]. The relation between the separated and original problem with partial observations in this case is a topic of current research.

APPROXIMATE SOLUTIONS

We return to controlled diffusions with complete observations. Explicit solutions to the problem of minimizing J are available in few instances. The best known example is the linear regulator; another is the portfolio selection problem [10, p. 160, 166]. One method of approximate solution is by discretization. This replaces the Bellman equation by difference equations, which are the dynamic programming equations for a corresponding controlled Markov chain [15, Chap. 9]. A quite different kind of approximation method, in case the noise coefficient σ is small, was described in [8]. A related perturbation technique was applied to a resource management problem in [18]. A method for approximate solution to nonlinear perturbations of the stochastic linear regulator was given in [24]. If the perturbation of the state dynamics is polynomial in the state, then the approximation can be implemented knowing only higher order moments of the (Gaussian) solution to the linear regulator.

MINIMUM EXIT PROBABILITY

Let D be a given region in R^n, with initial state $\xi(0)$ in D. We say that exit occurs if $\xi(t)$ reaches the boundary of D during the time interval $0 \leq t \leq T$. Instead of (2) we may take the exit probability as criterion to be minimized. This is reasonable if D is regarded as a region in which the system operates acceptably.

The following asymptotic estimate, for low noise intensities, was given in [11]. Let $\sigma = \sqrt{\varepsilon} I$, with I the identity matrix; and let q^ε denote the minimum exit probability. Then $-\varepsilon \log q^\varepsilon \to J^\circ$ as $\varepsilon \to 0$, where J° is the lower value of a certain differential game. This is analogous to an estimate of Ventcel-Freidlin type for uncontrolled diffusions [12, Chap. 14] [9].

OTHER PROBLEMS

Among optimization problems for diffusions which we have not discussed are optimal stopping [12] [14] [15], and impulsive control [1]. Finally, we should mention techniques of variational inequalities and quasivariational inequalities [1], which provide another framework in which to study a broad class of optimization problems.

REFERENCES

1. Bensoussan, A. and Lions, J. L. (1975) Sur le contrôle impulsionnel et les inequations quasivariationnelles d'evolution, C. R. Acad. Sci. Paris, Sér. A 280, 1049-1053.

2. Bismut, J-M. (1976) Théorie probabiliste du contrôle des diffusions, Memoirs Amer. Math. Soc. No. 167.

3. Bismut, J-M. (1977) Sur un problème de contrôle stochastique avec observation partielle, Preprint, Univ. de Paris-Sud, Orsay.

4. Blankenship, G. and Papanicolaou, G. C. (1978) Stability and control of stochastic systems with wide-band noise disturbances, Part I SIAM J. Appl. Math. 34, 437-476. Part II to appear in same journal.

5. Boel, R. and Varaiya, P. (1977) Optimal control of jump processes, SIAM J. Control and Optimiz. 15, 92-118.

6. Davis, M. H. A. and Varaiya, P. (1973) Dynamic programming conditions for partially observable stochastic systems, SIAM J. Control 11, 226-261.

7. Elliott, R. J. (1977) The optimal control of a stochastic system, SIAM J. Control and Optimiz. 15, 756-778.

8. Fleming, W. H. (1971) Stochastic control for small noise intensities, SIAM J. Control 9, 473-517.

9. Fleming, W. H. (1978) Exit probabilities and optimal stochastic control, Applied Math. and Optimiz. 4

10. Fleming, W. H. and Rishel, R. (1975) Deterministic and Stochastic Optimal Control, Springer-Verlag.

11. Fleming, W. H. and Tsai, C-P. (1978) Minimum exit probabilities and differential games, Proc. 3rd Kingston Conf. on Differential Games and Control Theory.

12. Friedman, A. (1976) Stochastic Differential Equations and Applications, vol II, Academic Press.

13. Haussmann, V. G. (1978) On the stochastic maximum principle, SIAM J. Control and Optimization 16, 236-251.

14. Krylov, N. V. (1977) Controlled Processes of Diffusion Type (in Russian), Izdat. "Nauka", Fiz-Mat.

15. Kushner, H. J (1977) Probability Methods for Approximations in Stochastic Control and for Elliptic Equations, Academic Press.

16. Kushner, H. J. (1978) Optimality conditions for the average cost per unit time problem, SIAM J. Control and Optimiz. 16, 330-346.

17. Liptser, R. S. and Shiryayev, A. N. (1977) Statistics of Random Processes I, II (English tr.) Springer-Verlag.

18. Ludwig, D. (1978) Optimal harvesting of a randomly fluctuating resource, SIAM J. Appl. Math.

19. Nisio, M. (1975) Some remarks on stochastic optimal controls, Proc. 3rd Japan-USSR Symp. Probability, Springer Lecture Notes in Math. No. 550.

20. Rishel, R. W. (1970) Necessary and sufficient dynamic programming conditions for continuous-time stochastic optimal control, SIAM J. Control 8, 559-571.

21. Rishel, R. W. (1977) Optimality for completely observed controlled jump processes, IEEE Trans. Auto. Control AC-22, No. 6., 906-908.

22. Ruzicka, J. (1977) On the separation principle with bounded controls, Appl. Math. Optimiz. 3, 243-261.

23. Segall, A. (1977) Optimal control of noisy finite-state Markov processes, IEEE Trans. Auto. Control AC-22, No. 2, 179-186.

24. Tsai, C-P. (1978) Perturbed stochastic linear regulator, SIAM J. Control and Optimiz. 16, 396-410.