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JUL 78 H MCDONALD, C K ZOLTANI

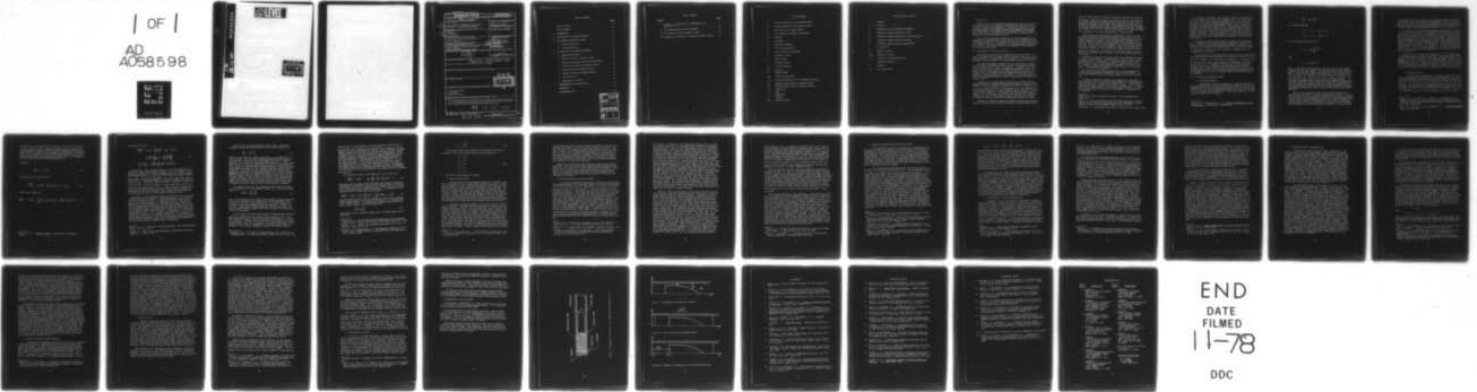
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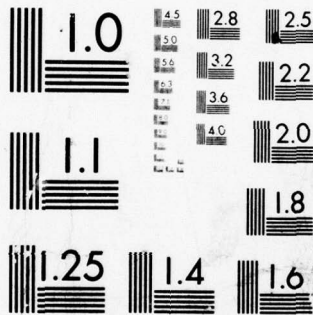
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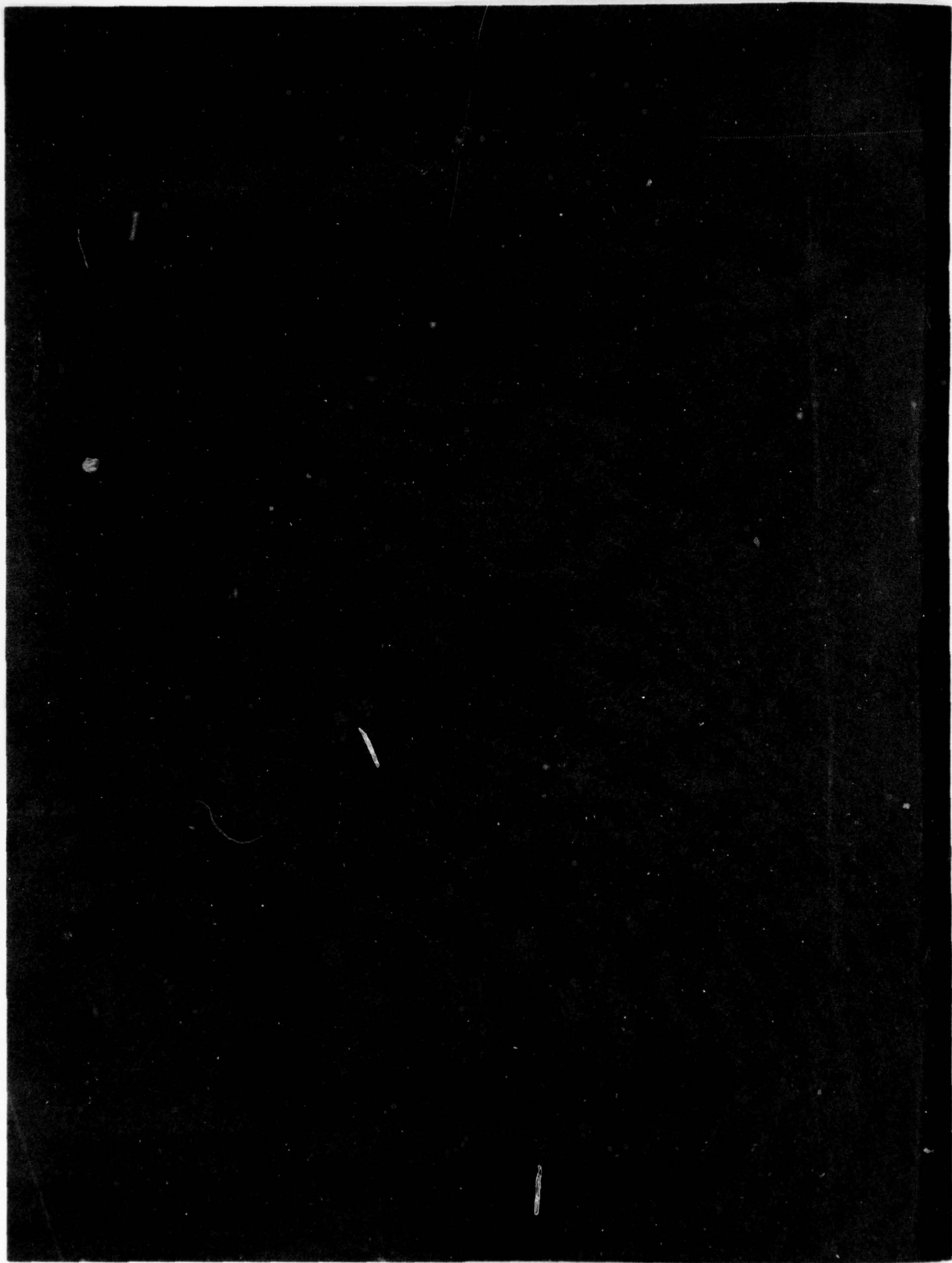
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LIST OF SYMBOLS

C_μ	constant appearing in the turbulence model
C_1	constant appearing in the turbulence model
$\bar{\epsilon}$	mean flow rate of strain tensor
f_j	j-th component of liquid or solid phase
h_o	enthalpy
h	gap height
H	stagnation enthalpy
k_a	acceleration parameter
k	kinetic energy of turbulence
l	distance of travel
l_t	mixing length
L	reference length
m_i	i-th gas component
p	pressure
Pr	Prandtl number
q_{RD}	radiation energy flux
r_i	chemical production rate, i-th species equation
Re	Reynolds number based on reference length
R_j	droplet source term for j-th particle class
s_{ij}	source term
t	time
T	temperature
u	velocity
W_D	molecular weight

LIST OF SYMBOLS (cont'd)

x	distance
y	distance
y_+	dimensionless normal distance coordinate
Γ_m	turbulent exchange coefficient of gas
Γ_p	turbulent exchange coefficient of the other phase
Γ_h	turbulent exchange coefficient of heat
ϵ	turbulence kinetic energy dissipation rate
n	$y/(4\nu t/Pr)^{1/2}$
μ_D	dynamic viscosity
μ_{eff}	effective viscosity
μ_T	turbulent viscosity coefficient
ν	kinematic viscosity
ρ	density
τ_w	wall shear stress

I. Introduction

Among the factors contributing to the wear and erosion of gun tubes, the leakage of the high temperature propellant gases past the moving projectile occupies an important role. Presently, there doesn't exist a rational basis for the estimation of the gas leakage for any given gun configuration. This report describes the physics of the leakage process and outlines a program for a quantitative determination of the leakage flow for smooth bore gun tubes.

In the following sections we describe the physical phenomena, the assumptions of the mathematical model, and give in an abbreviated form the governing equations. We then discuss the simplifications which have to be made to enable the development of an efficient algorithm which is then outlined. The report closes with recommendations for implementation of the outlined work.

II. Description of the Physical Process

Assume that a projectile in a smooth bore gun tube at time $t = 0$ starts to accelerate at the breech end due to the pressure rise of the gases generated in the propellant bed by combustion. The gases are compressible, viscid, turbulent and contain unburnt propellant fragments.

The ignition phenomena leading to the gas generation are complex and still need to be explored in fine detail. The sequence of events is pictured as follows: first, the base pad is ignited by the primer gases. The center core ignites and begins to spread the flame. The charge itself is then ignited, the propellant bag breaks and the bed expands to fill the chamber, thereby transforming the latter from a packed to a fluidized condition. See Figure 1. As the pressure rises due to the burning, the bore resistance is overcome and the projectile begins to move.

Initially the two phase mixture consisting of product gases and unburnt particles is moving subsonically with respect to the tube wall. However, it and the projectile continuously accelerate reaching a velocity of the order of 10^3 (m/s) at the gun tube exit. The temperature of the gases near the muzzle is around 2000° (K), while the wall is initially at ambient conditions. The gas motion is highly turbulent and unsteady. Due to the presence of unburnt particles and solid combustion products, the ratio of the specific heats of the mixture of particles and gases is lower than that for the combustion product gases alone.

Through the clearance between the projectile and the gun tube wall, which is of the order of 0.2 mm in a new tube, the high pressure gases

from the region behind the projectile leak past and outrace the moving projectile. Typically, as shown by Loder¹, for the 105 mm gun tube, at 0.5 calibers from the beginning of the rifling, which is also near the position of maximum wear, the groove due to the wear can be up to 2.28 mm (90 mils) measured land to land. The severity of the wear decreases as one goes from the breech toward the muzzle end of the tube. Interestingly enough, wear just as severe has been reported for smooth bore guns. For a summary of our current understanding of wear see Ward².

The nature of the flow around the projectile in the gun tube will be directly affected by the previous history of bore wear, i.e., the change in the area available for the passage of the two phase flow. The rate of leakage will depend on the pressures behind and ahead of the projectile and of course the clearance between the projectile and the gun tube. The pressure of course depends on the time and the position, while the clearance is position dependent. Hence, the leakage is time and position dependent. That leakage can be appreciable was already observed by Oswatitsch³, who found that the pressure ahead of the projectile in a gun tube can be as high as 10% of the pressure prevailing at its base.

The physical dimensions and time scales of the phenomena under consideration are as follows. The projectile is assumed to have a length to diameter ratio exceeding 20. It accelerates from rest and at the muzzle travels at a low supersonic velocity. Typical transit times for a medium to large caliber weapon are therefore of the order of 10 - 20 ms. The gap between the projectile and the barrel is of the order of 0.2 - 5% of the barrel diameter.

While the pressure behind the projectile is in the megapascal regime, locally encountered temperatures hover around 2000 [K]. Typical Reynolds numbers based on the barrel diameter are of the order of 10^8 .

The model adopted here assumes high pressure gases leaking past a moving projectile which in turn moves with respect to a smooth bore gun tube. The gases will be taken to be premixed and of a single phase. In reality of course, one has to deal with a two phase flow. To include some of the two phase effects into a single phase description, a ratio of the specific heat of the gas will be used which is lower than that of a pure gas.

¹Loder, R.K.; Ballistic Research Laboratory, Private Communication, August 1977.

²Ward, J.R., "A New Initiative in Gun Barrel Wear and Erosion", Proceedings of the Tri-Service Gun Tube Wear and Erosion Symposium, US Army Armament Research and Development Command, Dover, NJ, 1977.

³Oswatitsch, K., "Intermediate Ballistics", DVL Rpt. No. 358, Aachen, 1964.

The mixture is allowed to take part in a chemical transformation. A simple decomposition scheme $A \rightarrow B$ is adopted, where all the reactants are lumped into A and the reaction products into B. The latter are assumed to have attained chemical equilibrium. Finite rate chemistry is assumed to control the disappearance of the reactants with a rate parameter varying between 1 and 0, with 1 indicating the presence of reactants only and 0 the completion of the chemical change. The exothermicity of the reaction will be accounted for through the enthalpy change of the gases. Arrhenius kinetics, it is felt, describes the chemical evolution of the system adequately with the reaction rate constant and the activation energy taken as a composite of the most dominant species.

To adequately model the flow, one must adopt a viable turbulence model. Assuming the validity of Morkovin's hypothesis, which states that the effect of compressibility upon turbulence is negligible, excepting for an influence on the mean density of the gas, the $k-\epsilon$ model of turbulence of Launder and Jones⁴, proven for incompressible flow calculations, will be used.

The possibility of relaminarization of the flow, as it emerges from the narrow channel between the projectile and the gun tube wall, exists and will be admitted. The acceleration parameter $k_a = (\nu/u^2)(du/dx) > 10^{-6}$ for typical flows in gun tubes indicates the likelihood of relaminarization. Here ν is the kinematic viscosity and u the flow velocity. This turn of events is thought to come about because the turbulent boundary layer is unable to follow the acceleration and with the fall of the Reynolds number, viscous effects assume enough importance within the boundary layer to lead to the growth and the ultimate dominance of the sublayer.

III. The Proposed Mathematical Model

A. General Observations

Considerable insight and mathematical simplification results from an examination of the time constants of the dominant processes of the problem. In order to establish some idea of the time scale for the diffusion of heat, consider the highly simplified case of pure diffusion (i.e., no convection) of temperature T in one space dimension. The appropriate equation can be written

⁴Launder, B. E. and Spalding, D. B., Lectures in Mathematical Models of Turbulence, Academic Press, London, 1972.

$$\frac{\partial T}{\partial t} - \frac{\nu}{Pr} \frac{\partial^2 T}{\partial y^2} = 0$$

with boundary conditions

$$y \geq 0 \quad t < 0 \quad T(y,t) = T_0$$

$$y = 0 \quad t > 0 \quad T(0,t) = 0$$

and this equation has the solution

$$T = T_0 \frac{2}{\sqrt{\pi}} \int_0^{\eta} e^{-r^2} dr$$

where

$$\eta = y/(4\nu t/Pr)^{1/2}$$

Here ν is the usual laminar kinematic viscosity and Pr is the conventionally defined Prandtl number. The solution represents the transient growth of the cold wall temperature defect into a hot external environment. If the scale of length y in the definition of η is taken to be the tube radius and time scale the time for the gas to pass the projectile, then typically η would achieve values of fifty and higher. It can readily be deduced from the previously given temperature-time solution that using these typical values of η would result in thermal boundary layers which are extremely thin and of course transiently growing. In the problem of interest here, of course, such thin transiently varying thermal boundary layers control the wall heat transfer and cannot be neglected.

The fluid flow equations of motion given earlier contain time derivatives of the density, velocity and enthalpy of the gas. If these time derivatives were all negligible compared to the spatial derivatives, a quasi-steady flow approximation would be a reasonable assumption which could be used to reduce the computational labor. Order of magnitude considerations lead to the conclusion, for instance, that the temporal density gradient is important and cannot be neglected. Hence, as might

be supposed on intuitive grounds, the transient effects on the aerodynamic process within the barrel are not negligible either. Thus during most of the transit time the flow should exhibit significant transient effects on the aerodynamics and heat transfer and this is substantiated by (among others) the calculations of Anderson, et al (Reference 5).

It is probably self-evident that chemical reactions have a major role to play in the problem. The obvious one is of course the propellant burn after ignition and the prospect of gas phase reaction in the gap itself is feasible and of great concern. Further, should the burning be very rapid and the gas be in or near chemical equilibrium the proper determination of the gap heat transfer rates requires that the change in chemical equilibrium conditions due to changing flow environment be taken into account. Thus, in addition to the other previously mentioned factors we are led to consider a (simple) multicomponent reacting flow. For the leakage problem then, at least an axially symmetric, viscous (turbulent), time-dependent, reacting compressible flow must be considered.

The presence of the base corner gives rise to very significant flow gradient along the barrel, in addition to the conventional gradients that occur in most boundary layer type of problems and which occur normal to the wall. Diffusion, i.e., transport effects, upon heat mass and momentum in both the radial and the axial direction must be allowed for near the gap inlet. The pressure field at the gap entrance and exit is also not readily approximated or neglected, leading one to consider the full Navier-Stokes equations as being required to describe the flow.

B. Governing Equations

The flow under consideration is a turbulent chemically reacting multi-component mixture with heat and mass transport. The governing system of partial differential equations describing this process is based on the conservation laws of mass, momentum, energy and chemical species⁶. For simplicity these equations are expressed in vector notation below and all quantities are nondimensional. Velocities are normalized by U_D , density by ρ_D , enthalpy by h_D , temperature by T_D , molecular weights by W_D , pressure by $P_D = \rho_o R T_D Z_D$, ($Z_D = 10^3/W_D$), dynamic

⁵Anderson, L. W., Bartlett, E. P., Dahm, T. J. and Kendall, R. M., "Numerical Solution of the Non-Steady Boundary Layer Equations with Application to Convective Heat Transfer in Guns", Aerotherm Final Rpt. No. 70-22, October 1970.

⁶Bird, R. B., Stewart, W. E. and Lightfoot, E. N., Transport Phenomena, Wiley, New York, 1960.

viscosity by μ_D , radiation energy flux by q_{R_D} , and time by (L/U_D) where L is the reference length. Coupling between concentration and thermal gradients (Soret and Dufour effects), pressure gradient diffusion, body forces and bulk viscosity are all assumed to be negligible. In addition, Fick's law is presumed valid which implies equal binary diffusion coefficients for each pair of species in the mixture (see, e.g., Reference 7). The resulting set of ensemble-averaged equations is

Continuity

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \bar{u}) \quad (1)$$

Conservation of Gas Phase Species

$$\frac{\partial (\rho m_i)}{\partial t} = -\nabla \cdot (\rho \bar{u} m_i) + \frac{1}{Re} \nabla \cdot (\Gamma_m \nabla m_i) + r_i + \sum_j s_{i,j} \quad (2)$$

Conservation of Momentum

$$\frac{\partial (\rho \bar{u})}{\partial t} = -\nabla \cdot (\rho \bar{u} \bar{u}) - \frac{P_D}{\rho_D U_D^2} \nabla P + \frac{1}{Re} \nabla \cdot (2\mu_{eff} \bar{e}) - \frac{2}{3} \frac{1}{Re} \nabla [\mu_{eff} (\nabla \cdot \bar{u})] \quad (3)$$

⁷Williams, F. A., Combustion Theory, Addison-Wesley, Reading, MA, 1965.

Conservation of Energy

$$\begin{aligned} \frac{\partial(\rho H)}{\partial t} = & -\nabla \cdot (\rho \bar{u} H) + \frac{P_D}{\rho_D h_D} \frac{\partial p}{\partial t} + \frac{1}{Re} \nabla \cdot (\Gamma_h \nabla H) \\ & + \frac{1}{Re} \frac{U_D}{h_D} \nabla \cdot \left[\left(\mu_{eff} - \Gamma_h \right) \nabla \left(\frac{\bar{u} \cdot \bar{u}}{2} \right) \right] \\ & + \frac{1}{Re} \nabla \cdot \left[(\Gamma_m - \Gamma_h) \sum_i h_i \nabla m_i \right] - \frac{1}{Bo} \nabla \cdot \bar{q}_R - \sum_i r_i h_i \end{aligned} \quad (4)$$

ρ is the density, \bar{u} the velocity vector, m_i is the i^{th} component gas, f_j component of other phase (liquid or solid). p is the pressure, H the stagnation enthalpy, Re the Reynolds number based on reference length and reference velocity, Γ_h , Γ_m , Γ_p are the turbulent exchange coefficients of heat, gas and the other phase. $\bar{\epsilon}$ is the mean flow rate of strain tensor. A turbulence model is employed to define the effective velocity, μ_{eff} . Similarly, a chemistry model is employed to specify the production rate r_i for the chemistry, a phase change model could be employed for the source terms R_i and s_{ij} , and a radiation transport model again could be employed to specify $\nabla \cdot \bar{q}_R$. These terms are included for completeness but will not be used at least during the initial phase of the investigation, apart of course from the turbulence terms.

To account for the turbulent behavior in the solution of the ensemble-averaged Navier-Stokes equations, a turbulence model is introduced to define an effective viscosity. A review of turbulence models is available in the literature (see, e.g., References 4 and 8). Prandtl was perhaps the first to introduce a turbulence model when he postulated that the time-averaged shear stress and the time-averaged velocity gradient are proportional as in laminar flow, and that the length scale (the so-called mixing length) which enters the relationship is proportional to the turbulent shear region thickness. A disadvantage of the mixing length model is that it is an equilibrium model (i.e., turbulence is assumed to be produced and dissipated locally) and it requires an ad hoc mixing length distribution. Some of the shortcomings of the mixing length model have been overcome for many cases of interest by the introduction of various multiequation transport models of turbulence.

⁸Harlow, F.H., ed., "Turbulence Transport Modeling", AIAA Selected Reprint Series, Vol. XIV, 1973.

⁹Prandtl, L., "Bericht Über Untersuchungen Zur Ausgebildeten Turbulenz," ZAMM, Vol. 5, 1925, p. 136.

Many of the two-equation turbulence models employ the Prandtl-Kolmogorov formula for specification of the turbulent viscosity, μ_T ,

$$\frac{\mu_T}{\text{Re}} = C_\mu \rho k^{1/2} \ell_t \quad (6)$$

where k is the turbulence kinetic energy and ℓ_t is a length scale of the turbulence. This relation follows from dimensional arguments for turbulent flow described by the two parameters, k and ℓ_t . A major advantage of a two-equation turbulence model compared to a mixing length model is the fact that the length scale (as well as the turbulence kinetic energy) is determined from transport equations, whereas in mixing length models the length scale is determined from an ad hoc algebraic expression. Successful use of the mixing length model requires an a priori specification of the turbulence length scale. While a realistic assumption can be made for certain flows such as shear layers, in many flows of interest (such as internal recirculating flows) the choice of a proper turbulence length scale is not obvious. Since the turbulence length scale emerges from the solution in a two-equation model, these models are more likely to give accurate predictions over a wide range of geometric and flow conditions with the same empirical constants.

It should be noted that the two-equation models employ the eddy-viscosity formulation for the Reynolds stresses as in the mixing length model, i.e.,

$$\overline{\rho u_i' u_j'} = -\frac{\mu_T}{\text{Re}} \frac{\partial u_j}{\partial x_i} \quad (7)$$

Hence, this formulation still suffers from the physical shortcoming that there is zero Reynolds stress wherever the velocity gradient is zero. In addition, the eddy viscosity formulation is isotropic which may be incorrect in many three-dimensional and swirling flows. However, for practical calculations of complex turbulent internal flows there are no other available transport models which are as suitable or even as relatively well developed.

Various forms of the two-equation model of turbulence have been proposed since Kolmogorov¹⁰ first introduced the concept in 1942. Most investigators have chosen the kinetic energy of turbulence, k , as their first variable. However, there is a wide diversity of choice as to the second variable to be used. In general, each investigator chose a second variable which he felt was appropriate to the physical description of turbulence. For instance, Kolmogorov chose as his second variable a

¹⁰Kolmogorov, A.N., "Equations of Turbulent Motion of an Incompressible Turbulent Fluid", Izv. Akad. Nauk, SSR Ser. Phys. VI, No. 1-2, 56, 1942.

quantity which is proportional to the mean frequency of the most energetic motions, while Spalding¹¹ and Saffman¹² chose a quantity that represented the time-averaged square of the vorticity fluctuations. Another commonly chosen second variable has been the turbulence kinetic energy dissipation rate, ϵ , which was selected for this investigation. An advantage of using the dissipation rate of turbulence kinetic energy for the second equation is that the dissipation rate appears directly in the turbulence kinetic energy equation, and that an equation for ϵ can be readily developed. Since derivations of the equations for k and ϵ are lengthy and have previously been presented in the open literature, these derivations are not repeated in the present report. The appropriate transport equations for turbulent kinetic energy and energy dissipation rate, valid at high Reynolds numbers, are taken directly from Launder and Spalding¹³.

The turbulence kinetic energy equation in vector notation is

$$\frac{\partial(\rho k)}{\partial t} = -\nabla \cdot (\rho \bar{u}k) + \frac{1}{\text{Re}} \nabla \cdot \left(\frac{\mu_T}{\sigma_k} \cdot \nabla k \right) + \frac{\mu_T}{\text{Re}} (2\bar{\bar{e}}:\bar{\bar{e}}) - \rho \epsilon \quad (8)$$

where the first two terms on the right-hand side represent convection and diffusion of turbulence kinetic energy, respectively; the third and fourth terms represent the generation (due to shear forces) and dissipation of turbulence kinetic energy respectively. The equation for the dissipation rate of turbulence kinetic energy is

$$\frac{\partial(\rho \epsilon)}{\partial t} = -\nabla \cdot (\rho \bar{u}\epsilon) + \frac{1}{\text{Re}} \nabla \cdot \left(\frac{\mu_T}{\sigma_\epsilon} \nabla \epsilon \right) + C_1 \frac{\mu_T}{\text{Re}} \frac{\epsilon}{k} (2\bar{\bar{e}}:\bar{\bar{e}}) - C_2 \rho \frac{\epsilon^2}{k} \quad (9)$$

where the function of the terms is analogous to that in the turbulence kinetic energy equation. In Equations (8) and (9), $\bar{\bar{e}}$ is the mean flow rate of strain tensor. Using dimensional arguments the Prandtl-Kolmogorov formula, Equation (6), may be written as

$$\frac{\mu_T}{\text{Re}} = C_\mu \frac{\rho k^2}{\epsilon} \quad (10)$$

which implies that a turbulence length scale or "mixing length" may be defined as

¹¹Spalding, D. B., "The Prediction of Two-Dimensional, Steady Turbulent Flows", Imperial College, Heat Transfer Section Report EF/TN/A/16, 1969.

¹²Saffman, P. G., "A Model for Inhomogeneous Turbulent Flow", Proc. Roy. Soc., Vol. A317, 1970, p. 417.

¹³Launder, B. E. and Spalding, D. B., "The Numerical Computation of Turbulent Flows", Computer Methods in Applied Mechanics and Engineering, Vol. 3, 1974, p. 269.

$$\ell_t = C_\ell \frac{k^{3/2}}{\epsilon} \quad (11)$$

The constants appearing in Equations (8) through (10) have been evaluated by Jones and Launder¹⁴ and the values proposed are

$$\begin{aligned} C_\mu &= 0.09 \\ C_1 &= 1.55 \\ C_2 &= 2.0 \\ \sigma_k &= 1.0 \\ \sigma_\epsilon &= 1.3 \end{aligned} \quad (12)$$

IV. Discussion of the Numerical Treatment

A. General Observations

Having concluded in the previous section that a rigorous treatment of the physics of the leakage problem demands that the full Navier-Stokes equations be used, it now becomes necessary to discuss the problems involved in obtaining numerical solutions to these equations. The full Navier-Stokes equations do have the attractive features that at least in a formal sense their use minimizes the physical approximations required. However, both physical and numerical treatments are required to model on the one hand the turbulent transport and on the other to deal with the coupled nonlinear nature of the Navier-Stokes equations. Here the numerical problems which arise when the Navier-Stokes equations are solved under the conditions of the leakage problem are discussed.

The chosen numerical treatment is a critical factor in determining the economics or perhaps even the feasibility of performing the desired calculations. Use of a numerical algorithm not well suited to the problem at hand could result in a prohibitive run time to generate solutions to the required degree of accuracy. Consideration of the need to accurately define the very thin boundary layers, both thermal and momentum, leads to proposing the use of an implicit finite difference algorithm. Explicit and/or stability restricted schemes are usually forced to take extremely small time steps as a result of the refined spatial mesh. This unduly small time step is much smaller than would be required for transient accuracy. The implicit schemes usually do not suffer from this particular problem and can be very much more efficient for boundary region type of problems. Implicit schemes, however, have not been widely

¹⁴Jones, W. P. and Launder, B. E., "The Prediction of Laminarization with a Two-Equation Model of Turbulence", *Int. J. Heat Mass Transfer*, Vol. 15, 1972, p. 301.

used for transient mixed convection-diffusion problems and the possible problems which might arise in this area are discussed in detail. Further, the choice of an implicit scheme leads to the necessity of considering in detail equation linearization and coupling, and this will be done. A Taylor series expansion in time leads to a very powerful linearization and this concept is recommended. With an explicit scheme the choice of dependent variables can virtually eliminate the problem of linearization and coupling which is why in the literature this problem is often not, as yet, well discussed.

Common to any algorithm is the coordinate system and the boundary conditions. The choice of coordinate system and the subsequently used mesh can be a critical factor in the overall algorithm accuracy. Consequently both the coordinate system and the mesh will be discussed in detail. On the basis of this discussion it will be suggested that a body fitted coordinate system based upon the projectile would be both convenient and accurate.

B. Transient Accuracy, Stability Limits

As a general observation great care is required to obtain acceptable transient accuracy for long time integration of fluid flow equations in which the convective terms are significant. The problem is discussed for instance by Morton¹⁵ and Roache¹⁶. In the more widely used finite difference schemes two types of transient errors are commonly encountered, damping and dispersion. Considering the pure convection of a wave, damping would be reflected in the change with time of the wave amplitude while dispersion would be reflected in the relative phase change with time, i.e., the change with time of the wave propagation velocity. Time centering of the spatial derivatives which would require taking all spatial derivatives at the midpoint of the time step, can be employed to remove the numerical damping. However, dispersion can be much more difficult to treat adequately in order to achieve a desired level of accuracy due to the presence of phase errors. To date the more widely used schemes with good properties with regards to dispersion have been stability restricted in the manner described by Courant, Friedrichs and Lewy¹⁷. In this sense the maximum time step permissible in the simple compressible one-dimensional fluid flow problem is found to be restricted such that

¹⁵Morton, K. W., "Stability and Convergence in Fluid Flow Problems", Proc. Royal Soc., Vol. A323, No. 1553, June 1971, pp. 237-253.

¹⁶Roache, P. J., "Computational Fluid Dynamics", Hermosa Publishers, 1972.

¹⁷Courant, R., Friedrichs, K. O. and Lewy, H., "Uber die Partiellen Differenzgleichungen der Mathematischen Physik", Mathematische Annalen, Vol. 100, 1928, pp. 32-74.

$\Delta t < \Delta x / (|u| + a)$ where u is the convection velocity and a is the local speed of sound and Δx is the spatial mesh increment. Experience indicates that this type of restriction appears even in the multidimensional analogues. Since locally refined spatial meshes are required in the present problem to define the boundary layers and also as a result of the low local Mach numbers expected (high temperatures), the Courant-Friedrich-Lewy (CFL) requirement could be painfully restrictive. In such a situation the implication is that the physical processes are changing with time more slowly than the CFL limit and that the time truncation errors would be acceptable if a time step greater than the CFL limit were used and the numerical scheme could accept such a large time step. The counter argument sometimes given is that indeed the CFL limit represents the time scale of propagation of physical information; but this is clearly not generally correct since incompressible flow simulations of time-dependent low Mach number compressible flows are normally very good and in these analyses the speed of sound is infinite with a resulting allowable CFL limited time step of zero. Having said this, it should be pointed out that in many physical processes the relevant time scale of information propagation is indeed the local speed of sound and as such the CFL limit would not be restrictive (unless only the steady solution were being sought). For the particular problem under consideration unfortunately the CFL limit would be unduly restrictive certainly during the initial phases of the motion and a scheme not restricted by the CFL limit would be preferable. This leads to the previously mentioned recommendation of an implicit difference scheme.

Turning to these unconditionally stable schemes Morton¹⁵ shows the relatively poor dispersion characteristics of a Crank-Nicolson implicit scheme applied to the pure convection problem. Although not shown by Morton, the dispersion at a given wavelength increases progressively as the CFL limit is exceeded. Thus it appears that for pure convection with a Crank-Nicolson implicit scheme taking large time steps, a wave described by less than ten or so grid points would suffer unacceptable dispersion after some period of time. Thus accurate long time calculations of short wave length components of the motion would require a very refined spatial mesh. It would then seem that something of a dilemma exists. Schemes with good dispersion properties require many time steps to describe the motion due to the CFL restriction (the start of the motion seems particularly difficult with a CFL restricted scheme). Schemes not restricted by the CFL limit do not require as many time steps but do require good spatial resolution in order to have acceptable short wave length dispersion. Some observations are pertinent at this point however. First is that good spatial resolution is required in any event to define the boundary layers and shear layers in the particular problem under consideration. Next in the particular problem that is being considered, the time dependency is continuously imparted through the initial and boundary conditions. As a result the problem is more one of predicting the diffusion of a forced transient than trying to determine the phase lag of a wave in a slightly sheared flow after a

long propagation time. In addition, as Morton¹⁵ points out the better explicit schemes with good dispersion properties have certain other undesirable properties which require special treatment and which in a complicated multidimensional problem might prove difficult to implement. Lastly, apparently there have not been any significant attempts made to improve the dispersion characteristics of the implicit schemes, and should the potential gains warrant the labor, it would seem very worthwhile to investigate a modification to the implicit scheme similar to the one developed by Fromm¹⁸ for the Lax-Wendroff scheme or apply the ideas of Boris and Book¹⁹ which, although developed for treating large gradients in the flow, do reduce dispersive errors.

With the foregoing in mind it would be reasonable to proceed with an implicit scheme in view of the restrictions that might otherwise result in the use of the stability restricted explicit schemes with both low Mach numbers and refined spatial meshes. Dispersion might be regarded as a potential problem for the implicit scheme and therefore both mesh refinement studies and model problem evaluation must be carried out, both for the actual cases involved and for a model system more representative of the mixed convection-diffusion problem under consideration. Should critical flow features be found subject to undue dispersion, consideration should be given to either increased spatial resolution or to developing a correction for the implicit scheme following Fromm¹⁸ or Boris and Book¹⁹.

As was mentioned earlier, time centering or "Crank-Nicolson" averaging of the spatial derivatives produces a scheme which has no damping in the simple problem and is of formal second order accuracy in time. For the more complicated systems of equations which govern the fluid flow great care is required, for example, with the treatment of nonlinearities and cross derivatives to obtain formal second order temporal accuracy. When this is compounded with the difficulties of using a highly nonlinear turbulence and chemistry model coupled implicitly with the dependent variables in a consistent second order time-dependent manner, a great deal of work is clearly required to achieve formal second order transient accuracy in these non-model problems. At this early stage of the overall problem development it is doubtful if a great deal of extra effort required to achieve formal second order transient accuracy is warranted. Much, of course, can and should be done to improve the transient accuracy. After the fact and as part of the investigation of the dispersion problem an evaluation of the benefit of more complete second order accurate transient representation can be made.

¹⁸Fromm, J. E., "A Method for Reducing Dispersion in Convective Difference Schemes", *J. Comp. Physics*, Vol. 3, p. 176, 1968.

¹⁹Boris, J. P. and Book, D. L., "Minimizing Errors in Flux-Corrected Transport Algorithms", *NRL Report 3024*, April 1975.

C. Equation Coupling and Linearization Problems

In the previous section it was argued that a conditionally stable explicit scheme would require a very large number of time steps to integrate over the projectile transit time in view of the initial low Mach number and locally refined mesh used in this mixed convection-diffusion problem. This observation led to the proposed use of an implicit scheme and once an implicit scheme is envisaged, equation coupling and linearization must be considered. As mentioned earlier, with an explicit scheme the choice of dependent variables can mitigate greatly this particular problem such that it really has come to be thought of as primarily a feature of implicit methods.

The linearization problem leads naturally into the coupling problem so it is discussed here first of all. Both problems are reviewed in detail by McDonald and Briley²⁰ and Briley and McDonald²¹. It is argued by these authors that in order to get the most accuracy out of a given grid, the errors arising from representing nonlinear terms by linear combinations of terms at the unknown time level should be less than or equal to the discretization errors. If the linearization errors are greater than the temporal or spatial discretization errors then clearly the accuracy of the differencing is being squandered and iteration or some form of linearization improvement is called for. As a first option, iteration to reduce the linearization error is not to be recommended, since iteration only improves the linearization; yet computationally usually it costs as much for one iteration as it does to march one time step. Reducing the time step would be preferable to iteration since cutting back on the time step would improve both the transient accuracy and reduce the linearization error. To obtain a linearization which introduces errors of at most the same order as the temporal differencing, a Taylor series expansion about the known time level can be performed, and this is the approach adopted in References 22 and 23. The process results in, for instance, expanding a uv term about the known time level n to obtain a second order accurate linear expression at the unknown time level n+1,

²⁰McDonald, H. and Briley, W. R., "Three Dimensional Flow of a Viscous or Inviscid Gas", J. Comp. Physics, Vol. 19, No. 2, 1975.

²¹Briley, W. R. and McDonald, H., "Solution of the Multidimensional Compressible Navier-Stokes Equations by a General Implicit Method", J. Comp. Physics, Vol. 24, No. 4, August 1977.

²²Lindemuth, I. and Killeen, J., "Alternating Direction Implicit Techniques for Two-Dimensional Magnetohydrodynamic Calculations", J. Comp. Physics, Vol. 13, pp. 181-208.

²³Beam, R. M. and Warming, R. F., "An Implicit Factored Scheme for the Compressible Navier-Stokes Equations", AIAA Paper, 3rd Comp. Fluid Dynamics Conf., June 1977.

$$(uv)^{n+1} = u^n v^{n+1} + v^n u^{n+1} - u^n v^n + O(\Delta t^2)$$

and immediately a coupled system is obtained, since neither u nor v can be left at the old time level in order to generate an uncoupled system (i.e., one scalar equation for each dependent variable containing only that particular dependent variable at the unknown time level) without introducing a first order time truncation error. Thus the formal linearization process and consideration of the resulting errors indicates that from an accuracy point of view a coupled system of equations at the new time level must be treated. This can be efficiently accomplished by use of a block implicit scheme, particularly if the blocks are relatively small (e.g., equal to the number of governing partial differential equations, at most).

As a related topic the sequential solution of ad hoc uncoupled systems is sometimes advocated. This approach has merit in cases where a weak coupling between the governing equations exists and can be exploited. Blottner²⁴ has shown that in the case of the steady boundary layer equations such an ad hoc uncoupling can seriously degrade the solution accuracy and the sometimes more than ten circuits round the uncoupled system is required to achieve the accuracy level obtained in one pass with the coupled scheme. Consequently in spite of the increased labor involved in solving the block system a substantial net saving is obtained vis a vis the sequential iterative approach. In the present problem the coupling effects between equations can be much stronger than are observed in the conventional boundary layer equations and so the advantage of the block system approach would probably be considerable.

D. The Coordinate System and Related Topics

The projectile in the barrel has three distinct geometric regions of interest. Referring to Figure 2 a schematic is given which illustrates the three regions. Region I is the base region of the projectile while region II is the gap itself, shown much enlarged in the schematic. Region III is the projectile forebody and this region is important in the present problem mainly to set the exit conditions for the leakage flow from region II. Two convenient coordinate systems for regions I and II come immediately to mind. The first of these is the use of an axisymmetric variant of the blunt body conformal coordinate of Davis (see Ghia and Davis²⁵). In this two-dimensional case Ghia and Davis constructed a coordinate system for the region near the front of a blunt body in a uniform stream by a conformal transformation scheme.

²⁴Blottner, f. G., "Computational Techniques for Boundary Layers", AGARD Lecture Series 73, February 1975.

²⁵Ghia, U. and Davis, R. T., "Navier-Stokes Solutions for Flow Past a Class of Two-Dimensional Semi-Infinite Bodies", AIAA J., Vol. 12, No. 12, December 1974, pp. 1659-1665.

By use of an image body the barrel itself could be introduced and regions I and II described in a conformal coordinate system. At some point down the gap this coordinate system would have to be joined to a suitable forebody coordinate system, but this would not seem to be an insurmountable problem.

The alternative and recommended scheme for region I and II is to adopt a simple Cartesian mesh stretched in both r and z by an analytic stretching function such as that described by Roberts²⁶. The analytic stretching function could then be used to place a relatively dense mesh in the vicinity of the corner. As with the conformal transformation scheme the forebody coordinates would have to be adjoined to the gap coordinate. In view of the prior success which investigators have had with both aforementioned coordinates there is no telling argument for or against either system. The stretched Cartesian mesh is simpler to implement and as a consequence is the recommended approach at least for the initial stages of this work.

Joining of either coordinate systems to the forebody region III could be performed discretely with one-sided differencing used on either side of the join. Thus discontinuous derivatives of the coordinates could be permitted at the join line. While feasible, and indeed such discretely joined differencing meshes have been used in other investigations, in the present problem this join problem can be moved to the body nose by normalizing the radial mesh by the gap height $h(z)$. The mesh can then be continuous from region I into region III. Difficulty now occurs at the body nose where $\partial h/\partial z$ is discontinuous. The nose problem can now be treated by one-sided differencing of $\partial h/\partial z$ and the flow variables across the discontinuity. The principal attribute of the $h(z)$ normalization is that it provides a very simple method of treating arbitrary forebody geometries at the same time as conveniently blending region II and region III meshes. The singularity of $\partial h/\partial z$ at the body nose is a problem, however, that may require special treatment. The corner region mesh and its blend into region I is not a problem at least insofar as $h(z)$ is concerned since in region I $h(z)$ can be fixed at the corner value, $h(z_c)$.

The problem is very simply formulated in a projectile fixed coordinate system with a moving wall to represent the barrel. A schematic of this system is shown in Figure 3a with the upstream and downstream computational boundary denoted by a chain line. This particular coordinate

²⁶Roberts, G. O., "Computational Meshes for Boundary Layer Problems", Proceedings of the Second International Conference on Numerical Methods in Fluid Dynamics, Springer-Verlag, New York, p. 171, 1971.

system has the attribute that definition is retained in the gap region without difficulty as the flow develops in time. The alternative and for the time being, not recommended moving coordinate system could be introduced whereby the distance from the breech to the projectile base, $l(t)$, could be used to normalize the axial distance as is shown in Figure 3b. The new axial coordinate would be related to the physical distance by $\tilde{z}(z,t) = z/l(t)$ with $l(t)$ describing the position of the projectile. This time stretched coordinate reduces the axial definition in the gap as the solution develops in time and also continues to define regions of the flow where, from the gap leakage problem point of view, interest may no longer exist, or the flow may be adequately described by simpler concepts such as a boundary layer and core flow analysis.

In order to maximize the definition of the gap flow, a coordinate system attached only to the projectile base and not tied to both the projectile and the breech is recommended at the present time. Should the interaction between the gap and the breech conditions prove ultimately to be of critical importance, it is a straightforward matter to revert to the expanding mesh tied both to the projectile and the breech.

Corners in themselves present a major problem to finite difference schemes. This problem is discussed for instance by Mitchell²⁷ and by Whiteman²⁸. Roache¹⁶ gives a morphology of schemes for treating the corner singularity with finite difference procedures. Of particular note is the observation by both Mitchell and Whiteman that while finite difference solutions lose accuracy near re-entrant corners, mesh refinement reduces the zone of contamination. Whiteman was even able to prove uniform convergence for a family of finite difference solutions for a simple slit type problem. An alternative to mesh refinement is to imbed a finite series solution within the difference solution to represent the singularity. This method is also attractive and has been used with success in some problems. In view of the slow rate of convergence of the refined mesh finite difference solution, the imbedding technique should be investigated further to improve, if possible, its generality and ease of implementation. For the time being, however, the locally refined finite difference approach can be continued. Whiteman also advocated comparison of finite element with finite difference solution to see if there was a relative advantage for this type of singularity and to date this comparison has apparently not been performed, but would be worthwhile.

²⁷Mitchell, A. R., Computational Methods in Partial Differential Equations, John Wiley & Sons, 1969.

²⁸Whiteman, J. R., "Finite Difference Techniques for a Harmonic Mixed Boundary Problem Having a Reentrant Corner", Proc. Roy. Soc., London, A, Vol. 323, pp. 271-276, 1971.

E. Boundary Conditions Implementation

It has been plausibly argued, and will be adopted here, that the time history of the upstream boundary condition can be prescribed adequately for gap leakage purposes from one-dimension propulsion codes already in existence (see for instance Anderson et al.)⁵ Thus for the initial computation of gap leakage it is reasonable to assume that the time varying wall velocity will be given. No slip conditions will then be applied on the solid surface boundaries. The surface temperatures should in principle be determined in conjunction with a metal conduction program. In the initial studies the complexity of performing a metal conduction calculation at each time step can be avoided and the surface temperature and heat transfer bounded by the limits of adiabatic wall (zero thermal gradient at the wall) and surface temperature fixed at time zero value. Both of these limiting cases are readily implemented and should be evaluated. The wall pressure poses something of a problem and here it is suggested that the conventional steady boundary layer approximation of zero normal gradient of pressure should be adequate in the present problem, only in this instance the sum of the pressure gradient and the acceleration terms are equated to zero.

With the limited number of grid points available it is desirable to place the downstream boundary as close to the projectile as possible and retain gap definition. Three strategies then are open. First, place the boundary in the quiescent air outside the weapon with a highly stretched axial mesh and apply undisturbed flow as boundary conditions. Secondly, compute the flow ahead of the projectile by, say, an existing inviscid irrotational flow procedure and impose boundary conditions for the Navier-Stokes analysis close to the projectile nose based on these inviscid calculations. This leaves open the question, presumably answered by the author of the inviscid procedure, of the boundary conditions for the inviscid code. Here of course transformations and the use of a potential function ease the inviscid flow calculation labor considerably so it is probably feasible in this context to use quiescent air outside the breach as the inviscid flow downstream conditions. Thirdly, apply weak exit boundary conditions based upon linear extrapolation from interior grid points. Here the terminology "weak" refers to the constraint of the boundary condition upon the dependent variable value. Dirichlet boundary conditions, i.e., function specified, are "strong" in this terminology and the higher the order of the derivative boundary condition applied the "weaker" the constraint upon the dependent variable value at the boundary. The "weak" exit boundary conditions referred to here are in effect second derivatives set to zero with one-sided differences. In view of the complexity of the equation system involved it is not a priori clear that this third technique of linear extrapolation would work. Certainly all the suggestions are reasonable and the choice is simply a question of computing economies weighed against level of approximation and desired accuracy.

This third technique is probably the least inconvenient to apply and will be used unless the solutions indicate that this is either unstable or inaccurate. Availability of an inviscid code to compute the forebody flow field and/or the computational costs would indicate which of the two other strategies would be adopted should the linear extrapolation scheme be unacceptable.

The upstream boundary conditions are available from conventional propulsion and interior ballistics codes; see for example Anderson et al⁵, Dahm and Anderson²⁹, Celmins³⁰ and Gough³¹. The potential inaccuracies of these one-dimensional codes is a source of concern. For instance leakage flow, the subject of the detailed analysis under consideration, must obviously have an effect on the breech pressure level and hence the projectile velocity, as indeed so must the gap friction. Thus there should be a coupling between the local gap flow analysis and the overall interior ballistics predictions. However, at least in the initial phases of the detailed gap analysis, the coupling between the one-dimensional breech analysis and the gap will be ignored.

The boundary conditions for the turbulence model will be treated as follows. At the wall two options are open. In the first the grid point definition normal to the wall is either adequate or made to be adequate to define the viscous sublayer and hence boundary conditions of zero turbulence at the wall are physically reasonable. The difficulty with this approach is that apart from the required grid point definition, the physics of low Reynolds number turbulence must be modeled in a reasonable manner by governing turbulence equations. The alternative approach is to force the turbulence equations to give predicted levels at the first grid point away from the wall which are consistent, in some sense, with the well-known boundary layer of the wall. The law of the wall can be written for the axial velocity u for instance as

$$u^+ = f(y^+)$$

where

$$u^+ = u/u_\tau, \quad y^+ = yu_\tau/\nu \quad u_\tau = (\tau_w/\rho)^{1/2}$$

and y^+ is the distance normal to the wall and τ_w is the wall shear stress. The function f is a known universal relationship valid for smooth, rough and transpired turbulent boundary layers. Equivalent relationships are

²⁹Dahm, T. J. and Anderson, L. W., "Propellant Gas Convective Heat Transfer in Gun Barrels", Aerotherm Report No. 70-18, August 1970.

³⁰Celmins, A. K. R., "Theoretical Basis of the Recoilless Rifle Interior Ballistics Code 'RECRIF'", BRL Report No. 1931, September 1976.

³¹(AD #B013832L) Gough, P. S., "Numerical Analysis of a Two-Phase Flow with Explicit Internal Boundaries", Indian Head Contract Report 77-5, Naval Ordnance Station, Indian Head, MD, April 1977.

available for temperature species and turbulence quantities. The law of the wall can be applied much as given and the solution of the finite difference equations forced to satisfy the given equation. The given functional form can be differentiated and derivative conditions, i.e., "weaker" boundary conditions, used. No matter how implemented, the difficulty with this whole approach is that the validity of the law of the wall under the local conditions of the gun barrel gap leakage problem is questionable. The law of the wall matching technique has been largely favored in the past because it economizes on grid point numbers and also because presently little is known about low Reynolds number turbulence. Law of the wall-based constraints for turbulence model equations have been developed by Buggeln and McDonald³². This type of law of the wall constraint can be arranged to allow the effect of surface roughness to enter and it will be used in the numerical model.

The downstream boundary conditions for the turbulence model equations would, if possible, be of the very weak interior extrapolation type referred to earlier. As before, setting the second derivative of the dependent variable to zero should enforce rather weak constraints upon the solution. Should this turn out not to be the case then the indications are that the downstream boundary should be moved closer to the quiescent air outside the weapon. As far as upstream boundary conditions on the turbulence are concerned, little information is available. Here all that can be done until experimental guidance is obtained is to make predictions for various plausible upstream turbulence levels. For instance, turbulent energies varying between 0 and say, 100%, of the bulk gas velocity would seem reasonable. Further, a turbulent length scale varying between zero and the barrel radius seems to bound the possible scales involved. Within these bounds all that can be done is to examine the sensitivity of the solutions, in particular the wall heat transfer to these parametric variations. In view of the high acceleration levels near the gap inlet it is suspected that the gap fluid dynamics will be relatively insensitive to the breech turbulence levels.

F. Computer Storage and Related Problems

In developing the arguments earlier for a block implicit approach it has been claimed and assumed that the resulting linear system can be solved efficiently, i.e., with at least the same order effort as other competitive schemes, and that no unreasonably large storage demands are made. Here it is supposed that the investigation will result in a computer program which will run on a CDC 7600 generation machine. Prior work^{20,21} indicates that an alternating direction decomposition (ADI) or matrix factorization scheme as it is sometimes called, clearly can result

³²Buggeln, R.C. and McDonald, H., "Influence of Aerodynamic Phenomena on Pollution Formation in Combustion (Theoretical Studies)", Final Report EPA Contract No. 68-02-1873, November 1977.

in a very efficient technique for solving the linear equation system. Here we distinguish between ADI and matrix factorization terminology by reserving the term ADI to schemes where the intermediate level solutions represent consistent approximations to the governing partial differential equations. In this way physical boundary conditions can be applied at the intermediate levels, a critical factor contributing to the success of the schemes. Matrix factorization includes both ADI and the other schemes which do not possess this consistency property. Consistency is felt to be extremely important and inconsistent schemes are not recommended here. What is not widely recognized is that this ADI decomposition has a very profound beneficial effect upon the storage requirements. Indeed, using this decomposition usually only the number of lines in any one space dimension equal to the bandwidth of the spatial difference molecule, together with the number of time levels involved in the temporal difference molecule, need be in fast core memory at any point. As a practical matter then the core storage requirements can be kept to a minimum, making extensive use of peripheral storages. The line matrix block elimination usually provides sufficient time for buffered input-output transfers to be completed prior to the code requiring the stored arrays. Block data transfers rather than point by point transfer can and should be used to speed up the input-output process. Thus it is not expected that storage will be a major problem if an ADI scheme is adopted.

G. Available Codes

A number of documented codes exist which must be considered as potential candidates for the calculation of the leakage problem. In selecting and judging them there are a number of requirements which must be met: the code should be able to handle viscous turbulent flow and finite rate chemistry, thus resolving the reaction zone, and consequently exhibit accuracy over a range of Mach numbers. Desired added features include, but are not limited to, multiphase and multispecies capability, and ability to handle multidimensionality of the flow geometry. It is desirable that the code not exceed the large core storage capacity of the CDC 7600 at BRL, and running times per case should be less than one hour since the code is intended for parametric studies. The algorithm must have been checked for accuracy and stability under typical running conditions. Finally the code should be able to handle efficiently and economically stiff systems arising out of flows with realistic chemistry.

There are four broad categories of codes, documented and generally available, which have to be considered for the reactive leakage flow problem: the Los Alamos family of codes based on the ICE and ALE techniques, those derived from variants of MacCormack's scheme, Spalding's effort and MINT developed by Briley and McDonald.

Butler³³ has reported on calculations with different versions of ICE. Satisfactory results have also been claimed for treating a bank of gas dynamic lasers, flame propagation and the reactive flow inside a combustion engine. This method is applicable to the full two-dimensional, time-dependent Navier-Stokes equation with species transport, mixing and chemical reaction between the constituents. The RICE code, a reactive version of ICE, used in these calculations, is based on a semi-implicit technique which, by modifying the time step restriction, permits a more efficient calculation to be performed. The code and the calculational results can be criticized on a number of accounts, and in the present problem the principal problem would most likely be the undue stability restrictions on the time step. As a general observation with scant experimental data available for comparison, it is rather difficult to pass judgment on the physical accuracy of the reported results. Flow gradients are treated by numerical approximation and if this is not done sufficiently accurately, either as a result of poor choice of assumed functional form of the solution or by an inadequate mesh, then poor or false predictions of the physical process can result. It is felt that extensive validation with respect to measurements need to be performed before any of these codes can be considered for alternate applications.

MacCormack's scheme has been shown to yield acceptable results for reactive flows for a number of different geometries. The most efficient version of this explicit method uses a formulation whereby the chemistry is split off from the hydrodynamic part of the problem. Efficiency, accuracy and economy of storage have been claimed for this technique and demonstrated on a number of practical problems. MacCormack's approach runs into a number of difficulties and restrictions when applied to flow geometries where the nature of the flow undergoes a drastic change, such as a change from hyperbolic to parabolic type within a boundary layer. Running times for the problem of interest could be quite long due to the time step, stability restriction. Also, the technique, when tried on a two-phase problem, has not led to completely satisfactory results.

Spalding and his coworkers have been responsible for the creation of a number of reactive hydrocodes, some of which include the effect of turbulence on the reactive flow. However, from the point of view of the leakage problem, these codes have limited applicability since they primarily address the problem of steady flow at low Mach numbers. One time-dependent code in this family due to Gosman and Watkins³⁴ which might be

³³Butler, T. D. and O'Rourke, P. J., "A Numerical Method for Two-Dimensional Unsteady Reacting Flows", Proceedings of Sixteenth (International) Symposium on Combustion, Combustion Institute, Pittsburgh, PA, 1977.

³⁴Gosman, A. D. and Watkins, A. P., "A Computer Prediction Method for Turbulent Flow and Heat Transfer in Piston/Cylinder Assemblies", in Symposium on Turbulent Shear Flows, University Park, PA, April 1977.

suited for the present problem is not generally available. Even if this code were available the pressure treatment is somewhat awkward and approximate, making it an unlikely choice for the present problem.

The MINT code is based on a fully implicit technique and is a three-dimensional, time-dependent, compressible multicomponent code which has been exercised on a variety of combustor problems with satisfactory results²¹. It fulfills the criteria set forth at the beginning of this section and it can be adapted for the purpose of this analysis. The modifications needed include an adaptive coordinate system, a two equation turbulence model and characterization of the burning propellant bed.

Perhaps more importantly, present familiarity with this code and our confidence in its reliability of accurately modeling some aspects of two-phase flow phenomena suggest that MINT should be the departure point in the present modeling effort. It is anticipated that compression wave definition may pose some problems, but at this juncture it is felt that this shortcoming is outweighed by the advantages that this approach offers.

Several other developmental efforts should be mentioned at this juncture. Gough³⁵ is in the process of extending his one-dimensional two phase flow of the gun tube to two dimensions using the Lax-Wendroff algorithm. Experimental verification of his, as well as all other codes, needs to be performed. Finally, the Calspan model³⁶, which computes rather than takes a specified ignitor input, has been used to simulate a 155 mm howitzer with a modicum of success. However, like Gough's model it underpredicts the breech pressure as well as the muzzle velocity, leading one to question the physics as well as the mathematics of the code.

H. Experimental Validation

Little or no existing experimental evidence appears available for use in validating the code. This state of affairs must be rectified before confidence in the analysis can be obtained. Nonreacting experimental studies can be performed relatively cheap if the flow is steady. While not validating the transient capability of the analysis, such tests do prove a very necessary check since there would be little hope of treating the transient problem if the steady flow could not be accurately predicted. An insulated wall with a moderately heated gas would enable wall temperature levels to be predicted and compared to data for various gap heights and Reynolds numbers. With high response instrumentation

³⁵P. Gough Associates, Portsmouth, NJ, private communication by A. Horst, BRL.

³⁶Fisher, E.B., Graves, J.W. & Trigger, A.P., "Application of Flame Spread Model to Design Problems in the 155 mm Propelling Charge", 12th JANNAF Combustion Meeting, CPIA Pub. 273, pp. 199-214, 1975.

the wall or projectile could be translated to introduce some transients into the flow. Once again wall temperature would be a stringent test of predictive accuracy.

Reactive flow tests would be extremely difficult and probably wall temperature measurements at fixed locations on the bore of a weapon would be the most one could expect. Even this very modest type data would be difficult to obtain but very valuable in view of the realistic environment.

In both the hot and cold, steady and unsteady and experimental studies, mapping of the flow field would be highly desirable. Realistically speaking, however, such measurements are probably not feasible within the constraints of time and effort required to obtain them at present.

V. Recommendations for Future Work

Our principal recommendation is to adopt the MINT code to the needs of the interior ballistics modeling problem subject to the extensions and restrictions discussed in this report.

Future generalizations follow from our discussion. These include, but are not limited to an accurate description of the effect of the particles on the flow as well as consideration of particle-particle interactions. Realistic chemistry to take cognizance of the propellant bed dynamics as well as the interplay between burning rate and turbulence will have to become an integral part of the model.

The boundary treatment should reflect the heat losses through the wall which may influence the flow definition. With the availability of sufficient computer storage, three-dimensional effects such as balloting and swirling flow induced by the rifling of the gun tube should and can become an integral part of the model.

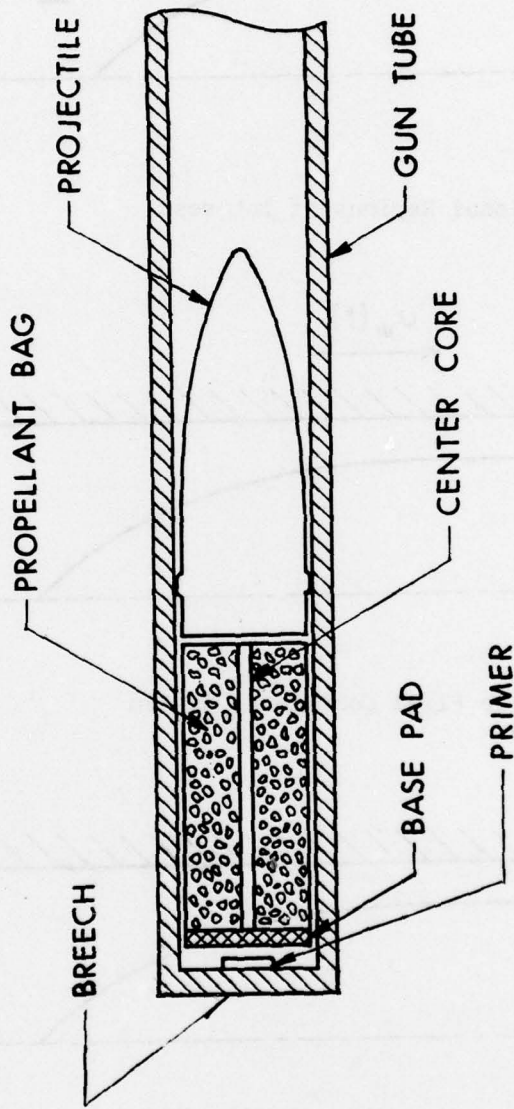


Figure 1. Schematic of Projectile in a Smooth Bore Gun Barrel at $t = 0$.

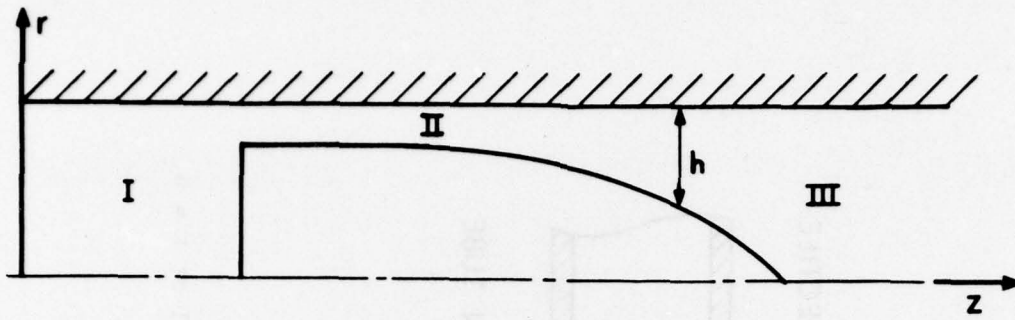


Figure 2. The Computational Regions of Interest

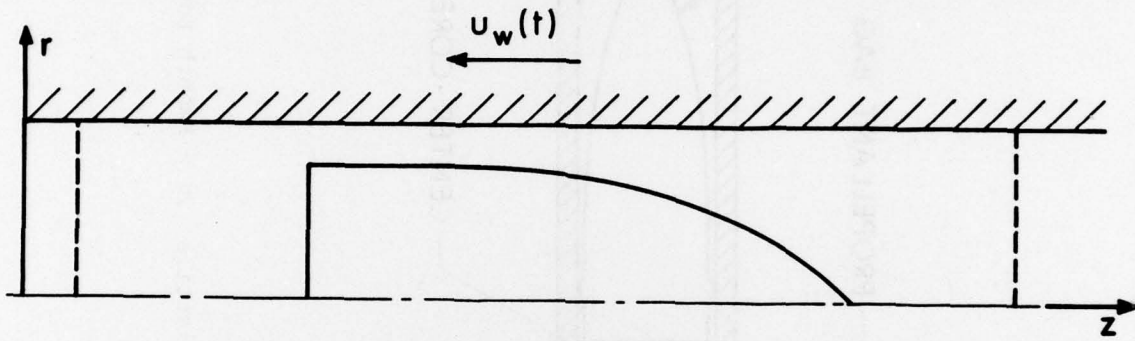


Figure 3a. The Projectile Fixed Coordinate System

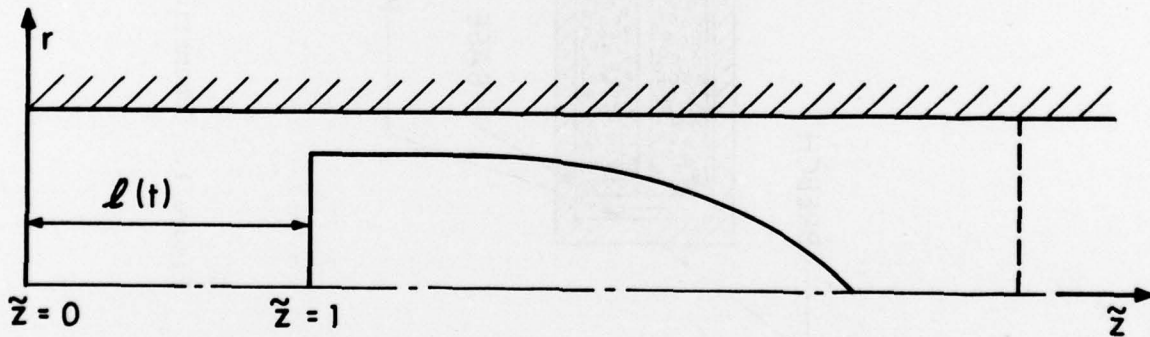


Figure 3b. Schematic of Projectile in a Moving Coordinate System

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