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# SIGNAL-TO-NOISE RATIO ANALYSIS OF TWO IMPLEMENTATIONS OF QUADRIPHASE DIRECT-SEQUENCE SPREAD-SPECTRUM MULTIPLE-ACCESS

FREDERICK DWIGHT GARBER







UNIVERSITY OF ILLINOIS - URBANA, ILLINOIS

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# REPORT DOCUMENTATION PAGE

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| | | |
|---|---|---|
| 1. REPORT NUMBER | 2. GOVT ACCESSION NO. | 3. RECIPIENT'S CATALOG NUMBER |
| 4. TITLE (and Subtitle) (6) SIGNAL-TO-NOISE RATIO ANALYSIS OF TWO IMPLEMENTATIONS OF QUADRIPHASE DIRECT-SEQUENCE SPREAD-SPECTRUM MULTIPLE-ACCESS | | 5. TYPE OF REPORT & PERIOD COVERED (9) Technical Report |
| | | 6. PERFORMING ORG. REPORT NUMBER (14) R-815; UILU-ENG-78-2208 |
| 7. AUTHOR(s) (10) Frederick Dwight Garber | | 8. CONTRACT OR GRANT NUMBER(s) (15) DAAB-07-72-C-0259, NSF-ENG-75-22621 |
| 9. PERFORMING ORGANIZATION NAME AND ADDRESS Coordinated Science Laboratory University of Illinois at Urbana-Champaign Urbana, Illinois 61801 | | 10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS |
| 11. CONTROLLING OFFICE NAME AND ADDRESS Joint Services Electronics Program | | 12. REPORT DATE (11) May 1978 |
| | | 13. NUMBER OF PAGES 58 |
| 14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) | | 15. SECURITY CLASS. (of this report) UNCLASSIFIED |
| | | 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE |

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

(12) 60p.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

78 08 15 029

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Spread-Spectrum Multiple-Access Communications
Quadriphase Sequences
Correlation Parameters

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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UILU-ENG 78-2208

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This work was supported in part by the Joint Services Electronics Program (U.S. Army, U.S. Navy and U.S. Air Force) under Contract DAAB-07-72-C-0259 and in part by the National Science Foundation under Grant NSF ENG 75-22621.

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BY

FREDERICK DWIGHT GARBER

B.S., Tri-State College, 1975


THESIS

Submitted in partial fulfillment of the requirements
for the degree of Master of Science in Electrical Engineering
in the Graduate College of the
University of Illinois at Urbana-Champaign, 1978

Thesis Adviser: Professor M. B. Pursley

Urbana, Illinois

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Two implementations of quadriphase direct-sequence spread-spectrum multiple-access (QDS/SSMA) communication systems are discussed and their performance is investigated. The average signal-to-noise ratio is related to the correlation parameters of the signature sequences for each system. The asymptotic behavior of the key aperiodic correlation parameters is investigated and preliminary numerical results are obtained. The results for QDS/SSMA are compared and contrasted with previous results on biphase direct-sequence spread-spectrum multiple-access systems.

## ACKNOWLEDGMENTS

The author is deeply grateful to Professor Michael B. Pursley for his patient guidance and for the many stimulating discussions offered throughout the course of this work.

Thankful consideration is also due to Mrs. Phillis A. Young for her expert and expedient typing.

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# CHAPTER I

## INTRODUCTION

Recently there has been considerable interest in a class of code-division multiple-access communication systems known as direct-sequence spread-spectrum multiple-access (DS/SSMA) systems. Previous investigations of such systems have primarily dealt with biphase DS/SSMA system models [1-5]. In this thesis, the analyses of quadriphase direct-sequence spread-spectrum multiple-access (QDS/SSMA) systems are presented.

There are a number of reasons for considering QDS/SSMA systems. Many of the important performance parameters of DS/SSMA systems exhibit a dependence on the length of the signature sequences. For biphase DS/SSMA systems these parameters, for the most part, change favorably with increasing sequence lengths [2], [3]. For many practical DS/SSMA systems, the bandwidth and data rate, and hence the sequence length, are constrained. However for a given bandwidth and data rate, QDS/SSMA systems can employ signature sequences which are twice the length of the sequences in a biphase system. Also, the consideration of QDS/SSMA using quadriphase signature sequences allows the investigation of larger classes of codes for use in DS/SSMA systems. Hence it is expected that the consideration of QDS/SSMA may result in improved system performance.

Many of the analytical results used to evaluate biphase DS/SSMA systems have been established for complex-valued sequences (e.g., [4], [16]). Moreover, certain classes of complex-valued sequences have been extensively studied in the radar literature and elsewhere (e.g., [6],[11], [18]). Thus it is anticipated that a number of performance parameters for QDS/SSMA systems may be evaluated using existing results.

Two implementations of QDS/SSMA systems will be considered. The first, described in Section 2.1, is a QDS/SSMA system with orthogonal biphase-coded carriers. The second is a QDS/SSMA system with quadriphase-coded carriers and is described in Section 2.2. The correlation receiver output signal-to-noise ratio for each system is expressed in terms of the key aperiodic correlation parameters identified in [4]. These aperiodic correlation parameters are investigated and preliminary numerical results are obtained for a special case of signature sequence assignment.

# CHAPTER 2

# SYSTEM MODELS AND ANALYSES

## 2.1. QDS/SSMA System with Orthogonal Biphase-Coded Carriers

### 2.1.1. System Model

The QDS/SSMA system we will consider in this section, shown in Figure 1 for K users, is an extension of the biphase DS/SSMA model analyzed in [2]. The k-th user generates a pair of data signals, $b_{2k}(t)$ and $b_{2k-1}(t)$, where each is a sequence of unit amplitude, positive and negative, rectangular pulses of duration T. The data signals represent alternate bits of the k-th user's binary information sequence $b_{k,\ell}$ and are given by

$$b_{2k}(t) = \sum_{\ell=-\infty}^{\infty} b_{k,2\ell}\, p_T(t-\ell T)$$

$$b_{2k-1}(t) = \sum_{\ell=-\infty}^{\infty} b_{k,2\ell-1}\, p_T(t-\ell T)$$

where $b_{k,\ell} \in \{+1,-1\}$ and $p_T(t) = 1$ for $0 \le t < T$ and $p_T(t) = 0$ otherwise. The k-th user is assigned two code waveforms, $a_{2k}(t)$ and $a_{2k-1}(t)$, each consisting of a periodic sequence of unit amplitude, positive and negative rectangular pulses of duration $T_c$. Thus, the code waveforms can be written as

$$a_{2k}(t) = \sum_{n=-\infty}^{\infty} a_n^{(2k)}\, p_{T_c}(t-nT_c)$$

$$a_{2k-1}(t) = \sum_{n=-\infty}^{\infty} a_n^{(2k-1)}\, p_{T_c}(t-nT_c)$$

where $(a_n^{(2k)})$ and $(a_n^{(2k-1)})$ are the discrete periodic signature sequences assigned to the k-th user. We assume that each of the k-th user's codes has period $N = T/T_c$ so that there is one code period per data symbol.

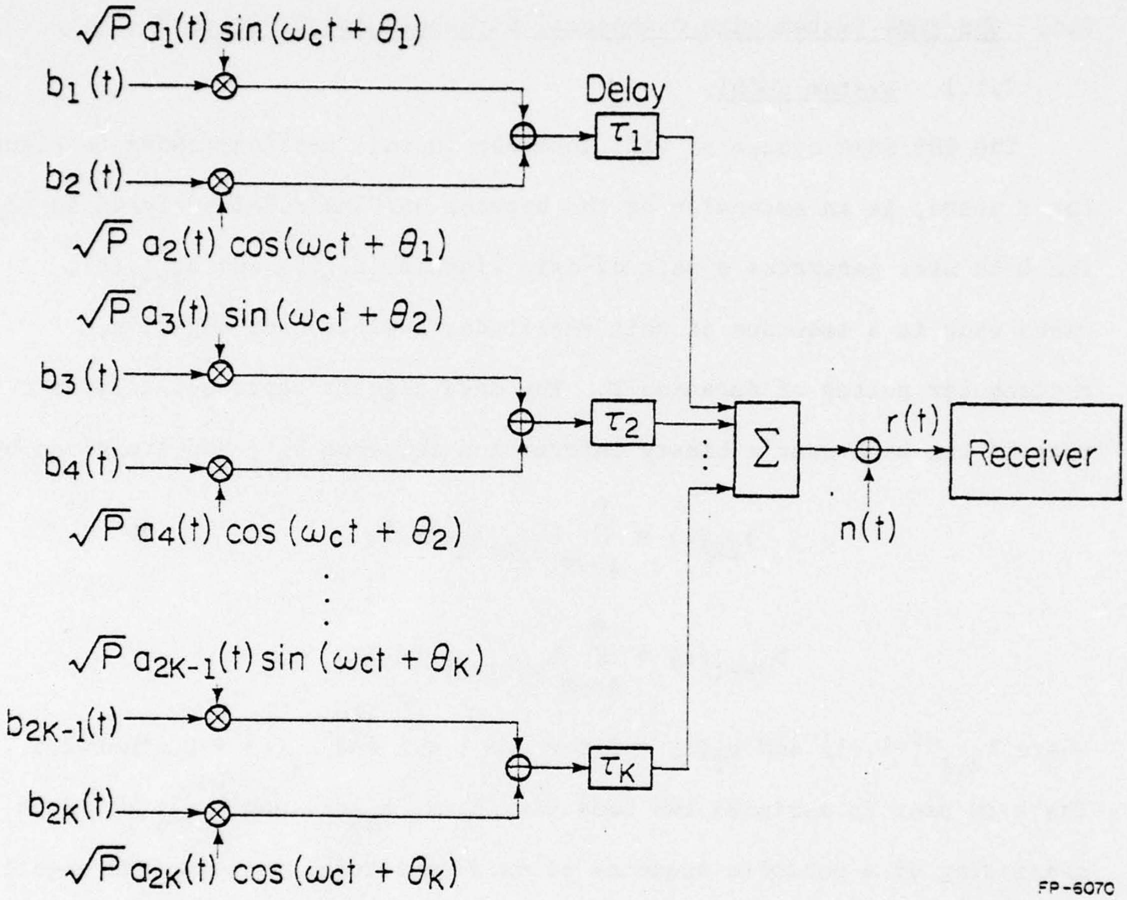


Figure 1a. QDS/SSMA system with orthogonal biphase-coded carriers.

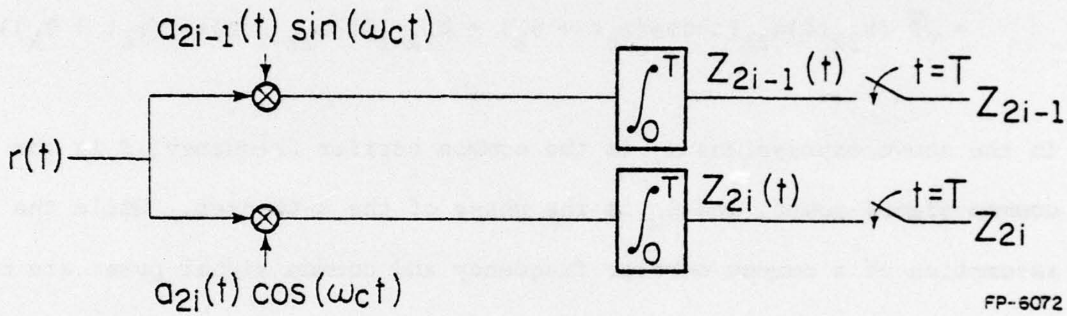


Figure 1b. Correlation receiver for the i-th user.

The k-th user's data signals are modulated onto the phase-coded carriers $c_{2k}(t)$ and $c_{2k-1}(t)$ given by

$$c_{2k}(t) = \sqrt{P}\, a_{2k}(t)\cos(\omega_c t + \theta_k) \qquad (2.1)$$

$$c_{2k-1}(t) = \sqrt{P}\, a_{2k-1}(t)\sin(\omega_c t + \theta_k) \qquad (2.2)$$

so that the transmitted signal for the k-th user is

$$s_k(t) = b_{2k}(t)c_{2k}(t) + b_{2k-1}(t)c_{2k-1}(t)$$

$$= \sqrt{P}\,(b_{2k}(t)a_{2k}(t)\cos(\omega_c t + \theta_k) + b_{2k-1}(t)a_{2k-1}(t)\sin(\omega_c t + \theta_k)). \qquad (2.3)$$

In the above expressions $\omega_c$ is the common carrier frequency, P is the common signal power, and $\theta_k$ is the phase of the k-th user. While the assumption of a common carrier frequency and common signal power are made to allow a more concise expression for the effects of multi-user interference, the results that follow can easily be modified to consider the more general case.

We will consider the system to be asychronous; i.e., we do not require that a common timing reference be made available to the K transmitters or that compensations be made for the various transmission pa[illegible] delays. Each signal, then, will experience a time delay $\tau_k$ as shown in Figure 1. The channel noise process n(t) is assumed to be an additive white Gaussian process with two-sided spectral density $N_0/2$.

The received signal, r(t), is given by

$$r(t) = \sum_{k=1}^{K} \sqrt{P}\,\{b_{2k}(t-\tau_k)a_{2k}(t-\tau_k)\cos(\omega_c t + \phi_k)$$

$$+ b_{2k-1}(t-\tau_k)a_{2k-1}(t-\tau_k)\sin(\omega_c t + \phi_k)\} + n(t)$$

where $\tau_k$ is the time delay for the k-th signal and $\phi_k = \theta_k - \omega_c\tau_k$. For the analysis of the interference effects on the i-th receiver we are concerned only with relative phase shifts modulo $2\pi$ and relative time delays modulo T, hence there is no loss of generality in assuming that $\theta_i = \tau_i = 0$ and considering only $0 \leq \tau_k < T$ and $0 \leq \theta_k < 2\pi$ for $k \neq i$.

The i-th receiver consists of a pair of synchronized correlation receivers. The even numbered branch of the i-th receiver is matched to the phase-coded carrier $c_{2i}(t)$, thus allowing a decision corresponding to the i-th user's data signal $b_{2i}(t)$. Similarly, the odd numbered branch of the i-th receiver is matched to $c_{2i-1}(t)$ and produces an estimate of $b_{2i-1}(t)$. If r(t) is the input to the i-th receiver, the output of the even numbered branch $Z_{2i}(t)$ at sample time t = T is expressed as

$$Z_{2i} = \int_0^T r(t)a_{2i}(t)\cos(\omega_c t)dt \quad .$$

The corresponding output $Z_{2i-1}(t)$ of the odd numbered branch of the i-th receiver at time t = T is

$$Z_{2i-1} = \int_0^T r(t)a_{2i-1}(t)\sin(\omega_c t)dt \quad .$$

We assume that $\omega_c \gg T^{-1}$ since this condition is always satisfied in a practical SSMA system and, given the frequency response of a realistic hardware implementation of the correlation receiver, we can ignore the double frequency components in the above integrands. The output $Z_{2i}$ of the even numbered branch of the i-th receiver becomes

$$Z_{2i} = \frac{\sqrt{P}}{2} \{b_{i,0}T + \sum_{\substack{k=1 \\ k\neq i}}^{K} [b_{k,-2}R_{2k,2i}(\tau_k) + b_{k,0}\hat{R}_{2k,2i}(\tau_k)]\cos \phi_k$$

$$+ \sum_{\substack{k=1 \\ k\neq i}}^{K} [b_{k,-1}R_{2k-1,2i}(\tau_k) + b_{k,1}\hat{R}_{2k-1,2i}(\tau_k)]\sin \phi_k\}$$

$$+ \int_0^T n(t)a_{2i}(t)\cos \omega_c t dt \qquad (2.4)$$

and the output $Z_{2i-1}$ of the odd numbered branch of the i-th receiver is

$$Z_{2i-1} = \frac{\sqrt{P}}{2} \{b_{i,-1}T + \sum_{\substack{k=1 \\ k\neq i}}^{K} [b_{k,-2}R_{2k,2i-1}(\tau_k) + b_{k,0}\hat{R}_{2k,2i-1}(\tau_k)]\sin \phi_k$$

$$+ \sum_{\substack{k=1 \\ k\neq i}}^{K} [b_{k,-1}R_{2k-1,2i-1}(\tau_k) + b_{k,1}\hat{R}_{2k-1,2i-1}(\tau_k)]\cos \phi_k\}$$

$$+ \int_0^T n(t)a_{2i-1}(t)\sin \omega_c t dt \qquad (2.5)$$

where $R_{k,i}$ and $\hat{R}_{k,i}$ are the continuous-time partial cross-correlation functions defined in [2] as

$$R_{k,i}(\tau) = \int_0^{\tau} a_k(t-\tau)a_i(t)dt ,$$

$$\hat{R}_{k,i}(\tau) = \int_{\tau}^{T} a_k(t-\tau)a_i(t)dt$$

for $0 \leq \tau \leq T$. For $0 \leq \ell T_c \leq \tau \leq (\ell+1)T_c \leq T$, it is pointed out in [2] that

$$R_{k,i}(\tau) = C_{k,i}(\ell-N)T_c + [C_{k,i}(\ell-N+1) - C_{k,i}(\ell-N)](\tau-\ell T_c) \quad (2.6)$$

and

$$\hat{R}_{k,i}(\tau) = C_{k,i}(\ell)T_c + [C_{k,i}(\ell+1) - C_{k,i}(\ell)](\tau-\ell T_c) \quad (2.7)$$

where $C_{k,i}$ is the discrete aperiodic cross-correlation function for the sequences $(a_j^{(k)})$ and $(a_j^{(i)})$ defined by

$$C_{k,i}(\ell) = \begin{cases} \sum_{j=0}^{N-1-\ell} a_j^{(k)} a_{j+\ell}^{(i)} , & 0 \leq \ell \leq N-1 \\ \sum_{j=0}^{N-1+\ell} a_{j-\ell}^{(k)} a_j^{(i)} , & 1-N \leq \ell \leq 0 \\ 0 , & |\ell| \geq N . \end{cases} \quad (2.8)$$

Note that $C_{k,i}(\ell) = C_{i,k}(-\ell)$, hence, the periodic cross-correlation function $\theta_{k,i}$ is given by $\theta_{k,i}(\ell) = C_{k,i}(\ell) + C_{k,i}(\ell-N)$ for $0 \leq \ell < N$. The odd cross-correlation function $\hat{\theta}_{k,i}$, so named by Massey and Uhran [7] since $\hat{\theta}_{k,i}(\ell) = -\hat{\theta}_{k,i}(N-\ell)$ for $0 \leq \ell < N$ is given by

$$\hat{\theta}_{k,i}(\ell) = C_{k,i}(\ell) - C_{k,i}(\ell-N).$$

The average probability of error and the average signal-to-noise ratio are two important measures of system performance. Although the former is difficult to compute, Yao [3] has obtained upper and lower bounds for biphase systems of interest. The latter, which involves much less computational effort, is considered in [2] and [1] for the biphase system and is discussed in the next section for the QDS/SSMA system with orthogonal biphase-coded carriers.

### 2.1.2. Average Signal-to-Noise Ratio

In this section we treat the phase shifts, time delays, and data symbols as mutually independent random variables and compute the signal-to-noise ratio by means of probabilistic averages with respect to these random variables. The interference terms appearing in (2.4) and (2.5) are thus random variables which we treat as additional noise. We assume that for $k \neq i$, $\phi_k$ is uniformly distributed on the interval $[0,2\pi]$ and that $\tau_k$ is uniformly distributed on $[0,T]$. We also assume that the data symbols take values +1 or -1 with equal probability.

We will first consider the output $Z_{2i}$ of the even numbered branch of the i-th receiver. By symmetry, we need only consider the case when $b_{i,0} = +1$, thus the desired signal component of $Z_{2i}$ is $\frac{1}{2}\sqrt{P}\,T$ while the variance of $Z_{2i}$ is

$$\begin{aligned}
\mathrm{Var}\{Z_{2i}\} = \frac{P}{4} \sum_{\substack{k=1\\k\neq i}}^{K} \frac{1}{T}\int_0^T & E\{[b_{k,-2}R_{2k,2i}(\tau) + b_{k,0}\hat{R}_{2k,2i}(\tau)]^2\cos^2\phi_k \\
& + [b_{k,-1}R_{2k-1,2i}(\tau) + b_{k,1}\hat{R}_{2k-1,2i}(\tau)]^2\sin^2\phi_k\}d\tau \\
& + \frac{N_0}{2}\int_0^T a_{2i}^2(t)\cos^2\omega_c t\, dt \\
= \left(\frac{P}{8T}\right) \sum_{\substack{k=1\\k\neq i}}^{K} \sum_{\ell=0}^{N-1} \int_{\ell T_c}^{(\ell+1)T_c} & [R_{2k,2i}^2(\tau) + \hat{R}_{2k,2i}^2(\tau) \\
& + R_{2k-1,2i}^2(\tau) + \hat{R}_{2k-1,2i}^2(\tau)]d\tau + \frac{N_0 T}{4}
\end{aligned}$$

where E denotes expected value.

Substituting for $R_{k,i}(\tau)$ and $\hat{R}_{k,i}(\tau)$ from (2.6) and (2.7) and evaluating the integral we have

$$\mathrm{Var}\{Z_{2i}\} = \frac{PT^2}{24N^3}\left(\sum_{\substack{k=1\\k\neq i}}^{K} r_{2k,2i} + r_{2k-1,2i}\right) + \frac{N_0T}{4} \tag{2.9}$$

where

$$r_{k,i} = \sum_{\ell=0}^{N-1}\{C^2_{k,i}(\ell-N) + C_{k,i}(\ell-N)C_{k,i}(\ell-N+1) + C^2_{k,i}(\ell-N+1)$$

$$+ C^2_{k,i}(\ell) + C_{k,i}(\ell)C_{k,i}(\ell+1) + C^2_{k,i}(\ell+1)\}.$$

In terms of the cross-correlation parameter $\mu_{k,i}(n)$ defined in [2] by

$$\mu_{k,i}(n) = \sum_{\ell=1-N}^{N-1} C_{k,i}(\ell)C_{k,i}(\ell+n),$$

we have the identity [2, Eq. (14)]

$$r_{k,i} = 2\mu_{k,i}(0) + \mu_{k,i}(1).$$

This may be seen by noting as in [2] that

$$\mu_{k,i}(0) = \sum_{\ell=1-N}^{N-1} C^2_{k,i}(\ell) = \sum_{\ell=0}^{N-1} C^2_{k,i}(\ell-N) + C^2_{k,i}(\ell)$$

$$= \sum_{\ell=0}^{N-1} C^2_{k,i}(\ell-N-1) + C^2_{k,i}(\ell+1)$$

and that

$$\mu_{k,i}(1) = \sum_{\ell=1-N}^{N-1} C_{k,i}(\ell)C_{k,i}(\ell+1)$$

$$= \sum_{\ell=0}^{N-1} C_{k,i}(\ell-N)C_{k,i}(\ell-N+1) + C_{k,i}(\ell)C_{k,i}(\ell+1) \quad .$$

Hence (2.9) becomes

$$\mathrm{Var}\{Z_{2i}\} = \frac{PT^2}{24N^3}\left(\sum_{\substack{k=1\\k\neq i}}^{K} 2\mu_{2k,2i}(0)+\mu_{2k,2i}(1)+2\mu_{2k-1,2i}(0)+\mu_{2k-1,2i}(1)\right) + \frac{N_0 T}{4} .$$

The signal-to-noise ratio at the output of the even numbered branch of the i-th correlation receiver is the desired signal component, $\frac{1}{2}\sqrt{P}\,T$, divided by the r. m. s. noise, $\sqrt{\mathrm{Var}\{Z_{2i}\}}$, and is given by

$$\mathrm{SNR}_{2i} = \{(6N^3)^{-1}\sum_{\substack{k=1\\k\neq i}}^{K}[2\mu_{2k,2i}(0)+\mu_{2k,2i}(1)+2\mu_{2k-1,2i}(0)+\mu_{2k-1,2i}(1)]+\frac{N_0}{2E_b}\}^{-\frac{1}{2}} \tag{2.10}$$

where $E_b = \frac{1}{2}\,PT$ is the energy per data bit. Recalling (2.5) and using a similar analysis, the signal-to-noise ratio at the output of the odd numbered branch of the i-th correlation receiver is seen to be

$$\mathrm{SNR}_{2i-1} = \{(6N^3)^{-1}\sum_{\substack{k=1\\k\neq i}}^{K}[2\mu_{2k,2i-1}(0)+\mu_{2k,2i-1}(1) + 2\mu_{2k-1,2i-1}(0)+\mu_{2k-1,2i-1}(1)] + \frac{N_0}{2E_b}\}^{-\frac{1}{2}} . \tag{2.11}$$

It was shown in [4] that for binary sequences

$$\sum_{\ell=1-N}^{N-1} C_{k,i}(\ell)C_{k,i}(\ell+n) = \sum_{\ell=1-N}^{N-1} C_k(\ell)C_i(\ell+n) ,$$

where the aperiodic autocorrelation function $C_{k,k}$ is denoted $C_k$. Hence $\mu_{k,i}(n)$ can be computed directly from the aperiodic autocorrelation functions for $(a_j^{(k)})$ and $(a_j^{(i)})$, so that knowledge of the cross-correlation function is not required.

Notice that for K = 1, (2.10) and (2.11) become

$$SNR_{2i} = SNR_{2i-1} = \sqrt{2E_b/N_0}$$

with associated bit error probability $P_e = 1 - \Phi(\sqrt{2E_b/N_0})$ where $\Phi$ is the standard Gaussian cumulative distribution function. Yao [3] showed that for practical values of N and K, $1 - \Phi(SNR_i)$ is a very accurate approximation to the average bit error probability.

For the biphase DS/SSMA system model analyzed in [2], the signal-to-noise ratio at the output of the i-th receiver, $SNR_i'$, is given by

$$SNR_i' = \{(6N^3)^{-1} \sum_{\substack{k=1 \\ k\neq i}}^{K} [2\mu_{k,i}(0) + \mu_{k,i}(1)] + \frac{N_0}{2E_b'}\}^{-\frac{1}{2}}$$

where $E_b' = PT$ is the energy per data bit for the biphase system. In [5] it was shown that the approximation

$$(6N^3)^{-1} \sum_{\substack{k=1 \\ k\neq i}}^{K} r_{k,i} \approx (K-1)/3N \qquad (2.12)$$

is very accurate for large values of the ratio N/K. In fact, for random signature sequences, (K-1)/3N was found to be the expected value of the left hand side of (2.12). In terms of (2.12) we have, as in [2],

$$SNR_i' \approx \{\frac{K-1}{3N} + \frac{N_0}{2E_b'}\}^{-\frac{1}{2}} .$$

Applying (2.12) to (2.10) and (2.11) we see that

$$SNR_{2i} \approx SNR_{2i-1} \approx \{\frac{2(K-1)}{3N} + \frac{N_0}{2E_b}\}^{-\frac{1}{2}} . \qquad (2.13)$$

If biphase signature sequences of the same period are being considered, the signal-to-noise ratio at each branch of the i-th receiver of the orthogonal biphase-coded QDS/SSMA system with K users is approximately that of a biphase DS/SSMA system with 2K users. The numerical results obtained for the biphase case can thus be used in the preliminary design of QDS/SSMA systems using orthogonal biphase-coded carriers.

For certain digital communication systems employing orthogonal biphase-coded carriers it is advantageous [8] to delay, by some time $t_0$, one modulated carrier with respect to the other to produce, in terms of (2.3)

$$s_k(t) = \sqrt{P}[b_{2k}(t)a_{2k}(t)\cos(\omega_c t + \theta_k) + b_{2k-1}(t-t_0)a_{2k-1}(t-t_0)\sin(\omega_c t + \theta_k)].$$

Such signals are known as offset quadriphase-shift-keyed [8], staggered quadrature amplitude modulated [9], or double binary PSK [10] signals. For certain choices of $t_0$, it has been shown [9], [10] that signals formed in this manner make more efficient use of available signal bandwidth than conventional QPSK. It should be noted that the above analysis holds for such systems and that the signal-to-noise ratios (2.10) and (2.11) are independent of a relative time delay, $t_0$.

## 2.2. QDS/SSMA System with Quadriphase-Coded Carriers

Using equations (2.10) and (2.11), the average signal-to-noise ratio for a QDS/SSMA system with orthogonal biphase-coded carriers can be computed if knowledge of the binary aperiodic correlation functions is available. If, however, for a class of quadriphase codes, only complex correlation data is available, it is not clear that any useful performance measures may be evaluated. It is felt that, at least for some quadriphase sequences, knowledge of the complex correlation parameters is not enough to specify the signal-to-noise ratios in (2.10) and (2.11). Any quadriphase sequence may be represented as a pair of biphase sequences. However, there are examples of quadriphase sequences that have a certain correlation property which cannot be represented as a pair of binary sequences which have the same type of correlation property. For example, there are quadriphase Barker sequences of length 15 [6], but there are no binary Barker sequences of that length.

The system considered in this section, shown in Figure 2, is a modified form of the system of Section 2.1. We will see that for this modified system, important performance measures can be computed from the complex correlation parameters.

2.2.1. System Model

The k-th user's code waveforms, $a_{2k}(t)$ and $a_{2k-1}(t)$, defined as in Section 2.1, are used to generate the quadriphase-coded carriers $c_{2k}(t)$ and $c_{2k-1}(t)$ given as

$$c_{2k}(t) = \sqrt{P}\cos(\omega_c t + \theta_k + a_k(t)) \qquad (2.14)$$

and

$$c_{2k-1}(t) = \sqrt{P}\cos(\omega_c t + \theta_k + a_k(t) + \pi/2) \qquad (2.15)$$

where $a_k(t) \in \{ \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \}$. The k-th user's data signals, $b_{2k}(t)$ and $b_{2k-1}(t)$, defined in Section 2.1 are modulated onto the quadriphase-coded carriers so that the transmitted signal $s_k(t)$ becomes

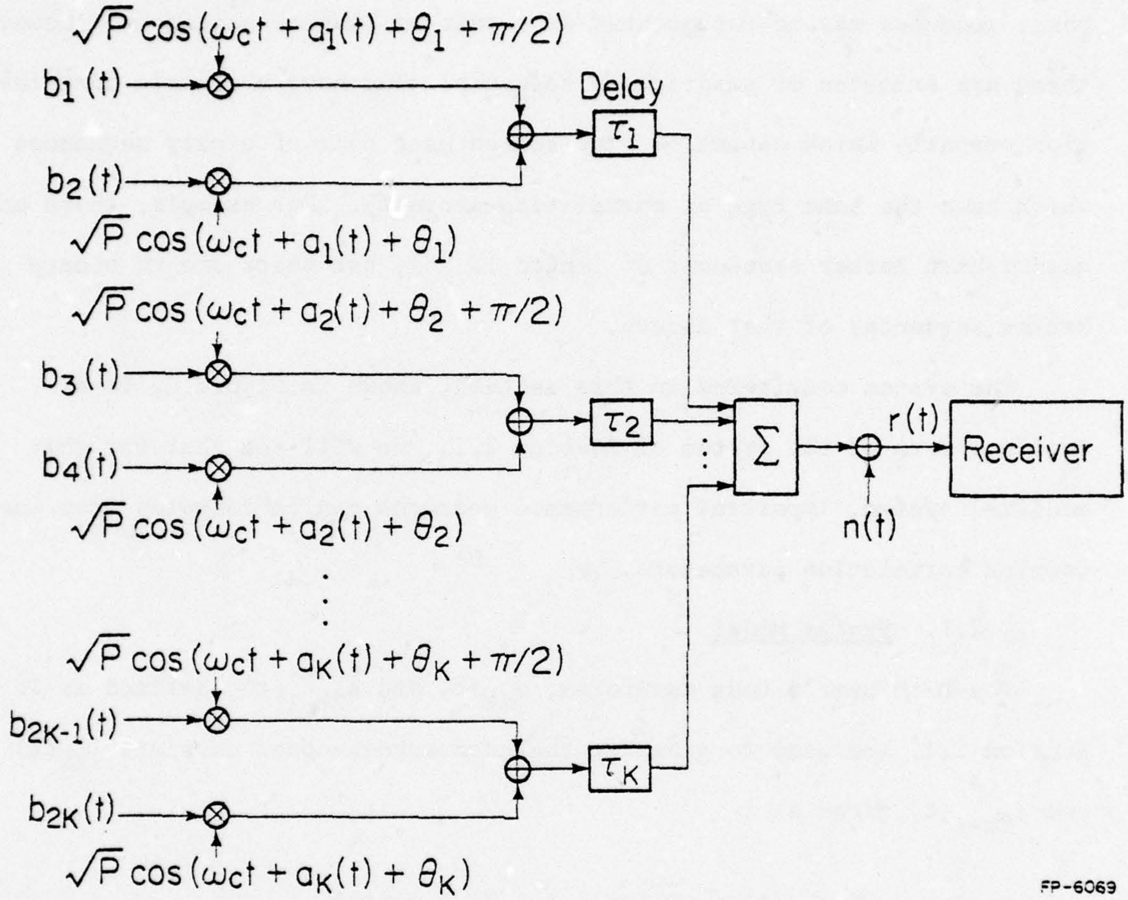


Figure 2a. QDS/SSMA system with quadriphase-coded carriers.

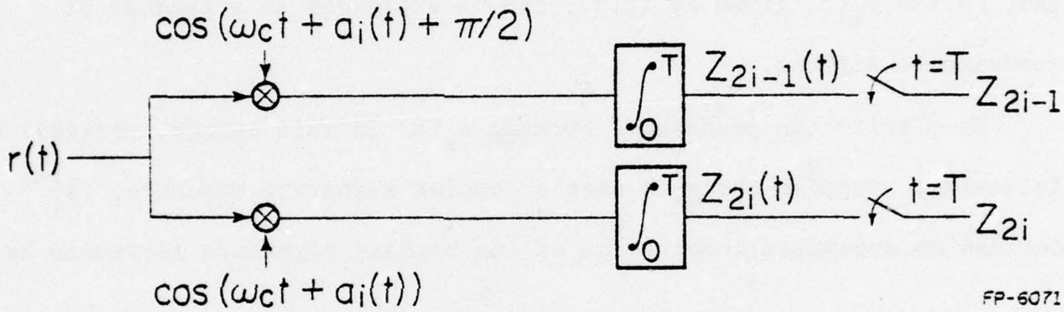


Figure 2b. Correlation receiver for the i-th user.

$$s_k(t) = b_{2k}(t)c_{2k}(t) + b_{2k-1}(t)c_{2k-1}(t)$$

$$= \sqrt{P}\, b_{2k}(t)\cos(\omega_c t+\theta_k+a_k(t)) + \sqrt{P}\, b_{2k-1}(t)\cos(\omega_c t+\theta_k+a_k(t)+\pi/2). \quad (2.16)$$

Note that (2.16) is not equivalent to (2.3). In Section 2.1, $s_k(t)$ is a combination of the biphase-coded carriers given by (2.1) and (2.2) and derives its quadriphase property through the orthogonal combination of biphase components. The signal defined in (2.16), however, is a linear combination of the quadriphase-coded carriers (2.14) and (2.15) and, unlike $s_k(t)$ given by (2.3), can be expressed as a product of quadriphase signals.

To clarify the purpose of forming $s_k(t)$ in this manner, consider the following. Suppose the k-th user's complex signature sequence, $(\tilde{a}_\ell^{(k)})$ is defined as a complex combination of its biphase signature sequences by

$$\tilde{a}_\ell^{(k)} = \sqrt{\tfrac{1}{2}}\,(a_\ell^{(2k)} + j a_\ell^{(2k-1)}) ,$$

where $j = \sqrt{-1}$, so that the complex code waveform is given by

$$\tilde{a}_k(t) = \sum_{\ell=-\infty}^{\infty} \tilde{a}_\ell^{(k)} p_{T_c}(t-\ell T_c) = \sum_{\ell=-\infty}^{\infty} \sqrt{\tfrac{1}{2}}\,(a_\ell^{(2k)} + j a_\ell^{(2k-1)}) p_{T_c}(t-\ell T_c)$$

$$= e^{j a_k(t)}$$

for $a_k(t) \in \{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\}$. The quadriphase-coded carriers (2.14) and (2.15) become

$$c_{2k}(t) = \mathrm{Re}\{\tilde{a}_k(t)\sqrt{P}\, e^{j(\omega_c t + \theta_k)}\}$$

where Re denotes the real part, and

$$c_{2k-1}(t) = -\mathrm{Im}\{\tilde{a}_k(t)\sqrt{P}\, e^{j(\omega_c t + \theta_k)}\}$$

where Im denotes the imaginary part. Similarly, if the k-th user's complex data signal is defined as

$$\tilde{b}_k(t) = \sum_{\ell=-\infty}^{\infty} \tilde{b}_{k,\ell} p_T(t-\ell T) = \sqrt{\tfrac{1}{2}} \sum_{\ell=-\infty}^{\infty} (b_{k,2\ell} + jb_{k,2\ell-1}) p_T(t-\ell T)$$

$$= e^{jb_k(t)}$$

where $b_k(t) \in \{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\}$, then $s_k(t)$ is given by

$$s_k(t) = \mathrm{Re}\{\tilde{b}_k(t)\tilde{a}_k(t)\sqrt{2P}\, e^{j(\omega_c t + \theta_k)}\}$$

$$= \sqrt{2P}\cos(\omega_c t + \theta_k + b_k(t) + a_k(t)) \ . \tag{2.17}$$

For the asynchronous system of Figure 2, the received signal r(t) is

$$r(t) = \sum_{k=1}^{K} \sqrt{2P}\cos(\omega_c t + \phi_k + b_k(t-\tau_k) + a_k(t-\tau_k)) + n(t) \ .$$

The i-th receiver is similar to the receiver of Section 2.1 with the even numbered correlation receiver matched to the quadriphase-coded carrier $c_{2k}(t)$ and the odd numbered correlation receiver matched to $c_{2k-1}(t)$. The output $Z_{2i}$ of the even numbered branch of the i-th receiver is written as

$$Z_{2i} = \int_0^T r(t)\cos(\omega_c t + a_k(t))dt.$$

Ignoring the double frequency terms in the integrand, we have

$$Z_{2i} = \int_0^T \sqrt{P/2}\cos(b_i(t))dt + \sqrt{P/2}\sum_{\substack{k=1\\k\neq i}}^{K}\int_0^T \cos(b_k(t-\tau_k) + a_k(t-\tau_k) - a_i(t) + \phi_k)dt$$

$$+ \int_0^T n(t)\cos(\omega_c t + a_i(t))dt$$

$$= \sqrt{P/2}\left\{\frac{b_{i,0}}{\sqrt{2}}T + \sum_{\substack{k=1\\k\neq i}}^{K} \mathrm{Re}\{[\tilde{b}_{k,-1}R_{k,i}(\tau_k) + \tilde{b}_{k,0}\hat{R}_{k,i}(\tau_k)]e^{j\phi_k}\}\right\}$$

$$+ \int_0^T n(t)\cos(\omega_c t + a_i(t))dt \qquad (2.18)$$

where $R_{k,i}$ and $\hat{R}_{k,i}$ are now the continuous-time complex cross-correlation functions defined by

$$R_{k,i}(\tau) = \int_0^{\tau} \tilde{a}_k(t-\tau)\tilde{a}_i^*(t)dt$$

$$\hat{R}_{k,i}(\tau) = \int_{\tau}^{T} \tilde{a}_k(t-\tau)\tilde{a}_i^*(t)dt$$

for $0 \leq \tau \leq T$ where $a^*$ denotes complex conjugate of a. Similarly, for the odd numbered branch of the i-th receiver, the output $Z_{2i-1}$ is

$$Z_{2i-1} = \sqrt{P/2}\left\{\frac{b_{i,-1}}{\sqrt{2}}T + \sum_{\substack{k=1\\k\neq i}}^{K} \mathrm{Im}\{[\tilde{b}_{k,-1}R_{k,i}(\tau_k) + \tilde{b}_{k,0}\hat{R}_{k,i}(\tau_k)]e^{j\phi_k}\}\right\}$$

$$- \int_0^T n(t)\sin(\omega_c t + a_i(t))dt \qquad (2.19)$$

For $0 \leq \ell T_c \leq \tau \leq (\ell+1)T_c \leq T$ the complex cross-correlation functions in (2.18) and (2.19) may be expressed as

$$R_{k,i}(\tau) = C_{k,i}(\ell-N)T_c + [C_{k,i}(\ell-N+1) - C_{k,i}(\ell-N)](\tau-\ell T_c) \qquad (2.20)$$

$$\hat{R}_{k,i}(\tau) = C_{k,i}(\ell)T_c + [C_{k,i}(\ell+1) - C_{k,i}(\ell)](\tau-\ell T_c) \qquad (2.21)$$

where the discrete, aperiodic complex cross-correlation function $C_{k,i}$ for the sequences $(\tilde{a}_\ell^{(k)})$ and $(\tilde{a}_\ell^{(i)})$ is defined in [4] by

$$C_{k,i}(\ell) = \begin{cases} \sum_{j=0}^{N-1-\ell} a_j^{(k)}[a_{j+\ell}^{(i)}]^*, & 0 \le \ell \le N-1 \\ \sum_{j=0}^{N-1+\ell} a_{j-\ell}^{(k)}[a_j^{(i)}]^*, & 1-N \le \ell < 0 \\ 0, & |\ell| \ge N . \end{cases} \quad (2.22)$$

2.2.2. Average Signal-to-Noise Ratio

We assume, as in Section 2.1, that the phase shifts and time delays are mutually independent, uniformly distributed random variables for $k \neq i$ and that the binary data signals are mutually independent random variables taking values +1 or -1 with equal probability for $k \neq i$. Considering first the output of the even numbered branch of the i-th receiver we find that the desired signal component of $Z_{2i}$ is $\frac{1}{2}\sqrt{P}\,T$ while the variance of the noise components of $Z_{2i}$ becomes

$$\mathrm{Var}\{Z_{2i}\} = \frac{P}{2} \sum_{\substack{k=1 \\ k\neq i}}^{K} \frac{1}{T} \int_0^T E\Big\{[\mathrm{Re}\{\tilde{b}_{k,-1} R_{k,i}(\tau) + \tilde{b}_{k,0} \hat{R}_{k,i}(\tau)\}]^2 \cdot [\mathrm{Re}\{e^{j\phi_k}\}]^2$$

$$+ [\mathrm{Im}\{\tilde{b}_{k,-1} R_{k,i}(\tau) + \tilde{b}_{k,0} \hat{R}_{k,i}(\tau)\}]^2 [\mathrm{Im}\{e^{j\phi_k}\}]^2\Big\} d\tau$$

$$+ \frac{N_0}{2} \int_0^T \cos^2(\omega_c t + a_i(t))dt$$

$$= \frac{P}{4T} \sum_{\substack{k=1 \\ k\neq i}}^{K} \sum_{\ell=0}^{N-1} \int_{\ell T_c}^{(\ell+1)T_c} |R_{k,i}(\tau)|^2 + |\hat{R}_{k,i}(\tau)|^2 d\tau + \frac{N_0 T}{4} .$$

We substitute for $R_{k,i}(\tau)$ and $\hat{R}_{k,i}(\tau)$ from equations (2.20) and (2.21) and evaluate the integral to obtain

$$\mathrm{Var}\{Z_{2i}\} = \frac{PT^3}{12N^3}\left(\sum_{\substack{k=1\\k\neq i}}^{K} r_{k,i}\right) + \frac{N_0T}{4} \tag{2.23}$$

where

$$\begin{aligned} r_{k,i} = \sum_{\ell=0}^{N-1} \{ & |C_{k,i}(\ell-N)|^2 + |C_{k,i}(\ell+1-N)|^2 \\ & + \tfrac{1}{2}[C_{k,i}(\ell-N)C^*_{k,i}(\ell+1-N) + C^*_{k,i}(\ell-N)C_{k,i}(\ell+1-N)] \\ & + |C_{k,i}(\ell)|^2 + |C_{k,i}(\ell+1)|^2 + \tfrac{1}{2}[C_{k,i}(\ell)C^*_{k,i}(\ell+1) + C^*_{k,i}(\ell)C_{k,i}(\ell+1)]\}. \end{aligned}$$

If we define the complex cross-correlation parameter $\mu_{k,i}(n)$ by

$$\mu_{k,i}(n) = \sum_{\ell=1-N}^{N-1} C_{k,i}(\ell)C^*_{k,i}(\ell+n) \tag{2.24}$$

then we can write $r_{k,i}$ as

$$r_{k,i} = 2\mu_{k,i}(0) + \tfrac{1}{2}[\mu_{k,i}(1) + \mu^*_{k,i}(1)].$$

The signal-to-noise ratio at the output of the even numbered branch of the i-th receiver, then is

$$\mathrm{SNR}_{2i} = \left\{(3N^3)^{-1}\sum_{\substack{k=1\\k\neq i}}^{K}[2\mu_{k,i}(0) + \mathrm{Re}\{\mu_{k,i}(1)\}] + \frac{N_0}{2E_b}\right\}^{-\frac{1}{2}}. \tag{2.25}$$

A similar analysis applied to the output of the odd numbered branch of the i-th receiver, $Z_{2i-1}$, yields a result identical to (2.25) so that for this system, $\mathrm{SNR}_{2i} = \mathrm{SNR}_{2i-1}$.

Notice that the signal-to-noise ratio for either branch of the i-th receiver may be evaluated without knowledge of binary correlation functions. Moreover, it has been shown [4] that the discrete complex aperiodic cross-correlation function in (2.22) satisfies

$$\sum_{\ell=1-N}^{N-1} C_{k,i}(\ell)C^*_{k,i}(\ell+n) = \sum_{\ell=1-N}^{N-1} C_k(\ell)C^*_i(\ell+n)$$

where the complex aperiodic autocorrelation function $C_{k,k}$ is denoted by $C_k$. Thus, the complex cross-correlation parameter defined in (2.24) may be computed directly from the complex aperiodic autocorrelation functions.

### 2.2.3. Mean-Square Difference Criterion

Correlation parameters of complex sequences have been considered in the radar literature for some time [11]. A radar performance measure that is related to complex correlation functions is the mean-square difference, where mean denotes time average. It is interesting to note that, when applied to the signals discussed in Section 2.2.1, the mean-square difference gives rise to the same complex correlation parameters considered in that section. Specifically, we express the signals given in (2.17) as a function of time $t$ and carrier phase $\theta_k$ by

$$s_k(t,\theta_k) = \sqrt{2P}\cos(\omega_c t + \theta_k + a_k(t) + b_k(t)).$$

We then define the *mean-square difference* $\mathcal{E}^2_{k,i}$ by

$$\mathcal{E}^2_{k,i} = \int_0^T [s_k(t-\tau_k,\theta_k) - s_i(t,0)]^2 dt$$

where $\tau_k$ is the time delay (modulo T) of the k-th signal and we assume that $s_i(t,0)$ defines our point of reference, i.e., $\tau_i = \theta_i = 0$.

In general it may be difficult to compute $\mathcal{E}_{k,i}^2$ since the analysis will involve both the essential baseband term as well as a double frequency term. To avoid this cumbersome analysis it is desirable to work in terms of a class of signals having only one sided spectra. Such signals are known as complex analytic-signals [12].

Most generally, the correct complex analytic-signal representation for a given waveform is composed of a real part which is, as desired, the original signal, and an imaginary part which is the Hilbert transform of the original signal. However, we are concerned only with signals which have the form of $s_k(t)$ in (2.17) where $a_k(t)$ and $b_k(t)$ are real signals applied to phase modulate the carrier. In this instance, if the carrier frequency, $\omega_c$, is sufficiently high so that there is negligible low-frequency energy, then the complex analytic-signal representation for $s_k(t,\theta_k)$ which we will denote $\tilde{s}_k(t,\theta_k)$ may be closely approximated as

$$\tilde{s}_k(t,\theta_k) \approx \sqrt{2P}\; e^{j(\omega_c t + \theta_k + a_k(t) + b_k(t))} .$$

For the analysis to follow, the above approximation will suffice. Thus we will assume that the complex analytic-signal representation of $s_k(t,\theta_k)$ is given by

$$\tilde{s}_k(t,\theta_k) = \tilde{b}_k(t)\tilde{a}_k(t)\sqrt{2P}\; e^{j(\omega_c t + \theta_k)} \tag{2.27}$$

where $\tilde{b}_k(t)$ is the k-th user's complex data signal and $\tilde{a}_k(t)$ is the k-th user's complex code waveform as defined in Section 2.1.1.

An important property of complex analytic-signals is that (2.26) can be replaced [12] by

$$\mathcal{E}^2_{k,i} = \tfrac{1}{2}\int_0^T |\tilde{s}_k(t-\tau_k,\theta_k) - \tilde{s}_i(t,0)|^2 dt$$

where $\tilde{s}_k(t-\tau_k,\theta_k)$ and $\tilde{s}_i(t,0)$ are the complex signal representations of $s_k(t-\tau_k,\theta_k)$ and $s_i(t,0)$, respectively. Expanding the above integral yields

$$2\mathcal{E}^2_{k,i} = \int_0^T |\tilde{s}_k(t-\tau_k,\theta_k)|^2 dt + \int_0^T |\tilde{s}_i(t,0)|^2 dt$$

$$- 2\int_0^T \mathrm{Re}\{\tilde{s}_k(t-\tau_k,\theta_k)\tilde{s}_i^*(t,0)\}dt$$

or equivalently, using (2.27)

$$\mathcal{E}^2_{k,i} = 2PT - \int_0^T \mathrm{Re}\{\tilde{b}_k(t-\tau_k)\tilde{a}_k(t-\tau_k)\sqrt{2P}\, e^{j(\omega_c t + \phi_k)}$$

$$\cdot\ \tilde{b}_i^*(t)\tilde{a}_i^*(t)\sqrt{2P}\, e^{-j\omega_c t}\}dt$$

$$= 2PT - \mathrm{Re}\{\chi_{k,i}(\tau_k,\phi_k)\}$$

where

$$\chi_{k,i}(\tau_k,\phi_k) = 2P\int_0^T \tilde{b}_k(t-\tau_k)\tilde{a}_k(t-\tau_k)\tilde{b}_i^*(t)\tilde{a}_i^*(t)e^{j\phi_k}dt$$

and where $\phi_k = \theta_k - \omega_c\tau_k$. The mean-square difference, $\mathcal{E}^2_{k,i}$, then depends only on the real part of $\chi_{k,i}(\tau_k,\phi_k)$ which is termed the time-phase cross-correlation function [13]. Straightforward calculation shows that for $0 \leq \ell T_c \leq \tau_k \leq (\ell+1)T_c \leq T$, $\chi_{k,i}(\tau_k,\phi_k)$ may be written as

$$\chi_{k,i}(\tau_k,\theta_k) = 2P\, \tilde{b}_{k,-1}\tilde{b}^*_{i,0}e^{j\phi_k}\{C_{k,i}(\ell-N)T_c+[C_{k,i}(\ell+1-N)-C_{k,i}(\ell-N)](\tau_k-\ell T_c)\}$$

$$+ 2P\, \tilde{b}_{k,0}\tilde{b}^*_{i,0}e^{j\phi_k}\{C_{k,i}(\ell)T_c+[C_{k,i}(\ell+1)-C_{k,i}(\ell)](\tau_k-\ell T_c)\}$$

where $C_{k,i}$ is the discrete aperiodic complex cross-correlation function given in (2.22). Thus we see that for the signals given by (2.17), the mean-square difference may be specified in terms of discrete, aperiodic correlation functions; a well-known result [11].

For some applications, such as radar resolution, the mean-square difference criterion is used to evaluate the ability to distinguish between a signal and a version of itself which has been shifted in time, frequency, or phase. For such applications, a "good" signal is one which makes the mean-square difference between itself and a perturbed version of itself as large as possible. An important measure of this criterion is the radar ambiguity function [11], [12].

When considering the average performance of DS/SSMA systems we have seen in Section 2.1.2 that it is desirable to design signals which minimize, in a statistical sense, the effects of the k-th signal on the i-th correlation receiver. A measure of these effects which is analogous to the radar ambiguity function is the cross-ambiguity function. It is defined [13] as the square of the magnitude of the time-phase cross-correlation function, i.e., $|\chi_{k,i}(\tau_k,\phi_k)|^2$. In terms of average system performance we wish to minimize the expected value of the cross-ambiguity function. Hence $E\{|\chi_{k,i}(\tau_k,\phi_k)|^2\}$ is a measure of the average interference effects of the signals being considered. Following the procedure used to compute $\mathrm{Var}\{Z_{2i}\}$ in Section 2.2.2 we find that

$$E\{|\chi_{i,i}(\tau_k,\phi_k)|^2\} = \frac{4P^2T^2}{3N^3}\{2\,\mu_{k,i}(0) + \tfrac{1}{2}[\mu_{k,i}(1) + \mu_{k,i}^*(1)]\}$$

where $\mu_{k,i}$ is the complex cross-correlation parameter defined in (2.24). Thus, the multi-user interference appearing in the signal-to-noise ratio in (2.25) may be completely specified in terms of the expected value of the cross-ambiguity function.

# CHAPTER 3

# ANALYSIS OF CORRELATION PARAMETERS

## 3.1. Random Sequences

For practical DS/SSMA systems the period $N$ of the signature sequences must be constrained for obvious reasons. However, in order to gain insight to the effects of multi-user interference it is useful to consider the asymptotic behavior of the interference terms defined in Chapter 2 for random signature sequences as the sequence length grows very large.

We define random sequences as sequences of independent, identically distributed random variables which take on each allowed value with equal probability. Hence for biphase sequences we mean sequences, $(w_n)$ for which $P\{w_n = +1\} = P\{w_n = -1\} = \frac{1}{2}$ and for quadriphase sequences we mean sequences of the form $\tilde{u}_n = \sqrt{\frac{1}{2}}\,(w_n + jz_n)$ where $(w_n)$ and $(z_n)$ are random biphase sequences and $j = \sqrt{-1}$.

Here and hereafter unless otherwise stated we will refer only to the correlation parameters generalized to complex-valued sequences given in Section 2.2. For real-valued sequences, these reduce to the corresponding functions given in Section 2.1.

As was previously noted, Roefs and Pursley [5] have considered the asymptotic behavior of correlation parameters for random biphase sequences and found that

$$E\{\sum_{\substack{k=1\\k\neq i}}^{K} r_{k,i}\} = (K-1)2N^2$$

for large values of the ratio N/K. This result can be extended to quadriphase sequences by considering random quadriphase sequences as a complex combination of random biphase sequences and applying the analysis to the resulting correlation parameters of the composite biphase sequences.

Recall that for the quadriphase sequences $(\tilde{u}_n)$ and $(\tilde{v}_n)$ we have

$$C_{\tilde{u},\tilde{v}}(\ell) = \begin{cases} \sum_{n=0}^{N-1-\ell} \tilde{u}_n[\tilde{v}_{n+\ell}]^*, & 0 \le \ell \le N-1 \\ \sum_{n=0}^{N-1+\ell} \tilde{u}_{n-\ell}[\tilde{v}_n]^*, & 1-N \le \ell < 0 \\ 0 & |\ell| \ge N \end{cases}$$

and

$$r_{\tilde{u},\tilde{v}} = 2\mu_{\tilde{u},\tilde{v}}(0) + \tfrac{1}{2}[\mu_{\tilde{u},\tilde{v}}(1) + \mu^*_{\tilde{u},\tilde{v}}(1)] \tag{3.1}$$

where

$$\mu_{\tilde{u},\tilde{v}}(n) = \sum_{\ell=1-N}^{N-1} C_{\tilde{u},\tilde{v}}(\ell) C^*_{\tilde{u},\tilde{v}}(\ell+n) \quad . \tag{3.2}$$

If the quadriphase sequences $(\tilde{u}_n)$ and $(\tilde{v}_n)$ are formed by complex combinations of the biphase sequences $(x_n)$, $(y_n)$, $(w_n)$, and $(z_n)$ by

$$\tilde{u}_n = \sqrt{\tfrac{1}{2}}\,(x_n + jy_n) \qquad \forall n$$

and

$$\tilde{v}_n = \sqrt{\tfrac{1}{2}}\,(w_n + jz_n) \qquad \forall n$$

then we have

$$C_{\tilde{u},\tilde{v}}(\ell) = \tfrac{1}{2}(C_{x,w}(\ell) + C_{y,z}(\ell)) + j\tfrac{1}{2}(C_{y,w}(\ell) - C_{x,z}(\ell)) \; .$$

The correlation parameters $\mu_{\tilde{u},\tilde{v}}(n)$ in (3.2) can be written as

$$\mu_{\tilde{u},\tilde{v}}(0) = \tfrac{1}{4}\sum_{\ell=1-N}^{N-1} C^2_{x,w}(\ell) + C^2_{y,z}(\ell) + C^2_{y,w}(\ell) + C^2_{x,z}(\ell)$$

$$+ 2C_{x,w}(\ell)C_{y,z}(\ell) - 2C_{x,z}(\ell)C_{y,w}(\ell) \qquad (3.3)$$

and

$$\mathrm{Re}\{\mu_{u,v}(n)\} = \tfrac{1}{4}\sum_{\ell=1-N}^{N-1} C_{x,w}(\ell)C_{x,w}(\ell+n) + C_{x,w}(\ell)C_{y,z}(\ell+n)$$

$$+ C_{y,z}(\ell)C_{x,w}(\ell+n) + C_{y,z}(\ell)C_{y,z}(\ell+n)$$

$$+ C_{y,w}(\ell)C_{y,w}(\ell+n) - C_{y,w}(\ell)C_{x,z}(\ell+n)$$

$$- C_{x,z}(\ell)C_{y,w}(\ell+n) + C_{x,z}(\ell)C_{x,z}(\ell+n) \quad . \qquad (3.4)$$

Following the development in [14] for biphase sequences we have for $0 \le \ell \le N-1$

$$C_{x,w}(\ell) = \sum_{n=0}^{N-1-\ell} x_n w_{n+\ell} \quad .$$

If $C_{x,w}(\ell) = d$, then we must have that $x_n = w_{n+\ell}$ for exactly $\tfrac{1}{2}(N-1+d)$ integer values of n in the range $0 \le n \le N-1-\ell$. Clearly we have $|d| \le N-\ell$, and it is easy to see that $N-\ell+d$ is an even valued integer. Thus, for each sequence $(w_n)$ there are

$$b(\ell,N,d) = \binom{N-\ell}{\tfrac{1}{2}(N-\ell+d)}$$

choices for the sequence values $(x_0,x_1,\ldots,x_{N-1-\ell})$ for which $C_{x,w}(\ell) = d$ is satisfied. There are $2^N$ choices for the sequence $(w_i)$ and $2^\ell$ choices for the sequence values $(x_{n-\ell},\dots,x_{N-1})$. Hence there are a total of

$$h(\ell,N,d) = 2^N 2^\ell b(\ell,N,d)$$

distinct pairs of sequences $(x_n)$, $(w_n)$ for which $C_{x,w}(\ell) = d$. A similar argument for $1-N \le \ell < 0$ yields

$$h(\ell,N,d) = 2^N 2^{|\ell|} b(|\ell|,N,d)$$

so that for $|d| \le N - |\ell|$ we obtain the probability mass function

$$P\{C_{x,w}(\ell) = d\} = \binom{N-|\ell|}{\tfrac{1}{2}(N-|\ell|+d)} 2^{-(N-|\ell|)} \quad . \tag{3.5}$$

The statistics useful in evaluating the average performance may be easily obtained using the moment generating function which for this probability mass function is given by

$$\begin{aligned} M(\alpha) &= E\{e^{\alpha C_{x,w}(\ell)}\} \\ &= \begin{cases} \prod_{n=0}^{N-\ell} E\{e^{\alpha(x_n w_{n+\ell})}\}, & 0 \le \ell \le N-1 \\ \prod_{n=0}^{N+\ell} E\{e^{\alpha(x_{n-\ell} w_n)}\}, & 1-N \le \ell < 0 \end{cases} \\ &= \prod_{n=0}^{N-|\ell|} \tfrac{1}{2}(e^{\alpha} + e^{-\alpha}) \\ &= [\cosh(\alpha)]^{N-|\ell|} \quad . \end{aligned}$$

Hence we find the first two moments of $C_{x,w}(\ell)$ to be

$$E\{C_{x,w}(\ell)\} = \frac{\delta M(\alpha)}{\delta\alpha}\Big|_{\alpha=0} = (N-|\ell|)[\cosh(\alpha)]^{N-|\ell|-1}\sinh(\alpha)\Big|_{\alpha=0} = 0$$

$$E\{C^2_{x,w}(\ell)\} = \frac{\delta^2 M(\alpha)}{\delta\alpha^2}\Big|_{\alpha=0} = 2\binom{N-|\ell|}{2}[\cosh(\alpha)]^{N-|\ell|-2}[\sinh(\alpha)]^2\Big|_{\alpha=0}$$

$$+ (N-|\ell|)[\cosh(\alpha)]^{N-|\ell|-1}\cosh(\alpha)\Big|_{\alpha=0} = N-|\ell| \ .$$

Also, it is easy to see that for $|\ell| \leq N-1$,

$$E\{C_{x,w}(\ell)C_{x,w}(\ell+1)\} = 0 \ .$$

If we apply these statistics to evaluate the expected value of the correlation parameters in (3.3) and (3.4) we find that

$$E\left\{\mathrm{Re}\{\mu_{\tilde{u},\tilde{v}}(1)\}\right\} = E\left\{\tfrac{1}{4}\sum_{\ell=1-N}^{N-1} C_{x,w}(\ell)C_{x,w}(\ell+1) + C_{x,w}(\ell)C_{y,z}(\ell+1)\right.$$

$$+ C_{y,z}(\ell)C_{x,w}(\ell+1) + C_{y,z}(\ell)C_{y,z}(\ell+1) + C_{y,w}(\ell)C_{y,w}(\ell+1)$$

$$\left. - C_{y,w}(\ell)C_{x,z}(\ell+1) - C_{x,z}(\ell)C_{y,w}(\ell+1) + C_{x,z}(\ell)C_{x,z}(\ell+1)\right\} = 0$$

while

$$E\{\mu_{\tilde{u},\tilde{v}}(0)\} = E\left\{\tfrac{1}{4}\sum_{\ell=1-N}^{N-1} C^2_{x,w}(\ell) + C^2_{y,z}(\ell) + C^2_{y,w}(\ell) + C^2_{x,z}(\ell)\right.$$

$$\left. + 2C_{x,w}(\ell)C_{y,z}(\ell) - 2C_{x,z}(\ell)C_{y,w}(\ell)\right\} = N^2 \ .$$

Application of the above results to (3.1) yields

$$E\{r_{\tilde{u},\tilde{v}}\} = 2N^2 \ .$$

It should be noted that the evaluation of the mathematical expectation of the asynchronous interference may be done directly in terms of the correlation parameters for random quadriphase sequences. In particular, for the random quadriphase sequences $(\tilde{u}_n)$ and $(\tilde{v}_n)$ we have

$$C_{\tilde{u},\tilde{v}}(\ell) = \sum_{n=0}^{N-1-\ell} \tilde{u}_n [\tilde{v}_{n+\ell}]^*$$

for $0 \le \ell \le N-1$. Let $\eta_1$ equal the number of integer values of n in the range $0 \le n \le N-1-\ell$ for which $\tilde{u}_n = [\tilde{v}_{n+\ell}]^*$ and let $\eta_2$ equal the number of integer values of n in the range $0 \le n \le N-1-\ell$ for which $(-j)\tilde{u}_n = [\tilde{v}_{n+\ell}]^*$. If $\mathrm{Re}\{C_{\tilde{u},\tilde{v}}(\ell)\} + \mathrm{Im}\{C_{\tilde{u},\tilde{v}}(\ell)\} = d'$ then it follows that

$$\eta_1 + \eta_2 = \tfrac{1}{2}(N-\ell+d') .$$

It is clear that $|d'| \le N-\ell$ and it can be shown that $N-\ell+d'$ is even. Hence, there are

$$q(\ell,N,d') = 2^{N-\ell} \binom{N-\ell}{\tfrac{1}{2}(N-\ell+d')}$$

choices for the sequence $(\tilde{u}_0,\tilde{u}_1,\ldots,\tilde{u}_{N-1-\ell})$ which satisfy $\mathrm{Re}\{C_{\tilde{u},\tilde{v}}(\ell)\} + \mathrm{Im}\{C_{\tilde{u},\tilde{v}}(\ell)\} = d'$. Since there are $4^N$ choices for the sequence $(\tilde{v}_i)$ and $4^\ell$ choices for the sequence $(\tilde{u}_{N-\ell},\ldots,\tilde{u}_{N-1})$, there are a total of $h'(\ell,N,d') = 4^N 4^\ell q(\ell,N,d')$ sequence pairs $(\tilde{u}_i)$, $(\tilde{v}_i)$ that satisfy $\mathrm{Re}\{C_{\tilde{u},\tilde{v}}(\ell)\} + \mathrm{Im}\{C_{\tilde{u},\tilde{v}}(\ell)\} = d'$. For $1-N \le \ell \le N-1$, we have

$$h'(\ell,N,d') = 4^N 4^{|\ell|} q(|\ell|,N,d')$$

so that we obtain the probability mass function

$$P\{Re\{C_{\tilde{u},\tilde{v}}(\ell)\} + Im\{C_{\tilde{u},\tilde{v}}(\ell)\} = d'\} = \binom{N-|\ell|}{\frac{1}{2}(N-|\ell| + d')} 2^{-(N-|\ell|)} \tag{3.6}$$

which is identical to (3.5) with d replaced by d'. If we evaluate the moment generating function for the probability mass function in (3.6) for the first two moments, we find that

$$E\left\{Re\{C_{\tilde{u},\tilde{v}}(\ell)\} + Im\{C_{\tilde{u},\tilde{v}}(\ell)\}\right\} = 0 \tag{3.7}$$

and

$$E\left\{[Re\{C_{\tilde{u},\tilde{v}}(\ell)\} + Im\{C_{\tilde{u},\tilde{v}}(\ell)\}]^2\right\} = N-|\ell| \; . \tag{3.8}$$

It is not immediately apparent that the above statistics yield the desired result. However, if we note that

$$P\left\{Re\{C_{\tilde{u},\tilde{v}}(\ell)\} + Im\{C_{\tilde{u},\tilde{v}}(\ell)\} = d'\right\} = P\left\{Re\{C_{\tilde{u},\tilde{v}}(\ell)\} - Im\{C_{\tilde{u},\tilde{v}}(\ell)\} = d'\right\}$$

then we have from (3.7)

$$E\left\{Re\{C_{\tilde{u},\tilde{v}}(\ell)\}\right\} + E\left\{Im\{C_{\tilde{u},\tilde{v}}(\ell)\}\right\} = E\left\{Re\{C_{\tilde{u},\tilde{v}}(\ell)\}\right\} - E\left\{Im\{C_{\tilde{u},\tilde{v}}(\ell)\}\right\} = 0$$

hence we see that

$$E\{C_{\tilde{u},\tilde{v}}(\ell)\} = E\left\{Re\{C_{\tilde{u},\tilde{v}}(\ell)\}\right\} + jE\left\{Im\{C_{\tilde{u},\tilde{v}}(\ell)\}\right\} = 0 \; .$$

Similarly, for the second moment in (3.8) we have

$$E\left\{[Re\{C_{\tilde{u},\tilde{v}}(\ell)\} + Im\{C_{\tilde{u},\tilde{v}}(\ell)\}]^2\right\} = E\left\{[Re\{C_{\tilde{u},\tilde{v}}(\ell)\} - Im\{C_{\tilde{u},\tilde{v}}(\ell)\}]^2\right\}$$

or equivalently

$$E\left\{Re\{C_{\tilde{u},\tilde{v}}(\ell)\}^2 + Im\{C_{\tilde{u},\tilde{v}}(\ell)\}^2 + 2Re\{C_{\tilde{u},\tilde{v}}(\ell)\}Im\{C_{\tilde{u},\tilde{v}}(\ell)\}\right\}$$
$$= E\left\{Re\{C_{\tilde{u},\tilde{v}}(\ell)\}^2 + Im\{C_{\tilde{u},\tilde{v}}(\ell)\}^2 - 2Re\{C_{\tilde{u},\tilde{v}}(\ell)\}Im\{C_{\tilde{u},\tilde{v}}(\ell)\}\right\}$$

so we find that

$$E\{C_{\tilde{u},\tilde{v}}(\ell)C^*_{\tilde{u},\tilde{v}}(\ell)\} = N-|\ell| \quad .$$

Again, it is easy to see that for $|\ell| \leq N-1$,

$$E\{C_{\tilde{u},\tilde{v}}(\ell)C^*_{\tilde{u},\tilde{v}}(\ell+1)\} = 0.$$

Finally, if we apply the above results to evaluate the expected value of the interference parameter $r_{\tilde{u},\tilde{v}}$ in (3.1) we have as before

$$E\{r_{\tilde{u},\tilde{v}}\} = 2N^2.$$

Hence, for a QDS/SSMA system with quadriphase coded carriers we have, for K users

$$E\{\sum_{\substack{k=1 \\ k\neq i}}^{K} r_{k,i}\} = (K-1)2N^2$$

where N is much larger than K and the k-th user's random quadriphase signature sequence is denoted by $(\tilde{u}_n^{(k)})$. In terms of this expectation, the signal-to-noise ratio (2.25) at the output of either branch of the i-th receiver for the QDS/SSMA system with quadriphase coded carriers becomes

$$SNR_{2i} = SNR_{2i-1} = \left\{\frac{2(K-1)}{3N} + \frac{N_0}{2E_b}\right\}^{-\frac{1}{2}}$$

which is identical to the result in (2.13) for the QDS/SSMA system with orthogonal biphase-coded carriers. Thus, when long random sequences are employed, the signal-to-noise ratios of both systems are equivalent and for either branch of the i-th receiver

$$SNR = \left\{\frac{2(K-1)}{3N} + \frac{N_0}{2E_b}\right\}^{-\frac{1}{2}}$$

regardless of which of the two QDS/SSMA system models is being considered.

## 3.2. Maximal-Length Shift-Register Sequences

For the implementation of a practical DS/SSMA system, the sequence length N as well as the signature sequences themselves must be fully specified. One class of sequences that have been considered for a variety of applications are Maximal-Length Shift-Register Sequences or m-sequences. The properties of m-sequences have been investigated for a number of years and an excellent tutorial as well as a survey of the literature appears in [15]. Roefs [14] has considered the performance of biphase DS/SSMA systems employing m-sequences. In this section, we will investigate the performance of QDS/SSMA systems where m-sequences are used as signature sequences.

### 3.2.1. Introduction to m-Sequences

A binary m-sequence $(\alpha_k)$ with period $N = 2^n-1$ is a sequence of elements from GF(2) which satisfies the linear recurrence relation

$$\alpha_k = \sum_{i=1}^{n} f_i \alpha_{k-i} \qquad \forall k$$

where the $f_i$ are the coefficients of a primitive polynomial, f(x) of degree n over GF(2) given by

$$f(x) = f_0x^n + f_1x^{n-1} + \ldots + f_{n-1}x + f_n.$$

A polynomial of degree n is primitive if it divides $x^m-1$ for $m = 2^n-1$ but not for any $m < 2^n-1$. The roots of a primitive polynomial of degree n are primitive elements of the extension field $GF(2^n)$. Thus every nonzero element of $GF(2^n)$ can be written as a power of any root of a primitive polynomial of degree n.

The polynomial f(x) specifies an n-stage linear feedback shift-register, as shown in Figure 3 for $f(x) = x^4 + x + 1$, where there is a feedback tap connected to the i-th stage of the register if $f_i = 1$ and no feedback tap connection otherwise.

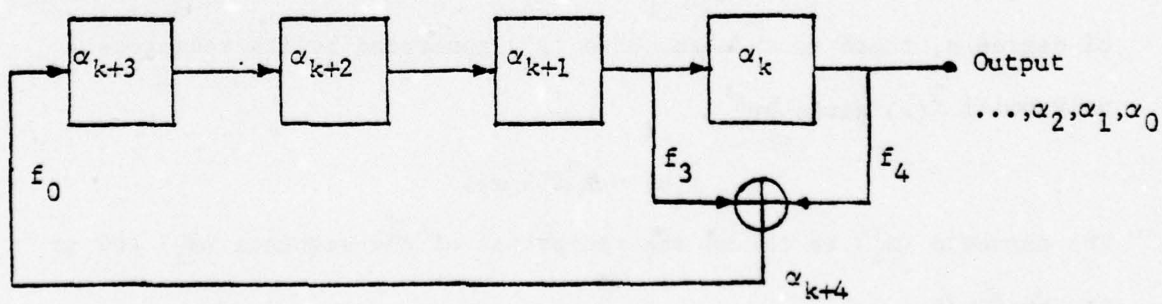


Figure 3. Shift register specified by $f(x) = x^4 + x + 1$.

If $(\alpha_{i+k})$ denotes the k-th cyclic shift of the sequence $(\alpha_i)$ then the set of m-sequences generated by the primitive polynomial f(x) of degree n consists of $(\alpha_i)$ and the $2^n-2$ distinct cyclic shifts of $(\alpha_i)$ given by $(\alpha_{i+k})$ for $k=1,\ldots,2^n-2$. To each sequence in this set there corresponds a unique initial shift register loading $(\alpha_0,\alpha_1,\ldots,\alpha_{n-1})$ consisting of not more than (n-1) zeros. Hence any m-sequence may be completely specified in terms of its primitive polynomial and its shift-register loading. For convenience we will represent the primitive polynomial and initial register loading for the sequence $(\alpha_i)$ in octal notation. For example if n=4 and

$$f(x) = x^4 + x + 1 = 0\cdot x^5 + 1\cdot x^4 + 0\cdot x^3 + 0\cdot x^2 + 1\cdot x + 1$$

then we have

$$f(x) = [010011] = 23 \quad .$$

Similarly, for the initial register loading, if

$$(\alpha_0,\alpha_1,\alpha_2,\alpha_3) = (1,0,1,1)$$

then

$$(\alpha_0,\alpha_1,\alpha_2,\alpha_3) = 54 \quad .$$

For each m-sequence $(\alpha_i)$ generated by the primitive polynomial f(x) of degree n, there is an m-sequence $(\alpha_i')$ generated by its reciprocal polynomial $\hat{f}(x)$ given by

$$\hat{f}(x) = x^n f(1/x).$$

The sequence $(\alpha_i')$ is called the reciprocal of the sequence $(\alpha_i)$ and is expressed as

$$\alpha_i' = \alpha_{N-1-i} \quad i=0,1,\ldots,N-1$$

where $N = 2^n-1$.

In the next section we consider biphase signature sequences $(a_k)$ which are obtained from binary m-sequences $(\alpha_k)$ by the relation

$$a_k = (-1)^{\alpha_k}, \quad k=0,\ldots,N-1$$

where N is the period of the binary m-sequence.

3.2.2. Numerical Results

In order to optimize the average performance of biphase DS/SSMA for a given set of m-sequences, it would be desirable to minimize, with respect to the cyclic shifts of each m-sequence, the biphase interference parameter $\sum_{\substack{k=1 \\ k\neq i}}^{K} r_{k,i}$. However, for typical values of N and K, this minimization requires a prohibitive amount of computation if the $r_{k,i}$ are computed directly for each cyclic shift of the k-th sequence.

In [16] it was pointed out that for most applications, the approximation

$$r_{k,i} \approx \hat{r}_{k,i} = 2\mu_{k,i}(0) = 2 \sum_{\ell=1-N}^{N-1} C_{k,i}^2(\ell)$$

is satisfactory even for moderate values of N. Also, it was shown [16] that

$$\mu_{k,i}(0) \leq N^2 + 2\{S_k S_i\}^{\frac{1}{2}}$$

where

$$S_k = \sum_{\ell=1}^{N-1} C_k^2(\ell) \quad .$$

Thus the autocorrelation parameter $S_k$, called the sidelobe energy [14], may be employed as a useful performance criterion for the selection of m-sequences which minimize the biphase interference $r_{k,i}$. Using a computer search, Roefs [17] has found sets of least sidelobe energy (LSE) m-sequences for periods 31, 63, 127, and 255. An m-sequence $(a_n^{(k)})$ is said to be a LSE sequence if for all $\overline{k}$

$$S_k \leq S_{\overline{k}}$$

where

$$(a_n^{(\overline{k})}) = (a_{n+i}^{(k)}) \quad , \; i=1,2,\ldots$$

are the cyclic shifts of the sequence $(a_n^{(k)})$. Clearly, if the m-sequence $(a_n^{(k)})$ is a LSE sequence, then its reciprocal, $(a_n^{(k')})$ defined in Section 3.2.1 is also a LSE sequence since $S_k = S_{k'}$. The LSE sequences found in [17] along with their reciprocals are given in Tables 1, 2, 3, and 4 for sequence lengths 31, 63, 127, and 255 respectively.

While it is clear that LSE sequences are good candidates for use as signature sequences in QDS/SSMA systems with orthogonal biphase-coded carriers; it is not apparant that quadriphase signature sequences formed from complex combinations of LSE sequences are desirable choices for use in QDS/SSMA systems with quadriphase-coded carriers. Not withstanding this fact, it is of interest to evaluate the performance of both QDS/SSMA systems when LSE sequences are employed.

Table 1. LSE m-sequences of length N=31

| Sequence # | Poly. | Loading | Reciprocal | Poly. | Loading |
|---|---|---|---|---|---|
| 1 | 45 | 46 | 1' | 51 | 50 |
| 2 | 67 | 60 | 2' | 73 | 24 |
| 3 | 75 | 74 | 3' | 57 | 44 |

Table 2. LSE m-sequences of length N=63

| Sequence # | Poly. | Loading | Reciprocal | Poly. | Loading |
|---|---|---|---|---|---|
| 1 | 103 | 02 | 1' | 141 | 37 |
| 2 | 133 | 42 | 2' | 155 | 54 |
| 3 | 147 | 61 | 3' | 163 | 53 |

Table 3. LSE m-sequences of length N=127

| Sequence # | Poly. | Loading | Reciprocal | Poly. | Loading |
|---|---|---|---|---|---|
| 1 | 211 | 620 | 1' | 221 | 234 |
| 2 | 217 | 024 | 2' | 361 | 774 |
| 3 | 235 | 300 | 3' | 271 | 104 |
| 4 | 247 | 134 | 4' | 345 | 304 |
| 5 | 277 | 464 | 5' | 375 | 034 |
| 6 | 357 | 570 | 6' | 367 | 010 |
| 7 | 323 | 754 | 7' | 313 | 070 |
| 8 | 203 | 334 | 8' | 301 | 554 |
| 9 | 325 | 254 | 9' | 253 | 224 |

Table 4. LSE m-sequences of length N=255

| Sequence # | Poly. | Loading | Reciprocal | Poly. | Loading |
|---|---|---|---|---|---|
| 1 | 455 | 674 | 1' | 551 | 006 |
| 2 | 453 | 777 | 2' | 651 | 220 |
| 3 | 435 | 604 | 3' | 561 | 770 |
| 4 | 537 | 550 | 4' | 765 | 170 |
| 5 | 545 | 146 | 5' | 515 | 554 |
| 6 | 543 | 214 | 6' | 615 | 522 |
| 7 | 607 | 702 | 7' | 703 | 566 |
| 8 | 717 | 346 | 8' | 747 | 170 |

For a given set of sequences, there are various ways in which pairs of sequences may be assigned to each of the K users of a QDS/SSMA system. In this section we first consider systems where each user is assigned a LSE sequence and its reciprocal. Thus, for the QDS/SSMA system with orthogonal biphase-coded carriers, which we will refer to as the orthogonal-biphase system, we assume that the k-th user phase-modulates the cosine and sine carriers with the signature sequences $(a_n^{(k)})$ and $(a_n^{(k')})$ respectively. Accordingly for the QDS/SSMA system with quadriphase-coded carriers, which will be referred to as the quadriphase-coded system, we assume the k-th user's complex signature sequence $(\tilde{a}_n^{(k)})$ is given by

$$\tilde{a}_n^{(k)} = \sqrt{\tfrac{1}{2}}\,(a_n^{(k)} + j a_n^{(k')}),\ n=0,\ldots,N-1.$$

This particular method of assignment was chosen for a preliminary evaluation of the two QDS/SSMA systems for a variety of reasons. In particular if $(a_n^{(k)})$ and $(a_n^{(k')})$ are reciprocal sequences, the biphase interference parameter $r_{k,i}$ and $r_{k',i}$ are identical. Thus, for the interference terms appearing in the signal-to-noise ratios, $SNR_{2i}$ and $SNR_{2i-1}$ for the orthogonal-biphase system , we have

$$r_{2k,2i} = r_{2k-1,2i} = r_{2k,2i-1} = r_{2k-1,2i-1}$$

and we may represent these simply by $r_{k,i}$. Also, it was pointed out [14] that for biphase sequences, the interference $r_{k,i}$ between a sequence and itself or its reciprocal is considerably larger. Hence, it would not be desirable to assign a particular sequence or a sequence and its recirpocal to two different users for the orthogonal-biphase system. Finally, the same assignment was made for the quadriphase-coded system in order to exhibit the differences inherent in the implementation of the two QDS/SSMA systems when all other considerations are the same.

For the LSE sequences of length 31 in Table 1, the interference parameters, $r_{k,i}$ are given in Table 5 and Table 6 for the orthogonal-biphase and the quadriphase-coded system respectively. Only a portion of the values are tabulated since for either system $r_{k,i} = r_{i,k}$. The self-interference parameters $r_{i,i}$, appearing on the diagonal are included here for completeness. For this method of sequence assignment, the interference values for the orthogonal-biphase system are the same as those calculated in [17] for the biphase DS/SSMA system. This is also true for lengths N = 63, 127 and 255.

The interference $r_{k,i}$ for the LSE sequences of Table 2 are given in Table 7 for the orthogonal-biphase system and in Table 8 for the quadriphase-coded system. Similarly Tables 9 and 10 list the interference parameters for the LSE sequences of length 127 in Table 3, and the interference parameters for the LSE sequences with period 255 in Table 4 are tabulated in Tables 11 and 12 for the orthogonal-biphase and quadriphase-coded system respectively. Notice that for lengths 31 and 63, the interference for the worst case of the quadriphase-coded system is less than the interference for any particular user of the orthogonal-biphase system.

Table 5. Interference parameter $r_{k,i}$ for the QDS/SSMA system with orthogonal biphase-coded carriers: N=31

| i = | 1 | 2 | 3 |
|---|---|---|---|
| k = 1 | 2182 | 1706 | 1890 |
| 2 | | 2414 | 1902 |
| 3 | | | 2206 |

Table 6. Interference parameter $r_{k,i}$ for the QDS/SSMA system with quadriphase-coded carriers: N=31

| i = | 1 | 2 | 3 |
|---|---|---|---|
| k = 1 | 3350 | 1642 | 1714 |
| 2 | | 3486 | 1742 |
| 3 | | | 3254 |

Table 7. Interference parameter $r_{k,i}$ for the QDS/SSMA system with orthogonal biphase-coded carriers: N=63

| i = | 1 | 2 | 3 |
|---|---|---|---|
| k = 1 | 9574 | 8262 | 7050 |
| 2 | | 9526 | 7706 |
| 3 | | | 9094 |

Table 8. Interference parameter $r_{k,i}$ for the QDS/SSMA system with quadriphase-coded carriers: N=63

| i = | 1 | 2 | 3 |
|---|---|---|---|
| k = 1 | 14278 | 7662 | 5962 |
| 2 | | 13094 | 7834 |
| 3 | | | 13742 |

Table 9. Interference parameter $r_{k,i}$ for the QDS/SSMA system with orthogonal biphase-coded carriers: N=127

| i = | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| k = 1 | 40158 | 33370 | 31638 | 32330 | 32018 | 33570 | 32450 | 32294 | 31898 |
| 2 | | 40222 | 30874 | 31406 | 31510 | 33190 | 32518 | 33306 | 31934 |
| 3 | | | 40454 | 32538 | 31202 | 32722 | 32898 | 33542 | 29722 |
| 4 | | | | 40326 | 32358 | 30598 | 30422 | 30346 | 32654 |
| 5 | | | | | 40310 | 33566 | 33086 | 32162 | 32710 |
| 6 | | | | | | 41230 | 35342 | 31714 | 33190 |
| 7 | | | | | | | 39902 | 33746 | 31750 |
| 8 | | | | | | | | 40134 | 32106 |
| 9 | | | | | | | | | 40262 |

Table 10. Interference parameter $r_{k,i}$ for the QDS/SSMA system with quadriphase-coded carriers: N=127

| i = | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| k = 1 | 59718 | 29802 | 27942 | 33434 | 32506 | 35482 | 30754 | 30550 | 31378 |
| 2 | | 63230 | 29826 | 31022 | 34550 | 29558 | 30774 | 32546 | 30974 |
| 3 | | | 65806 | 34826 | 32842 | 33298 | 31034 | 32326 | 31626 |
| 4 | | | | 61678 | 32214 | 29686 | 29558 | 28586 | 33398 |
| 5 | | | | | 61302 | 33534 | 34110 | 29698 | 30494 |
| 6 | | | | | | 61014 | 39846 | 32930 | 31214 |
| 7 | | | | | | | 58398 | 35986 | 29454 |
| 8 | | | | | | | | 59910 | 32034 |
| 9 | | | | | | | | | 62198 |

Table 11. Interference parameter $r_{k,i}$ for the QDS/SSMA system with orthogonal biphase-coded carriers: N=255

| | i = 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| k = 1 | 169694 | 133218 | 127770 | 130282 | 125758 | 130102 | 134830 | 127706 |
| 2 | | 167366 | 128718 | 127678 | 128882 | 131754 | 132162 | 126702 |
| 3 | | | 166366 | 131958 | 132586 | 135650 | 124474 | 136614 |
| 4 | | | | 167422 | 129658 | 125010 | 131210 | 130454 |
| 5 | | | | | 165126 | 136710 | 119646 | 132682 |
| 6 | | | | | | 164718 | 128374 | 136162 |
| 7 | | | | | | | 165942 | 127322 |
| 8 | | | | | | | | 162438 |

Table 12. Interference parameter $r_{k,i}$ for the QDS/SSMA system with quadriphase-coded carriers: N=255

| | i = 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| k = 1 | 255030 | 141354 | 133882 | 127642 | 126550 | 139766 | 140438 | 127850 |
| 2 | | 249230 | 126742 | 137854 | 143370 | 134394 | 136634 | 123830 |
| 3 | | | 254430 | 139758 | 140738 | 121074 | 131346 | 138926 |
| 4 | | | | 256478 | 130370 | 126866 | 134074 | 134790 |
| 5 | | | | | 246606 | 137206 | 118222 | 124178 |
| 6 | | | | | | 254718 | 129550 | 150810 |
| 7 | | | | | | | 252566 | 124074 |
| 8 | | | | | | | | 236350 |

Also, we see that for sequence length 127 the performance of the quadriphase-coded system is slightly better, in most instances, than that of the orthogonal-biphase system, while for sequence length 255, the quadriphase-coded system shows higher interference values than the orthogonal-biphase system for almost all cases.

It would be very natural to attempt to draw conclusions about the relative performance of the two systems based on the data just presented. However, these results represent only a preliminary investigation of QDS/SSMA systems and do not necessarily indicate what advantages might be realized by one system relative to the other for more carefully considered choices of signature sequences. For example, when considering biphase sequences for use in QDS/SSMA, it is undesirable to assign a sequence or a sequence and its reciprocal to two different users of the orthogonal-biphase system, while for the quadriphase-coded system this is not always the case.

In particular, for N=31 and K=3, an exhaustive computer search was conducted to find the best possible sequence assignments from a class of sequences with minimum odd autocorrelation, $\hat{\theta}_{kk}$. Such sequences are commonly called auto-optimal (AO) sequences and have previously been considered for use in biphase DS/SSMA systems in [7] and [14].

It was determined that for the orthogonal-biphase system, the optimum choice was to assign a different sequence and its reciprocal to each user. On the other hand, for the quadriphase-coded system, the minimum interference was obtained by assigning the same AO sequence for use as the real part of the signature sequences of two different users, while for only one user was a sequence and its reciprocal employed as the real and imaginary parts of the complex signature sequence. For this optimum sequence assignment the worst-case interference for the quadriphase-coded system was found to be twelve percent less than the interference for any of the users of the orthogonal-biphase system. If, in contrast, the AO sequences were assigned to the quadriphase-coded system in the same manner as for the orthogonal-biphase system, the quadriphase-coded system exhibited as much as fifteen percent more interference for any one user than did the orthogonal-biphase system.

Finally, it should be pointed out that for the quadriphase-coded system, we have only considered signature sequences which are complex combinations of biphase sequences with desired correlation properties. Sequences formed in this manner as well as other classes of quadriphase codes would need to be considered before the performance of QDS/SSMA systems with quadriphase-coded carriers can be properly evaluated.

# CHAPTER 4

# CONCLUSIONS

Two different implementations of QDS/SSMA were discussed and analyzed. The QDS/SSMA system with orthogonal biphase-coded carriers was seen to be a direct extension of biphase DS/SSMA systems. The average signal-to-noise ratio for this system was related to the aperiodic autocorrelation functions of the biphase signature sequences. The average signal-to-noise ratio for the QDS/SSMA system with quadriphase-coded carriers was related to the complex-valued aperiodic autocorrelation functions of the quadriphase signature sequences.

The asymptotic behavior of the aperiodic correlation parameters of random sequences was investigated for both systems. It was found that the expected value of the interference was the same for both QDS/SSMA systems and that this value was exactly twice the corresponding value for biphase DS/SSMA when random signature sequences of the same period are employed. It should be noted that if the energy per bit, the data rate, and the signal bandwidth are fixed, the QDS/SSMA system can employ signature sequences which are twice the length of the sequences employed by the biphase DS/SSMA system. Comparing the results presented in Section 2.1.2 (from [2]) for biphase DS/SSMA with the results obtained in Section 3.1 for QDS/SSMA, we see that for the more realistic constraints of equal bit energy, data rate, and signal bandwidth, the expected values of the signal-to-noise ratios are identical.

A preliminary numerical investigation for a special case of signature sequence assignment revealed that the QDS/SSMA system with quadriphase-coded carriers produced as much as twelve percent less interference than the QDS/SSMA system with orthogonal biphase-coded carriers. The data obtained for this special case is not sufficient to allow a prediction of the relative performance of the QDS/SSMA systems for larger classes of codes. However, preliminary results indicate that the quadriphase-coded system may achieve substantially lower interference values for most sequence lengths of interest.

It should be noted that while the average signal-to-noise ratio is an important measure of system performance, other performance parameters such as acquisition time, worst-case error probability, and immunity to multipath interference must be carefully studied before QDS/SSMA can be considered for use in a practical system. The analysis and preliminary results presented here indicate that further investigations of various performance parameters and methods of signature sequence design is warranted and it is anticipated that QDS/SSMA will become an attractive alternative to other forms of code-division multiple-access.

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