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A Transitive-Chain Theory of Syllogistic Reasoning

Martin J. Guyote and Robert J. Sternberg

Department of Psychology
Yale University
New Haven, Connecticut 06520

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(1)

A new theory of syllogistic reasoning, called the transitive-chain theory, is presented. The transitive-chain theory proposes that information about set relations is represented in memory by pairs of informational components. and (2) The theory further proposes that information about set relations is integrated by applying a small set of rules to transitive chains that are formed by rearranging informational components stored in memory. The method of rearranging informational components into transitive chains and the rules

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(cont) ~~that are~~ applied to these chains are specified. ~~in detail.~~

The transitive-chain theory is ~~then~~ compared to several earlier theories, each of which the random and complete combination theories of Erickson (1974), the conversion theory of Chapman and Chapman (1959), and the atmosphere theory of Woodworth and Sells (1935). Each of the theories is cast in terms of an information-processing model, ~~and then~~ mathematical models that quantify each of these information-processing models are presented. In a series of five experiments, the transitive-chain theory provides a good account of the response-choice data for syllogisms with various types of content, quantifiers, and logical relations (categorical and conditional). ~~The~~ results of these experiments offer tentative answers to five major issues in the ~~theory of~~ syllogistic reasoning theory:

(1) Generality of the processes used in syllogistic reasoning;
(2) Relationship of syllogistic reasoning ability to intelligence;
(3) Representation of ~~premise information~~ and
(4) Combination of premise information; and
(5) Sources of difficulty in syllogistic reasoning.

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A Transitive-chain Theory of Syllogistic Reasoning

Martin J. Guyote and Robert J. Sternberg

Yale University

Running head: Syllogistic Reasoning

Send proofs to: Martin J. Guyote
Department of Psychology
Box 11A Yale Station
Yale University
New Haven, Connecticut 06520

Abstract

A new theory of syllogistic reasoning, called the transitive-chain theory, is presented. The transitive-chain theory proposes that information about set relations is represented in memory by pairs of informational components. The theory further proposes that information about set relations is integrated by applying a small set of rules to transitive chains that are formed by rearranging informational components stored in memory. The method of rearranging informational components into transitive chains and the rules that are applied to these chains are specified in detail.

The transitive-chain theory is then compared to the random and complete combination theories of Erickson (1974), the conversion theory of Chapman and Chapman (1959), and the atmosphere theory of Woodworth and Sells (1935). Each of the theories is cast in terms of an information-processing model, and then mathematical models that quantify each of these information-processing models are presented. In a series of five experiments, the transitive-chain theory provides a good account of the response-choice data for syllogisms with various types of content, quantifiers, and logical relations (categorical and conditional). The results of these experiments offer tentative answers to five major issues in the theory of syllogistic reasoning:

(1) Generality of the processes used in syllogistic reasoning

(2) Relationship of syllogistic reasoning ability to intelligence

(3) Representation of premise information

(4) Combination of premise information

(5) Sources of difficulty in syllogistic reasoning

A Transitive-chain Theory of Syllogistic Reasoning

A categorical syllogism consists of three declarative statements, each of which describes a relation between two sets of items. The first two statements are called premises; the third statement is a conclusion drawn from the premises. The premises relate the items in the subject and predicate of the conclusion to a third set of items. In a syllogistic reasoning problem, the subject's task is to indicate whether the relation described between the subject (S) and predicate (P) of the conclusion is logically determined by their relation to the third term (M). An example of a categorical syllogism is

Some children are brats.
All Mouseketeers are children.
Some Mousketeers are brats.

Solution of this problem requires the individual to use information contained in the premises to infer the relation between Mousketeers and brats, and to compare this inferred relation to that described by the conclusion.

This article presents a general theory of syllogistic reasoning, called the transitive-chain theory. The article is divided into seven parts. First, the structure of categorical syllogisms is described. Second, five major issues in the theory of syllogistic reasoning are discussed. Third, the problem domain to which the theory is applied in this article is described. Fourth, the transitive-chain theory is presented, together with four competing models of syllogistic reasoning. Each of the theories is expressed in terms of an information-processing model of subjects' performance, and of mathematical models that quantify each of the information-processing models. Fifth, the methods used in five experiments performed to test these theories are outlined. Sixth, the results of these experiments are presented and discussed. Seventh, conclusions are drawn that provide tentative answers to the five major theoretical issues raised earlier.

STRUCTURE OF THE CATEGORICAL SYLLOGISM

In the sample syllogism given above, the first premise relates _P_ to _M_, and the second premise relates _S_ to _M_. This is true for all categorical syllogisms. There are, however, two important properties of categorical syllogisms that do vary across syllogisms.

The first variable is called the _mood_ of a syllogism. Mood refers to the quantification and polarity of the statements constituting the syllogism. Each of these statements may be either universal or particular in quantification, and either positive or negative in polarity. There are thus four types of statements: universal affirmatives (All _A_ are _B_), universal negatives (No _A_ are _B_), particular affirmatives (Some _A_ are _B_), and particular negatives (Some _A_ are not _B_), referred to as A, E, I, and O statements, respectively. Any natural set relation between a subject and a predicate can be expressed by one of these four statements. Classification of each of the three statements in a syllogism as A, E, I, or O uniquely defines the mood of the syllogism (for example, the mood of the "Mousketeers" syllogism above is IAI). Since any of the four types of statements is allowed for each of the three statements in a syllogism, there are 4^3, or 64 possible syllogistic moods.

The second variable is called the _figure_ of the syllogism. Figure refers to the order of the terms in the premises. Although the order of the premises in which the _S_, _M_, and _P_ terms appear is fixed, the order of the terms within each premise is not. Since each of two terms in each of two premises may appear either first or second, there are 2^2, or 4 possible figures. The variables of mood and figure combine to yield a total of 64 x 4, or 256 different syllogisms. Of these, only 24 are logically valid, that is, have a conclusion that _necessarily follows_ from the premises. In these 24 syllogisms, the conclusion of the syllogism is always true if the premises are true.

MAJOR ISSUES IN THE THEORY OF SYLLOGISTIC REASONING

The following section presents five major issues in the theory of syllogistic reasoning. In the course of the presentation, many of the important contributions to the voluminous literature on syllogistic reasoning are reviewed.

Generality of the Processes Used in Syllogistic Reasoning

Relationship Between Logic and Thought

Students of syllogistic reasoning have covered the full gamut in their range of opinions on the generality of the processes used in syllogistic and other forms of deductive reasoning. Henle's (1962) opening remarks on the relation between logic and thinking cite much of the relevant literature. One extreme position is represented by Boole (1854), who entitled a treatise on logic, _An investigation of the laws of thought_. Boole went so far as to claim that the laws of symbolic logic are equivalent to rules governing the operation of the mind in reasoning. The other extreme position is represented by Schiller (1930), who claimed that syllogistic reasoning "has nothing whatever to do with actual reasoning, and can make nothing of it" (p. 282).

Most students of syllogistic reasoning have taken a position falling somewhere between these two extremes. Although the mind does not operate according to the rules of symbolic logic, it seems undeniable that the processes used in solving syllogisms are general to many acts of reasoning. Two examples will illustrate this.

The first example involves the application of knowledge about the members of a category to a novel instance of that category, as in

> All Swedes have fair complexions.
> The person I am looking for is a Swede.
> Therefore, the person I am looking for has
> a fair complexion.

This is a case of a categorical syllogism in which one of the sets has only one member. Such applications of knowledge about categories of people and objects are widespread, and it is clear that a major purpose of inductive reasoning is the ability to apply the knowledge thus gained to novel situations.

Another example of the pervasiveness of syllogistic reasoning in everyday life is what Aristotle (1945) has termed the "practical syllogism," in which

> the conclusion drawn from the two premises becomes the action. For example, when you conceive that... on a particular occasion no man ought to walk, and you yourself are a man, you...remain at rest (p. 701).

After presenting some contemporary examples of the practical syllogism, Henle (1962) argues for the importance of the phenomenon:

> It might be argued that in the case of the practical syllogism we are dealing not with an implicit logical process, but with a learning process in which each response is cued off by the preceding one. There seem to be at least two grounds for rejecting such an interpretation. (a) There are cases in which the practical syllogism leads to a solution which is genuinely novel for the individual....(b) Our data obtained from conventional syllogisms suggest that a logical process often occurs even in cases in which the reasoning seems fallacious....In the interests of parsimony, therefore, a common explanation for the practical syllogism and the verbal one seems preferable. An interpretation in terms of an implicit logical process would fit both kinds of case (p. 374).

<u>Relationships Among Various Kinds of Syllogisms</u>

Several psychologists have studied the generality of the processes used in syllogistic reasoning across different kinds of syllogisms. Consider, for example, the relationship between reasoning with linear syllogisms (e.g., John is taller than Pete. Pete is taller than Bill. Who is tallest?) and with categorical syllogisms. Revlis (1975) has suggested that many of the same principles used to understand linear syllogistic reasoning can be applied to the understanding of categorical syllogistic reasoning; but he has not spelled out the nature of this correspondence. Whereas Revlis has emphasized the similarities between the two kinds of tasks, Wason and Johnson-Laird (1972) have emphasized the differences. They noted that although categorical syllogisms vary widely in difficulty, "even the easiest of [categorical] syllogisms tends to take a considerable time to solve in comparison to a three-term series [linear syllogism] problem. *These two facts imply that the process of combination is essentially a* series *of processes rather than a single act*" (p. 151).

As a second example of the relationships between different kinds of syllogisms, consider the relationship between reasoning with conditional syllogisms (e.g., If A then B. A. Therefore, B.) and with categorical syllogisms. Revlis (1975) has described in some detail analogies between sources of difficulty in categorical and conditional syllogistic reasoning, but as he admits, the analogy is speculative at the present time. His speculations are supported by the developmental data of Osherson (1974), who performed an extensive series of studies in which logical inference problems were presented in both categorical and conditional forms. Osherson's data suggested that differences between the two forms of presentation are minor, and hence that a single model of logical inference, which Osherson described in detail, can handle both kinds of reasoning. On the other hand, Roberge and Paulus (1971) treated categorical and conditional syllogisms as two levels of a single factor in a developmental study of deductive reasoning. They found both a significant main effect and significant interactions due to problem format, suggesting that different mechanisms may underlie reasoning applied to the two kinds of problems.

Relationships Among Various Kinds of Syllogism Content

Are the processes people used in solving syllogisms general across various kinds of premise content? The data of Osherson (1974) for logical inference problems suggest that they might well be. He used two different kinds of content, a make-believe world inhabited by creatures called toks, and a playground containing various kinds of play activities. He found that a single model applied equally well to both kinds of content. Many other investigators have found that content can have a dramatic effect upon the processes used in deductive reasoning, however, presumably because they used more varied contents.

The first investigation to study content effects in syllogistic reasoning was that of Wilkins (1928). She found that correlations between subtests with different contents were substantially lower than the reliabilities of the subtests, and suggested that "some individuals do relatively better with more abstract material than with more familiar and concrete material, and others do relatively better with familiar material" (p. 25). She concluded that the processes involved in solving syllogisms with abstract, unfamiliar, and counterfactual content are probably more similar to each other than are the processes involved in solving syllogisms with familiar, factually correct content.

Several investigations of generality of processes across contents have been concerned specifically with the effects of emotional appeals. Morgan and Morton (1944), for example, compared patterns of solution for categorical syllogisms with emotionally neutral premises (letters as terms) to patterns for syllogisms with emotionally charged premises, such as "Heydrich, the Nazi hangman, deserved a violent death." The authors found different patterns of errors for the two kinds of syllogisms, and concluded that (a) the processes involved in solving the two kinds of syllogisms are nonidentical, and (b) the processes that do overlap are assigned different weights in solving the two kinds of syllogisms. Lefford (1946) also used abstract and emotionally charged contents, and like Morgan and Morton, found "little relationship between the ability to reason accurately in nonemotional and emotional situations" (p. 150). Henle and Michael (1956), though, replicated the Morgan and Morton experiments with more careful controls, and found evidence that the earlier results may have been largely artifactual. Nevertheless, their evidence was still consistent with the hypothesis that attitudes influence reasoning processes.

Relationship of Syllogistic Reasoning Ability to Intelligence

Several of the studies cited above used individual-difference data to argue that different processes are involved in the solution of emotionally neutral and emotionally charged syllogisms. Individual-difference data have also been used to investigate the nature of the relationship between syllogistic reasoning ability and intelligence. Wilkins (1928) was a pioneer in these investigations as well as in the ones described above. She correlated scores on her syllogistic reasoning test with scores on the Thorndike College Entrance Test, finding a correlation coefficient of .58. As she noted, this correlation is an impressive one, given the restriction of range found in her sample of Columbia College undergraduates. She also correlated scores for individual kinds of syllogism content with the scholastic aptitude scores, and found that the correlation was highest for symbolic material (letters), and about the same for other kinds of content (factual, counterfactual, unfamiliar words).

Thurstone (1938) included a syllogisms test in a large battery of tests to be factor analyzed, and found that the syllogisms test showed its major loading on a spatial visualization factor. More recently, Frandsen and Holder (1969) found a correlation of .56 between deductive reasoning problems (including categorical syllogisms) and a spatial test, and of .63 between the deduction problems and a verbal reasoning test. A partial correlation of .34 between the spatial and deduction tests showed that spatial visualization made a statistically significant contribution to the simple correlation, independent of the effect of verbal reasoning. These data suggest that performance on syllogistic reasoning tasks can serve as one measure of general intelligence, and indeed, a syllogisms subtest can be found in the upper levels of the California Test of Mental Maturity.

Representation of Premise Information

Contributors to the substantial literature on categorical and conditional syllogistic reasoning have remained curiously silent on the issue of representation. Their silence stands in marked contrast to the vocality of contributors to the literature on linear syllogistic reasoning: In this literature, proponents of linguistic and spatial representations have hotly debated their respective positions. (See, for example, Clark, 1969a, 1969b, 1972; Huttenlocher, 1968; Huttenlocher & Higgins, 1971, 1972; Huttenlocher, Higgins, Milligan, & Kauffman, 1970.)

There seem to be two sketchily defined positions on the issue of representation in categorical and conditional syllogistic reasoning. One position, taken by Lippman (1972) and by Revlis (1975), is that subjects store the functional relations expressed by the premises of a syllogism in the form of linguistic deep-structural propositions. This position is essentially identical to that taken by Clark (1969a, 1969b) in describing how information might be represented in the solution of linear syllogisms.

The other position, mentioned in passing by Erickson (1974) and seemingly accepted tacitly by most workers in the field, is that subjects interpret premises "in a manner analogous to forming [Euler] diagrams" (p. 308). This position leaves open the possibility of a linguistic, spatial, or other representation. The only claim made is for a functional analogy between the representation of premise information and the familiar diagrams taught to most students of introductory logic.

Combination of Premise Information

The unwillingness of researchers in the field to commit themselves to a well specified position on the issue of representation has left these same researchers unable to specify in detail how information from two different premises is combined into a unified representation. This state of affairs is to be expected, since the precise nature of the combination algorithm of necessity will depend upon the way in which information is represented. Again, Erickson (1974) seems to express the position of many researchers when he states that subjects combine premise information "in a manner which can be modeled as the combination of [Euler] diagrams" (p. 309). Although he is clear in stating what the diagrams look like before and after combination, and in describing how subjects choose the particular diagrams they will combine, he specifies no algorithm by which the combination process actually takes place. Ceraso and Provitera (1971), also users of Euler diagrams, also fail to specify combination rules.

Revlis (1975), an advocate of a propositional representation for information, has claimed only that "the composite representation of the pair of premises results from an as yet unspecified deduction operation (either immediate or mediate deduction). The model specifies that the output of this stage is a single proposition relating the subject and predicate terms of the conclusion" (p. 115). In the next sentence, Revlis seems to blur the distinction between propositional and imagerial representation, stating that the combined representation "may be in the form of either a deep structure sentence (proposition) or [an Euler] diagram" (p. 115).

Sources of Difficulty in Syllogistic Reasoning

By far the largest proportion of the research effort that has been expended on studying syllogistic reasoning has been devoted to isolating the sources of difficulty subjects encounter in solving syllogisms. Four classes of hypotheses can be distinguished on the basis of whether they attribute errors primarily to failure in combining premise information, failure in encoding premise information, failure to accept the logical nature of the task, or response bias.

Combination of Premise Information

The first major breakthrough in understanding sources of difficulty in categorical syllogistic reasoning came from Woodworth and Sells (1935), who believed that subjects' main problems came not in encoding information but in combining it. Woodworth and Sells (1935), and subsequently Sells (1936) and Begg and Denny (1969), all suggested slightly different versions of what has come to be known as the atmosphere hypothesis. According to this hypothesis, the mood of a premise creates an atmosphere in a syllogism. If both premises are affirmative or negative, and universal or particular, then the atmosphere of the syllogism will be the same as the mood of the premises, and the subject will select a conclusion that has the same atmosphere as the two premises. If the two premises differ in mood, however, then the atmosphere of the conclusion selected will be particular if at least one of the two premises is particular, and negative if at least one of the two premises is negative. Thus, according to Woodworth and Sells, AA premises call for an A conclusion, AE or EE premises call for an E conclusion, AI or II premises call for an I conclusion, and AO, EI, EO, IO, or OO premises call for an O conclusion. The atmosphere hypothesis has generally accounted for some but not all of the errors subjects make in solving categorical syllogisms.

Chapman and Chapman (1959) have suggested that failure in combination of premise information may arise from what they refer to as "probabilistic inference." "By one kind of probabilistic inference, S reasons that things that have common qualities or effects are likely to be the same kinds of things, but things that lack common qualities or effects are not likely to be the same. In the syllogism, the available common characteristic is the middle term" (pp. 224-225). According to this hypothesis, then, subjects try to figure out the extent to which the subject and predicate of the conclusion share the characteristic expressed by the middle term. If, for example, no A's are B's and no B's are C's, subjects conclude that no A's are C's, since the middle term, B, is not shared. If some A's are B's and some C's are not B's, subjects conclude that some C's are not A's, since at least some C's do not share the B characteristic that is common to some A's. Although Chapman and Chapman believed that the probabilistic inference hypothesis was useful in accounting for certain error patterns in their data, the hypothesis has not received much attention in recent years.

A third hypothesis, proposed by Erickson (1974), is that subjects' combination of premise information is correct as far as it goes, but that it often does not go far enough. Erickson's random combination theory proposes that subjects never look at more than one possible combination of the premise representations they have encoded. Wason and Johnson-Laird (1972) have proposed a similar mechanism as a source of error in syllogistic reasoning. According to these authors, subjects propose a tentative conclusion for a categorical syllogism, and then attempt to check this answer by trying out some subset of the various possible combinations of set relations. But subjects are biased in favor of combinations that support rather than refute their tentative conclusion, so that they often fail to falsify a tentative conclusion that is in fact incorrect. Wason and Johnson-Laird's hypothesis is reported by the two authors to give a good account of observational data from their own extensive series of experiments; Erickson's random combination theory gives a very good quantitative account of response-choice data collected in his laboratory.

<u>Encoding of Premise Information</u>

A second class of hypotheses attributes errors in syllogistic reasoning to faulty encoding of premise information rather than to faulty reasoning with the premise information. The most well-known hypothesis, the conversion hypothesis, was mentioned in passing by both Wilkins (1928) and Woodworth and Sells (1935), but it was first investigated systematically by Chapman and Chapman (1959). According to this hypothesis, subjects "convert" premises, assuming that a premise immediately implies its converse. In conditional logic, this is known as affirming the consequent (Wason & Johnson-Laird, 1972), and conversion has been proposed as a source of errors in conditional as well as categorical syllogistic reasoning (see Taplin, 1971; Taplin & Staudenmeyer, 1973). Consider the statements "Some A are B" and "No A are B." Converting these statements to read "Some B are A" and "No B are A" has no effect upon the meaning of either statement. Consider the statements "All A are B" and "Some A are not B," however. Converting these statements does change their meanings. Thus, even impeccable logic will arrive at incorrect conclusions on the basis of these faulty encodings. The data of Chapman and Chapman (1959) are consistent with their notion of conversion, as are the data of Revlis (1975).

Several investigators have suggested that subjects read less rather than more into their encodings of premises. Ceraso and Provitera (1971) have suggested that subjects encode only one set relation for any premise. "All A are B" is encoded as equivalent sets, "Some A are B" and "Some A are not B" as overlapping sets, and "No A are B" as disjoint sets. Thus, only the encoding of the universal negative is complete. Ceraso and Provitera sought to show that when each premise is stated clearly so as to refer to just the one encoding the authors claim subjects use, the pattern of response choices would be the same as when each premise is stated in its standard form. Moreover, they sought to show that subjects reasoned correctly on this faulty data base. The data of Ceraso and Provitera were generally consistent with these hypotheses. Erickson (1974) has also assumed that subjects encode only one possible set relation for each premise. But whereas Ceraso and Provitera believed that all subjects always used the same set relation for a given premise, Erickson allowed for the possibility that the single encoding might vary over subjects or replications within subject. He made this allowance by assigning probabilities representing the likelihood that each possible set relation would be used in encoding a single premise. For example, Erickson proposed that subjects will encode "All _A_ are _B_" as either equivalent sets or a subset (_A_) of a set (_B_); but the probability of the former encoding (.75) is proposed to be higher than the probability of the latter encoding (.25).

Other investigators have pointed to still further sources of error in encoding. Woodworth and Sells (1935) were among the first to appreciate the ambiguity inherent in the word _some_. In its logical meaning, it denotes "some and possibly all," whereas in its everyday meaning, it is usually interpreted as connoting "some but not all." Most investigators, recognizing this problem, now explicitly instruct subjects in the logical meaning of _some_.

Lippman (1972) has emphasized the importance of syntax upon the encoding of premise material. She found that latencies and rated difficulties were greater for problems phrased in the passive voice or in the negative form than for problems phrased in the active voice or in affirmative form. In general, "the more direct the relationship between the surface and deep structure, the easier is problem solution" (p. 424). Revlis (1975) has also stressed the importance of syntactic factors in the solution of syllogisms, although his conclusions differed from Lippman's. An investigation of response-choice data for several data sets revealed that problems with negative premises were actually easier than were problems with affirmative premises. Revlis did find, however, that "reasoners are less likely to either accept an erroneous conclusion or reject a correct one when the conclusion is affirmative than when it is negative" (p. 100).

Henle (1962) has suggested three additional sources of error in solving syllogisms with everyday material that fall into the category of misencoding of premises. The first source of error is the restatement of a premise (or conclusion) so that its intended meaning is changed. For example, a premise such as "some vitamin deficiencies are dangerous to health" might be misconstrued as meaning that "vitamins are necessary or bad health results" (p. 371). The second source of error is omission of a premise. Subjects in this case fail to use all the information that is available to them. The third source of error is the insertion of a premise. "For example, premises may be added that are so commonplace, so much taken for granted, that they escape attention" (p. 372).

Failure to Accept the Logical Nature of the Task

Henle's (1962) most well known proposal regarding sources of error in syllogistic reasoning is that subjects fail to accept the logical nature of the reasoning task. This "means failure to distinguish between a conclusion that is logically valid and one that is factually correct or one with which the subject agrees" (p. 370). The effects of beliefs and attitudes on syllogistic reasoning are well documented. Janis and Frick (1943) tested two hypotheses: that when subjects agree with a conclusion, they will make more errors by judging an invalid conclusion to be valid than by judging a valid conclusion to be invalid; and that when subjects disagree with a conclusion, they will make more errors by judging a valid conclusion to be invalid than by judging an invalid conclusion to be valid. Both hypotheses were confirmed by their data. Morgan and Morton (1944), Lefford (1946), and Gordon (1953) all found that subjects make more errors on syllogisms with emotionally charged content than on syllogisms with emotionally neutral content, although the Morgan and Morton data are suspect because of methodological inadequacies (Henle & Michael, 1956).

Content effects are not limited to syllogisms that evoke a strong emotional response in the problem solver. Wilkins (1928) long ago found that it is much easier to reason with familiar material than with unfamiliar and symbolic material, even if the familiar material expresses statements that are counterfactual. Wason and Johnson-Laird (1972) also found large effects of content upon difficulty, and concluded that "when the material is abstract in our experiments the subjects tend to succumb to the effect of all the structural variables which we have considered. They will concentrate on what is mentioned in the premises; they will make illicit conversions; they will be blocked by negatives; they will be biased towards verification" (p. 243). In their view, then, the effect of content is indirect. It is not abstract content *per se*, but the effect of abstract content upon the way the subject handles the task, that results in a greater propensity to make errors.

Response Bias

Two response-bias hypotheses will be considered. The first, proposed by Woodworth and Sells (1935), is that of

> caution or wariness on the part of the subject in an experiment. It is evidently more incautious to accept a universal than a particular conclusion, and probably it is more incautious to accept an affirmative than a negative proposition. At any rate a larger percent of invalid particular conclusions are [sic] accepted than of universals, and of negative than of affirmative. (p. 452)

Revlis's (1975) response-bias hypothesis is quite different. According to Revlis, most studies of syllogistic reasoning have presented subjects with a preponderance of invalid syllogisms. The percentage of such items is usually about 70%. But problem solvers do not normally expect that most of the problems they encounter will have no determinate answer. They may therefore bring their answers into line with their expectations, choosing a disproportionately large number of determinate conclusions. Revlis believes that this tendency toward a response bias is enhanced by the instructions given in most syllogistic reasoning experiments, which he finds lead subjects toward accepting a determinate conclusion. In order to test this hypothesis, Moore and Revlis (Note 1) instructed subjects in one group of the proportion of determinate conclusions, instructed subjects in a second group of the proportion of indeterminate conclusions, and provided no information about response proportions to subjects in a third group. The authors found no effect of instructions upon performance on valid syllogisms, but found greatly improved performance for instructed subjects on invalid syllogisms. These data therefore provided support for a response bias interpretation of at least some errors in syllogistic reasoning tasks.

PROBLEM DOMAIN OF THE PRESENT STUDY

Table 1 presents sample problems for each of the five experiments conducted in the present study, and shows the domain to which at least some of the theories to be described apply. The problems in this table give some idea of the range of problem types to which we have so far applied the transitive-chain theory.

Insert Table 1 about here

In a first experiment, we presented subjects with conventional pairs of categorical premises, such as "No C̲ are B̲. All B̲ are A̲." Subjects were required to choose the best of the four standard conclusions (A, E, I, and O), or "None of the above," meaning that none of the four conclusions were judged to be logically valid. The problems in Experiment 2 used concrete terms in place of the capital letters of Experiment 1. Three different types of content were used. One third of the problems contained factual premises (that is, each premise expressed a factual relationship between two closely related terms). Another third of the problems contained counterfactual premises (that is, each premise expressed a counterfactual relationship between closely related terms). The remaining problems contained anomalous premises that expressed relationships between seemingly unrelated terms; these relationships could be either factual or counterfactual.

In the third experiment, we modified the problems in Experiment 1 by substituting the non-classical quantifiers <u>most</u> and <u>few</u> for each occurrence of the quantifier some. For example, the premises in syllogism 6 of Experiment 1 were: "Some B are C. All B are A." In Experiment 3, two problems were substituted for syllogism 6; one with the premises "Most B are C. All B are A;" and one with the premises "Few B are C. All B are A." In Experiment 4, the first premise of each problem was either a categorical or a conditional statement, and the second premise was a statement of fact. The subject's task was to judge the presented conclusion as valid or invalid. Note that if A is taken to be the set of states of the world in which event 'A' is true, B the set of states of the world in which event 'B' is true, and X is a particular state of the world, the categorical and conditional problems are logically equivalent. In our fifth experiment, premises consisted of two conditional statements cast in a mode stressing the temporal dependence between events, as in the statement, "If B occurs, then A occurs."

The various types of problems enabled us to test the validity and generalizability of the transitive-chain theory. These tasks should be kept in mind as the theory is presented and applied in each experiment.

FIVE THEORIES OF SYLLOGISTIC REASONING

Overview

Five theories of syllogistic reasoning will be presented.[1] Each theory consists of two parts: (a) assumptions about the internal representations used in encoding premise information, and (b) a specification of the combination rules used to integrate these internal representations. Specific models will be derived from each theory. Each model will also consist of two parts: (a) an information-processing model that describes in flow-chart form the processes used and the order in which they are executed in solving syllogistic reasoning problems, and (b) a mathematical model that quantifies the information-processing model expressed by the flow chart. All of the information-processing models include four stages: (a) an encoding stage, in which the premises are read and translated into internal representations that include information expressed by the premises, (b) a combination stage, in which the internal representations of the two premises are integrated to achieve a new representation, (c) a comparison stage, in which the representation formed during the combination stage is labelled, and this label is matched against the conclusion(s), and (d) a response stage, in which the subject responds on the basis of the match obtained between the label chosen during the comparison stage and the offered conclusion(s).

The five theories will be presented in five separate sections. Each section will include (a) a general overview of the workings of the theory, (b) the assumptions of the theory that are common to all of the problems to which the theory is applied (including representational assumptions of the theory and the combination rules used to integrate internal representations), and (c) a description of the information-processing models derived from the theory to predict performance in each experiment. The mathematical model corresponding to each information-processing model, the details of quantification, and the prediction equations used in the mathematical modelling are given in the Appendix. The theories will be presented with reference to the sample problems shown in Table 1. A concluding section will compare the information-processing models derived from the theories.

The Transitive-chain Theory

Overview

In the transitive-chain theory, information about set relations is represented in memory by pairs of informational components stored in symbolic form. Thus, a specific relation between set A and set B (e.g., A a subset of B) is represented by two components. One of these components specifies the number of members of set A that are also members of set B, and the other component specifies the number of members of set B that are also members of set A. These symbolic representations are integrated by forming transitive chains from the representations, and then applying one of two simple inferential rules to these chains. The application of these rules yields new components that are combined to form the integrated representation.

Representational Assumptions

Information about set relations is represented in memory by pairs of informational components that are stored in symbolic form. Each pair of components constitutes a symbolic representation of the relation between two sets. In the context of the transitive-chain theory, "symbolic representation" and simply "representation" will be used interchangeably to refer to a pair of components representing a single possible relation between two sets (e.g., equivalence). Five possible set relations and their corresponding symbolic representations are shown in Figure 1.

Insert Figure 1 about here

Each component of a pair can take any one of the following three forms:

(1) $x_1 \rightarrow Y$ (2) $x_1 \rightarrow Y$ (3) $x_1 \rightarrow -Y$

$x_2 \rightarrow Y$ $x_2 \rightarrow -Y$ $x_2 \rightarrow -Y$

In this notation, lowercase letters refer to disjoint, exhaustive partitions of a set. Uppercase letters refer to whole sets. The "arrow relation" indicates that the partition to the left of the arrow is a subset of the set to the right. Thus, component (1) indicates that both partitions of set X are subsets of set Y; since these two partitions are exhaustive, this means that all members of X are also members of Y. In component (2), one partition of X is a subset of Y and the other is a subset of not Y, indicating that some, but not all, members of X are also members of Y. Finally, component (3) conveys the information that neither partition of X is a subset of Y, or that no members of X are also members of Y. Suppose, for example, that X is the set of Communists and Y is the set of subversives. A member of the John Birch Society might claim that the proper relation between X and Y is expressed by component (1): All Communists are also subversives. A political middle-of-the-roader might prefer the relation given in component (2): Some Communists are subversives and other Communists are not. A member of the Communist party might claim (at least publicly) that component (3) is true: No Communists are subversives.

Each component contains information about the number of members of one set that are also members of another set. In the above example, each set of items is represented by two disjoint partitions, and this is assumed to be true for the representation of most of the premises presented in the present study. However, the choice of the number of partitions is arbitrary, and of course, the most accurate representation of a set would have as many partitions as there are members of the set. The notation described here uses two partitions for most representations for simplicity of presentation, and because two partitions are sufficient to depict universal and particular quantification of sets of any size. There were two occasions when it seemed reasonable to use a different number of partitions. The first is in Experiment 4, where a premise (or conclusion) such as "X is a B" is represented by

$$x \to B.$$

Note that the corresponding conditional premise "B" is represented the same way, and that in this case set B is a set of states of the world, and X is a particular state of the world. Since X is a set of size one, only one partition is necessary for its representation. The second occasion for not using two partitions is in Experiment 3, where the representation of the quantifiers most and few requires that each component include three partitions of a set. Thus, a relation in which most A are B and all B are A is represented by

$$\begin{array}{r|l} a_1 \to B & b_1 \to A \\ a_2 \to B & b_2 \to A \\ a_3 \to -B & b_3 \to A \end{array}$$

Pairs of components, as shown in Table 2, contain information about the inclusion of each of two sets in the other. For example,

$$\begin{array}{r|l} a_1 \to B & b_1 \to A \\ a_2 \to -B & b_2 \to A \end{array}$$

refers to a set relation in which B is a proper subset of A. The first component indicates that set A contains some members that belong to set B and others that do not, and the second component indicates that all members of B are also members of A. For convenience of presentation, the order of terms in the first component always matches the order of terms in the premise from which it was derived. The components will be referred to by the order of the terms within them. For example, any of the following three components will be referred to as a BA component:

$b_1 \rightarrow A$	$b_1 \rightarrow A$	$b_1 \rightarrow -A$
$b_2 \rightarrow A$	$b_2 \rightarrow -A$	$b_2 \rightarrow -A$

Insert Table 2 about here

As shown in Figure 1, a one-to-one correspondence can be drawn between each pair of symbolic components and each Euler diagram representing a different relation between two sets. One might therefore ask why the symbolic representations we have just outlined are needed. The most important disadvantage to Euler diagrams is that it is almost impossible to specify combination rules for them. No such performance rules have ever been specified for Euler diagrams. Consider an example in which A and C are both proper subsets of B, represented by two circles (corresponding to sets A and C) inside a larger circle corresponding to set B. Five relationships between A and C are possible, given these representations. But how are these relationships generated? Does one shrink one circle, enlarge another, move one or another--and in each case by how much--to generate all five possibilities? The following section will show, however, that the symbolic representation permits complete specification of the combination rules for integrating representations in a logical and plausible manner.

Combination Process

Overview. The subject starts with two symbolic representations. One of these representations gives the relationship between a set A and a set B, and the other gives the relationship between set B and a set C. The subject's task is to combine the information in these representations so that he or she may infer the relationship between set A and set C. Consider the following example:

	A B	B A
(AB-BA)	$animal_1 \rightarrow$ BIRD	$bird_1 \rightarrow$ ANIMAL
	$animal_2 \rightarrow$ -BIRD	$bird_2 \rightarrow$ ANIMAL
	B C	C B
(BC-CB)	$bird_1 \rightarrow$ ROBIN	$robin_1 \rightarrow$ BIRD
	$bird_2 \rightarrow$ -ROBIN	$robin_2 \rightarrow$ BIRD

The relationship between set A (animals) and set C (robins) is known when the subject has inferred the relation of $animal_1$ and $animal_2$ to ROBIN, and the relation of $robin_1$ and $robin_2$ to ANIMAL. The subject infers these relations by integrating transitive chains that are formed from the components of the two representations. A transitive chain can be formed from two components in which the first term in one component matches the second term in the other component. Thus, an AB component and a BC component form an AB-BC transitive chain. The integration of a transitive chain is accomplished by the following rules.

Rules for integrating transitive chains. Two simple inferential rules are used to integrate transitive chains:

1. Match in Pivot Component

$$x_i \rightarrow Y \;\&\; y_j \rightarrow Z \Longrightarrow x_i \rightarrow Z$$

2. Mismatch in pivot component

$$x_i \rightarrow -Y \;\; \& \;\; y_j \rightarrow Z \Longrightarrow x_i \rightarrow Z \;\; \underline{\text{or}} \;\; x_i \rightarrow -Z$$

The first rule states that if a partition x_i is a subset of Y and a partition y_j (where j may but need not equal i) is a subset of Z, then x_i is a subset of Z. This rule applies when the two middle terms match in polarity, that is, are both affirmative. The second rule states that if a partition x_i is a subset of not Y and a partition y_j (where j may but need not equal i) is a subset of Z, then x_i may be a subset of Z or not Z; one can't tell for sure. This rule applies when the two middle terms do not match in polarity, that is, the first is negative and the second is affirmative. Consider the following example:

(AB-BC)	$animal_1 \rightarrow BIRD$	$bird_1 \rightarrow ROBIN$
	$animal_2 \rightarrow -BIRD$	$bird_2 \rightarrow -ROBIN$

Rule 1 can be applied twice:

$$animal_1 \rightarrow BIRD \;\; \& \;\; bird_1 \rightarrow ROBIN \Longrightarrow animal_1 \rightarrow ROBIN$$

and

$$animal_1 \rightarrow BIRD \;\; \& \;\; bird_2 \rightarrow -ROBIN \Longrightarrow animal_1 \rightarrow -ROBIN$$

These two results are then stored in memory. Rule 2 can also be applied twice:

$$animal_2 \rightarrow -BIRD \;\; \& \;\; bird_1 \rightarrow ROBIN \Longrightarrow animal_2 \rightarrow ROBIN \;\; \underline{\text{or}} \;\; animal_2 \rightarrow -ROBIN$$

and

$$animal_2 \rightarrow -BIRD \;\; \& \;\; bird_2 \rightarrow -ROBIN \Longrightarrow animal_2 \rightarrow ROBIN \;\; \underline{\text{or}} \;\; animal_2 \rightarrow -ROBIN$$

Here both applications of the rule yield the same result, that $animal_2$ is a subset of BIRD or not BIRD. This result is stored only once since redundancies are not stored. From the application of rules 1 and 2, then, the subject knows that $animal_1$ is a subset of BIRD or not BIRD, and that $animal_2$

is also a subset of BIRD or not BIRD. Therefore, the subject stores the following components in memory:

$$
\begin{array}{llll}
(AC_1) & \text{animal}_1 \rightarrow \text{ROBIN} & (AC_2)\ \text{animal}_1 \rightarrow \text{ROBIN} & (AC_3)\ \text{animal}_1 \rightarrow \text{-ROBIN} \\
 & \text{animal}_2 \rightarrow \text{ROBIN} & \quad\quad\ \ \text{animal}_2 \rightarrow \text{-ROBIN} & \quad\quad\ \ \text{animal}_2 \rightarrow \text{-ROBIN}
\end{array}
$$

Note that the component

$$
\begin{array}{l}
\text{animal}_1 \rightarrow \text{-ROBIN} \\
\text{animal}_2 \rightarrow \text{ROBIN}
\end{array}
$$

is not stored because it is redundant with AC_2.

Combination of representations. The combination of two symbolic representations occurs in four steps. The combination process is illustrated below with reference to the following two representations:

$$
\begin{array}{lll}
(AB\text{-}BA) & \overset{A}{\text{animal}_1} \rightarrow \overset{B}{\text{BIRD}} & \overset{B}{\text{bird}_1} \rightarrow \overset{A}{\text{ANIMAL}} \\
 & \text{animal}_2 \rightarrow \text{-BIRD} & \text{bird}_2 \rightarrow \text{ANIMAL} \\
 & & \\
(BC\text{-}CB) & \overset{B}{\text{bird}_1} \rightarrow \overset{C}{\text{ROBIN}} & \overset{C}{\text{robin}_1} \rightarrow \overset{B}{\text{BIRD}} \\
 & \text{bird}_2 \rightarrow \text{-ROBIN} & \text{robin}_2 \rightarrow \text{BIRD}
\end{array}
$$

1. In this first step, the subject constructs transitive chains that yield AC and CA components as results. An important property of the symbolic representations is that exactly two such chains can always be formed. In the present example, the following two chains are formed and integrated:

$$
\begin{array}{lll}
(AB\text{-}BC) & \overset{A}{\text{animal}_1} \rightarrow \overset{B}{\text{BIRD}} & \overset{B}{\text{bird}_1} \rightarrow \overset{C}{\text{ROBIN}} \\
 & \text{animal}_2 \rightarrow \text{-BIRD} & \text{bird}_2 \rightarrow \text{-ROBIN} \\
 & & \\
(CB\text{-}BA) & \overset{C}{\text{robin}_1} \rightarrow \overset{B}{\text{BIRD}} & \overset{B}{\text{bird}_1} \rightarrow \overset{A}{\text{ANIMAL}} \\
 & \text{robin}_2 \rightarrow \text{BIRD} & \text{bird}_2 \rightarrow \text{ANIMAL}
\end{array}
$$

As we have seen, the integration of the AB-BC chain yields three AC components, listed above. In integrating the CB-BA chain, only Rule 1 can be applied; all of these applications yield the result that $robin_1$ is a subset of ANIMAL and that $robin_2$ is also a subset of ANIMAL. Thus, the integration of the CB-BA chain yields a single CA component:

$$
\begin{array}{lccc}
 & C & & A \\
(CA_1) & robin_1 & \rightarrow & ANIMAL \\
 & robin_2 & \rightarrow & ANIMAL
\end{array}
$$

2. In the second step of the combination process, the subject forms transitive chains in which each of the new components yielded by step 1 (AC_1, AC_2, AC_3, CA_1) is the first component in the chain. Exactly one such chain can always be formed for each new component derived in step 1. The chains formed with the AC components will be of the form AC-CB; a CA-AB chain will be formed with the CA component. The results of integrating these chains are checked to see if they are consistent with the components of the representations being combined. For example, the chain

$$
\begin{array}{lcccccc}
 & A & & C & C & & B \\
(AC_1\text{-}CB) & animal_1 & \rightarrow & ROBIN & robin_1 & \rightarrow & BIRD \\
 & animal_1 & \rightarrow & ROBIN & robin_2 & \rightarrow & BIRD
\end{array}
$$

yields the AB component

$$
\begin{array}{ccc}
A & & B \\
animal_1 & \rightarrow & BIRD \\
animal_2 & \rightarrow & BIRD
\end{array}
$$

This is inconsistent with the AB component in the original representation, and so the AC_1 component is rejected as impossible. A component is rejected whenever the chain formed with it does not yield at least one component that matches one of the components in the original representations. In the present example, only AC_1 is rejected in this step.

3. In the next step, transitive chains are formed in which each of the new components yielded by step 1 is the second component in the chain. Again, exactly one such chain can be formed with each new component; the chains formed with the AC components will be of the form BA-AC, and a BC-CA chain will be formed with the CA component. The results of these chains are again checked for consistency with the components in the original representations. In the present example, AC_3 is rejected because the chain

$$\text{(BA-AC)}\qquad \begin{array}{l} \overset{B}{\text{bird}_1} \rightarrow \overset{A}{\text{ANIMAL}} \\ \text{bird}_2 \rightarrow \text{ANIMAL} \end{array} \qquad \begin{array}{l} \overset{A}{\text{animal}_1} \rightarrow \overset{C}{\text{-ROBIN}} \\ \text{animal}_2 \rightarrow \text{-ROBIN} \end{array}$$

yields the BC component

$$\begin{array}{l} \overset{B}{\text{bird}_1} \rightarrow \overset{C}{\text{-ROBIN}} \\ \text{bird}_2 \rightarrow \text{-ROBIN} \end{array}$$

which is inconsistent with the BC component

$$\begin{array}{l} \overset{B}{\text{bird}_1} \rightarrow \overset{C}{\text{ROBIN}} \\ \text{bird}_2 \rightarrow \text{-ROBIN} \end{array}$$

in the original representation.

4. The final step in the combination process considers only those AC and CA components that have been retained in the second and third steps. All possible pairings of these components are formed to yield the final combined representation. In the present example, AC_1 was rejected in step 2 and AC_3 was rejected in step 3, so AC_2 is paired with CA_1 and the resulting representation

$$\text{(AC-CA)} \qquad \begin{array}{l|l} \overset{A}{\text{animal}_1} \rightarrow \overset{C}{\text{ROBIN}} & \overset{C}{\text{robin}_1} \rightarrow \overset{A}{\text{ANIMAL}} \\ \text{animal}_2 \rightarrow \text{-ROBIN} & \text{robin}_2 \rightarrow \text{ANIMAL} \end{array}$$

is stored.

Information-processing Model for Experiments 1, 2, 3, and 5

The first information-processing model derived from the transitive-chain theory will be used to predict performance in Experiments 1, 2, 3, and 5, in which subjects are given two premises and have to respond which of several conclusions is valid. A flow chart for this model is presented in Figure 2. This model will be described with reference to one of the problems in Table 1, in which the premises given to the subject are "No *C* are *B*. All *B* are *A*."

Insert Figure 2 about here

Encoding stage. The subject begins by reading and encoding each premise. The present model assumes that no errors occur during the encoding stage; that is, premises are always interpreted completely and correctly. The repesentations resulting from the encoding of "No *C* are *B*" and "All *B* are *A*" are shown in Figure 3; the representations resulting from the encoding of each type of premise in Experiments 1, 2, 3, and 5 are shown in Table 2.

Insert Figure 3 about here

Combination stage. Errors that arise in the combination stage occur because subjects do not necessarily combine every pair of representations that they encode. (In the present example, there are two pairs to combine, because the single representation for the first premise needs to be combined with two representations for the second premise.) The combination of each pair of representations is performed completely. However, the subject often fails to combine all possible pairs, presumably because of working memory limitations or a limit on the processing capacity that he or she can allocate to the problem. The number of pairs that the subject →

combines is equal to n_p, the number of premises combined (see Figure 2) The value of n_p is a function of the values of four parameters: p_1, p_2, p_3, and p_4. Each of these parameters represents the probability that n_p is equal to a certain value; thus, p_1 is the probability that n_p is equal to one, p_2 is the probability that n_p is equal to two, and so on. It is assumed that the subject always combines at least one pair of representations and never combines more than four pairs. Thus, the four parameters--p_1, p_2, p_3, and p_4--sum to one.

The subject's choice of which pair to combine first is determined by a natural preference for working with simple representations. Symbolic representations may be classified into three types on the basis of their simplicity (see Table 2). Representations with symmetrical components and no negatives have the simplest form (type I); representations with symmetrical components and at least one negative have a less simple form (type II); and representations with asymmetrical components have the most complex form (type III). All pairings of type I with type I and of type I with type II representations are combined first. If more than one such pairing exists, the subject chooses one randomly to combine first, then chooses one randomly from the remaining such pairs to combine next, and so on until he or she has combined all such pairs. All pairings of type II with type II representations are combined next. Again, the order of combination within this set of pairings is randomly determined. Finally, pairings of type III and other representations (any of types I, II, or III) are combined.

In the present example, the subject combines

$$\begin{array}{l|l} c_1 \to -B & b_1 \to -C \\ c_2 \to -B & b_2 \to -C \end{array} \quad \text{and} \quad \begin{array}{l|l} b_1 \to A & a_1 \to B \\ b_2 \to A & a_2 \to B \end{array}$$

first because the former is a type II representation and the latter is type I (see Figure 3). The subject next combines

$$\begin{array}{c|c} c_1 \rightarrow -B & b_1 \rightarrow -C \\ c_2 \rightarrow -B & b_2 \rightarrow -C \end{array} \quad \text{and} \quad \begin{array}{c|c} b_1 \rightarrow A & a_1 \rightarrow B \\ b_2 \rightarrow A & a_2 \rightarrow -B \end{array}$$

because the former is a type II representation and the latter is type III.

Comparison stage. During the comparison stage, the subject chooses a label that is consistent with the combined representations. If no label is consistent with the combined representations, the subject labels the combined representation indeterminate. However, the model must specify how a label is chosen when two labels are consistent with the combined representation. (No more than two labels are ever consistent.) The subject is assumed to have a preference or bias for two kinds of labels: strong labels and labels that match the atmosphere of the premises (as defined by Begg and Denny, 1969). The stronger of two labels is that label which refers to fewer possible relations. The four premise types, ranked in order from strongest to weakest, are E, A, O, and I. Two rules determine the atmosphere of the premises:

1. If the premises contain at least one negative, then the atmosphere of the premises will be negative; otherwise, it will be positive.

2. If the premises contain at least one particular, then the atmosphere of the premises will be particular; otherwise, it will be universal.

If each label satisfies one of these criteria, the subject chooses the label that matches the atmosphere of the premises with probability β_1, and chooses the stronger label with probability $1 - \beta_1$. Thus, the β_1 parameter reflects the subject's bias toward a label that matches the atmosphere of the premises. If one of the two labels is both the stronger and matches the atmosphere of the premises, it is chosen with probability β_2, and the remaining label is chosen with probability $1 - \beta_2$.

There is one additional source of error in the comparison stage. When the composite representation of a single pair of representations contains different initial components, the subject labels that composite representation indeterminate with probability equal to c. In the present example, the results of combining

$$\begin{array}{l|l} c_1 \to -B & b_1 \to -C \\ c_2 \to -B & b_2 \to -C \end{array} \quad \text{and} \quad \begin{array}{l|l} b_1 \to A & a_1 \to B \\ b_2 \to A & a_2 \to -B \end{array}$$

are

$$\begin{array}{l|l} a_1 \to -C & c_1 \to -A \\ a_2 \to -C & c_2 \to -A \end{array}, \quad \begin{array}{l|l} a_1 \to C & c_1 \to A \\ a_2 \to -C & c_2 \to -A \end{array}, \text{ and } \begin{array}{l|l} a_1 \to C & c_1 \to A \\ a_2 \to -C & c_2 \to A \end{array}$$

Since the initial components of the latter three representations are not the same, the subject may become confused as to whether a single label can be found that is consistent with all of these representations. Parameter c represents the probability of the subject mistakenly labeling a composite representation indeterminate in such a case.

Response stage. In the response stage, the subject chooses the response alternative that matches the label he or she has chosen. If an indeterminate label has been assigned to the composite representation, the subject responds that no valid conclusion exists for the pair of premises presented.

Latency parameters. A model of latencies for the problems in Experiment 1 was derived from the response-choice model by assigning parameters to the times taken to encode and combine symbolic representations. The mathematical model of latencies (presented in detail in the Appendix) includes five parameters: ENC_{I-II}, ENC_{III}, $COMB_{I-II}$, $COMB_{III}$, and CHECK. The ENC parameters measure the time taken to encode different types of representations. The COMB parameters measure the time taken to combine various pairs of representations. The roman numeral subscripts refer to the three types of representations described in the response-choice model. Thus, ENC_{I-II} measures the time taken to encode a type I or a type II representation, since only these representations have symmetrical components. Similarly, the subscripts for the COMB parameters refer to the combination of these different types of relations. $COMB_{I-II}$ is the time taken to combine two type I representations, a type I and a type II representation, or two type II representations, and $COMB_{III}$ is the time taken to combine a type III representation with any other representation. The CHECK parameter applies only to invalid syllogisms. Before a subject will label the relationship between _A_ and _C_ indeterminate, he or she is assumed to perform again the combination process that led to such a conclusion.

Experiment 4. A flow chart for this model is presented in Figure 4.

Insert Figure 4 about here

Encoding stage. The subject begins by reading and encoding each premise and the conclusion. As before, the model assumes that the encoding stage is error-free. In the sample problem described previously, ("All _A_ are _B_. _X_ is not a _B_. Therefore, _X_ is not an _A_."), the first premise is represented by .

$$\begin{array}{l|l} a_1 \to B & b_1 \to A \\ a_2 \to B & b_2 \to A \end{array} \quad \text{and} \quad \begin{array}{l|l} a_1 \to B & b_1 \to A \\ a_2 \to B & b_2 \to -A, \end{array}$$

the second premise by

$$x \to -B,$$

and the conclusion is represented by

$$x \to -A.$$

The representation of the corresponding conditional problem is assumed to be identical, as are the operations performed upon that representation.

Combination stage. In the next stage of processing, the subject *sets up a transitive* chain involving the representation of the second premise and the representations of the first premise. In the present example, these chains are of the form XB-BA. The number of representations combined is determined by parameters p_1 and p_2, where these parameters have the same meaning as in the previous model. There are only two such parameters in the present model, because the encoding of the first premise (which is always a universal) includes exactly two representations. The subject's choice of which pair to combine first is again determined by his or her preference for working with simple representations. In this example,

$$(1) \qquad \begin{array}{l|l} a_1 \to B & b_1 \to A \\ a_2 \to B & b_2 \to A \end{array}$$

is preferred over

$$(2) \qquad \begin{array}{l|l} a_1 \to B & b_1 \to A \\ a_2 \to B & b_2 \to -A \end{array}$$

because representation (1) is symmetrical. Thus, the BA component in representation (1) is used to form the first XB-BA chain:

(a) $x \rightarrow -B \quad b_1 \rightarrow A$

$\qquad\qquad\qquad\quad b_2 \rightarrow A$

The relationship between X and A cannot be determined from this chain, as the application of inferential rule (2) will show. When the transitive chain set up between representations of the two premises yields two possible results (in this case, X may or may not be an A), the subject has two choices. He or she can respond that the problem is invalid, or try to form a transitive chain involving the negation of the conclusion (in this example, $x \rightarrow A$) and the other component in the representation of the first premise. If the result of this chain is the negation of the second premise, the subject can respond that the conclusion is valid; otherwise, he or she responds that it is invalid (applying the rule of tollendo tollens).

The probability of forming this second chain (when necessary) depends on how many negatives are in the first premise. Parameter t_0 applies when there are no negatives in the first premise, t_1 when there is one negative, and t_2 when there are two negatives. Since in the present example the first premise contains no negatives, the subject negates the conclusion with probability t_0 and forms the following chain:

(b) $x \rightarrow A \quad a_1 \rightarrow B$

$\qquad\qquad\qquad a_2 \rightarrow B$

In this case the result

$$x \rightarrow B$$

is stored, which is the representation of the negation of the second premise. Now with probability p_2 the subject combines representation (2) with the representation of the second premise:

(c) $x \rightarrow -B \quad b_1 \rightarrow A$

$\qquad\qquad b_2 \rightarrow -A$

Once again, this first chain cannot be integrated, so the subject forms another chain (with probability $\underline{t}_0$) with the negation of the conclusion:

(d) $x \rightarrow A \quad a_1 \rightarrow B$

$\qquad\qquad a_2 \rightarrow B$

As with representation (1), the result of this chain is

$$x \rightarrow B,$$

which is the negation of the second premise. Therefore, the subject can respond that the conclusion offered in this problem is a valid one.

Response stage. If the chains formed with the first and second premise representations all yield components that match the representation of the conclusion, the subject responds that the conclusion is valid. If the chains formed with the representations of the first premise and conclusion all yield components that match the negation of the second premise, the subject also responds that the conclusion is valid. Otherwise, the subject responds that the conclusion offered is invalid. Processing is assumed to terminate when a subject derives a conclusion inconsistent with the given conclusion.

Latency parameters. Since the same number of representations is encoded for each problem, the time taken by the encoding and response stages are assumed to be constant, and all significant variation in solution times is provided by the combination stage. The mathematical model of latencies (described in detail in the Appendix) includes eight parameters: $\underline{p}_{1p}$, $\underline{p}_{1n}$, $\underline{p}_{2p}$, $\underline{p}_{2n}$, $\underline{s}_{1p}$, $\underline{s}_{1n}$, $\underline{s}_{2p}$, and $\underline{s}_{2n}$. These parameters all refer to the time taken to form and integrate a transitive chain.

Parameters with an initial _p_ refer to chains involving the first (or primary) component in the representation of the first premise, and parameters with an initial _s_ refer to chains involving the second component in the representation of the first premise. The subscript _1_ refers to chains involving the second premise, and the subscript _2_ refers to chains involving the negation of the conclusion. The _p_ and _n_ subscripts indicate whether the chain includes no negatives (_p_) or one or more negatives (_n_). Thus, for example, p_{1p} is the time taken to combine a chain involving the first component in the representation of the first premise and the representation of the second premise, when such a chain includes no negatives.

Erickson's Random and Complete Combination Theories

Overview

Erickson (1974) has proposed two theories of syllogistic reasoning: a random combination theory and a complete combination theory. These theories are presented together because they differ in only one stage of the information-processing models derived from the theories. Both theories assume that the representation and combination of premise information is isomorphic to the formation and combination of Euler diagrams. The two theories also assume that no more than one possible set relation is ever encoded for a single premise, and that errors are due to this incomplete encoding of premises and to the subject's choice of labels for the composite representation generated during the combination stage. The theories differ in whether they assume that errors occur during the combination stage. Since these theories have not been extended to conditional relations and non-classical quantifiers, they are tested only on the data of Experiments 1 and 2.

Representational Assumptions

The random and complete combination theories make weak assumptions about the nature of the internal representations used to encode premise information. They assume that whatever representations are used are isomorphic to Euler diagrams, and that the encoding process may be likened to the construction of Euler diagrams. The Euler diagram representations corresponding to each of the five possible set relations are shown in Figure 1.

Combination Process

These two theories do not specify the combination rules used to integrate representations. The theories assume that integration "is done in a manner which can be modeled as the combination of [Euler] diagrams" (p. 309).

Information-processing Models

A flow chart for the two information-processing models derived from the two theories is given in Figure 5. Although Erickson does not present flow charts for his models, the flow charts presented here seem to express correctly the sequence of operations in his models.

Insert Figure 5 about here

Encoding stage. The subject begins solution of a syllogism by encoding each premise. Although as many four Euler diagrams could be used to represent the information contained in a single premise (for example, "Some A are B" can refer to any of the first four representations in Table 2), the models assume that no more than one Euler diagram is ever used to represent the information in a single premise. Thus, only universal negative (E) premises are completely encoded.

The probability of choosing each particular relation to represent each type of premise is given in Table 3. The probabilities in this table are parameters of the mathematical models. It should be noted that this table is not identical to the analogous table presented in Erickson's paper. Although Erickson assumes that parameters $\underline{e}_3$, $\underline{e}_4$, $\underline{e}_7$, and $\underline{e}_9$ have values of zero, these parameters are estimated here to increase the predictive power of Erickson's models. Consider the example problem of Experiment 1 (see Table 1). The first premise, "No C are B," is encoded, with a probability of one, as referring to a disjoint relation between C and B. This is a complete representation of the first premise. However, the second premise is not encoded completely. The premise "All B are A" could refer to either of two possible relations between B and A. The subject chooses only one of these to represent the second premise. With probability $\underline{e}_1$ the subject chooses the equivalence relation, and with probability $\underline{e}_2$ the subset-set relation.

Insert Table 3 about here

Combination stage. The subject's next task is to combine the two representations that result from the encoding stage. Suppose that the disjoint relation is used to represent the first premise, and the subset-set relation is used to represent the second premise. Figure 6 illustrates the three possible combinations of these two representations.

Insert Figure 6 about here

The random combination model assumes that subjects never perform more than one possible combination of premise representations (therefore, the value of $\underline{n}_c$ in Figure 5 is one). When more than one combination is possible, as in the present example, the subject randomly chooses one to perform. In this example, the subject picks one of the three possible combinations that result from his choice of encodings. Since the subject's choice of which combination to perform is random, the probability of performing any particular combination in Figure 6 is 1/3.

The combination stage of the complete combination model, however, is different from its counterpart in the random combination model. In the complete combination model, the subject performs all possible combinations of the two representations he or she has encoded. In the present example, the subject performs all three possible combinations illustrated in Figure 6. In this model, then, the value of $\underline{n}_c$ in Figure 5 is equal to the number of possible combinations of the representations encoded (here, $\underline{n}_c$ is equal to three).

Comparison stage. During the comparison stage, the subject labels the composite representation resulting from the combination(s) he or she has performed. However, two labels may be consistent with this composite representation. In fact, this is always the case for the random combination model, since (a) the composite representation in the random combination model always consists of a single possible combination of the encoded representations, and (b) a single relation between A and C can always be labeled in one of two ways (e.g., a disjoint relation between A and C can be labeled "No A are C" or "Some A are not C"). When more than one label is consistent with the composite representation, the subject must choose one of these labels to describe the representation. The probability of choosing each particular label to describe each type of set relation is given in Table 4 (taken from Erickson, 1974). The probabilities in this table, like those in Table 3, are parameters of the mathematical models.

Insert Table 4 about here

In the random combination model, then, the composite representation consists of one of the three relations in Figure 6. With a probability of one, the label "No A are C" is chosen to describe the disjoint relation between A and C. With probability $\underline{d}_1$, the subset-set relation is labeled "Some A are C," and with probability $\underline{d}_2$, it is labeled "Some A are not C." With a probability of $\underline{d}_3$, the overlap relation is labeled "Some A are C," and with probability $\underline{d}_4$, it is labeled "Some A are not C."

In the complete combination model, the subject performs all three combinations that are possible for the representations encoded. Then, the subject chooses the label "Some A are not C" because it is the only label consistent with all of the relations illustrated in Figure 6.

Response stage. During the response stage, the subject chooses the response that matches the label he has just chosen. Thus, in the random combination model, the subject chooses the response "No A are C" after performing one of the possible combinations illustrated above; after performing either of the other combinations, he or she chooses one of two responses: "Some A are C" or "Some A are not C." In the complete combination model, the subject chooses the response "Some A are not C."

Model III

Erickson proposes a third model that differs from the previous models in its description of the combination stage. In this model, the subject sometimes performs just one combination of the encoded representations and sometimes performs all possible combinations of these representations. The probability of performing all possible combinations varies for different pairs of representations. Since each pair of premises may refer to any of five set relations, there are 25 different pairs of representations. This model also assumes that when the subject performs only one of the possible combinations, the probability of performing any particular combination depends on the two representations that are combined.

This model will not be considered further. There are two reasons for our disregarding it. First, as Erickson says, "Model III is frankly a rather descriptive model" (p. 322) that does not explain why subjects perform different combinations for various pairs of representations. Second, this model adds 18 parameters to those in either the random or complete combination models. Any success that the model might enjoy in predicting subjects' performance is of questionable worth because of the large number of free parameters.

Conversion and Atmosphere Theories

Overview

The conversion and atmosphere theories will be considered only briefly. Both are incompletely specified theories that seem hardly to do justice to the complexity of subjects' performance on syllogistic reasoning tasks.

Representational Assumptions and Combination Process

The atmosphere and conversion theories have nothing to say about the form in which premise information is represented in memory, or about the specific processes used to combine these representations. Therefore, only the information-processing model derived from each theory is described. As with Erickson's theories, the conversion and atmosphere theories are applied only to the data of Experiments 1 and 2.

Information-processing Model for the Conversion Theory

Encoding stage. In the conversion theory, both premises and their converses are assumed to be true. This has no effect on the interpretation of universal negative (E) and particular affirmative (I) premises, since logically "No *A* are *B*" is equivalent to "No *B* are *A*" and "Some *A* are *B*" is equivalent to "Some *B* are *A*." That is, each pair of statements refers to the same set of relations between the two terms. However, for universal affirmative (A) and particular negative (O) premises, the assumption that a premise's converse is true has an effect on the interpretation of the premise. In our example syllogism, "All *B* are *A*" is interpreted as implying that "All *A* are *B*" is also true. Thus, the subject stores only one possible relation (equivalence) between *B* and *A*, although the correct encoding of this premise would include the representations of two possible relations between *B* and *A* (*B* equivalent to *A* and *B* a proper subset of *A*).

Combination stage. The combination process is assumed to be complete and correct on the premises as encoded. Thus, in the present example, after encoding the first premise as a disjoint relation, and the second premise as an equivalence relation, the subject generates a combined representation in which A is disjoint with respect to C.

Comparison stage. During the comparison stage, the composite representation generated during the combination stage is labeled. If more than one label is consistent with the composite representation (as in this case), the subject chooses the label that matches the atmosphere of the premises. It should be noted that this rule is not intrinsic to the conversion hypothesis as formulated by Chapman and Chapman (1959), and it is possible to use other rules without changing the basic ideas of Chapman and Chapman's conversion hypothesis. The atmosphere rule is used here as a reasonable (and perhaps the most likely) way of choosing between labels equally consistent with the composite representation. In the present example, the subject labels the disjoint relation between A and C "No A are C."

Response stage. In the response stage, the subject chooses the response alternative that matches the label chosen during the comparison stage (in this case, "No A are C") with probability x, and each of the four remaining responses with probability $1/4(1 - x)$.

Information-processing Model for the Atmosphere Theory

Encoding stage. According to the atmosphere theory, subjects solving syllogistic reasoning problems do not encode relations but rather global properties of the premises. More specifically, the subject encodes the quantification (either universal or particular) and the polarity (either positive or negative) of each premise.

Combination stage. In keeping with the encoding stage, the subject, according to this theory, combines not relations but global properties to determine the atmosphere of the premises. Two combination rules are used.

If both premises are encoded as universal, then the atmosphere is universal; otherwise, it is particular. Second, if both premises are encoded as positive, then the atmosphere is positive; otherwise, it is negative. In the present example, both premises are universals, so the atmosphere is universal, and since one of the premises is negative, the atmosphere is negative.

Comparison stage. In the comparison stage, the subject chooses a label that matches the atmosphere of the premises as determined during the combination stage. In our example, the subject chooses the label "No A are C."

Response stage. In this last stage, the subject chooses the response alternative that matches the label just chosen with probability x, and each of the four remaining responses with probability $1/4(1 - x)$.

Comparison Among the Models

Table 5 summarizes the determinants of response choice in syllogistic reasoning according to each of the five information-processing models just described.

Insert Table 5 about here

Encoding Stage

The transitive-chain model makes the strong assumption that premises are always encoded completely and correctly. The random and complete combination models make the equally strong assumption that no more than one representation is ever used in the encoding of a single premise. The conversion model assumes that only universal negative and particular affirmative premises are encoded correctly. The atmosphere model assumes that subjects ignore the relations expressed by the premises, and encode only global properties of the premises. Thus, the encoding stage is a major source of error in all of the models except the transitive-chain model, where it is error-free. The assumption of error-free encoding by the transitive-chain model is not intuitively compelling; however, a recent study by Sternberg and Turner (Note 2) has provided impressive evidence in favor of this assumption. These researchers used a truth-table analysis similar to that used by Taplin and Staudenmayer (1973) and Staudenmayer (1975) with conditional statements to examine subjects' representations of a variety of premises. Sternberg and Turner found that the transitive-chain theory provided the best account of premise encodings as well as of the combination of those encodings.

<u>Combination Stage</u>

The transitive-chain model assumes that each pair of representations is completely combined, but that not all pairs are combined. The random combination model assumes that each pair of representations is incompletely combined, unless there is only one way of combining the representations. The complete combination, conversion, and atmosphere models assume that combination is complete and correct for the limited information that has been encoded.

<u>Comparison Stage</u>

All of the models except the atmosphere model agree that some variation in performance is due to the frequent necessity of choosing between two labels that are equally consistent with the composite representation derived during the combination stage. The transitive-chain model determines the appropriate label through two rules that reflect intuitive biases that subjects have. The conversion model uses only one of these biases (for a label matching the atmosphere of the premises) to choose between two labels. The random and complete combination models provide a set of parameters that simply describe the probability of choosing one label or another for each situation in which such a choice is necessary.

Finally, all of the models except the transitive-chain model predict that subjects should always indicate that a valid conclusion exists for premises that in fact have a valid conclusion. The transitive-chain model claims that subjects sometimes mistakenly respond that syllogisms with a valid conclusion are indeterminate.

The numbers of parameters estimated differed widely across models, an inevitable consequence of the different information-processing assumptions the models make. Thus, the transitive-chain model involved estimation of seven free parameters, the complete and random combination models involved estimation of thirteen free parameters apiece, and the atmosphere and conversion models each involved estimation of one free parameter. We were not particularly concerned with the differing numbers of parameters, however, for three reasons. First, our major concern was with comparing the historically important models in a way that did full justice to the initial conceptualizations, and these conceptualizations differ widely in their complexity and completeness. Second, we always predicted large numbers of data points (at least 100) in comparing models, thus minimizing the opportunity for capitalization upon chance variation in the data. Third, the fits of the models showed little correspondence to numbers of parameters in the models, suggesting that number of parameters was not an important determinant of fit.

METHOD

Five experiments were conducted to test the theories outlined above. The experiments had four major purposes:

1. To distinguish among the five theories described in the preceding section by measuring the performance of the models derived from the theories in accounting for response choices of subjects in syllogistic reasoning tasks.

2. To test the generality of the preferred theory across sessions, content types, quantifiers, and types of logical relations.

3. To determine how various processes in the preferred model(s) are related to individual differences in verbal, spatial, and abstract reasoning abilities.

4. To test models of solution times derived from the transitive-chain theory.

Subjects

Subjects in Experiment 1 were 49 Yale undergraduates; subjects in Experiments 2 and 5 were 50 Yale undergraduates. Subjects in Experiments 3 and 4 were 50 adults recruited from the New Haven area. The subjects in Experiments 2 and 5, and 25 of the subjects in Experiment 1, were selected from the introductory psychology subject pool; all remaining subjects were recruited by posted advertisements. Subjects were paid at the rate of $2.25 an hour ($2.00 in Experiment 1) or received credit in an introductory psychology course. No subject participated in more than one experiment, and none of the subjects had training in formal logic.

Materials

Stimuli

Experiment 1. The basic experimental stimuli were 38 categorical syllogism problems. Each problem consisted of two premises, followed by the same five conclusions: All A are C, No A are C, Some A are C, Some A are not C, and None of the above (see Table 1). These 38 problems included all 19 sets of premises for which a valid conclusion exists, and 19 randomly selected sets of premises for which no valid conclusion exists. Capital letters stood for sets of items. Two different sets of letters were used: A, B, C and S, M, P.

Experiment 2. The stimuli in Experiment 2 were a representative subset of 20 of the 38 problems used in Experiment 1 (10 valid and 10 invalid). However, concrete terms were used in all problems in Experiment 2. Three different types of content were used: factual, counterfactual, and anomalous. Each of the 20 types of problems was presented once with each content type; thus, there were 20 x 3, or 60 different problems.

Experiment 3. The stimuli were 65 categorical syllogism problems. These problems were constructed from the problems in Experiment 1 by substituting the quantifiers most and few for each occurrence of the quantifier some. Each problem consisted of two premises, followed by the same seven conclusions: All A are C, No A are C, Most A are C, Few A are C, Most A are not C, Few A are not C, and None of the above.

Experiment 4. The stimuli were the 64 problems shown in Appendix A. Each problem consisted of two premises and a conclusion drawn from the premises. Half of the problems dealt with conditional relations, and half dealt with categorical relations. Each conditional problem was paired with an isomorphic categorical problem.

Experiment 5. The stimuli were the same as in Experiment 1, except that conditional relations were substituted for the categorical relations in Experiment 1. For example, "If A occurs then B occurs" was substituted for "All A are B," "If A occurs then B does not occur" for "No A are B," "If A occurs then B sometimes occurs" for "Some A are B," and "If A occurs then B sometimes does not occur" for "Some A are not B."

Ability Tests

In Experiments 2, 3, 4, and 5, subjects received the Verbal Reasoning subtest, the Space Relations subtest, and the Abstract Reasoning subtest of Form S of the Differential Aptitude Test.

Apparatus

In Experiment 1, problems were presented at an Ontel computer terminal controlled by an IBM/370-158 computer at the Yale Computer Center. In Experiment 4, the problems were presented via a Gerbrands two-field tachistoscope that also measured the time taken to respond to each problem. In Experiments 2, 3, and 5, subjects were given printed booklets that contained two problems per page.

Procedure

Experiment 1

Subjects were tested on two separate days in experimental sessions of approximately 45 minutes each. In each session, subjects were asked to solve the same 38 problem types. The problems were shown on each day in a different random order that was unique for each subject. Different letters appeared as premise terms in each of the two sessions. The sessions in which the different sets of letters appeared were counterbalanced across subjects. Since the use of different sets of letters produced no difference in subjects' performance, the data have been pooled across letter sets.

Each subject was told that he or she would be given two statements on each trial of the experiment, and that his or her task was to select the response alternative that logically and necessarily followed from these two statements. The meaning of the quantifier <u>some</u> as used in classical logic was also explained.

At the beginning of each session, two different problems were selected at random from the complete set of 38 to serve as practice trials (with the constraint that one problem had a valid conclusion and one did not). These practice trials used a letter set different from that used in the session following the practice trials. At the end of the practice trials, the experimenter discussed the subject's performance with him or her to make sure that he or she understood the task. The experimental trials were then presented in random order in two blocks of 19 trials each. The subject was able to take as long as he or she wished after responding on a trial before signalling the computer to begin the next trial. The subject was given a five-minute break at the end of the first block of 19 trials. Reaction times were recorded for each trial, but the subject was not told that he or she was being timed. Subjects were told that they should respond to each problem as soon as they were sure of the correct answer. Subjects were given feedback concerning their performance only at the end of the experiment.

<u>Experiment 2</u>

Subjects were tested on two separate days. The first experimental session lasted about one hour, the second about 45 minutes. In the first session, subjects were asked to solve 60 syllogism problems. The problems were shown to each subject in a different random order, and the problems were not blocked according to content type. In the second session, all subjects received the same three reasoning tests in the same order: a verbal test, a spatial test, and an abstract reasoning test.

In both sessions, subjects were tested in groups of approximately six. At the beginning of the first session, subjects were given the same instructions given subjects in Experiment 1. As in Experiment 1, two problems were randomly selected from the set of 60 problems to serve as practice trials; however, abstract terms were used in the practice problems. At the end of the practice trials, the experimenter discussed the subjects' responses with them to make sure that they understood the task. The test booklets were then given to the subjects, who were instructed to take as long as they wished to complete each problem. Subjects were given a five-minute break at the end of 30 problems.

In the second session, the standard instructions for the Differential Aptitude Test were read aloud to subjects. Subjects were given 10 minutes to complete the verbal test, 12 minutes to complete the spatial test, and 11 minutes to complete the abstract reasoning test. (These time limits are shorter than those normally allowed.) Subjects were given a two-minute break after each test. Subjects were given feedback concerning their performance on the syllogisms only at the end of the experiment.

<u>Experiment 3</u>

The procedure was the same as in Experiment 2, with the following exceptions:

1. The booklets contained 65, rather than 60 problems.

2. The instructions contained no reference to a special interpretation of any of the quantifiers in the problems.

3. Subjects were given a five-minute break at the end of 33 trials.

Experiment 4

Subjects were tested on two separate days in experimental sessions of about 45 minutes each. In the first session, the 64 problems were presented tachistoscopically in four blocks of 16 trials each. The problems in each block were all of the same type (that is, either conditional or categorical). The problems were randomly assigned to each block; this assignment was performed separately for each subject. The order of the blocks was counterbalanced across subjects with the constraint that no successive blocks contained problems of the same type. A centisecond clock was started at the beginning of each presentation of a problem, and was stopped when the subject pressed either of two response keys. The solution time in centiseconds was then recorded by the experimenter. Subjects were tested individually in the first session. Each subject was told at the beginning of the first session that he or she would be given two statements and a conclusion on each trial, and that his or her task was to indicate whether the conclusion drawn from the premises was logically valid. Two problems were randomly selected from the set of 64 problems to serve as practice trials: one with a valid conclusion and one with an invalid conclusion. At the end of the practice trials, the experimenter discussed the subject's responses with him or her to make sure that he or she understood the task.

The procedure in the second session was the same as in Experiment 2.

Experiment 5

The procedure was the same as in Experiment 2, with the following exceptions:

1. The booklets contained 38, rather than 60 problems.

2. The instructions included examples of conditional, and not categorical statements. Subjects were told that sometimes should be interpreted as "sometimes, and possibly always."

3. Subjects were given a five-minute break at the end of 19 problems.

Design

Dependent Variables

Response choices to each problem were a dependent variable in all five experiments. Solution times to each problem were a dependent variable in Experiments 1 and 4. Scores on the three standardized mental ability tests were a dependent variable in Experiments 2-5.

Independent Variables

Subjects and syllogisms were independent variables in all five experiments, and were completely crossed in all experiments. Additional independent variables were session in Experiment 1, content type in Experiment 2, and logical relation (categorical or conditional) in Experiment 4; all of these variables were completely crossed with subjects and syllogisms.

RESULTS AND DISCUSSION

Overview

The results of the experiments will be presented in four major parts. First, we will present basic statistics for the response-choice and latency data. Second, we will describe the outcomes of mathematically modeling the response-choice and latency data. Third, we will discuss the parameter estimates for the preferred models. Fourth, we will examine individual differences in syllogistic reasoning.

Basic Statistics

Means and Standard Deviations

Table 6 presents basic statistics for the data sets that were used for mathematical modeling. Analyses of variance were conducted to test the significance of the difference in mean correct responses given to categorical and conditional problems in Experiment 4, and the difference in mean correct responses given to problems with different types of content in Experiments 1 and 2. Neither logical relation nor content type significantly affected mean correct responses (for Experiment 4, $F(1,126) = 1.08$, $p>.05$; for Experiments 1-2, $F(3,396) = 1.86$, $p>.05$).

Insert Table 6 about here

Reliabilities

The intercorrelations between the response-choice data sets are presented in Table 7. The diagonal elements in this table are odd-even reliabilities (calculated using the Spearman-Brown formula). Response choices in the two sessions of Experiment 1 correlated .974, and so the data from the first and second sessions have been combined.

Insert Table 7 about here

Response choices for problems with counterfactual and anomalous content correlated very highly with each other and with response choices for problems with abstract content. Response choices for problems with factual content correlated lowest with response choices for problems with all other content types. This pattern suggests that subjects' performance in Experiment 2 was affected in some manner by factual content. Evidence concerning the nature of this effect is presented in later sections. All of the correlations between different content types are high, however, suggesting that the data are reliable and may possibly be accounted for by a single model. Similarly, the high correlation between categorical and conditional problems in Experiment 4 suggests that a single model may account for performance with these two kinds of logical relations.

The odd-even reliabilities for the latency data in Experiments 1, 4a, and 4b were .94, .95, and .94, respectively. The correlation between latencies in the two sessions of Experiment 1 was .91, and the correlation between latencies for conditional and categorical problems in Experiment 4 was .93.

Mathematical Modeling of Response-choice Data

Parameter Estimation

Parameter estimation was done by nonlinear regression, using the BMD P3R computer program. This program obtains a least-squares fit to a general nonlinear function of several variables by means of Gauss-Newton iterations.

Model Fits

Table 8 presents model fits for the response-choice data sets in Experiments 1-5. These model fits are presented in terms of R^2 and RMSD. The former measure indicates the proportion of variance in the data accounted for by each model; the latter measure indicates the root-mean-square deviation of the observed from the predicted data points. Other means of assessing fit will be considered as well. The values for the "ideal model" were derived by predicting that all subjects would choose the logically correct answer to each problem. When more than one answer to a problem was logically correct, the ideal model predicted that an equal number of subjects would choose each correct answer.[2]

Insert Table 8 about here

The results of the experiments, considered either singly or as a whole, are unequivocal: The transitive-chain theory gives a better account of the response-choice data than does any competing theory. The fits of the alternative models are not even close to that of the transitive-chain theory on either measure of fit. Viewed in absolute terms, the transitive-chain theory also did very well. The model fits are uniformly high; R^2 was greater than .9 for almost every data set. (Predicted and observed values for the response-choice and latency data in Experiments 1 and 4 are given in Table A of the Appendix.)

Significance of Unexplained Variance

It is of some interest to determine whether the unaccounted for variance in the data is statistically significant. This was done by testing the statistical significance of correlations between pairs of residuals of observed from predicted values. These comparisons were made for each of the data sets. The correlations for the transitive-chain theory are presented in Table 9.

Insert Table 9 about here

Within-data-set comparisons were computed by modeling response choices separately for odd and even numbered subjects, calculating residuals of observed from predicted values for each set of subjects, correlating the residuals, and correcting the correlations by the Spearman-Brown formula. One-tailed significance tests were used for the correlations in order to maximize the probability of rejecting the models. The models were all rejected at the .05 level or better for all data sets.

Between-data-set comparisons were computed by modeling response choices separately for each data set, calculating residuals of observed from predicted values for each data set, and then correlating the residuals. The models were again rejected at the .05 level or better for all such comparisons made. Thus, although the high values of R^2 show that the models derived from the transitive-chain theory are close approximations to the true models of subjects' performance, we can conclude that these are not the true models of subjects' performance.

Discussion

Experiments 1 and 2. There are two major causes of the predictive failure of the complete and random combination models. First, these models poorly predict the proportion of subjects who correctly identify problems with no valid conclusion. The random combination model underestimates this proportion, and the complete combination model overestimates it. Second, neither model predicts the sizable proportion of subjects who incorrectly respond that certain problems have no valid conclusion. The data of Zax, which are presented in Erickson's (1974) article, and on which Erickson's models achieve their greatest success, mask these difficulties in Erickson's models. Zax included only valid syllogisms in his study, and did not offer subjects the option of responding that no valid conclusion existed for these problems. When fit to these data, the transitive-chain theory accounted for as great a percentage of the variance in the data (98) as Erickson's models.

Although clearly superior to the other models tested, the transitive-chain model could be rejected relative to the true model. We were unable to find in the pattern of residuals any clear clue to the source of the unexplained variance. One assumption of the theory that seemed suspect was that of error-free encoding of the premises in the encoding stage. Although the data from the previously noted Sternberg and Turner study (Note 2) and the good fits of the model suggested that subjects encoded the premises nearly completely, it still seemed possible that they sometimes made mistakes or encoded premises incompletely. To test this idea, we formulated a version of the model in which subjects were not assumed to encode premises completely. The number of representations encoded for any premise was determined by four parameters--$\underline{e}_1$, $\underline{e}_2$, $\underline{e}_3$, $\underline{e}_4$--where $\underline{e}_1$ is the probability of encoding exactly one representation per premise, $\underline{e}_2$ the probability of encoding exactly two representations, and so on. The subject was assumed to have the same preferences for encoding simpler representations, as described previously. This new model increased the value of R^2 by about .02 for each of the data sets and eliminated some of the significant correlations between pairs of residuals. However, the model was discarded because it seemed to buy little in exchange for an increase of more than 40% in the number of parameters.

<u>Experiment 3</u>. The results of Experiment 3 were similar to those of Experiment 1. The transitive-chain model accounted for a large proportion of the variance in the data, and yet there remained systematic unexplained variance in the data. One possible reason for the rejection of the model is that a more accurate representation of the quantifiers <u>most</u> and <u>few</u> requires more than three partitions of a set. Moreover, the representation of each of these quantifiers may require components in which the exact number of partitions mapped onto a set varies. In other words, "Most <u>A</u> are <u>B</u>" means that more than 50% and less than 100% of the members of <u>A</u> are also members of <u>B</u>, but does not specify a single value within this range. Given the "roughness" of the symbolic representations, the transitive-chain model does a remarkably good job of predicting subjects' performance.

<u>Experiment 4</u>. The high correlations between categorical and conditional problems on both dependent measures support the claim that the two kinds of relations are represented and combined in isomorphic ways. In addition, these correlations lead one to expect that the same model can account for both data sets. A look at the model fits for the response-choice data confirms this expectation.

<u>Experiment 5</u>. The transitive-chain model performed less well in predicting performance in this experiment than in the preceding experiments. We believe that the model didn't do as well on these problems because conditional relations referring to the temporal order of events are more complex than categorical relations. An example may help to clarify the difference. It is possible for the following two conditional statements both to be true: If <u>A</u> occurs then <u>B</u> occurs and If <u>B</u> occurs then <u>A</u> does

not occur. However, the analogous categorical statements, All A are B and No B are A, cannot both be true. Thus, a categorical relation between two objects has only two possible values (a member of one set does or does not correspond to a member of another set). A temporal relation between two events, however, can have three possible values, because one event may precede another, succeed another, or occur at the same time as another event.

At this point, we may ask why the transitive-chain theory was able to predict performance so well on the simpler conditional problems in Experiment 4. There are two possible reasons for this improved performance. The first reason is that subjects did not interpret the conditional relations in Experiment 3 as referring to a temporal ordering of events, but instead to the truth or falsity of certain states of the world. (Indeed, this might be expected to be the case, since the temporal cue words occurs and sometimes were not used in Experiment 3.) Given this interpretation, the categorical and conditional problems in Experiment 3 are indeed equivalent, as was pointed out above. Another possibility is that subjects interpreted the conditional relations in Experiment 3 as referring to temporal orderings of events, but that these problems were simple enough so that the process of determining the validity of the conditional problems was isomorphic to that used for determining the validity of the simpler categorical problems.

Conclusion

On the basis of the model fitting described in the above sections, the transitive-chain models were adopted as the preferred models for all types of premise content, quantifiers, and logical relations. Predicted versus observed proportions for the problems in Experiments 1 and 4 are shown in Appendix A. It can be seen that the observed values show good agreement with the predicted ones.

Mathematical Modeling of Latency Data

Parameter Estimation

Parameter estimation was done by linear regression, using the SPSS REGRESSION program.

Model Fits

Table 10 presents model fits for the latency data in Experiments 1 and 4. These model fits are presented in terms of R^2 and RMSD. Other means of assessing fit will be considered as well. In each case, the fit of the latency model is good, considering the reliability of the data. As one would expect from the correlations presented above, the transitive-chain model in Experiment 4 achieves comparable levels of fit for categorical and conditional problems.

Insert Table 10 about here

Significance of Unexplained Variance

The statistical significance of correlations between pairs of residuals was tested for the latency data, as it was for the response-choice data. The within-data-set comparisons are shown in Table 11. In every case, the correlation between residuals for odd- and even-numbered subjects was significant. Between-data-set comparisons were made between latencies for sessions 1 and 2 in Experiment 1, and between latencies for categorical and conditional problems in Experiment 4. In each case, correlations between residuals were significant at the .01 level. We can therefore conclude that these latency models are not the true models of subjects' performance, although, as with the response-choice models, they seem to provide good approximations to the true models.

Insert Table 11 about here

Discussion

Although the latency model in Experiment 1 provided a good account of the latency data in that experiment, there remained systematic variance in the data that was unaccounted for by the model. A look at the residuals suggests that a figure effect may be the cause of the model's rejection. In general, the model overestimates the time taken to solve problems in Figures I (Quantifier B are C, Quantifier A are B) and IV (Quantifier C are B, Quantifier B are A). Thus, subjects are faster than predicted on problems in which the terms form a forward or backward chain (as in the first and second examples above, respectively). One way in which this effect might be incorporated into the model is to assume that subjects prefer to combine the first two components of two representations, and then the last two components, rather than having to combine the first component of one representation with the last component of another. Thus, forming an AB-BC chain is easier with AB-BA and BC-CB representations than with AB-BA and CB-BC representations, since in the former case the two components needed are the first components in each representation.

In Experiment 4, the latency model replicated the success of the response-choice model in achieving comparable levels of fit for categorical and conditional problems. However, the response-choice model performs better than the latency model on the R^2 criterion. The major reason for this difference seems to be the difference in the reliabilities of the two measures.

PARAMETER ESTIMATES OF THE PREFERRED MODELS

Response-choice Models

Table 12 shows values of the response-choice parameters for the transitive-chain models as estimated for each data set. These estimates are all independent.

Insert Table 12 about here

The estimates of p_2, p_3, and p_4 were unreliable and thus not easily interpretable. The independent variables from which these parameters were estimated were highly correlated (.70 - .95), and this meant that the predictions of the model were not greatly affected by the specific values of these parameters. Therefore, these parameters are summed together in Table 12. The comparison of interest for the p parameters, therefore, is the value of p_1 versus the sum of p_2, p_3, and p_4. This comparison represents the probability of performing only one combination versus the probability of performing more than one combination.

The value of p_1 fluctuates within a fairly narrow range over wide variations in task content and format. It seems that much of the time, subjects are unwilling (or unable) to combine more than one pair of representations in solving these problems. There is some variation, however. The probability of performing more than one combination is greatest for problems with factual content. This result is a sensible one, suggesting that subjects store and manipulate factual information with greater ease than they store and manipulate other kinds of information. It is somewhat surprising that the value of $\underline{p}_1$ for Experiment 4 is similar to that of other experiments. One might expect that a chain in which one of the components includes a single element (for example, $X \rightarrow B$) would be easier to form and combine than a chain in which both components include two elements. There are two factors that might offset the relative simplicity of the chains formed in Experiment 4. First, the chains formed in this experiment usually contain more negatives than the chains in other experiments. Second, in Experiment 4, the subject sometimes must form a second chain involving the negation of the conclusion.

The values of the β parameters are relatively stable across experiments, as might be expected. β_1 *is much greater than .5, indicating that* subjects prefer a label that matches the atmosphere of the premises to one that is a stronger label. As would be predicted, β_2 is usually close to one, and is higher than β_1; when one label both matches the atmosphere of the premises and is the stronger label, it is almost always chosen.

On the basis of the $\underline{p}$ parameters, we would expect performance on the counterfactual and anomalous problems in Experiment 2 to be better than performance on the abstract problems in Experiment 1; other things being equal, the combination of more pairs of representations should lead to better performance, and this is not so: The $\underline{c}$ parameter provides a clue as to why. The $\underline{c}$ parameter, which represents the probability of incorrectly responding that a problem has no valid conclusion, is highest for counterfactual and anomalous problems. Since $\underline{c}$ in a sense represents excessive caution by the subject in evaluating the validity of problems, we might expect this parameter to be greater when the subject is dealing with counterfactual and anomalous statements. Thus, the advantage of concrete terms over abstract terms in counterfactual and anomalous problems is offset by the greater tendency in these problems to respond that no valid conclusion exists. Presumably, this tendency is influenced by the knowledge that subjects have stored about the concrete terms used in these problems: Their prior knowledge conflicts with the result of their logical operations, leading to uncertainty. The most extreme value of $\underline{c}$ is in Experiment 5. This may also reflect a greater caution used by subjects in working with the conditional relations in this experiment. It is reasonable to assume that the amount of caution used in dealing with a particular type of relation varies directly with the complexity of that relation. Thus, the increase in $\underline{c}$ may be explained by the increased complexity of the conditional relations in Experiment 5, which has been described elsewhere.

Finally, consider the pattern of the $\underline{t}$ parameters in Experiment 4. It seems that when there are two negatives in the first premise, the subject's processing capacity is used up in interpreting and encoding these negatives, leaving the subject without the additional capacity to form a second transitive chain using the conclusion.

Latency Models

Table 13 shows values of the latency parameters as estimated for the data in Experiments 1 and 4. Standard errors for the parameters are given in parentheses, and the starred values are significant at the given value. (CON is a constant estimated for each data set, and so has no significance value attached to it.)

Insert Table 13 about here

The parameter estimates for Experiment 1 are very reasonable in the context of the model. The simplicity of a symbolic representation affects both the time taken for its encoding and the time needed to combine it with another representation. The value of the CHECK parameter (approximately equal to the combination parameters) suggests that when the combination of two representations indicates an indeterminate relationship between A and C, the two representations are combined again as a check on the process. Finally, as might be expected, subjects are faster on both encoding and combination in the second session.

The pattern of parameter estimates for Experiment 4 is also very reasonable in the context of the latency model. The combination process is faster when the representation of the second premise rather than the conclusion is used in the chain, is faster when the initial component of the first premise is used, and is faster when no negatives are involved.

INDIVIDUAL DIFFERENCES IN SYLLOGISTIC REASONING

Correlations Between Ability Scores and Performance

In Experiments 2-5, subjects were given the Verbal Reasoning, Abstract Reasoning, and Space Relations subtests of the DAT. The ability test scores from each experiment were then subjected to a principal components analysis. Components with eigenvalues greater than or equal to one were retained, and were rotated to a VARIMAX solution. In each experiment, the above analysis yielded two ability factors, a verbal ability factor and a spatial-abstract ability factor. Table 14 shows correlations between the ability factor scores and the mean number of correct responses given to problems in each experiment. For all types of problems, the spatial-abstract factor, but not the verbal ability factor, correlated significantly with performance.

Insert Table 14 about here

Parameter Estimates for Individual Ability Groups

Subjects in each of the last four experiments were classified into four ability groups on the basis of their uncorrelated scores on the verbal and spatial-abstract ability factors: high verbal--high spatial-abstract, low verbal--high spatial-abstract, high verbal--low spatial-abstract, and low verbal--low spatial-abstract. Median cutoffs were used to assign each subject to one of these four groups. We then performed jackknife statistical procedures on these parameter estimates (see Mosteller and Tukey, 1969) in order to provide a best single estimate of the population value and standard error of each parameter. The procedures do not make any assumptions about the sampling distribution of the parameters, which are unknown. The best estimates for the parameters for each ability group in each experiment are given in Table 15 (standard errors for the parameters are given in parentheses).

Insert Table 15 about here

The pattern of correlations between the ability factor scores and the mean number of correct responses showed that spatial-abstract ability is significantly correlated with performance, whereas verbal ability is not. The patterns of values in Table 15 indicate why this is so. Differences in verbal ability did not result in significant differences in the values of any parameters in any experiments. However, spatial-abstract ability has a significant effect on the value of p_1 in all four experiments ($\alpha = .05$), as well as a significant effect on the t parameters in Experiment 4 ($\alpha = .05$). Both the p and t parameters are concerned with the number of transitive chains formed by subjects, and in both cases, spatial-abstract ability affects these parameters in a very reasonable way. Lower values of p_1 (indicating the combination of more pairs of representations) are associated with high spatial-abstract ability, as are higher values of the t parameters (indicating an increased probability of forming a second chain in these problems, when necessary). This relationship between spatial-abstract ability and the ability to form and integrate transitive chains provides evidence that the representations used in solving syllogisms may be spatial or abstract in nature, rather than verbal.

Summary

Experiments 1 and 2

The transitive-chain model provided a more accurate and comprehensive account of the data than did any of the competing models that have been proposed. The transitive-chain model was able to account for almost all of the systematic variance in four sets of data including problems with various types of abstract and concrete content. In addition, the model provided some understanding of the influence of different types of premise content and of the importance of different types of mental abilities on subjects' performance in syllogistic reasoning tasks. Finally, the model of latencies derived from the transitive-chain information-processing model provided a good account of the latency data collected in Experiment 1.

Experiment 3

The results of Experiment 3 showed that the processes of the transitive-chain model (and in particular, the biases assumed to operate in the combination and labelling of representations) are not specific to problems that include the quantifiers of classical logic. The pattern of individual differences in this experiment provided a replication of the results in Experiment 2. In doing so it strengthened the conclusions made in that experiment regarding the source of individual differences in syllogistic reasoning.

Experiments 4 and 5

The results of these experiments showed that a single theory could provide a good account of both response choices and latencies for problems involving categorical and conditional relations. The good fits obtained demonstrate the generality of the transitive-chain theory to conditional as well as categorical relations.

Finally, the analysis of individual differences in the present experiments replicated those of earlier experiments. Together, these results suggest that the most reliable source of individual differences in syllogistic reasoning is in the number of pairs of representations combined during the combination stage. Furthermore, the number of pairs combined was significantly correlated with spatial-abstract reasoning ability, suggesting that the representations combined may be of a spatial-abstract nature.

CONCLUSIONS

The transitive-chain theory successfully predicted subjects' performance on a wide variety of syllogistic reasoning problems. As the preferred theory of syllogistic reasoning, it provides some preliminary answers to the theoretical questions raised in the introduction. The conclusions of the present study will be presented in terms of these theoretical questions and the tentative answers provided by the theory.

Representation of Premise Information

The transitive-chain theory assumes that the categorical information contained in a premise is represented by one or more symbolic representations. Each representation corresponds to a possible relationship between two sets, and includes two distinct pieces of information, called components. These components may be combined with each other in various ways to yield different relationships between two sets.

The patterns of individual differences in four separate experiments provided some evidence for the use of a spatial-abstract representation (although not necessarily this one) in syllogistic reasoning, since spatial-abstract reasoning ability correlated significantly with the mean number of correct responses in these experiments. Specifically, the ability to combine pairs of representations varied significantly with spatial-abstract reasoning ability, but not with verbal reasoning ability.

Combination of Premise Information

The structure of the symbolic representations in the theory makes it possible for the first time to specify completely a performance algorithm for combining premise information. This algorithm includes two important processes. The first process is the formation of transitive chains; this process involves the rearrangement of components in the original representations. The second process is the application of two simple inferential rules to the transitive chains thus formed. This combination process is consistent with the assumptions of Erickson's theory, since its results are isomorphic to those obtained by combining Euler diagrams. But the precise specification of the rules of combination makes it possible to identify potential sources of error in the combination process.

Sources of Difficulty in Syllogistic Reasoning

The present research identified three major sources of difficulty in the solution of syllogistic reasoning problems. The most important of these is the processing capacity required to combine two symbolic representations. The high level of p_1 for almost all of the data sets attests to subjects' difficulty in combining representations. In almost every case, subjects were as likely to combine just one pair of representations as they were to combine more than one pair. The probability of combining more than one pair of representations seemed to be affected by two problem variables: the content of the premises, and the total number of pairs of representations to be combined.

Another source of error is subjects' preferences for working with simpler representations (that is, symmetrical representations and representations with no negatives). Since the values of the <u>p</u> parameters indicate that few pairs of representations are combined, we conclude that pairs of complex representations are rarely combined. As a result, there are many errors in problems where the results of combining complex representations are different from the results of combining simple representations.

A third source of error is found in biases subjects have in how they label the composite representation generated during the combination process. Three specific biases are identified by the theory. The first of these is a bias for strong labels, the second a bias for labels that match the atmosphere of the premises, and the third a tendency to label a composite representation indeterminate if it contains nonidentical initial components.

<u>Generality of the Processes Used in Syllogistic Reasoning</u>

<u>Relationships Among Various Kinds of Syllogism Content</u>

The present study found, as did Osherson (1974), that a single model could account for problems with various types of abstract and concrete content. However, we also found, as did Wilkins (1928), a substantial difference in performance between problems with concrete, factual content and problems with abstract, anomalous, or counterfactual content. This difference was due to a higher probability of combining more than one pair of representations when dealing with factual premises.

Relationships Among Various Kinds of Syllogisms

The results of Experiments 4 and 5 showed that a single theory can account for both categorical and conditional syllogisms, as hypothesized by Revlis (1975). The symbolic representations in the transitive-chain theory are capable of representing both kinds of information, and the same combination process can be applied to each set of representations. Moreover, the sources of difficulty in categorical and conditional syllogistic reasoning are highly similar. Finally, the same patterns of individual differences were found for both types of syllogisms: This pattern suggests that the processes used to solve both types of syllogisms are spatial-abstract.

Relationship of Syllogistic Reasoning Ability to Intelligence

The present work replicates the findings of Thurstone (1938) and Frandsen and Holder (1969) of a relationship between performance on syllogistic reasoning tasks and performance on tests of spatial ability. The present interpretation of this relationship is in terms of both the representation and processes used in syllogistic reasoning. In particular, the proposed representation is an abstract, symbolic one: Combination of information about set relations relations requires visualization of relationships between pairs of informational components expressed in this representation.

The present research may be viewed as a further step toward a general process theory of human intelligence. This process theory began with an account of analogical reasoning (Sternberg, 1977a, 1977b), was expanded upon with a theory of linear syllogistic reasoning (Sternberg, Note 3, Note 4), and now encompasses a theory of categorical and conditional syllogistic reasoning as well. The present direction this research is taking is toward a unified account of human reasoning, which will serve as a larger subtheory of intelligence than do any of the accounts proposed in the more specific task analyses that have been accomplished to date. Eventually, we hope to isolate a relatively small set of information-processing components that together constitute the building blocks for the execution of what we generally refer to as intelligent behavior.

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Footnotes

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[1] Immediately preceding the submission of this article for publication, P. N. Johnson-Laird and M. Steedman published a new theory of syllogistic reasoning, called the analogical theory. There are two major similarities between the transitive-chain theory and the analogical theory. First, a set of items is represented by an arbitrary number of distinct elements, and a relation between two items is represented by relations between the distinct elements representing each set. Second, the combination stage includes a heuristic substage in which some set of possible conclusions is generated, and a confirmation substage in which the conclusions generated during the heuristic substage are subject to possible falsification. However, there are also striking differences in both representation and combination in the two theories. In the analogical theory, only a single unitary representation of each premise is used, and the relation thus represented is unidirectional: For example, the representation of Some A are B tags an arbitrary number of a elements as equivalent to b elements, but does not indicate which b elements are equivalent to a elements. In the transitive-chain theory, however, the representation of Some A are B includes information about how many members of A are also members

of B, and information about how many members of B are also members of A. The combination process in the analogical theory is rather involved (with different combination rules for syllogisms with 0, 1, or 2 negative premises), and no specific combination algorithm is given, in contrast to the two simple rules used to combine representations in the transitive-chain theory. A fair comparison between the two theories would require derivation of mathematical models from the analogical theory in order to compare the relative adequacy of the theories in predicting subjects' performance.

[2]A variant of the ideal model was also explored in which a separate parameter was estimated for each instance in which more than one response to a problem was logically correct. The proportion of subjects choosing each of two logically correct answers was thus determined in part by the distribution of responses in the data. The improvements in fit obtained through the extra parameters were so small that these models were abandoned.

[3]A complete list of prediction equations for the transitive-chain model is available on request by writing to the authors.

Table 1

Examples of Test Problems Used in Experiments on Syllogistic Reasoning

Experiment	
1	No C are B.
	All B are A.
	______
	All A are C.
	No A are C.
	Some A are C.
	Some A are not C.
	None of the above.
2a (factual content)	No cottages are skyscrapers.
	All skyscrapers are buildings.
	______
	All buildings are cottages.
	No buildings are cottages.
	Some buildings are cottages.
	Some buildings are not cottages.
	None of the above.
2b (counter-factual content)	No milk cartons are containers.
	All containers are trash cans.
	______
	All trash cans are milk cartons.
	No trash cans are milk cartons.
	Some trash cans are milk cartons.
	Some trash cans are not milk cartons.
	None of the above.

Table 1 cont.

Experiment	Problem
2c (anomalous content)	No headphones are planets. All planets are frying pans. ____________ All frying pans are headphones. No frying pans are headphones. Some frying pans are headphones. Some frying pans are not headphones. None of the above.
3	Most B are C All B are A. ____________ All A are C. No A are C. Most A are C. Few A are C. Most A are not C. Few A are not C. None of the above.
4a (categorical)	All A are B. X is not a B. ____________ Therefore, X is not an A.
4b (conditional)	If A then B. Not B. ____________ Therefore, not A.

Table 1 cont.

Experiment	Problem
5	If C occurs, then B does not occur.
	If B occurs, then A occurs.
	...
	If A occurs, then C occurs.
	If A occurs, then C does not occur.
	If A occurs, then C sometimes occurs.
	If A occurs, then C sometimes does not occur.
	None of the above.

Table 2

Representations Resulting From the Encoding of Various Types of Premises

EXPERIMENTS 1, 2, and 5

Premise Type	Representation A,D	A,E	B,D	B,E	C,F
All A are B or If A occurs then B occurs	X	X			
Some A are B or If A occurs then sometimes B occurs	X	X	X	X	
Some A are not B or If A occurs then sometimes B does not occur			X	X	X
No A are B or If A occurs then B does not occur					X

Table 2 (contd.)

EXPERIMENT 3

Premise Type	G,K	G,L	G,M	H,K	H,L	H,M	I,K	I,L	I,M	J,N
	Representation									
All A are B	X	X	X							
Most A are B				X	X	X				
Few A are B							X	X	X	
Most A are not B							X	X	X	
Few A are not B				X	X	X				
No A are B										X

Note.-- $A = a_1 \rightarrow B$, $a_2 \rightarrow B$; $B = a_1 \rightarrow B$, $a_2 \rightarrow -B$; $C = a_1 \rightarrow -B$, $a_2 \rightarrow -B$; $D = b_1 \rightarrow A$, $b_2 \rightarrow A$; $E = b_1 \rightarrow A$, $b_2 \rightarrow -A$; $F = b_1 \rightarrow -A$, $b_2 \rightarrow -A$

$G = a_1 \rightarrow B$, $a_2 \rightarrow B$, $a_3 \rightarrow B$; $H = a_1 \rightarrow B$, $a_2 \rightarrow B$, $a_3 \rightarrow -B$; $I = a_1 \rightarrow B$, $a_2 \rightarrow -B$, $a_3 \rightarrow -B$; $J = a_1 \rightarrow -B$, $a_2 \rightarrow -B$, $a_3 \rightarrow -B$

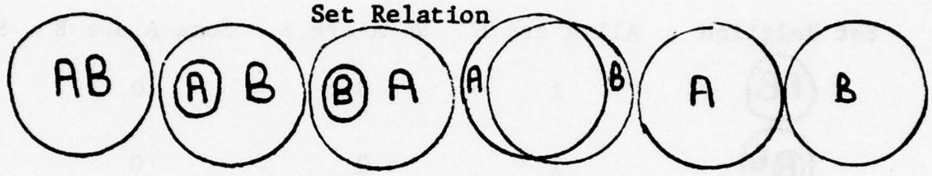
$K = b_1 \rightarrow A$, $b_2 \rightarrow A$, $b_3 \rightarrow A$; $L = b_1 \rightarrow A$, $b_2 \rightarrow A$, $b_3 \rightarrow -A$; $M = b_1 \rightarrow A$, $b_2 \rightarrow -A$, $b_3 \rightarrow -A$; $N = b_1 \rightarrow -A$, $b_2 \rightarrow -A$, $b_3 \rightarrow -A$

Table 3

Assumed Probability That Set Relation is Used as

Description of Statement in Erickson's Models

Statement | Set Relation

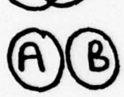


Statement					
All A are B	$\underline{e}_1$	$\underline{e}_2$	0	0	0
No A are B	0	0	0	0	1
Some A are B	$\underline{e}_3$	$\underline{e}_4$	$\underline{e}_5$	$\underline{e}_6$	0
Some A are not B	0	0	$\underline{e}_7$	$\underline{e}_8$	$\underline{e}_9$

Table 4

Assumed Probability That Statements Will be Used

to Label Set Relations in Erickson's Models

Set Relation	All A are B	No A are B	Some A are B	Some A are not B
	1	0	0	0
	1	0	0	0
	0	0	1 or $\underline{d}_1$*	0 or $\underline{d}_2$*
	0	0	1 or $\underline{d}_3$*	0 or $\underline{d}_4$*
	0	1	0	0

<u>Note</u>.-- Asterisks denote statements having two probabilities. The appropriate probability depends on whether the context of the premises is negative or positive. If at least one of the premises contains a negative, the second probability applies. Otherwise, the first probability applies.

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A TRANSITIVE-CHAIN THEORY OF SYLLOGISTIC REASONING.(U)

APR 78 M J GUYOTE, R J STERNBERG N00014-78-C-0025

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Table 5

Determinants of Response Choice in Syllogistic Reasoning

According to Alternative Models

Model	Stage: Encoding	Stage: Combination	Stage: Comparison
Transitive-Chain Model	Complete and correct	(1) Maximum of 4 possible pairs of representations are combined (2) Symmetrical representations and representations without negatives are combined before asymmetrical representations	(1) Choice of label that matches atmosphere of premises (2) Choice of label that matches atmosphere of premises and is stronger label (3) Mistaken labeling of combined representation as indeterminate
Complete Combination Model	Only 1 representation is encoded; determined by parameters of mathematical model	Complete and correct for single encoding	Choice of label determined by parameters of mathematical model
Random Combination Model	Only 1 representation is encoded; determined by parameters of mathematical model	Only one of the possible ways of combining 2 representations chosen at random	Choice of label determined by parameters of mathematical model

Table 5 (cont'd)

Model	Stage: Encoding	Stage: *Combination*	Stage: Comparison
Conversion Model	Both premises and their converses assumed true	Complete and correct on premises as encoded	Choice of label that matches the atmosphere of the premises
Atmosphere Model	Quantification and polarity of premises encoded	(1) Single negative creates negative atmosphere (2) Single particular creates particular atmosphere	(1) Negative atmosphere leads to preference for negative conclusion (2) Particular atmosphere leads to preference for particular conclusion

Table 6

Basic Statistics for Data Used in Modeling

Experiment	$\bar{X}$	S
	Response-choice Data	
1	.57	.20
2a (factual)	.66	.20
2b (counterfactual)	.57	.24
2c (anomalous)	.57	.22
3	.60	.22
4a (categorical)	.82	.20
4b (conditional)	.83	.24
5	.49	.24
	Latency Data	
1 (session 1)	45.72	8.58
1 (session 2)	33.22	7.92
1 (combined)	39.47	8.22
4 (categorical)	13.38	.72
4 (conditional)	13.51	.70
4 (combined)	13.45	.70

Note.-- Latency measures are expressed in seconds; response-choice measures are expressed in percent correct responses.

Table 7

Intercorrelations Among Response-choice Data Sets

	Experiment							
	1	2a	2b	2c	3	4a	4b	5
1	.99	.88	.96	.97	-	-	-	-
2a	-	.96	.92	.94	-	-	-	-
2b	-	-	.95	.97	-	-	-	-
2c	-	-	-	.95	-	-	-	-
3	-	-	-	-	.96	-	-	-
4a	-	-	-	-	-	.98	.97	-
4b	-	-	-	-	-	-	.98	-
5	-	-	-	-	-	-	-	.94

Table 8

Performance of Alternative Models in Predicting

Response-choice Data

Experiment	Model											
	Transitive Chain		Complete Combination		Random Combination		Conversion		Atmosphere		Ideal	
	R^2	RMSD	R^2	RMSD	R^2	RMSD	R^2	RMSD	R^2	RMSD	R^2	RMSD
1	.97	.05	.76	.14	.59	.18	.78	.13	.57	.18	.44	.29
2a	.91	.08	.67	.16	.34	.21	.46	.20	.29	.23	.68	.23
2b	.92	.08	.69	.15	.54	.18	.69	.15	.52	.18	.48	.29
2c	.89	.09	.66	.15	.53	.16	.61	.16	.46	.18	.51	.28
3	.94	.07										
4a	.97	.07										
4b	.95	.10										
5	.82	.15										

Table 9

Tests of Residuals: Predicted versus Observed Values

for Response-choice Data

<u>Within Data Sets</u>

(Internal Consistency Across Item Types)

Experiment	
1	.60**
2a	.58**
2b	.71**
2c	.69**
3	.54**
4a	.27*
4b	.25*
5	.66**

<u>Between Data Sets</u>

(Consistency Across Item Types)

Experiment

	1	2a	2b	2c	4a	4b
1	1.00	.42**	.71**	.68**	-	-
2a	-	1.00	.54**	.65**	-	-
2b	-	-	1.00	.74**	-	-
2c	-	-	-	1.00	-	-
4a	-	-	-	-	1.00	.31*
4b	-	-	-	-	-	1.00

* $p < .05$

** $p < .01$

Table 10

Performance of Transitive-chain Models

in Predicting Latency Data

Experiment	R^2	RMSD
1 (session 1)	.85**	331
1 (session 2)	.87**	290
1 (combined)	.88**	288
4a	.88**	25
4b	.84**	28
4 (combined)	.88**	24

* $p<.01$

** $p<.001$

Note.-- RMSD's are expressed in centiseconds.

Table 11

Tests of Residuals: Predicted versus Observed Values for Latency Data

Within Data Sets	
(Internal Consistency Across Item Types)	
Experiment	
1 (session 1)	.54*
1 (session 2)	.48*
1 (combined)	.52*
4a	.56*
4b	.44*
4 (combined)	.48*
Between Data Sets	
(Consistency Across Item Types)	
Experiment	
1 (sessions 1 and 2)	.52*
4 (categoricals and conditionals)	.49*

* $p < .01$

Table 12

Parameter Estimates for Transitive-chain Models in

Predicting Response-choice Data

Experiment	Parameter							
	p_1	$p_2+p_3+p_4$	β_1	β_2	c	t_0	t_1	t_2
1	.54	.46	.81	.92	.37	-	-	-
2a	.29	.71	.67	.95	.37	-	-	-
2b	.49	.51	.73	.94	.48	-	-	-
2c	.47	.53	.70	.92	.48	-	-	-
3	.40	.60	.64	.81	-	-	-	-
4a	.36	.64	-	-	-	.52	.48	.15
4b	.43	.57	-	-	-	.60	.61	.16
5	.57	.43	.76	.84	.61	-	-	-

Table 13

Parameter Estimates for Transitive-chain Models in

Predicting Latency Data

<u>Experiment 1</u>

	Parameter					
Data Set	ENC_{I-II}	ENC_{III}	$COMB_{I-II}$	$COMB_{III}$	CHECK	CON
Session 1	566 (398)	703*** (201)	527 (450)	714* (304)	858** (267)	466
Session 2	327 (197)	384*** (82)	505 (432)	623* (268)	690** (239)	387
Combined	435 (301)	532*** (138)	511 (457)	674* (288)	786** (242)	439

<u>Experiment 4</u>

	Parameter								
Data Set	$\underline{p}_{1p}$	$\underline{p}_{2p}$	$\underline{p}_{1n}$	$\underline{p}_{2n}$	$\underline{s}_{1p}$	$\underline{s}_{2p}$	$\underline{s}_{1n}$	$\underline{s}_{2n}$	CON
Conditionals	65* (29)	273*** (44)	159*** (26)	146*** (27)	135** (40)	157** (47)	120*** (26)	49 (28)	1160
Categoricals	72** (25)	387*** (46)	152*** (22)	209*** (31)	101** (36)	187*** (44)	103*** (21)	112** (32)	1152
Combined	70** (27)	325*** (41)	157*** (22)	175*** (27)	121** (35)	174*** (42)	113*** (22)	78** (27)	1154

Table 13 (cont'd)

* $\underline{p}<.05$

** $\underline{p}<.01$

*** $\underline{p}<.001$

Note.-- Parameter estimates are in centiseconds. Numbers in parentheses are standard errors.

Table 14

Correlations of Ability Factor Scores with Mean Number Correct Responses for Different Data Sets

	Factor Scores	
Experiment	Verbal	Spatial-Abstract
2a	.10	.42**
2b	.12	.36**
2c	.14	.45**
3	.14	.50**
4a	.15	.60**
4b	.14	.54**
5	.15	.35*

* $p<.05$

** $p<.01$

Table 15

Parameter Estimates for Subjects High and Low in

Verbal and Spatial-Abstract Abilities

		High Spatial-Abstract		Low Spatial-Abstract	
Parameter	Experiment	High Verbal	Low Verbal	High Verbal	Low Verbal
p_1	2	.26	.30	.50	.56
	3	.27	.31	.53	.49
	4	.36	.33	.53	.50
	5	.47	.46	.69	.66
β_1	2	.74	.69	.73	.77
	3	.64	.59	.65	.67
	5	.70	.75	.84	.67
β_2	2	.98	.97	.93	.96
	3	.84	.82	.80	.77
	5	.88	.92	.79	.80
c	2	.47	.42	.45	.47
	5	.65	.58	.71	.50
t_0	4	.64	.67	.43	.42
t_1	4	.60	.65	.39	.45
t_2	4	.21	.22	.10	.12

APPENDIX

This appendix presents details of the mathematical models used to predict subjects' performance in the experiments described above. All of the models for Experiments 1, 2, 3, and 5 will be presented with reference to syllogism 10: No C are B. All B are A.[3]

Transitive-chain Theory

Mathematical Model of Response Choices in Experiments 1, 2, 3, and 5

Seven parameters are estimated for this model. Parameters p_1, p_2, p_3, and p_4 represent the probabilities that n_p (see Figure 2) is equal to one, two, three, and four, respectively. Parameter β_1 represents strength of preference for a label that matches the atmosphere of the premises, given that one label matches the atmosphere of the premises but is weaker than the other label. Parameter β_2 represents strength of preference for a label that is both the stronger label and matches the atmosphere of the premises, given that one label both matches the atmosphere of the premises and is the stronger of two possible labels, or that one label fulfills one of these criteria and the other fulfills neither. Parameter c represents the probability of mistakenly labelling a composite representation indeterminate given that the composite representation includes different initial components.

With probability p_1, the subject combines only one pair of representations (see Figure 3):

$$\text{(CB-BC)}\quad \begin{array}{l|l} c_1 \to -B & b_1 \to -C \\ c_2 \to -B & b_2 \to -C \end{array} \qquad \text{and} \qquad \text{(BA-AB)}\quad \begin{array}{l|l} b_1 \to A & a_1 \to B \\ b_2 \to A & a_2 \to B \end{array}$$

This particular pair is always combined first because it pairs type I and type II representations. The combination of these two representations yields

$$\text{(AC-CA)} \qquad \begin{array}{l|l} a_1 \to -C & c_1 \to -A \\ a_2 \to -C & c_2 \to -A \end{array}$$

This representation may be labelled "No A are C" or "Some A are not C." "No A are C" is the stronger of these two labels, and it also matches the atmosphere of the premises. Therefore, it is chosen with probability β_2, and the label "Some A are not C" is chosen with probability $1 - \beta_2$. So far, the probabilities of the responses "No A are C" and "Some A are not C" are $p_1\beta_2$ and $p_1(1 - \beta_2)$, respectively.

With probability $1 - p_1$ (or $p_2 + p_3 + p_4$), the subject combines both possible pairs of representations:

$$\text{(1)} \quad \text{(CB-BC)} \quad \begin{array}{l|l} c_1 \to -B & b_1 \to -C \\ c_2 \to -B & b_2 \to -C \end{array} \quad \text{and} \quad \text{(BA-AB)} \quad \begin{array}{l|l} b_1 \to A & a_1 \to B \\ b_2 \to A & a_2 \to B \end{array}$$

$$\text{(2)} \quad \text{(CB-BC)} \quad \begin{array}{l|l} c_1 \to -B & b_1 \to -C \\ c_2 \to -B & b_2 \to -C \end{array} \quad \text{and} \quad \text{(BA-AB)} \quad \begin{array}{l|l} b_1 \to A & a_1 \to B \\ b_2 \to A & a_2 \to -B \end{array}$$

Combination of the second pair yields three possible relations between A and C:

$$\begin{array}{l|l} a_1 \to -C & c_1 \to -A \\ a_2 \to -C & c_2 \to -A \end{array}\ , \quad \begin{array}{l|l} a_1 \to C & c_1 \to A \\ a_2 \to -C & c_2 \to -A \end{array}\ , \text{ and } \quad \begin{array}{l|l} a_1 \to C & c_1 \to A \\ a_2 \to -C & c_2 \to A \end{array}$$

Since these representations contain different initial components, the subject labels this set of representations indeterminate with probability c. With probability $1 - c$ the subject labels the set of representations "Some A are not C," since this is the only label consistent with all of the representations (see Figure 3). So the probability of the response "No A

are $\underline{C}$" is $\underline{p}_1\beta_2$, the probability of the response "Some $\underline{A}$ are not $\underline{C}$" is $\underline{p}_1(1 - \beta_2) + (1 - \underline{p}_1)(1 - \underline{c})$, and the probability of the response "No valid conclusion exists" is $(1 - \underline{p}_1)\underline{c}$.

Mathematical Model of Latencies in Experiment 1

The subject first encodes three representations, one of each of the three types described above. With probability $\underline{p}_1$, the subject combines only the type I and type II representations. With probability $1 - \underline{p}_1$, the subject combines both the type I and type II representations and the type II and type III representations. Therefore, the time predicted to solve syllogism 10 is equal to $2\ ENC_{I-II} + ENC_{III} + COMB_{I-II} + (1 - \underline{p}_1)COMB_{III} +$ CON, where CON is a constant that includes the duration of the comparison and response stages.

Mathematical Model of Response Choices in Experiment 4

Five parameters are estimated for the mathematical model. Parameters $\underline{p}_1$ and $\underline{p}_2$ represent the probabilities that $\underline{n}_p$ (see Figure 4) is equal to one or two, respectively. Parameters $\underline{t}_0$, $\underline{t}_1$, and $\underline{t}_2$ represent the probabilities of forming a second transitive chain involving the first premise and the negation of the presented conclusion, when the first premise contains zero, one, or two negatives, respectively.

The method of deriving the prediction equations for Experiment 4 will be described with reference to problem 8: (a) If $\underline{A}$ then $\underline{B}$; not $\underline{B}$; Therefore, not $\underline{A}$; (b) All $\underline{A}$ are $\underline{B}$; $\underline{X}$ is not a $\underline{B}$; Therefore, $\underline{X}$ is not an $\underline{A}$.

With probability $\underline{p}_1$, the subject forms only one transitive chain involving the first and second premises:

(a) $x \rightarrow -B$ $\quad b_1 \rightarrow A$

$\qquad\qquad\qquad b_2 \rightarrow A$

The relationship of X to A cannot be determined from this chain. With probability $1 - t_0$ (since the first premise contains no negatives), the subject responds that the conclusion is invalid. With probability t_0, however, the subject forms an additional chain involving the negation of the conclusion and one of the components in the representation of the first premise:

$$\text{(b)} \qquad x \to A \qquad \begin{array}{l} a_1 \to B \\ a_2 \to B \end{array}$$

The integration of this chain yields the component $x \to B$. Since this component matches the negation of the second premise, the subject responds that the presented conclusion is valid.

With probability p_2, the subject goes through the routine described above, and if he has not already responded that the conclusion is invalid (with probability t_0), he forms a second chain involving the first and second premises:

$$\text{(c)} \qquad x \to -B \qquad \begin{array}{l} b_1 \to A \\ b_2 \to -A \end{array}$$

Once again, the relationship between X and A cannot be determined from this chain. The probability of forming another chain involving the negated conclusion and the first premise is $t_0p_2(t_0)$, since t_0p_2 is the probability of having gotten as far as combining chain (b). With probability t_0^2p, then, the following chain is formed:

$$\text{(d)} \qquad x \to A \qquad \begin{array}{l} a_1 \to B \\ a_2 \to B \end{array}$$

As with chain (b), the integration of chain (d) yields the component $x \to B$, and the subject responds that the presented conclusion is valid. Thus, the probability of the response "Valid" is $\underline{t}_0\underline{p}_1 + \underline{t}_0^2\underline{p}_2$, and the probability of the response "Invalid" is $1 - (\underline{t}_0\underline{p}_1 + \underline{t}_0^2\underline{p}_2)$.

Mathematical Model of Latencies in Experiment 4

The time taken to set up chains (a) and (c) is measured by $\underline{s}_{1n}$. With probability $\underline{p}_1 + (1 - \underline{t}_0)\underline{p}_2$ only chain (a) is formed, and with probability $\underline{t}_0\underline{p}_2$ both (a) and (c) are formed. Chain (b) alone is formed with probability $t_0(1-t_0p_2)$ and chains (b) and (d) are formed with probability $t_0^2\underline{p}_2$; the time taken to form these two chains is given by $\underline{p}_{2p}$. Therefore, the time predicted to solve problem 8 is equal to $\underline{s}_{1n} + \underline{t}_0\underline{p}_2\underline{s}_{1n} + \underline{t}_0\underline{p}_{2p} + \underline{t}_0^2\underline{p}_2\,\underline{p}_{2p}$ + CON, where CON is a constant that includes encoding and response times.

Random and Complete Combination Theories

Thirteen parameters are estimated for each of Erickson's theories. These parameters are shown in Tables 3 and 4. The parameters represent the probabilities of representing a given premise by particular set relations and of choosing various labels to represent particular composite representations.

In both theories, the subject represents the first premise ("No C are B") by a disjoint relation, and the second premise ("All B are A") by either an equivalence relation (with probability $\underline{e}_1$) or a subset-set relation (with probability $\underline{e}_2$). The combination of the equivalence and disjoint relations yields a disjoint relation between A and C. In both theories this relation is labelled "No A are C" and the subject chooses the response that matches this label. The combination of the subset-set and disjoint relations, however, yields three different relations (see Figure 6). In the complete combination theory, the subject performs all three combinations, and labels the result "Some A are not C," since this is the only label consistent with all three relations. In the random combination theory, with probability 1/3, the combined representation is a disjoint relation between *A* and *C*; as before, this relation is labelled "No *A* are *C*." The overlap and subset-set composite relations may be labelled as either "Some *A* are *C*" or "Some *A* are not *C*." The probability of the response "Some *A* are *C*" is equal to the probability of an overlap composite times the probability of labelling this composite "Some *A* are *C*" [$(1/3)e_2d_3$] plus the probability of a subset-set composite times the probability of labelling this composite "Some *A* are *C*" [$(1/3)e_2d_3$]. The probability of the response "Some *A* are not *C*" is equal to the probability of overlap and subset-set composites times the probability of labelling these composites "Some *A* are not *C*" [$(1/3)e_2d_4 + (1/3)e_2d_2$].

Atmosphere and Conversion Theories

The mathematical models for the atmosphere and conversion theories are quite simple. Only one parameter, x, is estimated for each model. In each case, this parameter represents the probability that the response predicted by the theory for a particular problem is chosen. The proportion of subjects predicted to choose each of the remaining responses is then $1/4(1 - x)$. For syllogism 10 (No *C* are *B*. All *B* are *A*.), both the atmosphere and conversion theories predict that the response "No *A* are *C*" should be preferred. The mathematical models for these two theories, then, predict that the probability of choosing this response is equal to x, and the probability of choosing any one of the remaining four responses is equal to $1/4(1 - x)$.

Table A

Predicted Versus Observed Values for the Response-choice and Latency Data in Experiments 1 and 4

Experiment 1

			Response Choices		Latencies	
	Problem		Predicted	Observed	Predicted	Observed
1	All B are C	a	0.92	0.97	3726	3153
	All A are B	b	0.0	0.0		
		c	0.08	0.02		
		d	0.0	0.0		
		e	0.0	0.0		
2	All B are C	a	0.54	0.52	3726	3871
	All B are A	b	0.0	0.0		
		c	0.32	0.33		
		d	0.0	0.02		
		e	0.14	0.11		
3	All B are C	a	0.05	0.0	4618	4150
	Some B are A	b	0.0	0.02		
		c	0.89	0.88		
		d	0.05	0.02		
		e	0.01	0.07		
4	All C are B	a	0.0	0.0	2662	2195
	No A are B	b	0.92	0.88		
		c	0.0	0.0		
		d	0.08	0.04		
		e	0.0	0.07		

Table A (cont'd)

Problem			Response Choices Predicted	Response Choices Observed	Latencies Predicted	Latencies Observed
5	All C are B	a	0.0	0.0	4086	4280
	Some A are not B	b	0.05	0.0		
		c	0.02	0.07		
		d	0.87	0.80		
		e	0.06	0.11		
6	Some B are C	a	0.05	0.0	4618	4917
	All B are A	b	0.0	0.0		
		c	0.89	0.97		
		d	0.05	0.0		
		e	0.01	0.02		
7	No B are C	a	0.0	0.02	4086	4303
	Some A are B	b	0.10	0.07		
		c	0.0	0.04		
		d	0.65	0.69		
		e	0.24	0.16		
8	No C are B	a	0.0	0.02	4086	4173
	Some B are A	b	0.10	0.14		
		c	0.0	0.02		
		d	0.65	0.47		
		e	0.24	0.35		
9	No B are C	a	0.0	0.02	2662	2897
	All A are B	b	0.92	0.90		
		c	0.0	0.0		
		d	0.08	0.04		
		e	0.0	0.02		

Table A (cont'd)

Problem			Response Choices Predicted	Response Choices Observed	Latencies Predicted	Latencies Observed
10	No C are B	a	0.0	0.02	2662	2780
	All B are A	b	0.50	0.52		
		c	0.0	0.0		
		d	0.33	0.28		
		e	0.17	0.16		
11	All B are C	a	0.0	0.0	3023	3225
	No A are B	b	0.50	0.59		
		c	0.0	0.02		
		d	0.04	0.09		
		e	0.46	0.28		
12	Some B are C	a	0.05	0.02	4807	4338
	All A are B	b	0.0	0.0		
		c	0.70	0.80		
		d	0.05	0.0		
		e	0.20	0.16		
13	Some B are not C	a	0.0	0.0	4353	4586
	All A are B	b	0.05	0.0		
		c	0.02	0.02		
		d	0.65	0.76		
		e	0.27	0.21		
14	All C are B	a	0.54	0.47	4056	4300
	All A are B	b	0.0	0.0		
		c	0.09	0.04		
		d	0.0	0.0		
		e	0.37	0.47		

Table A (cont'd)

Problem			Response Choices		Latencies	
			Predicted	Observed	Predicted	Observed
15	Some C are not B	a	0.0	0.0	4424	4317
	All A are B	b	0.05	0.02		
		c	0.02	0.02		
		d	0.60	0.61		
		e	0.33	0.33		
16	All B are C	a	0.0	0.02	3023	3302
	No B are A	b	0.50	0.47		
		c	0.0	0.07		
		d	0.04	0.09		
		e	0.46	0.33		
17	Some B are C	a	0.0	0.0	4448	3904
	No B are A	b	0.10	0.21		
		c	0.0	0.02		
		d	0.44	0.23		
		e	0.46	0.52		
18	No B are C	a	0.0	0.0	3830	4144
	Some B are not A	b	0.0	0.0		
		c	0.0	0.07		
		d	0.17	0.23		
		e	0.83	0.69		
19	All C are B	a	0.05	0.04	4807	4949
	Some B are A	b	0.0	0.04		
		c	0.70	0.54		
		d	0.05	0.02		
		e	0.20	0.33		

Table A (cont'd)

Problem		Response Choices Predicted	Response Choices Observed	Latencies Predicted	Latencies Observed
20 Some C are B	a	0.0	0.0	4448	4476
No B are A	b	0.10	0.11		
	c	0.0	0.02		
	d	0.44	0.38		
	e	0.46	0.47		
21 All B are C	a	0.05	0.07	4618	4774
Some A are B	b	0.0	0.0		
	c	0.89	0.85		
	d	0.05	0.04		
	e	0.01	0.02		
22 No B are C	a	0.0	0.0	2662	2338
All B are A	b	0.50	0.45		
	c	0.0	0.02		
	d	0.33	0.28		
	e	0.17	0.23		
23 No B are C	a	0.0	0.0	4086	4186
Some B are A	b	0.10	0.09		
	c	0.0	0.02		
	d	0.65	0.64		
	e	0.24	0.23		
24 Some B are not C	a	0.0	0.0	4086	3643
All B are A	b	0.05	0.0		
	c	0.02	0.07		
	d	0.87	0.83		
	e	0.06	0.09		

Table A (cont'd)

	Problem		Response Choices Predicted	Response Choices Observed	Latencies Predicted	Latencies Observed
25	All C are B	a	0.50	0.42	3726	3780
	All B are A	b	0.0	0.02		
		c	0.50	0.48		
		d	0.0	0.0		
		e	0.0	0.06		
26	All C are B	a	0.0	0.02	2662	2546
	No B are A	b	0.92	0.90		
		c	0.0	0.0		
		d	0.07	0.02		
		e	0.0	0.04		
27	Some C are B	a	0.05	0.02	4618	5138
	All B are A	b	0.0	0.0		
		c	0.89	0.88		
		d	0.05	0.0		
		e	0.01	0.09		
28	No C are B	a	0.0	0.0	2662	2975
	All A are B	b	0.92	0.85		
		c	0.0	0.04		
		d	0.07	0.07		
		e	0.0	0.02		
29	No C are B	a	0.0	0.0	4086	3991
	Some A are B	b	0.10	0.07		
		c	0.0	0.04		
		d	0.65	0.73		
		e	0.24	0.14		

Table A (cont'd)

Problem			Response Choices Predicted	Response Choices Observed	Latencies Predicted	Latencies Observed
30	All B are C	a	0.0	0.0	4448	4895
	Some B are not A	b	0.05	0.02		
		c	0.02	0.07		
		d	0.60	0.57		
		e	0.33	0.33		
31	All B are C	a	0.0	0.0	4424	4313
	Some A are not B	b	0.05	0.0		
		c	0.02	0.07		
		d	0.60	0.62		
		e	0.33	0.30		
32	No B are C	a	0.0	0.0	2606	2260
	No B are A	b	0.0	0.04		
		c	0.0	0.0		
		d	0.0	0.0		
		e	1.00	0.95		
33	Some B are not C	a	0.0	0.0	3830	3932
	No B are A	b	0.0	0.02		
		c	0.0	0.02		
		d	0.0	0.06		
		e	1.00	0.90		
34	All C are B	a	0.0	0.0	4353	4222
	Some B are not A	b	0.05	0.0		
		c	0.02	0.04		
		d	0.63	0.64		
		e	0.29	0.30		

Table A (cont'd)

Problem			Response Choices Predicted	Response Choices Observed	Latencies Predicted	Latencies Observed
35	All C are B	a	0.05	0.0	4807	5027
	Some A are B	b	0.0	0.0		
		c	0.70	0.69		
		d	0.05	0.04		
		e	0.20	0.26		
36	No C are B	a	0.0	0.0	3830	3819
	Some B are not A	b	0.0	0.07		
		c	0.0	0.02		
		d	0.17	0.19		
		e	0.83	0.71		
37	Some C are B	a	0.0	0.0	5521	5286
	Some B are not A	b	0.05	0.0		
		c	0.02	0.04		
		d	0.60	0.64		
		e	0.33	0.30		
38	Some C are B	a	0.05	0.02	4807	4598
	All A are B	b	0.0	0.0		
		c	0.70	0.72		
		d	0.05	0.04		
		e	0.20	0.20		

Note.-- a = All A are C; b = No A are C; c = Some A are C; d = Some A are not C; e = None of the above

Latencies are expressed in centiseconds.

Table A (cont'd)

Experiment 4

Problem	Conditional Form	Categorical Form		Response Choices Predicted	Response Choices Observed	Latencies Predicted	Latencies Observed
1	(A) If A then B.	(A) All A are B.	a	1.00	1.00	1267	1268
	A.	X is an A.	b	.00	.00		
	→ B.	→ X is a B.					
2	(A)	(A)	a	.40	.45	1379	1375
	B.	X is a B.	b	.60	.55		
	→ A.	→ X is an A.					
3	(A)	(A)	a	.00	.01	1224	1222
	A.	X is an A.	b	1.00	.99		
	→ not B.	→ X is not a B.					
4	(A)	(A)	a	.00	.11	1365	1382
	not B.	X is not a B.	b	1.00	.89		
	→ A.	→ X is an A.					
5	(A)	(A)	a	.00	.04	1355	1431
	not A.	X is not an A.	b	1.00	.96		
	→ B.	→ X is a B.					

Table A (cont'd)

Problem	Conditional Form	Categorical Form		Response Choices Predicted	Response Choices Observed	Latencies Predicted	Latencies Observed
6	(A)	(A)	a	.00	.05	1275	1278
	B.	X is a B.	b	1.00	.95		
	→ not A.	→ X is not an A.					
7	(A)	(A)	a	.22	.20	1434	1438
	not A.	X is not an A.	b	.78	.80		
	→ not B.	→ X is not a B.					
8	(A)	(A)	a	.41	.43	1549	1549
	not B.	X is not a B.	b	.59	.57		
	→ not A.	→ X is not an A.					
9	(B) If not A then B.	(B) All non-A are B.	a	.00	.02	1353	1345
	A.	X is an A.	b	1.00	.98		
	→ B.	→ X is a B.					
10	(B)	(B)	a	.00	.14	1267	1279
	B.	X is a B.	b	1.00	.86		
	→ A.	→ X is an A.					

Table A (cont'd)

Problem	Conditional Form	Categorical Form		Response Choices Predicted	Response Choices Observed	Latencies Predicted	Latencies Observed
11	(B)	(B)	a	.21	.16	1375	1352
	A.	X is an A.	b	.79	.84		
	→ not B.	→ X is not a B.					
12	(B)	(B)	a	.39	.45	1430	1459
	not B.	X is not a B.	b	.61	.55		
	→ A.	→ X is an A.					
13	(B)	(B)	a	1.00	1.00	1406	1414
	not A.	X is not an A.	b	.00	.00		
	→ B.	→ X is a B.					
14	(B)	(B)	a	.40	.51	1335	1351
	B.	X is a B.	b	.60	.49		
	→ not A.	→ X is not an A.					
15	(B)	(B)	a	.00	.17	1311	1332
	not A.	X is not an A.	b	1.00	.83		
	→ not B.	→ X is not a B.					

Table A (cont'd)

Problem	Conditional Form	Categorical Form		Response Choices Predicted	Response Choices Observed	Latencies Predicted	Latencies Observed
16	(B)	(B)	a	.00	.13	1362	1397
	not B.	X is not a B.	b	1.00	.87		
	→ not A.	→ X is not an A.					
17	(C) If A then not B.	(C) All A are non-B.	a	.00	.15	1311	1293
	A.	X is an A.	b	1.00	.85		
	→ B.	→ X is a B.					
18	(C)	(C)	a	.00	.13	1362	1344
	B.	X is a B.	b	1.00	.87		
	→ A.	→ X is an A.					
19	(C)	(C)	a	1.00	1.00	1406	1386
	A.	X is an A.	b	.00	.00		
	→ not B.	→ X is not a B.					
20	(C)	(C)	a	.40	.37	1335	1326
	not B.	X is not a B.	b	.60	.63		
	→ A.	→ X is an A.					

Table A (cont'd)

Problem	Conditional Form	Categorical Form		Response Choices Predicted	Response Choices Observed	Latencies Predicted	Latencies Observed
21	(C)	(C)	a	.21	.18	1375	1334
	not A.	X is not an A.	b	.79	.82		
	→ B.	→ X is a B.					
22	(C)	(C)	a	.39	.42	1430	1374
	B.	X is a B.	b	.61	.58		
	→ not A.	→ X is not an A.					
23	(C)	(C)	a	.00	.03	1353	1380
	not A.	X is not an A.	b	1.00	.97		
	→ not B.	→ X is not a B.					
24	(C)	(C)	a	.00	.03	1267	1248
	not B.	X is not a B.	b	1.00	.97		
	→ not A.	→ X is not an A.					
25	(D) If not A then not B.	(D) All non-A are non-B.	a	.06	.08	1260	1228
	A	X is an A.	b	.93	.92		
	→ B.	→ X is a B.					

Table A (cont'd)

Problem	Conditional Form	Categorical Form		Response Choices Predicted	Response Choices Observed	Latencies Predicted	Latencies Observed
26	(D)	(D)	a	.08	.09	1307	1292
	B.	X is a B.	b	.92	.91		
	→ A.	→ X is an A.					
27	(D)	(D)	a	.00	.04	1324	1313
	A.	X is an A.	b	1.00	.96		
	→ not B.	→ X is not a B.					
28	(D)	(D)	a	.00	.05	1267	1258
	not B.	X is not a B.	b	1.00	.95		
	→ A.	→ X is an A.					
29	(D)	(D)	a	.00	.00	1311	1322
	not A.	X is not an A.	b	1.00	1.00		
	→ B.	→ X is a B.					
30	(D)	(D)	a	.00	.19	1295	1313
	B.	X is a B.	b	1.00	.81		
	→ not A.	→ X is not an A.					

Table A (cont'd)

				Response Choices		Latencies	
Problem	Conditional Form	Categorical Form		Predicted	Observed	Predicted	Observed
31	(D)	(D)	a	1.00	1.00	1406	1407
	not A.	X is not an A.	b	.00	.00		
	→ not B.	→ X is not a B.					
32.	(D)	(D)	a	.40	.32	1335	1336
	not B.	X is not a B.	b	.60	.68		
	→ not A.	→ X is not an A.					

Note.-- Latencies are expressed in centiseconds.

a = Valid

b = Invalid

Figure Captions

Figure 1. Five possible set relations and their corresponding symbolic representations.

Figure 2. The transitive-chain model for Experiments 1, 2, 3, and 5.

Figure 3. Solution of a sample problem in the transitive-chain model.

Figure 4. The transitive-chain model for Experiment 4.

Figure 5. The random and complete combination models.

Figure 6. Solution of a sample problem in the random and complete combination models.

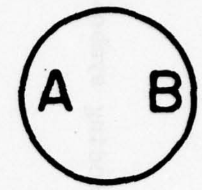
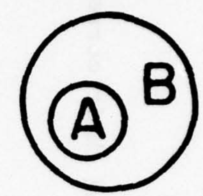
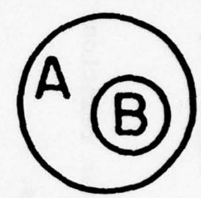
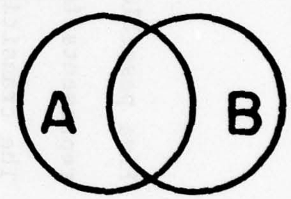
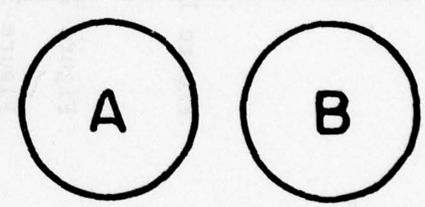
Set Relation	Symbolic Representation	Euler Diagram Representation
Equivalence	$a_1 \rightarrow B \mid b_1 \rightarrow A$ $a_2 \rightarrow B \mid b_2 \rightarrow A$	
Subset-Set	$a_1 \rightarrow B \mid b_1 \rightarrow A$ $a_2 \rightarrow B \mid b_2 \rightarrow -A$	
Set-Subset	$a_1 \rightarrow B \mid b_1 \rightarrow A$ $a_2 \rightarrow -B \mid b_2 \rightarrow A$	
Overlap	$a_1 \rightarrow B \mid b_1 \rightarrow A$ $a_2 \rightarrow -B \mid b_2 \rightarrow -A$	
Disjoint	$a_1 \rightarrow -B \mid b_1 \rightarrow -A$ $a_2 \rightarrow -B \mid b_2 \rightarrow -A$	

Figure 1

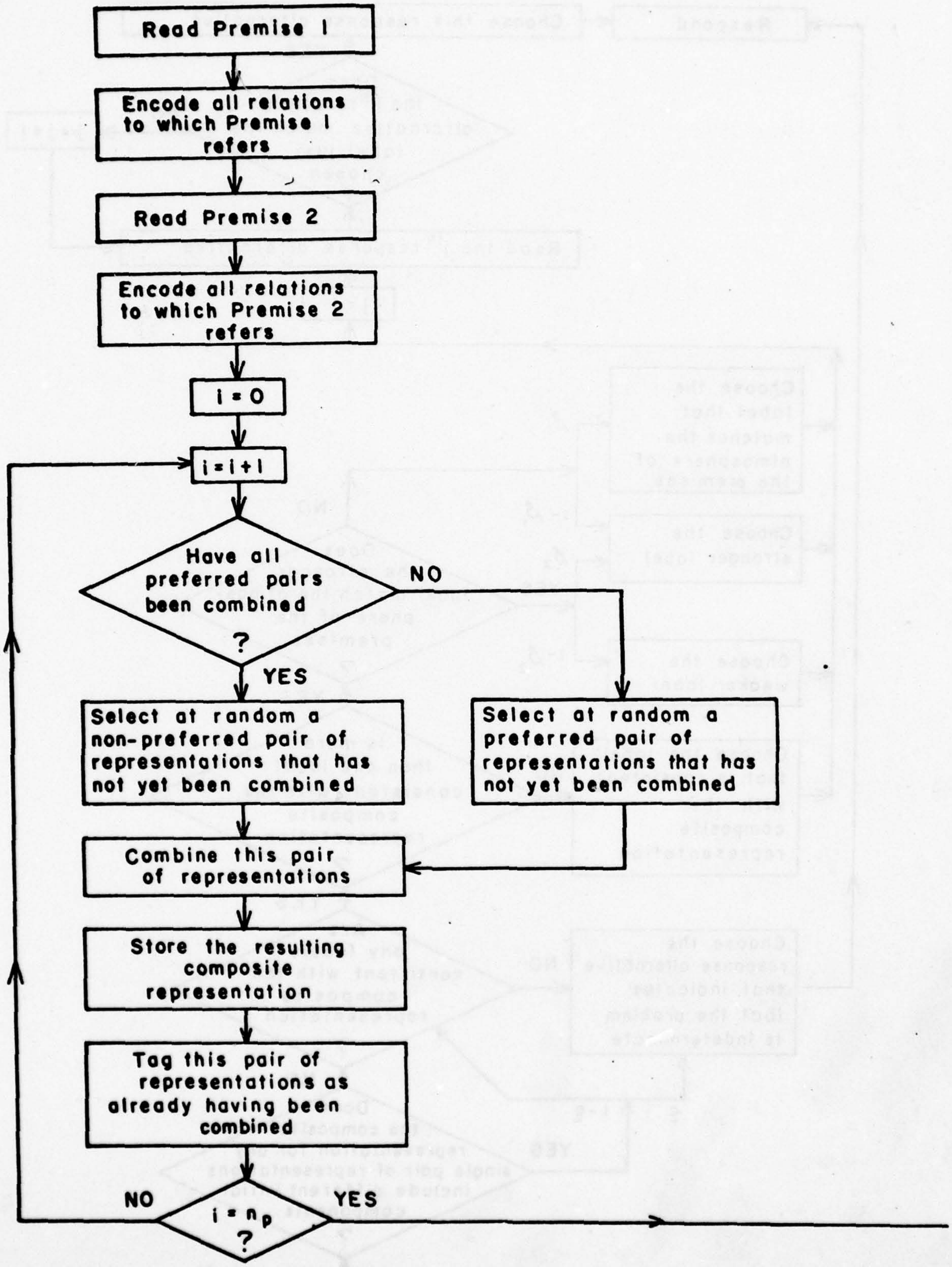

Read Premise 1
Encode all relations to which Premise 1 refers
Read Premise 2
Encode all relations to which Premise 2 refers
i = 0
i = i + 1
Have all preferred pairs been combined ?
NO
YES
Select at random a non-preferred pair of representations that has not yet been combined
Select at random a preferred pair of representations that has not yet been combined
Combine this pair of representations
Store the resulting composite representation
Tag this pair of representations as already having been combined
NO
YES
i = n_p ?

Figure 2

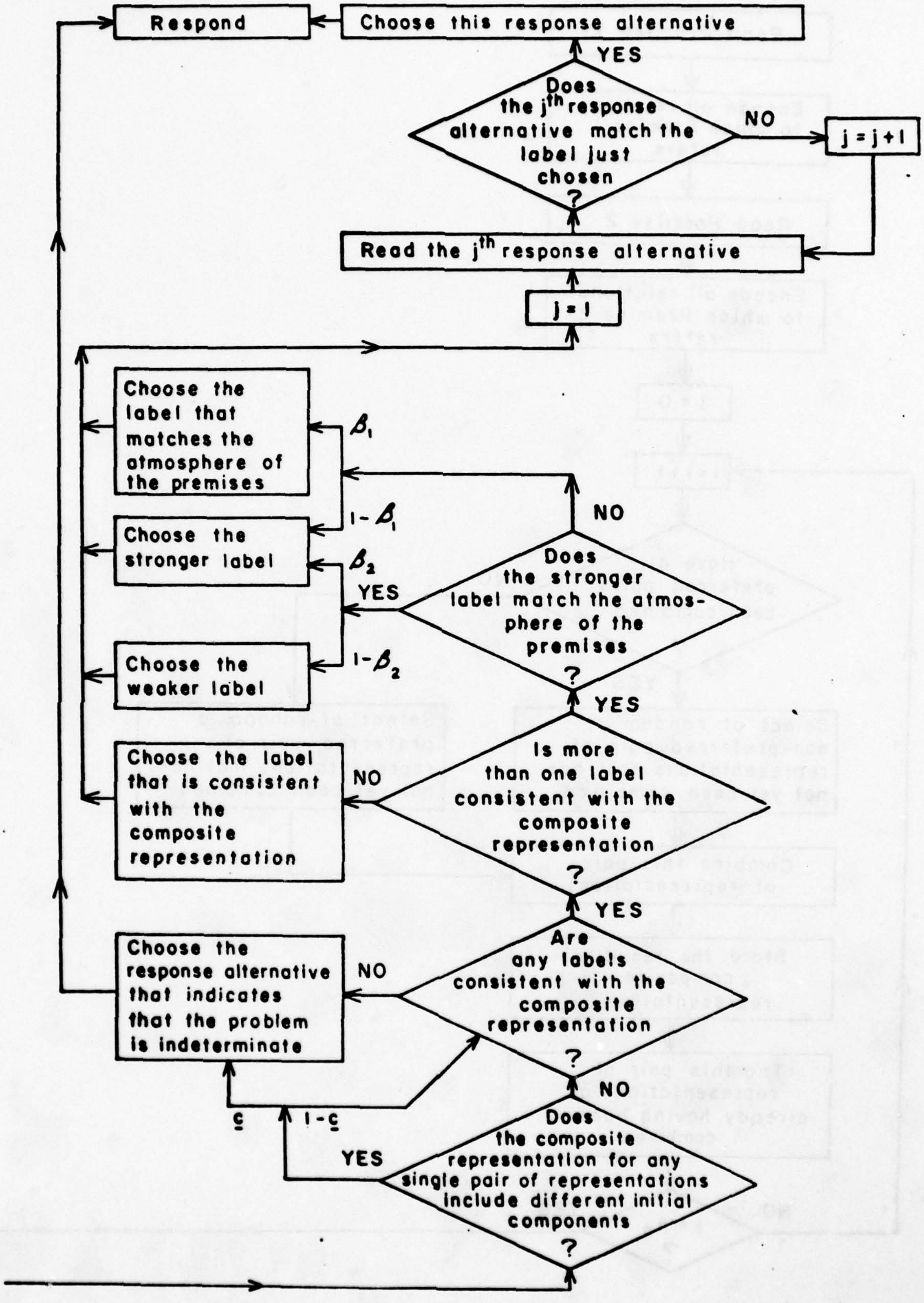
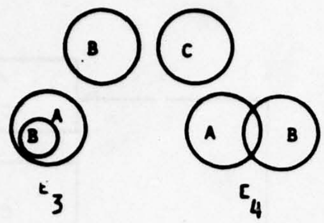


Figure 2 (Contd.)

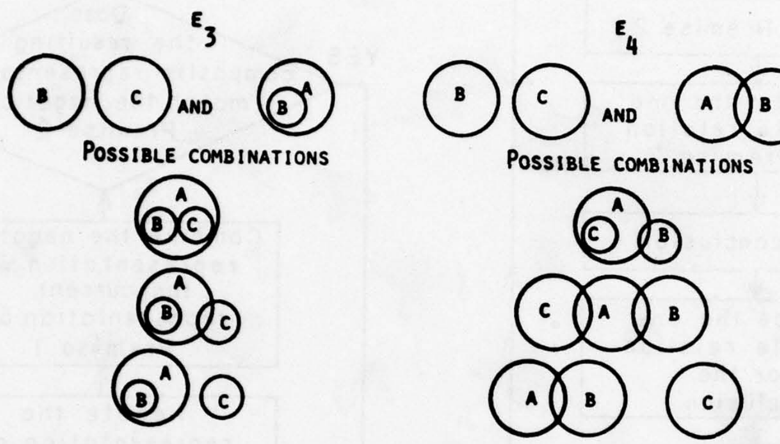
INTERPRETIVE STAGE:

NO B ARE C.

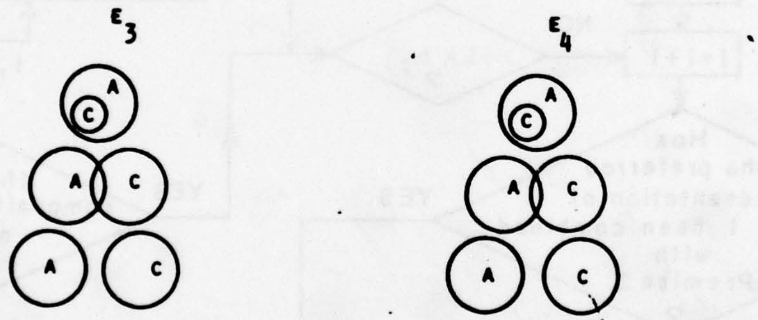
SOME A ARE B.



COMBINATION STAGE:



COMPARISON STAGE:



RANDOM

COMBINATION MODEL: ONLY ONE OF THE ABOVE COMBINATIONS IS PERFORMED.

"NO A ARE C" HAS $p = 1/3$.

"SOME A ARE C" HAS $p = (1/3)D_1 + (1/3)D_3$

"SOME A ARE NOT C" HAS $p = (1/3)D_2 + (1/3)D_4$

COMPLETE

COMBINATION MODEL: ALL OF THE ABOVE COMBINATIONS ARE PERFORMED.

THE ONLY LABEL CONSISTENT WITH ALL THREE COMBINATIONS IS "SOME A ARE NOT C," WHICH IS CHOSEN WITH $p = 1$.

Figure 3

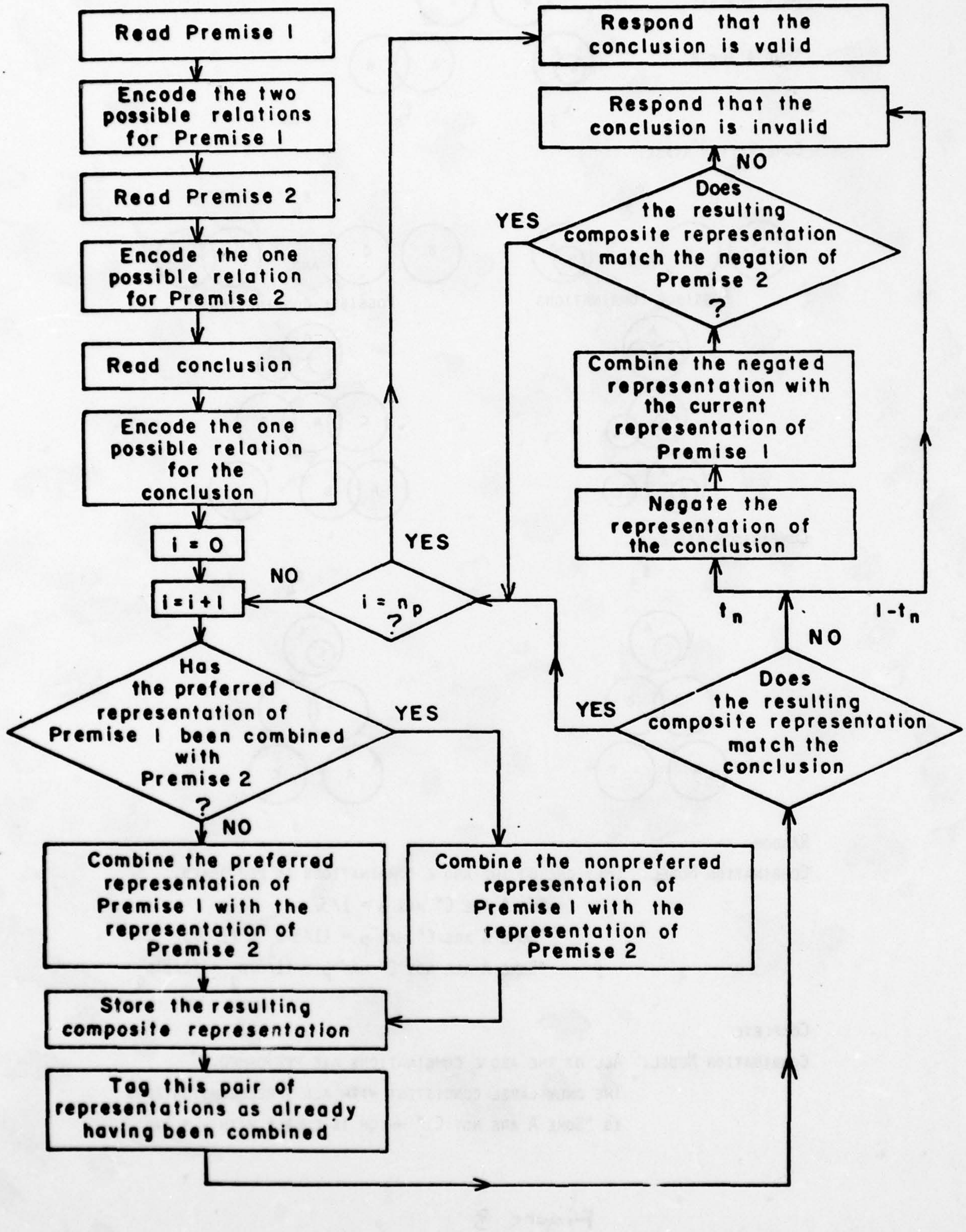

Read Premise 1
Encode the two possible relations for Premise 1
Read Premise 2
Encode the one possible relation for Premise 2
Read conclusion
Encode the one possible relation for the conclusion
i = 0
i = i + 1
Has the preferred representation of Premise 1 been combined with Premise 2 ?
YES
NO
Combine the preferred representation of Premise 1 with the representation of Premise 2
Combine the nonpreferred representation of Premise 1 with the representation of Premise 2
Store the resulting composite representation
Tag this pair of representations as already having been combined
i = n_p ?
NO
YES
Respond that the conclusion is valid
Respond that the conclusion is invalid
NO
Does the resulting composite representation match the negation of Premise 2 ?
YES
Combine the negated representation with the current representation of Premise 1
Negate the representation of the conclusion
t_n
$1-t_n$
NO
Does the resulting composite representation match the conclusion
YES

Figure 4

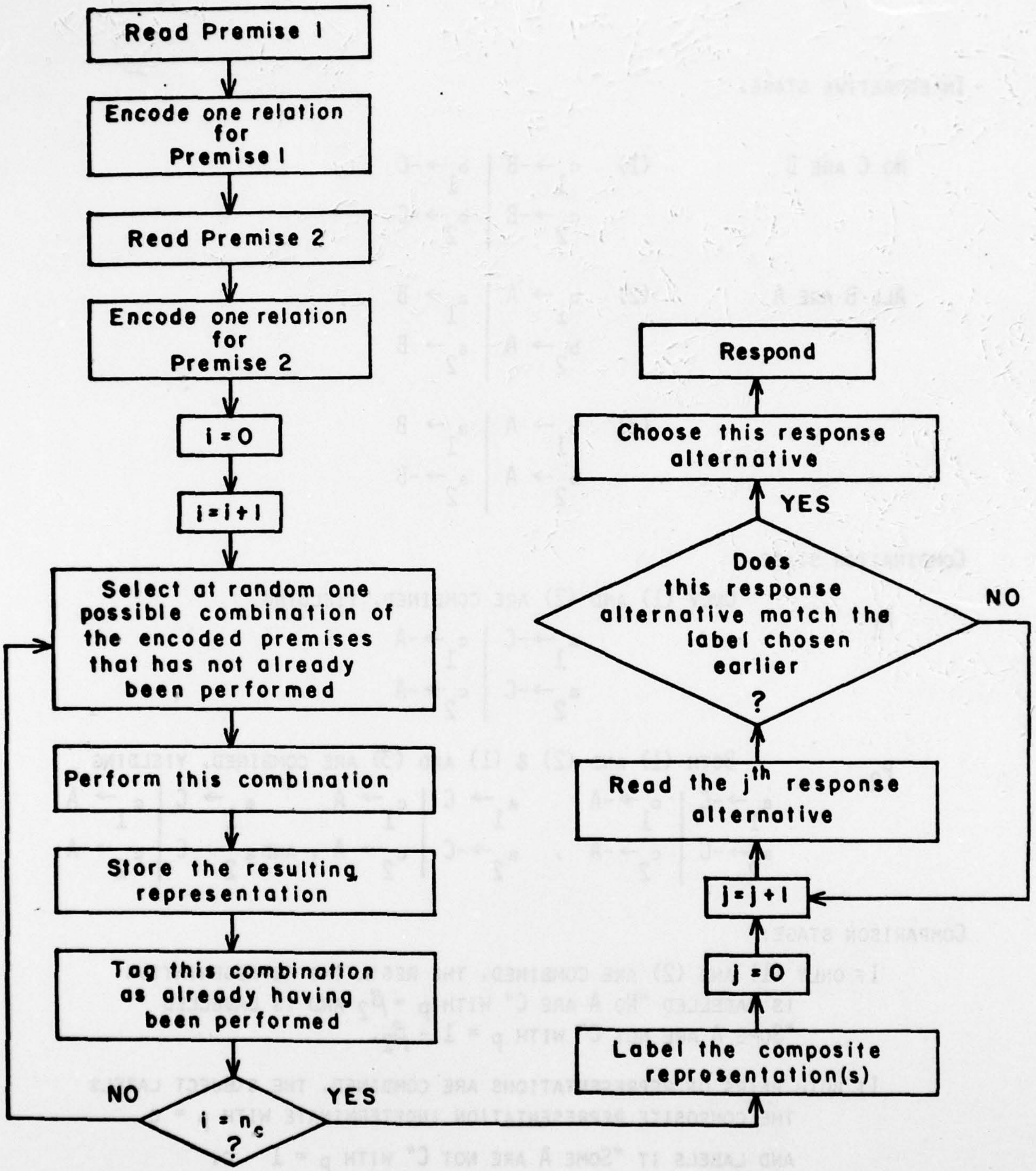

Read Premise 1
Encode one relation for Premise 1
Read Premise 2
Encode one relation for Premise 2
i = 0
i = i + 1
Select at random one possible combination of the encoded premises that has not already been performed
Perform this combination
Store the resulting representation
Tag this combination as already having been performed
NO
YES
i = n_c ?
Label the composite representation(s)
j = 0
j = j + 1
Read the j^{th} response alternative
Does this response alternative match the label chosen earlier ?
NO
YES
Choose this response alternative
Respond

Figure 5

INTERPRETIVE STAGE:

No C ARE B (1) $\begin{array}{l|l} c_1 \to -B & b_1 \to -C \\ c_2 \to -B & b_2 \to -C \end{array}$

ALL B ARE A (2) $\begin{array}{l|l} b_1 \to A & a_1 \to B \\ b_2 \to A & a_2 \to B \end{array}$

(3) $\begin{array}{l|l} b_1 \to A & a_1 \to B \\ b_2 \to A & a_2 \to -B \end{array}$

COMBINATION STAGE:

P_1 ONLY (1) AND (2) ARE COMBINED, YIELDING

$\begin{array}{l|l} a_1 \to -C & c_1 \to -A \\ a_2 \to -C & c_2 \to -A \end{array}$

P_2 BOTH (1) AND (2) & (1) AND (3) ARE COMBINED, YIELDING

$\begin{array}{l|l} a_1 \to -C & c_1 \to -A \\ a_2 \to -C & c_2 \to -A \end{array}$, $\begin{array}{l|l} a_1 \to C & c_1 \to A \\ a_2 \to -C & c_2 \to -A \end{array}$, AND $\begin{array}{l|l} a_1 \to C & c_1 \to A \\ a_2 \to -C & c_2 \to A \end{array}$

COMPARISON STAGE:

IF ONLY (1) AND (2) ARE COMBINED, THE RESULTING REPRESENTATION IS LABELLED "NO A ARE C" WITH $p = \beta_2$ AND IS LABELLED "SOME A ARE NOT C" WITH $p = 1 - \beta_2$.

IF BOTH PAIRS OF REPRESENTATIONS ARE COMBINED, THE SUBJECT LABELS THE COMPOSITE REPRESENTATION INDETERMINATE WITH $p = c$, AND LABELS IT "SOME A ARE NOT C" WITH $p = 1 - c$.

Figure 6

Navy

4 DR. JACK ADAMS
OFFICE OF NAVAL RESEARCH BRANCH
223 OLD MARYLEBONE ROAD
LONDON, NW, 15TH ENGLAND

1 Dr. Jack R. Borsting
Provost & Academic Dean
U.S. Naval Postgraduate School
Monterey, CA 93940

1 DR. MAURICE CALLAHAN
NODAC (CODE 2)
DEPT. OF THE NAVY
BLDG. 2, WASHINGTON NAVY YARD
(ANACOSTIA)
WASHINGTON, DC 20374

1 Dept. of the Navy
CHNAVMAT (NMAT 034D)
Washington, DC 20350

1 Chief of Naval Education and
Training Support)-(01A)
Pensacola, FL 32509

1 Dr. Charles E. Davis
ONR Branch Office
536 S. Clark Street
Chicago, IL 60605

1 Mr. James S. Duva
Chief, Human Factors Laboratory
Naval Training Equipment Center
(Code N-215)
Orlando, Florida 32813

5 Dr. Marshall J. Farr, Director
Personnel & Training Research Programs
Office of Naval Research (Code 458)
Arlington, VA 22217

1 DR. PAT FEDERICO
NAVY PERSONNEL R&D CENTER
SAN DIEGO, CA 92152

1 CDR John Ferguson, MSC, USN
Naval Medical R&D Command (Code 44)
National Naval Medical Center
Bethesda, MD 20014

Navy

1 Dr. John Ford
Navy Personnel R&D Center
San Diego, CA 92152

1 Dr. Eugene E. Gloye
ONR Branch Office
1030 East Green Street
Pasadena, CA 91101

1 CAPT. D.M. GRAGG, MC, USN
HEAD, SECTION ON MEDICAL EDUCATION
UNIFORMED SERVICES UNIV. OF THE
HEALTH SCIENCES
6917 ARLINGTON ROAD
BETHESDA, MD 20014

1 CDR Robert S. Kennedy
Naval Aerospace Medical and
Research Lab
Box 29407
New Orleans, LA 70189

1 Dr. Norman J. Kerr
Chief of Naval Technical Training
Naval Air Station Memphis (75)
Millington, TN 38054

1 Dr. Leonard Kroeker
Navy Personnel R&D Center
San Diego, CA 92152

1 CHAIRMAN, LEADERSHIP & LAW DEPT.
DIV. OF PROFESSIONAL DEVELOPMMENT
U.S. NAVAL ACADEMYY
ANNAPOLIS, MD 21402

1 Dr. James Lester
ONR Branch Office
495 Summer Street
Boston, MA 02210

1 Dr. William L. Maloy
Principal Civilian Advisor for
Education and Training
Naval Training Command, Code 00A
Pensacola, FL 32508

Navy

1 Dr. James McBride
Code 301
Navy Personnel R&D Center
San Diego, CA 92152

2 Dr. James McGrath
Navy Personnel R&D Center
Code 306
San Diego, CA 92152

1 DR. WILLIAM MONTAGUE
NAVY PERSONNEL R& D CENTER
SAN DIEGO, CA 92152

1 Commanding Officer
U.S. Naval Amphibious School
Coronado, CA 92155

1 Commanding Officer
Naval Health Research Center
Attn: Library
San Diego, CA 92152

1 CDR PAUL NELSON
NAVAL MEDICAL R& D COMMAND
CODE 44
NATIONAL NAVAL MEDICAL CENTER
BETHESDA, MD 20014

1 Library
Navy Personnel R&D Center
San Diego, CA 92152

6 Commanding Officer
Naval Research Laboratory
Code 2627
Washington, DC 20390

1 OFFICE OF CIVILIAN PERSONNEL
(CODE 26)
DEPT. OF THE NAVY
WASHINGTON, DC 20390

1 JOHN OLSEN
CHIEF OF NAVAL EDUCATION & TRAINING SUPPORT
PENSACOLA, FL 32509

Navy

1 Office of Naval Research
Code 200
Arlington, VA 22217

1 Scientific Director
Office of Naval Research
Scientific Liaison Group/Tokyo
American Embassy
APO San Francisco, CA 96503

1 SCIENTIFIC ADVISOR TO THE CHIEF OF NAVAL PERSONNEL
NAVAL BUREAU OF PERSONNEL (PERS OR)
RM. 4410, ARLINGTON ANNEX
WASHINGTON, DC 20370

1 DR. RICHARD A. POLLAK
ACADEMIC COMPUTING CENTER
U.S. NAVAL ACADEMY
ANNAPOLIS, MD 21402

1 Mr. Arnold I. Rubinstein
Human Resoureces Program Manager
Naval Material Command (0344)
Room 1044, Crystal Plaza #5
Washington, DC 20360

1 Dr. Worth Scanland
Chief of Naval Education and Training
Code N-5
NAS, Pensacola, FL 32508

1 A. A. SJOHOLM
TECH. SUPPORT, CODE 201
NAVY PERSONNEL R& D CENTER
SAN DIEGO, CA 92152

1 Mr. Robert Smith
Office of Chief of Naval Operations
OP-987E
Washington, DC 20350

1 Dr. Alfred F. Smode
Training Analysis & Evaluation Group (TAEG)
Dept. of the Navy
Orlando, FL 32813

Navy

1 CDR Charles J. Theisen, JR. MSC, USN
Head Human Factors Engineering Div.
Naval Air Development Center
Warminster, PA 18974

1 W. Gary Thomson
Naval Ocean Systems Center
Code 7132
San Diego, CA 92152

Army

1 ARI Field Unit-Leavenworth
P.O. Box 3122
Ft. Leavenworth, KS 66027

1 HQ USAREUE & 7th Army
ODCSOPS
USAAREUE Director of GED
APO New York 09403

1 DR. JAMES BAKER
U.S. ARMY RESEARCH INSTITUTE
5001 EISENHOWER AVENUE
ALEXANDRIA, VA 22333

1 DR. RALPH CANTER
U.S. ARMY RESEARCH INSTITUTE
5001 EISENHOWER AVENUE
ALEXANDRIA, VA 22333

1 DR. RALPH DUSEK
U.S. ARMY RESEARCH INSTITUTE
5001 EISENHOWER AVENUE
ALEXANDRIA, VA 22333

1 Dr. Milton S. Katz
Individual Training & Skill
Evaluation Technical Area
U.S. Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

1 Dr. Harold F. O'Neil, Jr.
ATTN: PERI-OK
5001 EISENHOWER AVENUE
ALEXANDRIA, VA 22333

1 Director, Training Development
U.S. Army Administration Center
ATTN: Dr. Sherrill
Ft. Benjamin Harrison, IN 46218

1 Dr. Joseph Ward
U.S. Army Research Institute
5001 Eisenhower Avenue
Alexandria, VA 22333

Air Force

1 Air Force Human Resources Lab
AFHRL/PED
Brooks AFB, TX 78235

1 Air University Library
AUL/LSE 76/443
Maxwell AFB, AL 36112

1 DR. G. A. ECKSTRAND
AFHRL/AS
WRIGHT-PATTERSON AFB, OH 45433

1 Dr. Alfred R. Fregly
AFOSR/NL, Bldg. 410
Bolling AFB, DC 20332

1 CDR. MERCER
CNET LIAISON OFFICER
AFHRL/FLYING TRAINING DIV.
WILLIAMS AFB, AZ 85224

1 Personnel Analysis Division
HQ USAF/DPXXA
Washington, DC 20330

1 Research Branch
AFMPC/DPMYP
Randolph AFB, TX 78148

1 Dr. Marty Rockway (AFHRL/TT)
Lowry AFB
Colorado 80230

1 Major Wayne S. Sellman
Chief, Personnel Testing
AFMPC/DPMYPT
Randolph AFB, TX 78148

1 Brian K. Waters, Maj., USAF
Chief, Instructional Tech. Branch
AFHRL
Lowry AFB, CO 80230

Marines

1 Director, Office of Manpower Utilization
HQ, Marine Corps (MPU)
BCB, Bldg. 2009
Quantico, VA 22134

1 DR. A.L. SLAFKOSKY
SCIENTIFIC ADVISOR (CODE RD-1)
HQ, U.S. MARINE CORPS
WASHINGTON, DC 20380

CoastGuard

1 MR. JOSEPH J. COWAN, CHIEF
PSYCHOLOGICAL RESEARCH (G-P-1/62)
U.S. COAST GUARD HQ
WASHINGTON, DC 20590

Other DoD

1 Dr. Stephen Andriole
ADVANCED RESEARCH PROJECTS AGENCY
1400 WILSON BLVD.
ARLINGTON, VA 22209

12 Defense Documentation Center
Cameron Station, Bldg. 5
Alexandria, VA 22314
Attn: TC

1 Dr. Dexter Fletcher
ADVANCED RESEARCH PROJECTS AGENCY
1400 WILSON BLVD.
ARLINGTON, VA 22209

1 Military Assistant for Human Resources
Office of the Director of Defense
Research & Engineering
Room 3D129, the Pentagon
Washington, DC 20301

1 Director, Research & Data
OSD/MRA&L (Rm. 3B919)
The Pentagon
Washington, DC 20301

1 Mr. Fredrick W. Suffa
MPP (A&R)
2B269
Pentagon
Washington, D.C. 20301

Civil Govt

1 Dr. Susan Chipman
Basic Skills Program
National Institute of Education
1200 19th Street NW
Washington, DC 20208

1 Dr. William Gorham, Director
Personnel R&D Center
U.S. Civil Service Commission
1900 E Street NW
Washington, DC 20415

1 Dr. Andrew R. Molnar
Science Education Dev.
and Research
National Science Foundation
Washington, DC 20550

1 Dr. Thomas G. Sticht
Basic Skills Program
National Institute of Education
1200 19th Street NW
Washington, DC 20208

1 Dr. Joseph L. Young, Director
Memory & Cognitive Processes
National Science Foundation
Washington, DC 20550

Non Govt

1 PROF. EARL A. ALLUISI
DEPT. OF PSYCHOLOGY
CODE 287
OLD DOMINION UNIVERSITY
NORFOLK, VA 23508

1 DR. MICHAEL ATWOOD
SCIENCE APPLICATIONS INSTITUTE
40 DENVER TECH. CENTER WEST
7935 E. PRENTICE AVENUE
ENGLEWOOD, CO 80110

1 1 psychological research unit
Dept. of Defense (Army Office)
Campbell Park Offices
Canberra ACT 2600, Australia

1 MR. SAMUEL BALL
EDUCATIONAL TESTING SERVICE
PRINCETON, NJ 08540

1 Dr. Nicholas A. Bond
Dept. of Psychology
Sacramento State College
600 Jay Street
Sacramento, CA 95819

1 Dr. John Seeley Brown
Bolt Beranek & Newman, Inc.
50 Moulton Street
Cambridge, MA 02138

1 Dr. John B. Carroll
Psychometric Lab
Univ. of No. Carolina
Davie Hall 013A
Chapel Hill, NC 27514

1 Dr. William Chase
Department of Psychology
Carnegie Mellon University
Pittsburgh, PA 15213

1 Dr. Micheline Chi
Learning R & D Center
University of Pittsburgh
3939 O'Hara Street
Pittsburgh, PA 15213

Non Govt

Dr. Kenneth E. Clark
College of Arts & Sciences
University of Rochester
River Campus Station
Rochester, NY 14627

Dr. Norman Cliff
Dept. of Psychology
Univ. of So. California
University Park
Los Angeles, CA 90007

Dr. Allan M. Collins
Bolt Beranek & Newman, Inc.
50 Moulton Street
Cambridge, Ma 02138

Dr. Meredith Crawford
5605 Montgomery Street
Chevy Chase, MD 20015

Dr. Donald Dansereau
Dept. of Psychology
Texas Christian University
Fort Worth, TX 76129

DR. RENE V. DAWIS
DEPT. OF PSYCHOLOGY
UNIV. OF MINNESOTA
75 E. RIVER RD.
MINNEAPOLIS, MN 55455

Dr. Ruth Day
Center for Advanced Study
in Behavioral Sciences
202 Junipero Serra Blvd.
Stanford, CA 94305

1 ERIC Facility-Acquisitions
4833 Rugby Avenue
Bethesda, MD 20014

1 MAJOR I. N. EVONIC
CANADIAN FORCES PERS. APPLIED RESEARCH
1107 AVENUE ROAD
TORONTO, ONTARIO, CANADA

Non Govt

1 Dr. Richard L. Ferguson
The American College Testing Program
P.O. Box 168
Iowa City, IA 52240

1 Dr. Victor Fields
Dept. of Psychology
Montgomery College
Rockville, MD 20850

1 Dr. Edwin A. Fleishman
Advanced Research Resources Organ.
8555 Sixteenth Street
Silver Spring, MD 20910

1 Dr. John R. Frederiksen
Bolt Beranek & Newman
50 Moulton Street
Cambridge, MA 02138

1 DR. ROBERT GLASER
LRDC
UNIVERSITY OF PITTSBURGH
3939 O'HARA STREET
PITTSBURGH, PA 15213

1 DR. JAMES G. GREENO
LRDC
UNIVERSITY OF PITTSBURGH
3939 O'HARA STREET
PITTSBURGH, PA 15213

1 Dr. Ron Hambleton
School of Education
University of Massechusetts
Amherst, MA 01002

1 Dr. Barbara Hayes-Roth
The Rand Corporation
1700 Main Street
Santa Monica, CA 90406

1 HumRRO/Ft. Knox office
P.O. Box 293
Ft. Knox, KY 40121

Non Govt

1 Library
HumRRO/Western Division
27857 Berwick Drive
Carmel, CA 93921

1 Dr. Earl Hunt
Dept. of Psychology
University of Washington
Seattle, WA 98105

1 Mr. Gary Irving
Data Sciences Division
Technology Services Corporation
2811 Wilshire Blvd.
Santa Monica CA 90403

1 Dr. Roger A. Kaufman
203 Dodd Hall
Florida State Univ.
Tallahassee, FL 32306

1 Dr. Steven W. Keele
Dept. of Psychology
University of Oregon
Eugene, OR 97403

1 Mr. Marlin Kroger
1117 Via Goleta
Palos Verdes Estates, CA 90274

1 LCOL. C.R.J. LAFLEUR
PERSONNEL APPLIED RESEARCH
NATIONAL DEFENSE HQS
101 COLONEL BY DRIVE
OTTAWA, CANADA K1A OK2

1 Dr. Frederick M. Lord
Educational Testing Service
Princeton, NJ 08540

1 Dr. Robert R. Mackie
Human Factors Research, Inc.
6780 Cortona Drive
Santa Barbara Research Pk.
Goleta, CA 93017

Non Govt

1 Dr. Richard B. Millward
Dept. of Psychology
Hunter Lab.
Brown University
Providence, RI 82912

1 Dr. Donald A Norman
Dept. of Psychology C-009
Univ. of California, San Diego
La Jolla, CA 92093

1 Dr. Melvin R. Novick
Iowa Testing Programs
University of Iowa
Iowa City, IA 52242

1 Dr. Jesse Orlansky
Institute for Defense Analysis
400 Army Navy Drive
Arlington, VA 22202

1 Dr. Seymour A. Papert
Massachusetts Institute of Technology
Artificial Intelligence Lab
545 Technology Square
Cambridge, MA 02139

1 MR. LUIGI PETRULLO
2431 N. EDGEWOOD STREET
ARLINGTON, VA 22207

1 DR. PETER POLSON
DEPT. OF PSYCHOLOGY
UNIVERSITY OF COLORADO
BOULDER, CO 80302

1 Dr. Frank Pratzner
Cntr. for Vocational Education
Ohio State University
1960 Kenny Road
Columbus, OH 43210

1 DR. DIANE M. RAMSEY-KLEE
R-K RESEARCH & SYSTEM DESIGN
3947 RIDGEMONT DRIVE
MALIBU, CA 90265

Non Govt

1 MIN. RET. M. RAUCH
P II 4
BUNDESMINISTERIUM DER VERTEIDIGUNG
POSTFACH 161
53 BONN 1, GERMANY

1 Dr. Mark D. Reckase
Educational Psychology Dept.
University of Missouri-Columbia
12 Hill Hall
Columbia, MO 65201

1 Dr. Joseph W. Rigney
Univ. of So. California
Behavioral Technology Labs
3717 South Hope Street
Los Angeles, CA 90007

1 Dr. Andrew M. Rose
American Institutes for Research
1055 Thomas Jefferson St. NW
Washington, DC 20007

1 Dr. Leonard L. Rosenbaum, Chairman
Department of Psychology
Montgomery College
Rockville, MD 20850

1 Dr. Ernst Z. Rothkopf
Bell Laboratories
600 Mountain Avenue
Murray Hill, NJ 07974

1 PROF. FUMIKO SAMEJIMA
DEPT. OF PSYCHOLOGY
UNIVERSITY OF TENNESSEE
KNOXVILLE, TN 37916

1 DR. WALTER SCHNEIDER
DEPT. OF PSYCHOLOGY
UNIVERSITY OF ILLINOIS
CHAMPAIGN, IL 61820

1 DR. ROBERT J. SEIDEL
INSTRUCTIONAL TECHNOLOGY GROUP
HUMRRO
300 N. WASHINGTON ST.
ALEXANDRIA, VA 22314

Non Govt

1 Dr. Robert Singer, Director
Motor Learning Research Lab
Florida State University
212 Montgomery Gym
Tallahassee, FL 32306

1 Dr. Richard Snow
School of Education
Stanford University
Stanford, CA 94305

1 DR. ALBERT STEVENS
BOLT BERANEK & NEWMAN, INC.
50 MOULTON STREET
CAMBRIDGE, MA 02138

1 DR. PATRICK SUPPES
INSTITUTE FOR MATHEMATICAL STUDIES IN THE SOCIAL SCIENCES
STANFORD UNIVERSITY
STANFORD, CA 94305

1 Dr. Kikumi Tatsuoka
Computer Based Education Research Laboratory
252 Engineering Research Laboratory
University of Illinois
Urbana, IL 61801

1 DR. PERRY THORNDYKE
THE RAND CORPORATION
1700 MAIN STREET
SANTA MONICA, CA 90406

1 Dr. Benton J. Underwood
Dept. of Psychology
Northwestern University
Evanston, IL 60201

1 DR. THOMAS WALLSTEN
PSYCHOMETRIC LABORATORY
DAVIE HALL 013A
UNIVERSITY OF NORTH CAROLINA
CHAPEL HILL, NC 27514

1 Dr. Claire E. Weinstein
Educational Psychology Dept.
Univ. of Texas at Austin
Austin, TX 78712

Non Govt

1 Dr. David J. Weiss
N660 Elliott Hall
University of Minnesota
75 E. River Road
Minneapolis, MN 55455

1 DR. SUSAN E. WHITELY
PSYCHOLOGY DEPARTMENT
UNIVERSITY OF KANSAS
LAWRENCE, KANSAS 66044