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THE PREDICTION OF VARIOUS SUPPLY MATERIAL AVAILABILITIES BY WEA--ETC(U)
JUL 78 R C MOREY, Z F LANSDOWNE, K EPSTEIN N00014-77-C-0806

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OFFICE OF NAVAL RESEARCH
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and N00014-78-C-0455

THE PREDICTION OF VARIOUS SUPPLY MATERIAL AVAILABILITIES BY WEAPON SYSTEM AS A FUNCTION OF PROCUREMENT AND REPAIR BUDGETS: A Computerized Weapon System Analyzer

152 p.

INVESTIGATORS:

Richard C. Morey, Ph.D. (Project Director and Principal Investigator)
Zachary F. Lansdowne, Ph.D (Associate Project Director)
Karen Epstein, M.A.
David P. Snyder, M.A.

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by

CONTROL ANALYSIS CORPORATION

800 Welch Road

Palo Alto, California 94304
(415) 326-2100

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1. INTRODUCTION AND SUMMARY

1.1 General Purpose of Effort

This report is the final deliverable under Contract Nos. N00014-77-C-0806 and N00014-78-C-0455 with the Navy Supply Command and the Office of Naval Research. The thrust of this effort has been to develop, document, validate, and install a working version of a set of computer programs designed explicitly to be able to build and execute resupply budgets, both procurement and repair, by weapon system. The material reported here is based on work initiated in July 1977 as a part of Contract N00014-72-C-0086 and continued under the above two contracts.

The goal of this effort has been then to be able to relate a weapon system's support budget to its level of readiness in the field. The budgets of interest are the procurement budgets for the consumable and repairable items which make up the weapon system, and the repair budgets (broken down by cogs) for the repairable items in the weapon system. As illustrations of the types of "what-if" questions that can now be addressed with the weapon systems analyzer, consider the following:

(1) For ASO's Weapon System F-14, what would be the level of support effectiveness realized (in terms of the fill-rate, average days delay, or time-weighted requisitions short) if one were to continue using the current reorder and repair trigger points? What are the budgets required to do this?

(2) If a 10% improvement in the fill-rate is desired for SPCC's grouping of items referred to as "Nuclear", how much would

it cost to achieve this improvement? What is the optimal division of this amount between additional procurement and repair activities.

(3) What is the impact of overriding the economic order quantities with 1 year's supply, or of altering the repair review periods from once every two weeks to the same frequency as procurement reviews, or putting all B08 items under a Level 4 Repair scenario or expanding the number of items receiving special repair management.

(4) If several weapon systems are to be given the same level of support effectiveness, what is the highest uniform service level that could result from redistribution of procurement and repair budgets?

(5) If the repair budget for all the repairable items in the 6R cog associated with weapon system F-14 were reduced (or increased), what would be the overall impact on the SMA for the 6R cog and for the entire weapon sysem?

The reader is referred to Figure 1.1 for an overview of how the weapon system analyzer can be used and how it interfaces with the current execution programs. It might be noted at this juncture that the original intent of the effort was to develop a program which could answer the above types of questions almost on an "on-line" basis. It was orginally envisioned to install the analyzer on a time-sharing system,* which could be accessed by either of the ICP's or budget planners at headquarters. The system could have the

*CAC utilized a similar approach in a previous effort performed for the Navy Supply System Command geared to improving in the management of the Navy's Stock Fund.

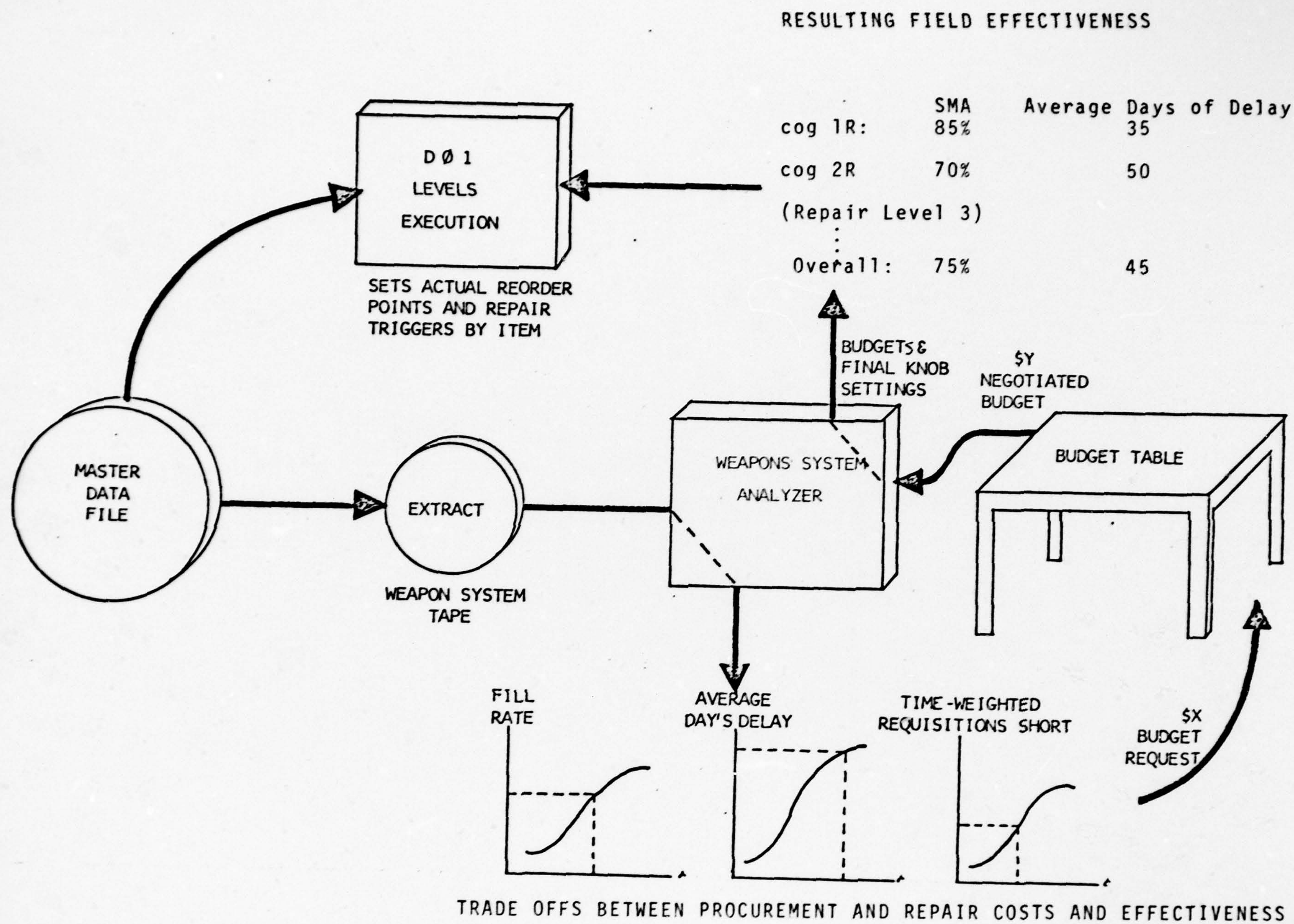

Figure 1.1 - Weapon System Analyzer, the Budget Building Process and Budget Execution.

capability of calling up any weapon system of interest,** and determining quickly, accurately, inexpensively, and defensibly, the impacts of various procurement and repair budget increases or decreases on such weapon system effectiveness measures as fill rates, average days of delay, time-weighted requisitions short, etc. It was in this spirit that CAC developed the Programs for IBM equipment suitable for running on time-sharing equipment or on SPCC's IBM 360/65. Indeed the programs have been successfully installed and tested for two typical weapons systems (one for each of the ICP's), and represent a capability for answering the "what-if" questions. However, midway thru the Project, it became apparent that available computer time on SPCC's 360 was very limited and that time-sharing was no longer a viable option (due to internal Navy budget pressures and policy restrictions). Hence the role of the programs has been switched to give more emphasis to aiding FMSO in incorporating desirable features of the model into its set of CARES program (which are run on Univac equipment where available time and access, especially for ASO, is greatly improved). As such, the CAC Weapon System Analyzer Programs will serve to validate the changes made by FMSO to their CARES programs, will continue to function as a backup option for analyses and will be available if the appropriate computer facilities materialize.

1.2 General Approach of Analyzer

Before getting into the types of outputs available, it might be mentioned that the analyzer works on an item by item basis, with

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**SPCC has about 61 weapon systems and ASO about fifty.

different sets of formulae utilized based on the particular type of item being analyzed. (These item types are discussed in detail in Section 2.0 but include high and low demand, consumable, repairable B08 items with repair levels 2, 3, 4, specially managed repair schemes such as FIRM,[1] CLAMP[2] HIGH BURNER, etc.) Section 2.2 provides some insight as to how the model handles the interactions of procurement and repair and their overall impact on SMA's. In this regard it is important to stress that the programs do not require any changes to the UICP Programs to be used, but are also capable of analyzing the impacts of proposed changes to the current rules; as such it can provide a valuable tool for laying the groundwork and defensibility of desirable changes.

Figure 1.2 provides a feeling for the level of analysis involved, and the general types of inputs utilized. Hence the program first determines the type of item it is analyzing (based upon coding schemes, computation of leadtime demands, the repair level scenario specified, etc.) and then utilizes an appropriate set of formulae to estimate the item's SMA, costs, average day of delay, etc. The overall cog and weapon system results are then weightings of the results by item, the weights being the fraction of the total cog's (or weapon system) requisitions due to that item.

In addition it should be stressed that the runs are reasonably fast, e.g., for SPCC's so-called "Nuclear System" and for ASO's "F-14" system, consisting of roughly 25,000 items and 17,000 items respectively, the

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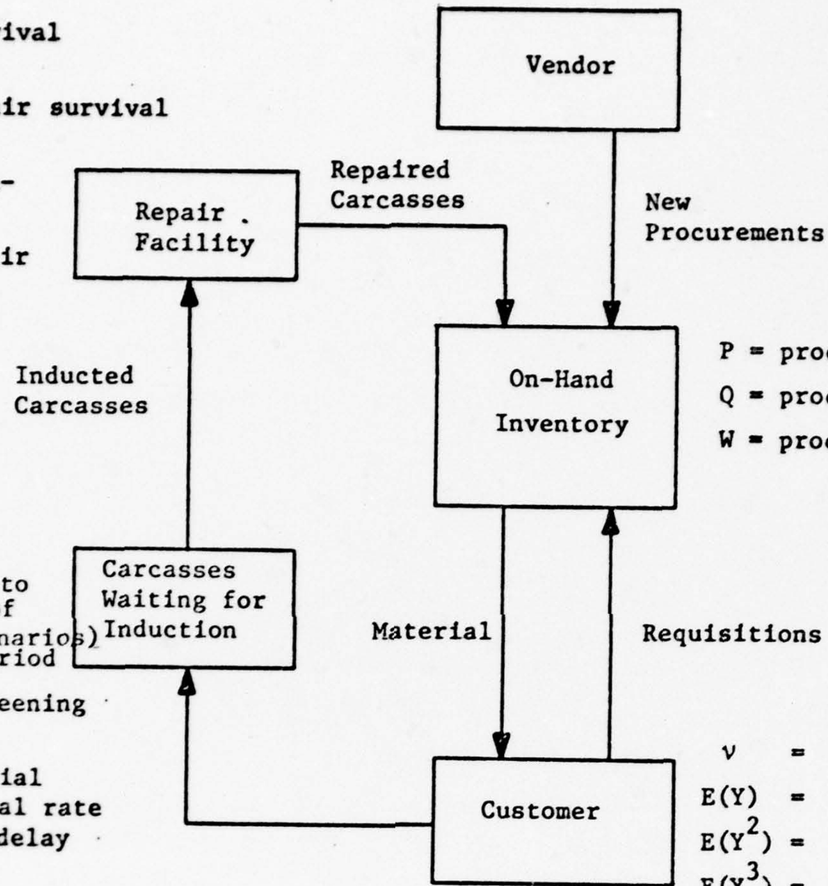
[1] Fleet Intensified Repairable Management (SPCC).

[2] Closed Loop Aeronautical Management Program (ASO).



$E(\beta_2)$ = mean repair survival rate

$V(\beta_2)$ = variance of repair survival rate

$E(R)$ = mean repair turn-around time

$V(R)$ = variance of repair turnaround time

RP = repair order-up-to point (function of repair level scenarios)

RW = repair review period

$E(\beta_1)$ = mean initial screening survival rate

$V(\beta_1)$ = variance of initial screening survival rate

C = carcass arrival delay

P = procurement reorder point

Q = procurement reorder quantity

W = procurement review period

ν = requisition arrival rate

$E(Y)$ = mean issue size

$E(Y^2)$ = second moment of issue size

$E(Y^3)$ = third moment of issue size

Figure 1.2 - Flows of Materials and Requisitions.

computer running times on an IBM 370/168 were between 2 - 3 CPU minutes. This included analyses for each cog of the impacts of 3 different min-max risk settings, and five different procurement shortage costs, together with summary calculations of the overall weapon system SMA and costs. The SPCC machine is a 360/65, which while admittedly three to four items slower, should still yield running times that are quite acceptable. In addition there are numerous options afforded the user which will speed up the calculations considerably at the expense of small amounts of accuracy.

1.3 Summary of Specific Program Outputs and the Type of Inputs Possible

The Programs represent an analyzer, in the spirit of FMSO's CARES PROGRAM but on a 4 digit cog basis, to predict impacts of many management controlled decision variables on many different measures of effectiveness for a given weapon system of interest. It also provides a number of useful auxillary descriptive statistics dealing with the composition of the weapon system, value of leadtime demand, value of annual demand, which items could not be processed because of missing information, etc.

1.3.1 Summary of Outputs

The key measures of interest, for a given procurement and repair scenario, include by cog the predicted:

(1) Procurement cost from the time of the "snapshot" until the end of the fiscal year and for subsequent year. (This is

explicitly modeled as a function of the procurement shortage cost knob (the so-called Lamda of Lagrangian analysis), the min-max risk settings, the UICP overrides used, the procurement review period, the distribution break-points* used, etc.) It includes planned requirements (before and after leadtime) and contracts due awarded, and recommended due awarded.**

(2) The predicted repair costs from the time of the "snapshot" until the end of the year and for the subsequent year. This is modeled explicitly as a function of the so-called B08 repair level scenario*** chosen, i.e., 1, 2, 3 or 4, one for each of the repair cogs involved, the repair shortage costs, any special repair management schemes such as CLAMP, HIGH BURNER or FIRM (i.e. special modules have been designed to yield SMA projections for items receiving one of the above repair schemes), the repair review time, etc. Hence there are separate results based on the user-selected repair level scenario desired. (The repair level scenarios of course, influence the level of the repair induction trigger and hence the resulting SMA's).

(3) The steady state fill rates by individual Mark categories and overall for the consumable cog, and by individual cog for each

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*This refers to the level of the demand during the leadtime for choosing one distribution over another, i.e. normal distribution or negative binomial, to represent the distribution of the leadtime demand.

**This can be easily modified to exclude planned requirements after leadtime, if desired.

*** The analysis assumes, for each B08 repair level scenario chosen, that the same repair level scenario was operating prior to the point in time being analyzed.

of the repairable cogs. This fill rate is a weighted fill rate based on the number of requisitions in each grouping. It has the interpretation of being the likelihood that a random requisition, chosen for a random item in that grouping, can be filled off the shelf, i.e. with no delay. (This computation has the option of either including or excluding non-replenishable demand, overrides on order quantities and safety levels, etc., and integrates the effectiveness of the procurement and repair functions.)

(4) The steady state average days of delay per requisition (averaged over all requisitions) and the average days of delay per requisition for delayed requisitions, again by Mark category, overall for the consumable cog, and for each repairable cog.

(5) The steady state average time weighted requisitions short per year (in requisition-days), again by Mark categories and overall for the consumable cog, and for each repairable cog.

(6) The dollar value of safety stock by Mark category and by cog, and the associated average days of safety stock (computed as the total dollar value of positive safety stocks divided by the average dollar daily demand for the Mark or cog of interest).

1.3.2 The Overall Weapon System SMA Computation

In addition to the above cog measures, which are displayed for a variety of shortage cost and min-max settings, there is an overall weapon system effectiveness calculation which utilizes one basic min-max interval for each cog (the user can select which of the min-max risk settings analyzed for that cog is to be the base case for that cog). The notion here is that for the ultimate decision maker, the overall SMA for the weapon system may not be sufficient; in addition he may well wish to have the assurance that each item has a lower bound on its SMA so that not only is the average SMA satisfactory but also that no one item is in very bad supply.

The program calculates and displays for a variety of SMA values the total procurement costs and the repair costs (thru the end of the fiscal year and thru an additional year) needed to insure that each of the 4 digit cogs (i.e. the weapon system by cog) represented has at least the SMA value specified. Hence, if a 75% level is specified, the program selects for each cog (using the base min-max risk setting specified by the user) the parameter settings (of the ones investigated) that will provide for that 4 digit cog at least a 75% SMA at minimum total cost. When it is not possible to achieve the overall fill rate sought for the weapon system (i.e., the existing SMA's are less than the SMA specified), the achieved fill rate will be computed using the fill rate closest to the one sought, and this SMA will be flagged as infeasible. It then combines these over all the cogs to yield the weapon system wide procurement and repair costs and the resulting weapon system SMA. This latter effectiveness is calculated by taking the 4 digit SMA associated with each cog, and then weighting them (by the number of requisitions in each 4 digit cog) to arrive at an overall weapon system SMA.

1.3.3 Ancilliary Statistics for the Weapon System of Interest

In addition the key ancilliary descriptive statistics for a given weapon system, which do not depend upon any particular management knob settings, include:

(1) Average dollar value of annual demand, by Mark category, over all consumables, and for each repairable cog;

(2) Average dollar value and days of leadtime demand (consumables only);

(3) A breakdown of the composition of the weapon system displaying, for each 4 digit cog, the numbers of items and numbers of annual requisitions involved;

(4) The numbers of items per cog receiving special repair, i.e. FIRM, CLAMP, etc.

(5) The numbers of items that were not processed, either because of missing information on the weapon system tape provided, or because the demand, both quarterly and end of leadtime, was equal to 0 .

2.0 OVERVIEW OF MODEL

2.1 The Different Types of Items Analyzed

To better appreciate the interworkings of the model, it is useful to consider the different types of items to which the model is applied. Note that the basic model is applied to each individual item to yield the item's specific SMA, cost, etc. and the results appropriately combined over a cog and over the weapon system.

The different types of items, each with its own UICP rules and model treatment, are:

(1) Mark 0 (very low demand) consumable items (the leadtime demand distribution in Levels is a Poisson distribution as is the case in the CAC Program).

(2) Low demand consumable (the low demand characteristic refers to the number of units demanded in the procurement leadtime; the distribution used in Levels, as in the CAC program, is the negative binomial).

(3) High demand consumable (the distribution used for the leadtime demand is the normal distribution as is the case in Levels)

(4) Low demand B08 repariable items, (repair levels 2, 3, or 4); the SMA calculation for these items must integrate the impact of both the procurement and repair settings. Note that repair level 1 is not included since from the weapon system tapes it is impossible to distinguish the high priority backorders (the only ones inducted under Repair Level 1) from backorders in general. The low demand characteristic makes itself felt both in the distributions of the leadtime demand and of the repair

turnaround time demand.

(5) High demand-B08 repairable items, repair levels 2, 3 and 4.

(6) Repairable items receiving special repair management; special computations have been developed for SPCC's FIRM program, and ASO's CLAMP and HIGH BURNER programs. (See Section 3 for a discussion of these programs.)

2.2 <u>Inputs Utilized and Interactions of Procurement and Repair Activities</u>

Figure 1.2 was earlier presented to display the interactions being modeled and the type of inputs required for the calculations. As discussed in detail in Appendix G, the mathematical models are realistic and flexible in that they allow the procurement review periods and repair review periods to vary, the size of requisition to be a random variable, can handle non-replenishable demand, allow more than one order from a vendor to be outstanding at any given time, model the time delay between receipt of requisition and receipt of returned carcasses, take into account explicitly the repair level scenario operating, etc.

It accomplishes this by first analytically decoupling the effectiveness calculations for the procurement activities from that of the repair activities, and then integrating the two. Essentially the model computes for a given procurement shortage cost, min/max setting, procurement leadtime (Mean and variance) demand rate, requisition size (mean, variance, and skewness), attrition rate, economic order quanty (or overrides) etc. the distribution of the number of units and requisitions that one expects to have backordered each procurement cycle, i.e. the interval between procurement buys. In doing this it takes into account the amount by which the inventory level may have fallen below the reorder point trigger before the buy is actually made (a function of the length of the procurement review period and the distribution of the requisition size). The assets included are all dues in, all carcasses including those in transit, and all RFI assets. It makes these calculations assuming that the repair side of the supply operation functions perfectly, i.e. no backorders due to insufficient inductions. See Figure 2.1 for the factors included in the procurement calculations and Appendix G for a detailed discussion.

At the same time the model analyzes the repair activities by computing the distribution of the number of backorders in a repair cycle, i.e. the interval between repair inductions, assuming that the procurement side of the house functions perfectly, i.e., no shortages due to insufficient buys. It does this using the repair induction triggers and RFI inventory objective rules specified, both for the case of the B08 items with Levels 2, 3, and 4,and for the case of specially managed repair schemes (namely CLAMP, FIRM, or HIGH BURNER). See Figure 2.2 for an overview of the repair process being modeled.

The model then, by computing the number of repair cycles in a procurement cycle, integrates the effectiveness calculations to arrive at the total distribution of the number of backordered units and requisitions in a procurement cycle. By comparing this amount with the total units demanded in a procurement cycle one can compute the SMA effectiveness measure. Also, by estimating when in the procurement cycle the backorders occurred, one can determine the average days of delay and the

Figure 2.1

PROCUREMENT REORDERING

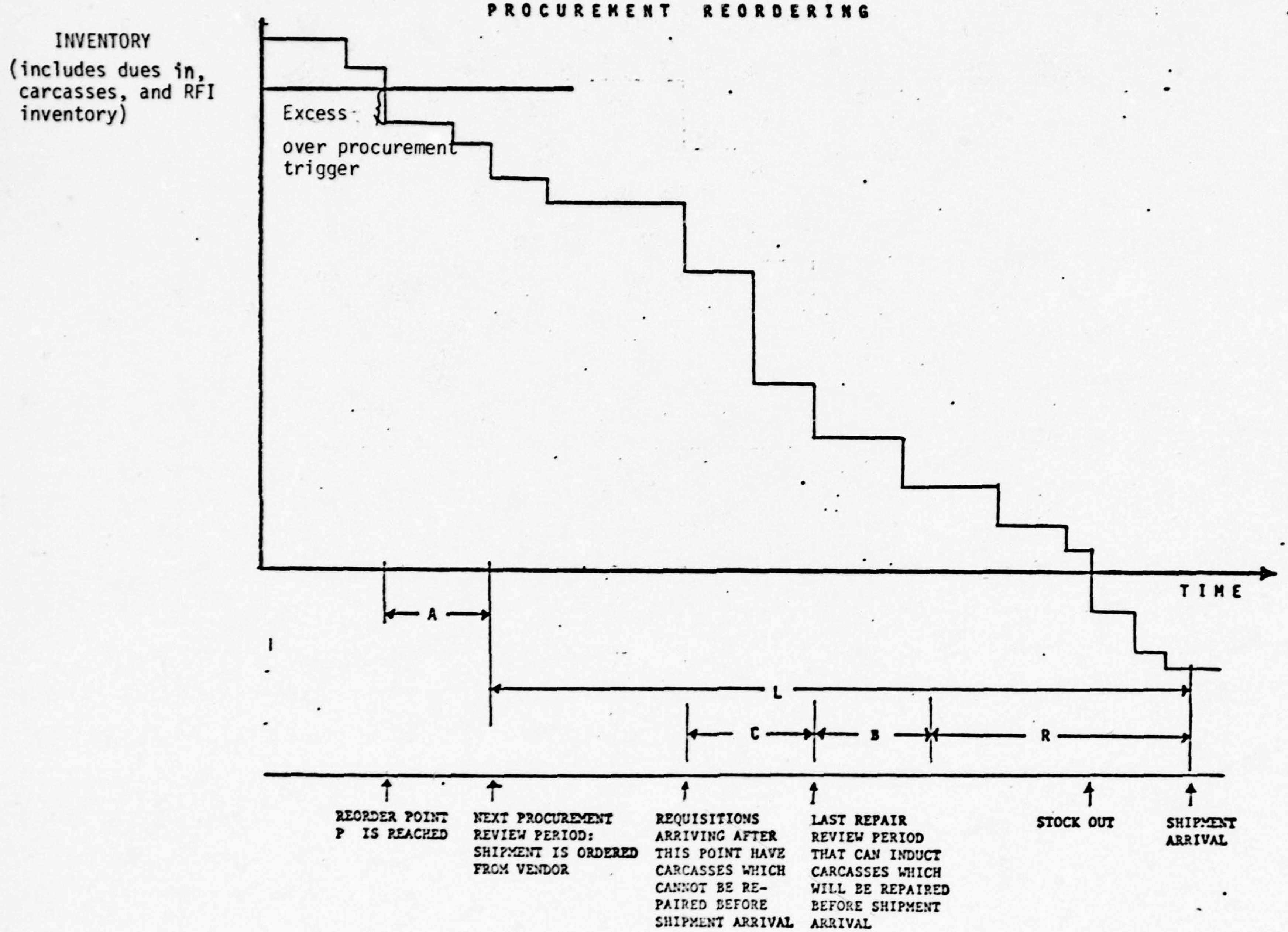
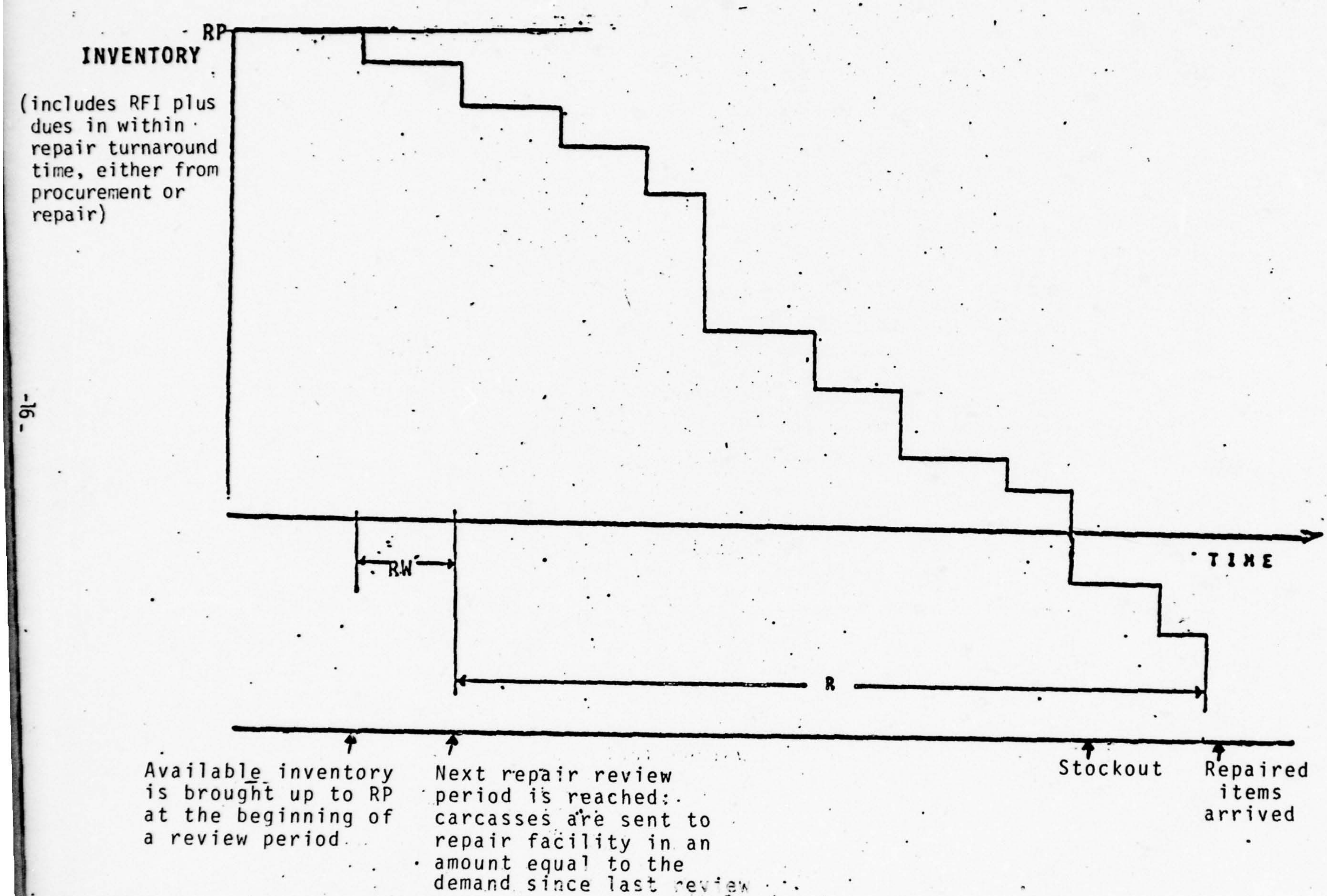


Figure 2.2

REPAIR INDUCTION OVER A TYPICAL REPAIR CYCLE



time weighted requisition short measures. Finally, using the procurement and repair triggers specified by the various user knobs, together with the various types of assets available at the point in time of the analysis, one can compute the procurement and repair costs to be incurred over any specified time period in the future,if the triggers are followed.

2.3 Sensitivity Findings

2.3.1 Factors Varied

This Section describes the impacts on the fill rate SMA of individually varying several physical and management decision parameters. It is presented at this time to help instill credibility in the results and suggest future areas for investigation. The cog in question is 2H (a repairable cog at SPCC) and the factors varied, for a fixed shortage procurement shortage and min-max risk factor, included:

i) the repair level scenarios, i.e. Level 2, 3 or 4.

ii) the procurement lot size (the possibilities investigaged were the Wilson EOQ quantity, and 3, 6, 9, 12, 15 and 18 months of attrition demand[1]).

iii) the procurement review period (the range was from 4 times a week[2], once a week[3], every 2 weeks, and every 4 weeks).

iv) the repair review period (the range investigated was the same as for the procurement review period; in this regard it is of interest to note that the current value is once every 2 weeks for SPCC, and reviews for repair induction "probes" once every week at ASO).

v) the carcass delay factor, representing the time delay between submission of requisitions and return of broken carcasses from the field. This was varied from 50 days, to 100 days. The longest program value for this is 100 days for SPCC and 30 days for ASO.

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[1]It is of interest to note that ASO now uses 12 months of attrition demand for all repairables to determine the procurement quantity.

[2]This is the value used at SPCC.

[3]This is the value used at ASO.

vi) the breakpoint or threshold used for the number of units demanded in the procurement leadtime to determine if the negative binomial or normal distribution is used to model the distribution of the leadtime demand. The values investigated are 4, 12 and 20.

vii) the inclusion or exclusion of the non-replenishable demand factor (the factor for 2H cog is 1.09).

Unless otherwise stated the fixed settings were:

a) procurement shortage cost of $400

b) min-max risk factors of (.15, .4), i.e. the constraints in the B08 Level risk computation is for an approximate SMA no smaller than 60% and no higher than 85%.

c) a carcass delay factor of 100 days

d) a breakpoint for the procurement leadtime distribution of 4.

e) a repair review period of once every 2 weeks, and a procurement review period of 4 times/week.

f) No non-replenishable demand factor, and the use of the Wilson lot size for the procurement quantity.

2.3.2 Findings

i) As the repair level scenarios varies from 2, 3 and 4, the SMA varies from .41, .55, and .87 respectively;

ii) As Q, the procurement quantity, varies from the Wilson lot size to various month of attrition, the SMA fill rate results are:

Procurement Quantity \ Repair Scenario	Repair Level 3	Repair Level 4
Wilson EOQ[1]	.5915	.8667
3 months	.5830	.8582
6	.5930	.8701
9	.6036	.8787
12	.6101	.8852
15	.6147	.8899
18	.6189	.8940

Hence we observe that the highest SMA's are associated with the largest buy quantity since then there are fewer exposures to a procurement stockout. On the other hand, there is more inventory carrying cost incurred.

[1] This is the current value for SPCC.

iii) As the breakpoint for the procurement leadtime distribution is varied from 4, 12, and 20, the SMA results are:

Distribution Breakpoint \ Repair Scenario	Level 3	Level 4
4[1]	.5915	.8667
12	.5286	.8099
20	.4937	.7859

Observe that the higher the breakpoint, the lower the SMA since the higher the breakpoint, the more often the negative binomial distribution is to be used, resulting in lower reorder points.

iv) As the procurement review period is varied

Procurement Review Period \ Repair Scenario	Level 3	Level 4
4 times/week[2]	.5915	.8667
once/week	.5908	.8660
once every 2 weeks	.5899	.8650
once every 4 weeks	.5880	.8631

[1] This was recently the value used by SPCC; 0 is the breakpoint for ASO.
[2] This is the value for SPCC.

Hence we observe that there is a very slight degradation in SMA, as is expected, if the procurement review period is increased, but that it is practically negligible for this repairable cog with low attrition factors since its SMA depends to a large extent on how timely repair inductions are made.

v) As the repair review period is varied, the results are somewhat more sensitive, namely:

Repair review period \ REPAIR SCENARIO	Level 3	Level 4
4 times per week	.6038	.8719
once per week	.5982	.8695
once every 2 weeks[1]	.5915	.8667
once every 4 weeks	.5789	.8604

Hence it appears that it is more warranted for repairable cogs to perform repair reviews frequently than it is for procurement reviews.

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[1]This is the amount value for SPCC.

vi) As the carcass delay factor, i.e. the delay between submission of requisitions and receipt of returned broken carcasses is varied one finds:

Carcass Delay Factor	Level 3	Level 4
50 days	.6059	.8810
100 days[1]	.5915	.8667

Hence as the carcass delay factor increases, the SMA falls.

vii) If the non-replenishment demand factor of 1.09 is included, the impacts, both for the Wilson EOQ and 1 year's attrition demand for the procurement quantity, are:

	Level 3 (EOQ)	Level 4 (EOQ)	Level 3 (1 year's attrition)	Level 4 (1 year's attrition)
No non-replenishment demand factor used	.5915	.8667	.6101	.8852
Non-replenishment demand factor used	.5646	.8387	.5915	.8655

Hence upon including the non-replenishment demand factor of 1.09 (i.e. increasing the demands by about 9%) leads to an overall reduction in SMA of about 3.5%. Note that the non-replenishment demand factor changes the calculation of the reorder points and induction triggers.

[1]This is the value suggested for use by SPCC, whereas ASO uses 30 days.

3.0 KEY REPAIR CONSIDERATIONS

This section is included to provide some technical background on the repair philosophies operating at the two ICP's, which is considerably more involved than is the case for procurement.[1] Some of this material has been included in two previous working reports, e.g., "THE PREDICTION OF SUPPLY MATERIAL AVAILABILITY FOR REPAIRABLES:"Suggested MODELING TREATMENT FOR CLAMP, FIRM, HIGH BURNER ITEMS FOR THE VARIOUS B08 REPAIR FUNDING SCENARIOS", Sept. 1977, and "MODIFICATIONS OF THE SUPPLY MATERIAL AVAILABILITY FORMULAE AND SOME SENSITIVITY FINDINGS", December, 1977 and is provided here in a summary form.

3.1 Overview of Repair Situation at the Two ICP's

SPCC has about sixty-thousand repairable items (in contrast with about 330 thousand consumable items) covering 25 repairable cogs, with a total annual demand in the last fiscal year of $270M and a FY77 repair budget of $73.6M (it is also of interest to note that currently SPCC has about 61 "weapon systems" under consideration covering about 106 thousand items). The great majority of the repairable items at SPCC are under the B08 philosophy which will be discussed subsequently. The other class of repairable items at SPCC, numbering about 2,200 now (or about 4% of their repairables), go under the classification of FIRM (Fleet) Intensified Repairables Management). This class of items was chosen for

[1] About the only difference in procurement philosophy (geared to attrition) between the 2 ICP's is that ASO overrides the FOQ for all repairables and instead buys one year's attrition; there is also a difference in the procurement review periods used, namely 4 times/week for SPCC and once a week for ASO.

special management because of a high CASREP situation and a critical mission essentially. This class of items is characterized by a quarterly negotiation in which the repair facilities are appraised of the repair quantities anticipated for the next quarter. The intent is to enable the repair facilities to take whatever measures are needed to guarantee that the capacity, and the bits and pieces necessary, will indeed be available to accomplish the needed repairs for the next quarter. For the FIRM items the broken carcasses are sent directly to the repair facility and can be inducted without any further bureaucratic approval or delay. Hence in some sense there is a continuous repair review. However, there can be a substantial delay, estimated to be on the average of the order of 100 days for SPCC, between the requisitions for RFI items and the actual receipt of the carcasses to be turned in; this is related to the fact that the broken carcasses are typically not turned in until the ship returns from its tour.

At ASO, there are roughly 50 thousand repairable items covering three cogs. Of these 50 thousand, about 35 thousand are B08 items, involving about 20% of the repair dollars. Another category, very similar to SPCC's FIRM program, is their so-called HIGH BURNER program, covering some 5 thousand items and 70% of the repair dollars. As with SPCC, the HIGH BURNER program is characterized by a quarterly negotiation in which the repair facilities agree to accomplish a stated number of repairs within the next quarter.

The remaining class of items at ASO goes by the name of CLAMP (Closed Loop Aeronautical Management Program) and comprises about 10 thousand items and 10% of the repair dollars. This class of items, operating under a performance incentive, is characterized by the fact that the repair facility, be it a Navy NARF or a private contractor, physically holds both the NRFI and RFI on hand assets and strives to be able to exchange a RFI item within 24 hours of the time that a non-RFI carcass is turned in. For this class of items the total amount of assets held by the repair facility is not influenced either by the procurement shortage cost knobs or repair knobs, but is preset to provide a 90% level of protection (over the repair turnaround time plus 15 days) against stockouts due to the repair side.[1] Hence the overall SMA for this class of items, taking into account procurement shortages as well, can be determined independent of any knobs and then combined with the SMA's of the other items in a given weapon system to determine the overall weapon system SMA.

3.2 The B08 Repair Levels

The basic philosophy in use by the ICP's is that, depending on the repair budget available in a given year for a given cog, the level of RFI assets at which one inducts carcasses into the repair channel or attempts to induct carcasses,[2] becomes a user's option. It should be understood that such tactics are only meaningful in the short term since in the steady state all of the carcasses should be eventually repaired (unless the item is being phased out). Hence the real tradeoff in the long term is between the lower inventory investment (if one operates with a lower induction point) and the

[1]Currently under discussion at ASO is the possibility of modifying the CLAMP rules to include some additional safety stock to cover vagaries in the procurement process.

[2]ASO utilizes so-called "probes" which are requests to induct items into repair and may or may not be accepted, based upon the backlog of the repair facility. SPCC has much more control over whether their inductions are accepted or not.

more unfilled requisitions that will result as a consequence of delaying the repair units.

However, it is clear that the actual induction triggers used for a given item can have a major impact on the item's SMA; indeed it can be shown that for highly[1] repairable items, the SMA level is much more sensitive to the repair induction points and repair review periods than to the corresponding quantities on the procurement side. This follows since there are typically many repair cycles in one procurement cycle so that inadequacies on the repair side have a much more pronounced effect than inadequacies on the procurement side (which is essentially geared only to attrition).

One of the kay thrusts of the effort performed has been to provide a set of computer programs which can generate SMA versus procurement budget curves and regain budget curves, depending on the repair level scenarios of interest. Consider now the various repair funding scenarios, namely repair levels 1,2,3 and 4.

3.2.1 Level 1 (High Priority Backorders)

It is not possible for the Programs to consider this scenario, the reason being that level 1 corresponds to only inducting carcasses when there are high priority backorders. This inability results from a data limitation associated with the data that is available on the weapon system tapes, namely one is not able to distinguish high priority backorders from backorders in general. Hence no analysis is possible for the repair level 1 scenario. In passing, however, it might be observed that this is not

[1] That is, those with low rates of attrition.

felt to be a serious limitation since the repair scenarios of most interest, in terms of the trade-offs between weapon systems SMA and procurement budget, should be the ones with fuller repair fundings; this is apparent since it is clearly most efficient to repair before procuring, both in terms of leadtime and dollars.

3.2.2 Level 2 (Immediate Requirements)

This corresponds to inducting carcasses only when there are backorders. More precisely, at the moment of each review, one examines any RFI assets (including any backorders as negative assets) plus those currently in the repair process (factored by a salvage rate) plus any procurements due in within the repair turnaround time. Only if this number is less than zero. does one induct carcasses and the number of carcasses inducted is that number (adjusted by the salvage factor) to bring the total assets in a RFI condition and in the repair channel back to the zero level. Note in this case no shortage cost knob for the repair problem is used to set the repair trigger so that the only decision parameters for this scenario are the procurement min/max risk factors and procurement shortage cost. Note also that the fill rates which result for this scenario, especially for highly repairable items, will tend to be very poor since carcasses are not inducted until there is indeed a bona-fide backorder.

3.2.3 Level 3 (Future Requirements Expected Within Repair Turnaround Time)

The carcass induction point in this case is the smaller of two quantities:

(1) The average demand during the repair turnaround time, denoted $\overline{D}_R$;

(2) The repair trigger point, calculated using the repair risk stockout equation in the UICP formula (based in turn on a repair shortage cost knob). We shall denote this trigger by T .

Note, if the repair risk setting provides any safety stock at all, then the average demand during the repair turnaround time will be smaller, i.e., (1) will be less than (2). However if the repair shortage cost knob is sufficiently small so that a negative safety stock situation arises, then (2) would be smaller than (1). Once again, one computes the RFI assets (including any backorders as a negative asset), plus those currently in the repair process (factored by salvage rate), plus any procurements due in within the repair turnaround time. If this number is less than the minimum of (1) and (2), one inducts a sufficient number of carcasses (accounting for salvage rate) to bring the total of assets, RFI or in the repair channel, back to the minimum of (1) and (2). If sufficient carcasses are not available[1] one inducts all that are available.

Note that for the most part the induction point for level 3 will be $\overline{D}_R$, again independent of the repair shortage cost knob. Also note that, aside from the small demand during the repair review period, the shipments received from the repair facility will on the average just balance out the backorders. Hence, the overall fill rates, even for Level 3, will not be very good for highly repairable items. This follows since even Level 3

[1] Note that if this occurs, the procurement side of the house is not functioning properly.

results in a slightly negative safety stock situation for the repair problem, and the realization that the overall SMA is largely a function of how timely repairs are actually made.

3.2.4 Level 4 (Future Requirements Beyond Repair Turnaround Time

This is the full repair funding scenario and is the only one where the computed economic repair quantity (a´ la Wilson) is used and where the repair shortage cost knob really makes itself felt. Using the same notation as in the level 3 computation for T and $\bar{D}_R$ and denoting the economic repair quantity by EROQ, then the induction point for level 4 is the sum of the level 3 induction point and the larger of:

(1) $(\bar{D}_R - T)^+$ plus 90 or 55 average days of supply[1]

(2) $EROQ + (T - \bar{D}_R)^+$,

where

$$X^+ = X \text{ if } X > 0$$
$$\quad\; 0 \text{ if } X \leq 0.$$

Essentially, then, for SPCC, e.g., if there is any safety stock provided by the repair trigger, i.e., if $T > \bar{D}_R$ (as will most likely be the case), then the carcass induction point for level 4, is the sum of the demand during the repair turnaround time (level 3), plus the larger of;

(1) three month's average demand, or

(2) the sum of the economic repair quantity plus the

---

[1]For SPCC, the number is currently 90 days and for ASO it is 55.

actual safety stock provided by the repair shortage cost UICP calculation.[2]

As before, one inducts a sufficient number of carcasses, adjusted for the salvage rate, to bring the assets up to this level; hence it is still an "inventory objective" induction philosophy which gives rise to smaller induction quantities than was the intent in the derivation of the economic repair quantity used. This has the impact of having more repair cycles in a procurement cycle than would result if the induction scheme were to induct a number of carcasses to bring the level of inventory back to the induction point plus the economic repair quantity. Hence since there are more repair cycles, there will be more exposures to stockout, and thus a lower SMA. The Principal Investigator feels strongly this is one of the areas where the UICP procedures need to be changed.

3.3 Overview of Special Repair Management Schemes

The basic notion here is that for each repairable item one can tell from information on the weapon system tape the particular repair philosophy being followed for that item. The program then utilizes the appropriate set of formulae for that item to compute its SMA and costs, and combines the SMA for that item with the SMA's for the other items in the cog using the weighted requisition concept.

3.3.1 Treatment for FIRM and HIGH BURNER

The above repair management schemes are characterized by:

(1) a quarterly negotiated repair capability;

[2] If the repair trigger gives rise to a negative safety stock, then the level 4 induction point is the repair trigger level (i.e., the level 3 induction point plus the larger of (1) the difference between $\bar{D}_R$ and the trigger plus 90 days supply, (2) the economic repair quantity.

(2) a continuous repair review (since carcases are sent immediately to the repair facility ypon receipt);

(3) the fact that there is no notion of an induction point (i.e., upon receipt at the repair facility, the contractor can begin repair without any formal authorization); and

(4) no use of an economic repair quantity.

Because of these reasons it is felt that a separate model for projecting the SMA versus procurement budget is desirable. The key features of this model, custom tailored to the FIRM and HIGH BURNER items, are:

(a) It is independent of any of the repair level scenarios; hence it will run quickly and inexpensively;

(b) Backorders and unfilled requisitions are assumed to be dependent on the management decision associated with the negotiated repair turnaround time (and any induction waiting time), and on the level at which the procurement reorder point is set. Utilizing the Levels rules, this procurement point is based on total assets, and includes those items that are RFI, NRFI, those currently in repair, those carcasses being forwarded, and any outstanding procurement orders.

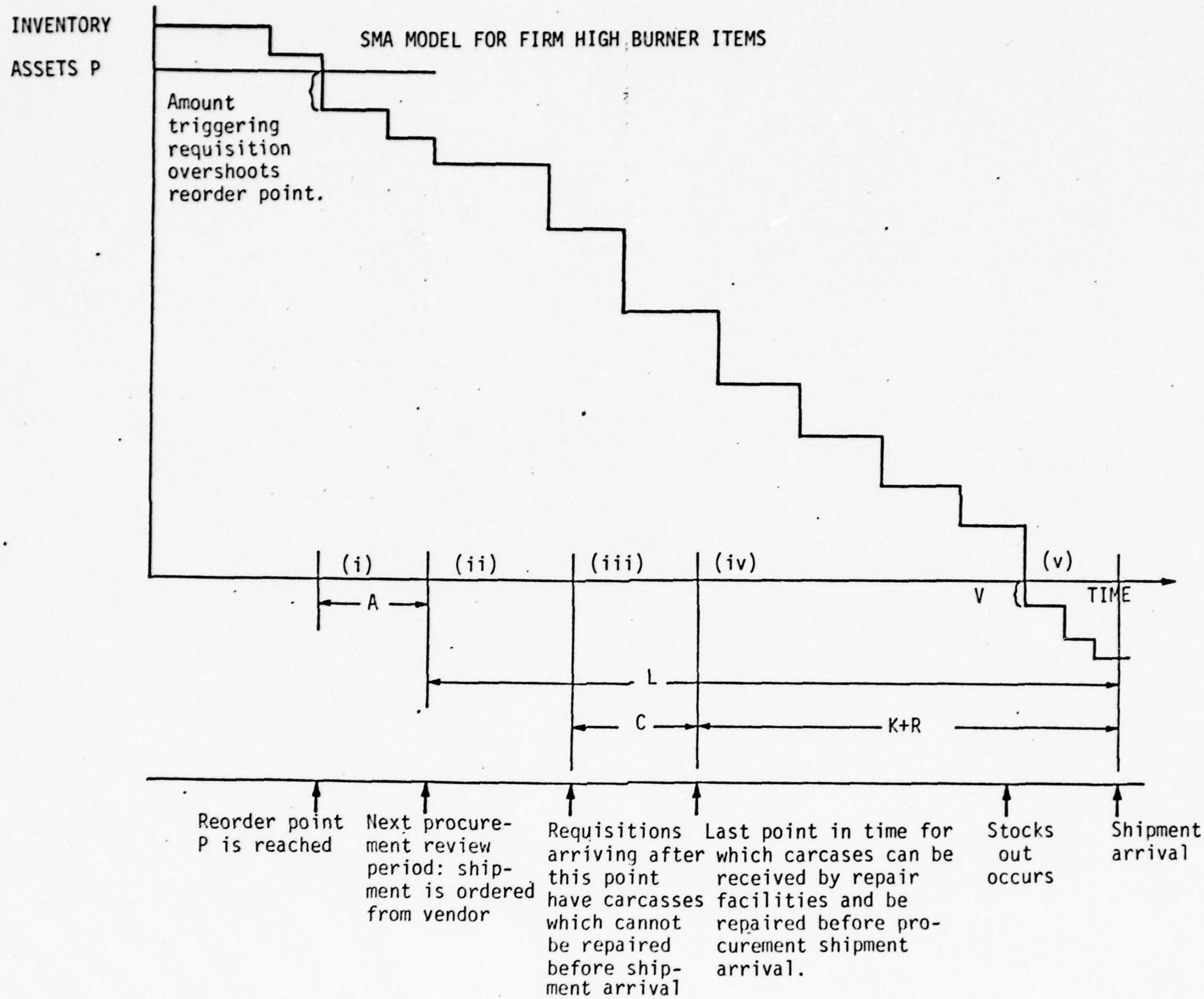
(c) It takes into account explicitly any lag between the receipt of requisitions and receipt of returned carcasses; the number in use for SPCC is 100 days; and 30 days for ASO. This can be charged through the use of an input card.

(d) It also takes into account an induction waiting period, denoted K, currently set at 15 days, added to the repair time.

Before looking at the details of the model, the motivation for a special treatment for this class of items, even though it accounts for only 4% of SPCC's repairables and 10% of ASO's repairables, is that it constitutes a group of items whose projection of SMA is critical to that of projecting the SMA for the weapon system. The model suggested has the added appeal that in addition to more accurately representing the actual situation, the computing effort is less than that for the B08 items and does not need to be repeated for different repair shortage costs, or repair level scenarios.

The appropriate modeling depiction for this set of assumptions is presented below (see Fig.3.1) where one notices the SMA is a function of the procurement reorder point (in turn a function of the procurement shortage cost knob and the procurement min/max risk factors), the repair turn-around time (mean and variance), the lag between requisitions and receipt of carcasses, and the induction waiting period; this is of course in addition to mean and variance of the demand rate, leadtime, attrition rates, etc. The basic idea is to realize that any carcasses received after point (iv) (see Fig.3.1) cannot be repaired in time to help fill requisitions during the critical exposure period in the procurement cycle being studied. Similarly any requisitions received after the point in time denoted (iii) will have to be filled out of RFI stocks or with repaired carcasses associated with requisitions arriving before (iii). Hence in terms of the fill rate, requisitions received during the elapsed time from (i) to (iii) have to be treated differently than those received after (iii). In particular the inventory level P must protect only against attrition during the period

Figure 3.1


INVENTORY
ASSETS P
SMA MODEL FOR FIRM HIGH BURNER ITEMS
Amount triggering requisition overshoots reorder point.
(i)
(ii)
(iii)
(iv)
(v)
A
V
TIME
L
C
K+R
Reorder point P is reached
Next procurement review period: shipment is ordered from vendor
Requisitions arriving after this point have carcasses which cannot be repaired before shipment arrival
Last point in time for which carcases can be received by repair facilities and be repaired before procurement shipment arrival.
Stocks out occurs
Shipment arrival

((i), (iii),) against total gross demand in the perid ((iii), (v)) and finally with the attrited units associated with that portion of the requisition remaining when the inventory first falls below P. Hence the idea is to use the mean and variance of this total demand and compare it with P to compute the average backorders accumulated over the period ((i), (v)). The detailed formulae are included in Appendix G.

3.3.2 Model Treatment for CLAMP Items

This group of items, accounting for 20% of ASO's repairables is actually the easiest to model. This is because the asset positions, both for RFI and non-RFI units, are actually physically held by the repair facility and are independent of any repair level funding scenarios and of any shortage cost knobs. Rather, the system on hand assets or on-hand inventory objective has been developed to yield a 90% protection over the repair turnaround time plus 15 days (which is retrograde/RFI pipeline time); this latter quantity may be 30 days for some items. This level of protection is to facilitate the one for one exchange program operating for the CLAMP[1] program. However, it should be noted that the system on-hand assets held by the contractor also have to protect against variabilities in the procurement leadtime, and hence the overall SMA will be less than the 90% figure.[2]

Table 3.1 is used for CLAMP items to determine the actual procurement reorder point in CLAMP, and this Table displays the system stock

[1]For the CLAMP items an additional requirement over and above that for HIGH BURNER, has been negotiated: namely that the repair facility is required to manage his repairs so that he can exchange a RFI item for a non-RFI item within 24 hours from the time of receipt of the carcass.

[2]This feature may be changed in the future.

to be used for each CLAMP item as a function of the average demand during the total turnaround time (including the 15 days retrograde during the RFI pipeline time).

The use of one level of service for this class of items again helps to reduce the total running time of the program which relates a weapon system SMA to its procurement budget. The SMA of the CLAMP items in the weapon system of interest will be computed once and its impact on the 4 digit COG's SMA determined by using the SMA of the CLAMP items and the per- cent of requisitions in the 4 digit COG that are associated with CLAMP managed items.

To make the SMA calculation for a CLAMP item, it is first necessary to compute the value of the reorder point P , determined from Table 3.1 (which is in turn calculated using a Poisson distribution with a mean equal to the average demand during the contractor's processing time plus 15 days, and a protection level of 90%). One then exercises the model developed for the FIRM and HIGH BURNER case, described in the previous section. Hence the difference between CLAMP and FIRM/HIGH BURNER is that the procurement reorder point is set in a different manner than is the case for the FIRM/HIGH BURNER case where it is set based on shortage cost knobs.

TABLE 3.1 REORDER POINT FOR CLAMP ITEMS

Demand During Turn-Around Time (including 15 day induction waiting time)	Total Contractor Float (Non-RFI and RFI)
0 to .1	0
.1 + to .5	1
.5 + to 1.1	2
1.1 + to 1.7	3
1.7 + to 2.4	4
2.4 + to 3.1	5
3.1 + to 3.8	6
3.8 + to 4.6	7
4.6 + to 5.4	8
5.4 + to 6.2	9
6.2 + to 7.0	10
7.0 + to 7.8	11
7.8 + to 8.6	12
8.6 + to 9.4	13
9.4 + to 10.2	14
10.2 + to 11.1	15
11.1 + to 11.9	16
11.9 + to 12.8	17
12.8 + to 13.6	18
13.6 + to 14.5	19
14.5 + to 15.3	20
15.3 + to 16.2	21
16.2 + to 17.1	22

(Table 3.1 continued...)

Demand During Turn-Around Time (including 15 days induction waiting time)	Total Contractor Float (Non-RFI and RFI)
17.1 + to 17.9	23
17.9 + to 18.8	24
18.8 + to 19.7	25
19.7 + to 20.5	26
20.5 + to 21.4	27
21.4 + to 22.3	28
22.3 + to 23.2	29
23.2 + to 24.1	30
24.1 + to 24.9	31
24.9 + to 25.8	32
25.8 + to 26.7	33
26.7 + to 27.6	34
27.6 + to 28.5	35
28.5 + to 29.4	36
29.4 + to 30.3	37
30.3 + to 31.2	38
31.2 + to 32.1	39
32.1 + to 33.0	40
33.0 + to 33.9	41
33.9 + to 34.8	42
34.8 + to 35.7	43
35.7 + to 36.6	44
36.6 + to 37.5	45
37.5 + to 38.4	46

(Table 3.1 continued...)

Demand During Turn-Around Time (including 15 day induction waiting time)	Total Contractor Float (Non-RFI and RFI)
38.4 + to 39.3	47
39.3 + to 40.2	48
40.2 + to 41.1	49
41.1 + to 42.0	50
42.0 + to 42.9	51
42.9 + to 43.9	52
43.9 + to 44.8	53
44.8 + to 45.7	54
45.7 + to 46.6	55
46.6 + to 47.5	56
47.5 + to 48.4	57
48.4 + to 49.3	58
49.3 + to 50.1	59

4.0 DESCRIPTION OF REPORTS

There are five reports produced by a run: options for the weapon system, parameter inputs by cog, data edit counts and a summary of the input tape, effectiveness tables (by mark category, repairable cog, combined consumable items, and combined repairable items), and overall weapon system fill rates and budgets. In addition, plots of fill rate versus budget can be requested for any or all of the effectiveness tables. Samples and explanations of these reports are included in this section. The first two outputs include default and "in use" values, as well input values if they are out of the acceptable range. All outputs in Section 4 are for a portion (510 items) of SPCC's Terrier Weapon System.

4.1 Options in Effect

The following is a sample output for a portion of the SPCC Terrier Weapon System, run on April 11, 1978.

SPCC TERRIER WEAPON SYSTEM 4/11/1978

OPTIONS IN EFFECT

	DEFAULT	IN USE	INPUT, IF DIFFERENT FROM IN USE.
TIME-WEIGHTED REQUISITIONS SHORT AND AVERAGE DAYS OF DELAY	1	1	0.
UNFILLED PORTION OF REQUISITION WHEN HIT REORDER LEVEL	1	1	0.
CARCASS RETURN MAD AND REPAIR SURVIVAL RATE MAD	0	0	0.
UNITS BACKORDERED AT START OF PROCUREMENT CYCLE	1	1	0.
REQUISITIONS SHORT AT START OF PROCUREMENT CYCLE	1	1	0.
DIFFERING BOB SCENARIOS BY COG	0	0	0.
RUN FOR ALL 3 BOB REPAIR SCENARIOS(2,3,4)	0	0	0.
REPAIR SCENARIO FOR ALL BOB ITEMS IN WEAPON SYSTEM	3	4	0.
CONTRACT DUE AWARDED (FOR ASSETS COMPUTATION)	1	1	0.
PR DUE COMMITTED (FOR ASSETS COMPUTATION)	1	1	0.
OUTPUT EFFECTIVENESS TABLES BY MARK CATEGORY	1	1	0.
OUTPUT PLOTS BY MARK CATEGORY	0	0	0.
OUTPUT PLOTS OF COMBINED CONSUMABLE ITEMS	1	0	0.
OUTPUT EFFECTIVENESS TABLES BY REP. COGS	1	1	0.
OUTPUT PLOTS BY REP. COGS	0	0	0.
OUTPUT PLOTS OF COMBINED REPAIRABLE ITEMS	1	0	0.
BREAKPOINT FOR PROCUREMENT IN SMA COMPUTATIONS	0	4	0.
BREAKPOINT FOR REPAIR IN SMA COMPUTATIONS	0	4	0.
CONSTANT FOR K=Z1 IN LEVELS FORMULA FOR P (CONS)	1	0.000	0.000
CONSTANT FOR K=Z1 IN LEVELS FORMULA FOR P (REP)	1	1.000	0.000
CONSTANT I IN LEVELS FORMULA FOR P	1	1	0

A 1 INDICATES INCLUDE IN THE COMPUTATIONS, A ZERO INDICATES OMIT FROM THE COMPUTATIONS

FIGURE 4. 1

Sample Options In Effect For
Terrier Weapon System

4.2 Parameter inputs

For each cog -- currently a maximum of 5 cogs are possible -- different parameters can be specified, again default values do exist. Figure 4.2 lists the parameters that should be specified for each cog, and gives the actual values that were used for cog 1H for the SPCC Terrier Weapon System. Note that items designated repairable existing in an otherwise consumable (the repair indicator is not set) which are found in an otherwise repairable cog, are included in an output with the single consumble cog, using cog's attributes.

PARAMETERS, BY COG

COG 1H	DEFAULT	IN USE	INPUT, IF DIFFERENT FROM IN USE.
PROCUREMENT SHORTAGE COST(1) (DOLLARS)	1.00	0.01	0.0
PROCUREMENT SHORTAGE COST(2) (DOLLARS)	5.00	1.00	0.0
PROCUREMENT SHORTAGE COST(3) (DOLLARS)	10.00	10.00	0.0
PROCUREMENT SHORTAGE COST(4) (DOLLARS)	100.00	100.00	0.0
PROCUREMENT SHORTAGE COST(5) (DOLLARS)	500.00	10000.00	0.0
REPAIR SHORTAGE COST(1) (DOLLARS)	0.10	0.01	0.0
REPAIR SHORTAGE COST(2) (DOLLARS)	0.50	1.00	0.0
REPAIR SHORTAGE COST(3) (DOLLARS)	1.00	10.00	0.0
REPAIR SHORTAGE COST(4) (DOLLARS)	100.00	100.00	0.0
REPAIR SHORTAGE COST(5) (DOLLARS)	100000.00	10000.00	0.0
MINIMUM RISK FACTOR(1)	0.10	0.0	0.0
MINIMUM RISK FACTOR(2)	0.0	0.01	0.0
MINIMUM RISK FACTOR(3)	0.20	0.05	0.0
MINIMUM RISK FACTOR(4)	0.0	0.0	0.0
MINIMUM RISK FACTOR(5)	0.0	0.0	0.0
MAXIMUN RISK FACTOR(1)	0.4000	1.00	0.0
MAXIMUN RISK FACTOR(2)	1.0000	0.40	0.0
MAXIMUN RISK FACTOR(3)	0.2500	0.50	0.0
MAXIMUN RISK FACTOR(4)	0.0	0.0	0.0
MAXIMUN RISK FACTOR(5)	0.0	0.0	0.0
INTEREST RATE, PROCUREMENT	0.1000	0.1000	0.0
INTEREST RATE, REPAIR	0.1000	0.1000	0.0
STORAGE COST	0.0100	0.0100	0.0
MIN SAFETY STOCK (IN MONTHS)	999.0000	999.0000	0.0
PROC. BREAKPOINT FOR DEMAND	0.0	0.0	0.0
REPAIR BREAKPOINT FOR DEMAND	0.0	0.0	0.0
LENGTH OF PROC. REVIEW (IN WEEKS)	0.25	0.25	0.0
LENGTH OF REPAIR REVIEW (IN WEEKS)	2.00	2.00	0.0
ORDER COST, MARKS 1 AND 2	102.00	69.16	0.0
REPAIR ADMINISTRATIVE COST	102.00	9.40	0.0
ORDER COST, LOW DEMAND	102.00	69.16	0.0
PROC. ORDER COST, NEGOTIATED	275.00	132.29	0.0
PROC. ORDER COST, ADVERTISED	325.00	132.29	0.0
MAX UNPRICED PURCHASE ORDER VALUE	8000.00	2200.00	0.0
DAYS OF SAFETY STOCK (CLAMP ITEMS)	15.	15.	0.0
INDUCTION WAITING PERIOD (DAYS)	15.	15.	0.0
CARCASS ARRIVAL DELAY (DAYS)	100.	100.	0.0
PORTION OF FISCAL YEAR REMAINING	1.	1.	0.0
OVERRIDE RECORDER QUANTITY WITH ONE YEAR ATTRITION	1.	1.	0.0
DAYS OF DEMAND FOR LEVEL 4 REPAIR TRIGGER	90.	90.	0.0
NON-RECURRING DEMAND CONSTANT, MARK 0	1.0000	1.0000	0.0
NON-RECURRING DEMAND CONSTANT, MARK 1	1.0000	1.0000	0.0
NON-RECURRING DEMAND CONSTANT, MARK 2	1.0000	1.0000	0.0
NON-RECURRING DEMAND CONSTANT, MARK 3	1.0000	1.0000	0.0
NON-RECURRING DEMAND CONSTANT, MARK 4	1.0000	1.0000	0.0

FIGURE 4.2

List of Parameter Inputs For Cog 1H For SPCC Terrier Weapon System

4.3 Data Edit Counts and Input File Overview

When all data for given weapon system have been entered and processed, counts of any edits performed and the reasons for not processing items are output. This enables the user to determine the validity of the effectiveness tables, based on the percentage of usable data. This may show cause for reviewing and updating the data, then re-running the weapon system. The input file overview contains information regarding the number of requisitions by cog and mark category as well as the number of items processed for each of these classifications, yielding a clear picture of which types and quantities of items exist in the weapon system.

SUMMARY FOR WEAPON SYSTEM

REPAIRABLE ITEMS BY 4-DIGIT COG

COG 4N	B08	FIRM/HIGH BURNER	CLAMP	TOTAL
NUMBER OF ITEMS	2.	0.	0.	2
NUMBER OF REQ'N	5.	0.	0.	5.35

TOTAL NUMBER OF REPAIRABLE ITEMS: 2

TOTAL NUMBER OF REPAIRABLE REQUISITIONS: 5.35

CONSUMABLE ITEMS

	MARK0	MARK1	MARK2	MARK3	MARK4	TOTAL
NUMBER OF ITEMS	3	82	0	3	1	89
NUMBER OF REQ'N	0.26	303.63	0.0	2.83	0.44	307.14

TOTAL NUMBER OF ITEMS PROCESSED: 91
TOTAL NUMBER OF REQUISITIONS: 312.49
SKIPPED CONSUMABLE ITEMS 402 SKIPPED REPAIRABLE ITEMS 17

419 SKIPPED ITEMS IN WEAPON SYSTEM OUT OF 510

FIGURE 4.3.a

Input File Overview For Terrier Weapon System

QUARTERLY DEMAND OF ZERO	392	
FREQUENCY IS ALSO ZERO	28	
DEMAND, END OF LEAD TIME	0	0
REQUISITION FREQUENCY	74	0
GREATER THAN DEMAND	3	
REGENERATIONS:		
QUARTERLY	1	1
END OF LEAD TIME	1	0
REPAIR SURVIVAL RATE	0	0
CARCASS RETURN RATE	1	1.
DEMAND DURING REPAIR TAT	0	
VARIANCES:		
REPAIR	1	0
PROCUREMENT	75	0
PRICES:		
REPLACEMENT	0	0
UNIT	0	0
REPAIR COST	0	0
MADS:		
CARCASS RETURN	0	0
REPAIR SURVIVAL RATE	0	0
NAVY TAT	0	0
COMMERCIAL TAT	0	0
PROC. LEAD TIME	0	0
SET UP COSTS:		
MANUFACTURER	0	0
REPAIR	0	0
PROCUREMENT PROBLEM TAT	0	0
OBSOLESCENCE RATE	0	0
PROCUREMENT LEAD TIME	0	0
LESS THAN ZERO	0	
ESSENTIALITY	0	0
ASSETS:		
RESERVATIONS	0	0
BACKORDERS	0	0
PPR DURING LT	0	0
PPR AFTER LT	0	0
RFI	0	0
NFFI	0	0
CONTRACT DUE	0	0
RECOMMENDED DUE	0	0
MAD OF DEMAND (SQUARED)	13	0

FIGURE 4.3.b

EDITS*ON THE INPUT DATA FOR TERRIER WEAPON SYSTEM

The first number indicates hitting the lower bound, the second number (when it exists) indicates hitting the upper bound.

*see Appendix A for more detail on the edits

4.4 Dollar Value of Annual Demand

The dollar value of annual demand is computed for each consumable mark category and for each repairable cog and output as shown in Figure 4.4. This value is used to convert the dollar value of safety stock and the dollar value of lead time demand to days. It is computed as the sum over all items in the mark category or cog of the annual demand multiplied by the unit price.

DOLLAR VALUE OF ANNUAL DEMAND

CSMA	MARK	0	651.58
CSMA	MARK	1	4377.62
CSMA	MARK	3	12258.36
CSMA	MARK	4	3430.00
SMAB08	COG 1H		1502.67

FIGURE 4.4

Dollar Value of Annual Demand For The Terrier Weapon System

4.5 Effectiveness Tables

Figure 4.5 is an example of the output effectiveness tables. Three min/max risk factor settings and five procurement/repair shortage cost settings are possible. For each of these 15 possible settings, fill rates, average days of delay (conditioned upon delayed requisitions, and unconditional), time weighted requisitions short, safety stock measures and lead time demands are computed.

MIN RISK FACTOR	MIN RISK FACTOR	PROC. SHORTAGE COST	PROC. BUDGET FIRST YEAR	PROC. BUDGET TWO YEARS	FILL RATE	AVE. DAYS OF DELAY FOR DELAYED REQUISITIONS	AVE. DAYS OF DELAY FOR ALL REQUISITIONS	AVERAGE REQUISITION DAYS SHORT PER YEAR	DOLLAR VALUE OF SAFETY STOCK	AVE. DAYS OF SAFETY STOCK	DOLLAR VALUE OF LEAD TIME DEMAND	DAYS OF LEAD TIME DEMAND
CONSUMABLE MARK 3												
0.0	1.00	0.	7574.	27453.	0.3573	411.30	264.3	303.6	0.0	0.	11646.	347.
0.0	1.00	1.	7574.	27453.	0.3573	411.30	264.3	303.6	0.0	0.	11646.	347.
0.0	1.00	10.	7574.	27453.	0.3573	411.30	264.3	303.6	0.0	0.	11646.	347.
0.0	1.00	100.	7574.	27453.	0.3573	411.30	264.3	303.6	0.0	0.	11646.	347.
0.0	1.00	10000.	11684.	31563.	0.8745	707.98	88.9	38.7	3059.9	91.	11646.	347.
0.01	0.40	0.	26294.	46173.	0.8415	354.43	56.2	32.9	6773.7	202.	11646.	347.
0.01	0.40	1.	26294.	46173.	0.8415	354.43	56.2	32.9	6773.7	202.	11646.	347.
0.01	0.40	10.	26294.	46173.	0.8415	354.43	56.2	32.9	6773.7	202.	11646.	347.
0.01	0.40	100.	26294.	46173.	0.8415	354.43	56.2	32.9	6773.7	202.	11646.	347.
0.01	0.40	10000.	28934.	48813.	0.9257	458.84	34.1	14.8	9413.7	280.	11646.	347.
0.05	0.50	0.	20154.	40033.	0.7231	346.04	95.8	73.9	633.7	19.	11646.	347.
0.05	0.50	1.	20154.	40033.	0.7231	346.04	95.8	73.9	633.7	19.	11646.	347.
0.05	0.50	10.	20154.	40033.	0.7231	346.04	95.8	73.9	633.7	19.	11646.	347.
0.05	0.50	100.	20154.	40033.	0.7231	346.04	95.8	73.9	633.7	19.	11646.	347.
0.05	0.50	10000.	22794.	42673.	0.9055	524.78	49.6	21.6	3273.7	97.	11646.	347.
CONSUMABLE MARK 4												
0.0	1.00	0.	3435.	7085.	0.0223	279.27	273.0	118.9	0.0	0.	3250.	346.
0.0	1.00	1.	3435.	7085.	0.0223	279.27	273.0	118.9	0.0	0.	3250.	346.
0.0	1.00	10.	3435.	7085.	0.0223	279.27	273.0	118.9	0.0	0.	3250.	346.
0.0	1.00	100.	3435.	7085.	0.0223	279.27	273.0	118.9	0.0	0.	3250.	346.
0.0	1.00	10000.	7565.	11215.	0.8864	62.11	7.1	3.1	880.1	94.	3250.	346.
0.01	0.40	0.	7005.	10655.	0.7832	77.90	16.9	7.4	320.1	34.	3250.	346.
0.01	0.40	1.	7005.	10655.	0.7832	77.90	16.9	7.4	320.1	34.	3250.	346.
0.01	0.40	10.	7005.	10655.	0.7832	77.90	16.9	7.4	320.1	34.	3250.	346.
0.01	0.40	100.	7005.	10655.	0.7832	77.90	16.9	7.4	320.1	34.	3250.	346.
0.01	0.40	10000.	7565.	11215.	0.8864	62.11	7.1	3.1	880.1	94.	3250.	346.
0.05	0.50	0.	6690.	10340.	0.7035	84.85	25.2	11.0	5.1	1.	3250.	346.
0.05	0.50	1.	6690.	10340.	0.7035	84.85	25.2	11.0	5.1	1.	3250.	346.
0.05	0.50	10.	6690.	10340.	0.7035	84.85	25.2	11.0	5.1	1.	3250.	346.
0.05	0.50	100.	6690.	10340.	0.7035	84.85	25.2	11.0	5.1	1.	3250.	346.
0.05	0.50	10000.	7565.	11215.	0.8864	62.11	7.1	3.1	880.1	94.	3250.	346.

FIGURE 4.5

Sample Output Effectiveness Tables For 2 Consumable Mark Categories in the Terrier Weapon System

It should be noted that the average days of delay for all reauistions is a monotonically decreasing function of the repair scenario i.e., going from repair level 2 to repair level 3 to repair level 4 results in a decrease in the unconditional average days of delay. This monotonic property does not hold for the average days of delay for delayed requisitions since it is a function of both the unconditional average days of delay and fill rate. Thus the average days of delay for delayed requisitions under repair level 3 may be greater than the corresponding delays under repair levels 2 and 4 if the decrease in fill rate is not in the same proportion as the decrease in unconditional days of delay when going from repair level 2 to level 3 to level 4.

4.6 Overall Weapon System

An array of 5 fill rates sought for the entire weapon system is input. For each of the elements of this array, a fill rate and the resulting procurement and repair budgets are computed and printed using the values associated with the first min/max risk factor setting for the given repairable cog or for the combined consumable items.

CONSUMABLE AND REPAIRABLE BUDGETS COMBINED

FILL RATE SOUGHT	FILL RATE ACHIEVED	PROCUREMENT BUDGET THROUGH PRESENT YEAR	PROCUREMENT BUDGET THROUGH SECOND YEAR	REPAIR COST THROUGH PRES. YEAR	REPAIR COST THROUGH TWO YEARS	TOTAL COST PRESENT YEAR	TOTAL COST TWO YEARS
0.7000	0.7383	25083.	49605.	958.	1311.	26041.	50916.
0.7500	0.7383	25083.	49605.	958.	1311.	26041.	50916.
0.8000	0.7789	25177.	49699.	958.	1311.	26135.	51010.
0.8500	0.8636	27872.	52393.	1243.	1905.	29115.	54298.
0.9500	0.9420	39680.	64205.	1243.	1905.	40923.	66110.

FIGURE 4.6

Combined Consumable and Repairable Budgets for the Terrier Weapon System, repair level 4

The total costs through the present fiscal year and through the following fiscal year are given. This table provides an indication of the budget necessary to achieve a prescribed fill rate and vice versa. An additional column, on the right, will indicate an infeasible fill rate achieved, when the fill rate sought is greater than all of the fill rates associated with the first min/max fisk factor setting.

4.7 Plots of Fill Rate vs. Budget

The final type of output available--suppressed in the default mode--is a plot.

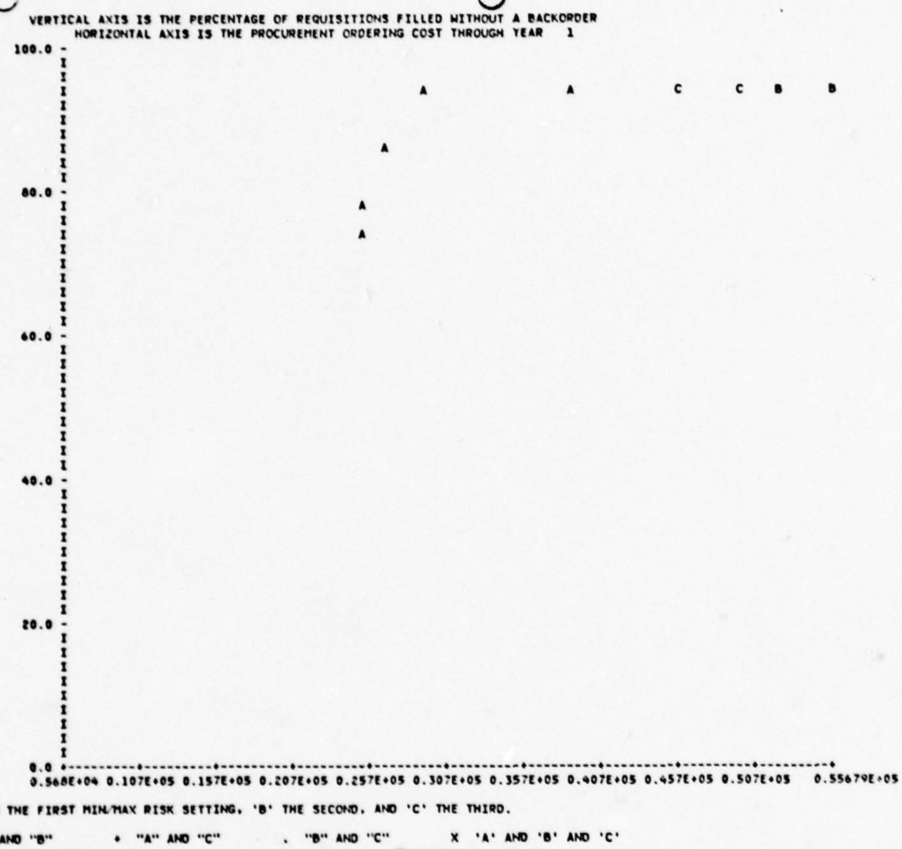


FIGURE 4.7

Sample Plot of Fill Rate vs. Budget for Consumable Items In Terrier Weapon System

Connecting the points corresponding to like min/max risk factor settings provides a visual indication of the change in budget and fill rate as the procurement and/or repair shortage costs "knobs" are turned.

5.0 OPERATION

This section deals with the operation of the model in a production mode.

5.1 Job Control Language

Figure 5.1 is the JCL (job control language) and input data for a run of the terrier weapon system on an IBM 370. The JCL may differ for the SPCC IBM 360 computer, but the format of the data remains the same. Note that this run compiles and link edits to an existing FORTRAN load module library named LDLIB. The //GO.FT01F001 DD statement specifies that the input data is on a disk volume. If the input data were on an unlabeled tape with serial number TERR01 the JCL for the //GO.FT01F001 DD statement would be modified as follows:

```
//GO.FT01F001 DD UNIT=TAPT,VOL=SER=TERR01,
//   DISP=(OLD,PASS),LABEL=(,NL,,IN),
//    DCB=(RECFM=FB,LRECL=290,BLKSIZE=8700)
```

In either case, unit one must be defined to hold the weapon system data as prescribed in Appendix B.

5.2 Selecting Options and Specifying Output

Each ICP (ASO and SPCC) may require different options for their respective weapon systems. Each run requires one card of input data. This will specify the ICP and the date, as well as the various run options and types of output desired. All options are described in detail in Appendix C in the common block OPTION. The FORTRAN format of this card is:

INPUT	FORMAT	
ICP	I1	0 if SPCC, 1 if ASO
Month, day, year	3I2	
Weapon system name	A8	
16 basic options	16I1	
Break points for procurement and repair to be used in the SMA calculations	2I2	
Fill rates sought for the weapon system (five are possible)	5F3.3	
Constants for use in computing the reorder level P	2F4.3,I2	

(See Figure 5.1)

```
//TERR JOB .TIME=(.10).REGION=200K
//  EXEC  FORTCLG.PARM.FORT='OPT=2.NOMAP.NOSOURCE'
//FORT.SYSIN  DD  *
C         CARES PROGRAM
          CALL START
          STOP
          END
        BLOCK DATA
        COMMON /PARAM/PLIST(46).COGS(5).PARAMS(230)
       COMMON /OPTION/ITWRS.ISKPF.IRMADS.IEU2.IES2.MIXLVL.LVL234.LVL.
     1     KCDA.KPROC.IMRK.ICCOGP.ICPLOT.IREP.
     2     IRCOGP.IRPLOT.IBRKSP.IBRKSR.IASO.IFIRST.ILAST.F39R.F39C.I39
        COMMON /BDATA/CNORM(130).YNORM(18).TABLES(70).TABLEC(61)
        COMMON /ARRAY/INDEX.NLAM.NRHO.NCOG.ICOG.NMARK.
     1  LMARK.LCOG.CSMA(75.15).SMAFHC(75.16).
     2  SMAB08(75.29).ICUT.REQ(15).NB(5).NBR(5).NBQ(5).NBQR(5).
     3  NORM(5).NORMQ(5).NORMR(5).NORMQR(5)
        REAL LCOG(25)
        INTEGER LMARK(10)
        DATA NB.NBQ.NBR.NBQR.NORM.NORMQ.NORMR.NORMQR/40*0/
        DATA LCOG/25*0.0/
        DATA IBRKSP.IBRKSR.LMARK/12*0/
        DATA CSMA/1125*0.0/
        DATA SMAB08/2175*0.0/
        DATA SMAFHC/1200*0.0/
        DATA YNORM/1.282.1.645.1.960.2.326.2.576.3.090.
     1       3.291.3.891.4.417..90..95..975..99..995..999..9995..99995.
     2       .999995/
        DATA CNORM/.5000..5040..5080..5120..5160.
     1      .5199..5239..5279..5319..5359..5398..5438..5478..5517..5557.
     2      .5596..5636..5675..5714..5753..5793..5832..5871..5910..5948.
     3      .5987..6026..6064..6103..6141..6179..6217..6255..6293..6331.
     4      .6368..6406..6443..6480..6517..6554..6591..6628..6664..6700.
     5      .6736..6772..6808..6844..6879..6915..6950..6985..7019..7054.
     6      .7088..7123..7157..7190..7224..7257..7291..7324..7357..7389.
     7      .7422..7454..7486..7517..7549..7580..7611..7642..7673..7704.
     8      .7734..7764..7794..7823..7852..7881..7910..7939..7967..7995.
     9      .8023..8051..8078..8106..8133..8159..8186..8212..8238..8264.
     A      .8289..8315..8340..8365..8389..8413..8438..8461..8485..8508.
     B      .8531..8554..8577..8599..8621..8643..8665..8686..8708..8729.
     C      .8749..8770..8790..8810..8830..8849..8869..8888..8907..8925.
     D      .8944..8962..8980..8997..9015/
        DATA TABLES/'0'.'A'.'B'.'C'.'D'.'E'.'F'.'G'.'H'.'J'.
     1      'K'.'L'.'1'.'M'.'N'.'P'.'Q'.'R'.'S'.'T'.'2'.'U'.'V'.'W'.'3'.
     2      'X'.'4'.'Y'.'5'.'$'.'6'.'7'.'8'.'9'.'Z'.0.0.1.0.2.0.3.0.4.0.
     3      5.0.6.0.7.0.8.0.9.0.10.0.11.0.12.0.13.0.14.0.15.0.16.0.17.0.
     4      18.0.21.0.24.0.27.0.30.0.33.0.36.0.42.0.48.0.54.0.60.0.
     5      66.0.72.0.84.0.96.0.108.0.120.0/
        DATA TABLEC/0.0..10..50.1.1.1.7.2.4.3.1.3.8.
     1      4.6.5.4.6.2.7.0.7.8.8.6.9.4.10.2.11.1.11.9.12.8.
     2      13.6.14.5.15.3.16.2.17.1.17.9.18.8.19.7.20.5.
     3      21.4.22.3.23.2.24.1.24.9.25.8.26.7.27.6.28.5.29.4.
     4      30.3.31.2.32.1.33.0.33.9.34.8.35.7.36.6.37.5.38.4.
     5      39.3.40.2.41.1.42.0.42.9.43.9.44.8.45.7.46.6.47.5.
     6      48.4.49.3.50.1/
          DATA ITWRS/1/.ISKPF/1/.IRMADS/0/.IEU2/1/.IES2/1/
          DATA IMRK/1/.IREP/1/.ICCOGP/0/.IRCOGP/0/.KCDA/1/.KPROC/1/
          DATA ICPLOT/1/.IRPLOT/1/.LVL/3/.LVL234/0/.MIXLVL/0/
          DATA COGS/5*0.0/.PARAMS/230*0.0/.IASO/0/
          DATA PLIST/1.0.5.0.10.0.100.0.500.0..10..50.1.0.100.0.
     1                   100000.0..1.0.0..20.0.0.0.0.
     2                   .4.1.0..25.0.0.0.0..10.
     3                   .10..01.999.0.0.0.0.0..0048..0385.3*102.0.
     4                   275.0.325.0.8000.0.15.0..041..274.1.0.0.0.
     5                   90.0.3.0.5*1.0/
           DATA REQ/15*0.0/
           END
//LKED.SYSLIB  DD  DSN=WYL.XF.K01.LDLIB.DISP=SHR.
//       VOL=SER=PUB005.UNIT=DISK
//GO.FT01F001 DD UNIT=DISK.VOL=SER=PUB005.DISP=SHR.
//        DSN=WYL.XF.K01.TERR
//GO.SYSIN DD *
0041178TERRIER 110110041110010004047007508008509501000000001
'1H  '..01.1.0.10.0.100.0.10000.0..01.1.0.10.0.100.0.10000.0.
0.05..01..1.2*0.0..50..4..4.2*0.0.
.10..10..01.999.0.0.0.0.0..0048..0385.
 69.16.9.4.69.16.132.29.132.29.2200.0.15.0.
.041..274.1.0.1.0.90.0.3.0.5*1.0/
'4N  '..01.1.0.10.0.100.0.10000.0..01.1.0.10.0.100.0.10000.0.
0.05..01..1.2*0.0..50..4..4.2*0.0.
.10..10..01.999.0.0.0.0.0..0048..0385.
 69.16.9.4.69.16.132.29.132.29.2200.0.15.0.
.041..274.1.0.1.0.90.0.3.0.5*1.0/
'00  '/
'00  '/
'00  '/
```

FIGURE 5.1

Job Control Language (JCL) and Input Data For A Run Of The Terrier Weapon System On An IBM 370

5.3 Parameter File

Associated with each of the cogs in a weapon system is a parameter list containing all attributes of a cog. These are described in detail in Appendix C in the common block PARAM. Defaults do exist for all the cog attributes; however, it is necessary to input the names of the cogs existing in the weapon system. This file is input with free-format -- all data need only be delimited with commas or spaces. The attributes for each cog should be separated by slashes. If fewer than 5 cogs exist in a weapon system, replace each non-existing cog with '00'/. In Figure 5.1 only two of the 5 possible cogs exist, thus 3 lines with '00'/ are used.

5.4 Creating a Load Module

To create a load module library the following JCL is required:

```
//LOAD  JOB ,TIME=(1.0),REGION==500K
// EXEC FORTLBLD,PARM.FORT='OPT=2',
//        PARM.PRELINK='R,S',DSN='WYL.XF.K01.LDLIB',
//        VOL='PUB005'
//FORT.SYSIN  DD *

      [ program

//LKED.SYSLMOD   DD   DISP=(NEW,KEEP),DCB=BLKSIZE=6233
```

FORTRAN Procedures may vary on different computers, thus this JCL may not always be correct.

When it is not possible to create such a library the following JCL should be used.

```
//LOAD JOB ,TIME=(1,0),REGION=SOOK
//  EXEC FORTCL,PARM.FORT='OPT=2'
//FORT.SYSIN DD *
      [ program



//LKED.SYSLMOD DD UNIT=DISK,VOL=SER=PUB005,
//    DSN=WYL.XF.K01.LOAD,
//    DISP=(NEW,CATLG).
```

To run the above, use:

```
//GO JOB
// EXEC FORTGO
//GO.FT01F001 DD...
//GO.SYSIN DD *
      [ input data
```

APPENDIX A:

EDITS AND CHECKS ON INPUT DATA

If a DEN does not fall within the given bounds, it is set to the appropriate bound (i.e., if it is less than the lower bound, it is set to the lower bound or if it is greater than the upper bound, it is set to the upper bound.) Any exceptions are noted.

- Quarterly demand $0 < B074 \leq 2{,}000{,}000$

  If B074 $\leq$ 0 and B023D > 0, B074 = B023D and the check is made on B074 again. If B074 fails the tests, the item is DROPPED from processing.

- Quarterly demand, end of lead time $0 < B023D \leq 2{,}000{,}000$

  If B023D falls outside the bounds, set it to B074.

- Quarterly requisition frequency $0 < A023B \leq 1{,}000{,}000$

  If A023B $\leq$ 0 set it to B074

- Regenerations B023F, B074A

  $0 <$ regenerations $\leq 1{,}000{,}000$

  if regenerations ≤ 0 set it to (1 - F007) x B074

- Repair survival Rate $0 < F009 \leq 1$

  if F009 $\leq$ 0 set it to .9

- Carcass Return Rate $0 \leq \beta_1 < 1$

  where $\beta_1 = \dfrac{B074A}{B074*F009}$

  if $\beta_1 \leq 0$ $\beta_1 = (1 - F007)/F009$

  if $\beta_1 \geq 1$ $\beta_1 = 1$

- Wear out rate $F007 = 1 - \beta_1 (F009)$

  this value replaces the data which was input

- Demand during repair TAT B023H > 0

  if B023H $\leq$ 0 set it to 1.4 x B074

- Variances: repair B019C, procurement B019A

  $0 < var \le 10{,}000{,}000$

  repair: if var ≤ 0 set it to $C(B023H)^P$

  procurement: if var ≤ 0 set it to $C(B074 \cdot B011A)^P$

  $$\left.\begin{array}{l} C = 4.112 \\ P = 1.402 \end{array}\right\} \text{SPCC} \qquad \left.\begin{array}{l} C = 1.0 \\ P = 1.0 \end{array}\right\} \text{ASO}$$

- Prices: B053 (unit price) B055 (replacement price)
  B055A (repair cost)

  $.01 \le DEN \le 1{,}000{,}000$

- MADS A019B, F009A, B012B, B012D, B011B

  $0 \le MAD \le 3{,}000{,}000$

  (carcass return, repair survival rate, commercial TAT, Navy TAT, PLT)

- Set up costs (repair & manufacturers) B058, B058A

  $0 \le COST \le 1{,}000{,}000$

- Procurement problem TAT $0 < B012F \le 12.0$

  if B012F ≤ 0 set it to 1.3 quarters

- Obsolescence rate $.01 \le B057 \le 1$

- Procurement lead time $.001 < B011A \le 12.0$

  if B011A ≤ 0 set it to 2.4 quarters

- Essentiality $.5 \le C008C \le 1.0$

- DENS for assets computations (reservations A013A, back orders A011, PPR during LT, PPR after LT, RFI, NFFI, contract due, recommended due)

  $$0 \le DEN \le \text{Limit} = \text{Max} \left\{\begin{array}{l} 100{,}000 \\ 16(B074) \end{array}\right.$$

- Mark category $\varepsilon \left\{0,1,2,3,4\right\}$

  if not, recompute as follows:

  B074 $\leq$ .25 => Mark = 0

  B074 $\leq$ 5 and B055 $\leq$ 50 or B074 (B055) < 75 => Mark = 1

  B074 > 5 and B055 $\leq$ 50 or B074 (B055) < 75 => Mark = 2

  B074 $\leq$ 5 and B055 > 50 or B074 (B055) $\geq$ 75 => Mark = 3

  B074 > 5 and B055 > 50 or B074 (B055) $\geq$ 75 => Mark = 4

- Shelf life C028 convert it to months

  if it is out of range, C028 = 9.99 months

- $(\text{MAD of demand})^2 \quad 0 < \text{A0192} + \text{A019A}^2 \leq 10{,}000{,}000$

  if $\text{A019}^2 + \text{A019A}^2 \leq 0$ set it to $\frac{\text{B074}^2}{\text{A023B}}$

Counts on each of the above edits are output (see Section 4.3) for the weapon system. These edits could be thought of as constituting a warning regarding the results of the program. If the edits are numerous, the SMA's may not be entirely comparable to empirical results. This may suggest a need to update the weapon system data.

APPENDIX B:

PREPARATION OF WEAPON SYSTEM INVENTORY TAPE

Each record consists of 290 characters - all the necessary information for processing that item. Data on the tape is stored in order of ascending FIIN.

Name of Data Item	Den	Number of Characters(1)	Alpha or Numeric(2)	Position In Record
COG	C003/C003W	4	A	1-4
(3) FAMILY GROUP CODE	C001A-C	5 (last bit is a zero)	A	5-9
(3) NATO CODE	C001E	2	A	10-11
FIIN/ACN	D046 or C002	7	A	12-18
(3) PROVISIONING INDICATOR	B067A	1(0)	N	19
SHELF LIFE	C028	1	A	20
MARK INDICATOR	from B067B-D	1(0)	N	21
REPAIRABLE INDICATOR	B067F	1(0)	N	22
PROCUREMENT METHOD CODE	D025E	1	A	23
(3) ACQUISITION ADVICE CODE	E089	1	A	24
(3) LOCAL ROUTING CODE	B002	3	A	25-27
OBSOLESCENCE RATE	B057	3(2)	N	28-30
(3) FEDERAL SUPPLY CLASS	C042	4	A	31-34
PROCUREMENT LEAD TIME	B011A	4(2)	N	35-38
PROCUREMENT VARIANCE	B019A	12(5)	N	39-50
UNIT REPLACEMENT PRICE	B055	10(2)	N	51-60
UNIT PRICE	B053	10(2)	N	61-70
QUARTERLY DEMAND FORECAST	B074	10(4)	N	71-80
QUARTERLY REGENERATIONS FORECAST	B074A	10(4)	N	81-90
SYSTEM REQUISITION AVERAGE	A023B	10(4)	N	91-100

Name of Data Item	Den	Number of Characters(1)	Alpha or Numeric(2)	Position In Record
MAD OF DEMAND (SQUARED)	$A019^2 + A019A^2$	12(5)	N	101-112
AVERAGE ITEM ESSENTIALITY	C008C	3(3)	N	113-115
RESERVED PLANNED REQUIREMENTS	A013A	8(0)	N	116-123
SYSTEM BACKORDER QUANTITY	A011	8(0)	N	124-131
PPR DUE DURING LEAD TIME	-	8(0)	N	132-139
PPR DUE AFTER LEAD TIME	-	8(0)	N	140-147
ON HAND READY FOR ISSUE	-	8(0)	N	148-155
ON HAND NOT FIT FOR ISSUE	-	8(0)	N	156-163
CONTRACT DUE AWARDED	-	8(0)	N	164-171
PR DUE COMMITTED	-	8(0)	N	172-179
MANUFACTURER'S SET UP COST	B058	8(0)	N	180-187
PROCUREMENT LEAD TIME MAD	B011B	3(1)	N	188-190
PROCUREMENT PROBLEM AVE. TAT	B012F	4(2)	N	191-194
(3) SPECIAL MATERIAL ID CODE	C003B	2	A	195-196
SHIPPER/RECEIVER COUNT	-	8(0)	N	197-204
WEAROUT RATE	F007	3(2)	N	205-207
REPAIR TAT MAD: NAVY NON-REPORTING/COMMERCIAL	B012B	3(1)	N	208-210
REPAIR TAT MAD: NAVY REPORTING	B012D	3(1)	N	211-213
REPAIR SURVIVAL RATE	F009	3(2)	N	214-216
MAD OF REPAIR SURVIVAL RATE	F009A	3(2)	N	217-219
NON-CREDITED GROUP REPAIR QTY	B021A	8(0)	N	220-227
SYSTEM REGENERATIONS, END OF LEAD TIME	B023F	8(1)	N	228-234
DEMAND DURING REPAIR PROBLEM TAT	B023H	8(1)	N	235-243
UNIT REPAIR COST	B055A	8(2)	N	244-251

Name of Data Item	Den	Number of Characters(1)	Alpha or Numeric(2)	Position In Record
REPAIR SET-UP COST	B058A	8(0)	N	252-259
DEMAND, END OF LEAD TIME	B023D	9(3)	N	260-268
MATERIAL CONTROL CODE	C003A	1	A	269
DRIPPER CODE (ASO ONLY)	B001E	1	A	270
CARCASS RETURN MAD	A019B	10(4)	N	271-280
REPAIR VARIANCE	B019C	10(4)	N	281-290

(1) The number of characters is the total number of characters, excluding any decimal point. The number in parenthesis (numeric only) is the number of characters to the right of the decimal point.

(2) An 'A' indicates that the data items consist of alphabetic characters. An 'N' indicates that the data items is a number.

(3) These data elements are not presently used in the model.

It is assumed that some editing has been done to the data on these tapes (e.g., family groupings).

APPENDIX C:

DICTIONARY OF VARIABLE AND ARRAY NAMES

C.1 Common Blocks

Table C.1 lists the common blocks and the subroutines in which they are used, and this table uses the following conventions:

Code

U - The entries are used and left unchanged

M - The entries are used and modified

E - The entries are used and modified by an equivalence statement

I - The entries are input from a data file.

blank - The common block is not used in that subroutine.

Common block:	BLANK	ARRAY	PARAM	OPTION	INPUT	TERMS	BDATA	size
Subroutine	224	18404	1124	96	212	84	1116	21,260
MAIN								192
START		M	I,E				I,E	9114
EDIT	I,M	M	M	M	I		U	9604
DONE		M		U			U	11414
SETCOG	U	M	E					662
CONS	M	M	M	U		M		1930
REPAIR	U	M	U	M				1282
B08	M	M	U	U		M		2866
FIRMHB	M	M	M	U		M		2606
CLAMP	M	M	M	U		M	U	2498
QUANT			U	U	U			3074
SSMA	U	U	U	U		U		7118
UNDOM								970
LPLOT								23598
LOWDEM								572
HIDEM							U	874
GNB								594
TAIL								914
BLOCK DATA		M	M	M			M	--

101,142 bytes

program code = 79,882 bytes

TABLE C.1:

Table of Common Blocks and the Subroutines in which they are used.

The following is a description of each common block and its entries:

1.	Blank Common	contains all input characteristics of the item being processed
	RIND	Repair indicator determined from the MCC (material control code) and if the UICP is ASO, the RCODE (dripper code)
	COG through REPVAR	Item attributes as read in from the tape
	BETA1	Carcass return rate computed from the inputs
	RD	Factor for non-recurring demand, default is 1.0
2.	/ARRAY/	Storage arrays for measures of effectiveness, numbers of requisitions, numbers of items, and pointers for the numbers of shortage cost and risk factor settings in use
	Index	Pointer to the row in the measure of effectiveness (SMA) arrays for the current shortage cost or risk factor setting
	NLAM	The number of procurement shortage cost setting used for a given cog (equal to the number of repair shortage cost settings)
	NRHO	The number of min/max stockout risk factor pairs used for a given cog.
	NCOG	Counter for the number of different repairable cogs processed.
	ICOG	Indicates which of the n(n=1,...,5) input cogs the present cog corresponds to.
	NMARK	The number of different consumable mark categories processed.

LMARK	10 by 1 array for the attibutes of the consumable mark categories (0,1,..,4) rows 1-5 contain the number of items for the category. Rows 6-10 contain the number of shortage cost and risk factor settings for each category
LCOG	Similar to LMARK, but for repairable items. A 25 by 1 array rows 1-5: name of the cog 6-10: # of B08 items, by cog 11-15: # of of firm/high burner items by cog 16-20: # of CLAMP items by 8 cog 21-25: # of shortage cost x risk factor by cog settings for each cog.
CSMA	75 by 15 storage matrix for the consumable items, see Table C.2.
SMAFHC	75 by 16 storage matrix for the FIRM/HIGH BURNER and CLAMP items, see Table C.2
SMAB08	75 by 29 storage matrix for the B08 items, see Table C.2

TABLE C.2: Array Descriptions For CSMA, SMAFHC, and SMAB08

	ARRAY NAME			
COLUMN	CSMA *	SMAFHC *	SMAB08 *	
1	Procurement cost through present fiscal year	Procurement cost through present fiscal year	Procurement cost through present fiscal year	
2	Procurement cost through following year	Procurement cost through following year	Procurement cost through following year	Repair Level 4 (columns 2–6)
3	Fill rate	Fill rate	Fill rate	
4	Average days of delay, delayed requisitions	Average days of delay, delayed requisitions	Average days of delay, delayed requisitions	
5	Average days of delay, all requisitions	Average days of delay, all requisitions	Average days of delay, all requisitions	
6	Time-weighted requisitions short per year	Time-weighted requisitions short per year	Time-weighted requistions short per year	
7	Dollar value of safety stock	Dollar value of safety stock	Fill rate	Repair Level 2 (columns 7–10)
8	Days of safety stock	Days of safety stock	Average days of delay, delayed requisitions	
9	Dollar value of lead-time demand	Repair cost through present fiscal year	Average days of delay, all requisitions	
10	Days of lead-time demand	Repair cost through following year	Time-weighted requisitions short per year	
11	Procurement shortage cost	Procurement shortage cost	Fill rate	Repair Level 3 (columns 11–14)
12	Minimum risk factor	Minimum risk factor	Average days of delay, delayed requisitions	
13	Maximum risk factor	Maximum risk factor	Average days of delay, all requisitions	
14	Number of requisitions	Number of requisitions	Time-weighted requisitions short per year	
15	Dollar value of annual demand	Cog	Dollar value of safety stock	
16		Dollar value of annual demand	Days of safety stock	
17			Repair cost through present fiscal year	Repair Level 4 (columns 17–18)
18			Repair cost through following year	
19			Procurement shortage cost	
20			Minimum risk factor	
21			Maximum risk factor	
22			Repair shortage cost	
23			Number of requisitions	
24			Cog	
25			Dollar value of annual demand	
26			Repair cost through present fiscal year	Repair Level 2 (columns 26–27)
27			Repair cost through following year	
28			Repair cost through present fiscal year	Repair Level 3 (columns 28–29)
29			Repair cost through following year	

* Each row of the matrix corresponds to a unique cog, shortage cost, and risk factor setting.

IOUT	Unit for outputs from program
REQ	15 by 1 array.
	rows 1-5 no. of requisitions for B08 items by Cog
	rows 6-10 no. of requisitions for firm/high burner items
	rows 11-15 no. of requisitions for CLAMP by cog items by cog.
NB,NBQ,NBP,NBQR	5 by arrays for counts, by cos, on the number of items using a negative binomial distribution. A Q indicates that the negative binomial distribution was used in computing the reorder and repair levels and triggers. An R indicates that this distribution was used in the repair problem. An array name with no Q in it, implies that the negative binomial distribution is used in computing the SMA's.
NORM, NORMQ, NORMR, NORMQR	5 by 1 arrays for counts, by cog, on the number of items using a normal distribution. (see description of NB arrays for further explanation)
3. /PARAM/	Parameters in use for the cog being processed
LAMP	the procurement shortage cost settings (maximum of 5)
LAMREP	The repair shortage cost settings (maximum of 5)
RHOMIV	The min risk factor settings (maximum of 3)
PROINT*	Interest rate for the procurement problem used in computing the holding cost
REPINT*	Interest rate for the repair problem
STORAG*	Storage cost
MSLS*	Maximum safety level (in months) for computing P, the reorder trigger

---

*Values are used solely in computing reorder levels and reorder triggers

BREAKP	Breakpoint for demand (normal/negative binomial) for the procurement problem
BREAKP	Breakpoint for demand (normal/negative binomial) for the repair problem
W	Length of the procurement review period (in years)
RW	Length of the repair review period (in years)
OCM12*	Mark 1 and 2 order costs for computing the procurement order cost.
REPADC*	Repair administrative cost (used to compute the basic economic repair quantity)
OCLOWD*	Low demand order cost, to compute the procurement order cost
POCNEG/POCADV*	Negotiated and advertised procurement order costs, to compute the procurement order cost
PURVMX*	Maximum purchase order value, to compute the procurement order cost
IDAY	Days of safety stock for CLAMP items--ASO only, default is 15 days
IW	Induction waiting period for FIRM, HIGH BURNER, and CLAMP items (default is 15 days)
C	Carcass arrival delay for FIRM, HIGH BURNER, and CLAMP items (default is 100 days)
PORT	Portion of the present fiscal year remaining for computing procurement and repair budgets (in years, default is 1)

*Values are used solely in computing reorder levels and reorder triggers

IOVRDQ	Flag to override the reorder quantity, Q, computation with one year of attrition if set to 12 (default is 12 repairable items only input is given in months of attrition).
L4DAY	Days of demand--safety stock--for level 4 B08 repair trigger computation
CLVL	Repair level a cog should be set to if cogs are allowed to differ in repair scenarios
RDM	5 by 1 array containing the non-recurring demand factor by mark category (default is 1.0)
COGS	5 by 1 array containing the cog names existing in this weapon system (stored in subroutine START, a maximum of 5)
PARAMS	A 230 by 1 array contains all of the above 46 values as input for each of the 5 cogs.

4. /OPTION/	Holds all options chosen for the current weapon system run.
ITWRS	Include/exclude time-weighted requisition short and average days of delay computation
ISKDF	Include/exclude computation for the mean and variance of the unfilled portion of a requisition when the reorder trigger is hit
IRMAD	Include/exclude computations involving the carcass return MAD + repair survival rate MAD.

IEU2(IES)	Include/exclude computations for the units (requisitions) backordered at the start of a procurement cycle.
MIXLVL[1,2]	Allows for cogs to be run at different repair levels (default is zero, i.e. this option is not used) If the value is one, CLVL is used as the repair level
LVL234[1,2]	Allows the data to be run for all 3 repair levels independently.
LVL[2]	Repair level for all items in the weapon system (default is 3)
KCDA[1]	Include/exclude contract due awarded from the computation of assets.
KPRDC[1]	Include/exclude PR due committed from the computation of assets.
IMRK[1]	output the plots of fill rate vs. budget by consumable mark category.
ICPLOT9[1]	output the plots of all rate vs. budget for the combined consumable items
IREP9[1]	output the tables of effectiveness by repairable cog.
IRCOGP[1]	output the plots of fill rate vs. buget (procurement and repair) for each repairable COG
IRPLOT[1]	output the plots of fill rate vs. budget (procurement and repair) for the combined repairable items

---

1 1 indicates include
0 indicates exclude

2 only one of LVL234, MIXLVL,LVL may be non-zero

IBRKSP,IBRKSR	Breakpoints in requisitions for the normal and negative binomial distributions. IBRSKP is for the procurement problem, IBRKSR is for the repair problem.
IASO	flag for which UICP the data belongs to IASO = 0 SPEC IASO = 0 ASO
IFIRST[3]	Pointer to the first repair level to be run
ILAST[3]	Pointer to the last repair level to be run.
F39R,F39C,I39	Constants to be used in computing the procurement trigger
5. /BDATA/	Contains 4 tables of data
NORM	cumulative standard normal distribution values.
TNORM	cumulative standard normal distribution values for large values of X $X \geq 1.2$
TABLES	conversion values for shelf life input.
TABLEC	table of reorder trigger values for various demands for CLAMP items
6. /TERMS/	terms used in computing the measures of effectives which are independent of the shortage cost and min/max risk factor settings.

[3] if IFIRST = ILAST => LVL234 = 0

NU	annual requisition rate
EY	average issue size
ER	expected repair turn-around time
EF	expected portion of a requistion left unfilled when the reorder trigger is hit
TERM1	annual demand (NU*EY)
EL	length of a procurement cycle
RECEY	reciprocal of EY
RECTRM	reciprocal of TERM1
BETA	reciprocal of attrition factor
ED3,VD3	expected value and variance of demand over leadtime plus procurement periodic delay, including the unfilled portion of the requisition (EF), less the items which can be repaired in this time.
ED5,VD5	expected value and variance of demand over leadtime, less the items which can be repaired in this time
ED6,VD6	expected value and variance of the demand over the repair turnaround time plus the repair review period
ED7,VD7	expected value and variance of demand over the repair turnaround time
SQVD3,SQVD5,SQVD6,SQVD7	square root of the variances as described above.

7. /INPUT/ contains the data exactly as input, with the exception of MARK and SHLFLF, which have been converted and recomputed if necessary. See BLANK COMMON for a description of each variable.

C.2 Data Elements

The following is a list of variable names with their corresponding data element number (den) and description.

VARIABLE NAME	DEN	DESCRIPTION
COG	C003	Cognizance symbol
NIIN	D046/C002	National Item Identification Number
SHLFLF	C028	Shelf life
MARK	B067B-D	Mark category
RII	B067F	repairable item indicator
PMC	D025E	procurement method code
ORT	B057	obsolescence rate
UPIR	B055	replacement price
UP	B053	unit price
PLT	B011A	procurement lead time
QSDF	B074	quarterly demand
QSRFIR	B074A	quarterly regenerations
PROVAR	B019A	procurement variance
SRA	A023B	system requisition frequency
AIE	C008C	average item essentially
RPR	A013A	reservations
SBQ	A011	backorders
PPRLT		PPR due during lead time
PPRFY		PPR due after LT, but before fiscal year
RFI		on hand RFI
NFFI		on hand NFFI

VARIABLE NAME	DEN	DESCRIPTION
CDA		contract due awarded
PRDC		PR due committed
MSUC	B058	manufacturer setup cost
PPTAT	B012F	procurement problem TAT
DMAD	$A019^2+A019A^2$	MAD of demand (squared)
SMIC	C003B	special material ID code
PLTMAD	B011B	procurement LT MAD
SHPREC		shipper/receiver count
WR	F007	wearout rate
CTATMD	B012B	commercial TAT MAD
NTATMD	B012D	Navy TAT MAD
RSR	F009	repair survival rate
MADRSR	F009A	repair survival rate MAD
CRMAD	A019B	carcass return MAD
REPVAR	B019C	repair variance
DRPTAT	B023H	demand during repair problem TAT
UCIR	B055A	repair cost
MCC	C003A	material control code
RSUC	B058A	repair setup cost
RFIRTG	B023F	qtly regenerations (end of LT
DELT	B023D	qtly demand (end of LT)
RCODE	B001E	dripper code

APPENDIX D:

DESCRIPTION OF SUBROUTINES

This section describes each subroutine's function and major arrays or variables involved (excluding those found in the common blocks). All subroutines are written in FORTRAN.

D.1 MAIN

Calls the routine START to get the program going.

D.2 SUBROUTINE START

- Initialize counters and input/output devices
- Read in the weapon system sponsor, the month, day, and year of the run
- Read in the options for the run
- Read in the parameters for each cog in the weapon system
- Validate inputs and echo them.

D.3 SUBROUTINE EDIT

- Read in items to be processed
- Edit the data, keeping counts on the edits
- Determine whether it is necessary to update the cog parameters in common /PARAM/
- Call the appropriate subroutine, depending on the repair indicator
- Compute and print the weapon system summary statistics

D.4 SUBROUTINE DONE

A. Function

- Combine all repairable items (B08, FIRM (HIGH BURNER), and CLAMP) into the array SMAB08
- Upon completion of the processing of the weapon system, output tables of effectiveness by consumable mark category, for combined consumable items, by repairable

cog, and for combined repairable items (once for each repair level)

- output the achieved weapon system fill rates and budgets corresponding to the fill rates sought for the weapon system

b. Major variables and arrays

REQN	reciprocal of the total number of requisitions processed
TOT	reciprocal of the number of requisitions associated with a given shortage cost and min/max risk factor setting
SMA	A 15 by 13 array to pass the fill rate and budget information to the plotting subroutine, LPLOT.
WSYS	A 15 by 6 array containing the achieved and sought fill rates for the weapon system and the associated procurement and repair budgets for the present and following fiscal years for each repair scenario

D.5 SUBROUTINE SETCOG

- Bring into common/PARAM/ the appropriate settings for the current cog
- Determine the number of min/max risk factor settings (NRHO) and shortage cost settings (NLAM) for the item

D.6 SUBROUTINE CONS

- Prepare the consumable items for processing
- Compute the values in common/TERMS/, which are independent of the shortage cost and min/max risk factor settings
- In a loop over all the shortage cost and min/max risk factor settings, call the subroutines to finish computing the measures of effectiveness.

D.7 SUBROUTINE REPAIR

- Set the repair level at which the item is to be run
- Call the appropriate subroutine depending on the repair management scheme (i.e., B08, FIRM, HIGH BURNER, or CLAMP)

D.8 SUBROUTINE B08

- Prepare the B08 items for processing
- Compute the values for the common block TERMS
- Set up a loop over all shortage cost and min/max risk factor settings to call the subroutines which compute the reorder levels, reorder triggers, and the measures of effectiveness

D.9 SUBROUTINE FIRMHB

- Prepare the FIRM or HIGH BURNER items for processing
- Compute the values for the common block TERMS
- Set up a loop over all shortage cost and min/max risk factor settings to call the subroutines which compute the reorder levels, reorder triggers, and the measures of effectiveness.

D.10 SUBROUTINE CLAMP

- Prepare the CLAMP items for processing
- Determine the reorder level (P) from the table TABLEC
- Compute the reorder quantity Q as one year's attrition demand
- Compute the values for the common block TERMS

D.11 SUBROUTINE QUANT

Goes through the levels computations as in SSDS D05 revision 5, 7/76.

- Compute the reorder levels for procurement > P
- Compute the reorder quantity for procurement B08 items only:
- Compute the reorder level for repair, RP
- Compute the reorder quantity for repair RQ.

D.12 SUBROUTINE SSMA

- Complete and store the measures of effectiveness computations as begun in CONS, B08, FIRMHB, and CLAMP,
- Compute and store the procurement order costs for the remainder of the present fiscal year and through the following fiscal year
- For repairable items, compute and store the repair costs for the remainder of the present fiscal year and through the following fiscal year
- Do all of the above for each repair level specified in the current run.

D.13 SUBROUTINE UNDOM

- Prepare the data for plotting by determining the minimum and the maximum budget for each plot
- Output titles for the plots.

D.14 SUBROUTINE LPLOT

- Convert the data to fit in the available graph area GR, a 52 by 101 array
- Output the plots with symbols unique to each min/max risk factor settings.

D.15 SUBROUTINE LOWDEM

- Computes the unconstrained reorder level for items determined to have a negative binomial demand distribution.

D.16 SUBROUTINE HIDTM

- Computes the unconstrained reorder level for items determined to have a normal demand distribution

D.17 FUNCTION GNB

- Computes the negative binomial inverse cumulative distribution summation for use in determining the number of units and requisitions backordered.

D.18 FUNCTION TAIL

- Determines the normal inverse cumulative distribution, summation tables CNORM and TNORM

APPENDIX E:

MATHEMATICAL COMPUTATIONS

E.1 Reorder Levels and Reorder Quantities

E.1.1 Computation of Reorder Level and Reorder Quantity for Procurement

(Levels formulae 4,19,22,23,37,38,39,40) all units of time are quarters.

1. Procurement order cost (POC)

   a) If an item is repairable or consumable Mark 3 or consumable Mark 4 and (demand (end of leadtime) X unit price) is less than or equal to

$$\frac{\left(\text{maximum unpriced purchased order value}\right)^2 \times \text{holding cost}}{8 \times \left(\text{manufacturer's setup cost} + \text{low annual demand order cost}\right)}$$

   then POC = low value annual demand order cost

   b) If an item is repairable or consumable Mark 3 or consumable Mark 4 and the inequality in (a) is false and the procurement method code (DEN D025E) is not zero, B, 1, or 2 then

   POC = negotiated procurement order cost

   otherwise POC = advertised procurement order cost

   c) If an item is not repairable and is Mark 0 or Mark 1 or Mark 2, then POC = Mark 1 and Mark 2 order cost

2.

$$\text{basic } Q = \min \begin{cases} \min \begin{cases} \text{net annual demand}^* \times \text{shelf life}^* \\ \dfrac{\text{net annual demand}}{\text{obsolescence rate}} \\ 5\text{x net annual demand} \end{cases} \\ {}^{1}\text{net qtly demand} \\ \sqrt{\dfrac{2(\text{manufacturer's setup cost} + \text{POC})\ \text{net annual demand}}{\text{Holding cost x unit price}}} \end{cases}$$

(Q = order quantity)

3. Variable procurement stockout risk

a) repairable items:

$$\hat{\rho} = \min \begin{cases} 999999 \\ \dfrac{\text{Holding cost x unit price x basic Q x demand}}{\lambda \text{ x frequency* x net annual demand x essentiality}} \end{cases}$$

where λ = procurement shortage cost, a system variable

b) consumable items:

$$\hat{\rho} = \min \begin{cases} 999999 \\ \dfrac{\text{Holding cost x unit price x demand}}{\lambda \text{ x essentiality x frequency*}} \end{cases}$$

4. Acceptable procurement stockout risk

$$\rho = \min \begin{cases} \text{max. risk allowed} \\ \max \begin{cases} \text{min. risk allowed} \\ \dfrac{\hat{\rho}}{1 + \hat{\rho}} \end{cases} \end{cases}$$

5. Procurement variable

a) repairable items:

Z1 = Lead time demand - Lead time regenerations + regenerations during procurement TAT

b) Consumable items:

Z1 = Lead time demand

6. Basic reorder level = X1

Compute X1: $P(Z > X1) \leq p < p(Z > X1 - 1)$

a) If $Z1 \geq$ Breakpoint (Breakpoint is a system constant)

$Z \sim N(Z1, \text{Procurement Variance})$

b) If $Z1 <$ Breakpoint

$$Z \sim \text{Negative Binomial} \begin{cases} q = \dfrac{\text{Procurement variance}}{Z1} \\[2ex] k = \dfrac{Z1}{p} \\[2ex] p = q - 1 \end{cases}$$

c) If the item is consumable Mark 0

$Z \sim \text{Poisson}\ (Z1)$

7. P = reorder level

$$p = \max \begin{cases} 0 \\ K^*Z1 \\ \min \begin{cases} Z1 + \left(\frac{M}{3}\right) \times \text{demand} \\ \max\ [X1,\ \text{shipper/receiver count}] \\ \text{net annual demand} \times \text{shelf life}^* + Z1 - I \\ \text{net annual demand/obsolescence rate} + Z1 - I \end{cases} \end{cases}$$

where K, M, and I are system constants

$$K = \begin{cases} 1 \text{ if repairable} \\ 0 \text{ if consumable SPCC} \\ 1 \text{ if consumable ASO} \end{cases}$$

I = 1

M = 999 months

Note: P for repairable items must be at least the lead time demand, similarly for ASO consumable items.

8. Q = reorder quantity

$$Q = \max \begin{cases} \text{net qtly demand} \\ \min \begin{cases} \text{basic } Q \\ \text{net annual demand x shelf life} - [P - Z1]^{+} \\ \text{net annual demand/obsolescence rate} - [P - Z1]^{+} \end{cases} \end{cases}$$

$$\text{where } [P - Z1]^{+} \text{ is the safety stock} = \begin{cases} P - Z1 & \text{if } P \geq Z1 \\ 0 & P \leq Z1 \end{cases}$$

E.1.2 <u>Computation of the reorder level and the reorder quantity for repair</u> (Levels formulae 12, 44, 45, 46, 47, 50)

1. Basic repair Quantity = basic RQ

$$= \text{Max} \begin{cases} \text{RW* x Regenerations} \\ \sqrt{\dfrac{8(\text{Repair setup cost} + \text{repair administrative cost}) \min [\text{demand}, \text{reg.}]}{\text{Holding cost x repair cost}}} \end{cases}$$

2. Variable stockout risk

$$\hat{\rho} = \min \begin{cases} 99 \\ \dfrac{\text{Holding cost x replacement price x basic RQ x demand}}{\text{4 x frequency* regenerations x } \lambda(\text{rep}) \text{ x essentiality}} \end{cases}$$

3. Stock out risk

$$\rho = \min \begin{cases} \text{max risk allowed} \\ \max \begin{cases} \text{min risk allowed} \\ \dfrac{\hat{\rho}}{1 + \rho} \end{cases} \end{cases}$$

4. Basic repair level = X2

Compute $X2 = P(D > X2) \leq \rho < P(D > X2 - 1)$

a) If the demand during repair TAT = DR $\geq$ Breakpoint

$D \sim N$ (DR, Repair Variance)

b) If DR < Breakpoint

$$D \sim \text{Neg. binomial} \begin{cases} q = \dfrac{\text{Repair variance}}{\text{DR}} \\ p = q - 1 \\ k = \dfrac{\text{DR}}{p} \end{cases}$$

5. Repair level RP:

$$RP = \max \begin{cases} 0 \\ \min \begin{cases} \max [X2 \text{ , shipper/receiver count}] \\ \text{annual demand x shelf life* + DR - 1} \\ \text{annual demand/obsolescence rate + DR - 1} \end{cases} \end{cases}$$

6. Repair quantity RQ

$$RQ = \max \begin{cases} 1 \\ \min \begin{cases} \text{basic RQ} \\ \text{annual demand x shelf life* - } [RP - DR]^+ \\ \dfrac{\text{annual demand}}{\text{obsolescence rate}} [RP - DR]^+ \end{cases} \end{cases}$$

*The term shelf life is only used if the shelf life is non zero (item is deteriorative)

*Net annual demand = 4(quarterly demand - quarterly regenerations)

* Net quarterly demand = (quarterly demand - quarterly regenerations)

* Frequency is the quarterly requisition frequency

* RW is the repair review period

E.2 Measures of Effectiveness

E.2.1 Consumable items

1) Use formulas 4, 19, 21, 37, 38, 39, and 40 in LEVELS to compute P and Q (E.1.1)

2) Compute fill rates, average days of delay (conditional and and unconditional) and time-weighted requisitions short as described in Appendix G, using the normal demand distribution or the negative binomial demand distribution as appropriate.

E.2.2 Repairable items, B08

1) Use formulas 4, 19, 21, 37, 38, 39, 40 of LEVELS to compute P and Q (E.1.1)

2) Use Formulas 12, 44, 46, 47, 50 of LEVELS to compute RP and RQ (E.1.2). Then compute the fill rate and the other measures of effectiveness as described in Appendix G, using the appropriate demand distribution.

E.2.3 Repairable items, FIRM or HIGH BURNER

1) Compute P and Q as in E.1.1

2) Compute the fill rate and the other measures of effectiveness as in Appendix G.

E.2.4 CLAMP items

1) Using the poisson table lookup, compute the reorder level P.

2) Compute reorder quantity, Q, of one year's attrition:

   Q = 4 x quarterly demand X wearout rate

3) Continue as for FIRM items to compute the various measures of effectiveness.

E.3 Procurement and Repair Costs

For all items, compute the procurement budgets through the present fiscal year, and through the following fiscal year using the reorder levels and quantities as follows:

ASSETS = RFI + NFI * RSR + CDA + PRDC - (RPR + SBQ + PPRLT)

Attrition since the last reorder was made

$$H1 = \begin{cases} 0 & \text{if } P \geq \text{ASSETS} \\ P+Q - \text{ASSETS} & \text{if ASSETS} > P \end{cases}$$

Annual attrition:

$$ATTR = \nu \times WR \times E(Y)$$

where ν is the number of annual requisitions, E(Y) is the average issue size, and WR is the wearout rate

Attrition over K years:

$$T = K(ATTR) + PPRFY$$

where PPRFY is the planned requirements due after the lead time, but before the fiscal year

The number of reorders in K years:

$$G1 = INT\left[\frac{T + H1}{E(O)}\right] \qquad G1 \geq 0$$

where E(O) is the expected order quantity during a procurement cycle.

The fraction of the procurement cycle between the arrival of an order and the first repair induction:

$$\varepsilon(RP) = \min\left(\frac{\text{Safety Stock} + E(O) - (RP - \nu E(Y)RW/2)}{E(O)/(1 - E(\beta_1)\,E(\beta_2))},\ 1 - E(\beta_1)\,E(\beta_2)\right)$$

$$WR = 1 - E(\beta_1)E(\beta_2)$$

where $E(\beta_1)$ is the expected carcass return rate, $E(\beta_2)$ is the expected repair survival rate, and RW is the repair review period.

Size of reorder quantity for immediate buy:

$$H2 = \begin{cases} P + Q - \text{ASSETS} & \text{if } P \leq \text{ASSETS} \\ 0 & \text{if ASSETS} > P \end{cases}$$

lead time attrition:

$$H3 = E(L)\ E(Y)\ (1 - E(\beta_1)\ E(\beta_2))$$

where E(L) is the expected procurement lead time

The number of procurement cycles during the remainder of the lead time attrition

$$G2 = \text{INT}\left[\frac{H2 - H1}{E(0)}\right]$$

Attrition since the last procurement arrival

$$H4 = (1 + G2)\ E(0) + H1 - H3$$

The number of procurement arrivals during K years

$$G3 = \text{INT}\left[\frac{T + H4}{E(0)}\right]$$

The fraction of the last repair cycle during which carcasses are included

$$H5(RP) = \min \begin{cases} \left[\dfrac{T + H4}{E(0)} - G3 - \varepsilon(RP)\right]^{+} / \ E(\beta_1)\ E(\beta_2) \\ 1 \end{cases}$$

The fraction of the first repair cycle during which carcasses are included

$$H6(RP) = \min \begin{cases} \left[\dfrac{H4}{E(0)} - \varepsilon(RP)\right]^{+} / E(\beta_1)\ E(\beta_2) \\ 1 \end{cases}$$

Procurement order costs over K years:

$$PC(K) = (\text{unit price})\ (H2 + G1) \times E(0)$$

Repair costs over K years:

$$RC(K) = \left[\frac{E(\beta_1)\ E(\beta_2)\ E(0)\ \text{repair cost}}{1 - WR}\right] \left[G3 + H5(RP) - H6(RP)\right]$$

Note that the above repair cost computation applies for the B08 items which have a repair induction point and a repair review period. In the case of the special repair management schemes, FIRM, HIGH BURNER and CLAMP, the procurement cost is the same as the above using the procurement reorder point arising from the Level Program (for FIRM and HIGH BURNER) and from the Table for CLAMP. The annual repair costs for these items can be calculated by simply summing up all repairable carcasses received in a year, i.e. $\beta_1\beta_2\ \nu E(Y)$ times the repair cost/unit.

The above formula formula for RC(K) assumes that the repair order-up-to point was equal to RP both before and after the beginning of the present fiscal year. Suppose, however, that the repair order-up-to point was RP_1 prior to the beginning of the present fiscal year and will be RP_2 over the next K fiscal years. In this case, the repair costs over K years will be given by

$$RC(K) = \left[\frac{E(\beta_i)\ E(\beta_2)\ E(0)\ \ \text{repair cost}}{1 - WR}\right] \left[G3 + H5(RP_2) - H6(RP_1)\right] .$$

APPENDIX F

NUMERICAL EXAMPLES

F.1 Consumable Item, High Demand Case

F.1.1 Characteristics:

Mark = 4	UPIR = 150	MSUC = 0
SHLFLF = 0	UP = 185	RPR = 191
NP1 = 1	PROVAR = 279.16845	PPRFY = 191
PMC = 3	QSDF = 3.6399	CDA = 230
SHPREC = 1	DELT = 3.999	
ORT = .12	DMAD = .00169	all other DEN'S
PLT = 4.0	SRA = 3.6399	are zero
PLTMAD = 1.6	A1E = .5	

F.1.2 Parameters:

storage cost = .01 interest rate = .1 HC = holding cost = .23

W = .0048 (proc. review period) λ = 1.00, ρmin = .01, ρmax = .5

V044 = 7500 V041 = 275 Break point = 0

Non-recurring demand factor = 1.0

F.1.3 Calculations:

since DELT(UPIR) $\geq$ V044^2 x HC/(8(V041 + MSUC))

599.85 $\geq$ 18039.21569

POC = V041 = 275

$$\text{basic Q} = \min \begin{cases} \min \begin{cases} 4(\text{DELT})/\text{ORT} = 133.3 \\ 20(\text{DELT}) = \underline{79.98} \end{cases} \\ \max \begin{cases} 1 \\ \text{DELT} = 3.999 \\ \sqrt{\dfrac{8\ (\text{POC} + \text{MSUC})\ \text{DELT}}{\text{HC x UPIR}}} \end{cases} = 15.97 \end{cases}$$

basic Q = 15.97

$$\hat{\rho} = \min \begin{cases} 999999 \\ \dfrac{HC \times UPIR \times QSDF}{SRA \times \lambda \times A1E} = .69 \end{cases}$$

$$\rho = \min \begin{cases} \rho\max = .5 \\ \max \begin{cases} \rho\min = .01 \\ \dfrac{\hat{\rho}}{1 + \hat{\rho}} = .4083 \end{cases} \end{cases} \qquad \rho = .4083$$

Z1 = QSDF x PLT = 14.5596 > 0 $\Longrightarrow$ use the normal distribution for demand

find X1: $P(Z > X1) \leq \rho < P(Z > X1 - 1)$

where $Z \sim N(14.5596, 279.16845)$

$X1 = .24 \times \sqrt{279.17} + 14.5596 = 18.57$

$$P = \max \begin{cases} 0 \\ K \times Z1 = 14.5596 \\ \min \begin{cases} Z1 + M/3\ (DELT) = 1346.227 \\ \max\ (X1,\ SHPREC) = 18.57 \\ 4(DELT)/ORT + Z1 - I = 146.8596 \end{cases} \end{cases}$$

K = 1
I = 1
M = 999 months

P = 19

$$Q = \max \begin{cases} 1 \\ DELT = 3.999 \\ \min \begin{cases} \text{basic } Q = 15.97 \\ 4\ DELT/ORT - [P - Z1]^{+} = 93.86 \end{cases} \end{cases}$$

Q = 16

$WR = 1$

$\nu = 4(SRA) = 14.5596$

$E(Y) = QSDF/SRA = 1$

$E(Y^2) = 1.57\ DMAD/SRA = .00072895$

$E(Y^3) = 3E(Y)E(Y^2) - 2(E(Y))^3 = -1.99$

$E(L) = PLT/4 = 1$

$V(L) = 1.57 \times PLTMAD^2/16 = .2512$

$E(F) = E(Y^2)/[2E(Y)] = .000364475$

$V(F) = [E(Y^3)/\ 3E(Y) - E(F)^2]^+ = 0$

$E(D1) = (W/2 + E(L))\nu \times E(Y) = 14.5945$

$V(D1) = (W/2 + E(L))\nu\ \ E(Y^2) + (W^2/12 + V(L))(\nu E(Y))^2 = 53.2609$

$E(D3) = E(D1) + E(F) = 14.5949$

$V(D3) = V(D1) + V(F) = 53.2609$

$K1 = (P - E(D3))/\sqrt{V(D3)} = .6036$

$\phi(K1) = .3325 \quad \Phi(K1) = .272 \quad G(K1) = .16098$

$E(U1) = \sqrt{V(D3)} \times G(K1) = 1.2238$

$E(D5) = E(D4) = E(L)\nu E(Y) = 14.5596$

$V(D5) = V(D4) = E(L)\nu E(Y^2) + V(L)(\nu E(Y))^2 = 53.2506$

$K2 = (P+Q-E(D5))/\sqrt{V(D5)} = 2.801$

$\phi(K2) = .00788 \quad \Phi(K2) = .00325 \quad G(K2) = 0$

$E(U2) = 0$

$E(U) = E(U1) - E(U2) = 1.2238 = E(N)$ since $E(Y) = 1$

$E(O) = W/2\ \nu\ E(Y) + E(F) + Q = 16.0353$

fill rate $= 1 - \frac{E(U)}{E(O)} = 92.37$

$E(S1) = \frac{V(D3)}{\nu E(Y)} h(K1) = .311675 \quad E(S2) = \frac{V(D5)}{\nu E(Y)} h(K2) = .01221$

E(S) = E(S1) - E(S2) = .2995

average days of delay for all requisitions:

6.8173

average days of delay for delayed requisitions:

93.005

time-weighted requisitions short, normalized to one year:

99.257

safety stock = $[P-Z1]^+$ = 4.4404

dollar value of annual demand = UP x ν x E(Y) = 2693.526

dollar value of safety stock - $[P-Z1]^+$ x UP = 821.5

$$\text{days of safety stock} = 365 \times \frac{\text{dollar value of safety stock}}{\text{dollar value of annual demand}} = 111.3$$

dollar value of lead-time demand = Z1 x UP=2693.526

$$\text{days of lead-time demand} = 365 \times \frac{\text{dollar value of lead time demand}}{\text{dollar value of annual demand}} = 365$$

```
SHLFLF=0.0  MARK=4  RII=0 FMC=3   CRT=0.12  DELT=        3.999
PLT= 4.00  PROVAR=    279.16846  UPIR=       150.00  UP=       185.00  QSDF=        3.6399
QSRFIR=         0.0      SRA=           3.6399  DMAD=         0.00265  AIE=0.500
RFR,SSQ,PPRLT,PPRFY,RFI          191         0         0       191        0
 NFFI,CDA,FRDC,MSUC          0       230        0          0  PLTMAD= 1.6  PPTAT= 0.0   SHPREC=  1
 NU=   14.55960  EY,EY2,EY3:      1.0000       0.0007      -1.9978
ER,VR:       0.0          0.0          EF,VF:       0.0          0.0
EL,VL:       1.0000       0.2512      ED1,VD1:            14.595          53.261
ED3,VD3:           14.5945         53.2609    ED5,VD5:           14.5596         53.2605
POC     275.0000   BECQ     15.9690    OMEGA1                0.7  Z1      14.5596   RHO1  0.408  X1     18.56960
P,Q           19          16
K1,GAUSS1,PHI1,GK1           0.6037           0.2730           0.3325           0.1677
K2,GAUSS2,PHI2,GK2           2.8009           0.0033           0.0079           0.0000
EU1,EU2,EU       1.2237       0.0001       1.2236
ES1,ES2,ES             0.314            0.012            0.302
```

MIN RISK FACTOR	MIN RISK FACTOR	PROC. SHORTAGE COST	PROC. BUDGET FIRST YEAR	PROC. BUDGET TWO YEARS	FILL RATE	AVE. DAYS OF DELAY FOR DELAYED REQUISITIONS	AVE. DAYS OF DELAY FOR ALL REQUISITIONS	AVERAGE REQUISITION DAYS SHORT PER YEAR	DOLLAR VALUE OF SAFETY STOCK	AVE. DAYS OF SAFETY STOCK	DOLLAR VALUE OF LEAD TIME DEMAND	DAYS OF LEAD TIME DEMAND
CONSUMABLE MARK 4												
.01	.50	100.00	35598.	38564.	0.9237	90.13	6.9	100.1	821.5	111.	2694.	365.

F.1.4 Procurement cost calculations

Assets = 39

Attrition = νEY = 14.5396

H1 = P+Q - ASSETS = -4

H2 = 0

T(1) = 205.5596 T(2) = 220.1192

G1(1) = 12 G1(2) = 13

PC(1) = $35598.4 PC(2) = $38564.9

F.2 B08 Item, Low Demand Case

F.2.1 Characteristics:

MARK = 0	RII = 1	SHLFLF = 0	PMC = 0
NP1 = 0	SHIPREC = 0	ORT = .1	PLT = 3.01
PROVAR - 207.0728	UPIR = 25.0	UP = 2515	UCIR = 12.34
REPVAR = .9737	QSDF = .3399	DELT = .34	SPA = .3391
QSRFIR = .30591	RFIREG = .3	DMAD = .24094	AIE = .5

MSUC = 0	RSUC = 60	PLTMAD = .1	DRPTAT = .5
RSR = .9	MADRSR = .04	BETA1 = 1	CRMAD = 3.2878
WR = .1			

F.2.2 Parameters

Storage cost = .01 interest = .1 HC = holding cost = .21

W = .0048 RW = .0385 ρ min = .01 ρ max = .5

λ = 100 λ(repair) = 100 Breakpoint = 20

VØ44 = 2200 VØ41 = 69.6 REPADC = 9.4 C = .274

F.2.3 Calculations:

since (DELT)UPIR) $\leq$ HCXV044^2/[8(V041 + MSUC)]

8.5 $\leq$ 1839.2

POC = V041 = 69.16

$$\text{basic Q} = \min \begin{cases} \min \begin{cases} 4(\text{DELT-RFIREG})/\text{ORT} = 1.6 \\ 20(\text{DELT-RFIREG}) = .8 \end{cases} \\ \max \begin{cases} 1 \\ \text{DELT - RFIREG} = .04 \\ \sqrt{\dfrac{8\ \text{POC (DELT-RFIREG)}}{\text{HC(UPIR)}}} = 2.05 \end{cases} \end{cases}$$

basic Q = .8

$$\hat{\rho} = \min \begin{cases} 999999 \\ \dfrac{\text{HC(UPIR)(basic Q)(QSDF)}}{\text{4 SRA (DELT-RFIREG) AIE}} = .52936072 \end{cases}$$

$$\rho = \min \begin{cases} \rho\max = .5 \\ \max \begin{cases} \rho\min = .01 \\ \dfrac{\hat{\rho}}{1+\hat{\rho}} = .346132 \end{cases} \end{cases} \qquad \rho = .4377$$

Z1 = QSDF x PLT - QSRFIR (PPTAT-PLT) = .6682434

find X1: P(Z>X1) $\leq \rho <$ P(Z>X1-1)

Z has a negative binomial distribution

with $q = \dfrac{\text{PROVAR}}{\text{Z1}} = 309.9$, $p = q-1 = 308.9$,

and $r = \dfrac{\text{Z1}^2}{\text{PROVAR - Z1}} = .002164$

$F(0) = q^{(-r)} .98766342 > \rho$

$F(K) = F(K-1)\ \dfrac{p(r+k-1)!}{q\ (k!)}$

$F(1) = .00213 \leq \rho$

$x1 = 1$

$$P = \max \begin{cases} 0 \\ Z1 = .66824 \\ \min \begin{cases} \max(X1, SHPREC) = 1 \\ 4(DELT-RFIREG)/ORT + Z1 - 1 = 30.806 \end{cases} \end{cases}$$

$P = 1$

Q = one year's attrition = 4xQSDFXWR = .13596

$Q = 1$

$$\text{basic repair } Q = \max \begin{cases} 1 \\ RW(RFIREG) = .01155 \\ \sqrt{\dfrac{8(REPADC + RSUC)\min(DELT, RFIREG)}{(HC)(UCIR)}} \end{cases}$$

basic repair Q = 8.0171

$$\hat{\rho} = \min \begin{cases} 99 \\ \dfrac{(HC)(UPIR)(\text{basic repair } Q)\ QSDF}{4(SRA)(RFIREG)\ \lambda(\text{repair})\ AIE} = .7015 \end{cases}$$

for the calculation of $\hat{\rho}$, SRA = max (SRA,QSDF) = QSDF

$$\rho = \min \begin{cases} \rho max = .5 \\ \max \begin{cases} \rho min = .01 \\ \dfrac{\hat{\rho}}{1 + \hat{\rho}} = .4123 \end{cases} \end{cases} \qquad \rho = .4123$$

find $X2 = P(D>X2) \leq \rho < P(D > X2-1)$

where D has a negative binomial distribution

$$q = \frac{REPVAR}{RDRPTAT} = 1.9474 \quad , \quad p = q - 1 = .9474,$$

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THE PREDICTION OF VARIOUS SUPPLY MATERIAL AVAILABILITIES BY WEA--ETC(U)
JUL 78 R C MOREY, Z F LANSDOWNE, K EPSTEIN N00014-77-C-0806
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$$\text{and } r = \frac{DRPTAT^2}{REPVAR - DRPTAT} = .52776$$

$$F(0) = q^{(-r)} = .703456242 > \rho$$

$$F(1) = F(0)\ rxp/q = .18061\text{-}12 \leq \rho$$

$$X2 = 1$$

$$RP = \max\begin{cases} 0 \\ \min \begin{cases} \max(X2, SHPREC) = 1 \\ 4DELT/ORT + DRPTAT - 1 = 13.1 \end{cases} \end{cases}$$

$$RP = 1$$

$$RQ = \max\begin{cases} 1 \\ \min \begin{cases} \text{basic repair } Q = 8.0171 \\ 4DELT/ORT - [RP - DRPTAT]^+ = 13.1 \end{cases} \end{cases}$$

$$RQ = 9$$

$$\nu = 4(SRA) = 1.3484$$

$$E(Y) = QSDF/SRA = 1.008306$$

$$E(Y^2) = 1.57 \times DMAD/SRA = 1.12214714$$

$$E(Y^3) = 3E(Y)E(Y^2) - 2[E(Y)]^3 = 1.3441352$$

$$E(L) = PLT/4 = .7525$$

$$V(L) = 1.57 \times PLTMAD^2/16 = .00098125$$

$$E(F) = WR\ E(Y^2)/[2E(Y)] = .05564516$$

$$V(F) = WR^2 E(Y^3)/[3E(Y)] - E(F)^2 = .001347158$$

$$E(R) = DRPTAT/[\nu E(Y)] = .367755222$$

$$V(R) = 1.57/16 \times CTATMD^2 = 0$$

$$E(D1) = [W/2 + E(L) - E(R) - RW/2 - C]\nu E(Y) = .127659322$$

$$V(D1) = [W/2 + E(L) - E(R) - RW/2 - C]\,\nu E(Y^2) + \left[\frac{W^2}{12} + V(L) + \frac{RW^2}{12}\right][\nu E(WY)]^2 = .14412$$

$$V(\beta_1) = [1.57x(CRMAD)^2 x4 - \beta_1^2\,\nu E(Y^2)]/[(\nu E(Y))^2/4 + \nu E(Y^2)]^+ = 33.602$$

$$V(\beta_2) = MADRSR^2 \times 1.57 = .002512$$

$$V(\beta_1\beta_2) = V(\beta_1)V(\beta_2) + \beta_1^2\,V(\beta_2) + \beta_2^2\,V(\beta_1) = 27.3044$$

where $\begin{cases} \beta_2 = RSR \\ \beta_1 = BETA1 \end{cases}$

$$E(D1)' = WRE(D1) = .0127659322$$

$$V(D1)' = (V(\beta_1\beta_2) + WR^2)V(D1) + V(\beta_1\ \beta_2)[E(D1)]^2 = 4.381528$$

$$E(D2) = (E(R) + RW/2 + C)\ \nu E(Y) = .898702574$$

$$V(D2) = (E(R) + RW/2 + C)\ \nu E(Y^2) = RW^2/12[\nu E(Y)]^2 = 1.000397449$$

$$E(D3) = E(D1)' + E(D2) + E(F) = .96711$$

$$V(D3) = V(D1)' + V(D2) + V(F) = 5.383$$

$$E(D4) = (E(L)-E(R)-RW/2-C)\nu E(Y) = .124396282$$

$$V(D4) = [E(L)-E(R)-RW/2-C]\nu E(Y^2) + [V(L)+RW^2/12](\nu E(Y))^2 = .140483$$

$$E(D4)' = WRE(D4) = .01243963$$

$$V(D4)' = (V(\beta_1\beta_2)+WR^2)V(D4)+V(\beta_1\beta_2)E(D4)^2 = 4.254734$$

$$E(D5) = E(D4)' + E(D2) = .9111422$$

$$V(D5) = V(D4)' + V(D2) = 5.2601$$

$$PC1 = V(D3)/E(D3) = 5.5663782$$

$$RC1 = E(D3)^2/[V(D3)-E(D3)] = .2117893$$

f(0)=.6951775

GNB(0,PC1,RC1)=.3048

GS(0,PC1,RC1)=.6952

PC2=V(D5)/E(D5)=5.7730835

RC2=E(D5)^2/[V(D5)-E(D5)]=.19089174

f(0)=.71557232

f(1)=.112936

GNB(1,PC2,RC2)=.45592

GS(1,PC2,RC2)=2.9752251

E(U1)+E(D3)-GNB(0,PC1,RC1)=.66231

E(U2)=E(D5)-GNB(1,PC2,RC2)=.455223

E(U)=E(U1)-E(U2)=.2076873

E(S1)=[V(D3)+[P-E(D3)]^2-GS(0,PC1,RC1)]/[2 E(Y)]=1.72447

E(S2)=[V(D5)+[P+Q-E(D5)]^2-GS(1,PC2,RC2)]/[2 E(Y)]=1.2763

E(S)=[E(S1)-E(S2)]$^+$=.4481817

E(O)=.5WR(W)νE(Y)+E(F)+Q=1.055971464

E(RO)=RWνE(Y)/β_2=.058]60658

E(M)=β_1E(O)/[WRxE(RO)]=181.561

E(D6)=(E(R)+RW)νE(Y)=.5523445

V(D6)=(E(R)+RW)νE(Y^2)=.6147061

E(D7)=E(R)νE(Y)=.49999

V(D7)=E(R)νE(Y^2)=.556464

PC1=V(D6)/E(D6)=1.113

RC1=E(D6)^2/[V(D6)-E(D6)]=4.8921844

PC2=V(D7)/E(D7)=1.113

RC2=E(D7)^2/[V(D7)-E(D7)]=4.4276

LEVEL 2

$RP_2=0$

$E(RU)=\beta_1\beta_2E(0)/WR=9.5037432$

$E(RS)=[E(R)+RW/2]\beta_1\beta_2E(0)/WR=3.677998$

$E(TU)=E(U)+E(RU)=9.7114$

fill rate=$1-E(TU)WR/E(0)=8.03\%$

average days of delay for all requisitions=$[E(S)+E(RS)]\times 365\times WR/E(0)=142.6$

average days of delay for delayed requisitions=

average days of delay for all requisitions/(1-fill rate)=155.0731

time-weighted requisitions short, normalized to one year=

average days of delay for all requisitions$\times\nu$=192.31253

```
SHLFLF=0.0  MARK=0  RII=1 PMC=0   ORT=0.10  DELT=        0.340
PLT= 3.01  PROVAR=    207.07820  UPIR=        25.00  UP=        25.50  QSDF=          0.3399
QSRFIR=          0.3059  SRA=          0.3371  DMAD=          0.37828  AIE=0.500
RFR,SSQ,PPRLT,PPRFY,RFI            0          0          0          0          0
 NFFI,COA,PROC,MSUC         0          0          0          0  PLTMAD= 0.1  PPTAT= 1.85  SHPREC=  0
WR=0.10  CTATMD,NTATMD=  0.0  0.0  RSR=0.90  MADRSR=0.04  RFIREG=      0.3
DRPTAT=          0.5000  UCIR=        12.34  RSUC=      60  CRMAD=      3.2878  REPVAR=       0.9737
 NU=      1.34840  EY,EY2,EY3:        1.0083       1.1221       1.3441
ER,VR:      0.3578       0.0          EF,VF:       0.0557       0.0013
EL,VL:      0.7525       0.0010      ED1,VD1:              0.128            4.381
ED3,VD3:           0.9671          5.3832      ED5,VD5:             0.9111           5.2601
 BETA1=1.000  ED6,VD6:           0.5523         0.6147  ED7,VD7:          0.5000           0.5565
POC      69.1600   BEOQ        0.8000     OMEGA1                0.5  21       0.6683   RHO1  0.654  X1      1.00000
P,Q          1          1
BEROQ       8.0171   OMEGA2              0.70   RHO2  0.588   X2       1.0000   RP,SQ          1          9
PC,RC            5.5662          0.2118
PC,RC            5.7731          0.1909
EU1,EU2,EU       0.6623       0.4552       0.2071
ES1,ES2,ES             1.724            1.276            0.448
```

Level 12

```
ERO,EM           0.0582        181.5043
ERU           9.501  ERS           3.677
```

Level 13

```
PC,RC            1.1129          4.8922
PC,RC            1.1129          4.4286
ERU1,ERU2,ERU             0.1449          0.1227          4.0324
ERS1,ERS2,ERS          0.0318          0.0676          2.5884
```

Level 14

```
PC,RC            1.1129          4.8922
PC,RC            1.1129          4.4286
ERU1,ERU2,ERU             0.0000          0.0000          0.0000
ERS1,ERS2,ERS          0.0000          0.0000          0.0
```

SUMMARY FOR REPAIRABLE ITEMS, LEVEL 2

MIN RISK FACTOR	MAX RISK FACTOR	PROC. SHORTAGE COST	PROC. BUDGET FIRST YEAR	PROC. BUDGET TWO YEARS	FILL RATE	AVE. DAYS OF DELAY FOR DELAYED REQUISITIONS	AVE. DAYS OF DELAY FOR ALL REQUISITIONS	AVERAGE REQUISITION DAYS SHORT PER YEAR	DOLLAR VALUE OF SAFETY STOCK	AVE. DAYS OF SAFETY STOCK	REPAIR COST FIRST YEAR	REPAIR COST TWO YEARS
COG 4N												
.01	.50	100.00	51.	51.	0.0804	155.09	142.6	192.3	8.5	89.	4.	21.

LEVEL 3 $RP_3 = 1$

using PC1 and RC1:

$f(0) = PC1^{(-RC1)} = .592293657$

$GNB(0, PC1, RC1) = .407706$

$GS(0, PC1, RC1) = .592293657$

$E(RU1) = E(D6) - GNB(0, PC1, RC1) = -.1446$

$E(RS1) = [V(D6) + (RP_3 - E(D6))^2 - GS(0, PC1, RC1)]/[2\nu E(Y)] = .082$

using PC2, RC2:

$f(0) = .622498071$

$GNB(0, PC2, RC2) = .3775$

$GS\ (0\ PC2, RTC2) = .6225$

$E(RU2) = .1227$

$E(RS2) = .668$

$E(RU) = [E(RU1) - E(RU2)]^{+}(EM) = 4.03$

$E(RS) = [E(M)(E(RS1) - E(RS2))] = 2.54$

$E(TU) = E(RU) + E\ (U) = 4.2265$

fill rate: $1 - E(TU)\ WR/E(0) = 59.976\%$

average days of delay for all requisitions = 103.357

average days of delay for delayed requisitions = 258.237

time weighted requisitions short, normalized to one year: 139.37

SUMMARY FOR REPAIRABLE ITEMS, LEVEL 3

MIN RISK FACTOR	MAX RISK FACTOR	PROC. SHORTAGE COST	PROC. BUDGET FIRST YEAR	PROC. BUDGET TWO YEARS	FILL RATE	AVE. DAYS OF DELAY FOR DELAYED REQUISITIONS	AVE. DAYS OF DELAY FOR ALL REQUISITIONS	AVERAGE REQUISITION DAYS SHORT PER YEAR	DOLLAR VALUE OF SAFETY STOCK	AVE. DAYS OF SAFETY STOCK	REPAIR COST FIRST YEAR	REPAIR COST TWO YEARS
COG 4N												
.01	.50	100.00	51.	51.	0.5994	261.43	105.0	141.6	8.5	89.	4.	21.

<u>LEVEL 4</u>

$$RP_4 = RP_3 + MAX \begin{cases} [DRPTAT - RP]^+ + 90 \times \nu \times E(Y)/365 = .335244 \\ [RP - DRPTAT]^+ + RQ = 9.5 \end{cases}$$

$RP_4 = 10$

using PC1, RC1

f(0) = .592293657

f(1) = .294186798

f(2) = .087993945

f(3) = .01754652614

f(4) = .003514889

f(5) = .000634648

f(6) = .000106232

f(7) = .000016782

f(8) = .000002532

f(9) = .00000036837

GNB(9, PC1, RC1) = .577858732

GS(9, PC1, RC1) = 89.69415171

E(RU1)=E(D6) - GNB = 0

$E(RS1) = [V(D6) + (RP_4 - E(D6))^2 - GS]/(2\nu E(Y)) = .065735827$

using PC2, RC2

f(0) = .622498071

f(1) = .279827033

f(2) = .077099362

f(3) = .016771104

f(4) = .003161793

f(5) = .000541066

f(6) = .000086145

f(7) = .0000130543

$f(8) = .00000189322$

$f(9) = .000002440605$

$GNB = .500338596$

$GS = 90.80679976$

$E(RU2) = 0$

$E(RS2) = .073427584$

$E(RU) = 0$

$E(RS) = 0$

$E(TU) = .2076873$

fill rate: $1 - E(TU)WR/E(U) = 98.03\%$

average days of delay for all requisitions = 15.49

average days of delay for delayed requisitions = 786.37

time-weighted requisitions short, normalized

to one year: 20.89

safety stock $[P - Z1]^+ = .3317566$

dollar value of annual demand = $UP \times \nu \times E(Y) = 34.67$

dollar value of safety stock = $UP \times [P-Z1]^+ = 8.4598$

days of safety stock = $\frac{\text{dollar value of safety stock}}{\text{dollar value of annual demand}} = 89.064$

SUMMARY FOR REPAIRABLE ITEMS, LEVEL 4

MIN RISK FACTOR	MAX RISK FACTOR	PROC. SHORTAGE COST	PROC. BUDGET FIRST YEAR	PROC. BUDGET TWO YEARS	FILL RATE	AVE. DAYS OF DELAY FOR DELAYED REQUISITIONS	AVE. DAYS OF DELAY FOR ALL REQUISITIONS	AVERAGE REQUISITION DAYS SHORT PER YEAR	DOLLAR VALUE OF SAFETY STOCK	AVE. DAYS OF SAFETY STOCK	REPAIR COST FIRST YEAR	REPAIR COST TWO YEARS
COG 4N												
.01	.50	100.00	51.	51.	0.9804	789.96	15.5	20.9	8.5	89.	4.	21.

F.2.4 Procurement Cost Calculations

Assets = 0

Attrition = $\nu E(Y)$ = .135959981

H1 = 0

H2 = P + Q - Assets = 2

T(1) = .135959981 T(2) = .271919962

G1(1) = 0 G1(2) = 0

PC(1) = \$51 PC(2) = \$51

F.2.5 Repair Cost Calculations

LEVEL 2

$$\varepsilon = \min\left(\left[\frac{[P-Z1]^{+} + E(0) - (RP + \nu E(Y)RW/2)}{E(0)/WR}\right]^{+},\quad WR\right) = .1$$

H3 = $E(L)\ \nu E(Y)\ \ WR$ = .10231

$$G2 = INT\left[\frac{H3-H1}{E(0)}\right] = 0$$

H4 = (1 + G2) E(0) + H1 - H3 = .954

G3 = INT[T + H4)/E(0)]

G3(1) = 1 G3(2) = 1

$$H5 = \min\left(\left[\frac{T+H4}{E(0)} - G3 - \varepsilon\right]^{+} /(1-WR)\ ,\ 1\right)$$

H5(1) = 0 H5(2) = .067355

$$H6 = \min\left(\left[\frac{H4}{E(0)} - \varepsilon\right]^{+} /(1-WR),\ 1\right) = .89234768$$

$$RC = \frac{(1-WR)UCIR\ E(0)}{WR}\ (G3 + H5-H6)$$

RC(1) = \$12.625 RC(2) = \$20.52

LEVEL 3

ε = .0324

H5(1) = 0 H5 (2) - .1425

H6 = .967458791

RC(1) = \$2.98 RC(2) = \$19.69

LEVEL 4

ε = 0

H5(1) = .035407081 H5(2) .1785

H6 = 1

RC(1) = \$4.152 RC(2) \$20.93

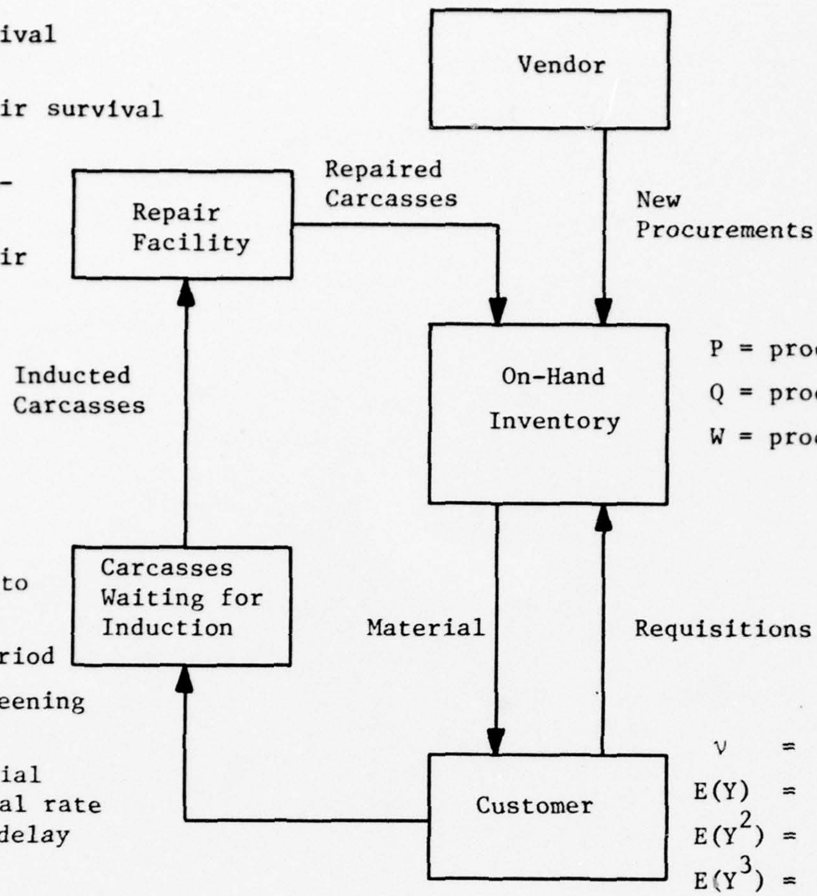
APPENDIX G

DERIVATION OF FORMULAS

There are several categories of items, and each of these has its own inventory control rules: there are either repairable or consumable items; either high or low demand cases; either B08 or FIRM programs for SPCC repairable items; and either B08, CLAMP, or HIGH BURNER programs for ASO repairable items. Because the formulas for computing the measures of service (fraction of requisitions satisfied without a backorder, time-weighted requisitions short, and average days delay) for all these different cases are similar, this Appendix will only give the derivations in detail for one of these cases: namely, for repairable items with high demand under the B08 program. At the end of this Appendix we will briefly discuss how these formulas should be modified to handle these other cases.

G.1 Input Parameters

The input parameters required by the model of repairable management under the B08 program are given in Figure G.1, and this diagram also illustrates their relationship. The demand process is assumed to be a Compound Poisson process, in which requisitions arrive according to the Poisson process with arrival rate ν (requisitions per year) and the issue sizes for different requisitions are independent and identically distributed with mean $E(Y)$, second moment $E(Y^2)$, and third moment $E(Y^3)$. After a delay of length C (years) from the time that a requisition arrives, the carcasses associated with that requisition will also arrive. The random variable β_1, with mean $E(\beta_1)$ and variance $V(\beta_1)$, is the fraction of the total issue size in the requisition for which there are carcasses that can survive an initial screening that takes place. The random variable β_2, with mean $E(\beta_2)$ and variance $V(\beta_2)$, is the fraction of carcasses surviving the initial screening that will also survive the repair process. The available inventory RI for the repair process is the RFI (ready for issue) inventory, plus repairable inducted carcasses, plus any on-order inventory that will arrive within the repair turnaround time. At the beginning of each repair review period of length RW, carcasses are inducted in the amount (if positive) of $(RP-RI)/E(\beta_2)$, where RP is the repair order-up-to-point. The random variable R, with mean $E(R)$ (years) and variance $V(R)$ (years^2), is the repair turnaround time, which is the interval between repair induction and receipt of the repaired carcasses as part of the RFI inventory. At the beginning of a review period of length W (years), the available inventory I for the procurement process is determined: I is the on-hand RFI inventory, plus on-order, plus repairable carcasses (either held or inducted). If I is less than the reorder point P,



$E(\beta_2)$ = mean repair survival rate

$V(\beta_2)$ = variance of repair survival rate

$E(R)$ = mean repair turn-around time

$V(R)$ = variance of repair turnaround time

RP = repair order-up-to point

RW = repair review period

$E(\beta_1)$ = mean initial screening survival rate

$V(\beta_1)$ = variance of initial screening survival rate

C = carcass arrival delay

P = procurement reorder point

Q = procurement reorder quantity

W = procurement review period

ν = requisition arrival rate

$E(Y)$ = mean issue size

$E(Y^2)$ = second moment of issue size

$E(Y^3)$ = third moment of issue size

Figure G.1

Flows of Materials and Requisitions for Repairable Items then new items are ordered from a vendor in the amount $Q + P - I$, where Q is the reorder quantity. The random variable L, with mean $E(L)$ (years) and variance $V(L)$ (years^2), is the procurement lead time, which is the interval between the reorder decision and receipt of the material as part of the RFI inventory.

Next, we briefly describe how each of these input parameters is obtained. The procurement reorder point P, reorder quantity Q, and repair order-up-to point RP are computed in the same manner as in the LEVELS documentation (Reference 1). The review period W for the procurement process is treated as a system constant, and a value corresponding to reviewing four times each week is generally used. The review period RW for the repair process is also treated as a system constant and is generally two weeks for SPCC items and one week for ASO items. Although adequate data for properly estimating the carcass arrival delay C is not currently available, the user can input any desired value as a system constant. The mean lead time $E(L)$, lead time variance $V(L)$, mean repair turnaround time $E(R)$, repair turnaround time variance $V(R)$, requisition frequency ν, mean repair survival rate $E(\beta_2)$, and repair survival rate variance $V(\beta_2)$ are all directly estimated from corresponding data elements that are available for each item, and this procedure needs no special explanation. While the remaining parameters ($E(Y)$, $E(Y^2)$, $E(Y^3)$, $E(\beta_1)$, $V(\beta_1)$) are also estimated from data elements that are available for each item, some special explanation is required, and this is given below.

a. <u>First and Second Moment of Issue Size</u>

Our approach for allowing a variable issue size is to model the demand with a Compound Poisson process. This assumes that requisitions arrive at the facility according to the Poisson process (i.e., the arrivals are random over time) and that the issue size is itself a random variable.

Let

$X(t)$ = actual demand for the item during a (fixed) interval of t years

Y_n = issue size for the n^{th} requisition

ν = mean rate at which requisitions arrive at the warehouse.

Thus,

$$X(t) = \sum_{n=1}^{N(t)} Y_n \quad ,$$

where $N(t)$ is the number of requisitions which arrive over the interval of length t. We assume that the random variables Y_n are independent and identically distributed with mean $E(Y)$ and second moment $E(Y^2)$. This implies that $X(t)$ is the sum of a random number of independent identically distributed random variables. It can be shown (see Theorem 2c, page 130, in Parzen [2]) that over a fixed interval of length t, the Compound Poisson process has mean

$$E[X(t)] = \nu t E(Y)$$

and variance

$$V[X(t)] = \nu t E(Y^2) \quad .$$

On the data tape, the "average quarterly demand" is represented by the den (data element number)[B074] and the "mad (mean absolute deviation) of quarterly demand" by the den [A019]. The foregoing formulas imply that the mean issue size can be estimated with

$$E(Y) = 4[B074]/\nu$$

and the second moment of issue size can be estimated with

$$E(Y^2) = 4(1.57)[A019]^2/\nu \quad ,$$

where we assume that the requisition forequency ν has already been estimated.*

b. Third Moment of Issue Size

The third central moment (about the mean) is defined as

$$\mu_3 = E\{[Y - E(Y)]^3\} \quad ,$$

which is a measure of asymmetry or skewness.

*For the normal distribution, the following relationship approximately holds:

$$1.57 \times (MAD)^2 = \text{Variance} \quad .$$

Symmetrical distributions, such as the normal, can be shown to have $\mu_3 = 0$, and our assumption is that the issue size distribution is symmetric. This implies that the third moment can be estimated from the first two moments using

$$E(Y^3) = 3\,E(Y)\,E(Y^2) - 2[E(Y)]^3 \quad .$$

d. Mean and Variance of Initial Screening Survival Rate

For the purpose of estimating the mean and variance of the initial screening survival rate, two data elements are available: The "average quarterly regenerations" is represented by the den [B074A] and the "carcass return mad" by [A019B]. We assume that the mean $E(\beta_2)$ and variance $V(\beta_2)$ of the repair survival rate have already been estimated.

The mean initial screening survival rate can be computed as

$$E(\beta_1) = \frac{4[B074A]}{E(\beta_2)\ \nu\, E(Y)} \quad ,$$

but the variance is more complex to estimate. The total number of carcasses that will be returned during a quarter is $H = \beta_1 X(\tfrac{1}{4})$, where $X(\tfrac{1}{4})$ is the demand over a quarter. Note that $X(\tfrac{1}{4})$ has mean $\nu E(Y)/4$ and variance $\nu E(Y^2)/4$. Thus H has conditional mean

$$E(H|\beta_1) = \beta_1 \nu E(Y)/4$$

and conditional variance

$$V(H|\beta_1) = \beta_1^2 \nu E(Y^2)/4 \quad .$$

Because the unconditional variance is equal to the mean of the conditional variance plus the variance of the conditional mean (see Page 53 in Parzen [2]),

$$\begin{aligned} V(H) &= E\{V(H|\beta_1)\} + V\{E(H|\beta_1)\} \\ &= \{V(\beta_1) + [E(\beta_1)]^2 \nu E(Y^2)/4 \\ &\quad + V(\beta_1)[\nu E(Y)/4]^2 \quad . \end{aligned}$$

Because V(H) is the carcass return variance, it can be obtained from the carcass return mad as follows: $V(H) = (1.57)[A019B]^2$. After substituting for V(H) and solving for $V(\beta_1)$, the resulting formula for the variance of the initial screening survival rate is

$$V(\beta_1) = \frac{(1.57)[A019B]^2 - [E(\beta_1)]^2 \nu E(Y^2)/4}{[\nu E(Y)/4]^2 + \nu E(Y^2)/4} \quad .$$

G.2 Renewal Process for Procurement Reordering

In the remainder of this Appendix, a number of formulas are derived which compute several measures of service as a function of the input parameters. This section uses a renewal theory result to estimate the mean and variance of the unfilled portion of the requisition causing the reorder point P to be reached in procurement reordering. The key observation here is that the expected issue size for the requisition hitting the reorder point will, in general, be larger than the mean issue size $E(Y)$. In other words, it is more likely for a requisition with a larger issue size to hit the reorder point, than it is for a requisition with a smaller issue size. At the beginning of a review period of length W, the system inventory is reviewed to determine whether a procurement is necessary. If the available inventory (on-hand, plus on-order, plus repairable carcasses) is less than the reorder point P, then sufficient stock is ordered to bring the total available inventory up to the level $P + Q$. This implies that the available inventory will again fall below the reorder point after a net demand (attrition plus new demands) of Q units. Define $Z_n = (1-\beta_1\beta_2)Y_n$ to be the net demand for the n^{th} requisition and

$$S_n = Z_1 + Z_2 + \ldots + Z_n$$

to be the total net demand for the first n requisitions following reordering. There exists a chance-dependent subscript N such that

G.3 Expected Number of Units Backordered Due to Insufficient Procurement

The purpose of these next two sections is to compute the expected number of units backordered during a procurement cycle. The beginning of a procurement cycle is defined to be just after a procurement shipment arrives and is added to the RFI inventory, and the end of a procurement cycle is defined to be just before the next shipment arrives. The fundamental basis of our approach is to split the total number of backorders into two categories: backorders due to insufficient procurement, which refers to shortages due to the reorder level P being too low; and backorders due to inducting insufficient carcasses, which refers to shortages due to the repair order-up-to point RP being too low. This section will address the procurement associated shortages, while the next section will address the repair associated shortages.

The procurement reordering process is illustrated in Figure G.2. It may not be possible to immediately reorder when the available inventory first falls below the reorder point P , as it is necessary to wait until the next procurement review. The random variable A is defined as the delay between when the reorder point P is first reached and the next review. We assume that A is distributed according to the uniform distribution with mean $W/2$ and variance $W^2/12$, where W is the length of the procurement review period. The lead time L is defined as the delay between when a shipment is ordered and when it arrives, where the mean $E(L)$ and variance $V(L)$ are included as input parameters. However, some of the demand that occurs during the lead time can be repaired and returned RFI prior to shipment arrival. The repair turnaround time R is the delay between when a carcass is inducted and when it can become part of the RFI inventory, where the mean $E(R)$ and variance $V(R)$ are

$$S_N \leq Q < S_{N+1} \quad ,$$

which means that the available inventory will again fall below the reorder point during the $(N+1)^{th}$ requisition. In renewal theory, the variable $F = S_{N+1} - Q$ is called the "excess life at time Q".

Suppose that the random variable Z (net demand for a requisition) has a cumulative distribution function (c.d.f) $L(\cdot)$ with mean

$$E(Z) = \int_0^\infty [1 - L(z)]\, dz \quad .$$

According to a result from renewal theory (see page 370 in Feller [3]), as $Q \to \infty$, the excess life F will have as its limiting density

$$h(f) = \frac{1}{E(Z)} [1 - L(f)] \quad .$$

Thus the k^{th} moment for F with respect to this limiting distribution is

$$E(F^k) = \int_{f=0}^\infty f^k h(f)\, df = \frac{1}{E(Z)} \int_{f=0}^\infty f^k [1 - L(f)]\, df \quad .$$

After integrating by parts, this expression becomes

$$E(F^k) = \frac{E(Z^{k+1})}{E(Z)(k+1)} .$$

It is convenient to compute $E(AT) = [1 - E(\beta_1) E(\beta_2)]$, which is the expected fraction of a requisition that can not be repaired. Because $Z = (1 - \beta_1 \beta_2) Y$, we will use the approximation

$$E(Z^k) = [E(AT)]^k E(Y^k) ,$$

which would be the exact expression if $V(\beta_1) = V(\beta_2) = 0$. Thus F has mean

$$E(F) = \frac{E(AT)\, E(Y^2)}{2\, E(Y)} ,$$

second moment

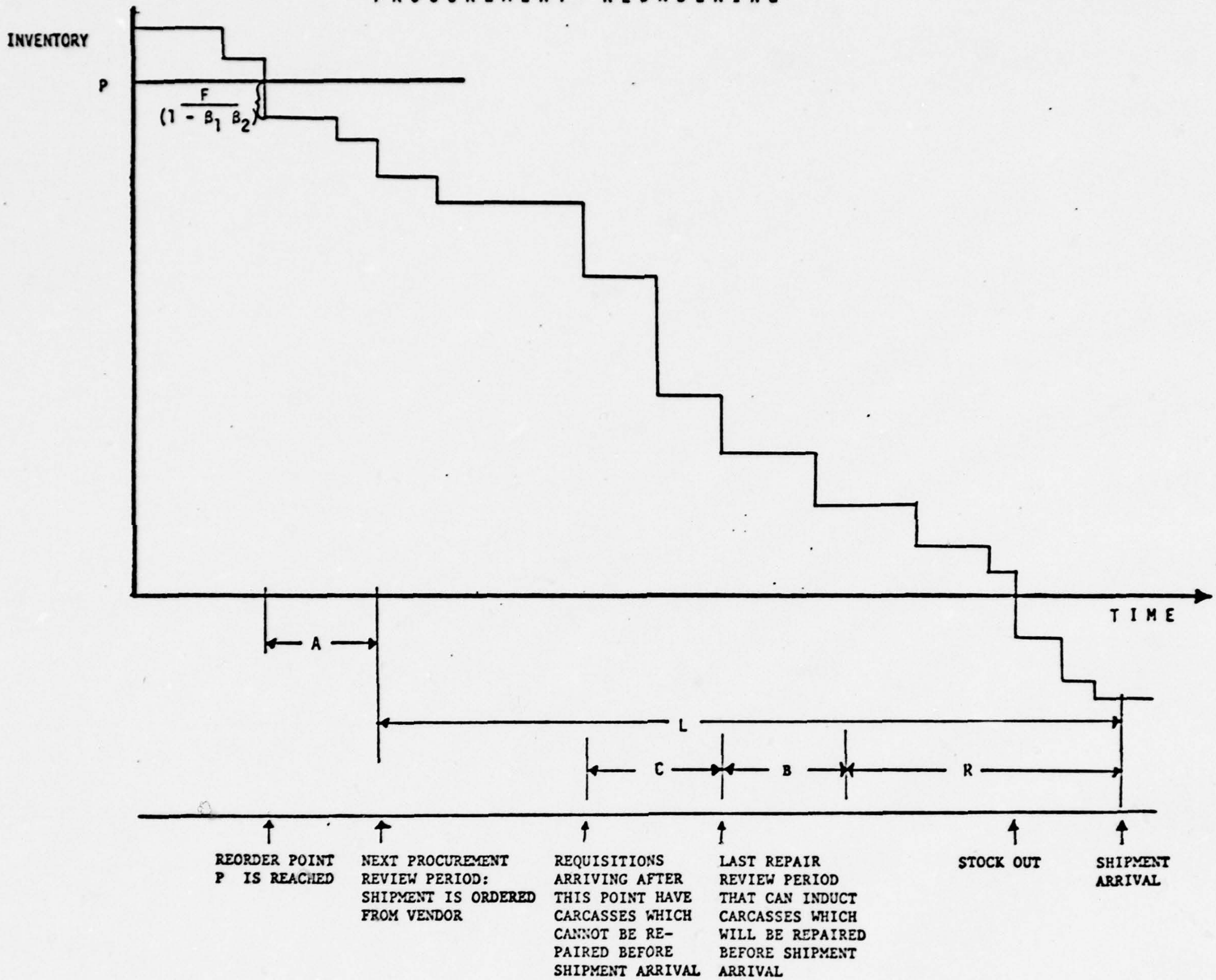
$$E(F^2) = \frac{[E(AT)]^2 E(Y^3)}{3\, E(Y)} ,$$

and variance

$$V(F) = \frac{[E(AT)]^2 E(Y^3)}{3\, E(Y)} - [E(F)]^2 .$$

Figure G.2

PROCUREMENT REORDERING



also included as input parameters. It follows that the only carcasses which can be repaired prior to shipment arrival are those which are inducted prior to the interval R before shipment arrival. But the inventory is reviewed for an induction decision only at the beginning of review periods with length RW . Thus the only carcasses which can be repaired are those which arrive prior to the last repair review prior to the interval R . The random variable B is defined to be the delay between this last repair review and the interval R . We assume that B is distributed according to the uniform distribution with mean RW/2 and variance $(RW)^2/12$. The input constant C is defined to be the delay between the arrival of a requisition and the arrival of its associated carcasses. Thus, only if a requisition arrives before the interval of length R + B + C prior to shipment arrival, can its carcasses be repaired before this arrival.

Let D_1 be the total demand over a random interval of length T = A + L - R - B - C , where we assume that T is nonnegative and that A, L, R, and B are independent. It follows that T has mean

$$E(T) = \frac{W}{2} + E(L) - E(R) - \frac{RW}{2} - C$$

and variance

$$V(T) = \frac{W^2}{12} + V(L) + V(R) + \frac{(RW)^2}{12} .$$

Note that formulas in Section G.1 give the mean and variance of demand over a fixed interval and that these formulas imply that D_1 has conditional mean

$$E(D_1|T) = \nu TE(Y)$$

and conditional variance

$$V(D_1|T) = \nu T E(Y^2) .$$

Because the unconditional variance is equal to the mean of the conditional variance plus the variance of the conditional mean (see page 55 in Parzen [2]),

$$V(D_1) = E\{V(D_1|T)\} + V\{E(D_1|T)\}$$

$$= \nu E(T) E(Y^2) + \nu^2 V(T)[E(Y)]^2 .$$

Because the unconditional mean is equal to the maan of the conditional mean

$$E(D_1) = E\{E(D_1|T)\}$$

$$= \nu E(T) E(Y) .$$

The next step is to derive the mean and variance of

$$D_1' = (1 - \beta_1\beta_2) D_1$$

in terms of the means and variances of D_1 , β_1 , and β_2 . It can be shown that $\beta_1\beta_2$ has mean

$$E(\beta_1\beta_2) = E(\beta_1) E(\beta_2)$$

and variance

$$V(\beta_1\beta_2) = V(\beta_1)\,V(\beta_2) + [E(\beta_1)]^2\,V(\beta_2) + [E(\beta_2)]^2\,V(\beta_1) \quad .$$

Also note that D_1' has conditional mean

$$E(D_1'|\beta_1\beta_2) = (1 - \beta_1\beta_2)\,E(D_1)$$

and conditional variance

$$V(D_1')|\beta_1\beta_2) = (1 - \beta_1\beta_2)^2\,V(D_1) \quad .$$

Since the unconditional variance is equal to the mean of the conditional variance plus the variance of the conditional mean:

$$V(D_1') = E\{V(D_1|\beta_1\beta_2)\} + V\{E(D_1'|\beta_1\beta_2)\}$$

$$= \{V(\beta_1\beta_2) + [1 - E(\beta_1)\,E(\beta_2)]^2\}V(D_1)$$

$$+ V(\beta_1\beta_2)[E(D_1)]^2 \quad .$$

Similarly, the unconditional mean is equal to the mean of the conditional mean or

$$E(D_1') = E\{E(D_1'|\beta_2\beta_2)\} = E(AT)\,E(D_1),$$

where $E(AT)$ was computed in Section G.2.

Define D_2 to be the total demand over an interval of length $R + B + C$, which is also a random interval. Using the same approach as above, it can be shown that D_2 has mean

$$E(D_2) = \left[E(R) + \frac{RW}{2} + C\right] \nu E(Y)$$

and variance

$$V(D_2) = \left[E(R) + \frac{RW}{2} + C\right] \nu E(Y^2)$$
$$+ \left[V(R) + \frac{(RW)^2}{12}\right] \nu^2 [E(Y)]^2 .$$

Let D_3 be the demand over the lead time plus procurement periodic delay, including the unfilled portion of the requisition when the reorder point is reached, less the number of units that can be potentially repaired and returned RFI during this interval. Thus

$$D_3 = (1 - \beta_1\beta_2) D_1 + D_2 + F ,$$

where the mean and variance of F were given in Section G.2. Assuming that these random variables are independent, D_3 has mean

$$E(D_3) = E(D_1') + E(D_2) + E(F)$$

and variance

$$V(D_3) = V(D_1') + V(D_2) + V(F) \quad .$$

Define the random variable U_1 to be the number of units backordered at the end of a procurement cycle (i.e. just before the arrival of the next shipment). In order to compute the expectation $E(U_1)$, we make the following assumptions: 1) orders do not cross; 2) only carcasses which can survive the repair process are counted in the available inventory; 3) the lead time exceeds the repair turnaround time, plus repair review period, plus carcass arrival delay; and 4) all repairable carcasses received prior to the interval of length $R + B$ before shipment arrival are repaired and returned RFI prior to shipment arrival. We anticipate that this last assumption would be valid for the level 3 and 4 repair scenarios under the B08 program (or of course for consumable items), but would not be valid for level 2 repair. Later in Section G.9 we will briefly indicate how our approach can be modified to handle the level 2 repair scenario. A decision to reorder is made whenever the available inventory I is below the reorder point P at the beginning of a review period. The foregoing assumptions imply that the on-hand RFI inventory just before the arrival of an order is equal to $P - D_3$. Thus the number of units backordered is equal to $U_1 = [D_3-P]^+$, and the expected value of U_1 can be computed if we know the distribution for D_3 .*

Because we are considering the high demand case, we will represent D_3 with a normal distribution. Thus the expected number of backorders can be computed as

$$E(U_1) = \int_{D=p}^{\infty} \frac{(D - P)}{\sigma} \varphi\left(\frac{(D - \mu)}{\sigma}\right) dB$$

where $\mu = E(D_3)$, $\sigma^2 = V(D_3)$, and

$$\varphi(r) = \frac{1}{\sqrt{2\pi}} \exp(- 1/2\, r^2) .$$

This expression can be evaluated (see page 446 in Hadley and Whitin [4]) to give the following as the expected number of units backordered at the end of a procurement cycle:

*We define $[x]^+ = x$ if $x \geq 0$, and $[x]^+ = 0$ if $x < 0$.

$$E(U_1) = [V(D)]^{1/2} g(K_1) ,$$

where

$$K_1 = \frac{P - E(D)}{[V(D)]^{\frac{1}{2}}} ,$$

$$\Phi(r) = \int_{x=r}^{\infty} \varphi(x)\, dx ,$$

and

$$g(K) = -K\, \Phi(K) - \varphi(K) .$$

Note that some of the backorders which are present at the end of a procurement cycle may have been carried over from a previous cycle. Define the random variable U_2 to be the number of backorders existing at the beginning of a procurement cycle (i.e. just after a procurement shipment arrives). If there were no additional backorders due to late repair inductions, then $E(U_1) - E(U_2)$ would be the total number of backorders occurring in a single procurement cycle. And if there were additional backorders due to late repair inductions we may still intrepret

$E(U_1) - E(U_2)$ to be the expected number of backorders due to insufficient procurement that occurred over the cycle. If we could assume that a procurement shipment arrival would eliminate all previous backorders, then $E(U_2) = 0$. We anticipate that this may be a sufficiently good assumption in practice to avoid the necessity for actually computing $E(U_2)$, and thus our program does give the user the option to bypass this calculation. The formulas for computing $E(U_2)$ are very similar to those described in detail for $E(U_1)$, and thus they will be listed with minimal further explanation. Let D_4 be the total demand over an interval with length $L - R - B - C$, and it has mean

$$E(D_4) = E(L) - E(R) - \frac{RW}{2} - C \quad \nu E(Y)$$

and variance

$$V(D_4) = \left[E(L) - E(R) - \frac{RW}{2} - C\right] \nu E(Y^2) + \left[V(L) + V(R) + \frac{(RW)^2}{12}\right] \nu^2 \{E(Y)\}^2 \quad .$$

Let $D_4' = (1 - \beta_1\beta_2)D_4$, and it has mean

$$E(D_4') = E(AT)\, E(D_4)$$

and variance

$$V(D_4') = \{V(\beta_1\beta_2) + [E(AT)]\}^2\, V(D_4)$$
$$+ V(\beta_1\beta_2)[E(D_4)]^2 \quad .$$

Let D_5 be the demand over the lead time, less the number of units that can be potentially repaired and returned RFI during this interval. Thus D_5 has mean

$$E(D_5) = E(D_4') + E(D_2)$$

and variance

$$V(D_5) = V(D_4') + V(D_2) \quad .$$

Note that $U_2 = [D_5 - P - Q]^+$, which implies that

$$E(U_2) = [V(D_5)]^{1/2}\, g(K_2) \quad ,$$

where

$$K_2 = \frac{P + Q - E(D_5)}{[V(D_5)]^{1/2}} \quad .$$

G.4 Expected Number of Units Short Due to Inducting Insufficient Carcasses

The available inventory RI for the repair problem is the RFI inventory, plus repairable inducted carcasses, plus any on-order inventory that will arrive within the repair turnaround time. The decision to induct carcasses is controlled by an order-up-to policy: if there are sufficient carcasses and if RI is less than RP at the beginning of a review period of length RW , then carcasses are inducted in the amount $(RP-RI)/E(\beta_2)$, where RP is the order-up-to repair point and $E(\beta_2)$ is the mean repair survival rate. At the beginning of a procurement cycle (i.e., just after a procurement shipment arrival), the on-hand RFI inventory is at its highest value. As requisitions from customers are received, this RFI inventory will fall and the inventory of repairable carcasses will increase until the RFI inventory first falls below the order-up-to point RP at the beginning of a review period. At this moment, the first repair cycle is initiated, which means that carcasses are sent to the repair facility and then are added to the RFI inventory after being repaired. From that time until the end of the procurement cycle (i.e. just before the next procurement shipment arrives), additional carcasses are inducted at each repair review as long as repairable carcasses are available.

We will define the beginning of a repair cycle (except for the first cycle) to be just after a shipment of repaired carcasses are delivered to the RFI inventory, and define the end of a repair cycle to be just before the next shipment arrives. The movement of these carcasses is controlled by the order-up-to point RP , and backorders will be incurred if this point is too low. Note that the backorders treated in Section G.3 were due to having insufficient system wide inventory because of insufficient procurements; while the backorders treated in this section are due to sending carcasses late to the repair facility even though there are sufficient carcasses within the system to meet the demand.

In our treatment of these repair backorders, we will make the approximation that all of the repair cycles have common characteristics: the same expected induction quantity; the same expected number of backorders at the end of a cycle; and (except for the first cycle) the same expected number of backorders at the beginning of a repair cycle. This approximation is not strictly valid for two reasons: the first induction quantity may be smaller than the others, as the RFI inventory may first reach the point RP in the middle of a review period, rather than at the beginning; and the final induction quantitites may also be smaller, as there may be insufficient carcasses available to induct the full desired amounts. Because the repair review period is fairly small (in practice RW is one week for ASO items and two weeks for SPCC items), in our judgement there will be sufficient number of repair cycles in a procurement cycle to enable this approximation of treating all repair cycles in the same way to be reasonably valid, and the increased accuracy resulting from a more careful treatment would be small compared to the increase required in computations.

Define the random variable RU_1 to be the number of backorders existing at the end of a repair cycle (i.e. just before a repair shipment arrival) and RU_2 to be the number of backorders existing at the beginning of a repair cycle (i.e. just following a repair shipment arrival). It follows that the expected backorders incurred during a typical repair cycle is $E(RU_1) - E(RU_2)$.

The formulas for computing $E(RU_1)$ and $E(RU_2)$ are similar to those described in detail in Section G.3, and thus they are listed below with minimal further explanation. Let D_6 be the total demand over the repair turnaround time R plus the repair review period RW , and it has mean

$$E(D_6) = [E(R) + RW]\, \nu\, E(Y)$$

and variance

$$V(D_6) = [E(R) + RW]\, \nu\, E(Y^2) + V(R)\, \nu\, [E(Y)]^2 \quad .$$

Note that $RU_1 = [D_6 - RP]^+$, which implies that

$$E(RU_1) = [V(D_6)]^{1/2} g(K_5) \quad ,$$

where

$$K_5 = \frac{RP - E(D_6)}{[V(D_6)]^{1/2}} \quad .$$

Let D_7 be the demand over the repair turnaround time R, and it has mean

$$E(D_7) = E(R)\,\nu\,E(Y)$$

and variance

$$V(D_7) = E(R)\,\nu\,E(Y^2) + V(R)\,\nu^2\,[E(Y)]^2 \quad .$$

Note that $RU_2 = [D_7 - RP]^+$, which implies that

$$E(RU_2) = [V(D_7)]^{1/2} g(K_4) \quad ,$$

where

$$K_4 = \frac{RP - E(D_7)}{[V(D_7)]^{1/2}} \quad .$$

G.5 Fraction of Requisitions Satisfied Without a Backorder

This section derives the formula for our first service measure, which is the fraction of requisitions satisfied without a backorder. In order to do this, it is necessary to make a key assumption: all repairable carcasses received prior to the interval of length $R + B$ before procurement arrival are repaired and returned RFI prior to procurement arrival. This implies that $E(U_1)$ is the expected number of backorders just prior to procurement arrival and $E(U_2)$ is the expected number of backorders just after procurement arrival. We believe that this is a valid assumption for levels 3 and 4 repair scenarios under the B08 program, but that it would not be valid for level 2 repair. Refer to Section G.9 for a brief discussion as to how our approach is modified for level 2 repair.

The first step is to estimate the expected number of repair shipments in a procurement cycle. Let $E(O)$ be the expected procurement reorder quantity, and it is equal to

$$E(O) = \tfrac{1}{2}E(AT)W\nu\, E(Y) + E(F) + Q \quad ,$$

where the first term corresponds to the demand over the procurement periodic delay (i.e. the time between when the reorder point P is first reached and the start of the next procurement review period), the second term corresponds to the unfilled portion of the requisition which caused the reorder point P to be reached, and the third term is the economic reorder quantity. The expected induction quantity corresponds to the demand over a repair review period and equals

$$E(RO) = (RW)\,\nu\,E(Y)/E(\beta_2) \;.$$

We may interpret $E(O)$ to be the expected attrition over a procurement cycle, $E(O)/E(AT)$ to be the expected demand over a procurement cycle, and $E(\beta_1)E(O)/E(AT)$ to be the expected number of carcasses received over a procurement cycle. Assuming that each repair cycle inducts carcasses in the amount $E(RO)$, then

$$E(M) = \frac{E(\beta_1)\ E(O)}{E(AT)\ E(RO)}$$

is our estimate of the expected number of repair shipments in a procurement cycle.

The expected number of backorders occurring between the beginning of the procurement cycle and the arrival of the first repair shipment is $E(RU_1) - E(U_2)$; the expected number of backorders occurring during each of the next $E(M) - 1$ repair cycles is $E(RU_1) - E(RU_2)$; and the expected number of backorders between the last repair shipment and the end of the procurement cycle is $E(U_1) - E(RU_2)$. Thus the expected total number of backorders in a procurement cycle is

$$E(TU) = E(U_1) - E(U_2) + E(M)\ [E(RU_1) - E(RU_2)] \quad .$$

Because $E(0)/E(AT)$ is the expected demand in a procurement cycle, $E(TU)E(AT)/E(0)$ is the fraction of units that are backordered. Thus, the fraction of units satisfied without a backorder is

$$\rho = 1 - \frac{E(TU)\ E(AT)}{E(0)} \quad .$$

If we assume that the expected number of requisitions backordered is $E(TU)/E(Y)$ and that the total number of requisitions in a procurement cycle is $E(0)/[E(AT)E(Y)]$, then this formula also would give the fraction of requisitions satisfied without a backorder.

G.6 Time-Weighted Units Short

Our second service measure is the expected time-weighted units short. The time-weighting is linear, so that a backorder lasting for two weeks counts as much as two backorders lasting for one week. Suppose, for example, that there are three backorders and the first one lasted for one week, the second for two weeks, and the third for three weeks; then the total time-weighted units backordered would be six unit-weeks. In this section we will estimate the expected time-weighted units backordered over a procurement cycle, and this will be computed by summing separate estimates for the procurement and repair problems.

The procurement side will be considered first, which means that we will be considering backorders caused by purchasing insufficient new stock. In Section G.3, the random variable U_1 was defined to be the number of units backordered at the end of a procurement cycle (i.e. just prior to a procurement shipment arrival). Some of these units will have remained backordered longer than others. The mean demand per year is $\lambda = \nu E(Y)$ and if we assume that this demand occurred uniformly during the period in which these U_1 units were backordered, then the time at which the first of these units was backordered occurred U_1/λ years prior to the end of the cycle;* also, the average time that a unit remained backordered is $\frac{1}{2}U_1/\lambda$, so that the total time-weighted units backordered is $S_1 = \frac{1}{2}(U_1)^2/\lambda$.

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*It might be thought that for repairable items the annual attrition $\lambda' = \nu E(Y)[1-E(\beta_1)E(\beta_2)]$ should be used in this calculation instead of λ , but this is not the case. Let T be the length of time prior to the end of the cycle that the first unit was backordered. It is reasonable to assume that T is less than the repair turnaround time R; thus all demand (not just attrition) during the interval T will be backordered at the end of the procurement cycle, so that the total units backordered will be $U_1 = T\lambda$, which implies that $T = U_1/\lambda$.

Our task now is to compute the expected value of S_1 . Section G.3 showed that $U_1 = [D_3 - P]^+$, where P is the reorder point and the demand D_3 has mean $\mu = E(D_3)$ and variance $\sigma^2 = V(D_3)$. If we assume that D_3 has the normal distribution, then S_1 has expectation

$$E(S_1) = \int_{D=P}^{\infty} \frac{(D-P)^2}{2\lambda\sigma} \varphi\left(\frac{D-\mu}{\sigma}\right) dD \ .$$

After integrating by parts, this expression becomes

$$E(S_1) = \int_{D=P}^{\infty} \frac{(D-p)}{\lambda} \Phi\left(\frac{D-\mu}{\sigma}\right) dD$$

which can be evaluated (see page 444 in Hadley and Whitin [4]) to give

$$E(S_1) = \frac{V(D_3)}{\nu E(Y)} h(K_1) \ ,$$

where

$$h(K) = \frac{1}{2} [1 + K^2] \Phi(K) - \frac{1}{2} K \phi(K)$$

and K_1 was computed in Section G.3.

Note that some of the U_1 backorders which are present at the end of the procurement cycle may have been carried over from a previous cycle. Section G.3 defined the random variable U_2 to be the number of backorders existing at the beginning of the procurement cycle (i.e. just after a procurement shipment arrival), so that $U_1 - U_2$ is the number of backorders that were actually incurred during the cycle. If a unit was first backordered in a previous cycle, then in order to compute the time-weighted units short associated with the current procurement cycle, it is necessary to subtract from S_1 the time interval between when the unit was first backordered and the beginning of the current cycle. Using the same assumptions as before, it follows that $S_2 = \frac{1}{2}(U_2)^2/\lambda$ is the portion of S_1 that was incurred prior to the beginning of the current procurement cycle. Thus $E(S_1) - E(S_2)$ is the expected time-weighted units short associated with a single procurement cycle. Using the same approach as before, it follows that

$$E(S_2) = \frac{V(D_5)\ h(K_2)}{\nu\ E(Y)}$$

where K_2 and $V(D_5)$ were computed in Section G.3.*

Next consider the repair side, which deals with those backorders incurred during a procurement cycle due to sending carcasses late to the repair facility. In Section G.4, the random variable RU_1 was

*Refer to Appendix H for a discussion on the relationship between these formulas and those derived by Hadley and Whitin [4] for a similar inventory control model.

defined to be the number of backorders existing at the end of a repair cycle (i.e. just before a repair shipment arrival) and RU_2 to be the number of backorders existing at the beginning of a repair cycle (i.e. just following a repair shipment arrival). As before, let $RS_1 = \frac{1}{2}(RU_1)^2/\lambda$ and $RS_2 = \frac{1}{2}(RU_2)^2/\lambda$ be the time-weighted units short associated with these quantities. It follows that $E(RS_1) - E(RS_2)$ is the expected time-weighted units short incurred during a typical repair cycle. Using the same approach employed for the procurement side, it can be shown that

$$E(RS_1) = \frac{V(D_6)}{\nu E(Y)} \cdot h(K_3)$$

and

$$E(RS_2) = \frac{V(D_7)\ h(K_4)}{\nu\ E(Y)} ,$$

where K_3, K_4, $V(D_6)$, and $V(D_7)$ were computed in Section G.4.

The expected time-weighted units short occurring between the beginning of the procurement cycle and the arrival of the first repair shipment is $E(RS_1) - E(S_2)$; the expected time-weighted units short occurring during each of the next $E(M)-1$ repair cycles is $E(RS_1) - E(RS_2)$; and the expected time-weighted units short occurring between the last repair shipment and the end of the procurement cycle is $E(S_1) - E(RS_2)$. Thus the expected total time-weighted requisitions short over the procurement cycle is

$$E(TS) = \{E(S_1) - E(S_2) + E(M)[\,E(RS_1) - E(RS_2)]\} \cdot 365/E(Y) ,$$

where the factor $365/E(Y)$ is used to convert the dimensions from unit-years to requisition-days.

G.7 Average Days Delay

Our final measure of service is the average number of days that a requisition is delayed. We assume that there is no delay for a requisition that is satisfied without being backordered. Define $E(DD_1)$ to be the conditional average delay in days for a requisition, given that it was backordered and define $E(DD_2)$ to be the unconditional average delay in days for a requisition. The amount $E(DD_1)$ refers to the average delay for a requisition among only those requisitions that were in fact backordered; while $E(DD_2)$ refers to the average delay among all requisitions. It follows that

$$E(DD_1) = \frac{E(TS)\ E(Y)}{E(TU)}$$

and

$$E(DD_2) = E(DD_1)\ (1 - \rho) \quad ,$$

where $E(TU)$ and ρ were computed in G.5, and $E(TS)$ was computed in G.6.

G.8 Approximations

If it is desired to compute the measures of service for a large number of items in a minimum amount of time, the computer program gives the user the option to skip over some of the preceding formulas in order to speed up the calculations. If this option is selected, then the program will automatically set $E(F) = 0$, $V(F) = 0$, $E(U_2) = 0$, and $E(S_2) = 0$, as well as skip over the formulas which compute these quantities. Of course, this procedure will result in some loss of accuracy. Next, we will discuss the affect upon accuracy of setting each of these quantities to zero.

a. $E(F) = 0$

The random variable F refers to the unfilled portion of the requisition which hit the procurement reorder point. The mean $E(F)$ affects both the mean net lead time demand $E(D_3)$ and the mean procurement order quantity $E(O)$. If the issue size is small or if there are a large number of requisitions during the lead time, then the affect of $E(F)$ on $E(D_3)$ will be small, and if the issue size is small compared to the economic order quantity Q , the effect of $E(F)$ upon $E(O)$ will also be small. Otherwise, it probably would be desirable to include $E(F)$ in the calculations. For example, if the sensitivity of the service measures is being analyzed with respect to changes in the economic order quantity Q , then it may be necessary to deal with values of Q that are sufficiently small so that $E(F)$ will have some impact.

b. $V(F) = 0$

The variance $V(F)$ only influences the variance $V(D_3)$ of the net lead time demand. Thus if the issue size variation is small or if there are a large number of requisitions during the lead time, then the affect of $V(F)$ will be small.

c. $\underline{E(U_2)}$

The random variable U_2 refers to the number of backorders existing at the beginning of a procurement cycle (i.e. just after a procurement shipment arrival). We anticipate that the usual values for the procurement reorder point P and economic order quantity Q are such that a shipment arrival will generally eliminate all existing backorders, so that $E(U_2)$ will be approximately zero. Of course, if it is desired to do sensitivity analyses with respect to changes in P or Q , then it may be necessary to compute $E(U_2)$.

d. $\underline{E(S_2)}$

The random variable S_2 refers to the time-weighted units short that are associated with the backorders existing at the beginning of a procurement cycle. Again, for typical values of P and Q , we anticipate that $E(S_2)$ will be approximately zero. But if it is desired to do sensitivity analyses with respect to changes in either P or Q , it may be necessary to compute $E(S_2)$.

G.9 Modifications for Other Cases

The preceding sections derived the formulas in detail for a repairable item with high demand under B08 management with levels 3 or 4 repair. This section will briefly discuss how these formulas should be modified to handle other cases.

a. Consumable items

Formulas that would be appropriate for a consumable item can be obtained simply by setting

$$E(R) = V(R) = E(\beta_1) = V(\beta_1) = E(\beta_2) = V(\beta_2) = 0$$

when computing $E(U_i)$ and $E(S_i)$ for procurement reordering in Sections G.3 and G.5, and setting $E(RU_i) = E(RS_i) = 0$ for the repair side.

b. Low demand items

The formulas in the previous sections assumed that the distribution for the random variables D_3, D_5, D_6, and D_7 could be represented by the normal distribution. This assumption would be appropriate for high demand items because of the Central Limit Theorem, but it would not be appropriate for low demand items. In the case of low demand items, our assumption is that these random variables can be represented by the negative binomial distribution. The negative binomial distribution has two useful properties: 1) if the random variable representing demand is Poisson distributed and the lead time has a gamma distribution, then the lead time demand has a negative binomial distribution (see page 203 in Hadley and Whitin [4]); and 2) The negative binomial distribution has two parameters which can be specified by knowing its mean and variance. Thus, our approach for handling low demand items is merely to replace the normal distribution with the negative binomial distribution when computing $E(U_i)$, $E(RU_i)$, $E(S_i)$, and $E(RS_i)$.

c. CLAMP, FIRM, and HIGH BURNER

The only difference in our approach between CLAMP, FIRM, and HIGH BURNER is in how the reorder point P is computed, rather than in how the service measures are computed. All of these special repair management programs are characterized by: 1) a continuous repair review (since carcasses are sent immediately to the repair facility upon receipt); and 2) the lack of an induction point (i.e., upon receipt at the repair facility, the contractor can begin work without any formal authorization). Our approach for handling these items is to assume that the only backorders that are incurred are due to insufficient procurement, rather than to a delay in inducting carcasses. This means that we can set $E(RU_i) = E(RS_i) = 0$ and bypass these calculations. Note that this is different from the approach for consumable items, because we still allow nonzero values for the mean and variance of R, β_1, and β_2 when computing $E(U_i)$ and $E(S_i)$.

d. Level 2 repair under B08

The following key assumption, made in Section G.5, enabled us to compute the total number of backorders in a procurement cycle: all repairable carcasses received prior to the interval of length $R + B$ before procurement arrival are repaired and returned RFI prior to the procurement arrival. While we anticipate this to be valid for levels 3 and 4 repair scenarios, we do not expect it to be valid for level 2 repair. Our approach for level 2 repair is to compute the expected number of backorders with

$$E(TU) = E(U_1) - E(U_2) + E(\beta_1)E(\beta_2)E(O)/E(AT) \quad ,$$

which assumes that the entire demand associated with repairable carcasses is backordered, and to compute the expected total time-weighted requisitions short over the procurement cycle with

$$E(TS) = \{E(S_1) - E(S_2) + [E(R) + \tfrac{1}{2}(RW)]E(\beta_1)E(\beta_2)E(O)/E(AT)\}365/E(Y) ,$$

which assumes that each unit of demand associated with repairable carcasses remains backordered over an average length of time equal to $E(R) + \frac{1}{2}(RW)$ years.

G.10 References

1. Supply System Design Specifications (SSDS) for Uniform Inventory Control Programs (UICP): Levels Computation for Consumables and Repairables, Department of the Navy, Naval Supply Systems Command, July 1976.

2. Emanuel Parzen, Stochastic Processes, Holden-Day, 1962.

3. William Feller, An Introduction to Probability Theory, vol. II, 2nd Ed., John Wiley & Sons, 1971.

4. G. Hadley and T. M. Whitin, Analysis of Inventory Systems, Prentice-Hall, 1963.

APPENDIX H

TIME-WEIGHTED UNITS SHORT

H.1 High Demand Case

Hadley and Whitin* considered the following lot size-reorder point inventory control model: whenever the on-hand inventory falls below the reorder point r, a new procurement equal to Q is reordered. Note that this is a continuous review situation in which the notion of a review period is not present. Let the mean annual demand be λ, the mean lead time demand be μ, and the standard deviation of lead time demand be σ. For the high demand case in which the lead time demand is represented with a normal distribution, Hadley shows that the annual expected time-weighted units short is equal to

$$B(Q,r) = \frac{1}{Q}\left[\beta(r) - \beta(r + Q)\right] ,$$

where

$$\beta(\nu) = \frac{1}{2}\left[\sigma^2 + (\nu - \mu)^2\right]\Phi\left(\frac{\nu - \mu}{\sigma}\right) - \frac{\sigma}{2}(\nu - \mu)\ \phi\left(\frac{\nu - \mu}{\sigma}\right)$$

$\phi(\cdot)$ is the normal density, and $\Phi(\cdot)$ is the "tail" c.d.f. Because there are λ/Q procurement cycles in a year, the expected time-weighted

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*Hadley and Whitin, Analysis of Inventory Systems, Prentice-Hall, 1963 pp. 193-194.

units short over a single procurement cycle must be equal to

$$E(S) = (Q/\lambda)\, B(Q,r) = [\beta(r) - \beta(r + Q)]/\lambda \quad . \qquad (1)$$

It is convenient to define

$$K_1 = \frac{r - \mu}{\sigma} \ , \quad K_2 = \frac{r + Q - \mu}{\sigma}$$

and

$$h(K) = \frac{1}{2}\left[1 + K^2\right] \Phi(K) - \frac{1}{2} K \phi(K) \quad .$$

It follows that $\beta(r) = \sigma^2 h(K_1)$ and $\beta(r + Q) = \sigma^2 h(K_2)$. Thus, Hadley's formula (1) becomes

$$E(S) = \frac{\sigma^2}{\lambda}\left[h(K_1) - h(K_2)\right] \quad . \qquad (2)$$

Next, we will show the equivalence between this formula and the one used in Appendix G (see Section G.6). Define U_1 to be the number of units short at the end of a procurement cycle (i.e., just before a procurement arrives) and $S_1 = 1/2(U_1)^2/\lambda$. Because $U_1 = [D - r]^+$ where the lead time demand D is normally distributed with mean μ and standard deviation σ ,* it follows that the expectation of S_1 can be computed with

*We define $[x]^+ = x$ if $x \geq 0$, and $[x]^+ = 0$ if $x < 0$.

$$E(S_1) = \int_{D=r}^{\infty} \frac{(D - r)^2}{2 \lambda \sigma} \phi \left(\frac{D - \mu}{\sigma} \right) dD \quad .$$

After integrating by parts, this expression becomes

$$E(S_1) = \int_{D=r}^{\infty} \frac{(D - r)}{\lambda} \phi \left(\frac{D - \mu}{\sigma} \right) dD \quad ,$$

which can be evaluated to give

$$E(S_1) = \frac{\sigma^2}{\lambda} h(K_1) \quad . \qquad (3)$$

Similarly, define U_2 to be the number of units short at the beginning of a procurement cycle (i.e., just after a procurement arrives) and $S_2 = 1/2(U_2)^2/\lambda$. Because $U_2 = [D - Q - r]^+$, it also can be shown using the same method that

$$E(S_2) = \frac{\sigma^2}{\lambda} h(K_2) \quad . \qquad (4)$$

Thus, we see that formulas (2) - (4) imply that

$$E(S) = E(S_1) - E(S_2) \quad , \qquad (5)$$

which is the formula that was derived and used in Appendix G. This shows that the approach taken in Appendix G is equivalent to that taken by Hadley, even though the derivation given in Appendix G is much simpler than Hadley's.

However, the formulas for $E(S_1)$ and $E(S_2)$ given in Appendix G are slightly different from those given above, because the model treated in Appendix G is different from the model treated by Hadley in two respects: (1) a procurement is not made when the inventory first reaches the reorder point, because it is necessary to wait until the beginning of the next review period; and (2) the amount ordered exceeds Q, because it also must include the demand during the delay between when the reorder point was first reached and when the procurement can be made. Thus, Hadley's formulas should not be used directly, because they were derived for a different (but admittedly very similar) model, whereas the formulas given in Appendix G are correct for the model treated there.

H.2 Low Demand Case

Hadley did not derive a formula corresponding to (1) for the low demand case in which the lead time demand is represented with the negative binomial distribution. In this section we will derive such a formula by using the relationship (5).

We have defined $S_1 = 1/2(U_1)^2/\lambda$, where $U_1 = [D - r]^+$. We assume that the lead time demand D is distributed with the negative binomial with mass function $f_{nb}(\cdot)$, mean μ, and variance σ^2. Thus

$$E(S_1) = \frac{1}{2\lambda}\sum_{j=r}^{\infty} (j - r)^2 f_{nb}(j)$$

$$= \frac{1}{2\lambda}\sum_{j=o}^{\infty} (j - r)^2 f_{nb}(j) - \frac{1}{2\lambda}\sum_{j=o}^{r-1} (j - r)^2 f_{nb}(j) \qquad (6)$$

$$= \frac{1}{2\lambda}\left[\sigma^2 + (r - \mu)^2\right] - \frac{1}{2\lambda}\sum_{j=o}^{r-1} (j - r)^2 f_{nb}(j) \quad .$$

APPENDIX I: COMMENTS ON THE FMSO COUNTERPROPOSAL FOR COMPUTING EXPECTED UNITS SHORT

I.1 Use of the UICP procurement variance calculations

In our judgement, the UICP procurement problem variance calculations are not rigorously correct, because two correlated random variables are treated as if they were independent at a key step in the calculations, as explained next. When computing the procurement problem variance (formula 36)*, the UICP formula correctly adds the variance in demand during the PTAT (procurement problem turnaround time) to the variance in attrition during the PLT (procurement lead time) less PTAT. The reason for this is that any demand occurring prior to the PTAT can be repaired and be returned RFI (ready for issue) prior to the end of the lead time; thus only the net attrition need be considered for demand occurring prior to the PTAT. The problem, however, is in the method used to compute the variance in attrition during the PLT less PTAT. This is done by first computing the variance in attrition per quarter (formula 32) by adding the variance in demand to the variance in RFI regenerations; but this procedure assumes that the amount of demand is independent of the number of RFI regenerations, which is not correct. Let D_1 be the total demand occurring during the PLT prior to the PTAT. Let β_1 be the initial screening survival rate and β_2 be the repair survival rate. Thus $D_2 = \beta_1\beta_2 \; D_1$ is the amount of RFI regenerations (i.e. the portion of D_1 that can be repaired), and $D_3 = (1-\beta_1\beta_2) \; D_1$ is the net attrition (i.e. the portion of D_1 that cannot be repaired). The UICP calculations (formula 32) in effect compute the variance of D_3 by adding the variance of D_1 to the

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*Supply System Design Specification for Uniform Inventory Control Program, Department of the Navy, Naval Supply Systems Command, Washington, D.C., July 1976.

variance of D_2 , which assumes that D_1 and D_2 are independent, and this results in the answer that the variance of D_3 is $[1 + (\beta_1\beta_2)^2]$ times the variance of D_1 . However, the correct answer is that the variance of D_3 is only $(1-\beta_1\beta_2)^2$ times the variance of D_1 . Thus, whenever the RFI regenerations are substantial (i.e. β_1 and β_2 are significantly larger than zero), the UICP formulas will substantially overestimate the true variance.

It is the purpose of the additional data elements used by the Appendex G formulas to enable the procurement problem variance to be computed in a rigorously correct way, so that it is not necessary to rely on the "less than correct" UICP result. Because the FMSO counterproposal* uses the UICP procurement problem variance, their method can be viewed as an approximation because it speeds up the running time while reducing accuracy.

I.2 Use of the same distribution for both summations when computing units short due to insufficient procurement

The approach in Appendix G is to compute the expected units short due to insufficient procurement with the expression

$$E(U) = \sum_{D_3=RP}^{\infty} (D_3-RP) \; Pr(D_3) - \sum_{D_5=RP+Q}^{\infty} (D_5-RP-Q) \; PR(D_5) \; ,$$

while the FMSO counterproposal is to use

$$E(U) = \sum_{X=RP-E(Def)}^{\infty} \{X - [RP-E(Def)]\} \; Pr(X) - \sum_{X=RP+Q}^{\infty} (X-RP-Q) \; PR(X) \; ,$$

where $E(Def)$ refers to the expected deficiency between the reorder point RP and the inventory position when a procurement is made. Because $D_5 = X$ and $D_3 = D_5 + Def$ where Def is the random deficiency, the FMSO

*Briefing on CARES given by FMSO in Washington, D.C. in May 1978.

counterproposal would be rigorously correct if the variance of Def were zero. But the variance of Def is not zero, and it consists of two components: one due to the possibility of having a multiple random issue size; and the other due to the random demand that occurs over the procurement review period. Nevertheless, whenever the variance of Def is small as compared with the variance of D_5 or X, then the FMSO counterproposal will be a valid approximation.

I.3 <u>Use of the same distribution for both summations when computing units short due to insufficient repair</u>

The approach in Appendix G is to compute the expected units short due to insufficient repair with the expression

$$E(RU) = \sum_{D_6=RL}^{\infty} (D_6-RL)\, Pr(D_6) - \sum_{D_7=RL}^{\infty} (D_7-RL)\, Pr(D_7)\ ,$$

while the FMSO counterproposal is to use

$$E(RU) = \sum_{Y=RL-E(RO)}^{\infty} \{Y-[RL-E(RO)]\}\, Pr(Y) - \sum_{Y=RL}^{\infty} (Y-RL)\, Pr(Y)\ ,$$

where $E(RO)$ is the mean repair induction quantity. Because $D_7 = Y$ and $D_6 = D_7 + RO$ where RO is the random repair induction quantity, the FMSO counterproposal would be rigorously correct if the variance of the repair induction quantity were equal to zero. However, this variance is not equal to zero, as it is equal to the variance of demand over the repair review period. Nevertheless, whenever this variance is small as compared with the variance of D_7 or Y, then the FMSO counterproposal will be a valid approximation, and this will occur whenever the repair review period is small compared with the repair turnaround time.