RADC-TR-78-69
Final Technical Report
March 1978

DETERMINATION OF RADAR PERFORMANCE DEGRADATION DUE TO TROPOSPHERIC DUCTS

Raytheon Company

LEVEL II

This research was supported by the Air Force In-House Laboratory Independent Research Fund

ROME AIR DEVELOPMENT CENTER
AIR FORCE SYSTEMS COMMAND
GRIFFISS AIR FORCE BASE, NEW YORK 13441

This report has been reviewed by the RADC Information Office (OI) and is releasable to the National Technical Information Service (NTIS). At NTIS it will be releasable to the general public, including foreign nations.

RADC-TR-78-69 has been reviewed and is approved for publication.

APPROVED: Koichi Mano
KOICHI MANO
Contract Monitor

APPROVED: Terence J. Elkins
TERENCE J. ELKINS, Acting Chief
Propagation Branch
Electromagnetic Sciences Division

APPROVED: Allan C. Schell
ALLAN C. SCHELL, Acting Chief
Electromagnetic Sciences Division

FOR THE COMMANDER: John P. Huss
JOHN P. HUSS
Acting Chief

This effort was funded totally by the Laboratory Directors' Fund.

If your address has changed or if you wish to be removed from the RADC mailing list, or if the addressee is no longer employed by your organization, please notify RADC (EEP), Hanscom AFB MA 01730.

Do not return this copy. Retain or destroy.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE	READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER: RADC-TR-78-69	2. GOVT ACCESSION NO. / 3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle): DETERMINATION OF RADAR PERFORMANCE DEGRADATION DUE TO TROPOSPHERIC DUCTS	5. TYPE OF REPORT & PERIOD COVERED: Final Technical Report 20 Apr - 31 Oct 77
	6. PERFORMING ORG. REPORT NUMBER: SR-10101
7. AUTHOR(s): G. H. Joshi	8. CONTRACT OR GRANT NUMBER(s): F19628-77-C-0145
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Raytheon Company, Missile Systems Division, Hartwell Road, Bedford MA 01730	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 61101F 23767702 17 77
11. CONTROLLING OFFICE NAME AND ADDRESS: Deputy for Electronic Technology (RADC), Hanscom AFB MA 01730, Monitor/Koichi Mano/EEP	12. REPORT DATE: March 1978
	13. NUMBER OF PAGES: 153
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office): 155 p.	15. SECURITY CLASS. (of this report): UNCLASSIFIED
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE: N/A

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

This research was supported by the Air Force In-House Laboratory Independent Research Fund

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Meteorological Ducts
Radar Performance Degradation
Wave Propagation
Geometrical Optics
Physical Optics
Diffraction Zone

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Wave propagation for anomalous meteorological conditions around Canton Island is studied by pursuing the geometrical optics and the physical optics approaches. The relative merits along with the limitations of these two approaches have been established. For ranges beyond line of sight the physical optics approach is applied to the bilinear and the tri-linear refractive index profiles. The numerical analysis to determine the real eigenvalues and the height gain functions has been carried out for the bilinear refractive index profile. This numerical analysis has been carried out for four bilinear

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

297620

refractive index profiles and three wavelengths.

EVALUATION

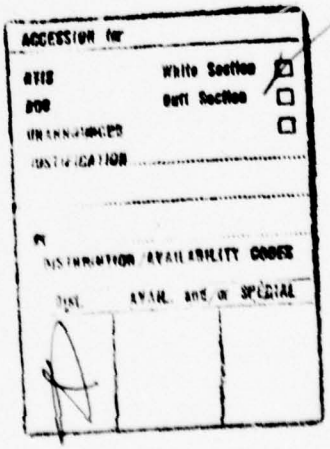
F19628-77-C-0145

1. This is the Final Report on the contract which over the period from 20 April to 31 October 1977 investigated analytically the radar performance degradation due to tropospheric ducts. With the case of low look-angle specially in mind, the physical-optics approach has been employed to study the wave propagation for the cases of bilinear and trilinear profiles of the atmospheric index of refraction.

2. The above work is of value to the Air Force since it will provide basic information which is needed for a proper evaluation of the radar performance related to the detection of low-altitude objects in the presence of the tropospheric ducts.

Koichi Mano

KOICHI MANO
Contract Monitor
Propagation Branch
Electromagnetic Sciences Division



PREFACE

The author would like to thank the project officers Dr. E. Altshuler, Dr. K. Mano, and Mr. M. Wong of Rome Air Development Center, for their continued interest and support throughout the project. He would also like to thank Mr. R. Roy of Raytheon for assistance in computing the solutions of the transcendental equation.

TABLE OF CONTENTS

		Page
1.	INTRODUCTION	7
	1.1 Statement of Problem	7
	1.2 Report Outline	11
2.	METEOROLOGICAL DATA FROM CANTON ISLAND	13
	2.1 Data Format	13
	2.2 Data Evaluation & Classification	13
3.	GEOMETRICAL VERSUS PHYSICAL OPTICS APPROACH	28
	3.1 Geometrical Optics Or Ray Trajectory Approach	28
	3.2 Physical Optics Or Guided Mode Approach	34
	3.3 Ray Trajectories For Four Refractive Index Profiles	36
4.	GUIDED MODE APPROACH - PHYSICAL OPTICS	42
	4.1 Propagation Through Stratified Media	42
	4.1.1 Propagation Through the Meteorological Duct With Bilinear M-Profile	44
	4.1.2 Trilinear Refractive Index Profile	51
	4.1.3 Propagation Through Ducts with Trilinear Profiles	54
	4.2 Solutions of the Transcendental Equation - The Bilinear Refractive Index Profile	57
	4.2.1 Newton-Raphson Technique	58
	4.2.2 Secant Method	59
	4.2.3 Zero Crossing Technique	59
	4.3 Numerical Analysis for Bilinear M-Profile	60
	4.3.1 The Zero Crossing Technique For Trapped Modes	61
	4.3.2 The Leaky Modes	75
	4.4 Height Gain Functions For The Trapped Modes	76

TABLE OF CONTENTS (CONT.)

		Page
5.	CONCLUSION/RECOMMENDATIONS	115
	5.1 Summary	115
	5.2 Recommendations	116
6.	REFERENCES	117 - 118
7.	APPENDIXES I and II - COMPUTER PROGRAMS	119 - 151
	I-1 thru I-17	119 - 136
	II-1 thru II-14	137 - 151

LIST OF ILLUSTRATIONS

		Page
Fig. (1-1)	M and N Profiles For Normal Atmosphere	9
Fig. (1-2)	M Profiles For (1) Normal, (2) Subrefractive and (3) Superrefractive Conditions	10
Figure (3-1) thru (3-4)	Typical Ray Trajectories	30 - 33
Fig. (3-5)	Tropospheric Duct Propagation	35
Fig. (3-6) thru (3-9)	Ray Trajectories For Canton Island - 1973	37 - 40
Fig. (4-1)	Canton Island Refractive Index Profiles For Four Days	43
Fig. (4-2)	Surface Duct - Bilinear Profile	52
Fig. (4-3)	Elevated Surface Duct - Trilinear Profile	52
Figure (4-4)	Elevated Duct - Trilinear Profile	52
Fig. (4-5) thru (4-16)	Plots of ϕ (z) as a Function of z	62 - 73
Fig. (4-17) thru (4-53)	Height Gain Functions For January, April, September and December	77 - 113

LIST OF TABLES

Table (2-1) thru (2-12)	Refractive Index Profiles For Canton Island For 1973	14 - 25
Table (4-1)	Trapped Wavelengths	60
Table (4-2)	Eigenvalues For Leaky Modes	76

1. INTRODUCTION

1.1 Statement of the Problem

The frequently occurring meteorological anomalies in certain geographical areas of the world can modify significantly the electromagnetic propagation. The modification in propagation affects the performance of the communication systems in such a fashion as to be able to communicate for distances significantly larger than what is expected during normal meteorological conditions. For ground based tactical missile systems and the long range surveillance radars, the impact is more significant. The radar systems experience range and angular errors for look angles which are greater than a couple of degrees. For elevation angles less than a couple of degrees, the systems can detect targets at significantly large distances. However the signals corresponding to these long ranges of detection are contaminated with the clutter, the primary and multiple time around. Further along with the extended detection ranges, for slightly higher elevation angles the target detection is impaired due to the existance of holes or the shadow zones.

The existence of regions of extended detection ranges, and of regions of shadow zones, are dependent on the meteorological conditions which tend to modify the refractive index profile and hence the propagation behavior. The refractive index η is related to the atmospheric pressure P, absolute temperature T, and the partial water vapor pressure e. The refractive index η is given in terms of different parameters such as N, M, and B units, etc. In this report we use only the N and M units to describe the refractive index and they are defined as follows:

$$N = (\eta - 1) \times 10^6 = \frac{k_1}{T} P + \frac{k_2 e}{T^2}$$

Here the constants k_1 and k_2 have the following empirical values.

$$k_1 = 77.6, \quad k_2 = 3.73 \times 10^5$$

For normal atmosphere N decreases uniformly with height imparting a downward bending for a ray path. On the other hand, the earth's curvature gives the appearance that the rays tilt upwards. This apparent upward tilt can be nullified by incorporating the curvature of the earth in the refractive index profile which leads to a straight trajectory for a flat earth. The media dependent bending of the ray then is described in terms of modified refractive index profile $\hat{\eta}$ or in terms of M - units, and they are defined as follows:

$$\hat{\eta} = \eta + \frac{z}{a}$$

and

$$M = (\hat{\eta} - 1) \times 10^6 = (\eta + \frac{z}{a} - 1) \times 10^6$$

Here z is the height and a is the radius of curvature of earth, ($a = 6.4 \times 10^6$ m).

Thus M and N units are related to each other in the following fashion:

$$M = \left(N + \left(\frac{z}{a} \times 10^6\right)\right)$$

For normal meteorological conditions, the plots of M and N units as a function of height reveal that there is a steady increase of M units with height as contrast to steady decrease of N units with height and this is displayed in Fig. (1-1).

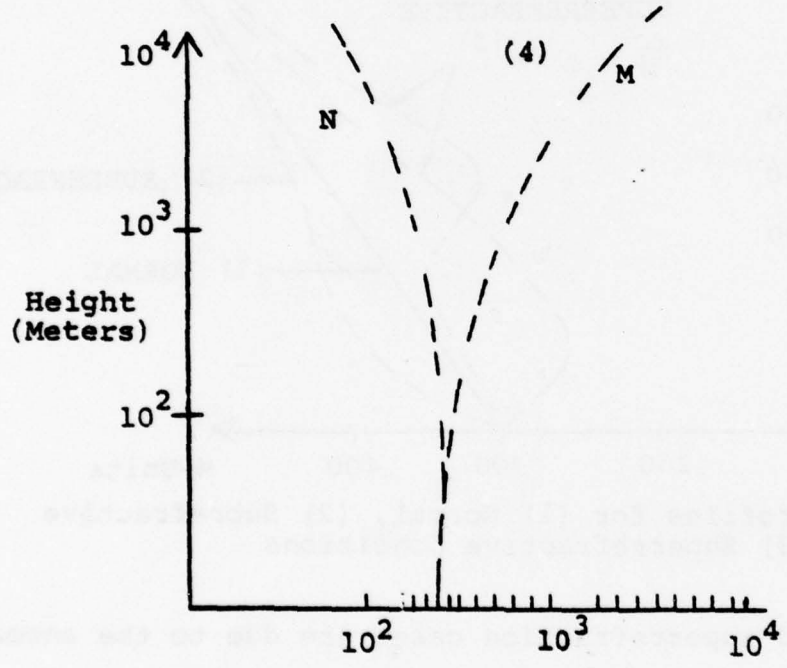


Figure (1-1) - M and N Profiles For Normal Atmosphere

During anomalous meteorological conditions the temperature and the water vapor content change in such a fashion as to cause significant deviations in the refractive index profiles and thus in the profiles of M and N as a function of height.

Some representative plots of M profiles have been reproduced in Fig. (1-2) and they, in general, fall in three major categories:

1. Normal refraction
2. Subrefraction
3. Superrefraction

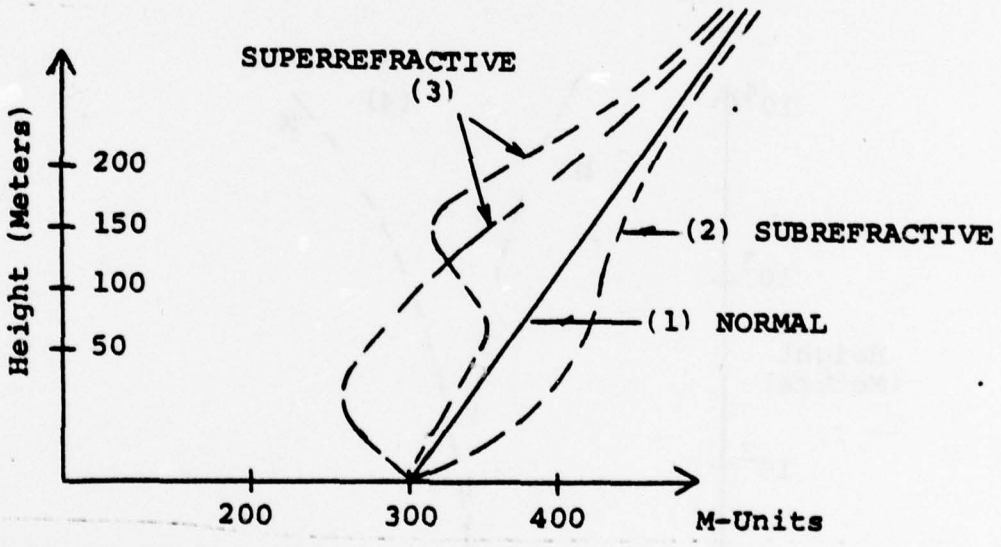


Figure (1-2) - M - Profiles for (1) Normal, (2) Subrefractive and (3) Superrefractive Conditions

The subrefraction and superrefraction cases are due to the anomalous meteorological conditions and this report deals with the electromagnetic propagation through such an anomalous medium.

1.2 The Outline of the Report

This section, Section 1, has identified the main topic of this report, namely, the determination of propagation through meteorological ducts.

For the propagation studies, the meteorological data for Canton Island was provided by ESD and Section 2 evaluates the data and classifies it in an appropriate fashion.

The electromagnetic propagation through the medium around Canton Island can be studied by geometrical optics which provides a reasonable characterization of the line of sight propagation. Section 3 discusses this approach and its limitations. The alternate approach, namely the physical optics, is also discussed in this section in order to highlight its application to the difraction region. Section 3 further provides the ray trajectories for four days of meteorological conditions so chosen as to cover the broad spectrum of variations in the characteristic parameters of the duct such as the duct width and the lapse rate of the M units.

The primary purpose of this investigation is to develop the physical optics approach for propagation through meteorological ducts and hence details of the analytical formulation of the guided mode approach is provided in Section 4 for two refractive index profiles, bilinear and trilinear. The numerical analysis based on several techniques such as Newton-Raphson to determine the characteristic wave numbers has been carried out for the bilinear case and is

discussed in Section 4. Also, the height-gain functions which describe the dependence of electromagnetic field with height for different modes have been included in Section 4 for the bilinear refractive index profile.

Finally, the conclusions and recommendations, the references, and the different appendixes are provided in Sections 5, 6 and 7, respectively.

2. METEOROLOGICAL DATA FROM CANTON ISLAND

For the purpose of anaysis, it was decided during the early stages of the contract that Raytheon will be provided with sample meteorological data from Canton Island (28°S, 171.6°W).

2.1 Data Format

The data was provided by U.S. Air Forces Environmental Technical Applications Centre at Scott Aire Force Base, Illinois and was in terms of the meteorological parameters (T, P, e). Here T, P, e, respectively, stand for the temperature, the atmospheric pressure and the partial water vapor pressure. The data provides for the typical refractive index profiles for the year 1973. This data is translated into N units, M units and the rate of change of N with height in tables (2-1) through (2-12). These tables are for twelve months, with two profiles per month. The criterion that is used to determine the presence or absence of a duct is based on the condition that the rate of change of N units with height is less than $10^6/r$ where r is the radius of the earth ($r = 6.4 \times 10^6$m).

2.2 Data Evaluation and Classification

Based on this criterion, the following observations are made regarding the nature of the refractive index profile:

1. The meteorological duct was present for all the days for which the data has been provided.

2JAN 12 17

H	N	M	DN/DH
.003	387.400	387.871	
.040	378.900	385.176	-229.730
1.137	293.800	472.209	-77.5752
1.473	278.400	509.531	-45.8333
3.121	228.700	718.422	-30.1578
4.386	189.000	877.216	-31.3834
4.972	174.300	954.466	-25.0853
5.250	161.900	985.688	-44.6043
5.850	155.500	1073.44	-10.6667
6.587	141.700	1175.28	-18.7246
6.742	138.500	1196.40	-20.6452
7.590	123.700	1314.66	-17.4528
9.720	95.500	1620.68	-13.2394
11.010	83.000	1810.60	-9.68992
12.500	70.100	2031.50	-8.65772
14.300	56.500	2300.34	-7.55556
16.610	42.300	2648.61	-6.14719

3JAN 0 22

H	N	M	DN/DH
.003	396.100	396.571	
.042	365.400	371.990	-787.179
.784	301.600	424.619	-85.9838
1.470	256.200	486.861	-66.1808
2.550	211.900	612.026	-41.0185
3.125	207.100	697.450	-8.34783
4.424	175.300	869.479	-24.4808
5.455	155.500	1011.45	-19.2047
5.860	147.900	1067.40	-18.7658
7.590	120.600	1311.56	-15.7803
9.730	94.800	1621.55	-12.0561
11.020	82.300	1811.47	-9.68992
11.358	79.400	1861.61	-8.57988
12.520	69.600	2034.14	-8.43373
14.330	56.500	2305.05	-7.23757
16.161	43.300	2579.15	-7.20918
16.703	40.600	2661.50	-4.98155
17.495	34.200	2779.37	-8.08081
18.774	26.600	2972.47	-5.94214
19.986	21.600	3157.64	-4.12541
20.660	18.800	3260.60	-4.15430
22.026	14.700	3470.84	-3.00146

TABLE (2-1)-Jan. 1973

TABLES (2-1) thru (2-12) - Refractive Index Profiles for Canton Island For 1973

3FEB 0 24

H	N	M	DN/DH
.003	394.900	395.371	
.076	351.600	363.525	-593.151
1.263	277.400	475.580	-62.5105
1.498	264.900	499.954	-53.1915
3.140	210.600	703.304	-33.0694
4.715	179.400	919.240	-19.8095
5.890	151.800	1076.01	-23.4894
7.620	121.700	1317.37	-17.3988
9.750	95.500	1625.39	-12.3005
11.030	82.800	1813.54	-9.92188
12.530	69.700	2035.81	-8.73333
14.340	56.100	2306.22	-7.51381
16.731	39.800	2665.09	-6.81723
18.240	30.000	2892.07	-6.49437
18.802	26.700	2976.96	-5.87189
20.660	19.100	3260.90	-4.09042
22.825	12.900	3594.42	-2.86374
23.192	11.900	3651.00	-2.72480
23.790	10.800	3743.74	-1.83946

5FEB 0 29

H	N	M	DN/DH
.003	397.800	398.271	
.081	353.300	366.010	-570.513
1.363	257.300	471.171	-74.8830
1.403	257.100	477.247	-5.00000
1.500	250.600	485.968	-67.0103
1.872	236.900	530.639	-36.8280
2.681	209.400	630.081	-33.9926
3.146	202.300	695.945	-15.2688
5.870	148.200	1069.27	-19.8605
6.662	134.900	1180.25	-16.7929
7.610	121.200	1315.30	-14.4515
9.750	95.600	1625.49	-11.9626
11.030	82.900	1813.64	-9.92188
12.520	69.700	2034.24	-8.85906
14.340	56.200	2306.32	-7.41758
16.710	40.400	2662.40	-6.66667
17.892	32.900	2840.37	-6.34518
18.032	32.200	2861.64	-5.00000
18.325	29.900	2905.31	-7.84983
18.723	27.200	2965.06	-6.78392
20.730	19.000	3271.79	-4.08570
21.775	15.900	3432.66	-2.96651
23.860	10.700	3754.62	-2.49400
24.530	9.700	3858.75	-1.49254
25.023	8.800	3935.21	-1.82556
26.450	7.100	4157.42	-1.19131
26.778	6.700	4208.49	-1.21951
27.311	6.100	4291.52	-1.12570
30.379	3.800	4770.63	-.749674

TABLE (2-2) - Feb. 1973

2MAR 0 29

H	N	M	DN/DH
.003	375.100	375.571	
.062	328.500	338.229	-789.831
1.479	258.100	490.173	-49.6824
2.179	229.600	571.511	-40.7143
3.124	201.600	691.793	-29.6296
5.860	147.800	1067.30	-19.6637
6.601	135.200	1170.98	-17.0040
7.600	120.900	1313.43	-14.3143
8.357	110.700	1422.01	-13.4742
9.740	95.300	1623.62	-11.1352
11.020	82.800	1811.97	-9.76563
12.520	69.600	2034.14	-8.80000
14.330	56.300	2304.85	-7.34807
16.249	43.600	2593.26	-6.61803
16.680	40.600	2657.89	-6.96056
17.590	34.500	2794.58	-6.70330
18.679	28.400	2959.36	-5.60147
19.822	22.300	3132.61	-5.33683
20.600	18.800	3251.19	-4.49871
22.130	14.100	3486.56	-3.07190
23.175	12.000	3648.43	-2.00957
23.770	10.600	3740.40	-2.35294

3MAR 0 15

H	N	M	DN/DH
.003	391.700	392.171	
.075	373.200	384.968	-256.944
1.494	286.000	520.426	-61.4517
3.142	220.400	713.417	-39.8058
4.613	185.400	909.235	-23.7933
5.870	155.800	1076.87	-23.5481
7.610	122.900	1317.00	-18.9080
9.750	94.900	1624.79	-13.0841
11.050	81.800	1815.68	-10.0769
11.625	76.800	1900.90	-8.69565
12.560	69.000	2039.81	-8.34225
14.390	55.800	2313.76	-7.21311
15.912	46.100	2542.88	-6.37319
16.757	40.100	2669.47	-7.10059
16.929	39.300	2695.66	-4.65116

TABLE (2-3) - March 1973

3APR 0 27

H	N	M	DN/DH
.003	394.900	395.371	
.075	349.000	360.768	-637.500
1.495	267.000	501.583	-57.7465
3.145	206.100	699.588	-36.9091
4.307	179.500	855.320	-22.8916
4.788	167.100	918.395	-25.7796
5.880	148.500	1071.14	-17.0330
7.610	121.400	1315.50	-15.6647
9.154	102.700	1539.07	-12.1114
9.740	95.500	1623.82	-12.2867
11.010	83.400	1811.00	-9.52756
12.500	69.900	2031.30	-9.06040
14.310	56.100	2301.51	-7.62431
16.680	40.400	2657.69	-6.62447
17.663	33.800	2805.34	-6.71414
17.871	31.700	2835.87	-10.0962
18.469	29.100	2927.11	-4.34783
18.700	27.500	2961.75	-6.92641
19.810	21.900	3130.33	-5.04505
20.700	18.800	3266.88	-3.48315
21.081	17.100	3324.96	-4.46194
23.950	10.400	3768.44	-2.33531

5APR 0 20

H	N	M	DN/DH
.003	383.600	384.071	
.085	331.300	344.638	-637.805
1.505	251.200	487.353	-56.4085
3.153	201.900	696.643	-29.9150
5.262	159.000	984.671	-20.3414
5.890	147.600	1071.81	-18.1529
7.630	121.300	1318.54	-15.1149
9.760	95.700	1627.16	-12.0188
11.030	83.200	1813.94	-9.84252
12.530	69.600	2035.71	-9.06667
14.340	56.100	2306.22	-7.45856
16.730	39.800	2664.94	-6.82008
18.760	28.000	2971.67	-5.81281
18.925	27.200	2996.76	-4.84848
20.380	20.100	3217.97	-4.87973
20.730	18.400	3271.19	-4.85714
23.960	10.500	3770.11	-2.44582
26.610	6.900	4182.33	-1.35849
27.898	5.500	4383.03	-1.08696
29.515	4.400	4635.66	-.680272

TABLE (2-4) - April, 1973

9MAY 0 32

M	N	M	DN/DM
.003	374.000	374.471	
.084	343.400	356.581	-377.778
.847	312.200	451.381	-38.8543
1.126	274.700	451.383	-156.904
1.504	258.200	494.196	-43.6508
2.186	220.000	563.010	-56.0117
3.158	197.200	692.728	-23.4568
4.348	187.800	870.053	-7.89916
4.789	170.700	922.151	-38.7755
5.890	152.500	1076.71	-16.5304
7.620	120.400	1316.07	-18.5549
9.740	95.300	1623.62	-11.8396
11.020	83.300	1812.47	-9.37500
12.500	70.300	2031.70	-8.78378
14.300	56.600	2300.44	-7.61111
15.563	48.000	2490.02	-6.80918
16.660	39.900	2654.05	-7.38377
17.786	33.000	2823.84	-6.12749
18.700	27.000	2961.25	-6.56455
19.515	23.300	3085.44	-4.53988
19.819	21.600	3131.44	-5.59211
20.730	18.400	3271.19	-3.51262

10MAY 0 23

M	N	M	DN/DM
.003	372.400	372.871	
.085	348.100	361.438	-296.341
.747	333.300	450.513	-22.3565
.972	300.600	453.118	-145.333
1.500	279.500	514.868	-39.9621
1.839	252.900	541.061	-78.4661
3.150	204.400	698.673	-36.9947
3.527	189.100	742.529	-40.5836
4.586	168.100	887.698	-19.8300
5.900	145.800	1071.58	-16.9711
7.620	120.400	1316.07	-14.7674
9.740	95.700	1624.02	-11.6509
11.010	83.300	1810.90	-9.76378
12.490	70.400	2030.23	-8.71622
15.281	49.500	2447.27	-7.48836
16.654	40.200	2653.41	-6.77349
19.681	22.000	3110.18	-6.01255
20.800	18.700	3282.47	-2.94906
24.026	10.400	3780.37	-2.57285
26.697	6.900	4195.98	-1.31037
30.167	3.900	4737.46	-.864553
31.433	3.300	4935.51	-.473934
32.904	2.600	5165.63	-.475867

TABLE (2-5) - May, 1973

3JUN 0 15

H	N	M	DN/DH
.003	364.600	365.071	
.083	336.700	349.724	-348.750
1.114	296.100	470.900	-39.3792
1.497	248.800	483.697	-123.499
1.814	237.800	522.438	-34.7003
1.950	266.200	572.178	208.824
3.136	216.400	708.476	-41.9899
3.231	195.500	702.483	-220.000
5.559	151.300	1023.57	-18.9843
5.850	155.200	1073.14	13.4021
6.282	147.600	1133.32	-17.5926
6.600	140.000	1175.62	-23.8994
7.580	120.600	1309.99	-19.7959
9.680	96.000	1614.91	-11.7143
9.799	94.400	1631.98	-13.4454

5JUN 0 23

H	N	M	DN/DH
.003	371.600	372.071	
.071	344.200	355.341	-402.941
.789	320.000	443.804	-33.7047
1.486	266.300	499.471	-77.0445
1.993	224.200	536.926	-83.0375
2.974	221.200	687.856	-3.05810
3.131	207.700	698.991	-85.9873
3.794	183.500	778.824	-36.5008
4.642	175.100	903.485	-9.90566
5.212	160.800	978.625	-25.0877
5.496	164.700	1027.09	13.7324
5.850	145.900	1063.84	-53.1073
6.105	149.700	1107.65	14.9020
6.368	143.000	1142.22	-25.4753
7.580	122.400	1311.79	-16.9967
8.247	115.400	1409.45	-10.4988
9.690	96.900	1617.38	-12.8205
10.960	83.500	1803.26	-10.5512
12.440	70.300	2022.28	-8.91892
14.230	57.000	2289.86	-7.43017
14.767	53.000	2370.12	-7.44879
15.446	47.600	2471.26	-7.95287
16.590	40.400	2643.57	-6.29371

TABLE (2-6) - June, 1973

3JUL 0 34

M	N	M	DN/DH
.003	353.300	353.771	
.075	328.600	340.368	-343.056
1.481	254.400	486.787	-52.7738
1.682	239.600	503.526	-73.6318
1.941	225.300	529.866	-55.2124
3.124	197.300	687.493	-23.6686
3.954	185.100	805.530	-14.6988
4.637	166.600	894.201	-27.0864
5.860	145.900	1065.40	-16.9256
6.905	131.000	1214.48	-14.2584
7.570	121.500	1309.32	-14.2857
8.033	115.100	1375.57	-13.8229
8.605	108.400	1458.63	-11.7133
8.883	104.700	1498.55	-13.3094
9.670	96.000	1613.34	-11.0546
10.802	84.900	1779.86	-9.80565
10.940	83.500	1800.12	-10.1449
12.420	70.700	2019.55	-8.64865

4JUL 0 31

M	N	M	DN/DH
.003	370.300	370.771	
.067	345.900	356.413	-381.250
.888	307.800	447.138	-46.4068
1.479	237.300	469.373	-119.289
2.070	222.100	546.908	-25.7191
2.563	210.200	612.365	-24.1379
3.122	197.400	687.279	-22.8980
5.140	157.400	963.928	-19.8216
5.860	146.300	1065.80	-15.4167
7.570	121.200	1309.02	-14.6784
8.799	105.600	1486.27	-12.6932
9.680	96.400	1615.31	-10.4427
10.940	84.100	1800.72	-9.76190
12.410	71.000	2018.28	-8.91156
14.200	56.900	2285.05	-7.87709
16.006	44.300	2555.83	-6.97674
16.540	39.900	2635.22	-8.23970
17.077	36.400	2715.99	-6.51769
17.338	33.300	2753.84	-11.8774
18.640	26.200	2951.04	-5.45315
20.001	20.800	3159.20	-3.96767
20.710	18.000	3267.65	-3.94922
23.147	12.300	3644.34	-2.33898
23.950	10.500	3768.54	-2.24159

TABLE (2-7) - July, 1973

2AUG 0 32

M	N	M	DN/DH
.003	371.600	372.071	
.093	347.100	361.693	-272.222
.910	303.100	445.890	-53.8556
1.500	267.200	502.568	-60.8475
1.570	235.400	481.752	-454.286
1.826	228.700	515.221	-26.1719
2.635	209.100	622.563	-24.2274
3.133	197.500	689.105	-23.2932
3.337	192.900	716.515	-22.5490
5.538	152.500	1021.46	-18.3553
5.850	147.000	1064.94	-17.6282
7.560	121.200	1307.45	-15.0877
9.660	96.900	1612.67	-11.5714
10.521	88.200	1739.07	-10.1045
10.920	83.700	1797.18	-11.2782
12.390	70.900	2015.04	-8.70748
13.595	61.400	2194.62	-7.88382
14.170	56.800	2280.24	-8.00000
15.531	47.100	2484.10	-7.12711

5AUG 0 32

M.	N	M	DN/DH
.003	360.700	361.171	
.044	335.300	348.481	-313.580
.549	321.000	407.145	-30.7527
1.223	278.700	470.603	-62.7596
1.483	256.200	488.900	-86.5385
3.111	196.100	686.253	-35.6880
3.620	180.800	754.821	-22.2004
4.627	167.800	893.832	-18.8679
5.820	146.800	1060.03	-17.6027
7.530	121.400	1302.95	-14.8538
8.974	103.300	1511.43	-12.5346
9.630	96.400	1607.46	-10.5183
10.600	86.800	1750.07	-9.89691
10.890	83.700	1792.47	-10.6897
12.360	71.000	2010.43	-8.63946
13.417	62.500	2167.79	-8.04163
14.150	56.700	2277.00	-7.91269
15.318	48.600	2452.18	-6.93493
16.235	41.000	2588.47	-8.28790
16.510	39.100	2629.72	-6.90909
16.992	36.400	2702.65	-5.60166
17.814	30.200	2825.43	-7.54258
18.630	26.200	2949.47	-4.90196

TABLE (2-8) - Aug., 1973

4SEP 0 42

H	N	M	DN/DH
.003	391.500	391.971	
.094	370.200	384.950	-234.066
.343	355.800	409.621	-57.8313
.707	321.500	432.437	-94.2308
1.501	293.400	528.925	-35.3904
1.905	262.700	561.617	-75.9901
2.515	223.700	618.334	-63.9344
2.876	210.800	662.079	-35.7341
3.125	219.800	710.150	36.1446
3.473	205.600	750.555	-40.8046
3.986	196.400	821.851	-17.9337
4.116	199.600	845.650	26.1538
4.380	176.500	863.774	-88.2576
5.380	159.600	1003.79	-16.9000
5.650	162.500	1049.05	10.7407
5.830	153.000	1067.80	-52.7778
6.457	146.400	1159.58	-10.5263
7.530	122.300	1303.85	-22.4604
8.330	111.600	1418.68	-13.3750
8.494	112.800	1405.61	7.31707
8.599	109.500	1458.79	-31.4286
8.726	109.300	1478.51	-1.57480
8.920	104.400	1504.05	-25.2577
9.620	96.600	1606.09	-11.1429
10.561	87.300	1744.45	-9.88310

5SEP 0 17

H	N	M	DN/DH
.003	386.700	389.171	
.102	353.900	369.905	-351.515
.819	301.900	430.411	-72.5244
1.151	286.700	467.306	-45.7831
1.997	246.700	560.053	-47.2813
2.232	242.700	592.928	-17.0213
2.484	223.900	613.669	-74.6032
3.139	197.600	690.147	-40.1527
4.799	164.800	917.821	-19.7590
5.849	146.900	1064.68	-17.0476
6.464	136.600	1150.88	-16.7480
7.565	121.300	1308.34	-13.8945
9.690	96.500	1616.98	-11.6706
10.607	87.200	1751.57	-10.1418
10.950	84.000	1802.19	-9.32985
12.420	70.800	2019.65	-8.97959
14.165	57.300	2279.96	-7.73639

TABLE (2-9) - Sept., 197

7OCT 0 37

M	N	M	DN/DM
.003	385.000	385.471	
.078	357.900	370.139	-361.333
.699	320.500	430.181	-60.2254
1.094	282.200	453.862	-96.9620
1.485	267.200	500.214	-38.3632
2.264	217.300	572.549	-64.0565
3.118	197.600	686.852	-23.0679
3.715	196.500	779.428	-1.84255
4.021	178.800	809.743	-57.8431
5.840	145.900	1062.27	-18.0869
6.290	139.600	1126.58	-14.0000
6.541	134.900	1161.26	-18.7251
7.560	121.500	1307.75	-13.1501
8.103	113.900	1385.36	-13.9963
9.670	96.000	1613.34	-11.4231
10.749	85.400	1772.05	-9.82391
10.940	83.700	1800.32	-8.90052
12.410	70.800	2018.08	-8.77551
14.200	57.000	2285.15	-7.70950
15.132	50.400	2424.79	-7.08155
15.458	47.400	2472.95	-9.20245
16.004	44.200	2555.42	-5.86061

TABLE (2-10) Oct., 1973

8OCT 0 19

M	N	M	DN/DM
.003	375.100	375.571	
.084	353.700	366.881	-264.198
.649	317.500	419.336	-64.0708
1.488	283.100	516.585	-41.0012
2.134	259.800	594.650	-36.0681
2.262	238.800	593.735	-164.062
2.480	212.300	601.442	-121.560
3.108	210.500	698.182	-2.86624
5.830	146.300	1061.10	-23.5856
7.550	121.000	1305.69	-14.7093
9.660	96.000	1611.77	-11.8483
10.686	85.900	1762.66	-9.84405
10.920	83.600	1797.08	-9.82906
12.400	70.500	2016.21	-8.85135
14.184	57.100	2282.74	-7.51121
14.806	52.700	2375.94	-7.07395
16.530	40.500	2634.25	-7.07657
16.762	38.000	2668.16	-10.7759
18.274	28.600	2896.01	-6.21693

5NOV 0 20

H	N	M	DN/DH
.003	371.900	372.371	
.067	347.100	357.613	-347.500
.696	317.000	426.211	-47.8537
1.468	274.700	505.047	-54.7927
1.938	239.700	543.795	-74.4681
3.107	197.400	684.925	-36.1848
5.830	106.300	1061.10	-18.7661
7.550	121.900	1306.59	-14.1860
8.883	104.400	1498.25	-13.1283
9.650	96.400	1610.60	-10.4302
10.727	85.500	1768.69	-10.1207
10.910	83.700	1795.61	-9.83607
12.390	70.600	2014.74	-8.85135
14.180	56.900	2281.91	-7.65363
16.530	40.300	2634.05	-7.06383
17.324	34.300	2752.64	-7.55668
18.592	27.000	2944.31	-5.75710
18.902	24.800	2990.75	-7.09677
20.590	18.800	3249.62	-3.55450
21.516	15.700	3391.82	-3.34773

TABLE (2-11) - Nov., 1973

6NOV 0 17

H	N	M	DN/DH
.003	363.600	364.071	
.067	342.500	353.013	-329.688
1.468	269.600	499.947	-52.0343
1.698	246.600	513.037	-100.000
2.429	218.700	599.839	-38.1669
3.095	204.900	690.543	-20.7207
5.810	150.200	1061.86	-20.1473
6.163	143.400	1110.45	-19.2635
7.530	121.000	1302.55	-16.3862
9.640	96.600	1609.23	-11.5640
10.715	85.600	1766.91	-10.2326
10.900	84.000	1794.34	-8.64865
12.380	70.500	2013.07	-9.12162
13.298	63.400	2150.02	-7.73420
14.170	56.400	2279.84	-8.02752
14.457	54.400	2322.88	-6.96864
16.540	40.100	2635.42	-6.86510

30EC 0 27

M	N	M	DN/DM
.003	374.000	374.471	
.084	355.200	368.381	-232.099
1.489	287.500	521.142	-48.1851
2.092	244.200	572.460	-71.8076
3.114	206.800	695.424	-36.5949
3.935	191.700	809.149	-18.3922
4.418	175.700	868.937	-33.1263
5.379	154.100	994.129	-22.4766
5.810	149.900	1061.58	-9.74478
6.845	134.600	1211.80	-14.5024
7.131	126.700	1245.64	-29.6992
7.520	121.400	1301.38	-13.6247
9.610	97.400	1605.32	-11.4833
10.499	87.900	1735.32	-10.6862
10.870	84.100	1789.73	-10.2426
12.340	71.300	2007.59	-8.70748
13.283	63.800	2148.06	-7.95334
14.110	57.300	2271.33	-7.85973

TABLE (2-12) - Dec., 1973

4DEC 0 33

M	N	M	DN/DM
.003	366.400	366.871	
.075	351.900	363.668	-201.389
.956	299.200	449.204	-59.8184
1.476	272.100	503.702	-52.1154
1.872	260.800	554.539	-28.5354
2.328	231.000	596.291	-65.3509
3.101	216.800	703.384	-18.3700
4.295	182.800	856.737	-28.4757
4.456	183.000	882.200	1.24224
4.980	161.800	943.222	-40.4580
5.780	152.500	1059.45	-11.6250
6.223	145.500	1121.96	-15.8014
6.338	138.200	1132.71	-63.4783
7.490	123.700	1298.97	-12.5468
8.500	108.600	1442.35	-14.9505
9.580	96.700	1599.92	-11.0185
10.442	88.300	1726.77	-9.74478
10.840	84.600	1785.53	-9.29648
12.300	71.200	2001.22	-9.17808
12.963	66.100	2100.15	-7.69231
14.080	57.200	2266.52	-7.96777
15.819	45.200	2527.39	-6.90052
16.400	40.100	2613.46	-8.77797

2. Eighty percent of the time there existed surface ducts.
3. Twenty percent of the time the duct profiles were more complex with possible existence of elevated ducts.
4. The limited number of data points (M-units versus altitude) force to characterize the surface ducts as bilinear.
5. The duct width, under the circumstances, is inaccurately determined. For the available data, the duct width is most likely the maximum value rather than the likely value.
6. The duct intensity, as determined by the rate of change of N with height, varies from -201/km to -789/km. The minimum intensity occurring in December and the maximum in March. The refractive index profiles based on the radiosonde data are not corrected for different measurement errors. The errors could either be sensor-related or could be the timelag in temperature and humidity measurements. Finally, the most conspicuous error is the lack of correlation between the first radiosonde data point at some altitude which is different than mean sea level and the sea level measurement.

7. The duct widths for the surface ducts, defined as the height at which the minimum for M occurs, varies from 40 meters for January 2nd to 102 meters for September 5th.
8. For the largest duct width of 102 meters, the minimum frequency that is trapped is approximately 350 MHz, and the minimum for the smallest duct of 40 meters, it is 1.43 GHz. This is based on a crude relationship between the maximum wave length trapped and the duct width.

$$\lambda_{max} \approx 0.014 d^{3/2}$$

(λ_{max}: cm; d: feet)

3. GEOMETRICAL VERSUS PHYSICAL OPTICS APPROACH

The electromagnetic propagation through a medium is governed by the refractive index profile of the medium; and it can be characterized either by the geometrical optics that is the ray treatment, or the physical optics that is the wave treatment. The distinction in the two approaches which was highlighted for optics originally, is equally valid for electromagnetic propagation.

3.1 Geometrical Optics or Ray Trajectory Approach

The geometrical optics approach is primarily based on the Fermat's principle which states that the line integral of the refractive index between two points is stationary. This leads to the well know Snell's law which states that $n_1 r_1 \cos \theta_1 = n_2 r_2 \cos \theta_2$. Here, n_1 is the refractive index at r_1 and θ_1 is the angle of inclination of the ray with the horizontal, and n_2 is the refractive index at r_2 and θ_2 is the angle of inclination at r_2.

Based on this, a set of ray trajectories for actual refractive index profiles has been plotted to highlight the salient features of the geometrical optics approach. The ray trajectories are for a specific location for four different days in the month of August at local noon. The antenna height is 50 meters, and the elevation angle is between 0 and 1 degree. All the four ray trajectories reveal most of the features of the ground duct, namely, the trapping of the rays at angles less than 1°, the shadow regions beyond 40 km range, and the modest refraction at or above 1° elevation.

These ray trajectories reveal the cases when the geometrical optics approach is not valid. It is well established that the geometrical optics or ray treatment is valid when two conditions are satisfied and those are:

$$\frac{1}{kn}\,\frac{\Delta n}{n} << 1, \qquad \frac{1}{kn}\,\frac{|\Delta J1/2|}{J1/2} << 1$$

Here J is the Jacobian and for detail explanation see Ref. (2), (Paes 52 - 58).

The first condition states that the relative variation of the refractive index per wavelength is smaller than unity. The second condition states that the relative density of the rays in a bundle, or the relative spacing between the rays, is less than a constant. This second condition is quite often not adequately emphasized when the geometrical optics approach is pursued. This second condition is often violated when the duct is present as is evident in the ray trajectories shown in Figures (3-1) thru (3-4). Further, the ray trajectores indicate that there exists a complete blank region with no electromagnetic energy at all, which in practice is not the case. Lastly, the geometrical optics approach is independent of frequency and polarization. This aspect prevents the geometrical optics approach from treating any scattering problems and thus the clutter which is dependent on frequency, polarization and roughness of ground or sea surface. For both communication and radar systems these considerations are of great importance, and the preferred approach is that based on physical

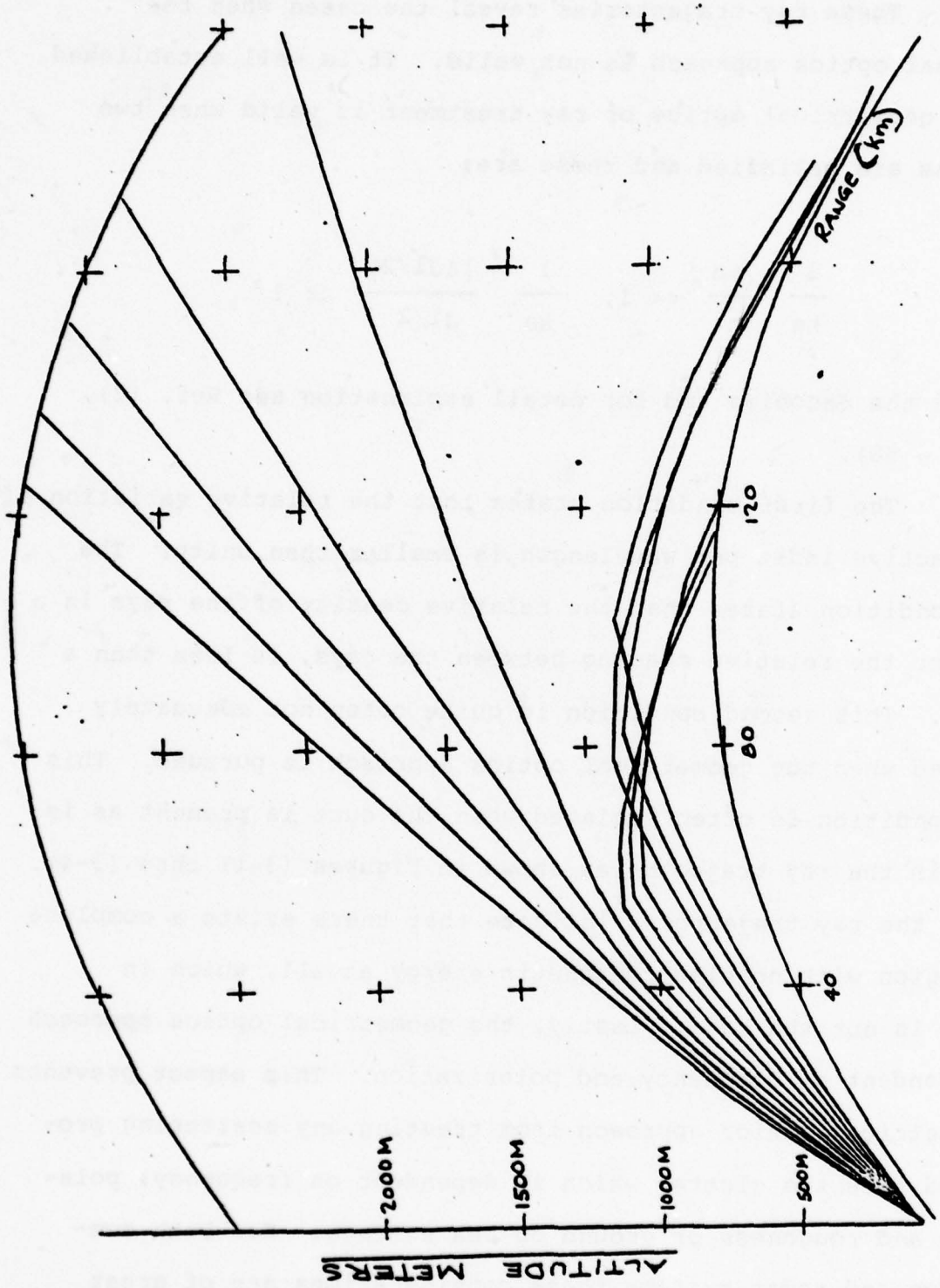


Figure (3-1) Typical Ray Trajectories for August 26

Figure (3-2) Typical Ray Trajectories for August 28

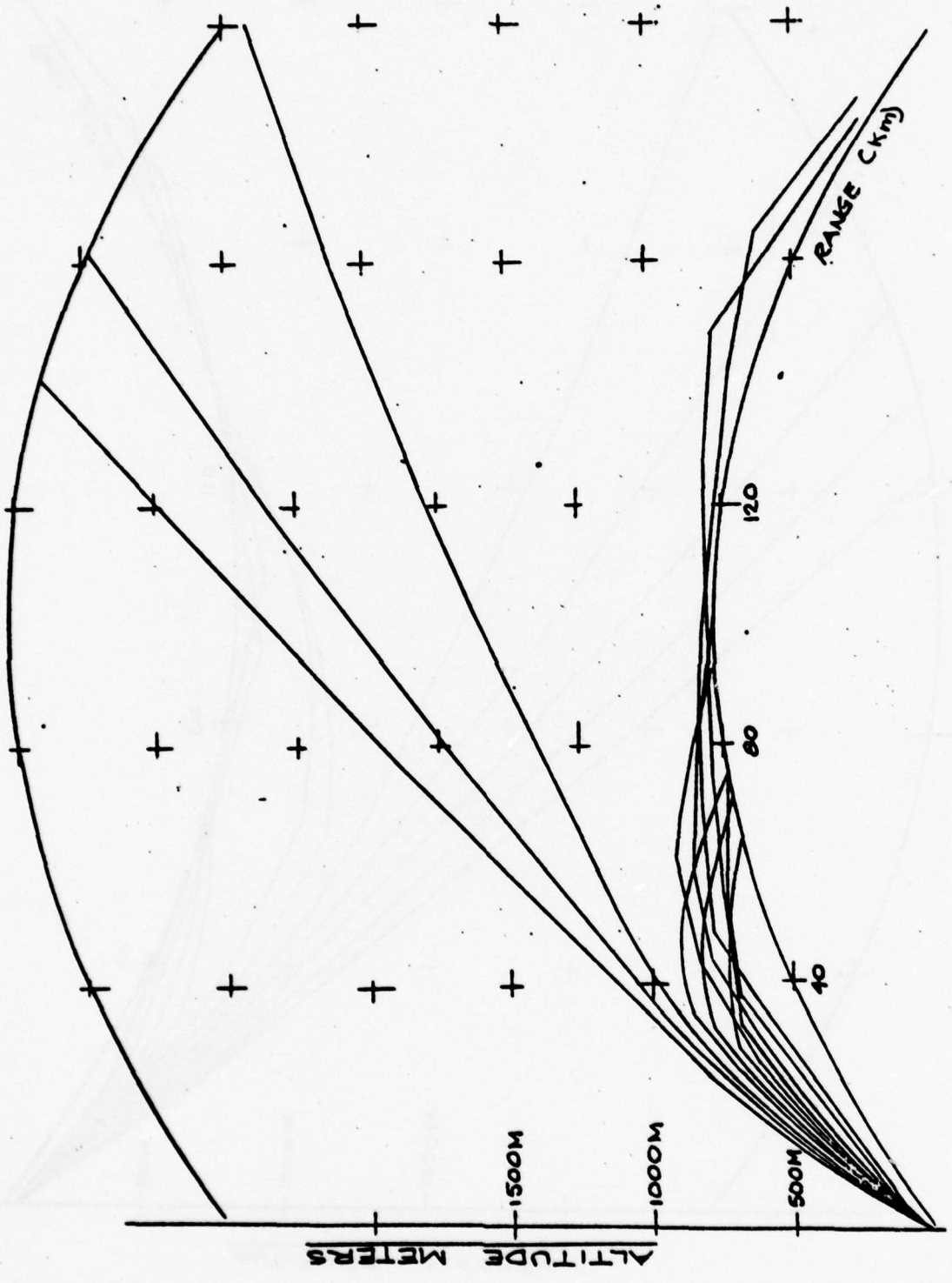


Figure (3-3) Typical Ray Trajectories for August 30

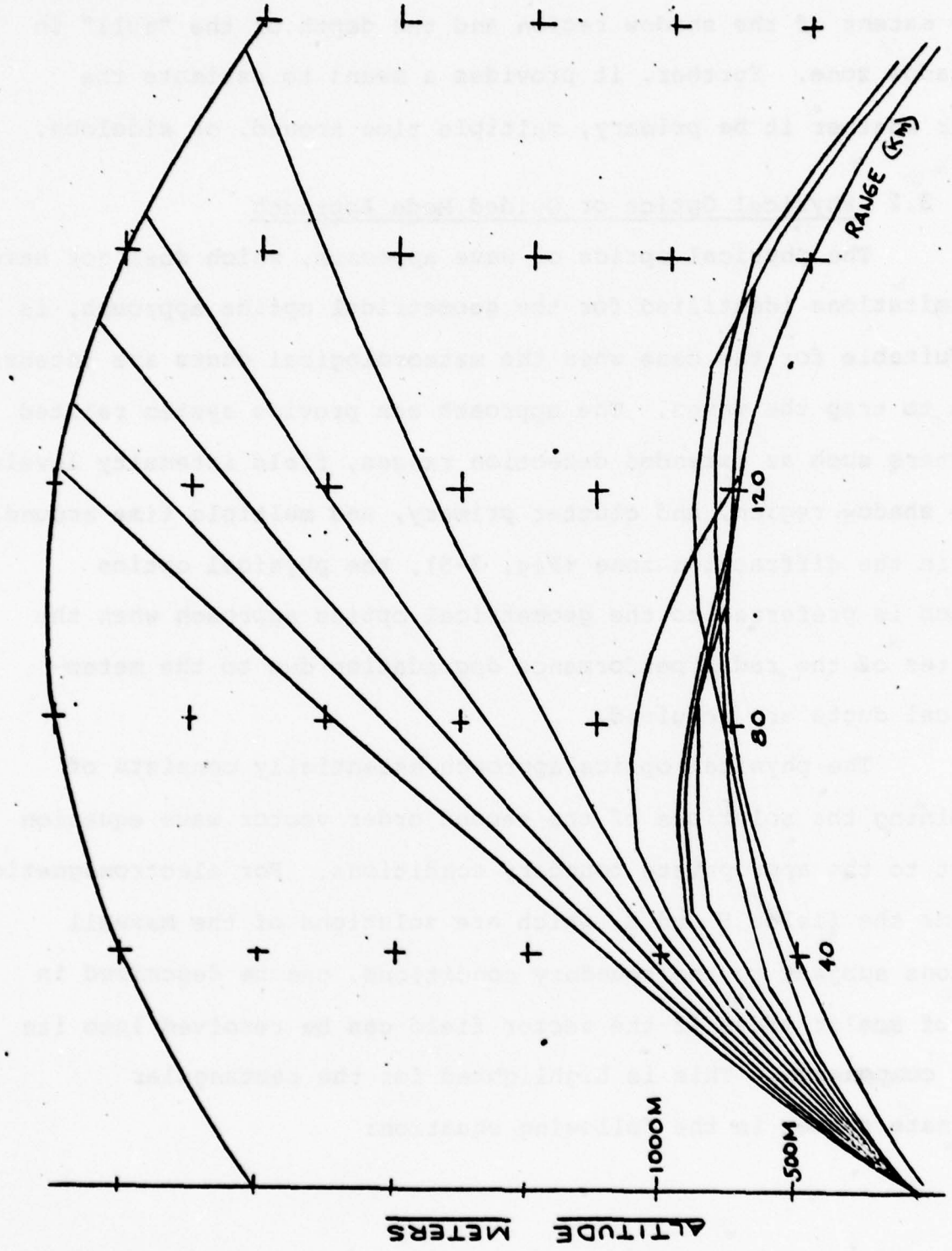


Figure (3-4) Typical Ray Trajectories for August 31

optics. The physical optics approach provides important information on the extent of the shadow region and the depth of the "null" in the shadow zone. Further, it provides a means to estimate the clutter whether it be primary, multiple time around, or sidelobe.

3.2 Physical Optics or Guided Mode Approach

The physical optics or wave approach, which does not have the limitations identified for the geometrical optics approach, is most suitable for the case when the meteorological ducts are intense enough to trap the waves. The approach can provide system related parameters such as extended detection ranges, field intensity levels in the shadow region, and clutter primary, and multiple time around. Thus, in the diffraction zone (Fig. 3-5), the physical optics approach is preferred to the geometrical optics approach when the estimates of the radar performance degradation due to the meteorological ducts are required.

The physical optics approach essentially consists of determining the solutions of the second order vector wave equation subject to the appropriate boundary conditions. For electromagnetic problems the fields $\underline{E}$ and $\underline{H}$, which are solutions of the Maxwell equations subject to the boundary conditions, can be described in terms of scalar modes if the vector field can be resolved into its scalar components. This is highlighted for the rectangular coordinate system in the following equation:

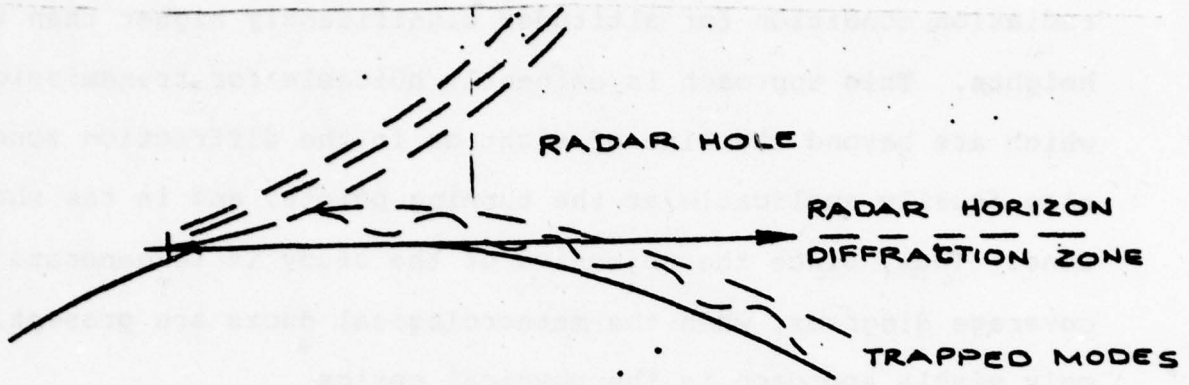


Figure (3-5) - Tropospheric Duct Propagation

$$\underline{\bar{\Phi}}\ (\underline{r}) = \sum_i a_i \phi_i\ (\underline{r}) = \sum_i a_i\ \phi_i\ (T)\ e^{jk_{xi}\cdot x}$$

Here, $\underline{\bar{\Phi}}\ (\underline{r})$ is the total vector electromagnetic field and is described in terms of the summation of the characteristic modes, $\phi_i\ (T)$; and T stands for the transverse coordinates y and z which are orthogonal to x, the direction of propagation. The longitudinal wave numbers k_{xi} are the characteristic values. The modes $\phi_i\ (T)$ are functions of the transverse coordinate, and they form a complete orthonormal set. The amplitude coefficient a_i thus are determined from the source conditions with the help of the orthogonality condition. Thus implicit in this approach are the characteristics of the source generating field such as frequency, polarization, antenna height, etc. Application of the boundary condition implies

the statement of the field characteristics (a) at the ground, (b) at the different regions of the M profile, and (c) the Sommerfeld radiation condition for altitudes significantly higher than the duct heights. This approach is eminently suitable for transmission paths which are beyond the line of sight or in the diffraction zone and also equally applicable at the turning points, and in the shadow zone. Thus, since the objective of the study is to generate radar coverage diagrams, when the meteorological ducts are present, the only viable approach is the physical optics.

3.3 Ray Trajectories For Four Refractive Index Profiles

The geometrical optics or the ray trajectory approach involves the use of Snell's Law to identify the propagation paths. Typical ray trajectories for four days of refractive index profiles from Canton Island are shown in Figs. (3-6 to 3-9). For January and April meteorological conditions, the existence of shadow zones are clearly noticeable. This should be evident from the strength of the duct as measured by dN/dz.

For the September and December meteorological conditions, the duct has been weak and a weak shadow zone is evident. It is to be noticed that the shadow zones of January and April will not be as pronounced as is displayed if finer intervals in the trajectory angles are chosen. Based on these trajectories, one would conclude that in the shadow zone there will be a total absence of propagation paths and hence a total absence of electromagnetic power. The question which is very pertinent for system performance evaluation is how valid is this interpretation;

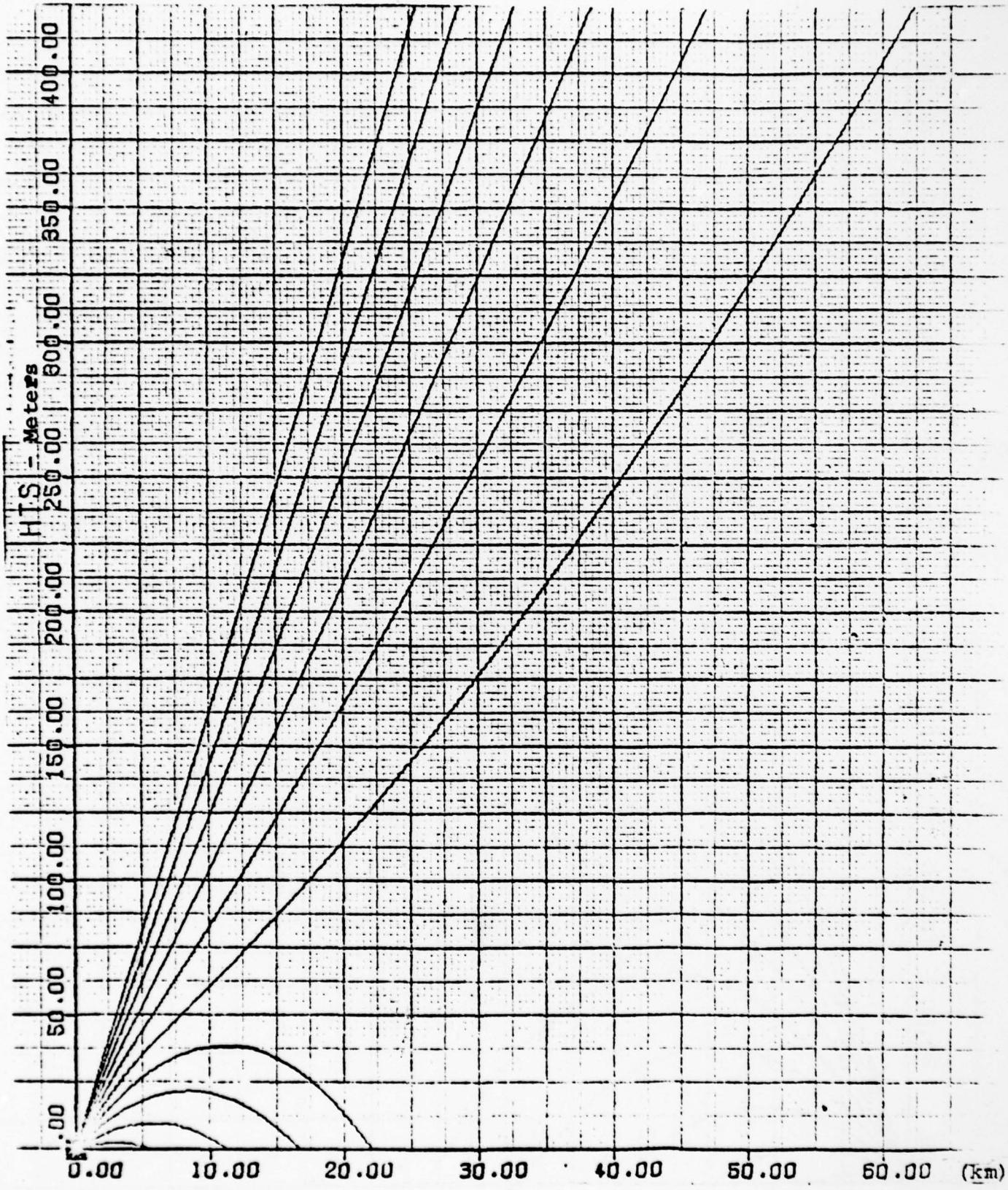


Fig. (3-6) Ray Trajectories for Canton Island - January, 1973

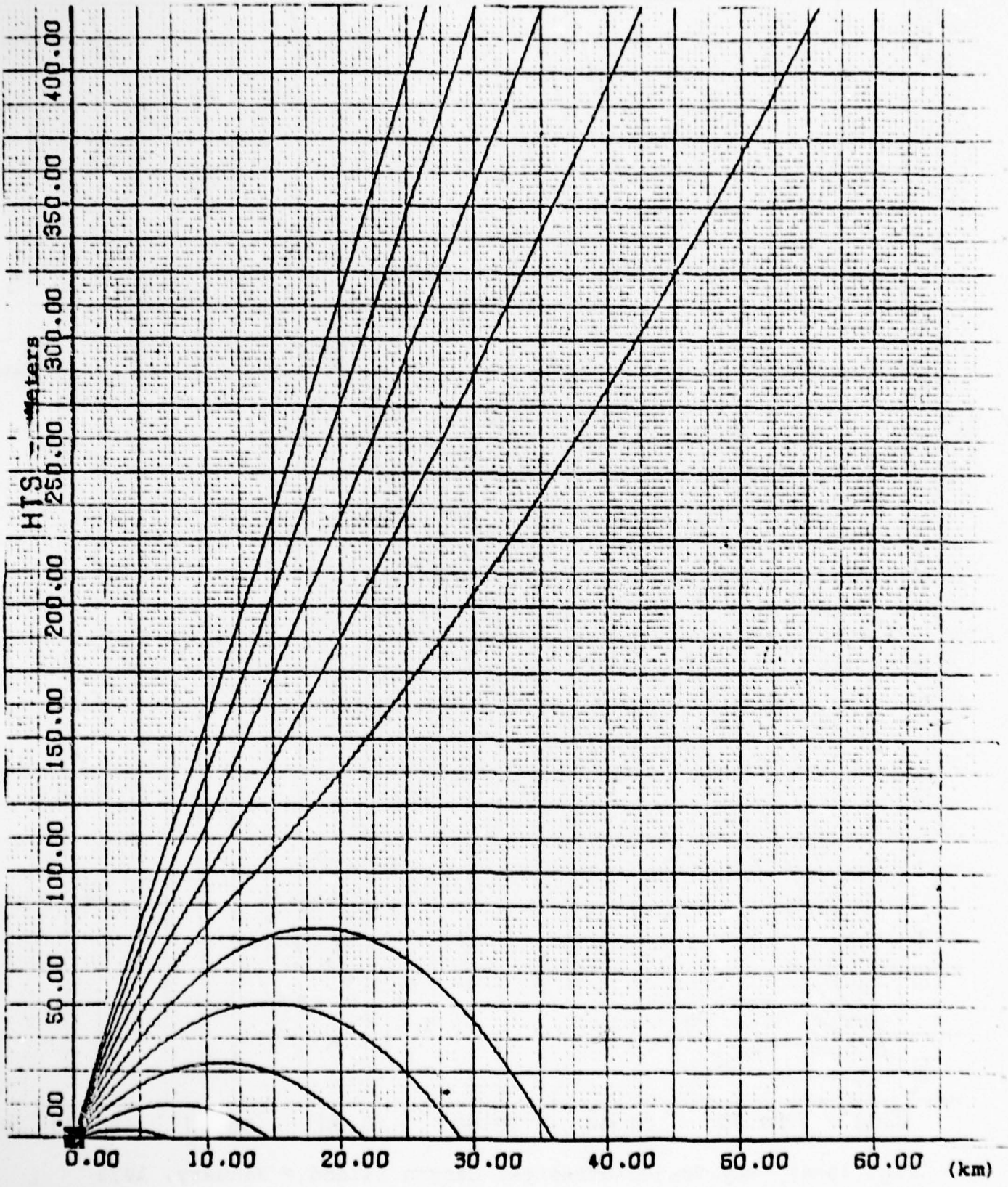


Fig. (3-7) Ray Trajectories For Canton Island - April, 1973

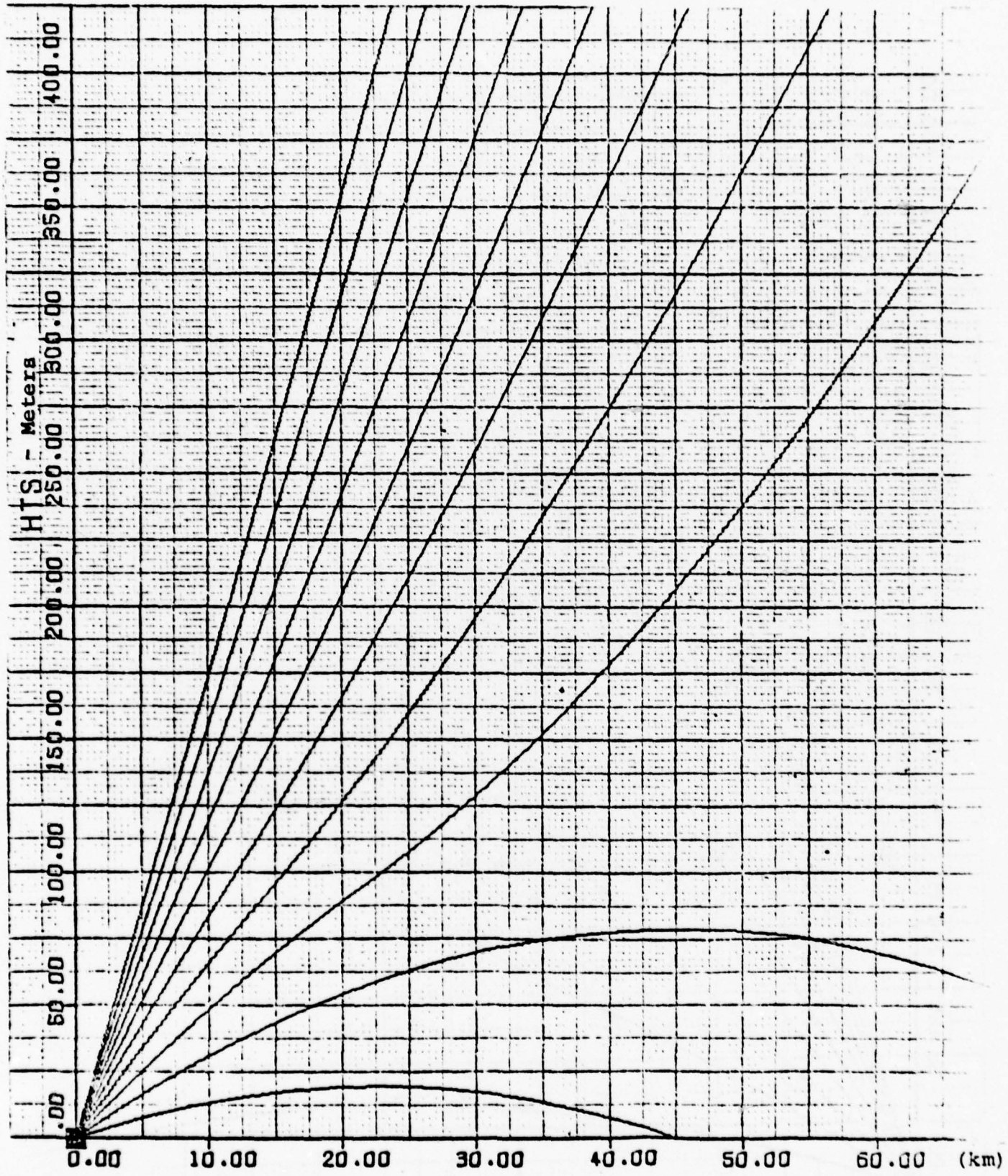


Fig. (3-8) Ray Trajectories For Canton Island - September, 1973

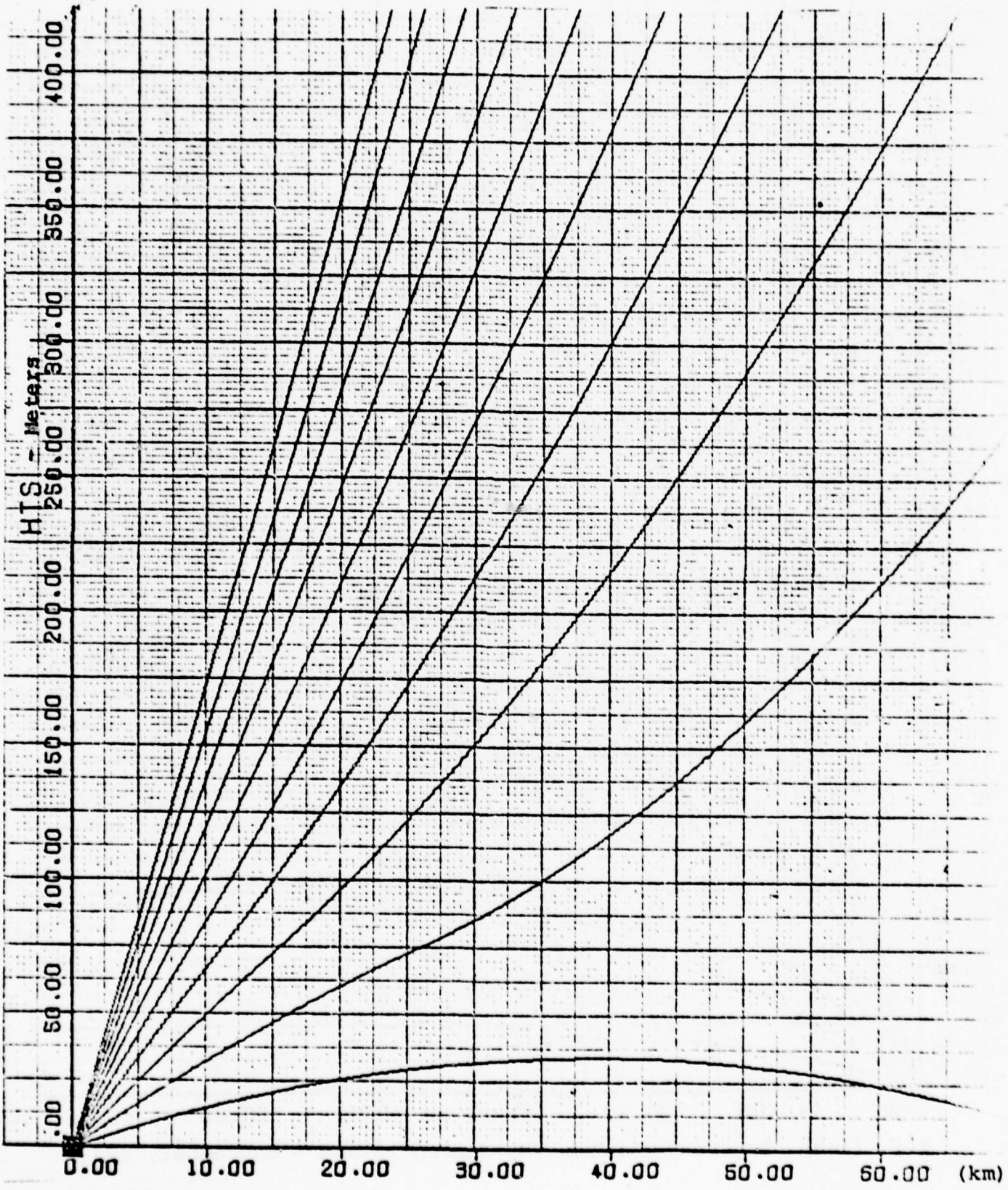


Fig. (3-9) Ray Trajectories For Canton Island - December, 1973

and is there a better way to estimate the field strength in the shadow zone.

In conclusion, it has been established that;

a. the geometrical optics approach is not valid in the diffraction zone;
b. it is inapplicable when the relative spacial variation of the refractive index per wavelength is larger than one;
c. when the fractional change in the spacing between the neighboring rays in a wavelength is not less than unity, then it is not applicable.

4. GUIDED MODE APPROACH - PHYSICAL OPTICS

It is thus evident that the guided mode approach is the most viable approach to establish radar performance in the diffraction zone and in the presence of meteorological ducts. The measure of the radar system performance is either (a) in terms of radar coverage diagrams depicting signal strengths or path loss as a function of ground range and altitude, or (b) the signal to noise ratio as a function of slant range. Such a performance evaluation is mandatory for the judicious hardware/software modifications to alleviate or minimize the deleterious effects of propagation degradation. Recognizing this, the goal of this phase of the program is to determine propagation behavior in the presence of bilinear and trilinear types of refractive index profiles. The important milestone of the next phase of the program is to establish the radar coverage diagrams for different meteorological conditions and for different systems parameters.

4.1 Propagation Through Stratified Medium

For the propagation analysis, it was assumed that the medium is stratified in the vertical z direction. The stratification of the medium dictated by temperature, pressure and partial water vapor pressure variations with altitude is described in terms of the refractive index profile, which in turn is represented by the N-profile or the M-profile. The profiles which have been considered are the bilinear and trilinear. This is based on the evaluation of the Canton Island meteorological data for 1973.

The M-profile for January, April, September and December are produced in Fig. (4-1) and they will be considered in the numerical analysis.

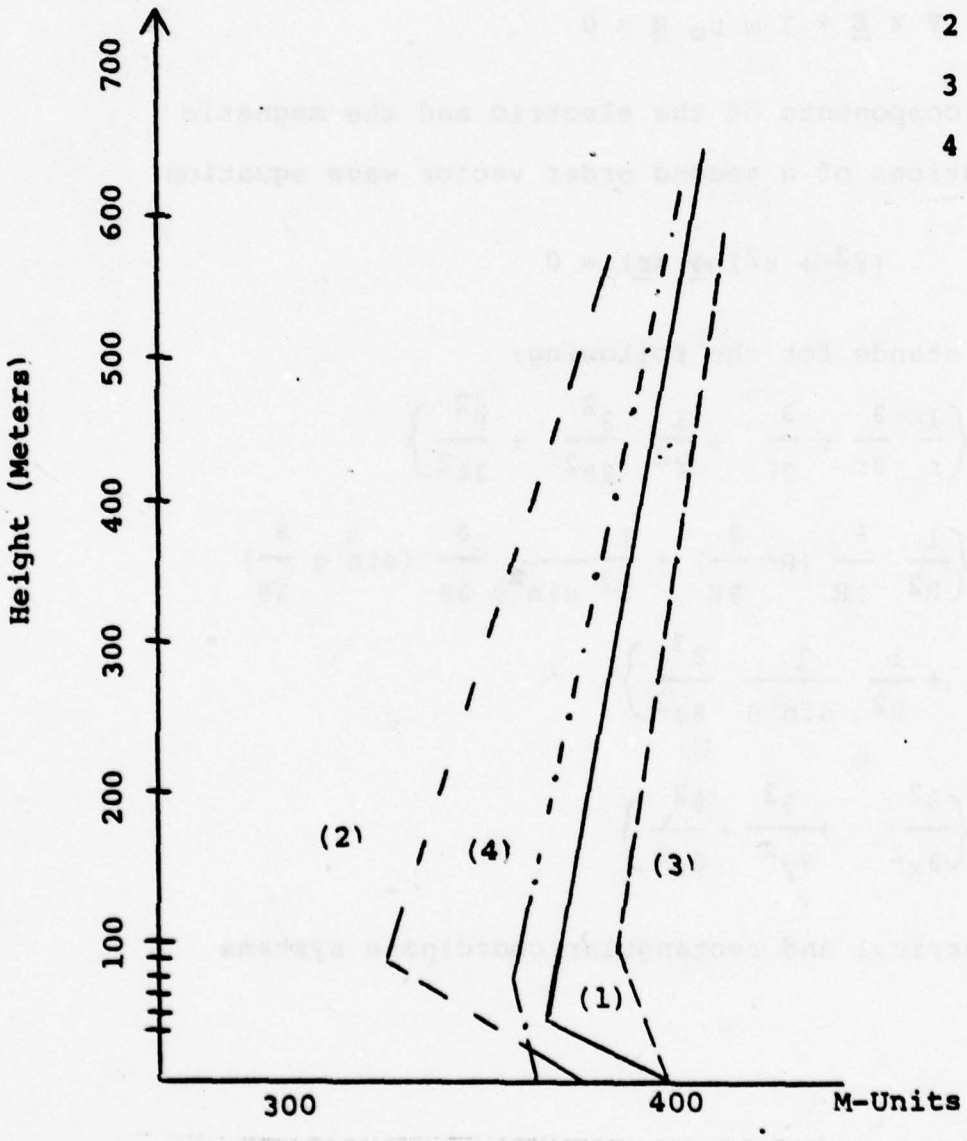


Fig. (4-1) - Canton Island Refractive Index Profiles for Four Days

4.1.1 Propagation Through The Meteorological Duct With Bilinear M Profile

The electromagnetic propagation through any medium is governed by Maxwell's equations.

$$\nabla \times \underline{H} - j\,\omega\,\epsilon_o\,\underline{E} = 0$$

and

$$\nabla \times \underline{E} + j\,\omega\,\mu_o\,\underline{H} = 0$$

and the individual components of the electric and the magnetic fields are the solutions of a second order vector wave equation

$$(\nabla^2 + k^2)\,\underline{\Phi}\,(\underline{r}) = 0$$

Here, Laplacian ∇^2 stands for the following:

$$\nabla^2 \rightarrow \left\{\frac{1}{r}\,\frac{\partial}{\partial r}\,r\,\frac{\partial}{\partial r} + \frac{1}{r^2}\,\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}\right\}$$

$$\rightarrow \left\{\frac{1}{R^2}\,\frac{\partial}{\partial R}\,(R^2\,\frac{\partial}{\partial R}) + \frac{1}{R^2 \sin^2\theta}\,\frac{\partial}{\partial \theta}\,(\sin\theta\,\frac{\partial}{\partial \theta})\right.$$

$$\left. + \frac{1}{R^2}\,\frac{1}{\sin\theta}\,\frac{\partial^2}{\partial \phi^2}\right\}$$

$$\rightarrow \left\{\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right\}$$

in cylindrical, spherical and rectangular coordinate systems respectively.

The wave number $k = k_o \eta$, k_o is the free space wave number, $k_o = \omega/c$, ω is the angular frequency and c is the velocity of light in free space. η is the refractive index of the medium.

The vector field $\underline{\Phi}$ stands for the $\underline{E}$ and $\underline{H}$ field

$$\underline{\Phi} \rightarrow \begin{pmatrix} \underline{E}\ (\underline{r}) \\ \underline{H}\ (\underline{r}) \end{pmatrix} e^{j\omega t}$$

For a point source above ground, such as the dipole, the wave equation is more suitably described in terms of the spherical coordinate system. For the homogeneous atmosphere the mode solution of the wave equation was first obtained by Watson [18] and is given by

$$E_y = e^{j\omega t} \sum_m \frac{A_m}{x^{1/2}} e^{-jk_{xm} \cdot x} \quad u_{m1}(z)\ u_{m2}(z)$$

Here, $u_{m1}(z)$ and $u_{m2}(z)$ are the height gain functions at the transmitter and the receiver, respectively; the subscript 1 stands for transmitter and 2 for the receiver. The transmitter and receiver locations have been left unspecified. For a ground radar system, the transmitter position is fixed at h and then $u_{m1}(z) \equiv u_m(h)$ is the specified antenna characteristics.

A_m are the amplitude coefficients of the modes; x is the horizontal range. Finally, the height gain function $u_m(z)$ is the solution of the one dimensional wave equation.

$$\left\{\frac{d^2}{dz^2} + k_o^2\,\hat{\eta}^2 - k_x^2\right\} E_y(z) = 0$$

The x dependence has been assumed as $e^{-jk_x \cdot x}$ and $E_y(z) \equiv u_{m2}(z)$. The z dependence will be deleted from hereon.

The refractive index $\hat{\eta}$ is assumed to be stratified only in the vertical z direction. For the chosen four days of M profile

$$\frac{dM}{dz} = -p \qquad z < d$$

$$= +q \qquad z > d$$

d is the duct width.

The scalar wave equation for E_y is thus given by

$$\left\{\frac{d^2}{dz^2} + k_o^2\,(1 + 2\,M(z)) - k_x^2\right\} E_y = 0$$

Thus for the M-profile, the wave equation takes the following form:

$$M(z) = M_o - pz \qquad z < d$$

$$M_{min} + q(z-d) \qquad z > d$$

$$\left\{ \frac{d^2}{dz^2} + B1 - C1\, z \right\} Ey = 0 \qquad z < d$$

$$\left\{ \frac{d^2}{dz^2} + B_2 + C_2\, z \right\} Ey = 0 \qquad z > d$$

Here $B_1 = k_o^2 (1 + 2 M_o) - k_x^2$

$C1 = 2k_o^2\, p$

$B_2 = k_o^2 (1 + 2 (M_o - pd - qd)) - k_x^2$

$C_2 = 2k_o^2\, q$

The wave equations are further transformed by the following change of variables

$$\xi_< = \frac{B_1}{C_1^{2/3}} - C_1^{1/3} z$$

$$\xi_> = \frac{B_2}{C_2^{2/3}} + C_2^{1/3} z$$

Thus substituting for $\xi_{<_{>}}$ in the wave equations, we have a well known Stokes equation

$$\left\{\frac{d2}{d\xi^2} + \xi\right\}_{<_{>}} E_{y<_{>}} = 0$$

$$\xi_< d = \xi_{1d} \quad , \quad \xi_> d = \xi_{2d}$$

$$\xi_{1d} = \frac{B_1}{C_1{}^{2/3}} - C_1{}^{1/3}\, d$$

$$\xi_{2d} = \frac{B_2}{C_2{}^{2/3}} + C_2{}^{1/3}\, d$$

$$\xi_< = \xi_{1d} - C_1{}^{1/3}\,(z-d)$$

$$\xi_> = \xi_{2d} + C_2{}^{1/3}\,(z-d)$$

and since

$$(B_1 - C_1 d) = (B_2 + C_2\, d) \quad , \qquad z = d$$

$$C_1{}^{2/3}\,\xi_{1d} = C_2{}^{2/3}\,\xi_{2d}$$

Finally,

$$\xi_{<o} = \xi_{10} = \frac{B_1}{C_1{}^{2/3}}$$

The Stokes equation now has to be solved with the help of the following boundary conditions:

$$E_{y<} = E_{y>} \quad , \quad \frac{dE_y}{dz}\Big|_{<} = \frac{dE_y}{dz}\Big|_{>} \qquad z = d$$

For perfectly conducting earth

$$E_y = 0 \qquad z = 0$$

E_y for $z \gg d$ is an outgoing wave. This condition represents the Sommerfeld radiation condition. The Stokes equation is well known and it has solutions in terms of Airy Integral Functions $A_i(\xi)$ and $B_i(\xi)$ and they are given by

$$E_{y<} = a\, A_i(-\xi_<) + b\, B_i(-\xi_<)$$

$$E_{y>} = C\left[A_i(-\xi_>) + j\, B_i(-\xi_>)\right]$$

Here, a, b and c are the coefficients to be determined by applying the boundary conditions. Note that up until now, we have not introduced any modal designate.

Application of the boundary conditions lead to the following transcendental equation

$$\frac{\left\{A_i\ (-\xi_{1d}) - \frac{A_i(-\xi_{10})}{B_i(-\xi_{10})} \cdot B_i\ (-\xi_{1d})\right\}\left\{A_i'(-\xi_{2d}) + j\ B_i'\ (-\xi_{2d})\right\}}{A_i\ (-\xi_{2d}) + j\ B_i\ (-\xi_{2d})}$$

$$= -\left(\frac{p}{q}\right)^{1/3}\left\{A_i'\ (-\xi_{1d}) - \frac{A_i(-\xi_{10})}{B_i(-\xi_{10})} \cdot B_i'\ (-\xi_{1d})\right\}$$

Here the prime refers to differentiation with respect to ξ, and

$$\frac{d}{dz}\Big|_< = \frac{d}{d\xi} \cdot \frac{d\xi}{dz}\Big|_< = -\ C_1^{1/3}\ \frac{d}{d\xi}\Big|_<$$

$$\frac{d}{dz}\Big|_> = \frac{d}{d\xi} \cdot \frac{d\xi}{dz}\Big|_> = +\ C_2^{1/3}\ \frac{d}{d\xi}\Big|_>$$

The solutions of the transcendental equations are the eigenvalues and for the fixed frequency, the wave number k_{xi} is related to the i^{th} eigenvalue ξ_{1di} by

$$k_{xi}^2 = k_o^2\ (1 + 2\ M_{min}) - \xi_{1di} \cdot C_1^{2/3}$$

$$k_{xi} \approx \left(k_o\ (1 + M_{min}) - \left(\frac{p}{q}\right)^{2/3} \cdot \frac{\xi_{1di}}{L}\right)$$

and

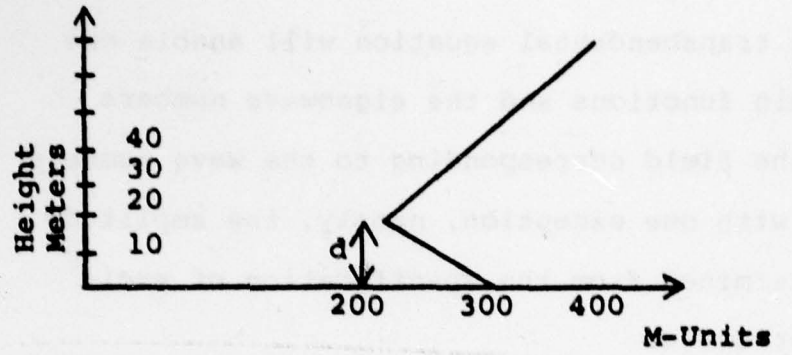
$$L = \left(\frac{2\ k_o}{q^2}\right)^{1/3}$$

Thus, the solutions of the transcendental equation will enable one to determine the height gain functions and the eigenwave numbers along the x axis. Thus, the field corresponding to the wave numbers are completely determined with one exception, namely, the amplitude coefficients which are determined from the specification of excitation or antenna characteristics.

4.1.2 Trilinear Refractive Index Profiles

The selective meteorological data from Canton Island (Tables 2-1 through 2-12) has revealed that although, because of the nature of the measurements, majority of the cases were representable as having bilinear refractive index profiles; there were occasions when the profiles were more representable as trilinear and this in particular when the ducts were elevated. This necessitated developing an analytical propagation model for the trilinear refractive index profile. In principal, the approach is identical to the one developed for bilinear case and it entails additional boundary conditions representing matching of the transverse impedances at the intermediate regions of M-profile.

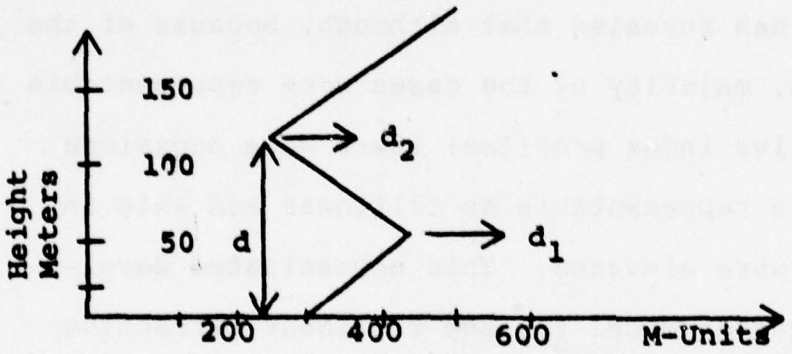
In order to display the basic features of the trilinear refractive index profile, we have reproduced in Figs. (4-2, 3, 4) some well known refractive index profiles. The Fig. (4-2) is obviously a surface duct with the bilinear M-profile, Fig. (4-3) and (4-4) are in general categorized as elevated ducts with trilinear M-profiles.



$$\frac{dM}{dz} = -p \qquad z < d$$

$$= +q \qquad z > d$$

Fig. (4-2) - Surface Duct - Bilinear Profile

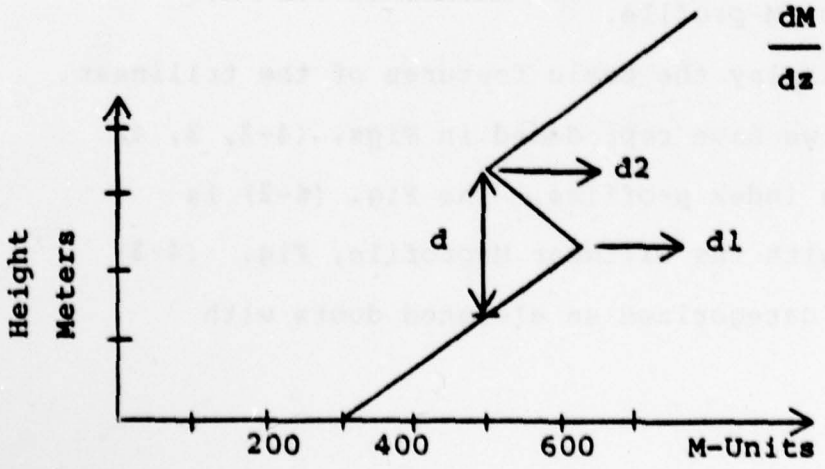


$$\frac{dM}{dz} = +q_1 \qquad z < d_1$$

$$= -p \qquad d_1 < z < d_2$$

$$= +q_2 \qquad z > d_2$$

Figure (4-3) - Elevated Surface Duct - Trilinear Profile



$$\frac{dM}{dz} = +q \qquad z < d_1$$

$$= -p \qquad d_1 < z < d_2$$

$$= +q \qquad z > d_2$$

Figure (4-4) - Elevated Duct - Trilinear Profile

In all the three cases, the duct width is defined by d. In Figs. (4-3) and (4-4) note the difference in the elevated surface duct and the elevated duct. It is that at the duct apex the M_{min} is less than the M at the surface for the elevated surface duct case, and M_{min} is greater than M at the surface for the elevated duct. It is also to be recognized that in practice, the slope of the M-profile will not be that sharp, however, the measurement techniques perhaps would not provide closely spaced points to round off the M curves.

4.1.3 Propagation through Ducts with Trilinear Profiles

The scalar wave equation for E_y for the trilinear refractive index profiles is given by:

$$\left\{ \frac{d^2}{dz^2} + k_o^2 \left[1+2M(z)\right] - k_x^2 \right\} E_y = 0$$

$$M(z) \rightarrow M_o+q_1 z \qquad z<d_1$$

$$\rightarrow M_o+q_1 d_1-p(d_2-d_1)+p(d_2-z) \qquad d_1<z<d_2$$

$$\rightarrow M_o+q_1 d_1-p(d_2-d_1)+q_2(z-d_2) \qquad z>d_2$$

or

$$\left\{ \frac{d^2}{dz^2} + B_1 + C_1 z \right\} E_y = 0 \qquad z<d_1$$

$$\left\{ \frac{d^2}{dz^2} + B_2 - C_2 z \right\} E_y = 0 \qquad d_1<z<d_2$$

and

$$\left\{ \frac{d^2}{dz^2} + B_3 + C_3 z \right\} E_y = 0 \qquad z>d_2$$

Here B_1, C_1; B_2, C_2; and B_3, C_3 are given by

$$B_1 = k_o^2(1+2M_o)-k_x^2$$

$$C_1 = 2k_o^2 q_1$$

$$B_2 = k_o^2\left(1+2(M_o+q_1 d_1+p d_1)\right)-k_x^2$$

$$C_2 = 2k_o^2 p$$

$$B_3 = k_o^2\left(1+2(M_o+q_1 d_1-p(d_2-d_1)-q_2 d_2)\right)-k_x^2$$

$$C_3 = 2k_o^2 q_2$$

Let

$$\xi_{<1} = \frac{B_1}{C_1^{2/3}} + C_1^{1/3} z \qquad \frac{d\xi}{dz}\Big|_{<} = +C_1^{1/3}$$

$$\xi_{<12>} = \frac{B_2}{C_2^{2/3}} - C_2^{1/3} z \qquad \frac{d\xi}{dz}\Big|_{<12>} = -C_2^{1/3}$$

$$\xi_{>2} = \frac{B_3}{C_3^{2/3}} + C_3^{1/3} z \qquad \frac{d\xi}{dz}\Big|_{>} = C_3^{1/3}$$

These substitutions in the wave equations lead to

$$\left\{\frac{d^2}{d\xi^2_{\substack{1\\2\\3}}} + \xi_{\substack{1\\2\\3}}\right\} E_{y_{\substack{1\\2\\3}}} = 0$$

This again is the well known Spokes equation and has solutions in terms of Airy Integral functions.

Analogous to the bilinear case, the field description in the three different regions is as follows:

$$E_{y_1} = a_1\left\{A_i(-\xi_{<1}) - \frac{A_i(-\xi_{10})}{B_i(-\xi_{10})} B_i(-\xi_{<1})\right\}$$

$$E_{y_2} = a_2\{A_i(-\xi_{<12>}) + a_3 B_i(-\xi_{<12>})\}$$

$$E_{y_3} = a_4\{A_i(-\xi_{>2}) + jB_i(-\xi_{>2})\}$$

The E field has to satisfy the following continuity relations:

$$\left.\begin{aligned} E_{y_1} &= E_{y_2} \\ \frac{dE_{y_1}}{dz} &= \frac{dE_{y_2}}{dz} \end{aligned}\right\} \quad \text{at } z = d_1$$

$$\left.\begin{aligned} E_{y_2} &= E_{y_3} \\ \frac{dE_{y_2}}{dz} &= \frac{dE_{y_3}}{dz} \end{aligned}\right\} \quad \text{at } z = d_2$$

These boundary conditions enable us to eliminate the different amplitude coefficients.

From now on we are specializing to the case of $q_1 = q_2 = q$. Consider first the continuity relations for field and its derivative at $z = d_1$ which lead to the following equations:

$$a_1 \left\{A_i(-\xi_{1d1}) - \frac{A_i(-\xi_{10})}{B_i(-\xi_{10})} \cdot B_i(-\xi_{1d1})\right\} =$$

$$a_2 \left\{A_i(-\xi_{2d1}) + a_3 B_i(-\xi_{2d1})\right\}$$

and

$$a_1 \left\{A_i'(-\xi_{1d1}) - \frac{A_i(-\xi_{10})}{B_i(-\xi_{10})} \cdot B_i'(-\xi_{1d1})\right\} =$$

$$-\left(\frac{p}{q}\right)^{1/3} \cdot a_2 \left\{A_i'(-\xi_{2d1}) + a_3 B_i'(-\xi_{2d1})\right\}$$

or

$$\left\{\frac{A_i(-\xi_{1d1}) - \frac{A_i(-\xi_{10})}{B_i(-\xi_{10})} \cdot B_i(-\xi_{1d1})}{A_i'(-\xi_{1d1}) - \frac{A_i(-\xi_{10})}{B_i(-\xi_{10})} \cdot B_i'(-\xi_{1d1})}\right\} =$$

$$-\left\{\left(\frac{p}{q}\right)^{-1/3} \cdot \frac{\left\{A_i(-\xi_{2d1}) + a_3 B_i(-\xi_{2d1})\right\}}{\left\{A_i'(-\xi_{2d1}) + a_3 B_i'(-\xi_{2d1})\right\}}\right\}$$

Let the L.H.S. $\equiv P/Q$.

then

$$a_3 = -\left\{\frac{A_i(-\xi_{2d1}) + \frac{P}{Q}\left(\frac{p}{q}\right)^{1/3} \cdot A_i'(-\xi_{2d1})}{B_i(-\xi_{2d1}) + \frac{P}{Q}\left(\frac{p}{q}\right)^{1/3} \cdot B_i'(-\xi_{2d1})}\right\}$$

Likewise, the continuity relations at $z = d_2$ lead to the following

$$a_4 \{Ai(-\xi_{>}d2)+jBi(-\xi_{>}d2)\} =$$

$$a_2 \{Ai(-\xi_2 d2)+a_3 Bi(-\xi_2 d2)\}$$

and

$$a_4 \{Ai'(-\xi_{>}d2)+jBi'(-\xi_{>}d2) =$$

$$-a_2 \left(\frac{p}{q}\right)^{1/3} \cdot \{Ai'(-\xi_2 d2)+a_3 Bi'(-\xi_2 d2)\}$$

Thus the transcendental equation is given by

$$\left\{ \frac{Ai(-\xi_{>}d2)+jBi(-\xi_{>}d2)}{Ai'(-\xi_{>}d2)+jBi'(-\xi_{>}d2)} \right\} \cdot \left(\frac{q}{p}\right)^{-1/3} =$$

$$-\left\{ \frac{Ai(-\xi_2 d2)+a_3 Bi(-\xi_2 d2)}{Ai'(-\xi_2 d2)+a_3 Bi'(-\xi_2 d2)} \right\}$$

where

$$a_3 = -\frac{\left\{Ai(-\xi_2 d1)+\frac{P}{Q}\left(\frac{p}{q}\right)^{1/3} \cdot Ai'(-\xi_2 d1)\right\}}{\left\{Bi(-\xi_2 d1)+\frac{P}{Q}\left(\frac{p}{q}\right)^{1/3} Bi'(-\xi_2 d1)\right\}}$$

4.2 The Solutions of the Transcendantal Equation - The Bilinear Refractive Index Profile

The transcendental equation which is arrived at by satisfying the boundary conditions at z=d, physically represents the matching of the transverse impedances at z=d.

$$Z_{z<} = Z_{z>}$$

or equivalently $\phi(Z) \equiv (Z_{z<} - Z_{z>}) = 0$

Here
$$\mathcal{Z}_z = \frac{E_y}{H_x} \propto \frac{E_y}{\frac{dE_y}{dz}}$$

and $\mathcal{Z}_{z<}$, $\mathcal{Z}_{z>}$ are the transverse impedances for z<d and z>d, respectively.

Because of the nature of the transcendental equation, a closed form solution does not exist and an iterative, numerical procedure to locate the zeros of the function $\phi(\mathcal{Z})$ has to be adopted. Three different approaches have been explored, and they are:

a. The Newton-Raphson Method

b. The Secant Method

c. The Zero Crossing Method

4.2.1 The Newton-Raphson Technique

This technique, essentially, provides a procedure to obtain an approximate solution of the transcendental equation which, after several iterations, approaches the exact solution and it states

$$\mathcal{Z}_1 = \mathcal{Z}_0 - \frac{\phi(\mathcal{Z}_0)}{\phi'(\mathcal{Z}_0)}$$

Here, $\mathcal{Z}_0$ is the first trial solution of the transcendental equation $\phi(\mathcal{Z}) = 0$. $\phi'(\mathcal{Z}_0)$ is the derivative of ϕ with respect to $\mathcal{Z}_0$ and $\mathcal{Z}_1$ is the next better solution to be used for second iteration and so on. Thus, the ith solution is obtained from (i-1)th and is

$$\mathcal{Z}_i = \mathcal{Z}_{i-1} - \frac{\phi(\mathcal{Z}_{i-1})}{\phi'(\mathcal{Z}_{i-1})}$$

If the transcendental equation has n solutions then this procedure has to be repeated n times.

The Newton-Raphson iterative technique to locate the zeros of the transcendental equation is effective only when the trial solution is reasonably close to the actual solution. The Newton-Raphson procedure thus has severe limitations.

4.2.2 The Secant Method

This iterative approach is analogous to the Newton-Raphson approach and the iterative solutions are given by

$$\mathcal{Z}_i = \mathcal{Z}_{i-1} - \frac{(\mathcal{Z}_i - \mathcal{Z}_{i-1})\,\phi(\mathcal{Z}_{i-1})}{\phi(\mathcal{Z}_i) - \phi(\mathcal{Z}_{i-1})}$$

Note that in the limit when $\mathcal{Z}_i$ approaches the exact solution $\mathcal{Z}_{i-1}$, then this expression is identical to the Newton-Raphson expression. Alternatively, if $\mathcal{Z}_i - \mathcal{Z}_{i-1} << \delta$ and if $\phi(\mathcal{Z}_{i-1}) - \phi(\mathcal{Z}) << \epsilon$ then the technique is identical with the Newton-Raphson technique. Here δ and ϵ are very small quantities.

4.2.3 The Zero-Crossing Technique

Another technique which is applicable to determine the solutions of the transcendental equation is the zero crossing of $\phi(\mathcal{Z})$, when $\phi(\mathcal{Z})$ is plotted as a function of $\mathcal{Z}$. This procedure is extremely effective for the trapped modes or when the ξ_{1di} are pure real, positive or negative numbers.

In any case, for all three cases, the computer programs for the Airy Integral Functions and the Hankel Functions are required.

4.3 Numerical Analysis For Bilinear M-Profile

It was decided that four meteorological profiles will be chosen in such a fashion that the two extremes of the characteristic parameters such as the duct height and duct intensity shall be chosen.

For the 3rd of January, the intensity of the duct was close to the maximum; for the 4th of December, the duct intensity was minimum. For the 4th of September the duct height was close to the largest while January 3rd it was close to the smallest. For the 5th of April the duct was moderately intense. For all four days, the profile was describable as bilinear. Finally, in order to extract the frequency dependence, three wavelengths were chosen, and they were 10, 20 and 50 cm.

Based on the expression for the maximum wavelengths that are trapped by ducts of height d, the following observation is made:

Table (4-1) - Trapped Wavelengths

Date	d (meters)	λ max (cm)
3rd Jan.	42	22.65
5th Apr.	85	65.20
4th Sept.	94	75.82
4th Dec.	75	54.04

a. For 10 and 20 cm wavelengths the field is trapped for all the four cases.

b. For 50 cm wavelengths the field is not trapped for Jan. 3rd case, however, it is trapped for the remaining three days.

4.3.1 The Zero-Crossing Technique For Trapped Modes

For the four days of M-profile data and for three wavelengths 10, 20 and 50 cm, the transcendental function $\phi(\xi)$ is plotted for various real values of ξ varying from -5 to +5. These plots are displayed in Figs. (4-5 to 4-16). The zero-crossing values of ξ, or equivalently ξ_{1d}, for which $\phi(\xi) = 0$ are the solutions of the transcendental equation. Consider now the January 3rd profile and $\lambda = 10$ cm case (Fig. 4-5). The zero-crossings take place for the following ξ_{1d} values:

$$\xi_{1d}=-1.25,\ 0.68,\ 2.8,\ 5.25$$

First observe that the zero-crossing at -1.67 is not a true crossing as is evident from the computations for finer spacings of the ξ_{1d} values. The $\phi(\xi)$ will continue to decrease approaching $-\infty$ for ξ_{1d} values in the vicinity of -1.65. Likewise for $\xi_{1d} \rightarrow -1.68$, the $\phi(\xi)$ will increase approach $+\infty$; hence no zero-crossing for $\xi_{1d} = -1.67$ is possible.

The first ξ_{1d} for which $\phi(\xi) = 0$ is -1.25 and this negative root needs to be interpreted. Consider the field Ey associated with this root and observe how it behaves as the height increases from z=0 to z=d and z>>d.

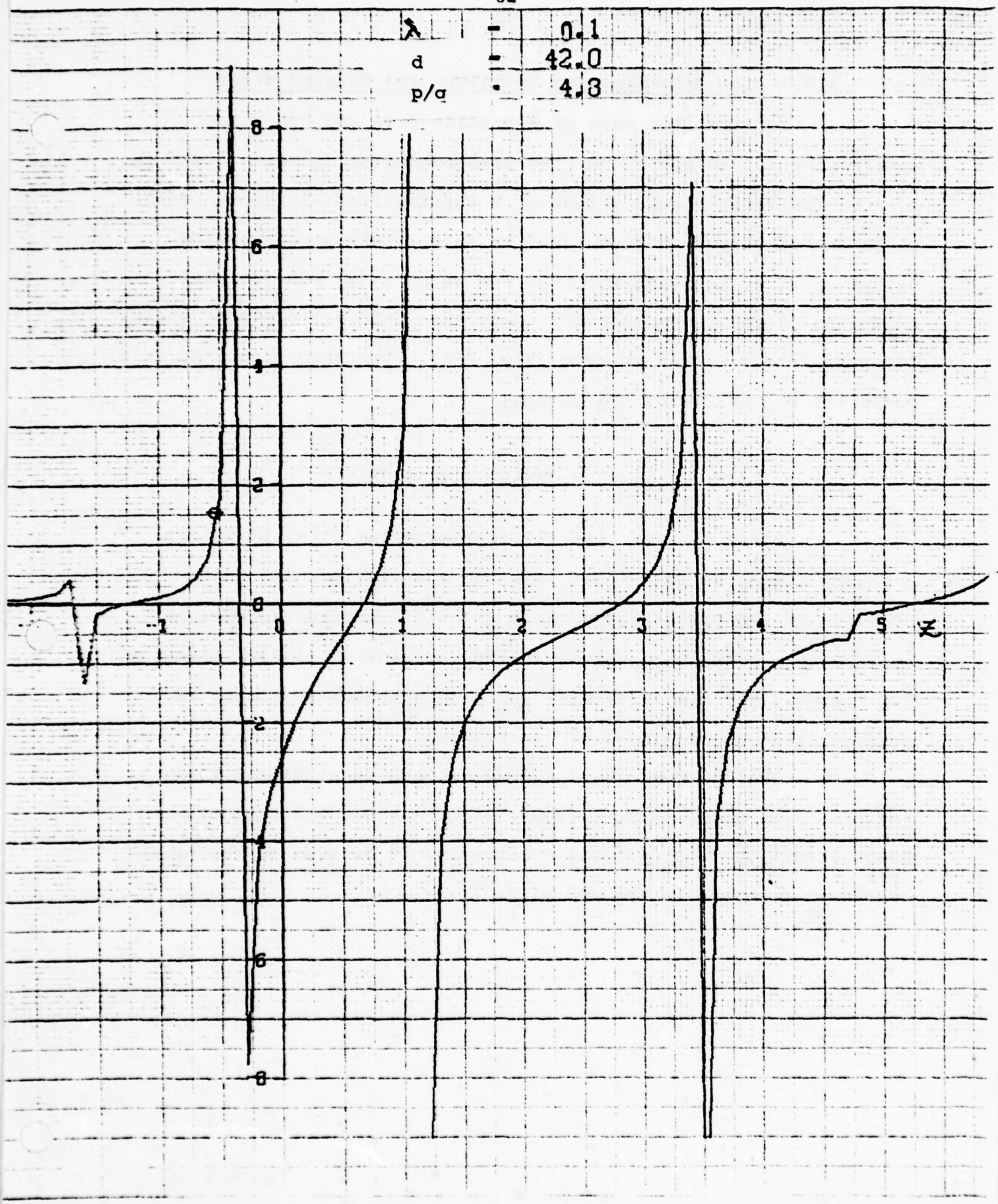


FIG. (4-5) PLOTS OF $\Phi(Z)$ AS A FUNCTION OF Z

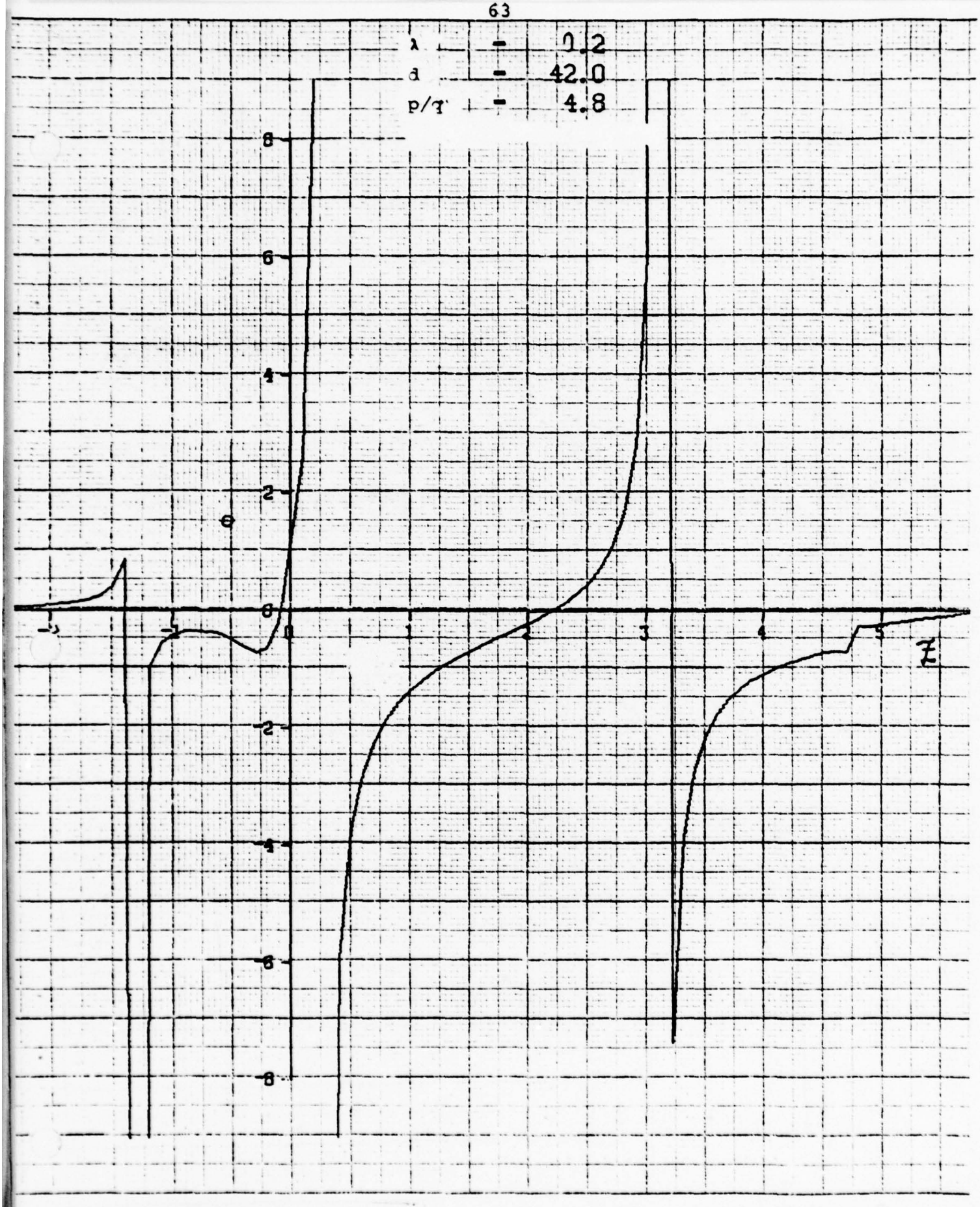


FIG. (4-6) PLOTS OF $\mathcal{J}(Z)$ AS A FUNCTION OF Z

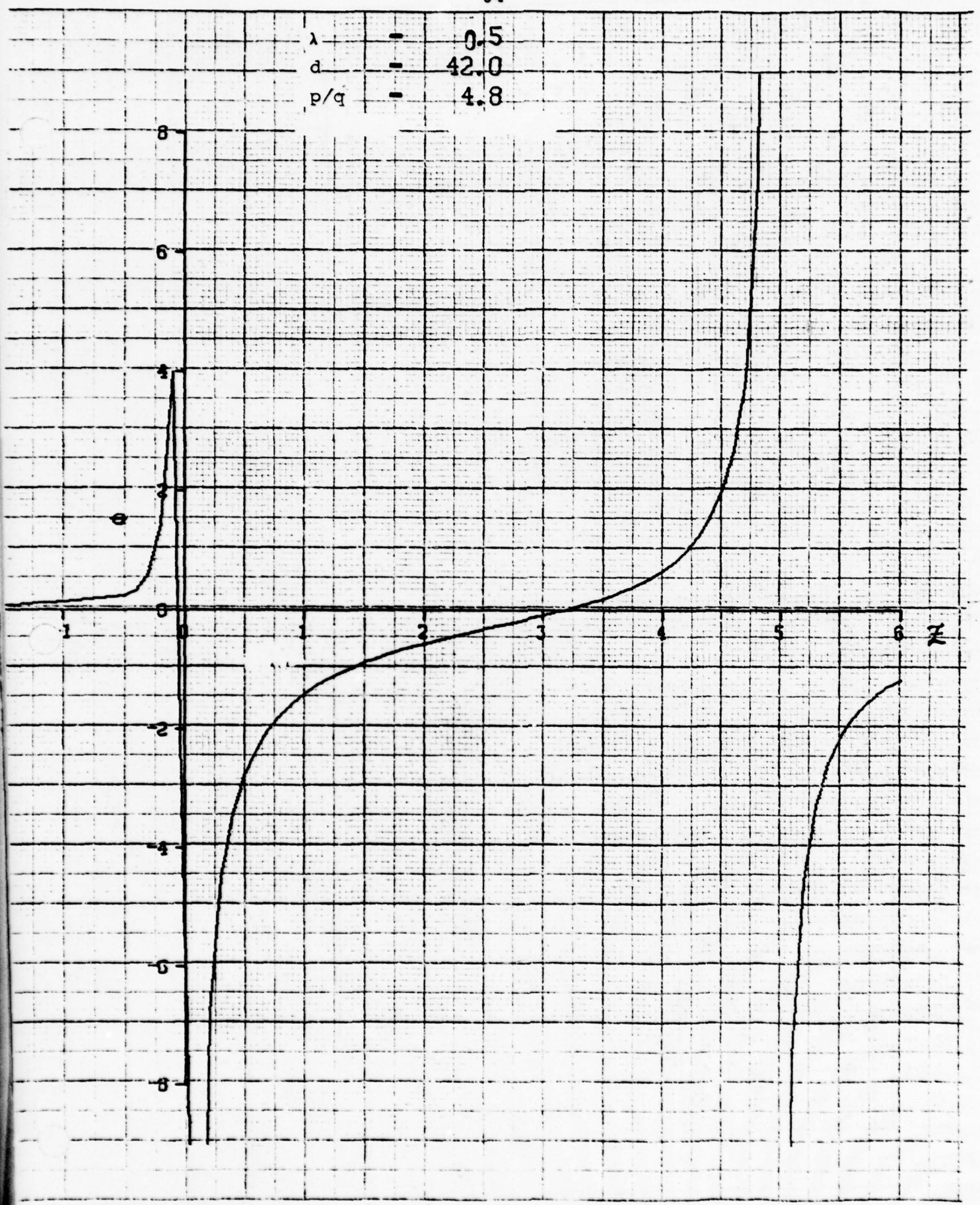


FIG. (4-7) PLOTS OF $\mathcal{J}(Z)$ AS A FUNCTION OF Z

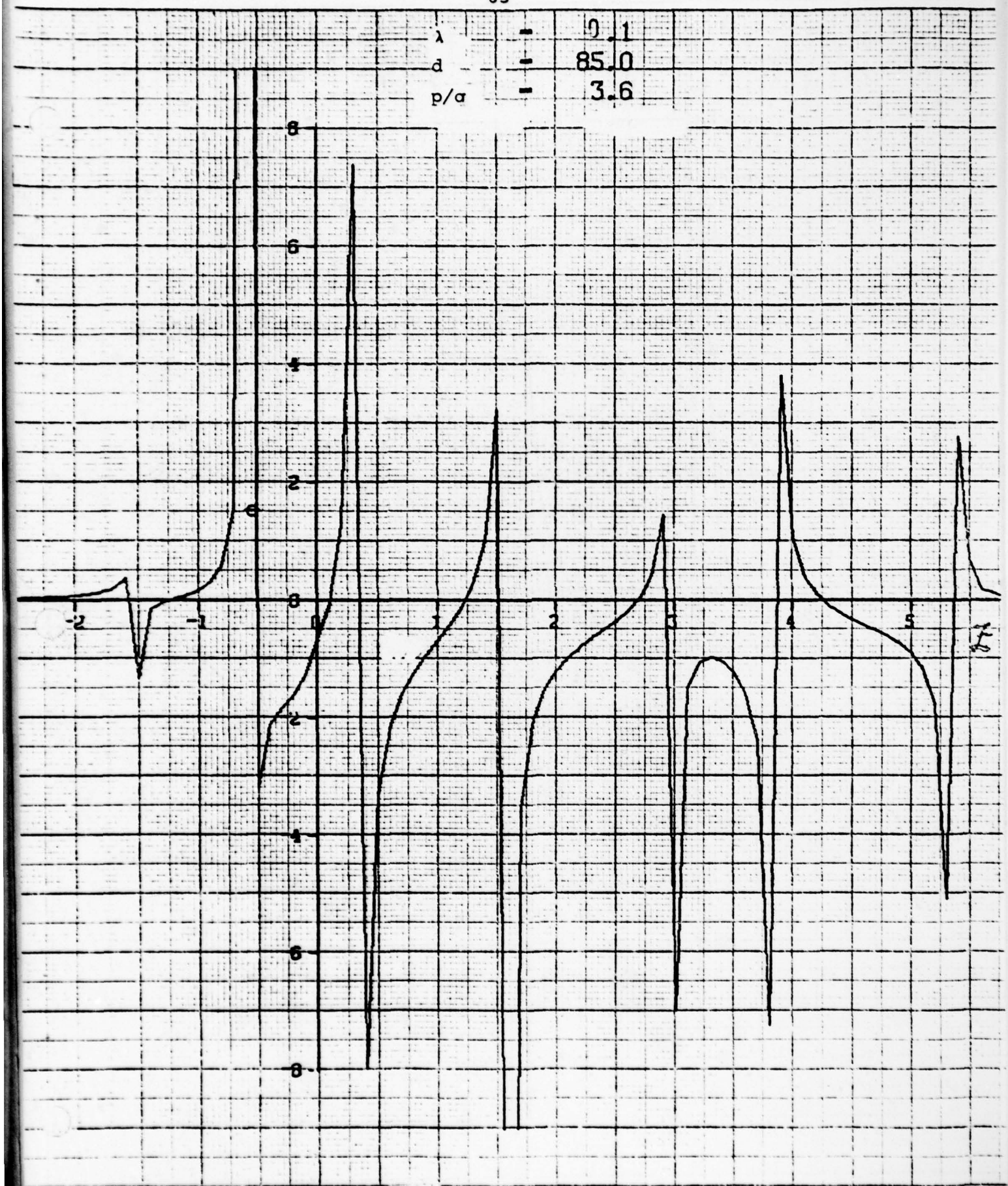


FIG. (4-8) PLOTS OF $\mathcal{G}(Z)$ AS A FUNCTION OF Z

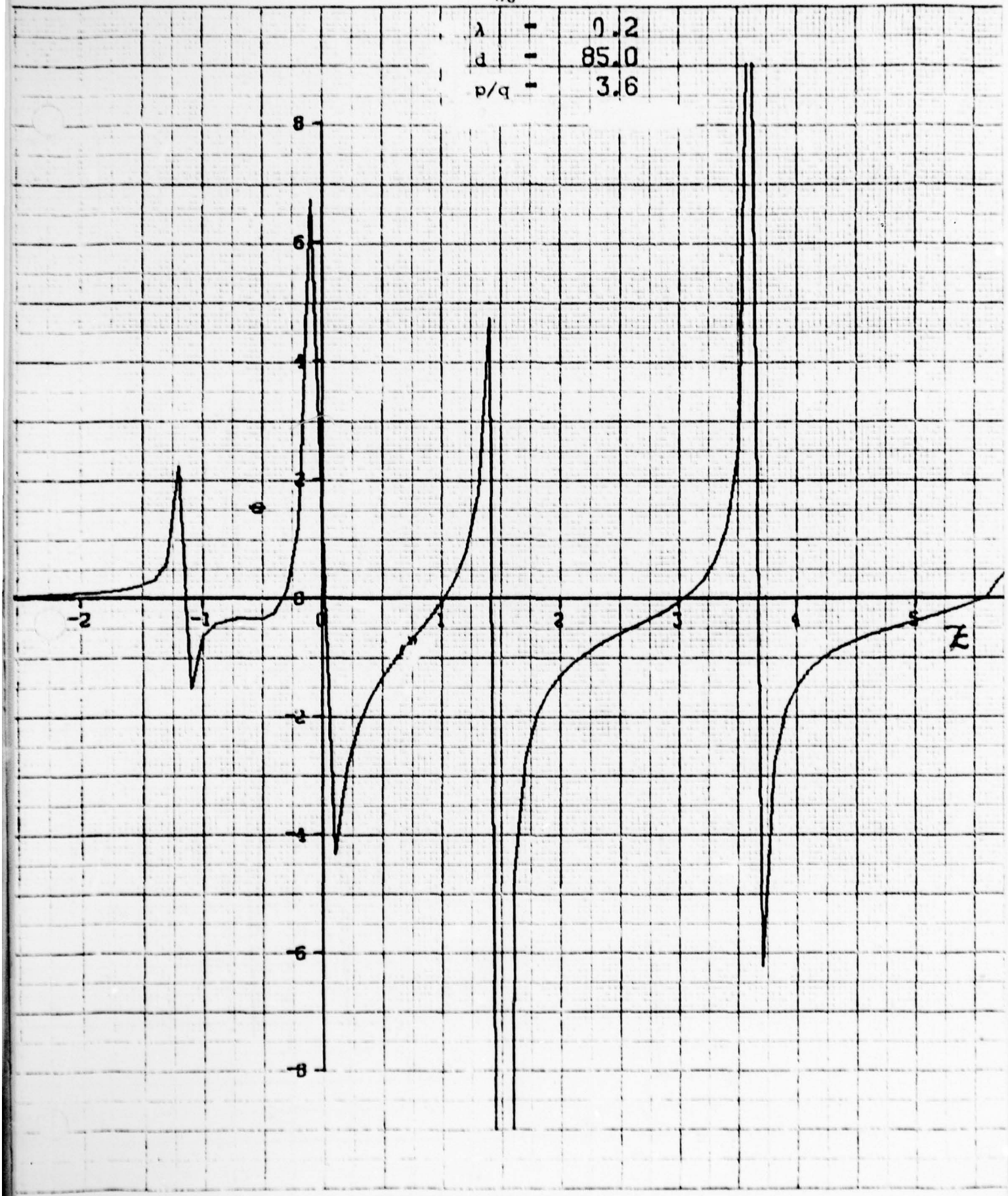


FIG. (4-9) PLOTS OF $\mathcal{G}(Z)$ AS A FUNCTION OF Z

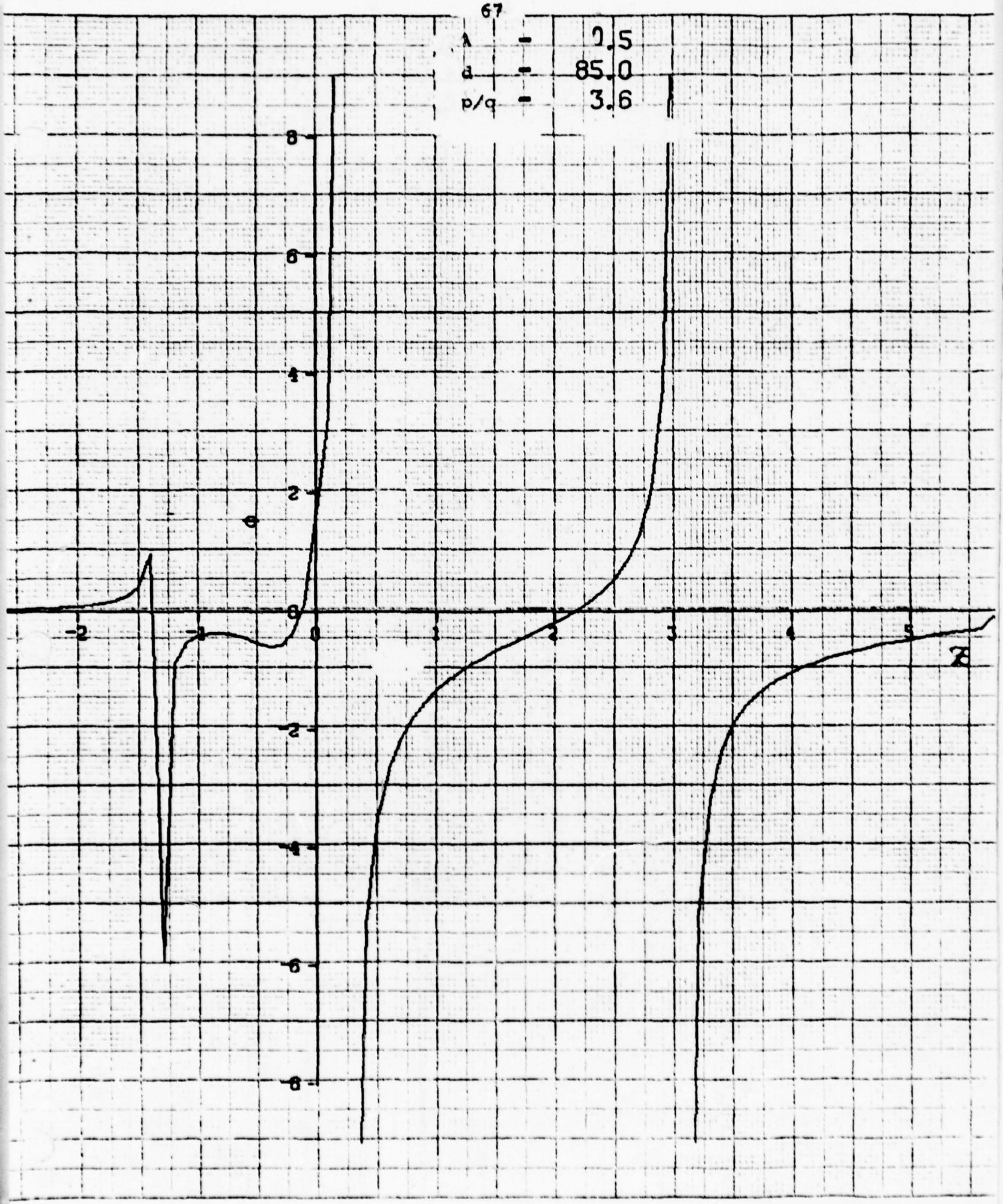


FIG. (4-10) PLOTS OF $f(Z)$ AS A FUNCTION OF Z

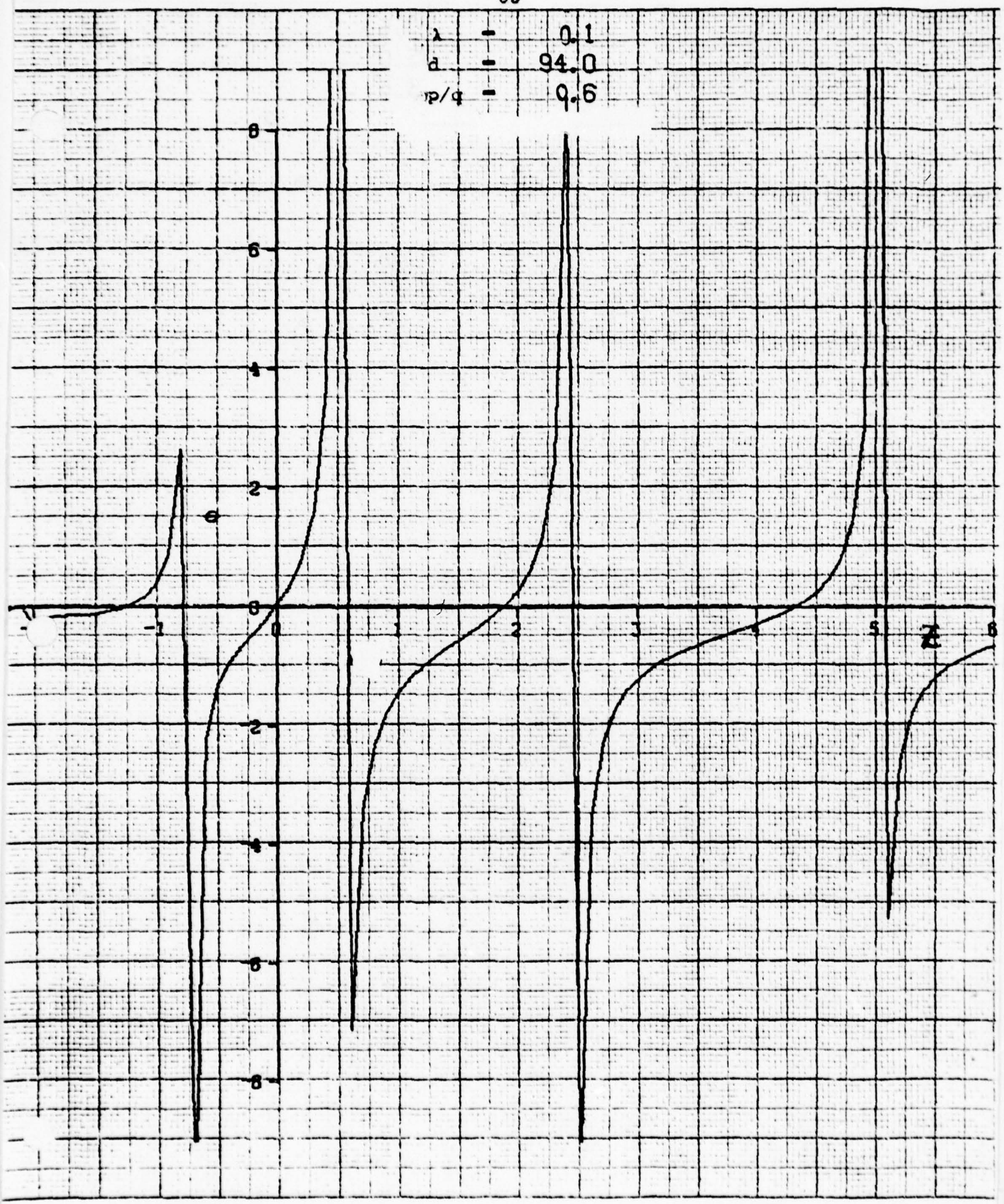


FIG. (4.11) PLOTS OF $\Phi(Z)$ AS A FUNCTION OF Z

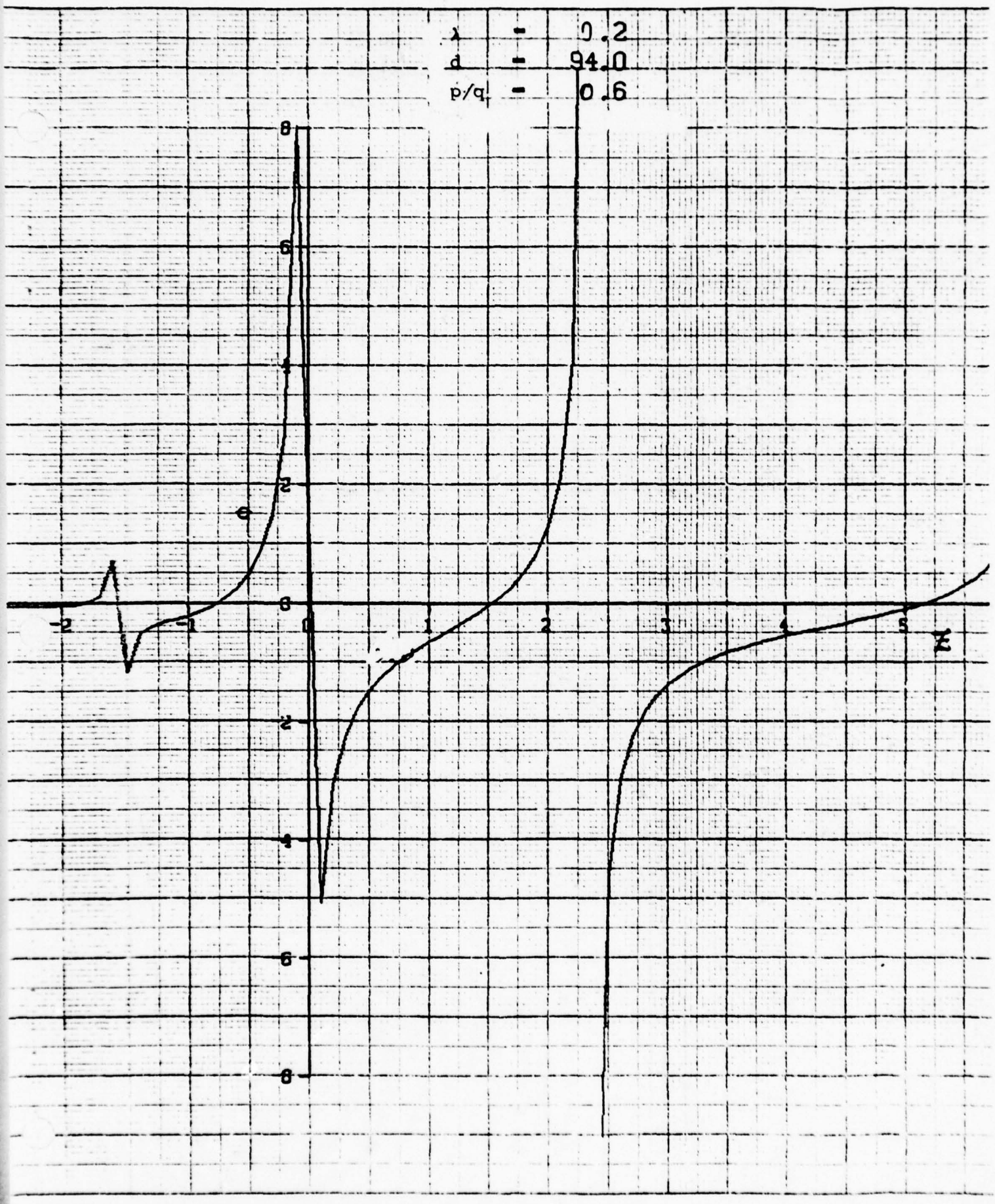


FIG. (4-12) PLOTS OF $\mathcal{G}(Z)$ AS A FUNCTION OF Z

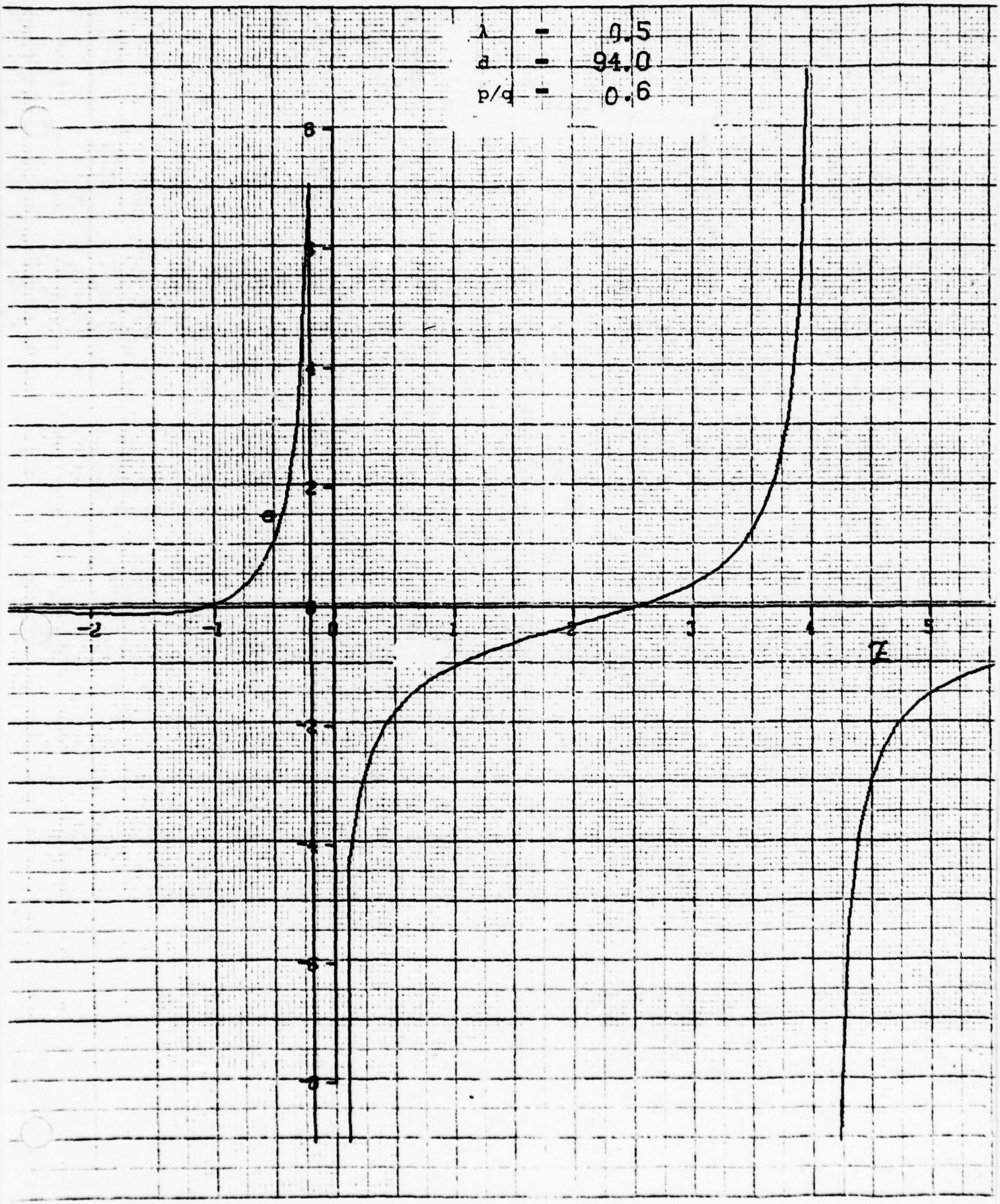


FIG. (4-13) PLOTS OF $\mathcal{J}(Z)$ AS A FUNCTION OF Z

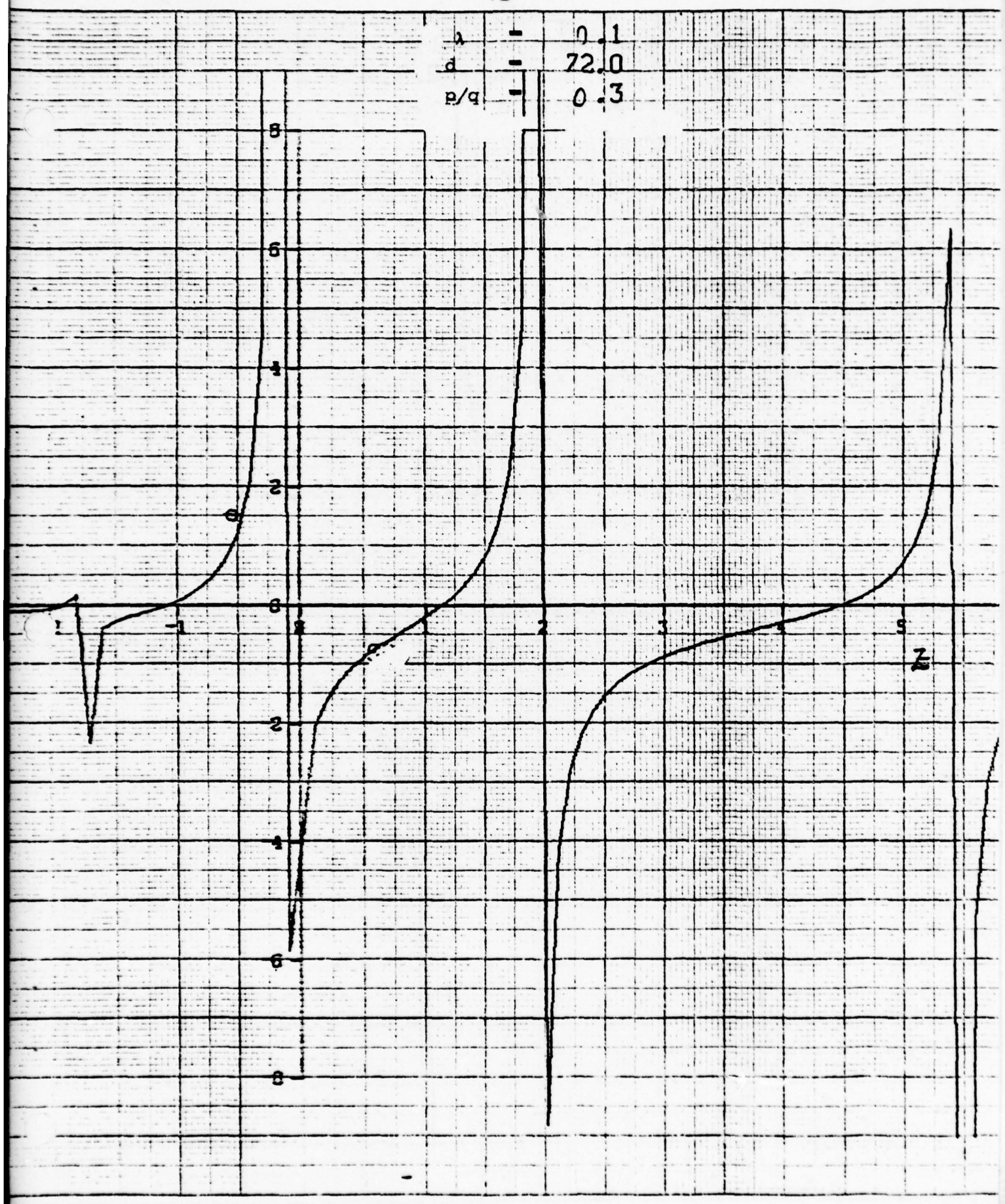


FIG. (4-14) PLOTS OF $\Phi(Z)$ AS A FUNCTION OF Z

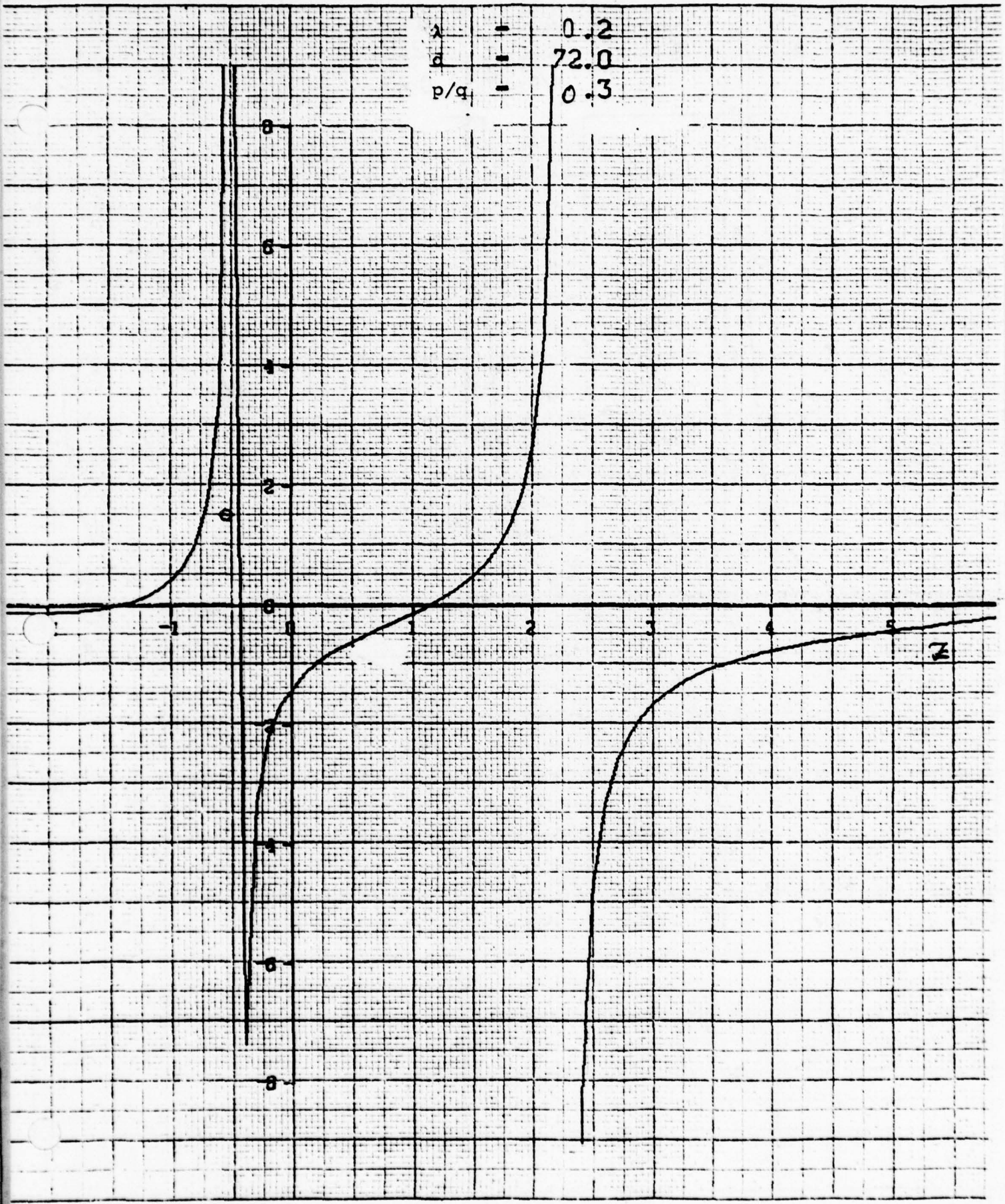


FIG. (4-15) PLOTS OF $\mathcal{G}(Z)$ AS A FUNCTION OF Z

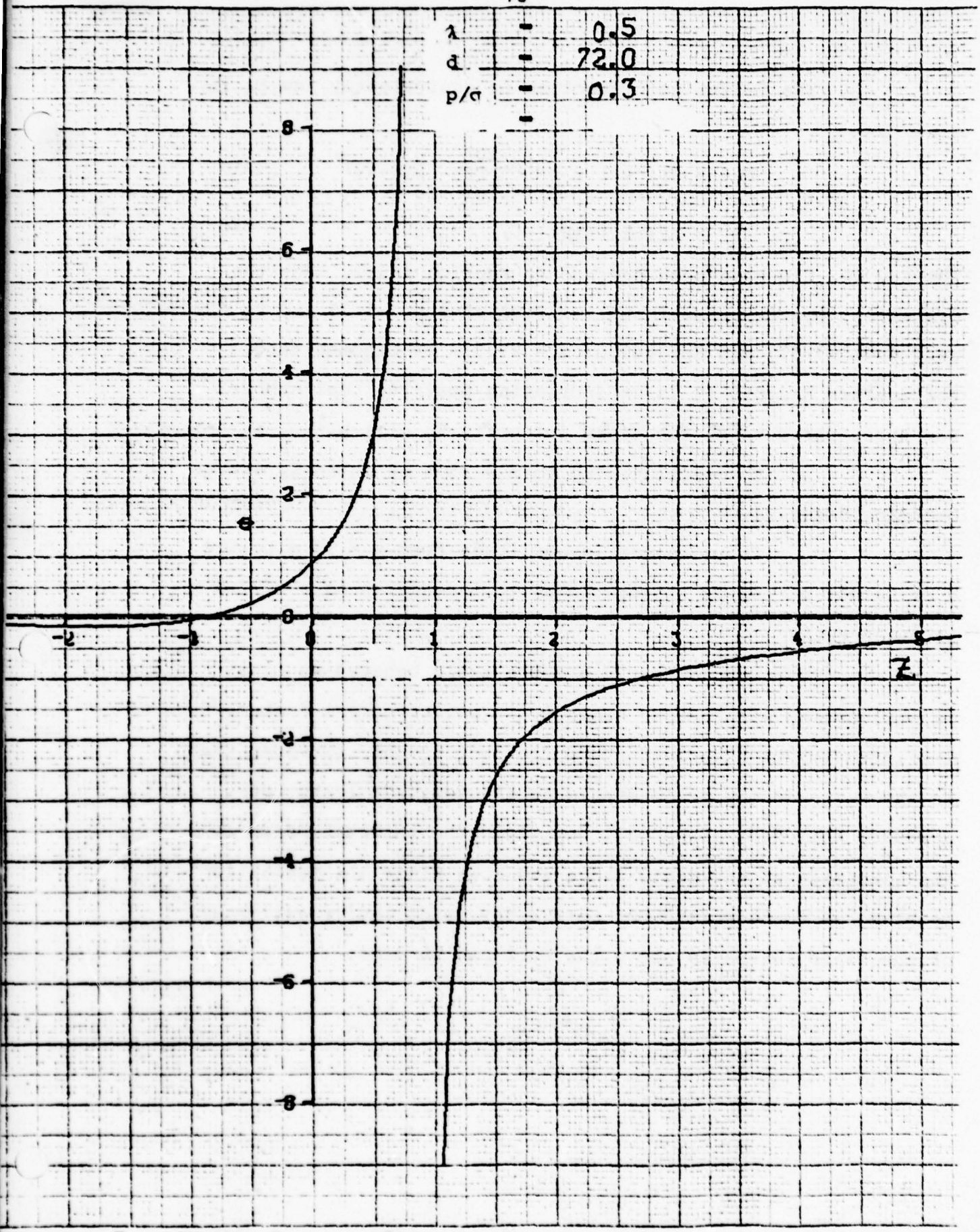


FIG. (4-16) PLOTS OF $\mathcal{S}(Z)$ AS A FUNCTION OF Z

The height gain function or equivalently the E_y is given by

$$E_{y<} = A_i(-\xi_<) - \frac{A_i(-\xi_{10})}{B_i(-\xi_{10})} B_i(-\xi_<) \qquad z<d$$

$$= A_i(-\xi_>) + jB_i(-\xi_>) \qquad z>d$$

The $\xi_<$, $\xi_>$, ξ_{10} have been defined in Section 4.1.1. For the ξ_{1d} value, the $\xi_<$ range in values from +5·9 to -1·25 and $\xi_>$ approaches zero from -3·57; and for height z = 77.8 m it becomes zero. For greater heights it is an increasing positive real number. For these parameters, the field intensity E_y will behave as follows:

a. E_y oscillates up to z = 35 m at which $\xi_<$ starts becoming negative.

b. It increases in amplitude from z = 35 m to the duct height z = 42 m. It continues to inrease up to z = 77.8 m at which $\xi_> \to 0$

c. Beyond z = 77.8 m the field behaves as if it is attenuating with height since $\xi_>$ is positive real number. The intensity of the field above the duct height is higher than in the duct and may be construed as the dominant leaky mode. However, this fact alone will not yield the information as to the intensity of the field above the duct since the excitation problem has to be worked out before any quantitative estimates of the field intensity above the duct can be made.

4.3.2 The Leaky Modes

Up until now we sought solutions of the transcendental equation which were positive or negative real numbers. The determination of the solutions of the transcendental equation for complex values is much more involved and time consuming, and the resources of this contract do not permit detailed analysis. Nonetheless, some already developed approximate expressions are used to determine the complex eigenvalues or solutions characterizing leaky modes.

The expression which is used to determine the leaky modes is given by (2)

$$A_m = \{\xi_m + g + \frac{1}{12} Sg^4 - \frac{1}{90} S\xi_m g^6 + \frac{(13S^2-5S)}{1260} g^7 + \frac{1}{1260} S\xi_m^2 g^8 + \ldots\ldots\}$$

Here, $g=\frac{d}{H}$, $H=(k_o^2 q)^{-1/3}$, $S=1-s^3$, $s^3=\frac{p}{q}$ and ξ_m are the zeros of the Hankel function of second kind and one third order and are given by

$$\xi_1 = -1.169+j2.025$$
$$2 \quad -2.044+j3.54$$
$$3 \quad -2.76+j4.78$$

The expression for the eigenvalue $A_m=\alpha_m+jB_m$ is valid for small values of $g(g\leq 1)$. That is, the duct height is comparable to the scale height H and for these values of g the eigenvalues approach the zeroes of the Hankel function of the second kind and one third order.

Recognizing that for λ=100 cm the field will not be trapped by any of the ducts; λ=100 cm was chosen for calculations. Thus for λ=100 cm and for two days of duct profile, the complex eigenvalues are presented in the table as follows:

Table (4-2) Eigenvalues for Leaky Modes

λ=100 cm	Jan. $g = 0.545$	Dec. $g = 0.6646$
A_1	-0.574+j2.02	-0.48+j2.021
2	-1.448+j3.53	-1.354+j3.53
3	-2.163+j4.78	-2.07+j4.77

Use of the same equation for λ=50 cm leads to results which do not appear to be valid. For λ=50 cm, g = 1.378 for Jan. 3rd duct and g=1.476 for Dec. 4th duct and thus the expression for g > 1 does not seem to hold.

4.4 The Height Gain Functions for the Trapped Modes

As discussed before, the variation of the field with height, also termed the height gain function, will reveal whether or not there exists field outside the duct. For this reason, height gain functions for individual modes have been computed and they are shown in Figs. (4-17 through 4-53).

The height gain functions are the plots of modal E field i.e., they are the plots of the following expressions:

$$E_{y<} = A_i(-\xi_<) - \frac{A_i(-\xi_{10})}{B_i(-\xi_{10})} \cdot B_i(-\xi_<) \qquad z<d$$

$$E_{y>} = A_i(-\xi_>) + jB_i(-\xi_>) \qquad z>d$$

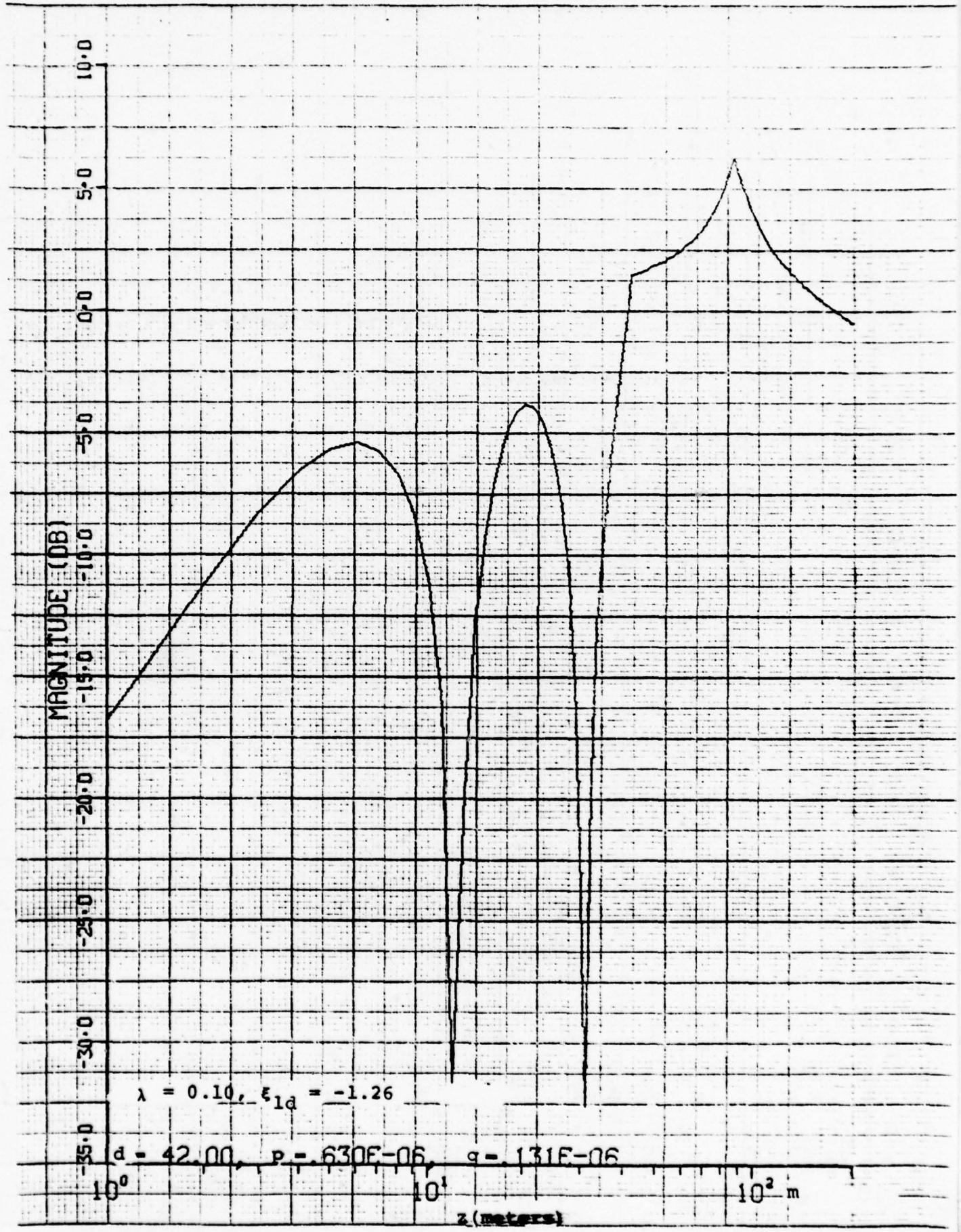


Fig. (4-17) Height-Gain Functions - January

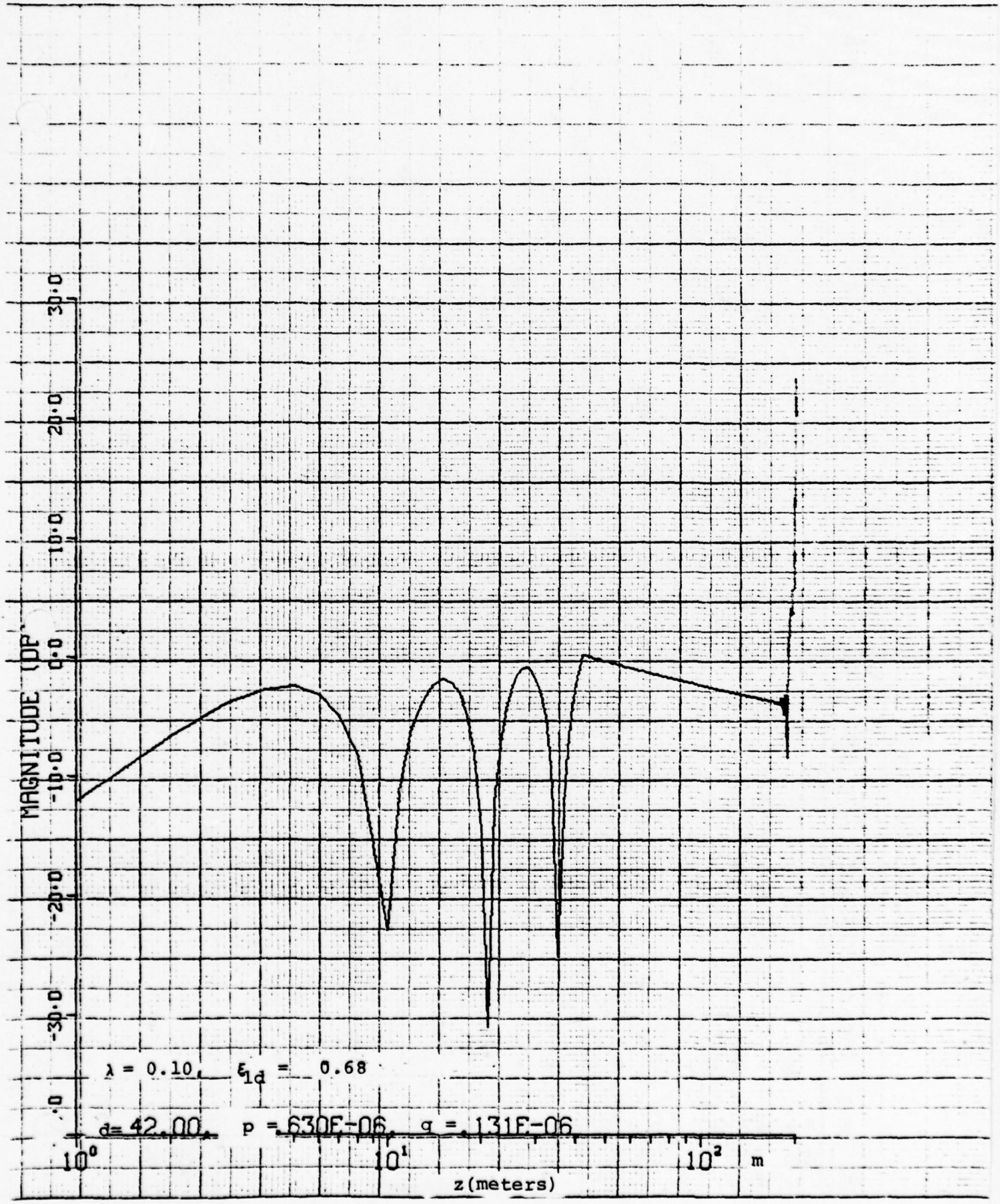


Fig. (4-18) Height-Gain Functions - January

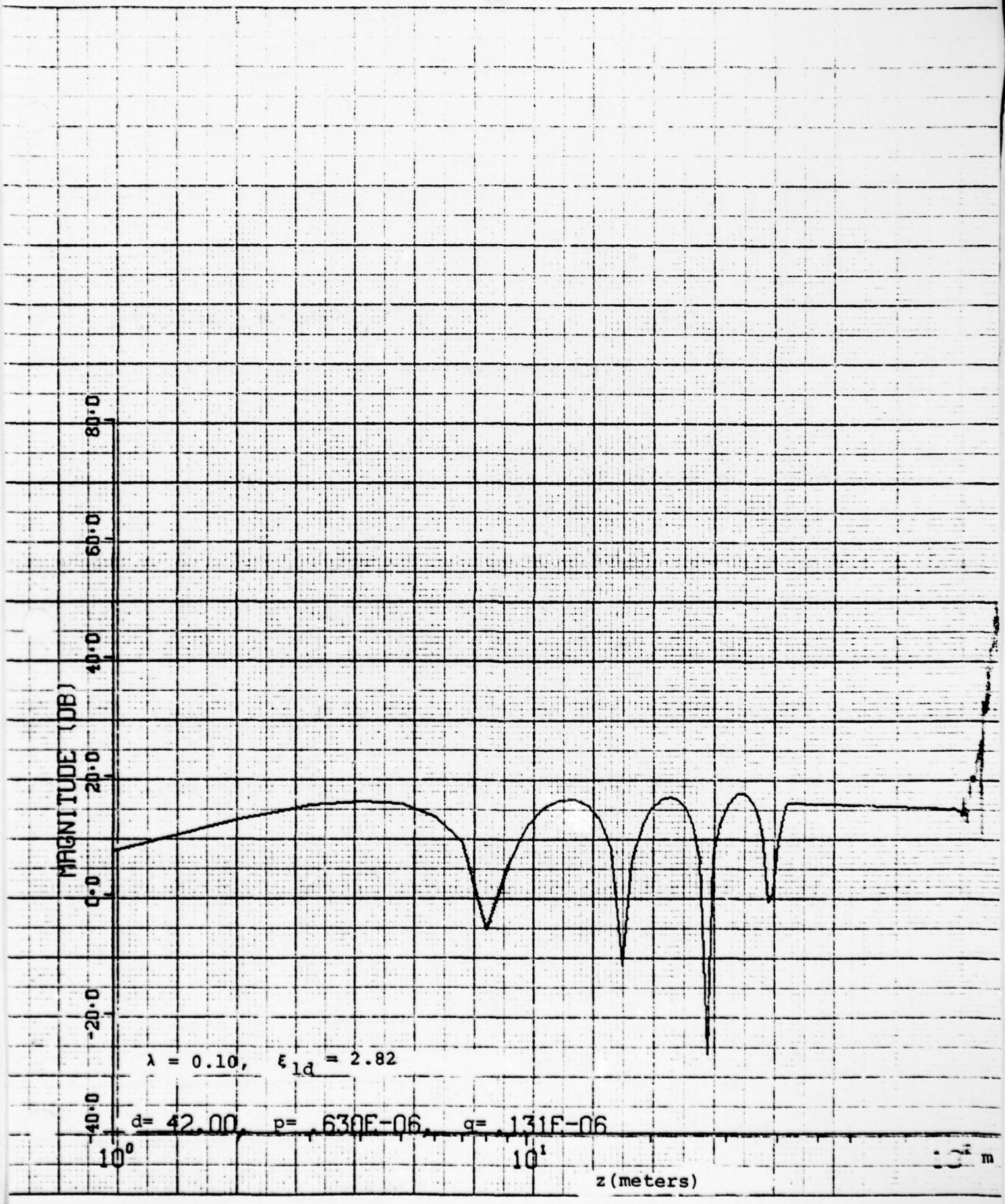


Fig. (4-19) Height-Gain Functions - January

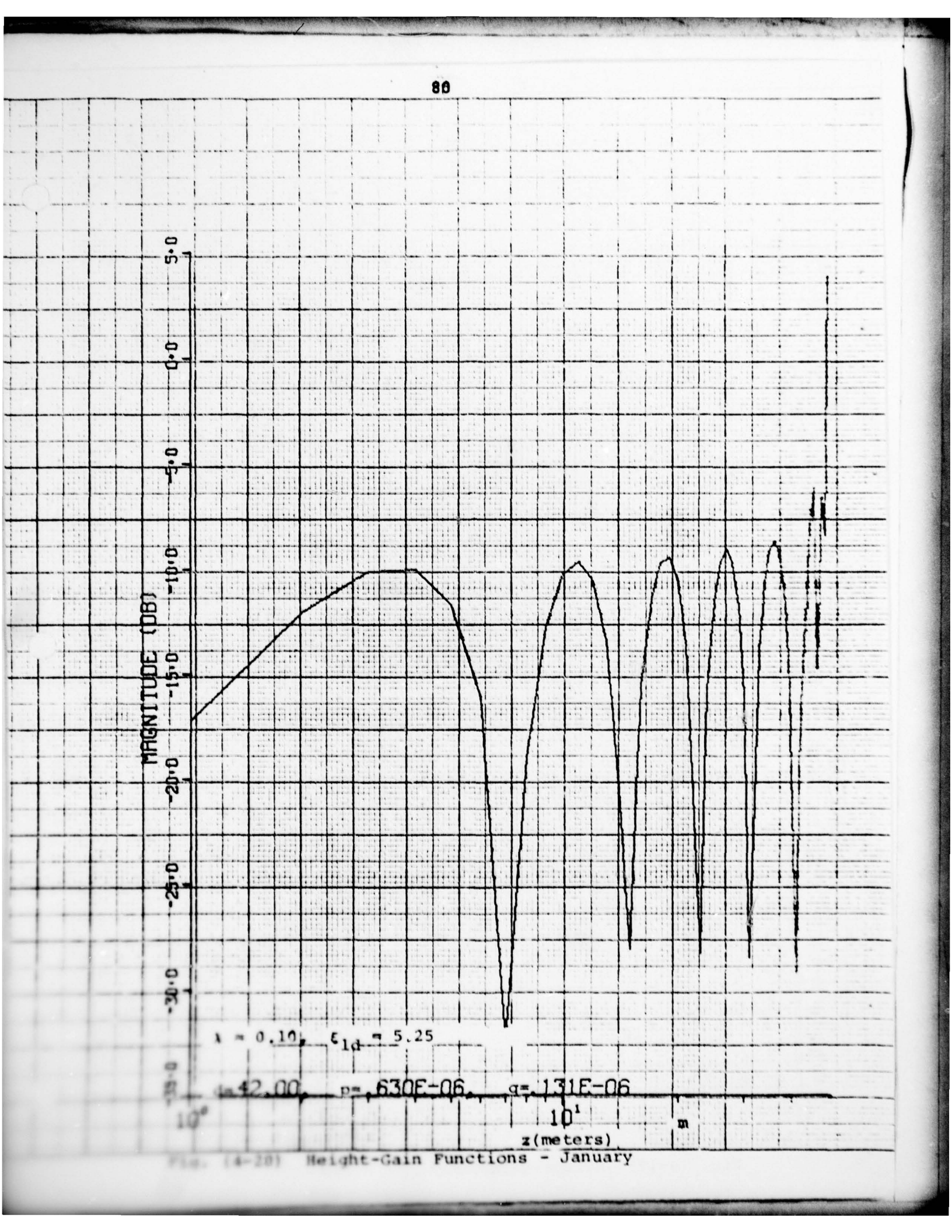


Fig. (4-20) Height-Gain Functions - January

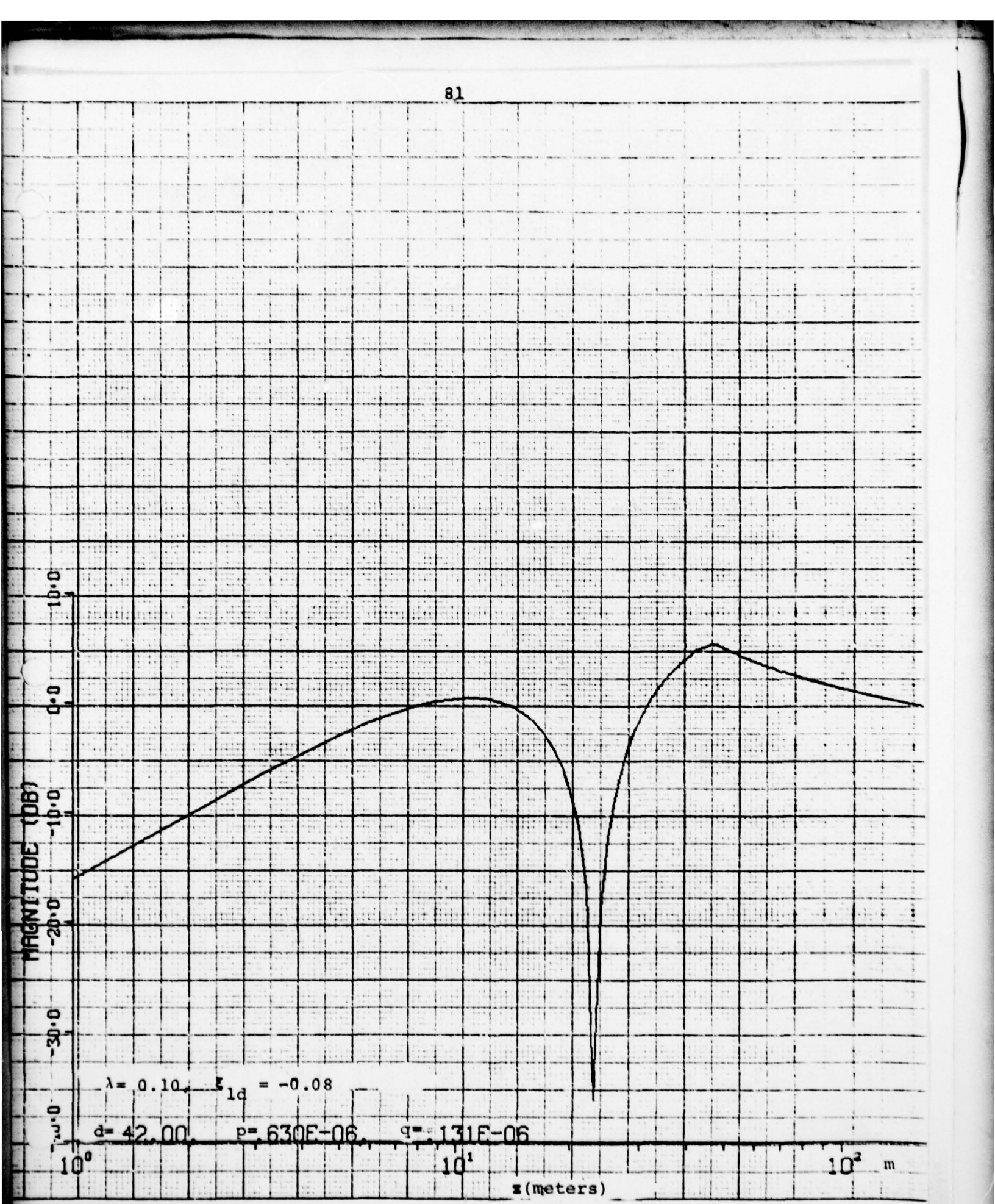


Fig. (4-21) Height-Gain Functions - January

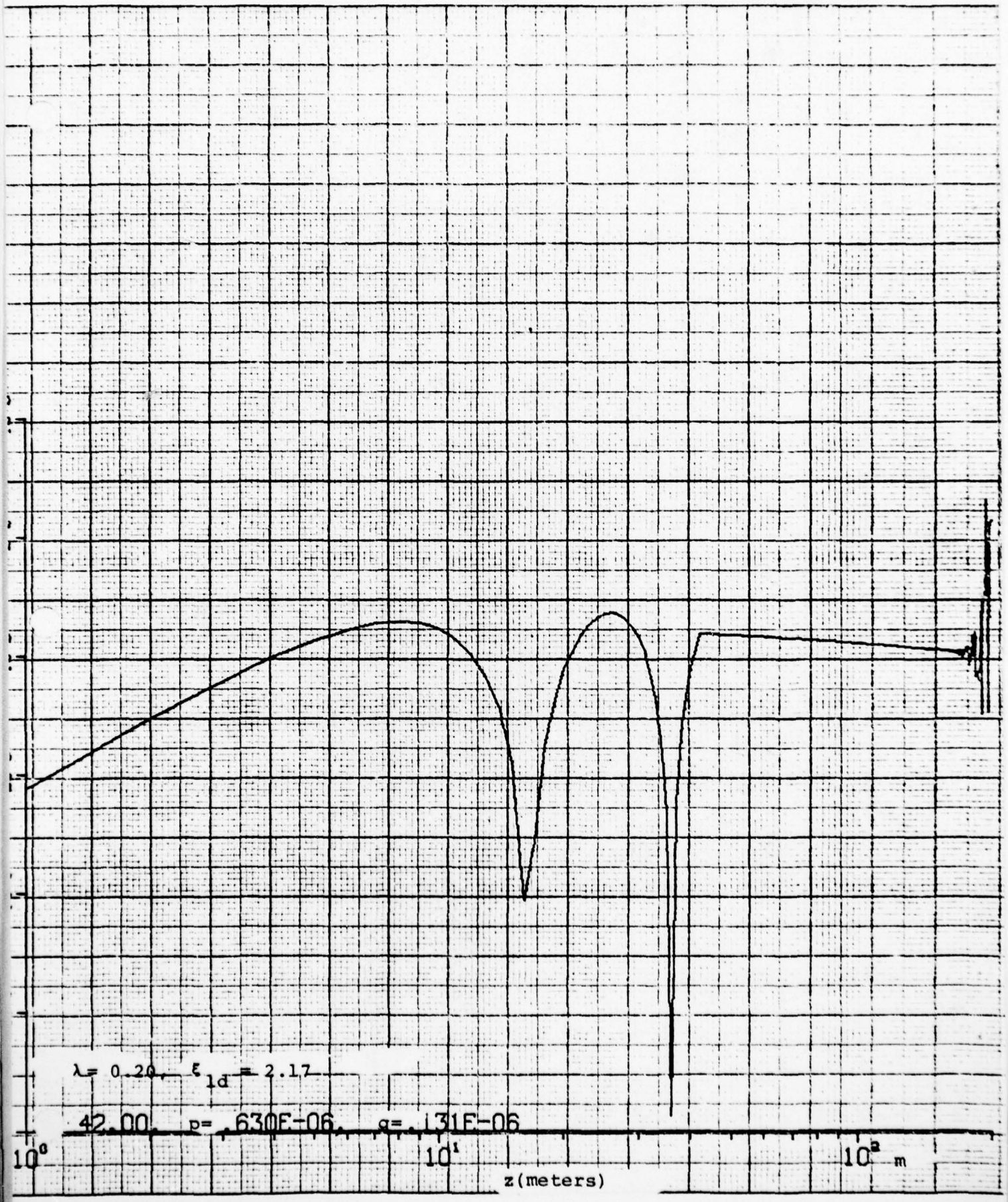


Fig. (4-22) Height-Gain Functions - January

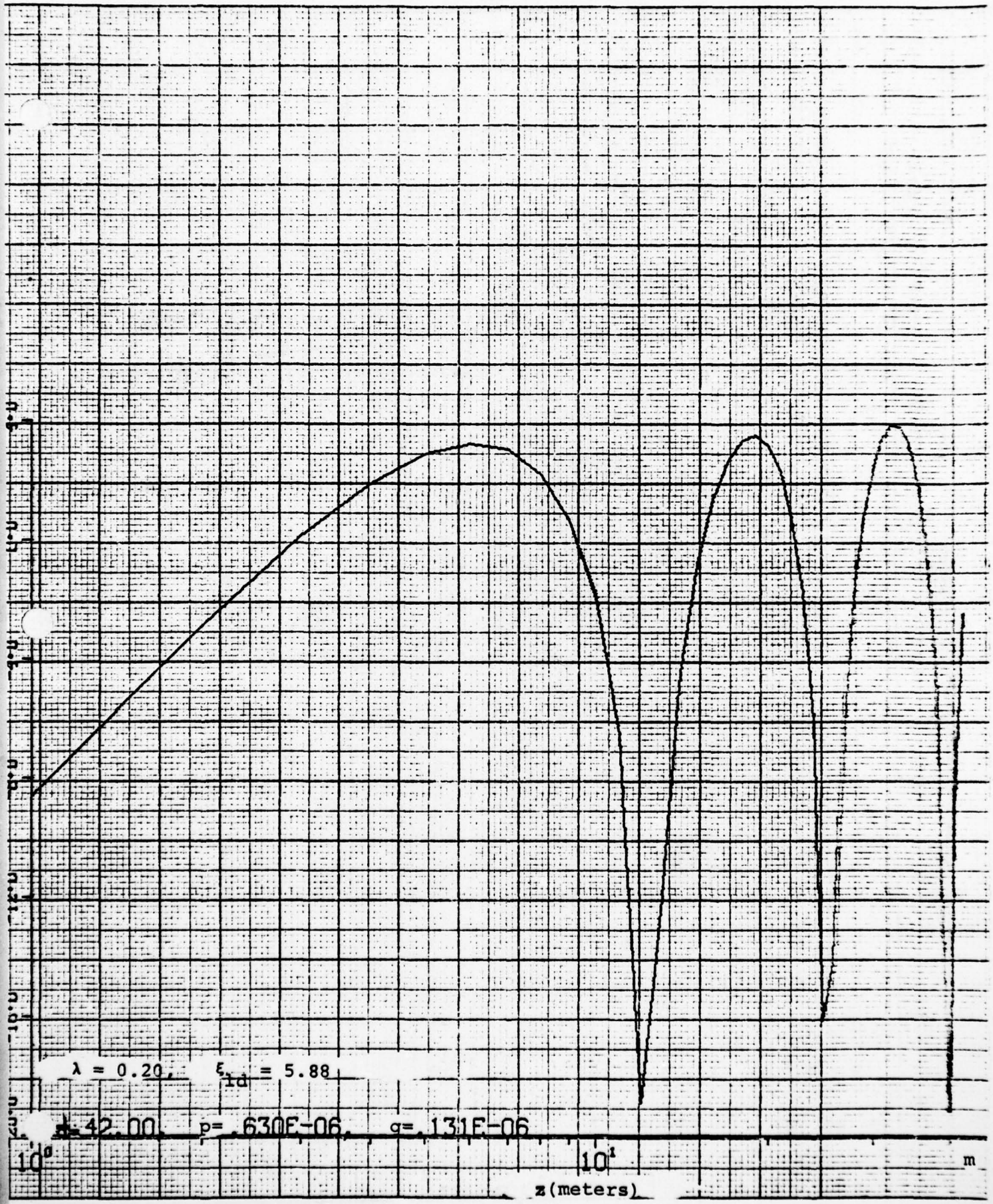


Fig. (4-23) Height-Gain Functions - January

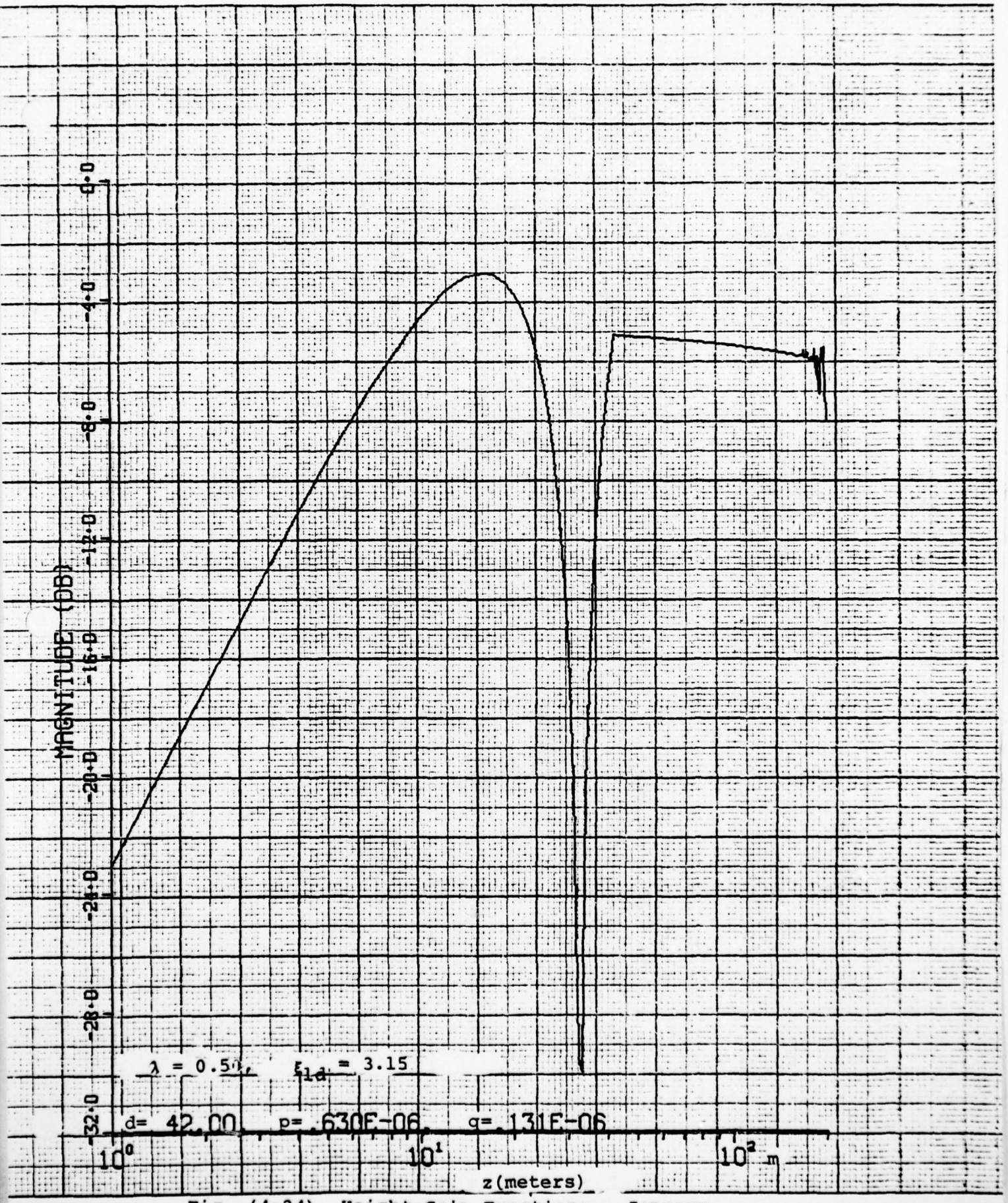


Fig. (4-24) Height-Gain Functions - January

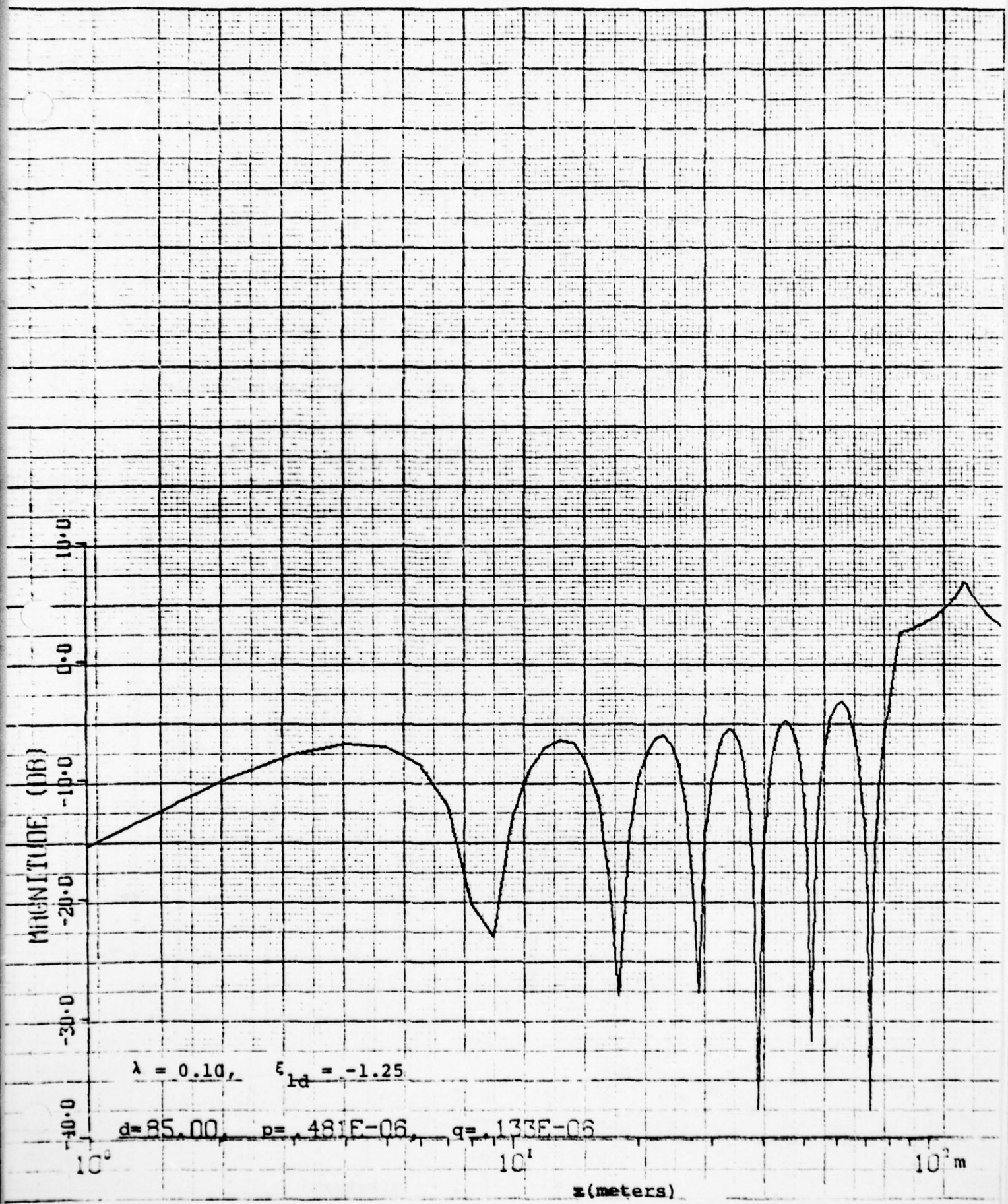


Fig. (4-25) Height-Gain Functions - April

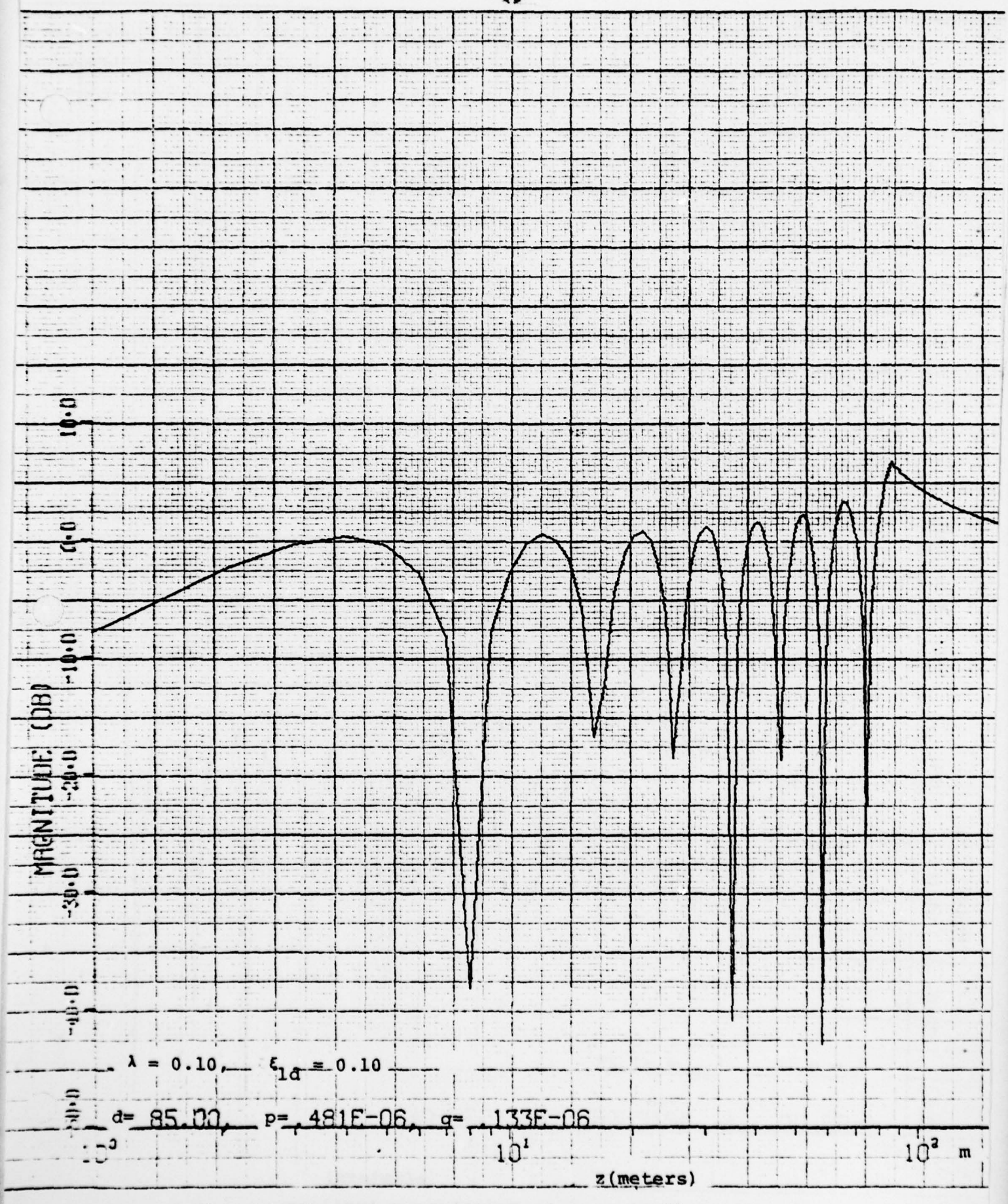


Fig. (4-26) Height-Gain Functions - April

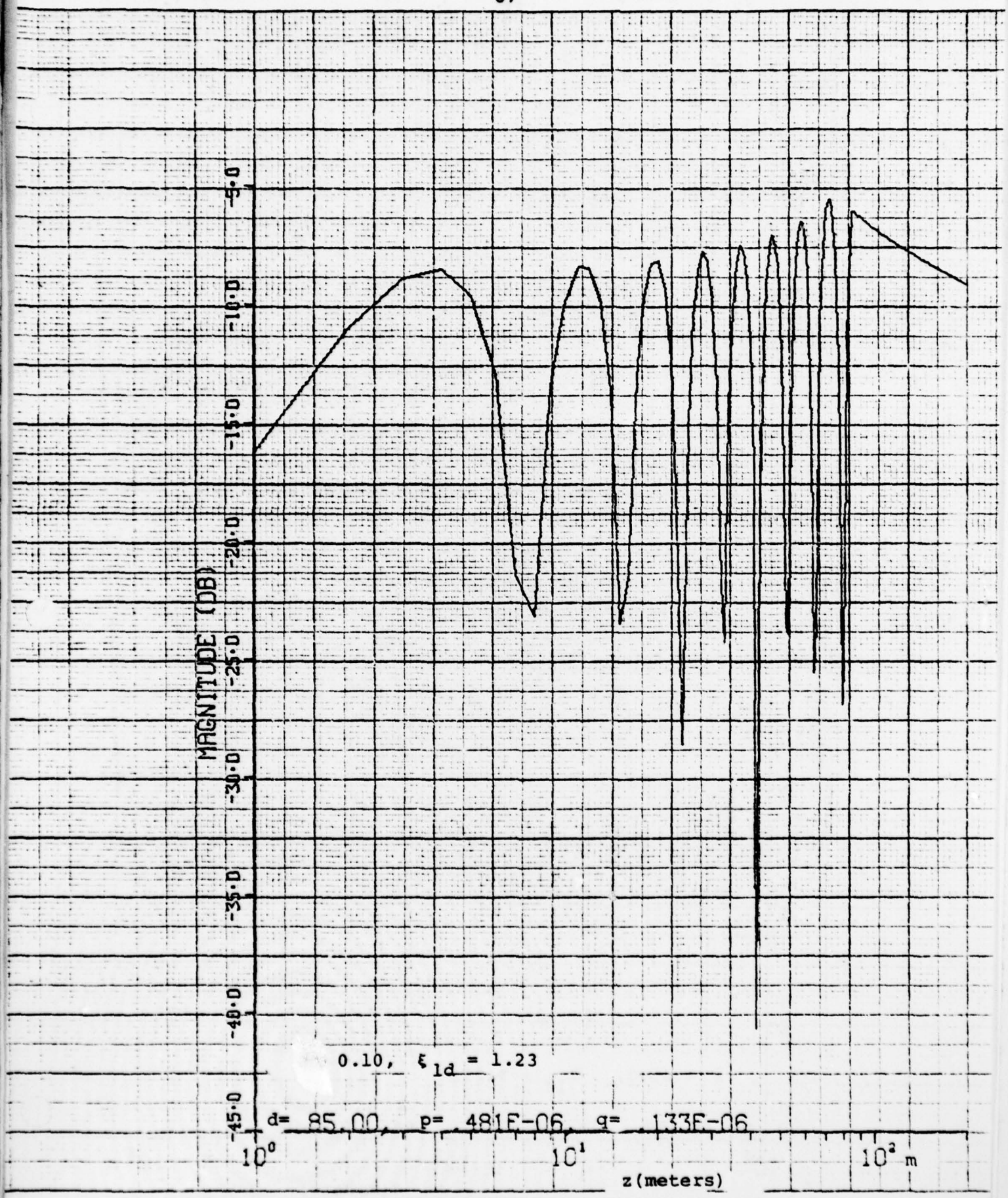


Fig. (4-27) Height-Gain Functions - April

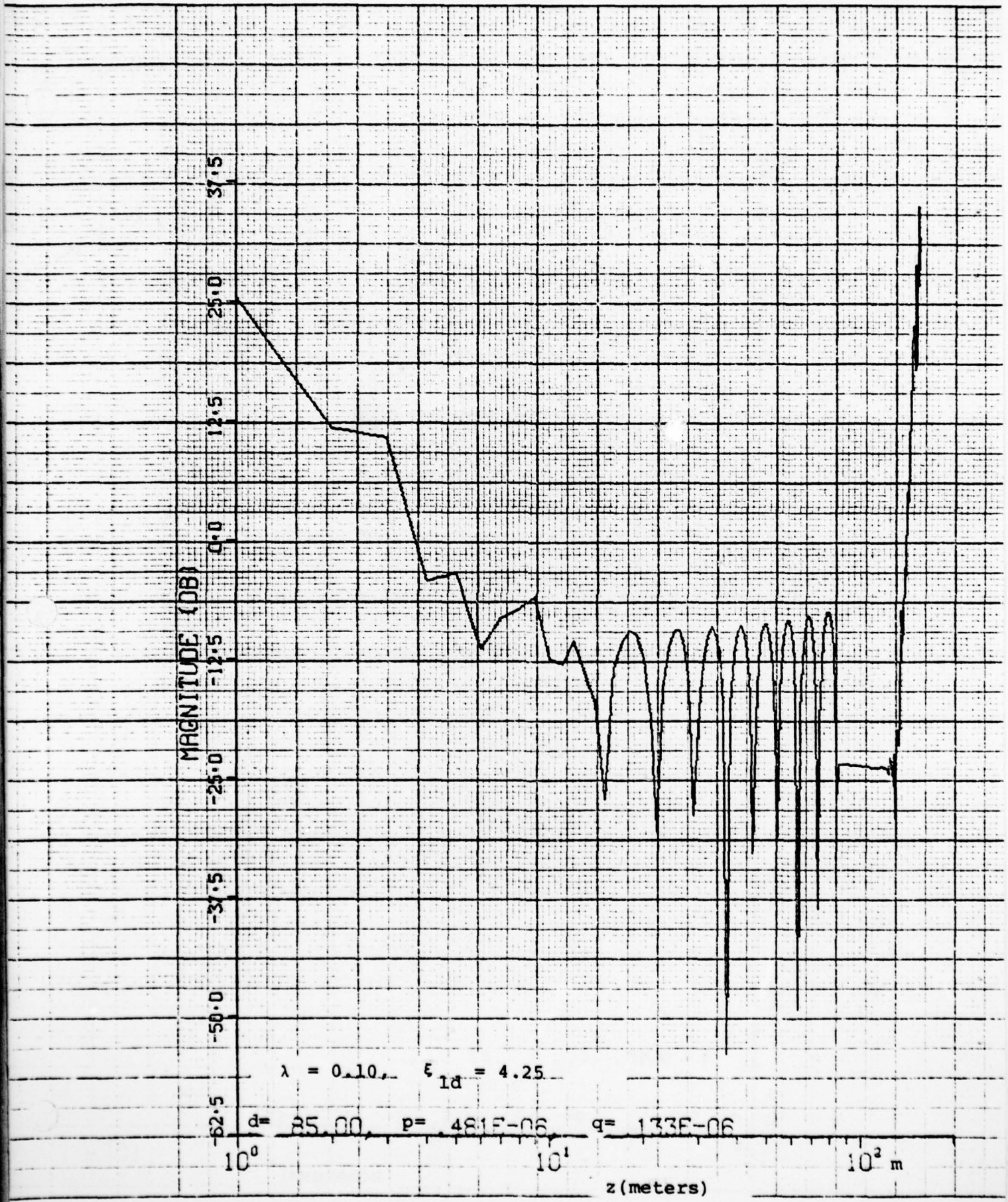


Fig. (4-28) Height-Gain Functions - April

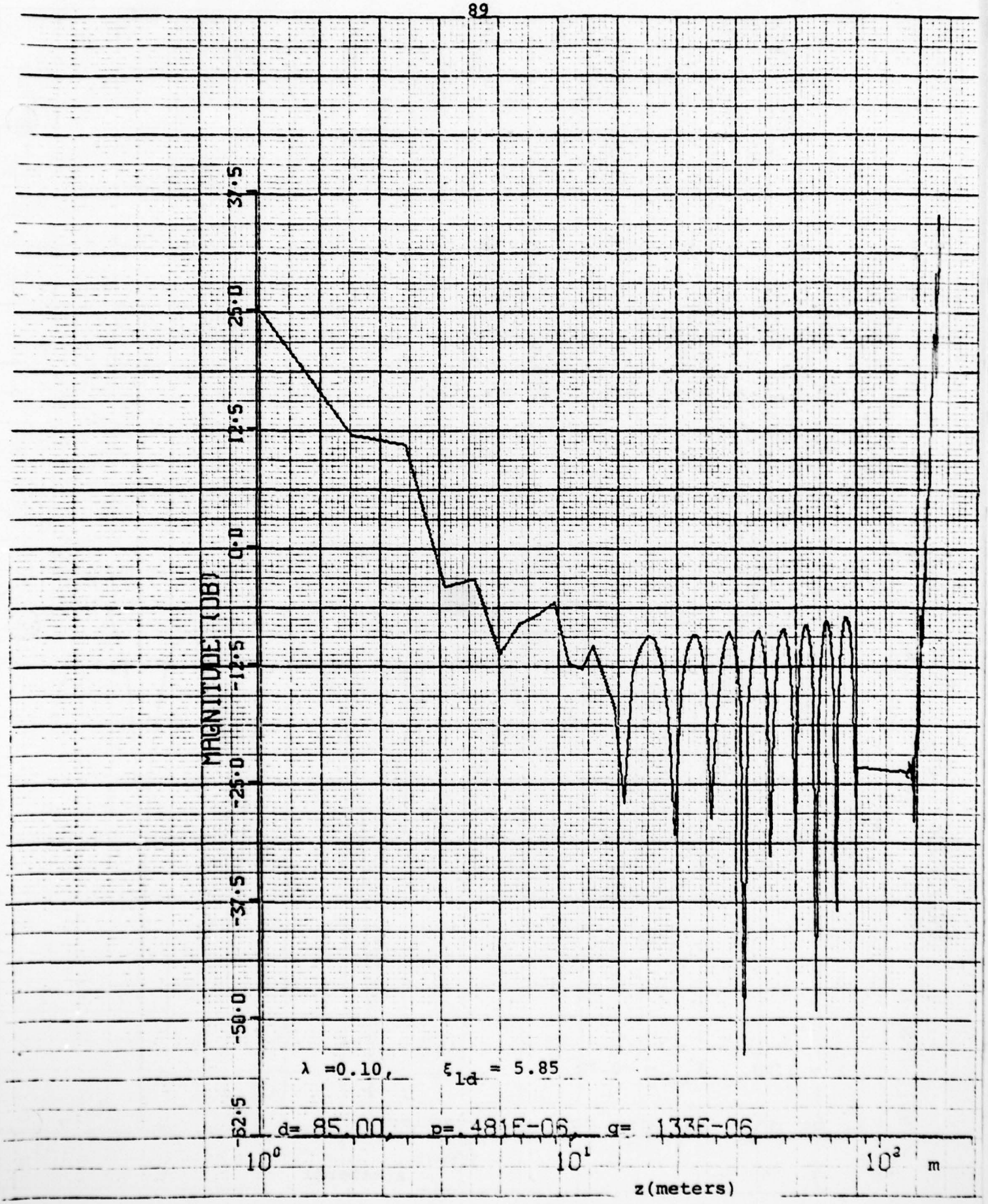


Fig. (4-29) Height-Gain Functions - April

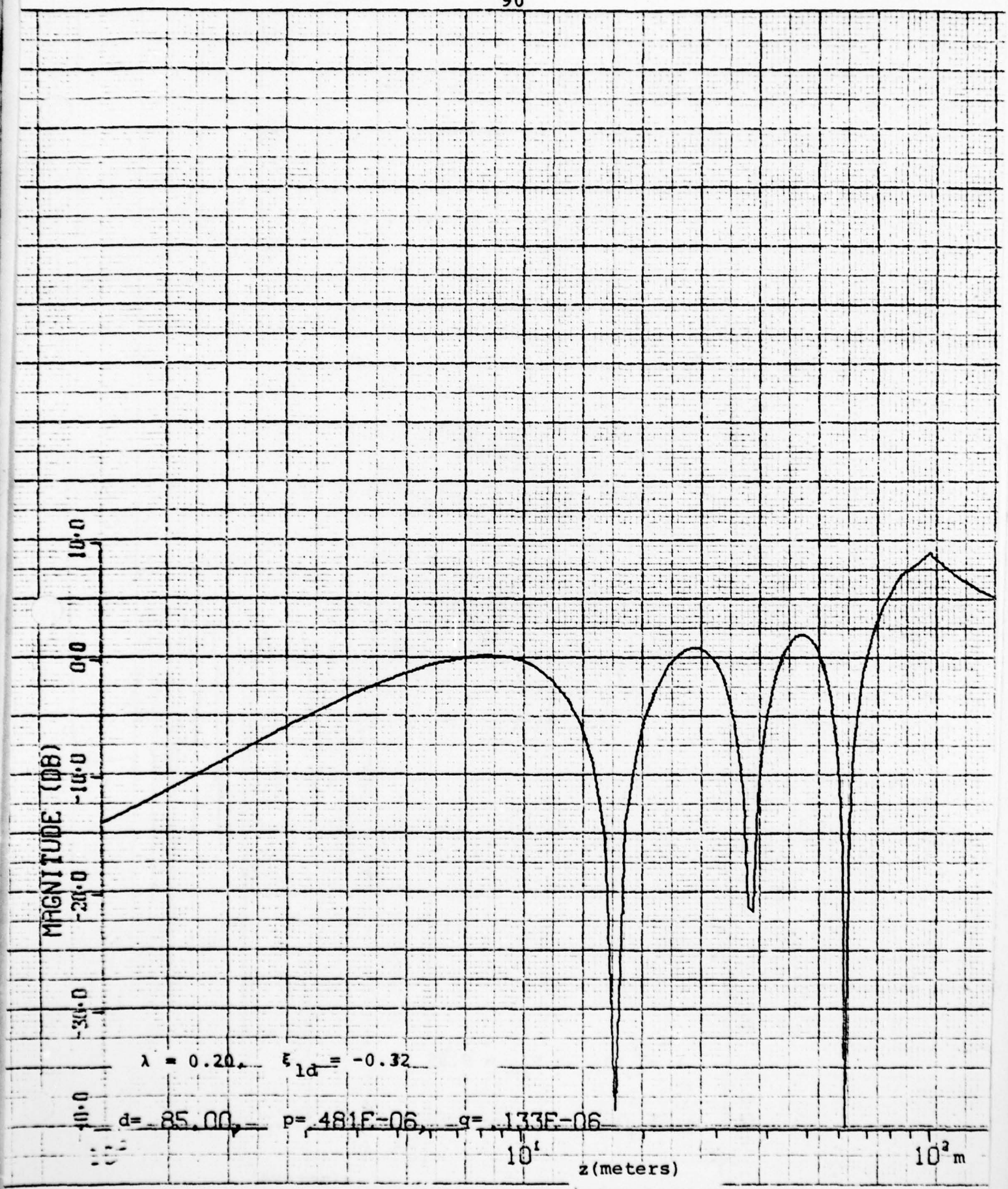


Fig. (4-30) Height-Gain Functions - April

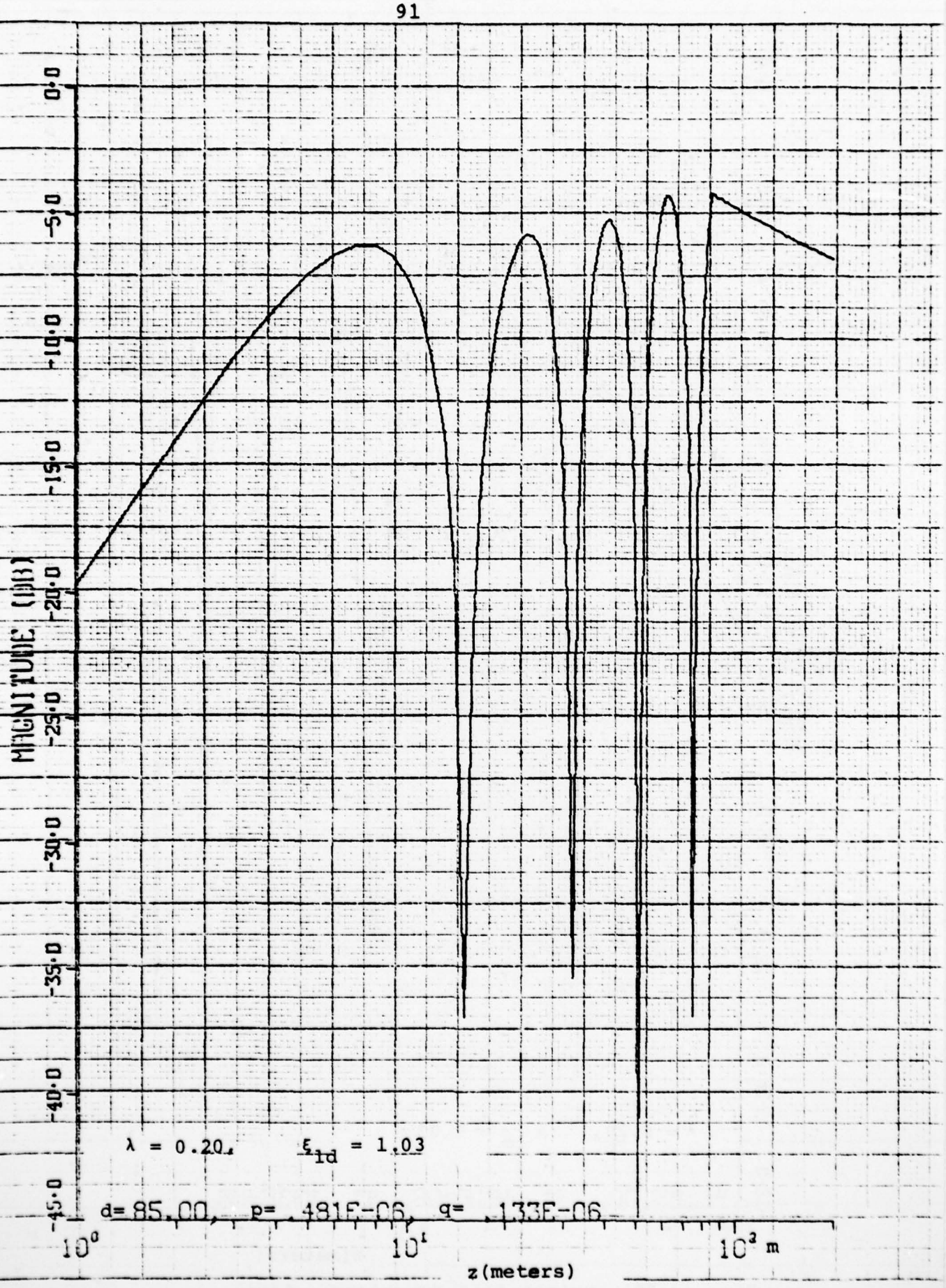


Fig. (4-31) Height-Gain Functions - April

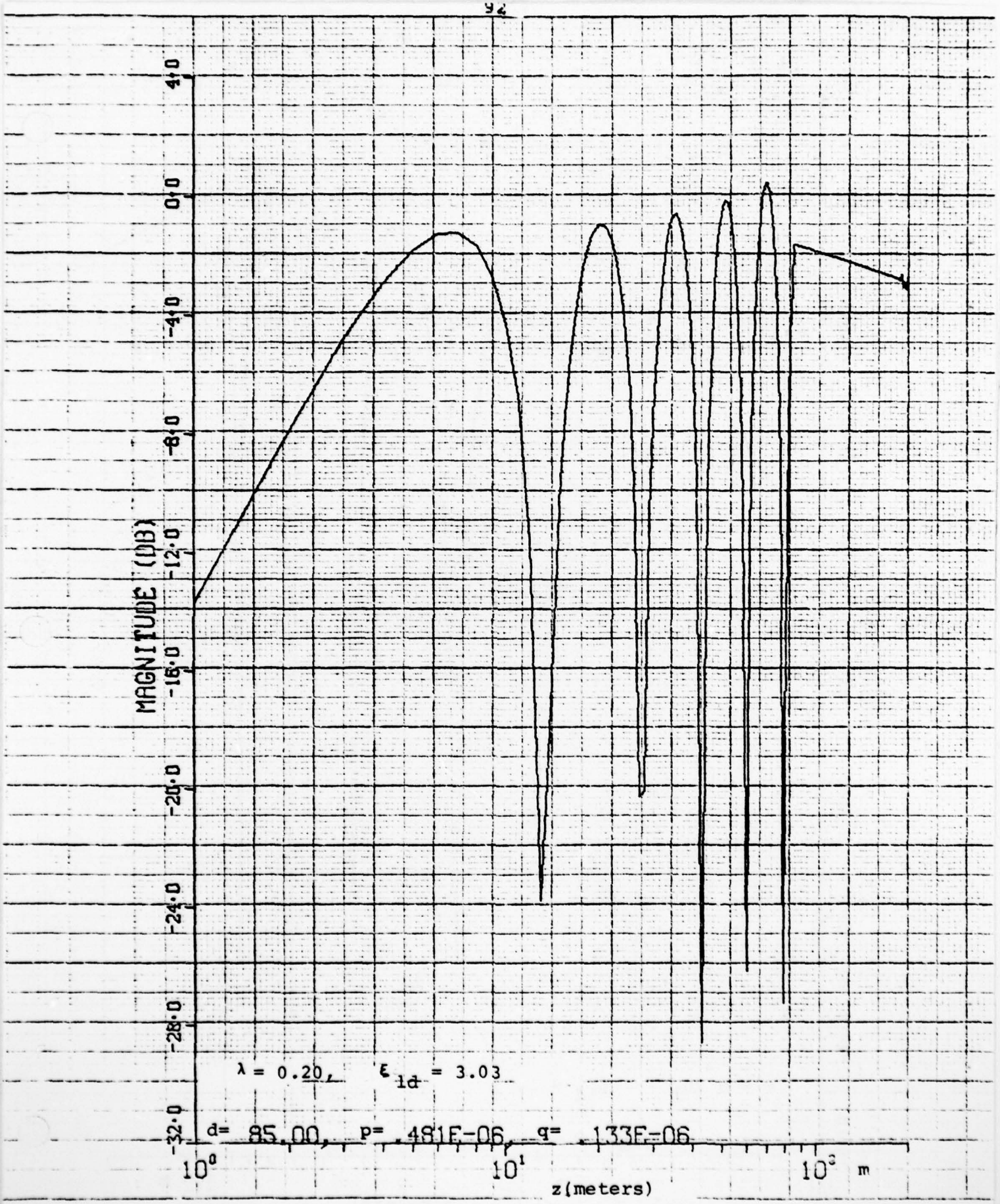


Fig. (4-32) Height-Gain Functions - April

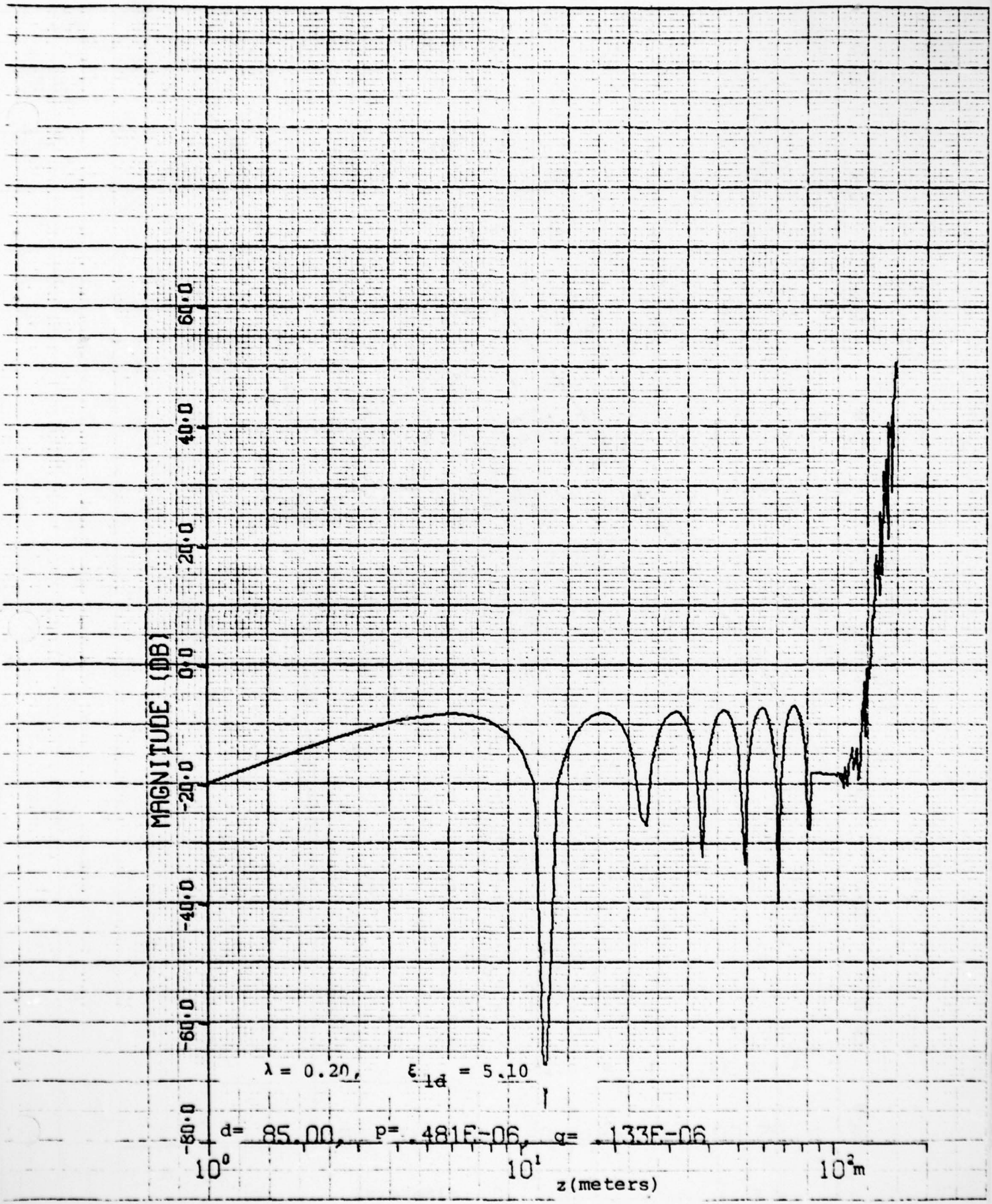


Fig. (4-33) Height-Gain Functions - April

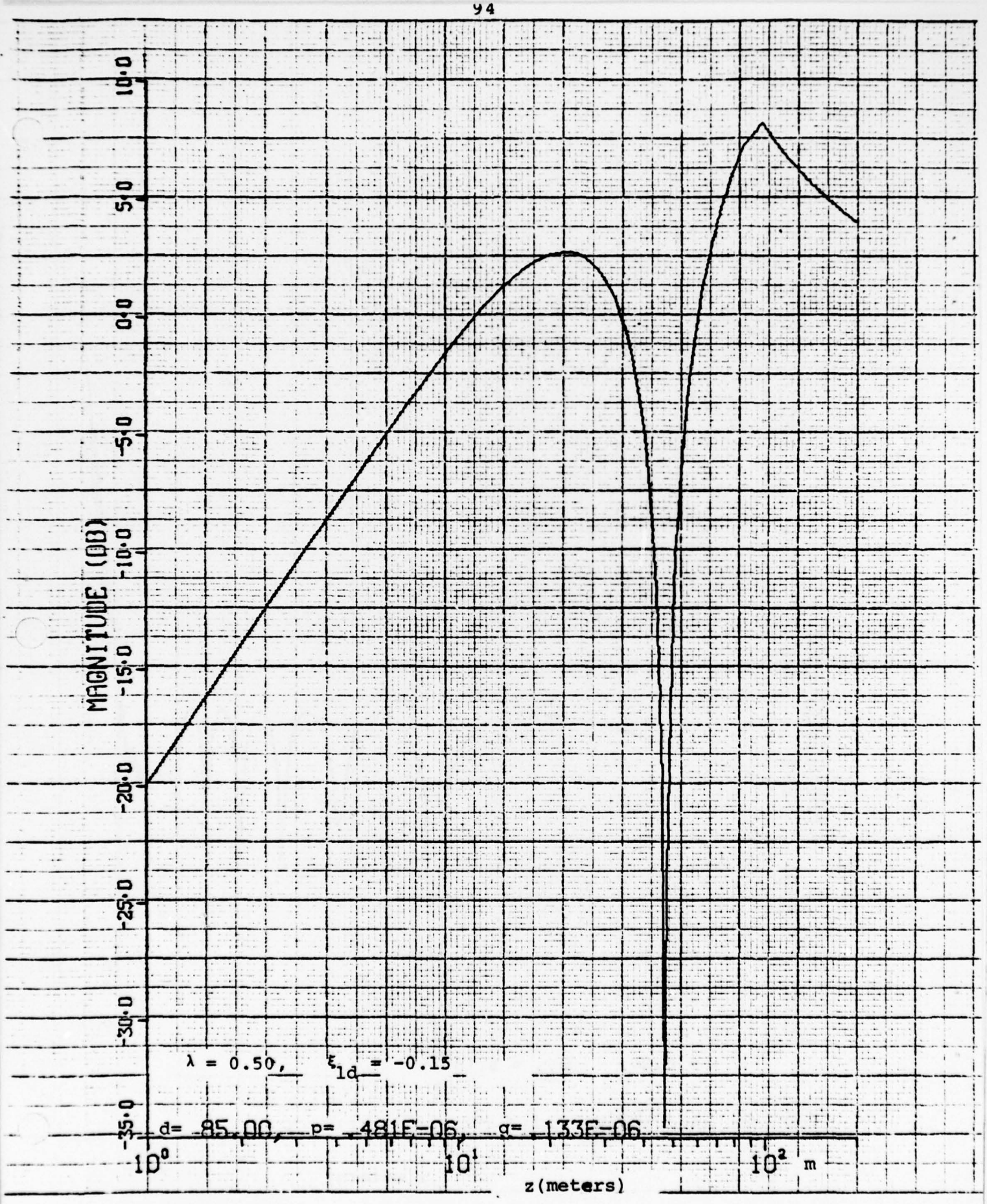


Fig. (4-34) Height-Gain Functions - April

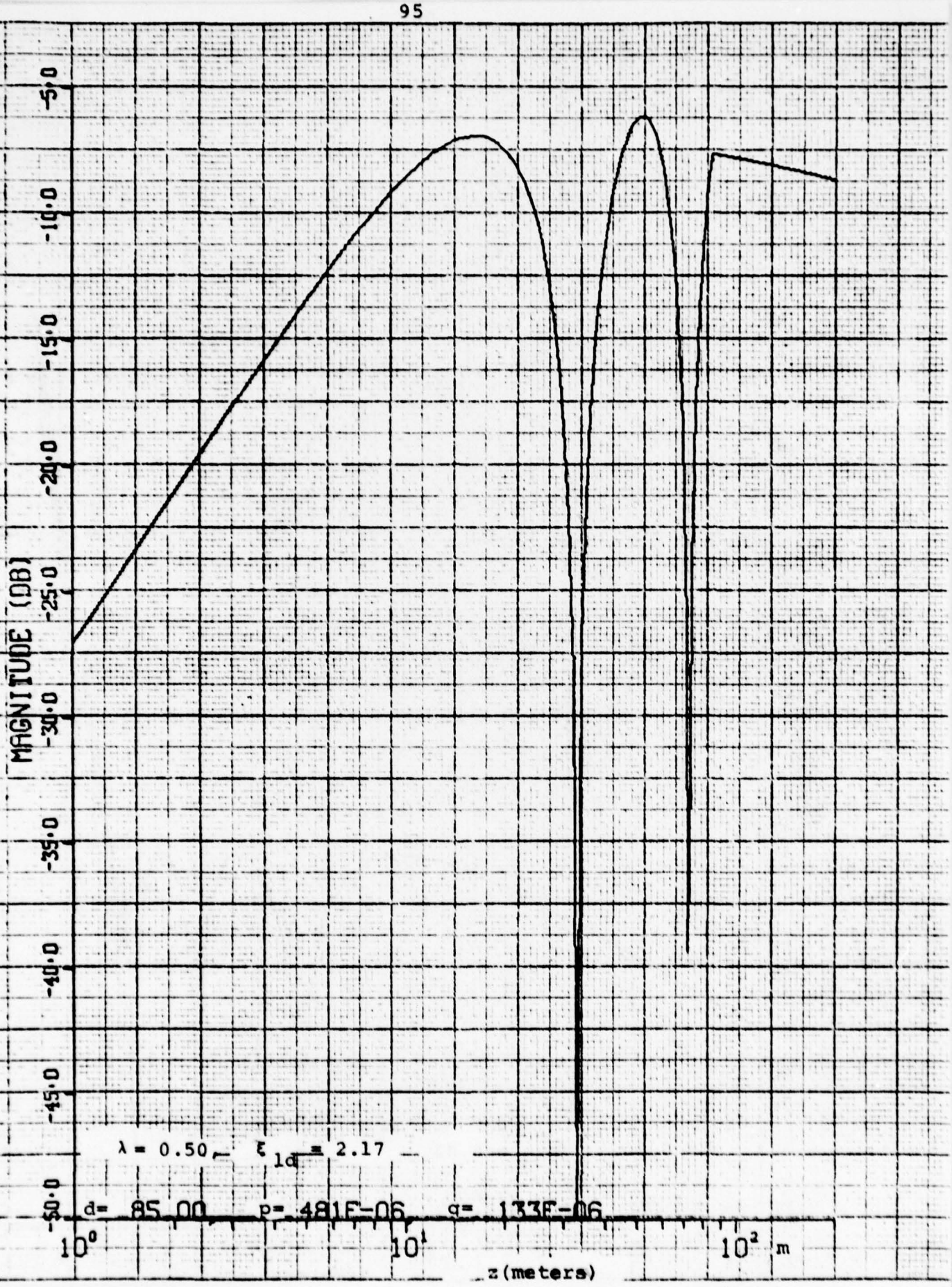


Fig. (4-35) Height-Gain Functions - April

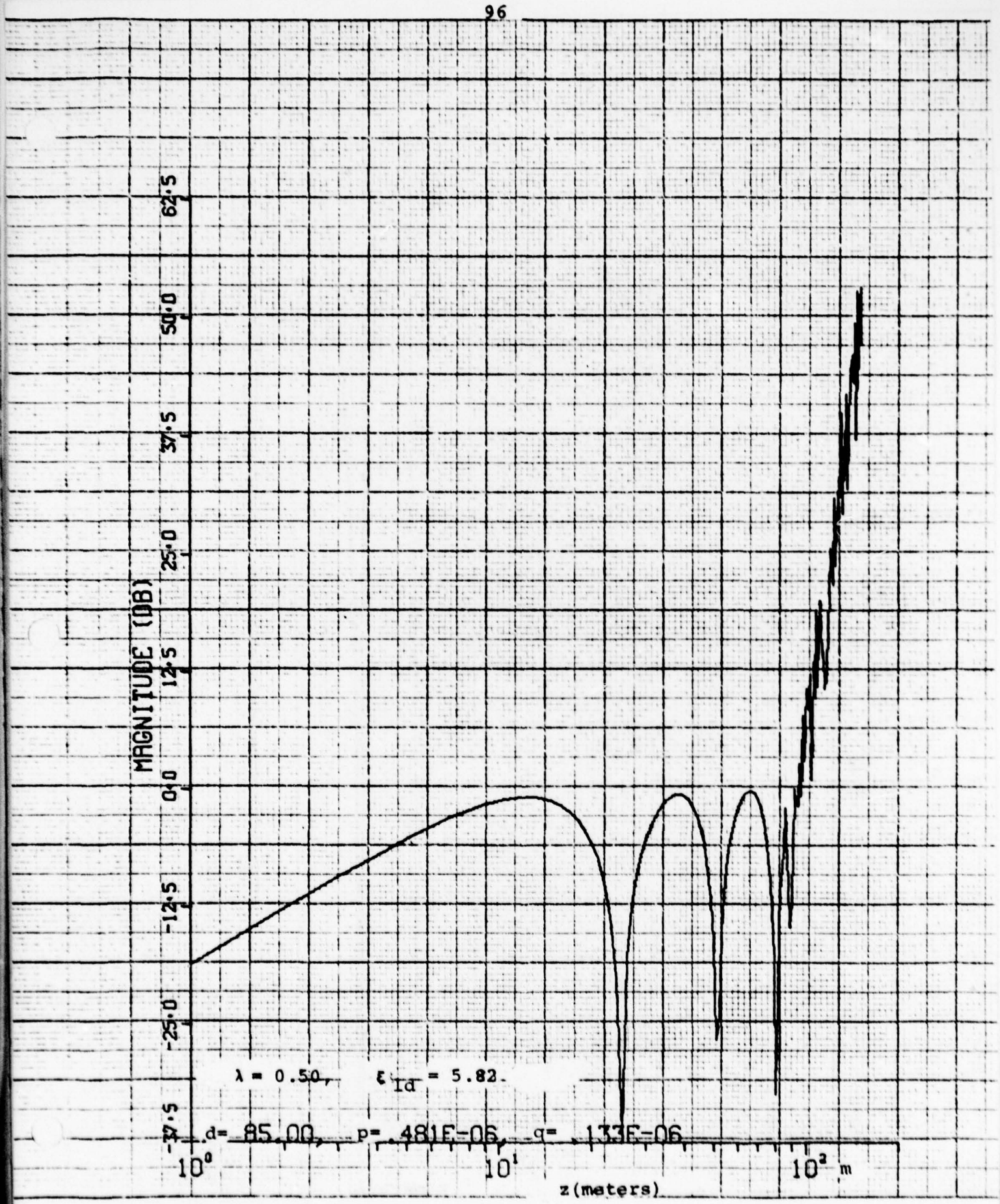


Fig. (4-36) Height-Gain Functions - April

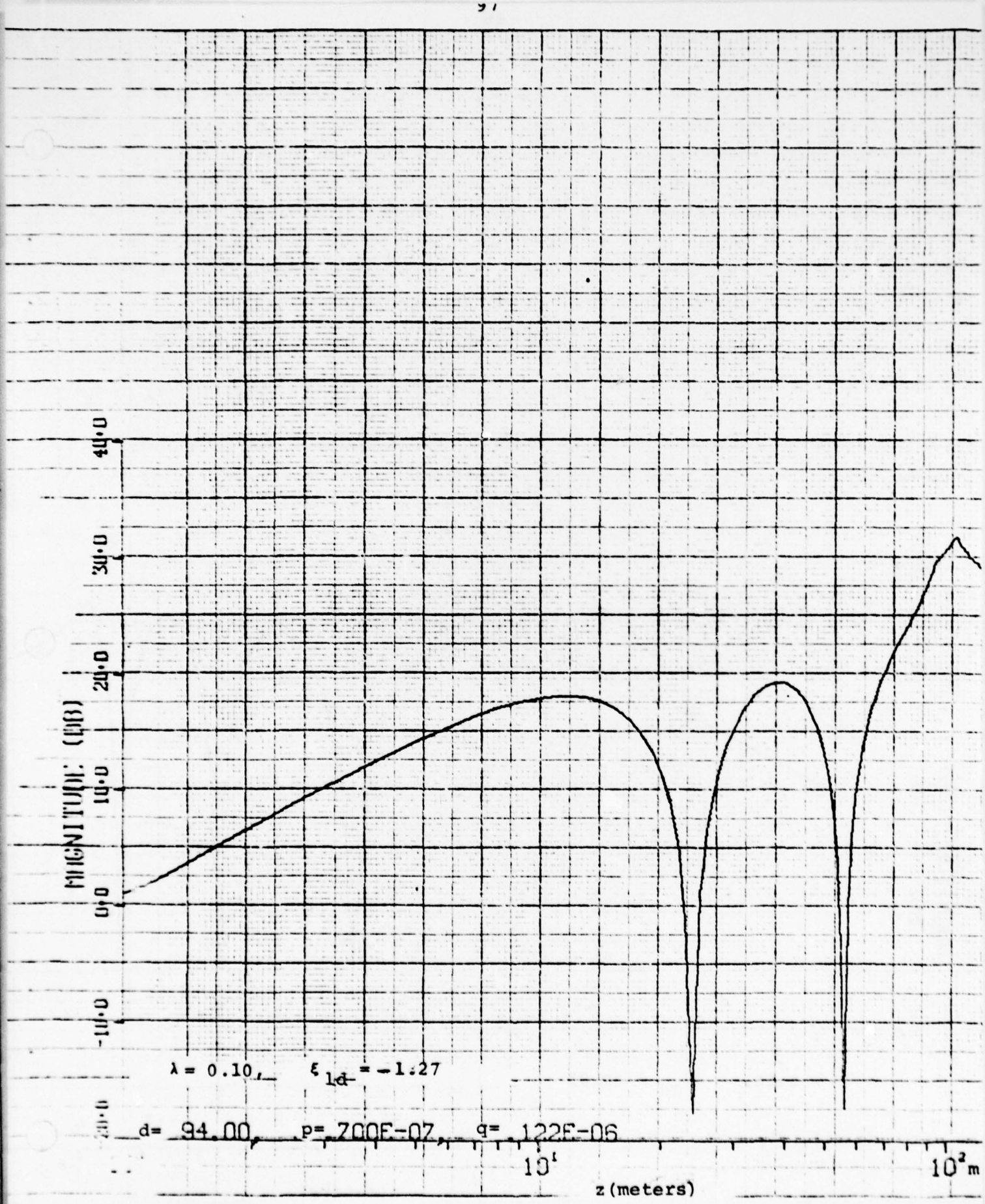


Fig. (4-37) Height-Gain Functions - September

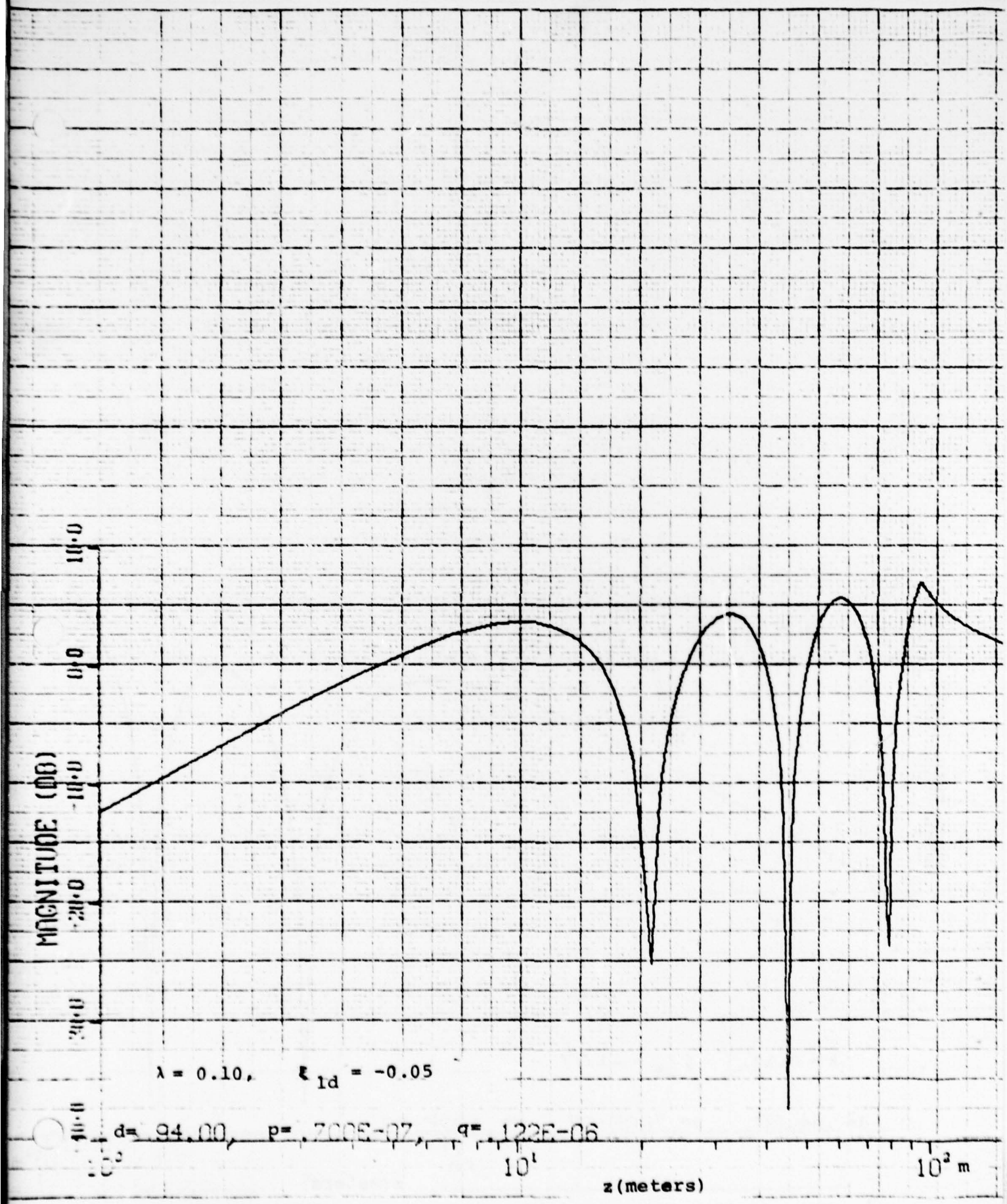


Fig. (4-38) Height-Gain Functions - September

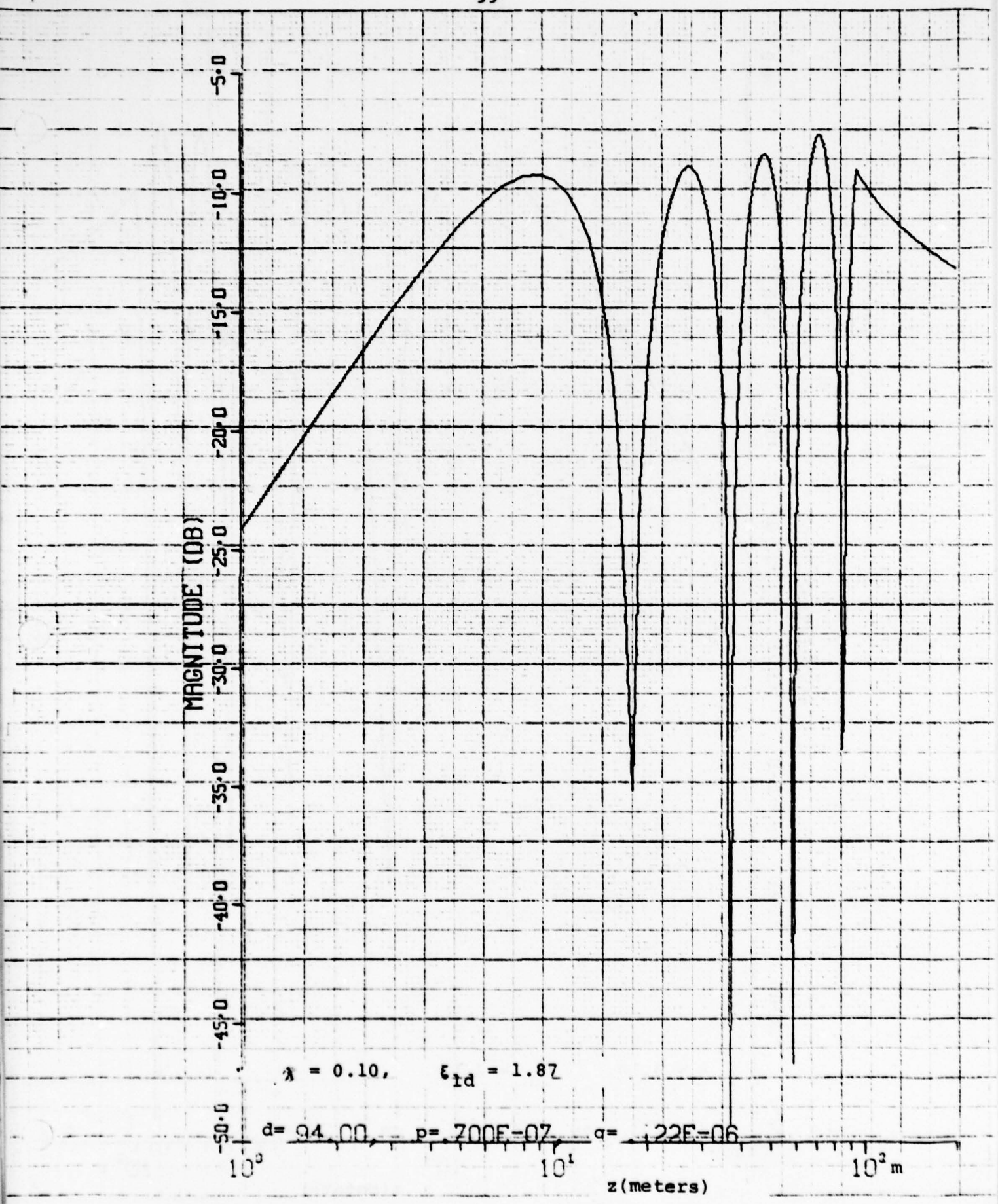


Fig. (4-39) Height-Gain Functions - September

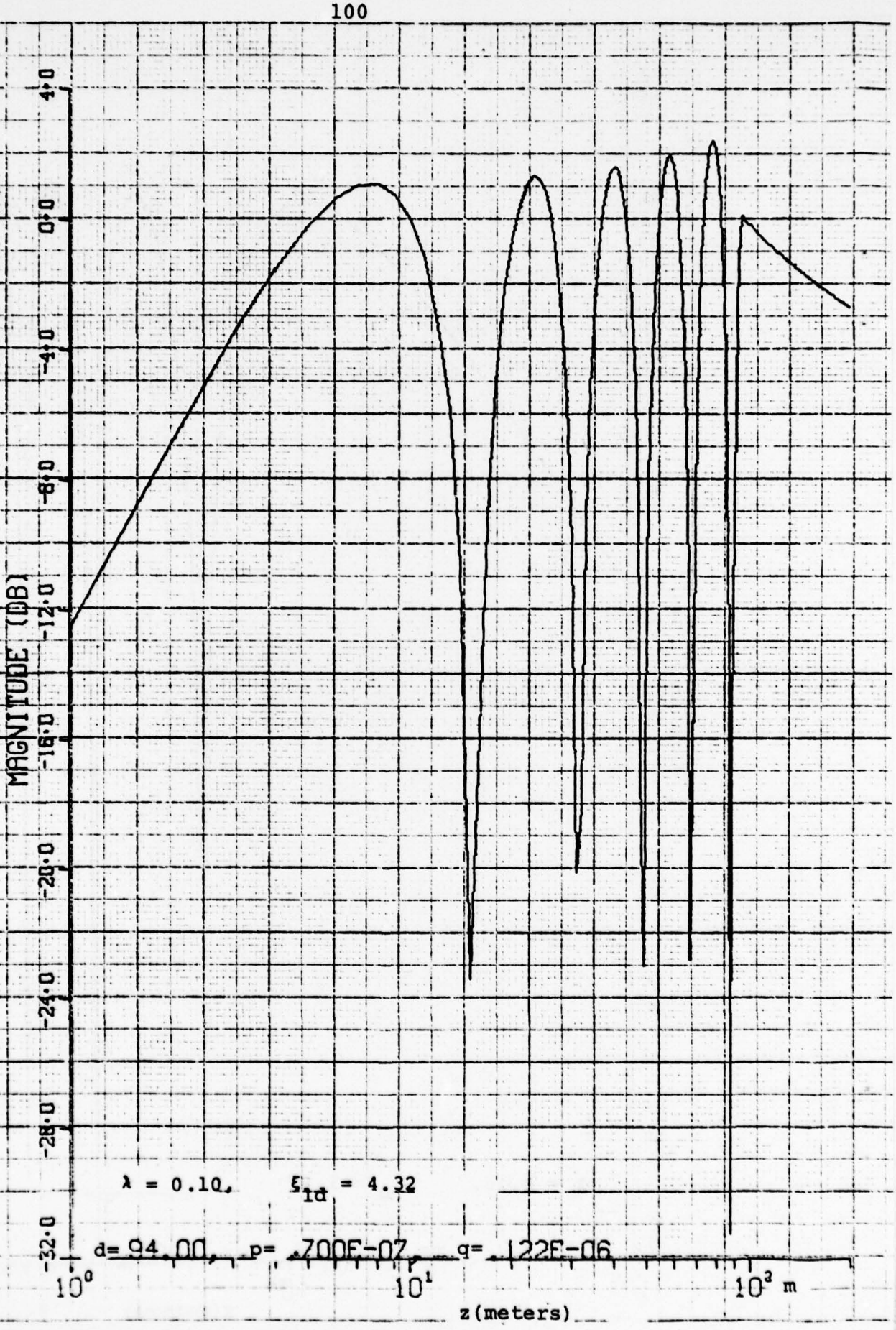


Fig. (4-40) Height-Gain Functions - September

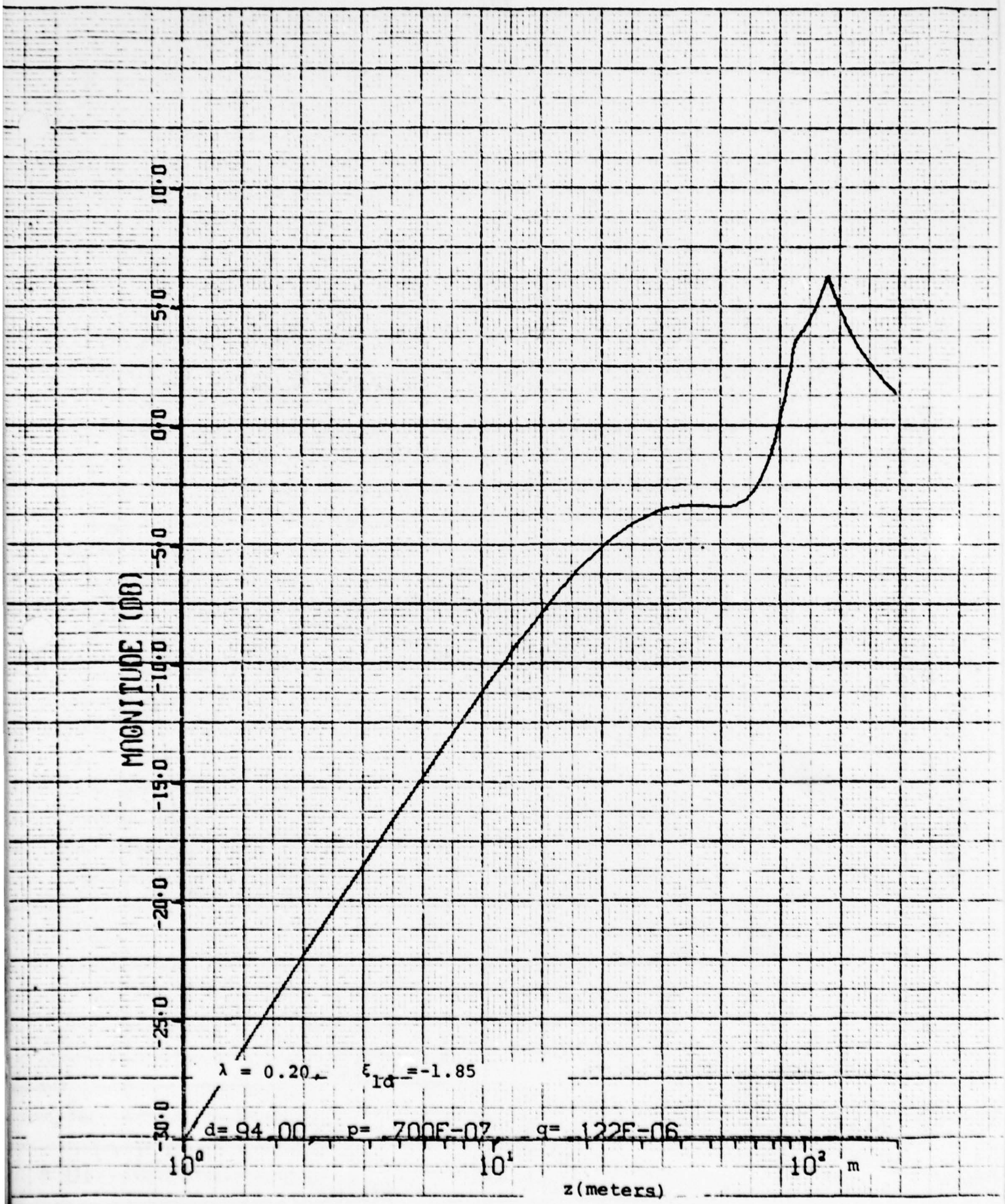


Fig. (4-41) Height-Gain Functions - September

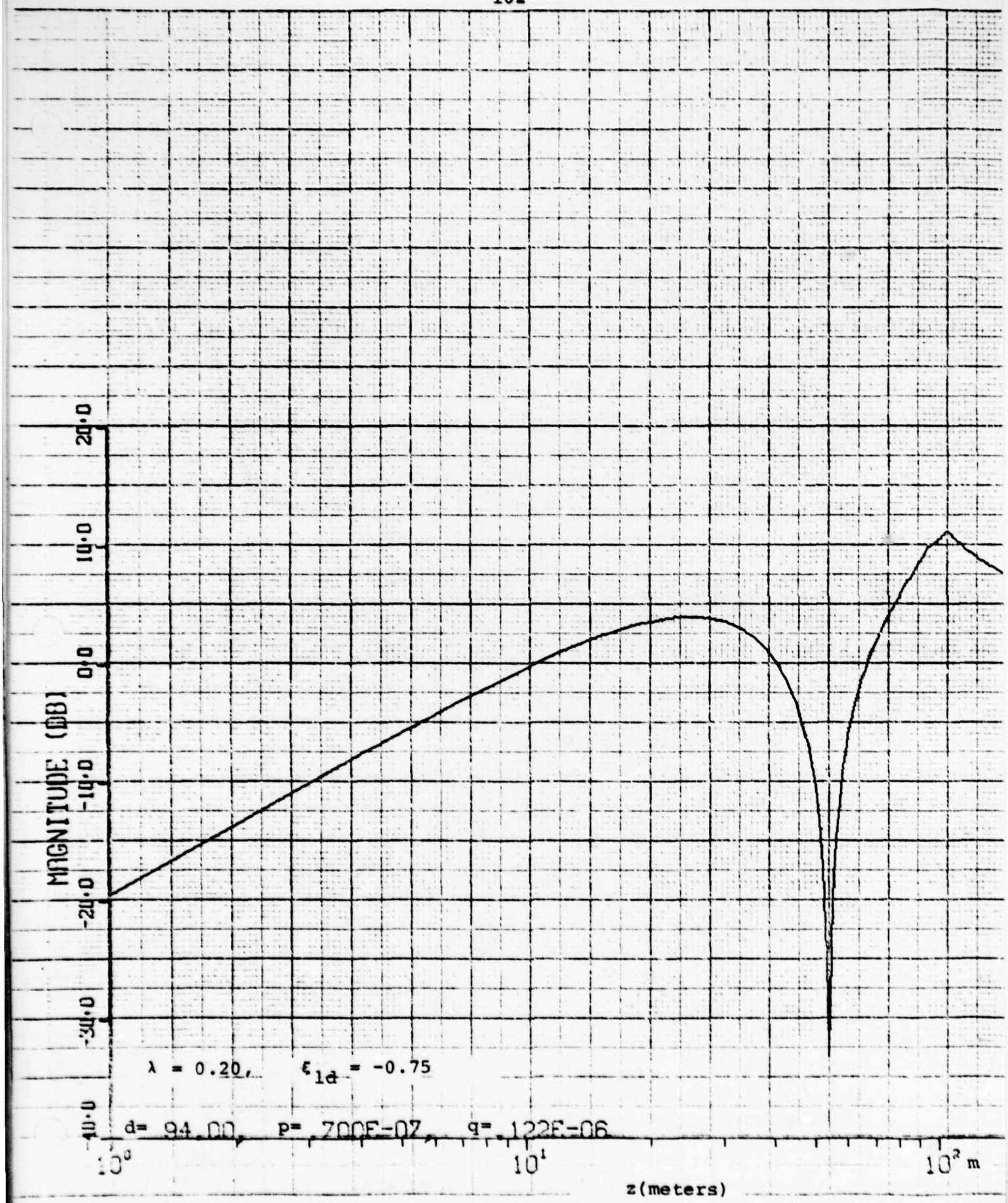


Fig. (4-42) Height-Gain Functions - September

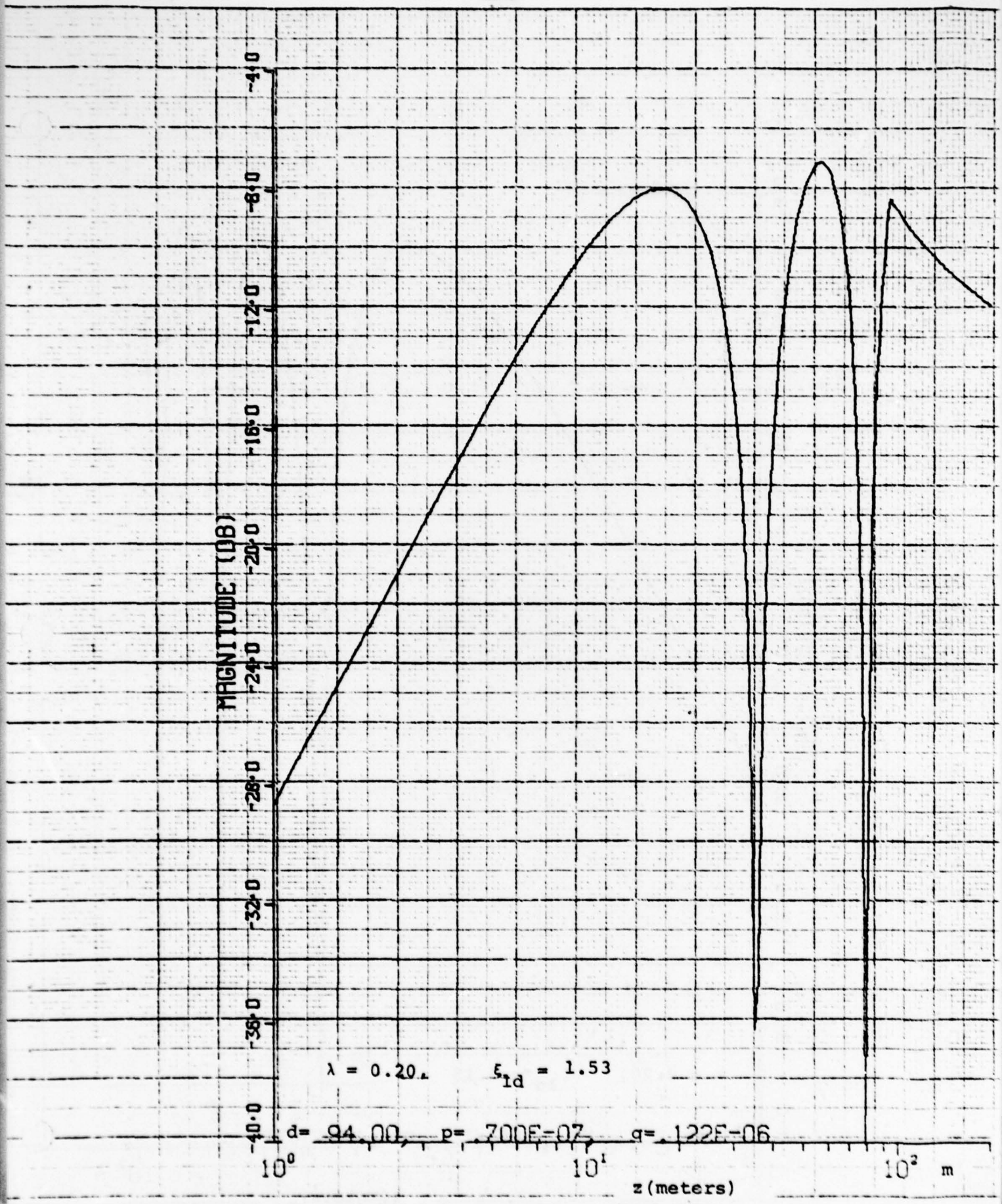


Fig. (4-43) Height-Gain Functions - September

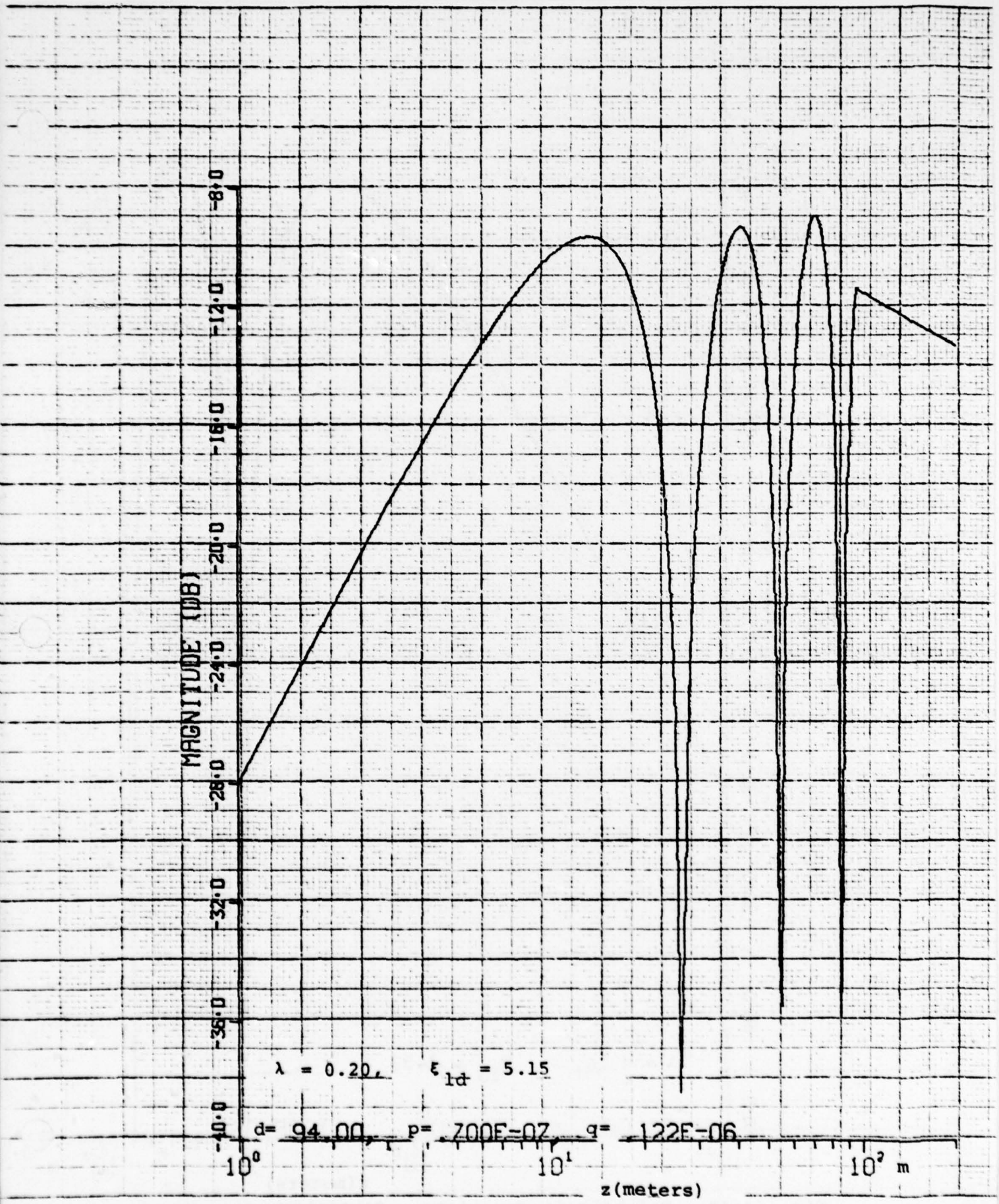


Fig. (4-44) Height-Gain Functions - September

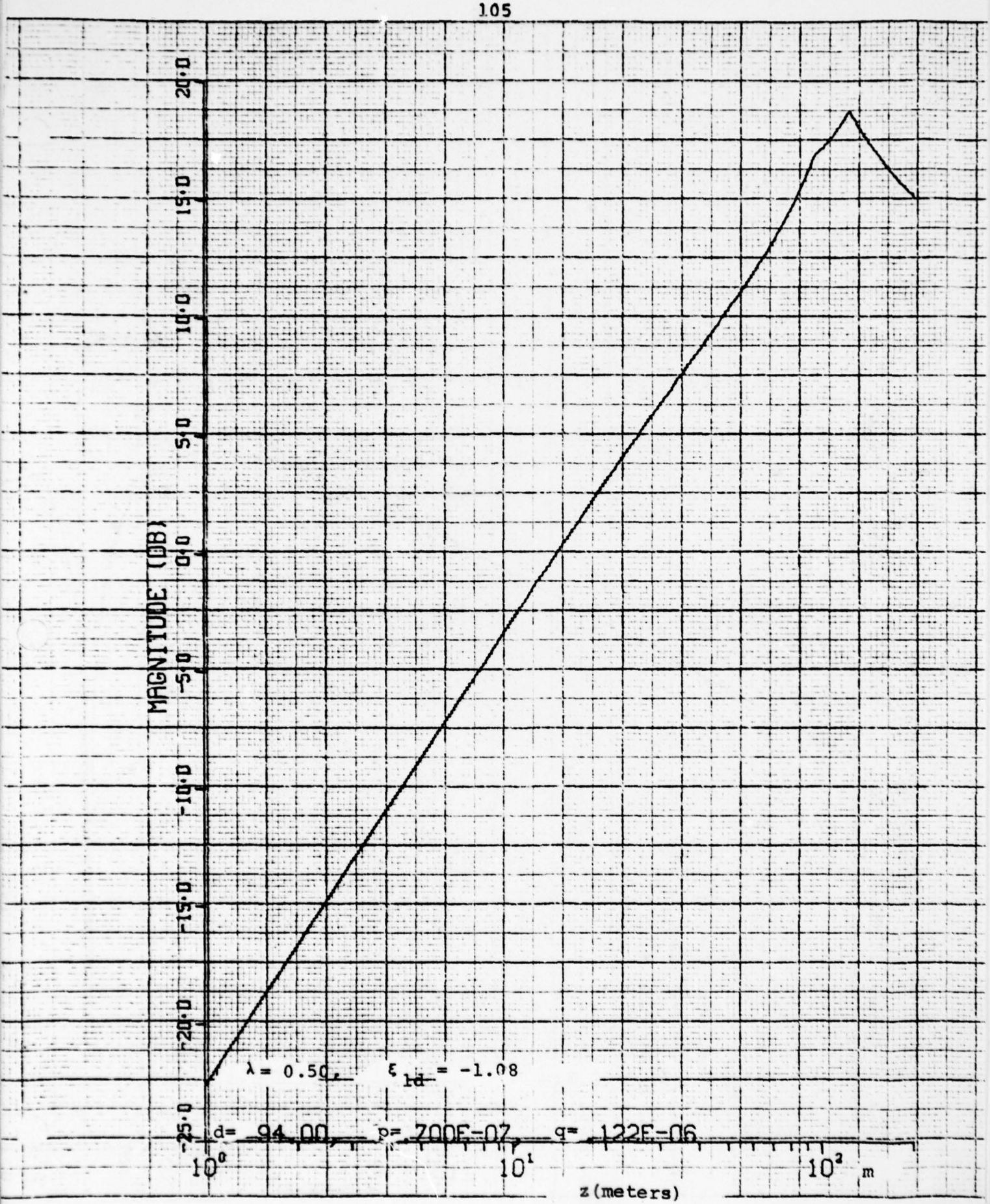


Fig. (4-45) Height-Gain Functions - September

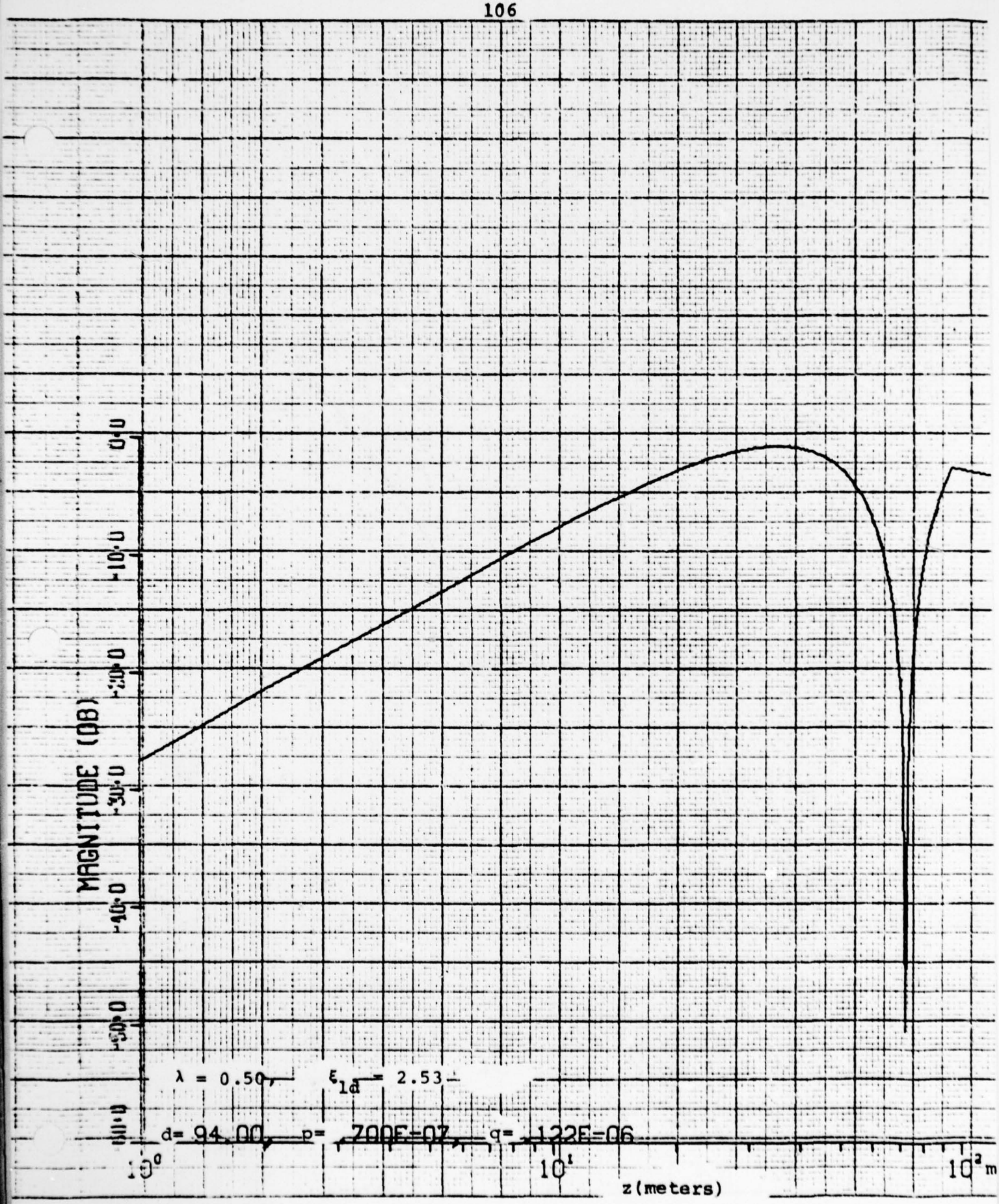


Fig. (4-46) Height-Gain Functions - September

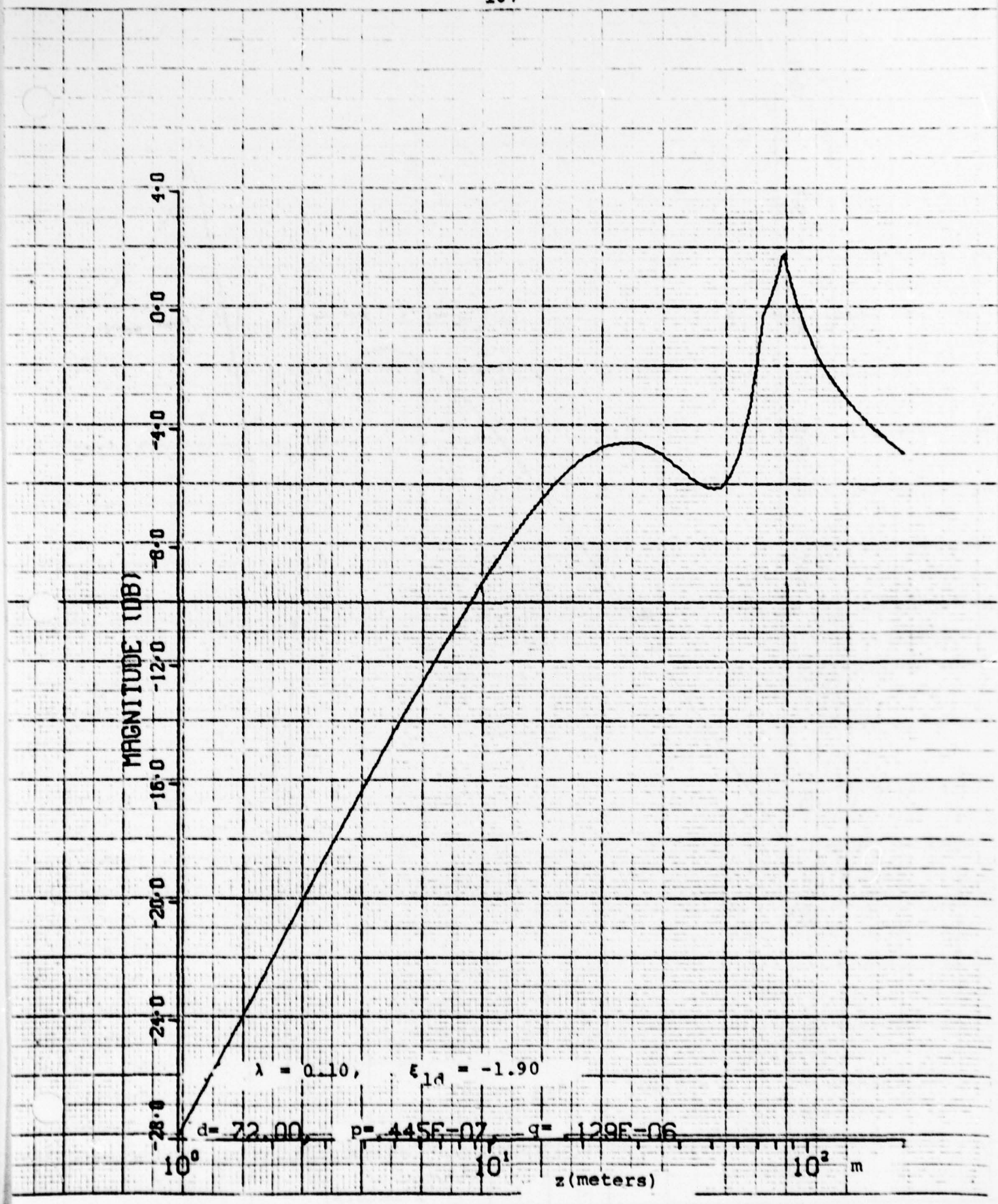


Fig. (4-47) Height-Gain Functions - December

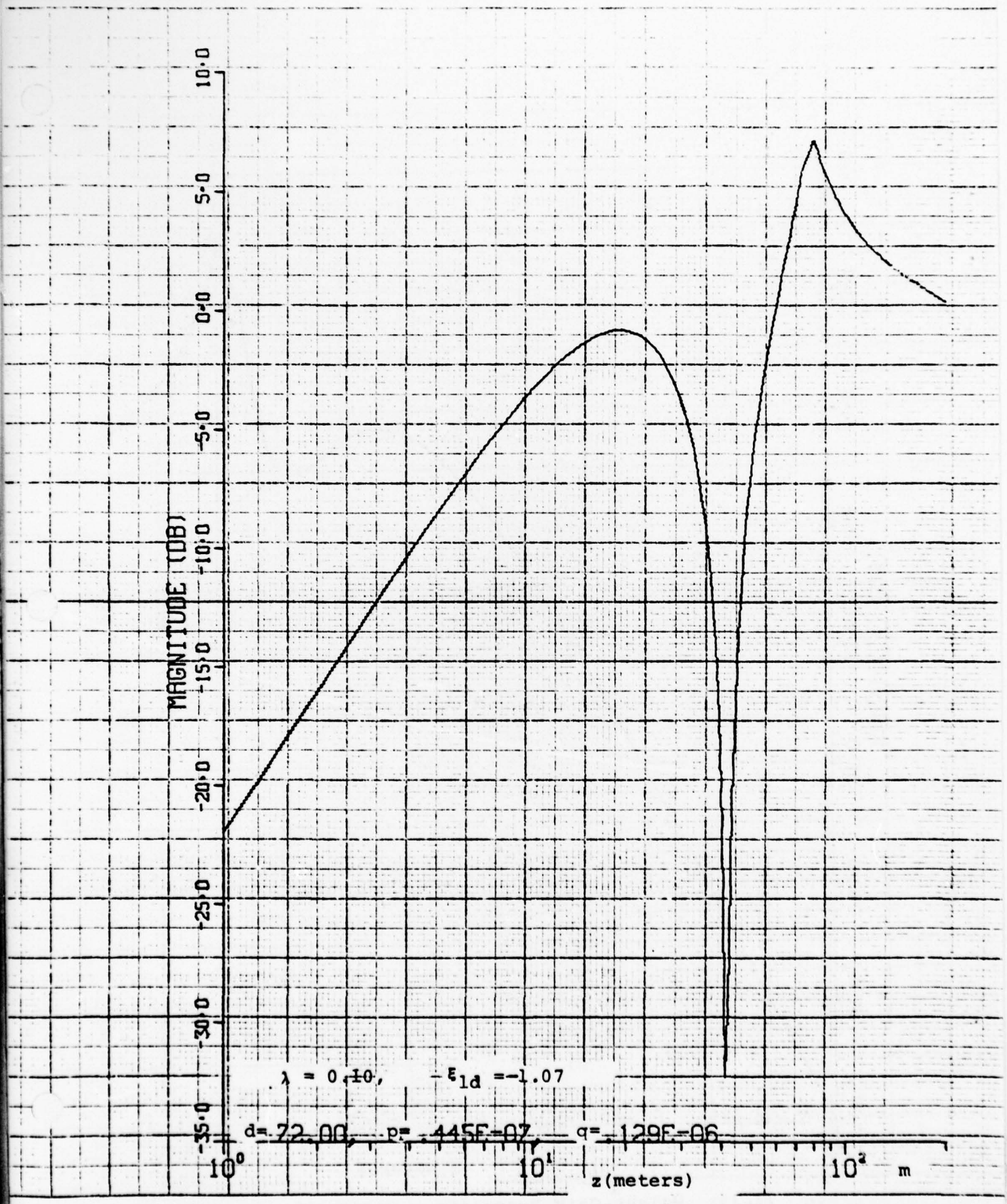


Fig. (4-48) Height-Gain Functions - December

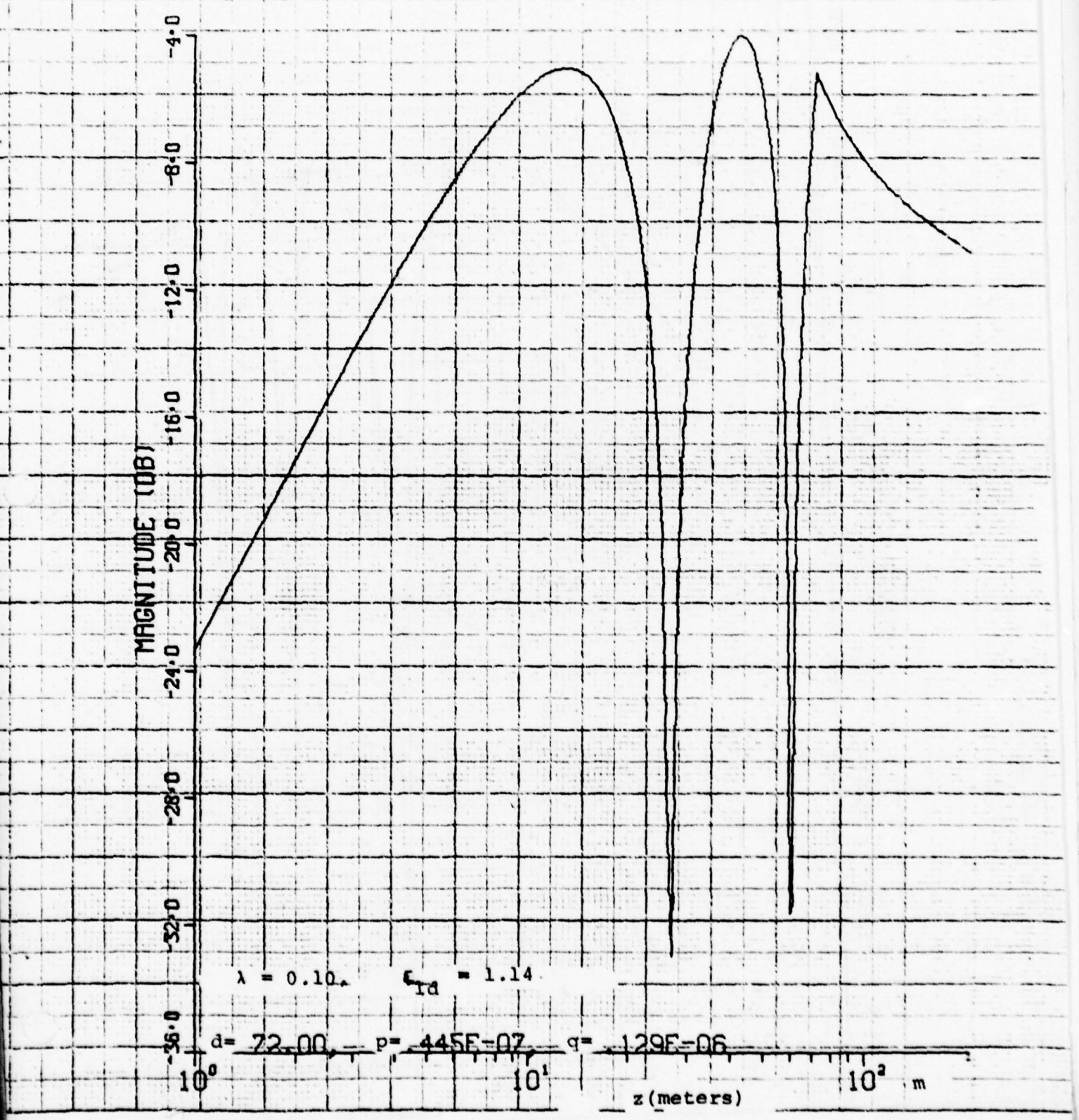


Fig. (4-49) Height-Gain Functions - December

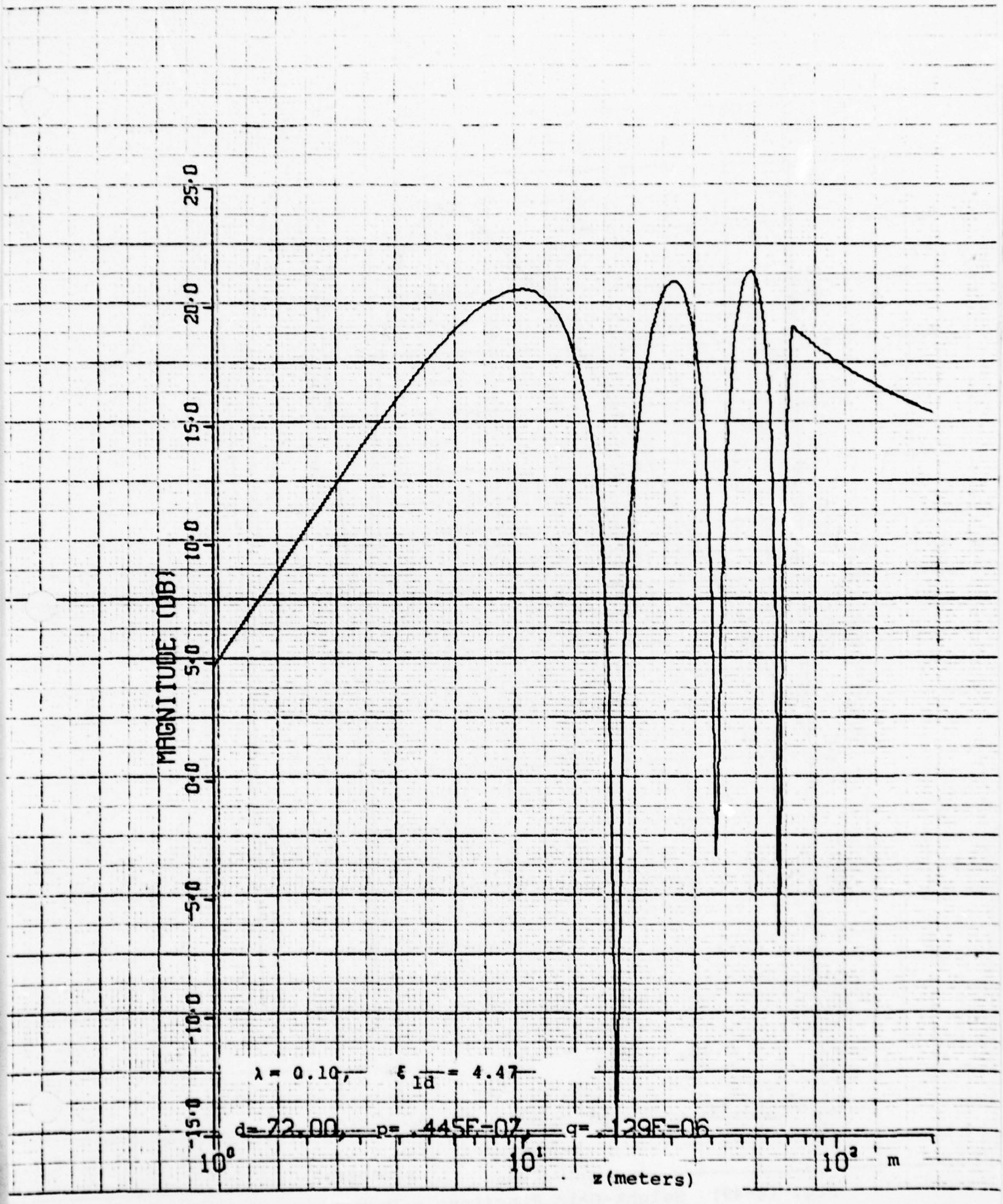


Fig. (4-50) Height-Gain Functions - December

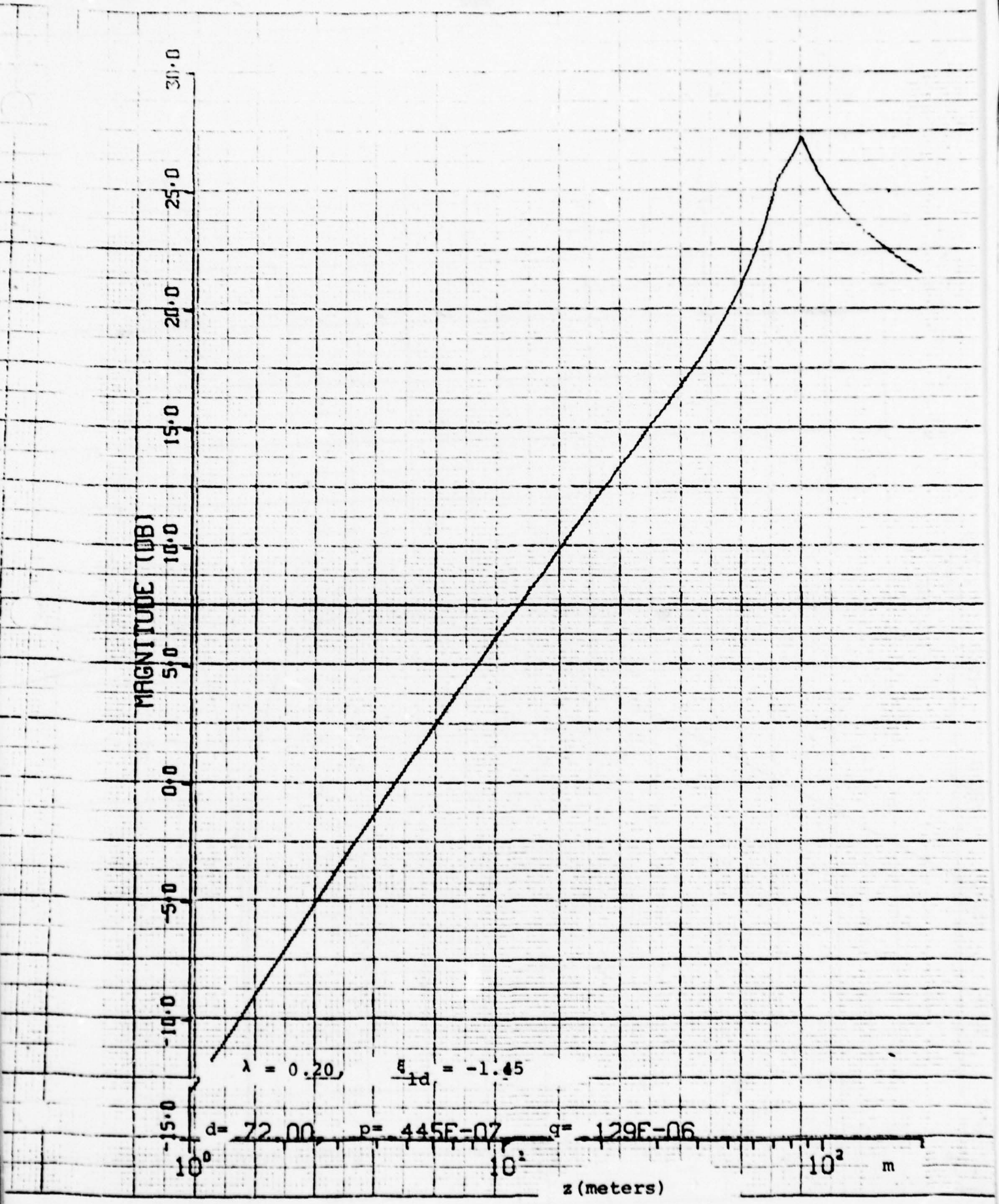


Fig. (4-51) Height-Gain Functions - December

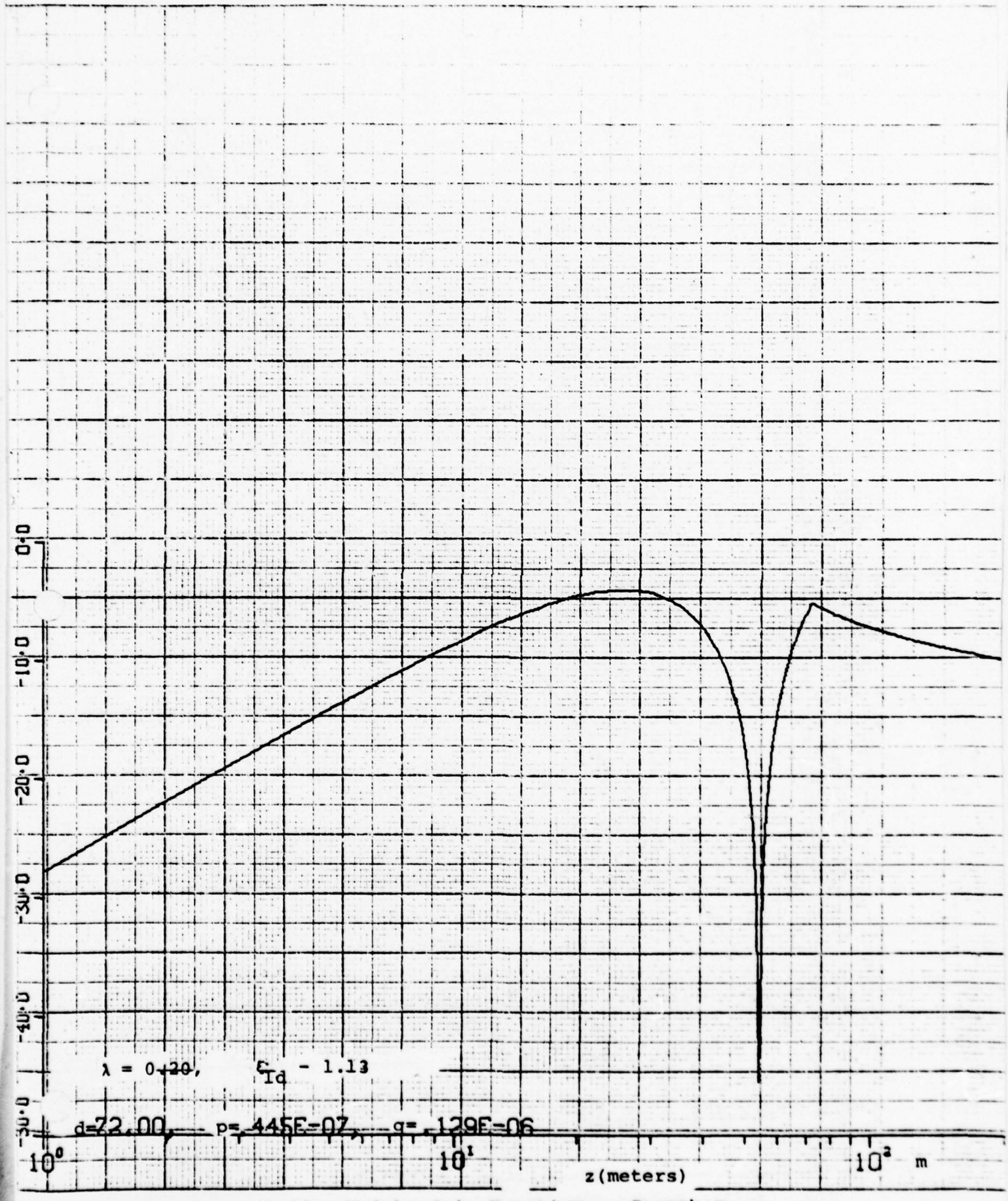


Fig. (4-52) Height-Gain Functions - December

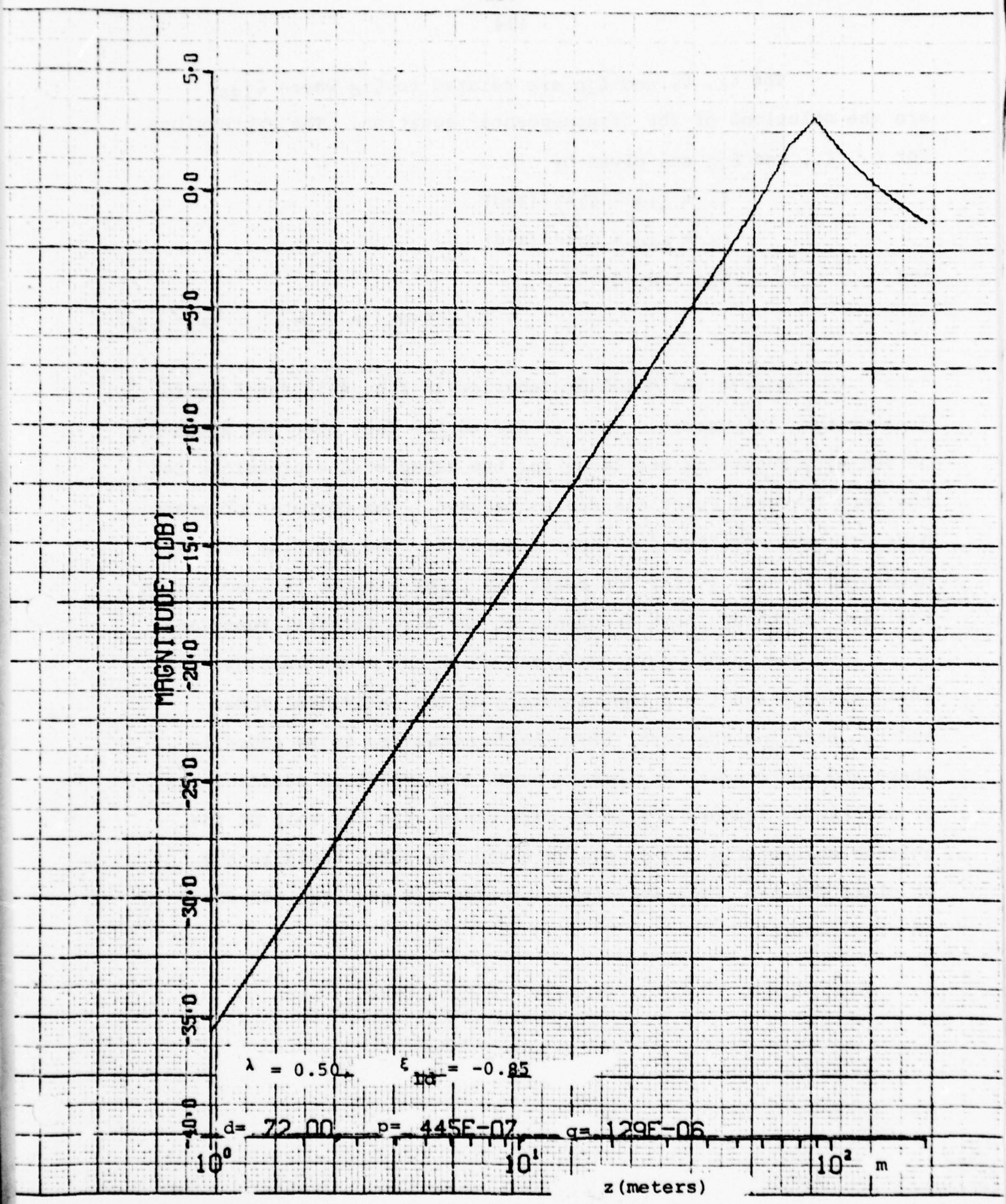


Fig. (4-53) Height-Gain Functions - December

The $\xi_<$, $\xi_>$ and ξ_{10} are related to ξ_{1d} where ξ_{1ds} are the solutions of the transcendental equation. The expressions for $\xi_<$, $\xi_>$, and ξ_{10} are given by

$$\xi_< = \xi_{1d} - C_1^{1/3}(z-d)$$

$$\xi_> = \xi_{zd} + C_2^{1/3}(z-d)$$

and

$$\xi_{10} = B_1/C_1^{2/3}$$

Observe the peculiar behavior in Fig. 4-18 for heights greater than 100 meters. This is due to the fact that the arguments of the Airy Functions are large and the asymptotic expressions for the Airy Integral Functions are to be used. The computer programs have not been modified for large arguments. The peculiar behavior displayed in Fig. (4-28) for small and large heights is also due to the same reason of large arguments of the Airy Integral Functions.

The height gain functions which are shown in Figs. (4-17 through 4-53) are the behaviors of the individual modes and to determine the total field these modes are to be added in phase and amplitude. But before that, the amplitude coefficients or excitation factors are to be determined with the help of the antenna characteristics or equivalently the total field in the yz plane for x=0. The excitation problem has not been addressed in this report.

5. SUMMARY / RECOMMENDATIONS

5.1 Summary

In this phase of the program, the electromagnetic propagation through a medium, which was characteristic of meteorological conditions around Canton Island, was considered in depth. The meteorological conditions around Canton Island were represented by bilinear and trilinear refractive index profiles.

For the propagation analysis, both the geometrical optics or ray trajectory approach and the physical optics or the guided wave approach were considered. The relative merits of these approaches were highlighted. The ray trajactories for four different days of meteorological data were also generated.

The guided mode approach was pursued to determine the propagation behavior for both the bilinear and trilinear refractive index profiles.

The numerical analysis to determine the modal solutions and the corresponding height-gain functions was carried out only for the bilinear profile. The modal solutions were obtained by iterative numerical analysis of the transcandental equation for the trapped modes only.

For the leaky modes, a previoushy developed approximate expression was used. The validity of the axpression for duct heights smaller than the frequency dependent scale height was established.

Although leaky modes are indicative of the field above the duct height, for quantative assessment of the field intensity in the shadow zone, the excitation problem has to be solved. This has not been addressed in this phase of the program.

5.2 Recommendations

The usefulness of the guided wave propagation analysis hinges on the ability to generate radar coverage diagrams. The radar coverage diagrams in the presence of different kinds of meteorological ducts would enable one to evaluate the performance degradation of ESD sponsored surveillance radar systems.

For the generation of radar coverage diagrams, the following tasks need to be completed.

1. An effective analytical technique to determine the complex eigenvalues representing leaky modes.
2. Determining the amplitude coefficients of the trapped and leaky modes by addressing the excitation problems or by specifying the antenna characteristics.
3. Applying these techniques to different meteorological conditions and different radar systems parameters.

These tasks can constitute the second phase of the program.

6. REFERENCES

1. B.R. Bean and E.J. Dutton - Radio Meteorology, Dover Publication, 1968

2. D.E. Kerr - Propagation of Short Radio Waves, Radiation Laboratory Series Vol. 13, McGraw-Hill, 1951

3. A. Zancla - (Editor) - Modern Topics in Microwave Propagation and Air-Sea Interaction, Proceedings of the NATO Advanced Study Institute, Sorento, Italy, June 5-14, 1973 - D. Reidel Publishing Company

4. J.R. Wait - Electromagnetic Waves in Stratified Media, Macmillan Company, 1962

5. P. Beckman and A. Spizzichino - The Scattering of Waves from Rough Surfaces, Macmillan Company, 1963

6. D. Barton - Radars Vol. 5 - Radar Clutter ARTECH House, 1975

7. M. Abramowitz and L.A. Stegun (Edited by) Handbook of Mathematical Functions National Bureau of Standards - Applied Mathematics Series - 55, June 1964

8. K.G. Budden - The Wave Guide Mode Theory of Wave Propagation Prentice Hall, 1961

9. J.H. Richter and H.V. Hitney - The Effect of the Evaporation Duct on Microwave Propagation, NELC Report TR-1949, April 1975

10. H.V. Hitney - Propagation Modeling in the Evaporation Duct, NELC Report - TR-1947, April 1975

11. H. Jeske and K. Brooks - Comparison of Experimental on Duct Propagation above the Sea with the Mode Theory of Booker and Walkinshaw, Radio Science Vol. 1, August 1966

12. H.W. Fruchtenicht - Notes on Duct Influences on Line-of-Sight Propagation, IEEE Transactions on Antennas and Propagation Vol. AP-22, No. 2 - March 1974

13. J.K. DeRosa - Refractive Multipath in Low Angle Radar Tracking, IEEE International Radar Conference, 1975

14. G.H. Joshi - Propagation Modeling in the Presence of Meteorological Ducts, Raytheon Technical Memo No. GHJ:76:09, August 1976

15. L.M. Brekhovshikh - Waves in Layered Media, Academic Press, 1960

REFERENCES (CONT)

16. Tables of the Modified Hankel Functions of Order One-Third and of Their Derivatives, The Annals of the Computation Laboratory of Harvard University, Vol. II, Harvard University Press, 1945

17. J.C.P. Miller - The Airy Integral - Mathematical Tables, University Press Cambridge, 1946

18. G.N. Watson - The Diffraction of Electric Waves by the Earth, Proc. Roy. Soc. A, g5, 83, 1919

19. G.N. Watson - The Transmission of Electric Waves Round the Earth, Proc. Roy. Soc. A, g5, 546, 1919

APPENDIX I

Computer program to determine the roots of the transcendental equation.

Appendix I
I-1


```
      C 09-22-77  /JOB7/
        PROGRAM JOB7(INPUT,OUTPUT,TAPE2,PLFILE=65)
      C
        COMMON /XFACT/XFACT(150),FT(43),GT(43),FTP(43),GTP(43),
     +  FMULT,GMULT,JHOL(2),XP1(120),YP1(120),IHOL(100)
      C
        COMPLEX S2D,S2DT1,S2DT2
      C
        DOUBLE PRECISION C1,FMULT,FT,FTP,GMULT,GT,GTP
        DOUBLE PRECISION PA3,S00,S000,S11,XDEN,XFACT,XK0,XNUM
        DOUBLE PRECISION S0001,S0002,XNUM1,XNUM2,XDEN1,XDEN2
      C
        DIMENSION D(4),P(4),PA(4),XL(3)
      C
        DATA D/42.,85.,94.,72./
        DATA P/.630E-6,.481E-6,.07E-6,.0445E-6/
        DATA PA/4.8,3.61,.574,.344/
        DATA XL/.1,.2,.5/
        DATA JHOL/10HAPPROX    ,10HEXACT     /
      C
        CALL MGAMMA(2./3.,FMULT,IER1)
        CALL MGAMMA(4./3.,GMULT,IER2)
        FMULT=(2.**(1./3.))/FMULT
        GMULT=(2.**(1./3.))/(GMULT*3.**(2./3.))
      C
        PI=4.*ATAN(1.)
        CALL CFACT
        CALL CUMPRS
        CALL NOBRDR
        CALL INTAXS
        CALL PAGE(15.,10.75)
        CALL PHYSOR(1.5,1.5)
        CALL YAXANG(0.)
        CALL CROSS
        CALL MX1ALF(*STANDARD*,1H*)
        CALL MX2ALF(*L/CGREEK*,1H!)
      C
        PRINT (2,90)
     90 FORMAT(1H1,///,
     +*   LAMBDA         D              P              P/A           S11*,
     +*            S2D*,/,
     +*   ------         -              -              ---           ---*,
     +*            ---*)
      C
        DO 300 IXL=1,1
      C
        DO 300 IDER=1,1
      C
        DO 300 ILOOP=1,1
        S2D=2.3301*CEXP(CMPLX(0.,2.*PI/3.))
        PRINT 95,XL(IXL),D(ILOOP),P(ILOOP),PA(ILOOP),S2D
     95 FORMAT(1H1,//,
     + * LAMBDA          = *,F10.6,/,
     + * D               = *,F10.6,/,
     + * P               = *,F10.6,/,
     + * P/A             = *,F10.6,/,
     + * S2D             = *,1H(,F10.6,*  +J*,F10.6,1H),////)
```

```
      PA3=PA(ILOOP)**(1./3.)
      XK0=2.*PI/XL(IXL)
      C1=P(ILOOP)*XK0**2
      S00=CABS(S20)/PA3**2
      PRINT 96,JHOL(IDER)
   96 FORMAT(//,2X,A10,/,
     +  "  I    S20         ",13X,"        S00            S11")
      PRINT 97
   97 FORMAT("  -    ---         ",13X,"        ----           ----")
C
  100 DO 200 I=1,1
      ICNT=0
C
      DO 120 J=1,121
      S00=-6.+(J-1)*.1
      IF(S00.EQ.0)GO TO 120
      IF(ISUB1(S00,S20,S000,PI,PA3,D(ILOOP),C1,XNUM,XDEN)
     + .NE.0)GO TO 300
      PRINT 121,S00,XNUM
  121 FORMAT(" S00=",G15.6," XNUM=",G15.6)
      ICNT=ICNT+1
      XP1(ICNT)=S00
      YP1(ICNT)=XNUM
  120 CONTINUE
C     IF(IDER.EQ.2)GO TO 150
      GO TO 300
      IF(ISUB1(.99999*S00,S20T1,S0001,PI,PA3,D(ILOOP),C1,XNUM1,XDEN1)
     + .NE.0)GO TO 300
      IF(ISUB1(1.00001*S00,S20T2,S0002,PI,PA3,D(ILOOP),C1,XNUM2,XDEN2)
     + .NE.0)GO TO 300
      IF(IDER.EQ.1)S11=S00-XNUM/((XNUM2-XNUM1)/(.00002*S00))
  150 IF(IDER.EQ.2)S11=S00-XNUM/XDEN
      PRINT 111,I,S20,S000,S11
  111 FORMAT(
     + I4,1H(,2G15.8,1H),3G15.8)
  200 S00=S11
C
      PRINT (2,250)XL(IXL),D(ILOOP),P(ILOOP),PA(ILOOP),S11,S20,
     + JHOL(IDER)
  250 FORMAT(5G12.6,1H(,2G12.6,1H),2X,A10)
  300 CALL IPLOT(ICNT,XL(IXL),D(ILOOP),PA(ILOOP))
      CALL DONEPL
      PRINT(2,311)
  311 FORMAT(1H1)
  999 ENDFILE 2
      END
```

CARD NR.	SEVERITY	DETAILS	DIAGNOSIS OF PROBLEM
65	I		LOWER LIMIT .GE. UPPER LIMIT. ONE TRIP LOOP.
67	I		LOWER LIMIT .GE. UPPER LIMIT. ONE TRIP LOOP.
69	I		LOWER LIMIT .GE. UPPER LIMIT. ONE TRIP LOOP.
60	I		LOWER LIMIT .GE. UPPER LIMIT. ONE TRIP LOOP.
84	I		THERE IS NO PATH TO THIS STATEMENT.

SYMBOLIC REFERENCE MAP (R=3)

ENTRY POINTS DEF LINE REFERENCES

3370 JOB7 2

VARIABLES	BN	TYPE	RELOCATION									
4223	C1	DOUBLE			REFS	9	74	84	86	DEFINED	60	
4270	D	REAL	ARRAY		REFS	13	51	74	84	86	95	98
					DEFINED	15						
1204	FMULT	DOUBLE		XFACT	REFS	4	9	21	23	DEFINED	23	
454	FT	DOUBLE	ARRAY	XFACT	REFS	4	9					
730	FTP	DOUBLE	ARRAY	XFACT	REFS	4	9					
1206	GMULT	DOUBLE		XFACT	REFS	4	9	22	24	DEFINED	24	
602	GT	DOUBLE	ARRAY	XFACT	REFS	4	9					
1056	GTP	DOUBLE	ARRAY	XFACT	REFS	4	9					
4205	I	INTEGER			REFS	90	DEFINED	60				
4206	ICNT	INTEGER			REFS	78	79	80	98	DEFINED	69	78
4263	IDER	INTEGER			REFS	62	88	89	95	DEFINED	67	
4257	IER1	0 INTEGER			REFS	21						
4200	IER2	0 INTEGER			REFS	22						
1572	IHOL	INTEGER	ARRAY	XFACT	REFS	4						
4264	ILOOP	INTEGER			REFS	3051	58	60	74	84	86	3093
						2090	DEFINED	49				
4262	IXL	INTEGER			REFS	51	54	95	98	DEFINED	45	
4267	J	INTEGER			REFS	72	DEFINED	71				
1210	JHOL	INTEGER	ARRAY	XFACT	REFS	4	62	95	DEFINED	19		
4270	P	REAL	ARRAY		REFS	13	51	60	95	DEFINED	16	
4300	PA	REAL	ARRAY		REFS	13	51	58	95	98		
					DEFINED	17						
4225	PA3	DOUBLE			REFS	10	61	74	84	86		
					DEFINED	58						
4261	PI	REAL			REFS	50	59	79	84	86		
					DEFINED	26						
4227	S00	DOUBLE			REFS	10	73	74	76	79	84	86
						2088	89	DEFINED	61	72	93	
4231	S000	DOUBLE			REFS	10	74	90				
4243	S0001	DOUBLE			REFS	11	84					
4245	S0002	DOUBLE			REFS	11	86					
4233	S11	DOUBLE			REFS	10	90	93	95	DEFINED	88	89
4215	S20	COMPLEX			REFS	7	51	61	74	90	95	
					DEFINED	50						
4217	S20T1	COMPLEX			REFS	7	84					
4221	S20T2	COMPLEX			REFS	7	86					
4235	XDEN	DOUBLE			REFS	10	74	89				
4253	XDEN1	DOUBLE			REFS	11	84					
4255	XDEN2	DOUBLE			REFS	11	86					
0	XFACT	DOUBLE	ARRAY	XFACT	REFS	4	10					
4237	XK0	DOUBLE			REFS	10	60	DEFINED	59			
4304	XL	REAL	ARRAY		REFS	13	51	59	95	98		
					DEFINED	18						
4241	XNUM	DOUBLE			REFS	10	74	76	80	88	89	
4247	XNUM1	DOUBLE			REFS	11	84	88				
4251	XNUM2	DOUBLE			REFS	11	86	88				
1212	XPI	REAL	ARRAY	XFACT	REFS	4	DEFINED	79				
1402	YPI	REAL	ARRAY	XFACT	REFS	4	DEFINED	80				

THIS PAGE IS BEST QUALITY PRACTICABLE
FROM COPY FURNISHED TO DDC

```
     PROGRAM JOOT       73/74   OPT=1 TRACE    1              FTN 4.6+452          77/09/22. 13.59.33       PAGE   8

FILE NAMES          MODE
   0  INPUT
1050  OUTPUT    FMT                   WRITES      51         62         85        76        90
3204  PLFILE
2130  TAPE2     FMT                   WRITES      30         95        100     MOTION      102

EXTERNALS           TYPE    ARGS            REFERENCES
        ATAN        REAL       1  LIBRARY     26
        CABS        REAL       1  LIBRARY     61
        CEXP        COMPLEX    1  LIBRARY     50
        CFACT                  0              27
        LINPHB                 0              28
        CHUBS                  0              34
        DONEPL                 0              99
        INTAXS                 0              30
        IPLOT                  4              98
        IOUN1       INTEGER    9              74      84        86
        NGAMMA                 3              21      22
        NX1ALF                 2              35
        NX2ALF                 2              36
        NOORDR                 0              29
        PAGE                   2              31
        PHYSOR                 2              32
        YAXANS                 1              33

INLINE FUNCTIONS    TYPE    ARGS        DEF LINE   REFERENCES
        CMPLX       COMPLEX    2  INTRIN              50

STATEMENT LABELS                    DEF LINE   REFERENCES
 4021   90      FMT                    39         30
 4053   95      FMT                    52         51
 4101   96      FMT                    63         62
 4114   97      FMT                    66         65
    0  100           INACTIVE          68
 4140  111      FMT                    91         90
 3550  120                             81         71        73
 4130  121      FMT                    77         76
    0  150           INACTIVE          89
    0  200                             93         68
 4162  250      FMT                    97         95
 3666  300                             98         45      47        49       74       83      84      86
 4171  311      FMT                   101        100
    0  999           INACTIVE         102

LOOPS  LABEL      INDEX      FROM-TO     LENGTH     PROPERTIES
 2640   300     * IXL          45          8B              EXT REFS   NOT INNER
 2740   300     * IDER         47          8B              EXT REFS   NOT INNER
 3000   300     * ILOOP        49          8B              EXT REFS   NOT INNER
 4200   200     * I            68          8B              EXT REFS     EXITS    NOT INNER
 3530   120     * J            71  81     23B              EXT REFS     EXITS

COMMON BLOCKS    LENGTH     MEMBERS - BIAS NAME(LENGTH)
       XFACT      990            0 XFACT   (300)          300 FT     (86)          386 BT     (86)
                               472 FTP     (86)           558 BTP    (86)          644 FMULT  (2)
                               646 BMULT   (2)            648 JMOL   (2)           650 XP1    (120)
                               770 YP1     (120)          890 IMOL   (100)
```

Appendix I
I-5

PROGRAM JOB7 73/76 OPT=1 TRACE. FTN 4.6+492 77/09/22. 13.59.33 PAGE 9

STATISTICS
PROGRAM LENGTH 7348 670
BUFFER LENGTH 33618 1777
CM LABELED COMMON LENGTH 17348 990
520000 CM USED

```
      FUNCTION IABIJ(ARG,AI,BI,AIP,BIP)
C
      COMMON /XFACT/XFACT(150),FT(43),GT(43),FTP(43),GTP(43),
     +  FMULT,GMULT,JHOL(2),XPI(120),YPI(120),IHOL(100)
C
      DOUBLE PRECISION A,AI,AIP,ARG,B,BI,BIP,F,FMULT
      DOUBLE PRECISION FP,FT,FTP,G,GMULT,GP,GT,GTP,XFACT
C
      DATA C1,C2,SQRT3/.355028053E0,.258819045E0,1.732050808E0/
C
C     PRINT 9,' ARG=',ARG
      IABIJ=1
      A=1.
      B=1.
      F=1.
      G=ARG
      FP=0.
      GP=1.
      FTMAX=GTMAX=0
C
  100 DO 200 K=1,43
      A=A*(3*K-2)
      B=B*(3*K-1)
      FT(K)=A*ARG**(3*K)/XFACT(3*K)
      GT(K)=B*ARG**(3*K+1)/XFACT(3*K+1)
      IF(DABS(FT(K)).GT.ABS(FTMAX))FTMAX=DABS(FT(K))
      IF(DABS(GT(K)).GT.ABS(GTMAX))GTMAX=DABS(GT(K))
      F=F+FT(K)
      G=G+GT(K)
      FTP(K)=(3.*K)*A*ARG**(3*K-1)/XFACT(3*K)
      GTP(K)=(3.*K+1)*B*ARG**(3*K)/XFACT(3*K+1)
      FP=FP+FTP(K)
      GP=GP+GTP(K)
C     PRINT 191,K,A,B,FT(K),GT(K),F,G,FTP(K),GTP(K),FP,GP
  191 FORMAT(I3,1X,10D11.3)
  200 CONTINUE
      DO 202 I=1,5
      IF(1000.*DABS(FT(38+I)).LT.ABS(FTMAX).AND.
     +   1000.*DABS(GT(38+I)).LT.ABS(GTMAX))GO TO 202
      GO TO 204
  202 CONTINUE
      GO TO 220
  204 PRINT 205,ARG,F,G
  205 FORMAT(' MORE ACC. REQUIRED IN AIRY CALC.'/' ARG= ',D18.3/(
     +  ' F= ',D15.6,/,' G= ',D15.6)
      PRINT 206,(I,FT(I),GT(I),I=1,43)
  206 FORMAT(I5,2D15.6)
      IABIJ=-1
      GO TO 999
  220 AI=C1*F-C2*G
      BI=SQRT3*(C1*F+C2*G)
      AIP=C1*FP-C2*GP
      BIP=SQRT3*(C1*FP+C2*GP)
 C210 PRINT 211,K,A,B,F,G,FP,GP,AI,BI,AIP,BIP
  211 FORMAT(' K,A,B,F,G,FP,GP,AI,BI,AIP,BIP='/(
     +  I3,2X,11D10.3))
  999 RETURN
```

```
FUNCTION IABIJ       73/74   OPT=1 TRACE                        FTN 4.6+452          77/09/22. 13.59.33        PAGE    2

                END

SYMBOLIC REFERENCE MAP (R=3)

ENTRY POINTS       DEF LINE      REFERENCES
    5  IABIJ           1             57

VARIABLES      SN   TYPE          RELOCATION
  435  A              DOUBLE                          REFS        6        22        24        30    DEFINED      15       28
    0  AI             DOUBLE             F.P.         REFS        6   DEFINED         1        50
    0  AIP            DOUBLE             F.P.         REFS        6   DEFINED         1        52
    0  ARG            DOUBLE             F.P.         REFS        6        16        24        25        30       31       43
                                                   DEFINED        1
  437  B              DOUBLE                          REFS        6        23        25        31    DEFINED      14       23
    0  BI             DOUBLE             F.P.         REFS        6   DEFINED         1        51
    0  BIP            DOUBLE             F.P.         REFS        6   DEFINED         1        53
  362  C1             REAL                            REFS       50        51        52        53    DEFINED       9
  363  C2             REAL                            REFS       50        51        52        53    DEFINED       9
  441  F              DOUBLE                          REFS        6        28        43        50        51
                                                   DEFINED       15        28
 1204  FMULT          DOUBLE             XFACT        REFS        3         6
  443  FP             DOUBLE                          REFS        7        32        52        53    DEFINED      17       32
  454  FT             DOUBLE    ARRAY    XFACT        REFS        3         7      2026        28        30       46
                                                   DEFINED       24
  451  FTMAX          REAL                            REFS       26        38   DEFINED        19        26
  730  FTP            DOUBLE    ARRAY    XFACT        REFS        3         7        32   DEFINED        30
  445  G              DOUBLE                          REFS        7        29        43        50        51
                                                   DEFINED       16        29
 1206  GMULT          DOUBLE             XFACT        REFS        3         7
  447  GP             DOUBLE                          REFS        7        33        52        53    DEFINED      18       33
  602  GT             DOUBLE    ARRAY    XFACT        REFS        3         7      2027        29        38       46
                                                   DEFINED       25
  452  GTMAX          REAL                            REFS       27        38   'DEFINED        19        27
 1056  GTP            DOUBLE    ARRAY    XFACT        REFS        3         7        33   DEFINED        31
  450  I              INTEGER                         REFS     2030      3046   DEFINED        37        46
  434  IABIJ          INTEGER                      DEFINED       12        48
 1572  IHOL           INTEGER   ARRAY    XFACT        REFS        3
 1210  JHOL           INTEGER   ARRAY    XFACT        REFS        3
  453  K              INTEGER                         REFS       22        23      3024      3025      2026     2027       28
                                                            29     4030      4031        32        33    DEFINED      21
  364  SORTS          REAL                            REFS       51        53   DEFINED         9
    0  XFACT          DOUBLE    ARRAY    XFACT        REFS        3         7        24        25        30       31
 1212  XP1            REAL      ARRAY    XFACT        REFS        3
 1402  YP1            REAL      ARRAY    XFACT        REFS        3

FILE NAMES          MODE
  OUTPUT            FMT                 WRITES       43        46

INLINE FUNCTIONS    TYPE    ARGS        DEF LINE   REFERENCES
  ABS               REAL      1  INTRIN              26        27      2030
  DABS              DOUBLE    1  INTRIN            2026      2027      2030
```

821240

```
      FUNCTION IADIJ       73/74   OPT=1 TRACE                        FTN 4.6+452        77/09/22, 13.59.53        PAGE    5

STATEMENT LABELS                  DEF LINE   REFERENCES
     0  100          INACTIVE      21
   365  191      FMT  NO REFS      35
     0  200                        36          21
   260  202                        41          37         38
   265  204                        43          40
   376  205      FMT               44          43
   420  206      FMT               47          46
   423  211      FMT  NO REFS      55
   310  220                        50          42
   352  999                        57          49

LOOPS  LABEL     INDEX        FROM-TO     LENGTH      PROPERTIES
   34  200        K           21 36       201B             EXT REFS
  237  202        I           37 41        20B        OPT    EXITS
  272             I           46 46        11B             EXT REFS

COMMON BLOCKS    LENGTH       MEMBERS - BIAS NAME(LENGTH)
 XFACT           990              0 XFACT    (300)           300 FT        (86)           386 GT      (86)
                                472 FTP      (86)            558 GTP       (86)           644 FMULT   (2)
                                646 GMULT    (2)             648 JMOL      (2)            650 XPI     (120)
                                770 YPI      (120)           890 IMOL      (100)

STATISTICS
  PROGRAM LENGTH                  472B        314
  CM LABELED COMMON LENGTH      1736B         990
       52000B CM USED
```

```
      SUBROUTINE CFACT
C
      COMMON /XFACT/XFACT(150),FT(43),GT(43),FTP(43),GTP(43),
     +  FMULT,GMULT,JHOL(2),XP1(120),YP1(120),IHOL(100)
C
      DOUBLE PRECISION FMULT,FT,FTP,GMULT,GT,GTP,XFACT
C
      XFACT(1)=1
      DO 100 I=2,150
  100 XFACT(I)=XFACT(I-1)*I
C
  999 RETURN
      END
```

SYMBOLIC REFERENCE MAP (R=3)

ENTRY POINTS

		DEF LINE	REFERENCES
2	CFACT	2	13

VARIABLES

		SN	TYPE		RELOCATION								
1204	FMULT		DOUBLE		XFACT	REFS	4	7					
456	FT		DOUBLE	ARRAY	XFACT	REFS	4	7					
730	FTP		DOUBLE	ARRAY	XFACT	REFS	4	7					
1206	GMULT		DOUBLE		XFACT	REFS	4	7					
602	GT		DOUBLE	ARRAY	XFACT	REFS	4	7					
1056	GTP		DOUBLE	ARRAY	XFACT	REFS	4	7					
22	I		INTEGER			REFS	3911	DEFINED	10				
1572	IHOL		INTEGER	ARRAY	XFACT	REFS	4						
1210	JHOL		INTEGER	ARRAY	XFACT	REFS	4						
0	XFACT		DOUBLE	ARRAY	XFACT	REFS	4	7	11	DEFINED	9	11	
1212	XP1		REAL	ARRAY	XFACT	REFS	4						
1402	YP1		REAL	ARRAY	XFACT	REFS	4						

STATEMENT LABELS

			DEF LINE	REFERENCES
0	100		11	10
0	999	INACTIVE	13	

LOOPS

LOOPS	LABEL	INDEX	FROM-TO	LENGTH	PROPERTIES
13	100	I	10 11	58	INSTACK

COMMON BLOCKS

	LENGTH	MEMBERS - BIAS NAME(LENGTH)								
XFACT	990	0	XFACT	(300)	300	FT	(86)	386	GT	(86)
		472	FTP	(86)	558	GTP	(86)	644	FMULT	(2)
		646	GMULT	(2)	648	JHOL	(2)	650	XP1	(120)
		770	YP1	(120)	890	IHOL	(100)			

STATISTICS

```
PROGRAM LENGTH                238      19
CM LABELED COMMON LENGTH    17368     990
      520008 CM USED
```

```
      FUNCTION IH2H2P(ARG,H2,H2P)
C
      COMMON /XFACT/XFACT(150),FT(43),GT(43),FTP(43),GTP(43),
     +  FMULT,GMULT,JHOL(2),XPI(120),YPI(120),IHOL(100)
C
      DIMENSION HFT(43),HGT(43),HFTP(43),HGTP(43)
C
      COMPLEX ARG,HFT,HGT,HFTP,HGTP,F,G,FP,GP,
     +  FT,GT,FTP,GTP,HC,HPC
C
      DOUBLE PRECISION A,B,FMULT,GMULT
      DOUBLE PRECISION H2,H2P,XFACT
C
      DATA SQRT3/1.732050808/
C
C     PRINT 9,' ARG=',ARG
      IH2H2P=1
      A=1.
      B=1.
      F=1.
      G=ARG
      FP=0.
      GP=1.
      HFTMAX=0.
      HGTMAX=0.
C
  100 DO 200 K=1,43
      A=A*(3*K-2)
      B=B*(3*K-1)
      HFTK=((-1.)**K)*A/XFACT(3*K)
      HFT(K)=HFTK*ARG**(3*K)
      HGTK=((-1.)**K)*B/XFACT(3*K+1)
      HGT(K)=HGTK*ARG**(3*K+1)
      IF(CABS(HFT(K)).GT.ABS(HFTMAX))HFTMAX=CABS(HFT(K))
      IF(CABS(HGT(K)).GT.ABS(HGTMAX))HGTMAX=CABS(HGT(K))
      F=F+HFT(K)
      G=G+HGT(K)
      HFTPK=((-1.)**K)*(3.*K)*A/XFACT(3*K)
      HFTP(K)=HFTPK*ARG**(3*K-1)
      HGTPK=((-1.)**K)*(3.*K+1)*B/XFACT(3*K+1)
      HGTP(K)=HGTPK*ARG**(3*K)
      FP=FP+HFTP(K)
      GP=GP+HGTP(K)
C     PRINT 191,K,A,B,HFT(K),HGT(K),F,G,HFTP(K),HGTP(K),FP,GP
  191 FORMAT(I5,1X,10D11.3)
  200 CONTINUE
      DO 202 I=1,5
      IF(1000.*CABS(HFT(3*I+1)).LT.ABS(HFTMAX).AND.
     +   1000.*CABS(HGT(3*I+1)).LT.ABS(HGTMAX))GO TO 208
  202 CONTINUE
      GO TO 204
  208 CONTINUE
      GO TO 220
  204 PRINT 205,ARG,F,G
  205 FORMAT(' MORE ACC. REQUIRED IN IH2H2P CALC.',/,' ARG =',2E10.3,/,
     +  ' F =',2E15.6,/,' G =',2E15.6)
      PRINT 206,(I,HFT(I),HGT(I),I=1,43)
  206 FORMAT(I5,4E15.6)
```

```
                      IH2H2P=-1
                      GO TO 999
     00         220   F=FMULT*F
                      G=GMULT*G
                      FP=FMULT*FP
                      GP=GMULT*GP
                      HC=G-CMPLX(0.,1.)*(G-2.*F)/SQRT3
     03               HPC=GP-CMPLX(0.,1.)*(GP-2.*FP)/SQRT3
                      H2=CABS(HC)
                      H2P=CABS(HPC)
                C210  PRINT 211,K,A,B,F,G,FP,GP,H2,H2P
                211   FORMAT(' K=',A,B,F,G,FP,GP,H2,H2P='/
     70               +  13,2X,9D10.3)
                999   RETURN
                      END
```

SYMBOLIC REFERENCE MAP (R=3)

ENTRY POINTS DEF.LINE REFERENCES
5 IH2H2P 1 71

VARIABLES SN TYPE RELOCATION

471	A	DOUBLE			REF	11	28	30	38	DEFINED	18	28
0	ARG	COMPLEX		F.P.	REF	8	21	31	33	39	41	55
					DEFINED	1						
473	B	DOUBLE			REF	11	29	32	40	DEFINED	19	29
455	F	COMPLEX			REF	8	36	53	60	64		
					DEFINED	20	36	60				
1204	FMULT	DOUBLE		XFACT	REF	3	11	60	62			
461	FP	COMPLEX			REF	8	42	62	65	DEFINED	22	42
					62							
454	FT	COMPLEX	ARRAY	XFACT	REF	3	8					
730	FTP	COMPLEX	ARRAY	XFACT	REF	3	8					
457	G	COMPLEX			REF	8	37	53	61	2968		
					DEFINED	21	37	61				
1206	GMULT	DOUBLE		XFACT	REF	3	11	61	63			
463	GP	COMPLEX			REF	8	43	63	2965	DEFINED	23	43
					63							
602	GT	COMPLEX	ARRAY	XFACT	REF	3	8					
1056	GTP	COMPLEX	ARRAY	XFACT	REF	3	8					
465	HC	COMPLEX			REF	8	66	DEFINED	64			
505	HFT	COMPLEX	ARRAY		REF	6	8	2934	36	48	96	
					DEFINED	31						
500	HFTK	REAL			REF	31	DEFINED	30				
475	HFTMAX	REAL			REF	34	48	DEFINED	24	34		
761	HFTP	COMPLEX	ARRAY		REF	6	8	42	DEFINED	39		
502	HFTPK	REAL			REF	39	DEFINED	38				
633	HGT	COMPLEX	ARRAY		REF	6	8	2935	37	48	96	
					DEFINED	33						
501	HGTK	REAL			REF	33	DEFINED	32				
476	HGTMAX	REAL			REF	35	48	DEFINED	25	35		
1107	HGTP	COMPLEX	ARRAY		REF	6	8	43	DEFINED	41		
503	HGTPK	REAL			REF	41	DEFINED	40				

```
      FUNCTION IH2H2P      73/74   OPT=1 TRACE                     FTN 4.6+452       77/09/22. 13.59.33        PAGE    3

 VARIABLES     SN   TYPE                RELOCATION
   467  HPC         COMPLEX                          REFS        8        67    DEFINED       65
     0  H2          DOUBLE                  F.P.     REFS       12   DEFINED          1       66
     0  H2P         DOUBLE                  F.P.     REFS       12   DEFINED          1       67
   504  I           INTEGER                          REFS     2048      3056    DEFINED       47       56
  1572  IHOL        INTEGER     ARRAY      XFACT     REFS        3
   456  IH2H2P      INTEGER                       DEFINED       17        56
  1210  JHOL        INTEGER     ARRAY      XFACT     REFS        3
   477  K           INTEGER                          REFS       28        29       2030       2031       2032       2033       2034
                                                     2035       36        37       3038       2039       3040       2041         48
                                                       43  DEFINED        27
   402  SORT3       REAL                             REFS       64        65    DEFINED       14
     0  XFACT       DOUBLE      ARRAY      XFACT     REFS        3        12         30         32         38         40
  1212  XP1         REAL        ARRAY      XFACT     REFS        3
  1002  YP1         REAL        ARRAY      XFACT     REFS        3

 FILE NAMES         MODE
        OUTPUT      FMT                   WRITES       53         56

 EXTERNALS          TYPE    ARGS          REFERENCES
        CABS        REAL      1 LIBRARY     2034        2035       2040       66         67

 INLINE FUNCTIONS   TYPE    ARGS    DEF LINE   REFERENCES
        ABS         REAL      1 INTRIN               34         35       2040
        CMPLX       COMPLEX   2 INTRIN               64         65

 STATEMENT LABELS               DEF LINE   REFERENCES
     0  100         INACTIVE        27
   403  191     FMT  NO REFS        45
     0  200                         46         27
   271  202                         51         47         48
   276  204                         53         50
   414  205     FMT                 54         53
   436  206     FMT                 57         56
   441  211     FMT  NO REFS        69
   321  220                         60         52
   360  999                         71         59

 LOOPS  LABEL     INDEX     FROM-TO     LENGTH     PROPERTIES
    34  200       0 K        27 46       213B            EXT REFS
   251  202       0 I        47 51        23B            EXT REFS    EXITS
   303            0 I        56 56        11B            EXT REFS

 COMMON BLOCKS   LENGTH     MEMBERS - BIAS NAME(LENGTH)
        XFACT      998             0 XFACT   (300)           300 FT      (86)            386 GT      (86)
                                 472 FTP     (86)            558 GTP     (86)            644 FMULT   (2)
                                 646 GMULT   (2)             648 JHOL    (2)             650 XP1     (120)
                                 770 YP1     (120)           890 IHOL    (108)

 STATISTICS
   PROGRAM LENGTH                   1246B      678
   CM LABELED COMMON LENGTH         1736B      990
          52000B CM USED
```

```
      1              FUNCTION ISUB1(S00,S2D,S000,PI,PA3,DILOOP,C1,XNUM,XDEN)
                     C
                     COMPLEX S2D
                     DOUBLE PRECISION H2,H2P,EYP,EYM,S00,S000,A00,B00
      5              DOUBLE PRECISION A00P,B00P,A000,B000,A000P,B000P
                     DOUBLE PRECISION XNUM,XDEN,C1,PA3
                     C
                     ISUB1=0
                     S2D=S00*CEXP(CMPLX(0.,2.*PI/3.))*PA3**2
     10              IF(IH2H2P(S2D,H2,H2P).NE.1)GO TO 970
                     EYP=H2/H2P
                     S000=S00+DILOOP*C1**(1./3.)
                     IF(IABIJ(-S00,A00,B00,A00P,B00P).NE.1)GO TO 970
                     IF(IABIJ(-S000,A000,B000,A000P,B000P).NE.1)GO TO 970
     15              EYM=(A00-(A000/B000)*B00)/(A00P-(A000/B000)*B00P)
                     XNUM=EYM-PA3*EYP
                     C    XDEN=1.+PA3-S000EYM**2-PA3*S2D*EYP**2
                     XDEN=(1.+S00*EYM**2)+PA3*(1.+S2D*EYP**2)
                     GO TO 999
     20              C
                   970 ISUB1=-1
                     C
                   999 RETURN
                     END
```

SYMBOLIC REFERENCE MAP (R=3)

ENTRY POINTS — DEF LINE — REFERENCES

5	ISUB1	1	23

VARIABLES	SN	TYPE	RELOCATION									
212	A00	DOUBLE		REFS	4	13	15					
216	A00P	DOUBLE		REFS	5	13	15					
222	A000	DOUBLE		REFS	5	14	2*15					
226	A000P	DOUBLE		REFS	5	14						
214	B00	DOUBLE		REFS	4	13	15					
220	B00P	DOUBLE		REFS	5	13	15					
224	B000	DOUBLE		REFS	5	14	2*15					
230	B000P	DOUBLE		REFS	5	14						
0	C1	DOUBLE	F.P.	REFS	6	12	DEFINED	1				
0	DILOOP	REAL	F.P.	REFS	12	DEFINED	1					
210	EYM	DOUBLE		REFS	4	16	18	DEFINED	15			
206	EYP	DOUBLE		REFS	4	16	18	DEFINED	11			
202	H2	DOUBLE		REFS	4	10	11					
204	H2P	DOUBLE		REFS	4	10	11					
201	ISUB1	INTEGER		DEFINED	8	21						
0	PA3	DOUBLE	F.P.	REFS	6	9	16	18	DEFINED	1		
0	PI	REAL	F.P.	REFS	9	DEFINED	1					
0	S00	DOUBLE	F.P.	REFS	4	9	12	13	18			
				DEFINED	1							
0	S000	DOUBLE	F.P.	REFS	4	14	DEFINED	1	12			
0	S2D	COMPLEX	F.P.	REFS	3	10	10	DEFINED	1	9		
0	XDEN	DOUBLE	F.P.	REFS	6	DEFINED	1	18				

```
      FUNCTION ISUB1        73/74   OPT=1 TRACE                              FTN 4.6+452        77/09/22, 15.59.33        PAGE    2

VARIABLES      SN  TYPE                RELOCATION
    0  XNUM        DOUBLE                 F.P.          REFS        6   DEFINED        1     -16-

EXTERNALS              TYPE    ARGS            REFERENCES
        CEXP       COMPLEX       1  LIBRARY        9
        IABIJ      INTEGER       5                13          14
        IM2M2P     INTEGER       3                10

INLINE FUNCTIONS       TYPE    ARGS          DEF LINE  REFERENCES
        CMPLX      COMPLEX       2  INTRIN                    9

STATEMENT LABELS                       DEF LINE   REFERENCES
   144  970                               21         10          13          14
   145  999                               23         19

STATISTICS
   PROGRAM LENGTH                       232B      154
      52000B CM USED
```

Appendix I
I-14

233

THIS PAGE IS BEST QUALITY PRACTICABLE
FROM COPY FURNISHED TO DDC

```
              SUBROUTINE IPLOT(ICNT,XLIXL,DILOOP,PAILOOP)
      C
              COMMON /XFACT/XFACT(150),FT(43),GT(43),FTP(43),GTP(43),
          +   FMULT,GMULT,JHOL(2),XP1(120),YP1(120),IHOL(100)
      C
      C
              DOUBLE PRECISION XFACT,FT,GT,FTP,GTP,FMULT,GMULT
              CALL TITLE("S",100,"                    S",100,
          +   "               ILS",100,12.,8.)
              XMSG=7.
              YMSG=9.
              YMSG=YMSG-.25
              ENCODE(100,111,IHOL)XLIXL
        111   FORMAT("IL*    = ",F6.1,"S")
              CALL MESSAG(IHOL,100,XMSG,YMSG)
              YMSG=YMSG-.25
              ENCODE(100,121,IHOL)DILOOP
        121   FORMAT("D    = ",F6.1,"S")
              CALL MESSAG(IHOL,100,XMSG,YMSG)
              YMSG=YMSG-.25
              ENCODE(100,131,IHOL)PAILOOP
        131   FORMAT("P/A = ",F6.1,"S")
              CALL MESSAG(IHOL,100,XMSG,YMSG)
              CALL GRAPH(-6.,1.,-8.,2.)
              CALL CURVE(XP1,YP1,ICNT,0)
              CALL ENDPL(0)
      C
        999   RETURN
              END
```

SYMBOLIC REFERENCE MAP (R=3)

ENTRY POINTS		DEF LINE	REFERENCES
4	IPLOT	1	28

VARIABLES

	VARIABLES	SN	TYPE		RELOCATION								
0	DILOOP		REAL		F.P.	REFS	17	DEFINED	1				
1204	FMULT		DOUBLE		XFACT	REFS	3	7					
454	FT		DOUBLE	ARRAY	XFACT	REFS	3	7					
730	FTP		DOUBLE	ARRAY	XFACT	REFS	3	7					
1206	GMULT		DOUBLE		XFACT	REFS	3	7					
602	GT		DOUBLE	ARRAY	XFACT	REFS	3	7					
1056	GTP		DOUBLE	ARRAY	XFACT	REFS	3	7					
0	ICNT		INTEGER		F.P.	REFS	25	DEFINED	1				
1572	IHOL		INTEGER	ARRAY	XFACT	REFS	3	15	19	23	DEFINED	13	17
							21						
1210	JHOL		INTEGER	ARRAY	XFACT	REFS	3						
0	PAILOOP		REAL		F.P.	REFS	21	DEFINED	1				
0	XFACT		DOUBLE	ARRAY	XFACT	REFS	3	7					
0	XLIXL		REAL		F.P.	REFS	13	DEFINED	1				
142	XMSG		REAL			REFS	15	19	23	DEFINED	11		
1212	XP1		REAL	ARRAY	XFACT	REFS	3	25					
143	YMSG		REAL			REFS	12	15	16	19	20	25	

Appendix I
I-15

VARIABLES SN TYPE RELOCATION

				DEFINED	11	12	16	20
1002 YP1	REAL	ARRAY	XFACT	REFS	5	25		

EXTERNALS	TYPE	ARGS	REFERENCES		
CURVE		8	25		
ENDPL		1	26		
GRAPH		4	24		
MESSAG		8	15	19	23
TITLE		8	8		

STATEMENT LABELS			DEF LINE	REFERENCES
100	111	FMT	16	13
111	121	FMT	18	17
122	131	FMT	22	21
0	999	INACTIVE	28	

COMMON BLOCKS	LENGTH	MEMBERS = BIAS NAME(LENGTH)					
XFACT	990	0 XFACT	(300)	300 FT	(86)	386 GT	(86)
		472 FTP	(86)	558 GTP	(86)	644 FMULT	(2)
		646 GMULT	(2)	648 IMUL	(2)	650 XP1	(120)
		770 YP1	(120)	890 IMUL	(100)		

STATISTICS

PROGRAM LENGTH	1548	108
CM LABELED COMMON LENGTH	17360	990

520000 CM USED

FWA OF THE LOAD 111
LWA+1 OF THE LOAD 57415

TRANSFER ADDRESS -- JOBY 5437

PROGRAM AND BLOCK ASSIGNMENTS.

BLOCK	ADDRESS	LENGTH	FILE	DATE	PROCESSOR	VER LEVEL	HARDWARE	COMMENTS
/XFACT/	111	1736						
JOBY	2047	4317	LGO	77/09/22	FTN	4.6 452	666X I	PROGRAM OPT=1 TRACE
IABIJ	6366	472	LGO	77/09/22	FTN	4.6 452	666X I	FUNCTION OPT=1 TRACE
CFACT	7060	23	LGO	77/09/22	FTN	4.6 452	666X I	SUBROUTINE OPT=1 TRACE
IH2M2P	7103	1246	LGO	77/09/22	FTN	4.6 452	666X I	FUNCTION OPT=1 TRACE
IBIIA1	10351	232	LGO	77/09/22	FTN	4.6 452	666X I	FUNCTION OPT=1 TRACE
IPLOT	10603	154	LGO	77/09/22	FTN	4.6 452	666X I	SUBROUTINE OPT=1 TRACE
MGAMMA	10757	573	UL=INSLIB	77/08/26	FTN	4.6 428	666X I	OPT=1
UERTST	11552	76	UL=INSLIB	77/08/26	FTN	4.6 428	666X I	OPT=1
MONIT	11650	41	UL=INSLIB	77/08/26	FTN	4.6 428	666X I	OPT=1
ALFNET	11711	3	UL=DISSPLA	76/03/29	FTN	4.5 410	666X I	OPT=1
/GWPLOT/	11716	74						
/GW191/	12010	50						
/GW171/	12060	72						
/GW1712/	12152	43						
/GGALK1/	12215	5						
/GGLEGD/	12222	13						
/GGALFD/	12235	4						
/GWALFP/	12241	4						
/GGAPP/	12245	2						
/GGSIS/	12247	70						
/GGMPR2/	12337	11						
/GWANGX/	12350	24						
BGNPL	12374	1154	UL=DISSPLA	76/03/29	FTN	4.5 410	666X I	OPT=1
COMPRS	13550	11	UL=DISSPLA	76/03/29	FTN	4.5 410	666X I	OPT=1
CROSS	13561	4	UL=DISSPLA	76/03/29	FTN	4.5 410	666X I	OPT=1
CURVE	13565	22	UL=DISSPLA	76/03/29	FTN	4.5 410	666X I	OPT=1
DONEPL	13607	24	UL=DISSPLA	76/03/29	FTN	4.5 410	666X I	OPT=1
/GGCR2/	13633	2						
ENDPL	13635	624	UL=DISSPLA	76/03/29	FTN	4.5 410	666X I	OPT=1
GRAPH	14461	162	UL=DISSPLA	76/03/29	FTN	4.5 410	666X I	OPT=1
INTAXS	14643	24	UL=DISSPLA	76/03/29	FTN	4.5 410	666X I	OPT=1
IGGASC	14667	43	UL=DISSPLA	76/03/29	FTN	4.5 410	666X I	OPT=1
IGGA6	14732	25	UL=DISSPLA	76/03/29	COMPASS	3. 2-410		
/GGENTP/	14757	17						
IGGEOF	14776	52	UL=DISSPLA	76/03/29	FTN	4.5 410	666X I	OPT=1
IGGFNT	15050	710	UL=DISSPLA	76/03/29	FTN	4.5 410	666X I	OPT=1
IGGFRG	15760	14	UL=DISSPLA	76/03/29	FTN	4.5 410	666X I	OPT=1
/GGDAT/	15774	67						
IGGJOB	16063	767	UL=DISSPLA	76/03/29	FTN	4.5 410	666X I	OPT=1
IGGJIB	17052	133	UL=DISSPLA	76/03/29	FTN	4.5 410	666X I	OPT=1
IGGLC4	17205	16	UL=DISSPLA	76/03/29	FTN	4.5 410	666X I	OPT=1
IGGNBL	17223	101	UL=DISSPLA	76/03/29	FTN	4.5 410	666X I	OPT=1
/GGLUST/	17324	7						
IGGBLU	17333	117	UL=DISSPLA	76/03/29	FTN	4.5 410	666X I	OPT=1

APPENDIX II

Computer program for the Hankel (16) Function of second kind and one third order.

```
C  09-15-77  /J087/
      PROGRAM J087(INPUT,OUTPUT,TAPE11)
C
      COMMON /XFACT/XFACT(150),FT(43),GT(43),FTP(43),GTP(43),
     +  FMULT,GMULT,JHOL(2)
C
      COMPLEX S2D,S2DT1,S2DT2
C
      DOUBLE PRECISION C1,FMULT,FT,FTP,GMULT,GT,GTP
      DOUBLE PRECISION PA3,S00,S000,S11,XDEN,XFACT,XK0,XNUM
      DOUBLE PRECISION S0001,S0002,XNUM1,XNUM2,XDEN1,XDEN2
C
      DIMENSION D(4),P(4),PA(4),XL(3)
C
      DATA D/42.,85.,94.,72./
      DATA P/.630E-6,.481E-6,.07E-6,.0445E-6/
      DATA PA/4.8,3.61,.574,.344/
      DATA XL/.1,.2,.5/
      DATA JHOL/10HAPPROX    ,10HEXACT     /
C
      CALL MGAMMA(2./3.,FMULT,IER1)
      CALL MGAMMA(4./3.,GMULT,IER2)
      FMULT=(2.**(1./3.))/FMULT
      GMULT=(2.**(1./3.))/(GMULT*3.**(2./3.))
C
      PI=4.*ATAN(1.)
      CALL CFACT
C
      PRINT (11,90)
   90 FORMAT(1H1,///,
     +"   LAMDA          D          P          P/A          S11",
     +"          S2D",/,
     +"   ------          -          -          ---          ---",
     +"          ---")
C
      DO 300 IXL=1,3
C
      DO 300 IDEN=1,2
C
      DO 300 ILOOP=1,1
      S2D=2.3381*CEXP(CMPLX(0.,2.*PI/3.))
      PRINT 95,XL(IXL),D(ILOOP),P(ILOOP),PA(ILOOP),S2D
   95 FORMAT(1H1,//,
     + " LAMDA        = ",F10.6,/,
     + " D            = ",F10.6,/,
     + " P            = ",F10.6,/,
     + " P/A          = ",F10.6,/,
     + " S2D          = ",1H(,F10.6," ,",F10.6,1H),////)
      PA3=PA(ILOOP)**(1./3.)
      XK0=2.*PI/XL(IXL)
      C1=P(ILOOP)*XK0**2
      S00=CABS(S2D)/PA3**2
      PRINT 96,JHOL(IDEN)
   96 FORMAT(//,2X,A10,/,
     +"     I     S2D          ",13X,"     S000          S11")
      PRINT 97
   97 FORMAT("     -     ---          ",13X,"     ----          ---")
```

```
      C
  100 DO 200 I=1,1
      C
      DO 120 J=1,100
      S00=-5.+(J-1)*.1
      IF(S00.EQ.0)GO TO 120
      IF(ISUB1(S00,SZ0,S000,PI,PA3,D(ILOOP),C1,XNUM,XDEN)
     + .NE.0)GO TO 300
      PRINT 121,S00,XNUM
  121 FORMAT(" S00=",G15.6," XNUM=",G15.6)
  120 CONTINUE
      IF(IDER.EQ.2)GO TO 150
      IF(ISUB1(.99999*S00,SZ0T1,S0001,PI,PA3,D(ILOOP),C1,XNUM1,XDEN1)
     + .NE.0)GO TO 300
      IF(ISUB1(1.00001*S00,SZ0T2,S0002,PI,PA3,D(ILOOP),C1,XNUM2,XDEN2)
     + .NE.0)GO TO 300
      IF(IDER.EQ.1)S11=S00-XNUM/((XNUM2-XNUM1)/(.00002*S00))
  150 IF(IDER.EQ.2)S11=S00-XNUM/XDEN
      PRINT 111,I,SZ0,S000,S11
  111 FORMAT(
     + I4,1H(,2G15.8,1H),3G15.8)
  200 S00=S11
      C
      PRINT (11,250)XL(IXL),D(ILOOP),P(ILOOP),PA(ILOOP),S11,SZ0,
     + JHOL(IDER)
  250 FORMAT(5G12.6,1H(,2G12.6,1H),2X,A10)
  300 CONTINUE
      PRINT(11,311)
  311 FORMAT(1H1)
  999 ENDFILE 11
      END
```

CARD NR. SEVERITY DETAILS DIAGNOSIS OF PROBLEM

CARD NR.	SEVERITY	DETAILS	DIAGNOSIS OF PROBLEM
40	I		LOWER LIMIT .GE. UPPER LIMIT, ONE TRIP LOOP.
50	I		LOWER LIMIT .GE. UPPER LIMIT, ONE TRIP LOOP.

SYMBOLIC REFERENCE MAP (R=3)

ENTRY POINTS		DEF LINE	REFERENCES
3212	JOOT	2	

VARIABLES		SN	TYPE		RELOCATION							
3774	C1		DOUBLE			REFS	9	64	70	72	DEFINED	61
4040	D		REAL	ARRAY		REFS	13	42	64	70	72	81
						DEFINED	15					
1204	FMULT		DOUBLE		XFACT	REFS	4	9	21	23	DEFINED	23
454	FT		DOUBLE	ARRAY	XFACT	REFS	4	9				
730	FTP		DOUBLE	ARRAY	XFACT	REFS	4	9				
1206	GMULT		DOUBLE		XFACT	REFS	4	9	22	24	DEFINED	24

VARIABLES	SN	TYPE	RELOCATION										
602	GT	DOUBLE	ARRAY	XFACT	REFS	4	9						
1056	GTP	DOUBLE	ARRAY	XFACT	REFS	4	9						
4036	I	INTEGER			REFS	76	DEFINED	59					
4034	IDER	INTEGER			REFS	53	69	74	75	81			
					DEFINED	38							
4030	IER1	INTEGER			REFS	21							
4031	IER2	INTEGER			REFS	22							
4035	ILOOP	INTEGER			REFS	3042	49	51	64	70	72	3081	
					DEFINED	40							
4033	IXL	INTEGER			REFS	42	50	81	DEFINED	36			
4037	J	INTEGER			REFS	62	DEFINED	61					
1210	JHOL	INTEGER	ARRAY	XFACT	REFS	4	53	81	DEFINED	19			
4044	P	REAL	ARRAY		REFS	13	42	51	81	DEFINED	16		
4050	PA	REAL	ARRAY		REFS	13	42	49	81	DEFINED	17		
3776	PA3	DOUBLE			REFS	10	52	64	70	72			
					DEFINED	49							
4032	PI	REAL			REFS	41	50	64	70	72			
					DEFINED	26							
4000	S00	DOUBLE			REFS	10	63	64	66	70	72	3078	
					75	DEFINED	52	62	79				
4002	S000	DOUBLE			REFS	10	64	76					
4014	S0001	DOUBLE			REFS	11	70						
4016	S0002	DOUBLE			REFS	11	72						
4004	S11	DOUBLE			REFS	10	76	79	81	DEFINED	84	88	
3766	S2D	COMPLEX			REFS	7	42	52	64	76	81		
					DEFINED	41							
3770	S2DT1	COMPLEX			REFS	7	70						
3772	S2DT2	COMPLEX			REFS	7	72						
4006	XDEN	DOUBLE			REFS	10	64	75					
4024	XDEN1	DOUBLE			REFS	11	70						
4026	XDEN2	DOUBLE			REFS	11	72						
0	XFACT	DOUBLE	ARRAY	XFACT	REFS	4	10						
4010	XK0	DOUBLE			REFS	10	51	DEFINED	30				
4054	XL	REAL	ARRAY		REFS	13	42	50	81	DEFINED	18		
4012	XNUM	DOUBLE			REFS	10	64	66	74	75			
4020	XNUM1	DOUBLE			REFS	11	70	74					
4022	XNUM2	DOUBLE			REFS	11	72	74					

FILE NAMES		MODE								
0	INPUT									
1054	OUTPUT	FMT		WRITES	42	53	56	66	76	
2130	TAPE11	FMT		WRITES	29	81	85	NOFION	87	

EXTERNALS	TYPE	ARGS		REFERENCES		
ATAN	REAL	1	LIBRARY	26		
CABS	REAL	1	LIBRARY	52		
CEXP	COMPLEX	1	LIBRARY	41		
CFACT		0		27		
ISUB1	INTEGER	9		64	70	78
MGAMMA		3		21	22	

INLINE FUNCTIONS	TYPE	ARGS	DEF LINE	REFERENCES
CMPLX	COMPLEX	2	INTRIN	41

STATEMENT LABELS		DEF LINE	REFERENCES
3576 90	FMT	30	29
3630 95	FMT	43	42

```
       PROGRAM J087        73/74    OPT=1 TRACE                    FTN 4.6+452         77/09/16. 09.40.39        PAGE     4

   STATEMENT LABELS                 DEF LINE   REFERENCES
     3656  96        FMT              54         53
     3671  97        FMT              57         56
        0 100               INACTIVE  59
     3721 111        FMT              77         76
     3350 120                         68         61       63
     3705 121        FMT              67         66
     3431 150                         75         69
        0 200                         79         59
     3737 250        FMT              83         81
     3467 300                         84         36       38        40        64        70        72
     3746 311        FMT              86         85
        0 999               INACTIVE  87

   LOOPS  LABEL     INDEX       FROM-TO     LENGTH    PROPERTIES
    3251  300     $ IXL          36 84       224B              EXT REFS  NOT INNER
    3253  300     $ IDER         38 84       217B              EXT REFS  NOT INNER
    2400  300     $ ILOOP        40            6B              EXT REFS  NOT INNER
    3540  200     $ I            58            6B              EXT REFS    EXITS   NOT INNER
    3335  120     $ J            61 68        16B              EXT REFS    EXITS

   COMMON BLOCKS   LENGTH       MEMBERS - BIAS NAME(LENGTH)
         XFACT       650                0 XFACT   (300)               300 FT      (86)              386 GT      (86)
                                      472 FTP     (86)                558 GTP     (86)              644 FMULT   (2)
                                      646 BMULT   (2)                 648 JMOL    (2)

   STATISTICS
     PROGRAM LENGTH                   653B      427
     BUFFER LENGTH                  32000B    16608
     CM LABELED COMMON LENGTH        1212B      650
            52000B CM USED
```

```
      FUNCTION IABIJ(ARG,AI,BI,AIP,BIP)
C
      COMMON /XFACT/XFACT(150),FT(43),GT(43),FTP(43),GTP(43),
     +  FMULT,GMULT,JHOL(2)
C
      DOUBLE PRECISION A,AI,AIP,ARG,B,BI,BIP,F,FMULT
      DOUBLE PRECISION FP,FT,FTP,G,GMULT,GP,GT,GTP,XFACT
C
      DATA C1,C2,SQRT3/.3550280538,.2588194037,1.732050808/
C
C     PRINT 9,0 ARG=0,ARG
      IABIJ=1
      A=1.
      B=1.
      F=1.
      G=ARG
      FP=0.
      GP=1.
      FTMAX=GTMAX=0
C
  100 DO 200 K=1,43
      A=A*(3*K-2)
      B=B*(3*K-1)
      FT(K)=A*ARG**(3*K)/XFACT(3*K)
      GT(K)=B*ARG**(3*K+1)/XFACT(3*K+1)
      IF(DABS(FT(K)).GT.ABS(FTMAX))FTMAX=DABS(FT(K))
      IF(DABS(GT(K)).GT.ABS(GTMAX))GTMAX=DABS(GT(K))
      F=F+FT(K)
      G=G+GT(K)
      FTP(K)=(3.*K)*A*ARG**(3*K-1)/XFACT(3*K)
      GTP(K)=(3.*K+1)*B*ARG**(3*K)/XFACT(3*K+1)
      FP=FP+FTP(K)
      GP=GP+GTP(K)
C     PRINT 191,K,A,B,FT(K),GT(K),F,G,FTP(K),GTP(K),FP,GP
  191 FORMAT(I3,1X,10D11.3)
  200 CONTINUE
      DO 202 I=1,5
      IF(1000.*DABS(FT(38+I)).LT.ABS(FTMAX).AND.
     +   1000.*DABS(GT(38+I)).LT.ABS(GTMAX))GO TO 202
      GO TO 204
  202 CONTINUE
      GO TO 220
  204 PRINT 205,ARG,F,G
  205 FORMAT(0 MORE ACC. REQUIRED IN AIRY CALC.0,/,0 ARG= 0,D16.3,//,
     +  0 F= 0,D15.6,//,0 G= 0,D15.6)
      PRINT 206,(I,FT(I),GT(I),I=1,43)
  206 FORMAT(I5,2D15.6)
      IABIJ=-1
      GO TO 999
  220 AI=C1*F-C2*G
      BI=SQRT3*(C1*F+C2*G)
      AIP=C1*FP-C2*GP
      BIP=SQRT3*(C1*FP+C2*GP)
C210  PRINT 211,K,A,B,F,G,FP,GP,AI,BI,AIP,BIP
  211 FORMAT(0 K,A,B,F,G,FP,GP,AI,BI,AIP,BIP=,/,
     +  I3,2X,11D10.3)
  999 RETURN
```

```
      FUNCTION IABIJ      73/74    OPT=1 TRACE                    FTN 4.6+452       77/09/16. 09.40.39        PAGE     3

                          END

           SYMBOLIC REFERENCE MAP (R=3)

  ENTRY POINTS     DEF LINE     REFERENCES
      5  IABIJ         1           57

  VARIABLES       SN  TYPE            RELOCATION
     435  A           DOUBLE                       REFS        6       22      24      30  DEFINED      13      22
       0  AI          DOUBLE               F.P.    REFS        6  DEFINED       1      50
       0  AIP         DOUBLE               F.P.    REFS        6  DEFINED       1      52
       0  ARG         DOUBLE               F.P.    REFS        6       16      24      25      30      31      43
                                                DEFINED        1
     437  B           DOUBLE                       REFS        6       23      25      31  DEFINED      14      23
       0  BI          DOUBLE               F.P.    REFS        6  DEFINED       1      51
       0  BIP         DOUBLE               F.P.    REFS        6  DEFINED       1      53
     362  C1          REAL                         REFS       50       51      52      53  DEFINED       9
     363  C2          REAL                         REFS       50       51      52      53  DEFINED       9
     441  F           DOUBLE                       REFS        6       28      43      50      51
                                                DEFINED       15       28
    1204  FMULT       DOUBLE               XFACT   REFS        3        6
     443  FP          DOUBLE                       REFS        7       32      52      53  DEFINED      17      32
     454  FT          DOUBLE      ARRAY    XFACT   REFS        3        7    2026      28      38      46
                                                DEFINED       24
     451  FTMAX       REAL                         REFS       26       38  DEFINED      19      26
     730  FTP         DOUBLE      ARRAY    XFACT   REFS        3        7      32  DEFINED      38
     445  G           DOUBLE                       REFS        7       29      43      50      51
                                                DEFINED       16       29
    1206  GMULT       DOUBLE               XFACT   REFS        3        7
     447  GP          DOUBLE                       REFS        7       33      52      53  DEFINED      18      33
     602  GT          DOUBLE      ARRAY    XFACT   REFS        3        7    2027      29      38      46
                                                DEFINED       25
     452  GTMAX       REAL                         REFS       27       38  DEFINED      19      27
    1056  GTP         DOUBLE      ARRAY    XFACT   REFS        3        7      33  DEFINED      31
     454  I           INTEGER                      REFS     2030     3046  DEFINED      37      46
     434  IABIJ       INTEGER                   DEFINED       12       48
    1210  JHOL        INTEGER     ARRAY    XFACT   REFS        3
     453  K           INTEGER                      REFS       22       23    3024    3025    2026    2027      38
                                                          29     4030    4031      32      33  DEFINED      21
     364  SQRT3       REAL                         REFS       51       53  DEFINED       9
       0  XFACT       DOUBLE      ARRAY    XFACT   REFS        3        7      24      28      30      31

  FILE NAMES           MODE
         OUTPUT        FMT                  WRITES        43       46

  INLINE FUNCTIONS     TYPE    ARGS           DEF LINE   REFERENCES
         ABS           REAL      1   INTRIN                  26      27    2038
         DABS          DOUBLE    1   INTRIN               2026    2027    2038

  STATEMENT LABELS                    DEF LINE   REFERENCES
       0  100            INACTIVE        21
     365  191       FMT  NO REFS         35
       0  200                            36        21
```

STATEMENT LABELS

				DEF LINE	REFERENCES	
260	202			41	37	38
265	204			43	40	
376	205	FMT		44	43	
420	206	FMT		47	46	
423	211	FMT	NO REFS	55		
310	220			50	42	
352	999			57	49	

LOOPS

LABEL	INDEX	FROM-TO	LENGTH	PROPERTIES	
34 200	* K	21 36	201B		EXT REFS
237 202	* I	37 41	24B	OPT	EXITS
272	* I	46 46	11B		EXT REFS

COMMON BLOCKS

	LENGTH	MEMBERS - BIAS NAME(LENGTH)							
XFACT	650	0 XFACT	(300)	300 FT	(86)	386 ST	(86)		
		472 FTP	(86)	558 GTP	(86)	644 FMULT	(2)		
		646 GMULT	(2)	648 JMOL	(2)				

STATISTICS

PROGRAM LENGTH 4728 314

BN LABELED COMMON LENGTH 12128 650

520008 CM USED

```
      1          C
                 SUBROUTINE CFACT
                 C
                 COMMON /XFACT/XFACT(150),FT(43),ST(43),FTP(43),STP(43),
     2           *  FMULT,GMULT,JHOL(2)
                 C
                 DOUBLE PRECISION FMULT,FT,FTP,GMULT,ST,STP,XFACT
                 C
                 XFACT(1)=1
     10          DO 100 I=2,150
          100    XFACT(I)=XFACT(I-1)*I
                 C
          999    RETURN
                 END
```

SYMBOLIC REFERENCE MAP (R=3)

ENTRY POINTS

		DEF LINE	REFERENCES
2	CFACT	2	13

VARIABLES

		SN	TYPE		RELOCATION								
1204	FMULT		DOUBLE		XFACT	REFS	4	7					
454	FT		DOUBLE	ARRAY	XFACT	REFS	4	7					
730	FTP		DOUBLE	ARRAY	XFACT	REFS	4	7					
1206	GMULT		DOUBLE		XFACT	REFS	4	7					
602	GT		DOUBLE	ARRAY	XFACT	REFS	4	7					
1056	GTP		DOUBLE	ARRAY	XFACT	REFS	4	7					
22	I		INTEGER			REFS	3011	DEFINED	10				
1210	JHOL		INTEGER	ARRAY	XFACT	REFS	4						
0	XFACT		DOUBLE	ARRAY	XFACT	REFS	4	7	11	DEFINED	9	11	

STATEMENT LABELS

			DEF LINE	REFERENCES
0	100		11	10
0	999	INACTIVE	13	

LOOPS

LOOPS	LABEL	INDEX	FROM-TO	LENGTH	PROPERTIES
13	100	I	10 11	5B	INSTACK

COMMON BLOCKS

COMMON BLOCKS	LENGTH	MEMBERS - BIAS NAME(LENGTH)					
XFACT	650	0 XFACT (300)	300 FT (86)	386 ST (86)			
		472 FTP (86)	558 STP (86)	644 FMULT (2)			
		646 GMULT (2)	648 JHOL (2)				

STATISTICS

PROGRAM LENGTH	236	14
CM LABELED COMMON LENGTH	12120	650

520000B CM USED

```
      FUNCTION IH2H2P(ARG,H2,H2P)
C
      COMMON /XFACT/XFACT(150),FT(43),GT(43),FTP(43),GTP(43),
     +  FMULT,GMULT,JHOL(2)
C
      DIMENSION HFT(43),HGT(43),HFTP(43),HGTP(43)
C
      COMPLEX ARG,HFT,HGT,HFTP,HGTP,F,G,FP,GP,
     +  FT,GT,FTP,GTP,HC,HPC
C
      DOUBLE PRECISION A,B,FMULT,GMULT
      DOUBLE PRECISION H2,H2P,XFACT
C
      DATA SQRT3/1.732050808/
C
C     PRINT *,' ARG=',ARG
      PRINT *,'          ARG                    H2=',
     +' H2P='
      DO 300 J8=1,13
      DO 300 I8=1,13
      X=-7.+I8
      Y=-7.+J8
      IF(X.EQ.0.AND.Y.EQ.0)GO TO 300
      ARG=CMPLX(X,Y)
      IH2H2P=1
      A=1.
      B=1.
      F=1.
      G=ARG
      FP=0.
      GP=1.
      HFTMAX=0.
      HGTMAX=0.
C
  100 DO 200 K=1,43
      A=A*(3*K-2)
      B=B*(3*K-1)
      HFTK=((-1.)**K)*A/XFACT(3*K)
      HFT(K)=HFTK*ARG**(3*K)
      HGTK=((-1.)**K)*B/XFACT(3*K+1)
      HGT(K)=HGTK*ARG**(3*K+1)
      IF(CABS(HFT(K)).GT.ABS(HFTMAX))HFTMAX=CABS(HFT(K))
      IF(CABS(HGT(K)).GT.ABS(HGTMAX))HGTMAX=CABS(HGT(K))
      F=F+HFT(K)
      G=G+HGT(K)
      HFTPK=((-1.)**K)*(3.*K)*A/XFACT(3*K)
      HFTP(K)=HFTPK*ARG**(3*K-1)
      HGTPK=((-1.)**K)*(3.*K+1)*B/XFACT(3*K+1)
      HGTP(K)=HGTPK*ARG**(3*K)
      FP=FP+HFTP(K)
      GP=GP+HGTP(K)
C     PRINT 191,K,A,B,HFT(K),HGT(K),F,G,HFTP(K),HGTP(K),FP,GP
  191 FORMAT(I3,1X,10D11.3)
  200 CONTINUE
      DO 202 I=1,5
      IF(1000.*CABS(HFT(30+I)).LT.ABS(HFTMAX).AND.
     +  1000.*CABS(HGT(30+I)).LT.ABS(HGTMAX))GO TO 202
```

```
                      GO TO 204
                  26  CONTINUE
              60      GO TO 220
                 204  PRINT 205,ARG,F,G
                 205  FORMAT(' MORE ACC. REQUIRED IN IH2H2P CALC.',/,' ARG= ',2E10.3,/,
                     +  ' F= ',2E15.6,/,' G= ',2E15.6)
                      PRINT 206,(I,MPT(I),MGT(I),I=1,43)
              65      FORMAT(I5,4E15.6)
                      IH2H2P=-1
                      GO TO 999
                 220  F=FMULT*F
                      G=GMULT*G
              70      FP=FMULT*FP
                      GP=GMULT*GP
                      HC=G-CMPLX(0.,1.)*(G-2.*F)/SQRT3
                      HPC=GP-CMPLX(0.,1.)*(GP-2.*FP)/SQRT3
                      H2=CABS(HC)
              75      H2P=CABS(HPC)
                 C210 PRINT 211,K,A,B,F,G,FP,GP,H2,H2P
                 211  FORMAT(' K,A,B,F,G,FP,GP,H2,H2P=',/,
                     +  I3,2X,9D10.3)
                      PRINT 301,ARG,HC,HPC
              80      CONTINUE
                 301  FORMAT(3(1H(,2E15.6,2H) ))
                      STOP
                 999  RETURN
                      END
```


SYMBOLIC REFERENCE MAP (R=3)

ENTRY POINTS DEF LINE REFERENCES
5 IH2H2P 1 83

VARIABLES	SN	TYPE		RELOCATION									
534	A	DOUBLE			REFS	11	36	38	46	DEFINED	26	36	
0	ARG	COMPLEX		F,P,	REFS	8	29	39	41	47	49	61	
					79	DEFINED	1	24					
536	B	DOUBLE			REFS	11	37	40	48	DEFINED	27	37	
520	F	COMPLEX			REFS	8	44	61	68	72			
					DEFINED	28	44	68					
1204	FMULT	DOUBLE		XFACT	REFS	3	11	68	70				
524	FP	COMPLEX			REFS	8	58	70	73	DEFINED	38	60	
					70								
454	FT	COMPLEX	ARRAY	XFACT	REFS	3	8						
730	FTP	COMPLEX	ARRAY	XFACT	REFS	3	8						
522	G	COMPLEX			REFS	8	45	61	69	2072			
					DEFINED	29	45	69					
1206	GMULT	DOUBLE		XFACT	REFS	3	11	69	71				
526	GP	COMPLEX			REFS	8	51	71	2073	DEFINED	31	51	
					71								
602	GT	COMPLEX	ARRAY	XFACT	REFS	3	8						
1056	GTP	COMPLEX	ARRAY	XFACT	REFS	3	8						
530	HC	COMPLEX			REFS	8	74	79	DEFINED	73			

VARIABLES	SN	TYPE		RELOCATION							
554	HFT	COMPLEX	ARRAY		REFS	6	8	2042	44	56	60
					DEFINED	39					
547	HFTK	REAL			REFS	39	DEFINED	38			
544	HFTMAX	REAL			REFS	42	56	DEFINED	32	62	
1030	HFTP	COMPLEX	ARRAY		REFS	6	8	50	DEFINED	67	
551	HFTPK	REAL			REFS	47	DEFINED	46			
702	HGT	COMPLEX	ARRAY		REFS	6	8	2043	45	56	60
					DEFINED	41					
550	HGTK	REAL			REFS	41	DEFINED	40			
545	HGTMAX	REAL			REFS	43	56	DEFINED	33	63	
1156	HGTP	COMPLEX	ARRAY		REFS	6	8	51	DEFINED	69	
552	HGTPK	REAL			REFS	49	DEFINED	48			
532	HPC	COMPLEX			REFS	8	75	79	DEFINED	73	
0	H2	DOUBLE		F.P.	REFS	12	DEFINED	1	74		
0	H2P	DOUBLE		F.P.	REFS	12	DEFINED	1	75		
553	I	INTEGER			REFS	2056	3064	DEFINED	55	64	
517	IH2M2P	INTEGER			DEFINED	25	66				
541	I8	INTEGER			REFS	21	DEFINED	20			
1210	JHOL	INTEGER	ARRAY	XFACT	REFS	3					
540	J8	INTEGER			REFS	22	DEFINED	19			
546	K	INTEGER			REFS	34	37	2038	2039	2048	2041	2048
						2043	44	45	3046	2047	3048	2049	50
						51	DEFINED	35					
426	SQRT3	REAL			REFS	72	73	DEFINED	14			
542	X	REAL			REFS	23	24	DEFINED	21			
0	XFACT	DOUBLE	ARRAY	XFACT	REFS	3	12	38	40	46	48	
543	Y	REAL			REFS	23	24	DEFINED	22			

FILE NAMES	MODE						
OUTPUT	MIXED	WRITES	17	61	64	79	

EXTERNALS	TYPE	ARGS		REFERENCES				
CABS	REAL	8	LIBRARY	2042	2043	2056	74	75

INLINE FUNCTIONS	TYPE	ARGS		DEF LINE	REFERENCES		
ABS	REAL	8	INTRIN		42	43	2056
CMPLX	COMPLEX	2	INTRIN		24	72	73

STATEMENT LABELS				DEF LINE	REFERENCES		
0	100		INACTIVE	35			
433	191	FMT	NO REFS	53			
0	200			54	35		
305	202			59	55	56	
312	204			61	58		
444	205	FMT		62	61		
466	206	FMT		65	64		
471	211	FMT	NO REFS	77			
335	220			68	60		
374	300			80	19	20	23
505	301	FMT		81	79		
405	999			83	67		

LOOPS	LABEL		INDEX	FROM-TO	LENGTH	PROPERTIES		
23	300	*	J8	19 80	3578	EXT REFS	EXITS	NOT INNER
25	300	*	I8	20 80	3528	EXT REFS	EXITS	NOT INNER
50	200	*	K	35 54	2138	EXT REFS		
265	202	*	I	55 59	238	EXT REFS	EXITS	

Appendix II
II-12


```
FUNCTION IN2M2P     73/74    OPT=1 TRACE              FTN 4.6+452     77/09/16. 09.40.30     PAGE    4

LOOPS  LABEL    INDEX     FROM-TO    LENGTH    PROPERTIES
 317            I         64 64      11B                   EXT REFS

COMMON BLOCKS   LENGTH      MEMBERS - BIAS NAME(LENGTH)
   XFACT         650                0 XFACT  (300)      300 FT    (86)       386 GT     (86)
                                  472 FTP    (86)       558 GTP   (86)       644 FMULT  (2)
                                  646 GMULT  (2)        648 JMUL  (2)

STATISTICS
   PROGRAM LENGTH                  1327B     727
   CM LABELED COMMON LENGTH        1212B     650
         52000B CM USED
```

```
      1              FUNCTION ISUB1(S00,S2D,S000,PI,PA3,DILOOP,C1,XNUM,XDEN)
                     C
                     COMPLEX S2D
                     DOUBLE PRECISION H2,H2P,EYP,EYM,S00,S000,A00,B00
      5              DOUBLE PRECISION A00P,B00P,A000,B000,A000P,B000P
                     DOUBLE PRECISION XNUM,XDEN,C1,PA3
                     C
                     ISUB1=0
                     S2D=S000*CEXP(CMPLX(0.,2.*PI/3.))**PA3**2
     10              IF(IH2H2P(S2D,H2,H2P).NE.1)GO TO 970
                     EYP=H2/H2P
                     S000=S00+DILOOP*C1**(1./3.)
                     IF(IABIJ(-S00,A00,B00,A00P,B00P).NE.1)GO TO 970
                     IF(IABIJ(-S000,A000,B000,A000P,B000P).NE.1)GO TO 970
     15              EYM=(A00-(A000/B000)*B00)/(A00P-(A000/B000)*B00P)
                     XNUM=EYM+PA3*EYP
                     C      XDEN=1.+PA3-S000*EYM**2+PA3**2*S2D*EYP**2
                     XDEN=(1.+S000*EYM**2)+PA3*(1.+S2D*EYP**2)
                     GO TO 999
     20              C
                  970 ISUB1=-1
                     C
                  999 RETURN
                     END
```

SYMBOLIC REFERENCE MAP (R=3)

ENTRY POINTS

		DEF LINE	REFERENCES
5	ISUB1	1	23

VARIABLES

	SN	TYPE	RELOCATION								
212	A00	DOUBLE		REFS	4	13	15				
216	A00P	DOUBLE		REFS	5	13	15				
222	A000	DOUBLE		REFS	5	14	2*15				
226	A000P	DOUBLE		REFS	5	14					
214	B00	DOUBLE		REFS	4	14	16				
220	B00P	DOUBLE		REFS	5	13	16				
224	B000	DOUBLE		REFS	5	14	2*15				
230	B000P	DOUBLE		REFS	5	14					
0	C1	DOUBLE	F.P.	REFS	6	12	DEFINED	1			
0	DILOOP	REAL	F.P.	REFS	12	DEFINED	1				
210	EYM	DOUBLE		REFS	4	16	18	DEFINED	15		
206	EYP	DOUBLE		REFS	4	16	18	DEFINED	11		
202	H2	DOUBLE		REFS	4	10	11				
204	H2P	DOUBLE		REFS	4	10	11				
201	ISUB1	INTEGER		DEFINED	8	21					
0	PA3	DOUBLE	F.P.	REFS	6	9	16	18	DEFINED	1	
0	PI	REAL	F.P.	REFS	9	DEFINED	1				
0	S00	DOUBLE	F.P.	REFS	4	9	12	13	18		
				DEFINED	1						
0	S000	DOUBLE	F.P.	REFS	4	14	DEFINED	1	12		
0	S2D	COMPLEX	F.P.	REFS	3	10	18	DEFINED	1	0	
0	XDEN	DOUBLE	F.P.	REFS	6	DEFINED	1	18			

Appendix II
II-14

MISSION
of
Rome Air Development Center

RADC plans and conducts research, exploratory and advanced development programs in command, control, and communications (C^3) activities, and in the C^3 areas of information sciences and intelligence. The principal technical mission areas are communications, electromagnetic guidance and control, surveillance of ground and aerospace objects, intelligence data collection and handling, information system technology, ionospheric propagation, solid state sciences, microwave physics and electronic reliability, maintainability and compatibility.

Printed by
United States Air Force
Hanscom AFB, Mass. 01731