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(6) Statistical Methods in Search

by

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Abstract

The work under ONR contract N00014-77-C-0511 is described. This work consists of the development of a number of statistical techniques useful in solving search problems. Among these are the employment of Bayesian, minimax, and maximum likelihood inferential techniques in the estimation of the position of a moving target during a search. An application of potential theory to search problems is also considered. This document summarizes the results detailed in Johns Hopkins Technical Reports numbered 278, 280, 283, 286, 291, 295, 297, and 301.

FINAL REPORT

ONR CONTRACT N00014-77-C-0511

STATISTICAL METHODS IN SEARCH

1. Introduction

The work performed under ONR contract N00014-77-C-0511 from September 1, 1977 through June 30, 1978 has been addressed to developing new methods for planning and analyzing search for moving targets. These methods were developed for application to ASW search in the DS and SAI missions.

Recent work at various fleet locations, including COMSUBPAC and COMSUBLANT, has centered around the development of programs written for desk-top calculators such as the Wang 2200 and Techtronix 4051 to address the ASW search problem. In particular, approaches have been developed for computing target location distributions, called probability maps, on these small calculators. Previously such technology was only available on large, high-speed computers. These newly developed programs have been used at sea and on shore for the solution of search problems inherent to the DS and SAI missions.

These new calculator programs have made probability maps available for the first time to the at-sea commander. For example, the versions of the so-called 'analytic' programs designed for use in the DS mission have been used successfully in both planning and real-time analysis in many Pacific Fleet exercises since 1975. These DS programs represented the first real access of a Commander at sea to a complex search information processing system.

The key to this new capability lies in the fact that the 'analytic' programs are implemented on portable desk-top calculators, rather than on large stationary computers, as are the so-called 'Monte Carlo' programs. This portability has proved extremely useful in many operational search problems. On the other hand, this flexibility has not been achieved without cost. In general terms, this cost has been in the reduced versatility of the 'analytic' programs relative to the 'Monte Carlo' programs.

The 'analytic' search programs differ from the 'Monte Carlo' search programs in the way in which probability maps are computed. Instead of obtaining a probability map from a complex sampling procedure, the 'analytic' programs compute it directly. The complexity of the sampling procedure inherent in the 'Monte Carlo' programs has resulted in their implementation on large, high-speed computers.

While less demanding in terms of programming requirements, the current 'analytic' programs rest on a highly specialized body of mathematical assumptions. These have been shown to have wide applicability in ASW search problems; however, they are still significantly more restrictive than those underlying the 'Monte Carlo' programs. As a result, the 'analytic' programs are considered less versatile than the 'Monte Carlo' programs. The work that has been performed under ONR Contract N00014-77-C-0511 has been addressed to enhancing the versatility of search programming implement on small desk-top calculators. The sole purpose of this enhancement is to make useful tactical decision aids available on portable computing systems for the use of the on-scene commander.

Two major objectives of the work have been:

1. Generalization of existing modeling and inferential techniques in order that the desk-top 'analytic' programs may be applied to a wider range of search problems; and
2. Automation of a number of the features of the current 'analytic' programs in an attempt to reduce the role of the analyst in the solution of operational search problems.

These objectives have been addressed through technical progress in three important areas:

1. Generalization of the models for target motion and models for sensor operation employed in the 'analytic' programs;

2. Development of statistical procedures for estimating the position of the target without stipulating a prior distribution for the target; and

3. Development of statistical procedures for optimal real-time sensor allocation.

These technical advances have been made within the context of the development of general statistical methods to address search problems. The Bayesian statistical analysis which currently provides the inferential structure of both the 'analytic' and 'Monte Carlo' programs is only one of many possible statistical techniques for making inference about the position of a target during a search. In an attempt to address the objectives of this contract, the application of other known statistical techniques to the problem of target localization was investigated. The result of this investigation is a body of statistical tools for estimating the position of a target during a search which significantly generalizes currently employed inferential techniques.

These new tools will be briefly described in sections 3, 4, 5 and 6. An introduction to the use of statistics in the theory of search will be presented in section 2. A compendium of the abstracts of the technical reports referred to in sections 3, 4, 5 and 6 will be given in section 7. Each statistical technique referred to in sections 3, 4, 5 and 6 is in the form of an algorithm which may be either incorporated into existing 'analytic' programs or written into a new program for use on a desk-top calculator. In each case, suggestions for the applicability of each technique will be made.

2. The Use of Statistics in the Theory of Search

The problem of target localization may be viewed as a statistical estimation problem. In a typical search problem, a target is assumed to be moving in a region known as a search space. Uncertainties are typically associated with both the initial position of the target and the manner in which the target moves from the initial position through the search space.

A searcher is one who chooses a sequence of random variables to sample in order to make inference about the position of the target during the search. One random variable is chosen at each stage of the search. For example, a random variable with outcomes detection and non-detection may be associated with the search of a subregion of the search space called a cell. Then, in choosing a sequence of cells to be searched the searcher is really choosing a sequence of binary random variables to be sampled. The sequence of random variables chosen from time 0 through to time t is called the experiment associated with time (or stage) t.

At each time t the searcher may observe the outcomes of the experiment associated with time t. This amounts to observing or recollecting the outcomes of all the random variables chosen for sampling between time 0 and time t. When a random variable is associated with the search of a cell this is equivalent to observing either a detection or non-detection during the search of that cell.

The searcher then has two problems to address. His immediate problem is to use the outcomes of the experiment associated with time t to construct an estimate for the position of the target at time t. This is a problem in statistical inference. A more subtle problem, but one of equal importance, involves the choice of the experiment associated with time t. This is actually a problem which must be addressed prior to the construction of the estimate. It is a problem in experimental design.

In point of fact, these two problems are intimately connected. This is because the searcher must choose the experiment associated with time t to optimize some property of the target location estimator which he constructs at time t. This criterion for optimality usually involves a payoff function which places more value on estimates which are close to the actual target position and less value on estimates which are more distant. The object of search planning then is, for any time t to choose an experiment associated with time t and an estimator for target location at time t which maximizes the average payoff to the searcher.

To summarize, there are six basic components of the statistical inference problem which confronts the searcher at time t.

i) $\Theta = \{\theta\}$ - Search Space. This is the set of possible locations for the target during the search. It is assumed that the target starts from one of the elements of Θ and at any time during the search is still located at some element of Θ .

ii) $E_t = \{e_t\}$ - Family of Experiments. This is the collection of all experiments associated with time t , denoted by e_t, which the searcher may use in order to estimate the position of the target at time t. An element e_t of E_t is just a sequence of random variables.

iii) $Z_t = \{z_t\}$ - Sample Space. This is the set of all possible outcomes of the experiments associated with time t. Having chosen an element e_t of E_t the searcher observes an outcome z_t in Z_t .

iv) $\mu : \Theta \times \Theta \to \mathbb{R}$ - Payoff Function. The value $\mu(\theta, a)$ is the payoff to the searcher if he estimates the target's position at time t to be $a \in \Theta$ and it is actually $\theta \in \Theta$.

v) $G = \Theta \to [0,1]$ - Prior Distribution. The value $G(\theta)$ is the prior probability that the target starts at time 0 at location $\theta \in \Theta$.

vi) $F: E_t \times Z_t \times \Theta \times \Theta \to [0,1]$ - Observation Distribution. The value $F_{e_t}(z_t, \theta_t | \theta_0)$ is the probability of the event: target is located at $\theta_t \in \Theta$ at time t and outcome z_t has been obtained, given the target started at location $\theta_0 \in \Theta$ at time 0 .

The problems of statistical inference and experimental design discussed above involved in predicting target location at time t may then be regarded as a game between the searcher and the target. The essential elements of the game are the six basic components of the estimation problem listed above. Different forms of statistical inference amount to different rules by which the target and the searcher are assumed to play this game.

During the work under this contract, three forms of statistical inference were considered: Bayesian inference, minimax estimation, and maximum likelihood estimation. Within the context of the search problem, these different forms of inference result from:

i) different assumed mechanisms for the target's choice of starting position, and

ii) different payoff functions.

The game between the searcher and the target at any time t may be represented graphically as in Figure 1.

Figure 1

The Game Between the Searcher and Target

Move #:	1	2	3	4	Payoff
Move by:	Target	Searcher	Target	Searcher	Referee
Choices:	$\theta_0 \in (H)$	$e_t \in E_t$	$z_t \in Z_t$ $\theta_t \in (H)$	$a \in (H)$	$\mu(\theta_t, a)$
Mechanism for Choice	i) $G(\cdot)$ ii) Free	Free	$F_{e_t}(\cdot,\cdot \mid \theta_0)$	Free	Fixed

The game may be considered as a four move game, with the target and the searcher alternating moves.

In the first move the target chooses an initial position. In Bayesian inference it is assumed that this choice is made randomly, using the prior distribution G as the randomization mechanism. In both minimax estimation and maximum likelihood estimation, no such assumption is made. Rather, it is assumed that the target's initial position is chosen completely arbitrarily.

The second move is the searcher's. To address the problem of estimating the target's position at time t, he chooses an experiment associated with time t, denoted e_t. The searcher is free to choose any element of E_t.

In the thrid move the target makes two choices:

i) an outcome $z_t \in Z_t$, and

ii) a location $\theta_t \in \Theta$.

Once again it is assumed that the target is not free to choose any pair (z_t, θ_t) but rather must choose the pair randomly from the distribution $F_{e_t}(\cdot,\cdot|\theta_0)$. Note that the distribution of (z_t, θ_t) is conditioned upon the choices θ_0 and e_t which were determined earlier in the game.

The fourth move is the searcher's. Having observed outcome z_t he must choose an element a of Θ as his estimate for the location of the target. The searcher is free to choose any element of Θ.

After the target and searcher have completed two moves each, a referee pays the searcher an amount $\mu(\theta_t, a)$, where θ_t is the position of the target at time t known only to the referee and a is the searcher's estimate of that position. It is assumed that the mechanism for calculating the payoff was fixed before the game started.

In Bayesian inference and minimax estimation the payoff function is typically a function which decreases with increasing distance between real and estimated target position. In maximum likelihood estimation the payoff function is related to the asymptotic variance of the estimate of the target's position. These ideas will be made precise in the next sections.

In section 3 the Bayesian approach to this estimation problem will be discussed in greater detail. In particular the technical reports written to address specific search problems within the Bayesian context will be briefly discussed. Applications for which the Bayesian approach to estimation is appropriate will also be suggested.

In section 4 the minimax estimator for target location will be addressed in greater detail. Technical reports using this statistical technique to address specific search problems will be discussed and applications of this inferential technique mentioned.

In section 5 the maximum likelihood approach to estimation of target location will be further developed within the context presented here. Discussion of material addressed to specific search problems within this inferential framework will be presented along with suggestions for the applicability of this approach.

In section 6 an application of a different branch of mathematics called potential theory to the search problem is discussed. Applications for this technique are mentioned. All technical reports discussed in these four sections are abstracted in section 7.

3. Bayesian Search

In this section the Bayesian solution to the estimation problem introduced in section 2 will be discussed in detail. Use of Bayesian inference to address specific search problems has been considered in four technical reports. These reports will be briefly discussed below. Applications for the results of these reports will also be addressed.

A. The Bayesian Estimation Problem

As was mentioned in section 2, the general estimation problem inherent to a search may be characterized as a four-move game with a payoff at the end. The differences in this game under different forms of statistical inference are differences in the mechanisms for choice in move #1 and in the characterization of the payoff. Consider the Bayesian's version of the estimation game displayed in Figure 2.

Figure 2

The Game Between a Bayesian Searcher and a Target

Move #:	1	2	3	4	Payoff
Move by:	Target	Searcher	Target	Searcher	Referee
Choices:	$\theta_0 \in \Theta$	$e_t \in E_t$	$z_t \in Z_t$ $\theta_t \in \Theta$	a	$\mu(\theta_t, a)$
Mechanism for Choice	$G(\cdot)$	Free	$F_{e_t}(\cdot,\cdot\|\theta_0)$	Free	Fixed
Knowledge	$G(\cdot)$	$G(\cdot)$	θ_0 $F_{e_t}(\cdot,\cdot\|\theta_0)$	$G(\cdot)$ e_t $F_{e_t}(\cdot,\cdot\|\theta_0)$ z_t	θ_t a

The Bayesian searcher assumes that, in move #1, the target chooses its initial position θ_0 using the prior distribution G. In move #2 the Bayesian search must choose an experiment associated with time t, e_t. He does this assuming that he knows the prior distribution G with which the target has chosen its initial position θ_0. In the third move the target must choose a position at time t, θ_t and produce an observation z_t. The target is assumed to know both its initial position θ_0 and the form of the observation distribution $F_{e_t}(\cdot,\cdot|\theta_0)$. In the fourth move the Bayesian searcher must choose an estimate a of θ_t. He does this with a knowledge of the prior distribution G; the experiment which he chose at move #2, e_t; the observation distribution $F_{e_t}(\cdot,\cdot|\theta_0)$; and the observation z_t. The referee, knowing both θ_t and a pays the searcher an amount $\mu(\theta_t,a)$. The problem for the Bayesian searcher is how to choose experiment e_t and, given observation z_t, how to choose estimate a, so as to maximize his expected payoff.

B. <u>The Solution to the Bayesian Estimation Problem.</u>

The solution to the Bayesian's optimization problem is a two-step solution:

i) For any given experiment $e_t \in E_t$, choose an estimator $a(\cdot)$ to maximize the expected payoff under experiment e_t.

This expected payoff, denoted by $r_{e_t}(a)$, is given by:

$$r_{e_t}(a) = \int_{\Theta} G(d\theta) \int_{\Theta} \int_{Z_t} F_{e_t}(dz, d\phi \mid \theta)\mu(\phi, a(z)) .$$

If a^* maximizes $r_{e_t}(\cdot)$, then a^* is called a Bayes Estimator for θ_t under experiment e_t . The value $r_{e_t}(a^*)$ is known as the Bayes payoff under experiment e_t .

ii) Once a Bayes estimator a^* has been selected for each experiment $e_t \in E_t$, the optimal experiment is to be chosen. The optimal experiment associated with time t , denoted e_t^*, is that member of E_t which maximizes $r_{e_t}(a^*)$. The value $r_{e_t^*}(a^*)$ is then called the Bayes payoff.

C. Technical Reports

Four technical reports were written under this contract to employ this Bayesian solution to specific search problems. These reports are as follows:

1. Search for a Moving Target and the Exponential Formula.
2. Statistical Methods in the Theory of Search.
3. Parametric Search Modeling.
4. On Multiplicative Functionals on Diffusion Processes.

These technical reports are abstracted in section 7.

D. Discussion of Technical Reports

All four of these technical reports apply Bayesian inference to the problem of search for a continuously moving target in Euclidean n-space. It is assumed that the search is conducted by many sensors moving continuously in $\mathbb{R}^n$. Methods for representing the Bayes estimator for the target's position at any time during the search are developed. These representations are based upon particular methods for parameterizing the model for target motion and the operation of the sensors conducting the search. The technical reports include examples of the use of the representations in solving specific search problems. The algorithms presented in the technical reports may be implemented on small desk-top calculators.

E. Applications of Bayesian Inference

i) General Considerations: The assumptions inherent to the Bayesian solution of the target localization problem presented in the technical reports listed above limit this technique's usefulness in two important ways. First, in order to use Bayesian inference successfully, the assumption that the target chooses its initial position from a known prior distribution must hold. This is a key assumption in the Bayesian framework and one which is difficult to justify tactically.

Typically there is little reason to believe that a target has chosen its initial position randomly from a particular distribution and almost never a reason for the searcher to contend that he knows the form of this distribution. Arguments can be made that the prior distribution should reflect the searcher's knowledge rather than an objective selection mechanism used by the target. These arguments result, however, in inferential models for the searcher's speculations rather than models for the target's actual location.

A second, more technical problem inherent in the representations developed in the technical reports involve their complexity. What may realistically be programmed on a small desk-top computer are approximations to the Bayes estimators derived in the technical reports. Practically, this means that when implemented on small calculators, only simple models for target motion--such as diffusions--and simple models for sensor operation--such as ones involving only direct path and one convergence zone--should be employed. Of course, when implemented on larger computers, more complex models of the type suggested in the technical reports may be employed.

ii) Specific Considerations. The results obtained in these four technical reports generalize the methodology underlying DS and SAI 'analytic' programs currently in use at the SUBPAC TAG. Specifically, these technical reports significantly generalize the types of models for target motion and models for sensor operation which may be considered by programs of the 'analytic' type. Improved procedures for representing the 'analytic' solution to the Bayes estimation problem have also been developed here. These improvements could be incorporated into existing DS and SAI 'analytic' programs. For the specifics of these generalizations the reader is referred to the technical reports.

4. Minimax Search

In this section the minimax solution to the estimation problem introduced in section 2 will be discussed in detail. Use of minimax estimation to address specific search problems has been considered in two technical reports. These technical reports will be briefly discussed below. Potential applications for these results will also be discussed.

A. The Minimax Estimation Problem

Once again we return to the general target localization problem outlined in section 2. It was shown that this problem may be characterized as a four-move game with a payoff at the end. Now consider a version of this game based upon minimax estimation. Such a version is displayed in Figure 3.

The game proceeds as in the case of the Bayesian searcher, except that at move #1 the target chooses an initial position θ_0 freely, without the aid of a prior distribution. Consequently, when the searchers chooses an experiment e_t, he does so without any information regarding the choice of the initial position which the target has made. Thus, use of a minimax estimation philosophy may be employed under a relaxation of the assumptions used by the Bayesian to generate his solutions to the search problem.

Yet, as in the case of the Bayesian searcher, a searcher using minimax estimation must still decide at move #4 how to choose an estimate for the target's position at time t, and at move #2 decide

Figure 3

The Game Between a Minimax Searcher And a Target

Move #	1	2	3	4	Payoff
Move by:	Target	Searcher	Target	Searcher	Referee
Choices:	$\theta_0 \in Ⓗ$	$e_t \in E_t$	$z_t \in Z_t$ $\theta_t \in Ⓗ$	a	$\mu(\theta_t, a)$
Mechanism for Choice	Free	Free	$F_{e_t}(\cdot,\cdot\|\theta_0)$	Free	Fixed
Knowledge:	--	--	θ_0 $F_{e_t}(\cdot,\cdot\|\theta_0)$	e_t $F_{e_t}(\cdot,\cdot\|\theta_0)$ z_t	θ_t a

upon an experiment to be employed.

What makes his problem more difficult than the Bayesian's is that he must make these decisions without the luxury of assuming he knows the random mechanism used by the target for generating his initial position.

B. The Minimax Solution to the Estimation Problem

As before, the minimax solution to this problem is a two-step solution.

i) For a given experiment $e_t \in E_t$, choose an estimator $a(\cdot)$ to maximize the minimum expected payoff under experiment e_t. The minimum expected payoff, denoted by $R_{e_t}(a)$ is given by,

$$R_{e_t}(a) = \min_{\theta} \int_{Ⓗ} \int_{Z_t} F_{e_t}(dz, d\phi|\theta)\mu(\phi, a(z)) .$$

If $\bar{a}$ maximizes $R_{e_t}(\cdot)$, then $\bar{a}$ is called a minimax estimator for θ_t under experiment e_t. The value $R_{e_t}(\bar{a})$ is known as the value of the estimation game under experiment e_t.

ii) Once a minimax estimator $\bar{a}$ has been selected for each experiment $e_t \in E_t$, the optimal experiment is to be chosen. The optimal experiment associated with time t (in this case), denoted $\bar{e}_t$, is that member of E_t which maximizes $r_{e_t}(\bar{a})$. The value $r_{\bar{e}_t}(\bar{a})$ is then called the value of the estimation game.

C. Technical Reports

Two technical reports were written under this contract to employ the minimax estimation techniques in the solution of specific search problems. These reports are as follows:

1. Search for a Stationary Target Under Minimax Estimation
2. Search for a Moving Target Under Minimax Estimation.

These technical reports are abstracted in section 7.

D. Discussion of Technical Reports

Both of these technical reports apply minimax estimation to particular search problems. In both cases the target is assumed to choose its initial position at an unknown posiiton in the search space. In these technical reports a discrete search space is considered. The case of a stationary target (or a moving target which moves very slowly with respect to changes in the search) is considered in the first technical report. In the second technical report the target is assumed to move discretely through the search space according to a Markov process. Representations for the minimax estimator for target position and the value of the estimation game are derived in each case. As a part of the solution in each case, a prior distribution for the target is also derived which would be the most difficult for a Bayesian searcher to consider. This distribution is called the least favorable prior distribution.

E. Applications of the Minimax Estimator

i) General Considerations: The minimax estimator may be used under a relaxation of the assumptions used to derive the Bayesian solution to the target location estimation problem. The key assumption lacking in the derivation of the minimax estimator is that of a prior distribution for the target's initial position which is known to the searcher. However, as is pointed out in the technical reports, there is a Bayesian interpretation for the optimality of the minimax estimator.

The minimax estimator for a target's position at any time during a search is in fact a Bayes estimator under a very specific prior distribution for the target's initial location. Since no prior distribution is assumed by the minimax technique, it automatically proposes a prior distribution. The prior distribution which it suggests is the one which minimizes the maximum payoff to the searcher. This is the least favorable prior distribution mentioned above. The minimax estimator is then a

Bayesian estimator with respect to this prior distribution.

Since the minimax estimator chooses as its implied prior distribution for the target's initial position, the one which produces the most difficult estimation problem for the searcher, the minimax estimator is inherently more conservative than the Bayes estimator. This conservatism means that the estimator will admit more possibilities for the target's position at any time t (and be consequently less definitive) than a Bayes estimator in the same search. This loss of precision is a direct consequence of the relaxation of assumptions.

The conclusion here is that the minimax estimator should be used only when a prior distribution for the target's initial position is not indicated and then only if it is of interest to guard assiduously against making mistakes in estimation. The minimax estimator artificially creates a prior distribution to guard against large mistakes. The use of prior distributions in this way tends to deemphasize the importance of the observations. We shall see in the next section that another technique, called maximum likelihood, may be used in similar search problems (ones with no apparent prior) to emphasize the role of the observations rather than the role of the real or implied prior assumptions in making inference about target location.

ii) Specific Considerations: The results obtained in these two technical reports are totally new results. However, the modeling of target motion and sensor operation employed therein are of the same type as is currently employed in the SUBPAC TAG'S Markov chain SAI program written for the Wang 2200. This program currently uses a Bayesian estimator for target position. The algorithms provided in the technical reports could be written into a subroutine and appended to the aforementioned SUBPAC program. The benefit to be derived from such an addition would be in the expansion of the type of tactical SAI situation for which the programmed techniques would be applicable. The current program, employing Bayesian techniques, requires detailed information about the target's prior distribution. In many tactical situations such information is unavailable. The algorithms provided by the technical reports listed above do not require stipulation of a prior distribution for the target. Yet they produce decision aids for the search planner of the same general type as the Bayesian, Markov chain SAI program.

5. Maximum Likelihood Search

In this section the maximum likelihood solution to the estimation problem introduced in section 2 will be addressed in detail. Use of maximum likelihood methods to address a specific search problem is addressed in one technical report. This technical report will be briefly discussed in subsection D. Potential applications for these results will also be discussed.

A. The Maximum Likelihood Estimation Problem

Return again to the four-move estimation game described in section 2. Consider now a version of this game based upon maximum likelihood estimation. Such a version would appear identical in its rules to the game depicted in Figure 3.

As in the case of minimax estimation, at move #1 the target chooses an initial position θ_0 freely, without any enforced randomization mechanism. When the searcher chooses an experiment e_t, he does so without any information regarding the target's choice of θ_0. Once again, this game is one which is much more difficult for the searcher since he has less information upon which to base his choices.

Yet, as in the case of minimax estimation and Bayesian estimation, the searcher must decide at move #4 how to choose an estimate for the target's position at time t, and at move #2 decide upon an experiment to be employed.

B. The Maximum Likelihood Solution to the Estimation Problem

As in the previous two cases, the maximum likelihood solution to this problem is a two-step solution.

i) For a given experiment $e_t \in E_t$ and the observation $z_t \in Z_t$, choose as an estimate of target position at time t the position $a \in Ⓗ$ which maximizes the likelihood function. For $e_t \in E_t$ and $z_t \in Z_t$, the likelihood function, for a stationary target, denoted $f_{e_t}(z_t|\cdot)$ is given by

$$f_{e_t}(z_t|a) = F_{e_t}(z_t, a|a) \quad , \text{ for } a \in Ⓗ .$$

If for $e_t \in E_t$ and $z_t \in Z_t$, $\tilde{a}$ maximizes $f_{e_t}(z_t|\cdot)$, then $\tilde{a}$ is called the maximum likelihood estimate for θ_t under e_t, for observation z_t.

ii) The optimal experiment associated with time t (in this case), denoted $\tilde{e}_t$, is that member of E_t which maximizes the minimum Fisher information.[1] Here for $e_t \in E_t$, the minimum Fisher information, denoted $H(e_t)$ is given by

$$H(e_t) = \min_{\theta} \int_{Z_t} \left| \nabla f_{e_t}(z|\cdot) \Big|_{\theta} \right|^2 F_{e_t}(dz, \theta|\theta) .$$

The rationale behind this two-step solution is described in the technical report described below.

C. Technical Report

One technical report was written under this contract on the employment of maximum likelihood methods in search. This report is entitled Maximum Likelihood Search. This technical report is abstracted in section 7 .

D. Discussion of Technical Report

In the above technical report the maximum likelihood estimator for target position is examined in the case of a stationary target. The search is conducted by a set of sensors, each one of which has associated with it an instantaneous probability of detection functions. The asymptotic distribution of the maximum likelihood estimator for the target's position is obtained in terms of these detection function Within this context, it is demonstrated that Fisher information arises as the natural criterion for selection of the optimal experiment.

E. Applications of the Maximum Likelihood Estimator

i) General Considerations: The maximum likelihood estimator, like the minimax estimator may be used under a relaxation of the assumptions used to derive the Bayesian solution to the estimation problem. This relaxation is that no prior distribution is assumed for the target.

Like the minimax estimator for the target's location, the maximum likelihood estimator also has a Bayesian interpretation.

When the search space has finite size (area or number of cells), the maximum likelihood estimator is a Bayes estimator under a very specific prior distribution for the target. Since no prior distribution is assumed by the maximum likelihood technique, like the minimax technique, it automatically proposes a prior distribution. The prior distribution which the maximum likelihood technique suggests is one which represents a completely neutral prior opinion about the location of the target. It should be construed as representing complete ignorance about the location of the target.

Therefore, the maximum likelihood estimator strives to deemphasize the role of the prior assumptions in making inference about the target's location. The role of the observations then becomes correspondingly more important. Consequently, while the minimax estimator is less sensitive to the observations than the Bayes estimator, the maximum likelihood estimator is more sensitive. Small changes in the observations may produce large changes in the estimated target position. This is due to the fact that the maximum likelihood estimator relies totally on the observations for its conclusions. In the same vein, the Bayes estimator balances the observations against specific prior assumptions; and the minimax estimator balances the observations against the risks involved in estimating the target's position incorrectly.

The conclusion here is that the maximum likelihood estimator should be used only when both of the following two conditions are met: i) when a prior distribution for the target's location is not justified and ii) when objectivity in estimating the target's position is considered to be more important than the penalties involved in incorrect estimation.

ii) Specific Considerations: The results obtained in the above technical report are totally new. However, the modeling of sensor operation used in the report are of the same type as is currently employed in the 'analytic' SAI and DS programs. These programs currently use Bayes estimators for target location. The maximum likelihood estimator discussed in the technical report could be written into a subroutine and appended to either program. Such an addition would be useful for dealing with tactical situations in which the target moves significantly more slowly than the sensors and in which no prior distribution is indicated.

It should be noted that these techniques have much wider applicability than just as possible add-ons to existing 'analytic' programs. The maximum likelihood techniques described in the technical report referenced could be slightly generalized and applied to submarine vs. submarine search problems. This is because in local tactical problems, a prior distribution for the target's location is rarely justified. In such cases the data must provide all the information about the target's location. Maximum likelihood methods are designed for this purpose.

6. The Search Potential

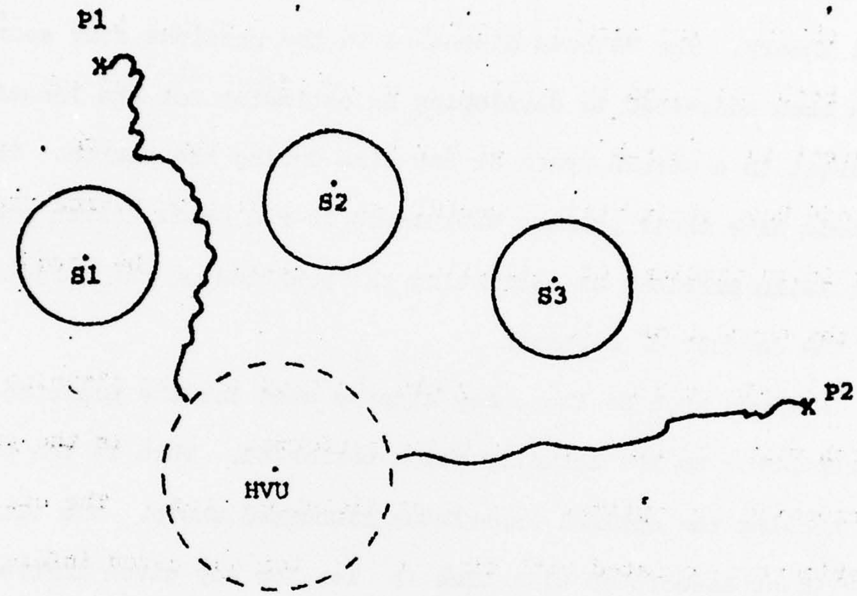
In this section an application of a totally different branch of mathematics to the search problem is discussed--namely potential theory. The methods discussed in the previous four sections have been addressed to developing an estimator for the location of a target in a search space at any time during the search. These methods have their primary usefulness in real-time search problems; that is, in problems of estimating the position of the target during the conduct of a search.

Methods such as these may also be used for the planning of a search prior to its actually being performed. This is the problem of choosing the optimal experiment discussed above. The optimal experiment associated with time t is, for any given inferential technique, the set of random variables to be sampled (or search plan) which makes the estimator for target location at time t most efficient.

Other criteria for choice of search plan also suggest themselves. Often a search involves an attempt on the part of the target to achieve an objective. The role of the searcher is then to detect and neutralize the target before it achieves its objective. Consider the tactical situation outlined in Figure 4.

Figure 4

A Search Involving a Terminal Payoff



A target begins a maneuver at position P1. The goal of this maneuver is to penetrate the screen of searching units, located at position S1, S2, and S3, and to arrive at some position on the dotted circle surrounding the HVU, the high value unit. Upon arriving at the dotted circle the target receives a payoff. If the purpose of the target's penetration is to attack the HVU, then the payoff might be the probability with which the attack is successful.

It may be the case that some positions on the dotted circle are advantageous positions from which to launch an attack. These positions would have a higher payoff than those from which an attack is more difficult. But we assume that the target's primary problem is reaching the dotted circle and once on the circle, it takes the payoff associated with the point of initial contact.

The penetrating target may not reach the circle at all. We assume that each of the searching units S1, S2, and S3 remains stationary relative to the HVU (which may move); but detects the target according to a detection rate which is a function of the position of the target relative to the sensor. We assume that if the target is detected during penetration, it is somehow neutralized and therefore receives no payoff.

From Figure 4 it seems apparent that some starting positions for the target are better than others. For example, a target starting from position P1 has more defenses to penetrate than a target starting from position P2. Consequently, this target has more of a chance of being neutralized than a target starting from position P2. However, as was pointed out above, merely having a high probability of successful penetration does not guarantee a high payoff. This is because a target starting from position P2, for example, may arrive at a position on the dotted circle with a low payoff. Therefore, the best starting positions for a penetrating target are those which balance the penetration probability against the terminal payoff. The best initial positions are those which give a reasonably high penetration probability and a good payoff.

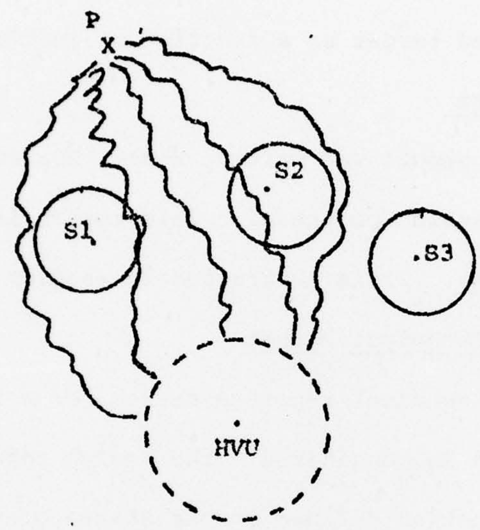
One way to judge the effectiveness of a defensive plan as displayed in Figure 4 would be to determine where all the best initial positions for a penetrating target might be, and to design a defense which makes the expected payoff to a target starting from one of these positions as low as possible. The solution of this problem will be discussed below. One technical report has been written on this topic. Possible applications for this technique are also discussed below.

A. Computing the Search Potential

The problem of evaluating the expected payoff to a penetrating target may be considered within the context of stochastic processes. One of the principle sources of uncertainty in the payoff to the target is the uncertainty involved in the target's motion. Only one path for each target was drawn in Figure 4. However, in an actual tactical situation many paths are possible between a given starting position and the payoff circle about the HVU. Some possible penetration paths are drawn in Figure 5. This implies that a stochastic model for target motion might be a realistic one.

Figure 5

A Stochastic Model For Penetration



As is indicated in Figure 5, for a given starting position P, many penetration paths are possible. Different penetration paths have different values of expected payoff. This is due to the fact that different paths have different probabilities of being detected and different payoffs.

The problem outlined in the previous section reduces to computing a particular kind of average. For each possible starting position P outside the defenses, the average payoff over all possible paths originating at that point and ending on the payoff circle must be computed. If it is assumed that the target moves according to a homogeneous Markov process and that the sensors are stationary relative to the HVU, this average is easily computed. If this average is plotted against initial target position, a function called the <u>search potential</u> results. This function gives the potential payoff to an undetected target as a function of initial position.

B. <u>Technical Report</u>

One technical report was written under this contract on the employment of the search potential. This report is entitled <u>The Search Potential</u>. It is abstracted in section 7.

C. <u>Discussion of Technical Report</u>

In the above technical report a search for a target in discrete Markov motion is considered. The search potential function is defined as a function defined on the states of the Markov chain with non-zero prior probability, which is the expected payoff to an undetected target upon being absorbed in the terminal states of the chain. A method for computing this function is presented and three examples are worked.

D. <u>Application of the Search Potential</u>

i) <u>General Considerations</u>: The two key assumptions in the above technical report involve the homogeneity of the assumed Markov motion for the target and the stationarity of the sensors relative to the HVU. These assumptions restrict the usefulness of the technique to situations in which the sensors are screening the high value unit. In such situations the screening units and the high value unit move in unison and the penetration tactics of the target are likely to depend on its position in the search space, but not on the time of penetration.

Thus, the most fruitful area of application for these techniques is in direct support. The search potential would provide a way of evaluating the screen of a task force in terms of its ability to prevent a penetrating target from carrying out its mission.

ii) Specific Considerations: The algorithm presented in the technical report may be programmed on a desk top calculator. Programs for direct support search planning and evaluation are currently implemented on the Wang 2200. The program ASP developed at the SUBPAC TAG is written specifically for direct support applications. The algorithm presented above could also be written for the Wang 2200 and included as part of the ASP programs. This would provide the ASP programs with a tool specifically designed to evaluate HVU screens--a tool which ASP does not currently have.

7. Abstracts of Technical Reports

1. Search for a Moving Target and The Exponential Formula

(JHU Technical Report No. 278)

Abstract

A target is assumed to move according to a Wiener Process in $\mathbb{R}^1$. The probability of detecting the target is computed in terms of the search effort which accumulates along the target's path. Under regularity assumptions this probability is given by the expectation of an exponential functional of the process. The problem treated here is that of determining the probability of detecting the target in a given cell of finite Lebesque measure. In stationary searches this probability is often approximated using the exponential formula evaluated at the total accumulated search effort in the cell. It is shown here that the cell failure probability in a search for a Wiener target is asymptotically proportional to $T^{-1/2}$ rather than $\exp[-\rho T]$, where T is accumulated time spent searching in the cell. The asymptotic failure probability is also shown to be a function only of cell size, not cell position in $\mathbb{R}^1$. In a similar fashion it is shown the cell failure probability in a search for a Wiener target in $\mathbb{R}^2$ is independent of cell location and asymptotically proportional to $(c \log T + 1)^{-1}$, $c > 0$.

2. Statistical Methods in the Theory of Search

(JHU Technical Report No. 280)

Abstract

A target is assumed to move according to a continuous stochastic process in Euclidean $\mathbb{R}^n$. A searcher makes a selection of a search strategy from among a set of alternatives. The state of the searcher's knowledge during the course of the search is modeled as a two-state continuous-time Markov chain, called the detection process. The two states are assumed to be "out of contact" and "in contact". The transition intensity of the detection process at any time t is assumed to depend upon the search strategy chosen, the target's position at time t, and t itself. It is shown that the Bayes estimator for target location at any time during the search is determined by a family of probability measures derived from the search, called the coverage distributions. Techniques for approximating the Bayes estimator based upon known properties of the coverage distributions are discussed. The methodology developed is discussed in terms of an example.

3. Parametric Search Modeling

(JHU Technical Report No. 283)

Abstract

A target moves according to a continuous stochastic process in Euclidean $\mathbb{R}^n$. A search is conducted by chosing a search strategy and by observing events of a detection process. Methods for representing the posterior distribution for target location at any time during the search are discussed. The particular methods for parameterizing the models for target motion and the transition vector of the detection process which yield tractable representations are introduced. The methodology is discussed in terms of two examples.

4. On Multiplicative Functionals on Diffusion Processes

(JHU Technical Report No. 286)

Abstract

A class of Gaussian Diffusion processes is considered. A multiplicative functional is defined on such a process and gives rise to a generalized transition function. This generalized transition function satisfies a modified version of the Kolmogorov backward equation of the diffusion process. A constructive method for generating a solution to this modified backward equation, with the required final condition, is presented. The methodology is discussed in terms of an example.

5. Search For a Stationary Target Under Minimax Estimation

(JHU Technical Report No. 291)

Abstract

A target is assumed hiding at an unknown position in a finite search space. No prior probability distribution for target location is assumed. A search is defined to be the observation of a sequence of random variables. Expressions for the minimax estimator for target location, the least favorable prior distribution for target location, and the value of the estimation game at any stage of the search are derived. The methodology is illustrated in term of an example.

6. Search for a Moving Target Under Minimax Estimation

(JHU Technical Report No. 295)

Abstract

A target is assumed to choose its starting position in a search at an unknown position in a finite search space. No prior probability distribution for the target's initial location is assumed. During the search the target is assumed to move from position to position in the search space according to a Markov process. A search is defined to be the observation of a sequence of random variables. Representations for the minimax estimator for target location at any stage of the search, the least favorable prior distribution for the target, and the value of the estimation game are presented.

7. The Search Potential

(JHU Technical Report No. 297)

Abstract

A search for a particle in discrete Markov motion is considered. The states of the particle's motion are assumed to be either transient or absorbing. An operator called the search potential operator is defined. This operator maps real-valued functions defined on the absorbing states into real-valued functions defined on the transient states. If a function defined on the absorbing states is construed to be the payoff to an undetected particle upon being absorbed, then the search potential operator maps it into the conditional expectation of payoff as a function of starting state. Three examples are provided.

8. Maximum Likelihood Search

(JHU Technical Report No. 301)

Abstract

A target is assumed located at an unknown position in $\mathbb{R}^n$ No prior probability distribution for the target is assumed. A search is defined to be the observation of a sequence of Bernoulli random variables. The maximum likelihood estimator for target location is examined. In particular, the asymptotic distribution of the maximum likelihood estimator is derived and use of Fisher Information is made for the optimal selection of the sequence of Bernoulli random variables to be sampled.

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