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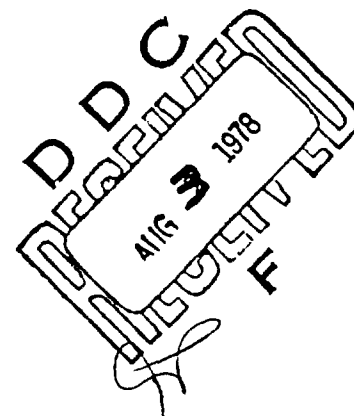
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**AERODYNAMIC DESIGN AND ANALYSIS OF PROPELLERS FOR
MINI-REMOTELY PILOTED AIR VEHICLES**

Volume II - Ducted Propellers

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May 1978

Final Report for Period June 1976 - March 1978

Approved for public release;
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Prepared for

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APPLIED TECHNOLOGY LABORATORY
U. S. ARMY RESEARCH AND TECHNOLOGY LABORATORIES (AVRADCOM)
Fort Eustis, Va. 23604

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APPLIED TECHNOLOGY LABORATORY POSITION STATEMENT

This report, Volume II of a two-volume report, presents the design and analysis of two small-diameter ducted propellers for use on mini-remotely piloted vehicles (mini-RPVs). These ducted propeller thrustors have been designed for use with two-cycle, 20-hp engines with 8000- and 5860-rpm output speeds. In addition, an open pusher propeller was optimized for the lower speed engine. Detailed airfoil data were presented for each blade design as an aid to blade fabrication.

Mr. James Gomez of the Propulsion Technical Area, Technology Applications Division, served as Project Engineer for this effort.

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER 18 USAAMRDL TR-77-45B	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER 9
4. TITLE (and Subtitle) 6 AERODYNAMIC DESIGN AND ANALYSIS OF PROPELLERS FOR MINI-REMOTELY PILOTED AIR VEHICLES, VOLUME II, DUCTED PROPELLERS		5. TYPE OF REPORT & PERIOD COVERED Final Report, June 1976 to March 1978
7. AUTHOR 16 Henry V. Borst		8. PERFORMING ORG. REPORT NUMBER
9. PERFORMING ORGANIZATION NAME AND ADDRESS Henry V. Borst & Associates 203 W. Lancaster Avenue Wayne, Pa. 19087		10. CONTRACT OR GRANT NUMBER(s) 15 DAAJ02-76-C-0031
11. CONTROLLING OFFICE NAME AND ADDRESS U.S. Army Aviation Systems Command P.O. Box 209, Main Office St. Louis, Missouri 63166		12. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 16 62209AHF262209AH76 17 00 150 EK
13. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Applied Technology Laboratory U.S. Army Research and Technology Laboratories (AVRADCOM) Fort Eustis, Virginia 23604		14. REPORT DATE 11 May 1978
15. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited. 12 75p.		16. NUMBER OF PAGES 76
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		18. SECURITY CLASS. (of this report) Unclassified
18. SUPPLEMENTARY NOTES Volume II of a two-volume report.		19. DECLASSIFICATION/DOWNGRADING SCHEDULE
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Propellers, Propulsion, Remotely Piloted Vehicles, Aerodynamics, Reynolds Number, Ducted Fans		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This report presents the design and analysis of two open propellers and two ducted propellers for use on advanced Remotely Piloted Vehicles, RPV's. One of the two open propellers was designed for use on a direct-drive engine with a maximum rpm of 8000. The other open propeller was designed for a geared engine of the same power output, but with a maximum rpm of 5860. Two ducted propellers were designed for the same engines. The open		

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and ducted propellers were designed based on a procedure that was established for determining the lowest power and rpm to meet the performance requirements at any operating condition. The geometric characteristics of the four propellers designed based on this procedure are presented so that the blades of these propellers can be fabricated.

The performance of the four-bladed open propeller installed on the low-speed engine is superior to that of the two-bladed open propeller on the high speed engine when operating at the cruise and dash conditions. At cruise condition the performance of the ducted propeller on the high-speed engine is equal to, or better than, any of the other configurations analyzed. At the cruise and dash conditions the performance of the ducted propeller on the low-speed engine is below that of the other two open propellers and the ducted propeller that were analyzed.

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INTRODUCTION

Propellers for mini-remotely piloted vehicles must be designed for peak performance at relatively low forward speeds when operating on engines that have a high output speed. The noise level produced by the propellers must be low to avoid detection of the aircraft during its operation. Low noise levels are generally achieved by reducing propeller tip speed and by increasing blade number. Such changes may not be possible and still meet the thrust and efficiency requirements of the airplane within the power rpm characteristics of the engine. This is especially true if fixed-blade-angle propellers are desired to eliminate complexity.

Volume I documents the work that was done to develop the necessary methods of analysis so that the performance of mini-RPV propellers could be predicted and optimum configurations could be developed. Because of the small size of propellers needed for these RPV applications, it was necessary to determine corrections to account for operation at low Reynolds numbers. The corrections needed were developed to modify the airfoil data normally used for analyzing full-scale propellers for the effects of Reynolds number, so that the performance of the small propellers could be calculated with good accuracy. Using the newly developed method of analysis, optimum propellers were designed for the Aquila RPV and the advanced RPV, both using direct-drive engines. To obtain the required performance, it was necessary to operate the propellers at relatively high rotational speeds with correspondingly high tip speeds. Further, the propellers were designed for peak cruise velocity at the maximum available cruise power. Although the performance of these propellers is good at all the design conditions, including the maximum endurance cruise condition, it would appear that the rpm is high so that the noise level would be excessive.

Based on the performance of a ducted propeller designed for similar operating conditions, suitable configurations were sized for the design conditions of the advanced RPV configuration. The results of this analysis, given in Volume I, indicate that large improvements in performance can be obtained when using properly designed ducted propellers. These ducted propellers will have a lower rotor diameter than the open propeller configuration, with a corresponding reduction in tip speed to about 30 to 35% that of the open propeller.

An engine with the same power output, but with reduced rotational speed, would also make possible reduced propeller tip speeds and thus lower noise levels. To determine the possible advantages of the use of such engines with either open or ducted propellers and also to investigate the effects of

operating at lower cruise speed, a Phase II study was undertaken. In this study it was required to design optimum open and ducted propellers installed on advanced RPV's using engines with a maximum propeller rpm of either 8000 or 5860, as installed on advanced RPV's. The propellers were to be designed for peak performance when operating at the 60-knot launch and landing conditions, and also have good performance at the 75-knot maximum endurance-cruise speed.

DESIGN OPERATING CONDITIONS

ADVANCED RPV DESIGN OPERATING CONDITIONS

The design operating conditions for the advanced RPV, equipped with either the direct-drive engine or an engine using a gear ratio of 0.7325, are:

<u>Mode</u>	<u>Condition</u>	<u>Power Setting</u>	<u>Climb Rate</u>	<u>True Airspeed</u>
Launch	4000 ft/95°F	Maximum	610 fpm	60 kt
Recovery	4000 ft/95°F	Maximum	200 to 610 fpm	60 kt
Cruise	4000 ft/95°F		0	75 kt
Dash	4000 ft/95°F	Maximum	0	100 kt (min)

The thrust horsepower required for the advanced RPV is given in Figure 1 for the launch, cruise, and landing conditions. The increase in thrust horsepower noted for the landing condition reflects the drag increase due to the deployment of the spoiler flaps.

ENGINE POWER CHARACTERISTICS

The advanced RPV is equipped with either a 20-horsepower direct-drive engine or a 20-horsepower engine which has a gear ratio of 0.7325. The maximum propeller rpm for each engine is 8000 and 5860, respectively. The full power characteristics for both engines as a function of crankshaft speed are given in Figure 2. Each engine is equipped with an alternator, which reduces the output power to the level shown in Figure 2.

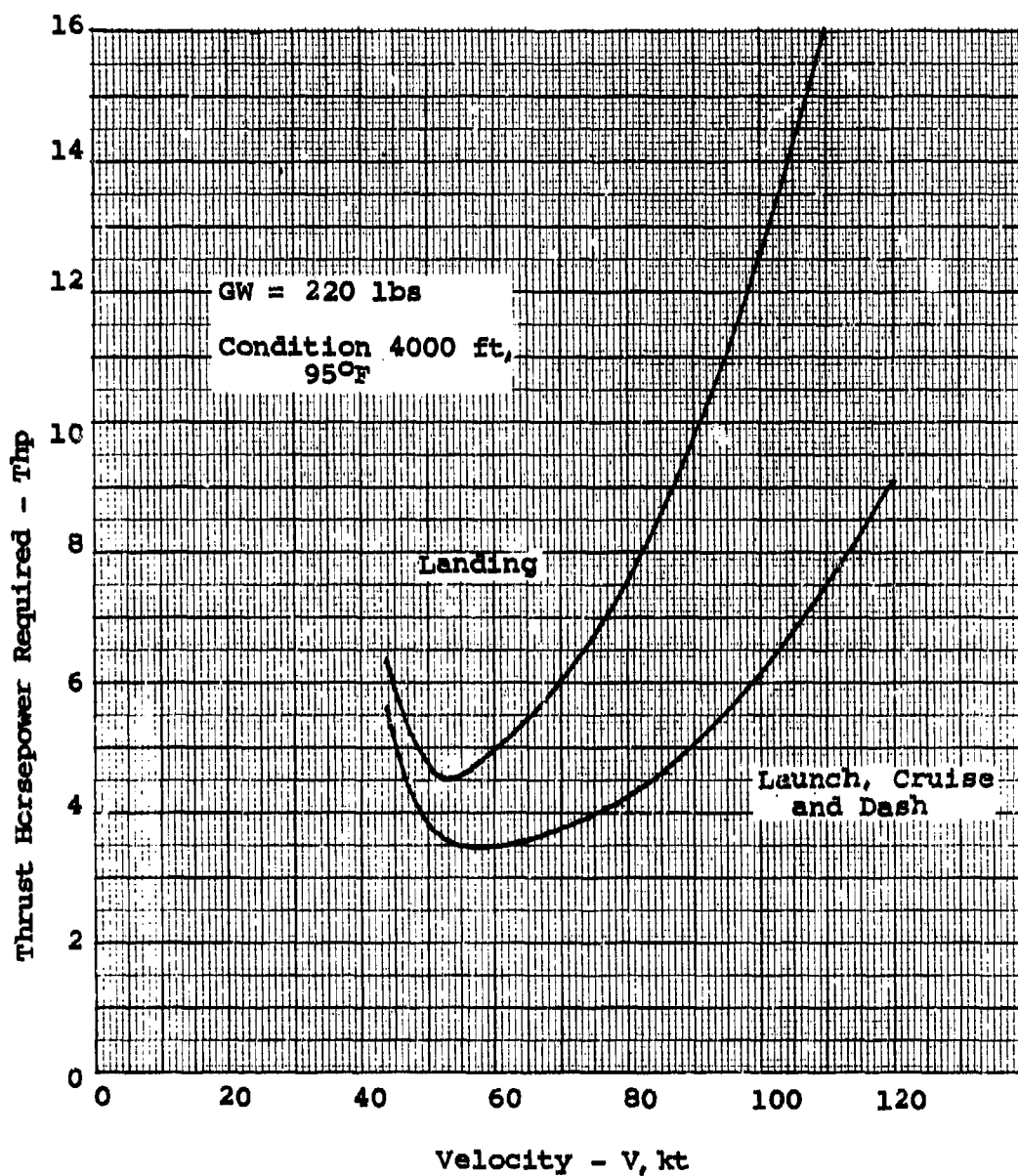


Figure 1. Thrust Power Required vs Velocity for Advanced RPV.

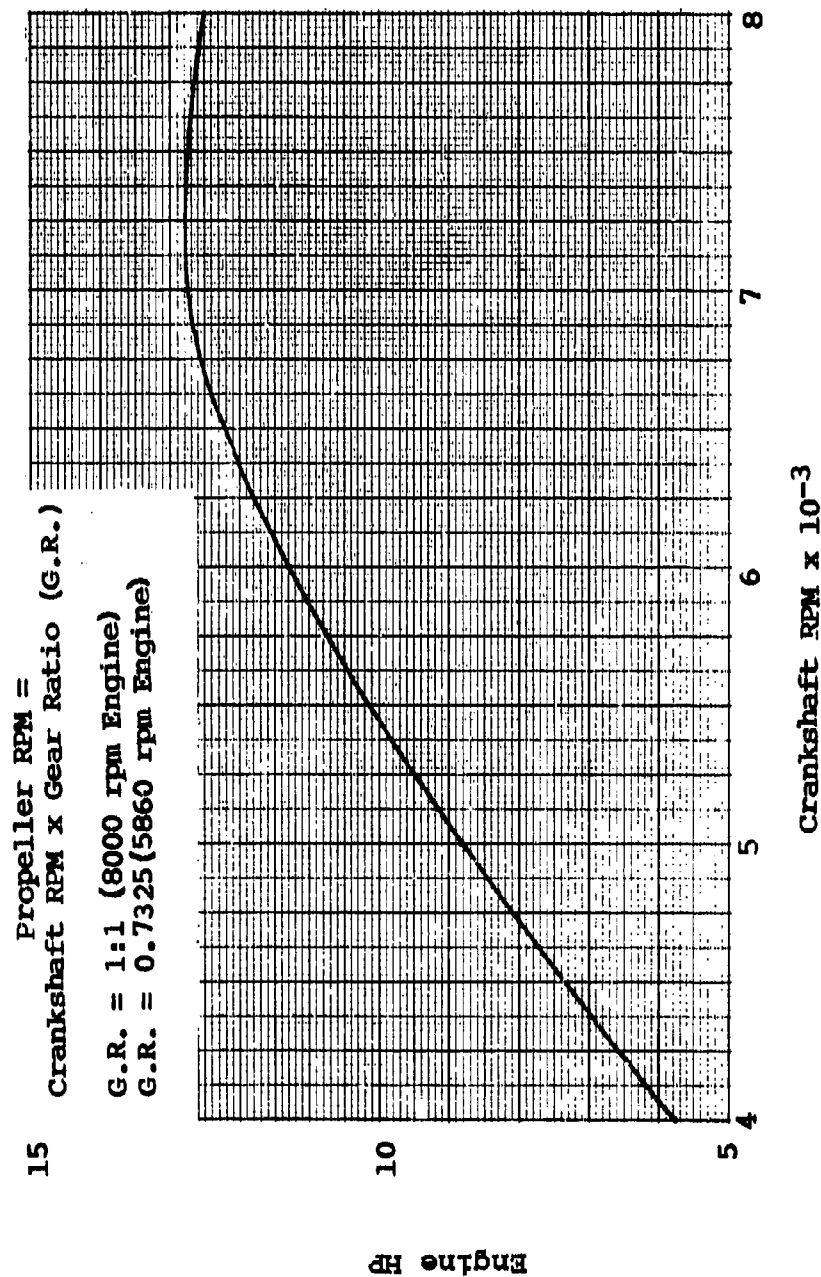


Figure 2. RPV Engine HP Available at Full Throttle
With Alternator Installed.

METHODS OF ANALYSIS

PROPELLERS

The methods used for aerodynamic design and analysis of propellers for mini-RPV's are described in Volume I of this report and in Reference 1. Strip methods are used to calculate the performance of any given propeller design. These methods depend on theory for finding the induced losses, and two-dimensional airfoil data for finding the profile losses. These methods and airfoil data have been used for many years to calculate performance of propellers operating at high Reynolds numbers. As described in Volume I, corrections to the profile drag losses were developed to account for the lower Reynolds number operation encountered with mini-RPV propellers. With these corrections for Reynolds number, the basic strip analysis method as programmed for high-speed computers, B-87, is accurate within $\pm 3\%$, based on comparisons of test and calculated propeller performance as shown in Volume I.

DUCTED PROPELLERS

Because of the interaction between the propeller and the duct, the methods used for designing and calculating the performance of ducted propellers are much more involved than those for open propellers. To use the strip analysis for calculating performance, it is also necessary to find the induced or three-dimensional losses developed on the rotor operating within the duct and then find the profile losses on each blade section based on two-dimensional airfoil data.

With an extension of the Theodorsen² theory of propellers, Wright³ and Gray⁴ developed the necessary coefficients for

- ¹ Borst, H.V., et al, SUMMARY OF PROPELLER DESIGN PROCEDURES AND DATA, Vols. I, II, and III, USAAMRDL Technical Report 73-34A,B, and C, H.V. Borst & Associates, Eustis Directorate, U.S. Army Air Mobility Research & Development Laboratory, Fort Eustis, Virginia, Nov. 1973, AD 774831, 774836, and 776998.
- ² Theodorsen, T., THEORY OF PROPELLERS, McGraw Hill, 1948.
- ³ Wright, T., EVALUATION OF THE DESIGN PARAMETERS FOR OPTIMUM HEAVILY LOADED DUCTED FANS, Journal of Aircraft, Vol. 7 No. 6, Nov.-Dec. 1970.
- ⁴ Gray, F.B., and Wright, T., A VORTEX WAKE MODEL FOR OPTIMUM HEAVILY LOADED DUCTED FANS, Journal of Aircraft, Vol. 7 No. 2, Mar.-Apr. 1970.

calculating the performance of optimum loaded ducted propellers.

These coefficients were calculated for the ducted propeller operating with the optimum load distribution in the same way as was done for open propellers. This is the same as assuming the rotor is operating with the trailing vortices forming a rigid wake. In compressors, this is the equivalent to the case of a vortex-free flow, which also gives an optimum load distribution. The $K(x)$ coefficients were calculated assuming the Kutta condition holds at the duct exit. Also, when calculating $K(x)$ using the theory, the duct was replaced by vortices along the mean camber line in a manner such as to cancel all normal velocity components developed by the propeller.

When calculating the performance of the rotor operating in a duct, the $K(x)$ coefficients developed by Gray and Wright⁴ are used in the same manner as the $K(x)$ coefficients of open propellers as determined by Theodorsen.² Thus, the induced angle change for applying two-dimensional airfoil data and determining the induced efficiency of ducted propellers is calculated in the same manner as for open propellers presented in Reference 1, except the $K(x)$ factors of References 3 and 4 are used.

Before the above procedures can be used for calculating the forces and moments on the rotor operating in the duct, it is necessary to find the velocity ahead of the rotor disk. This is the velocity induced by the duct when the rotor is developing thrust. This velocity changes with both the free-stream velocity and the disk loading. The duct-induced velocity is calculated based on the vortex theory developed by Kaskel, et al,⁵ which accounts for duct shape and size, rotor thrust, blade number, advance ratio, and rotor location within the duct. The pressure distribution and the duct thrust can also be calculated using the procedures calculated by Kaskel, et al.⁵

Thus, for an assumed duct shape and size, the performance of a ducted propeller is calculated as follows:

¹ Borst, et al.

² Theodorsen.

³ Wright.

⁴ Gray and Wright.

⁵ Kaskel, A.L., Ordway, D.E., Hough, G.R., and Ritter, A., A DETAILED NUMERICAL EVALUATION OF SHROUD PERFORMANCE FOR FINITE-BLADED DUCTED PROPELLERS, Therm Advanced Research, Division of Therm, Ithaca, N.Y., TAR-TR 639, Dec. 1963.

1. Assume a rotor thrust; calculate the velocity induced at the rotor face by the duct, using the method and data of Reference 5.
2. At an assumed blade angle, calculate the thrust and power developed by the rotor using ducted propeller values of $K(x)$ for calculating the induced angle of attack and suitable two-dimensional airfoil data.
3. Adjust blade angle to a value so that the thrust developed by the rotor is equal to the assumed value in Step 1.
4. Knowing rotor thrust and the flight condition, calculate the duct thrust and skin friction drag.
5. Calculate the efficiency from the equation:

$$\eta = \frac{(T_D + T_R + D_{SF})V_0}{550 \text{ HP}}$$

where

T_D = duct thrust

T_R = rotor thrust

D_{SF} = duct skin friction drag

V_0 = free-stream velocity

HP = installed horsepower.

SELECTION OF OPTIMUM CONFIGURATION

When selecting the optimum propeller or ducted fan configuration for an airplane with several critical design flight conditions, it is necessary to establish the relative merits and requirements of each condition. Further, it is necessary to determine what types of performance compromises might be involved at secondary design conditions, if the propeller were optimized only for the primary design condition. This is done by determining the optimum performance at each operating condition for comparison with practical propellers that may be designed for peak performance at any other flight condition.

⁵ Kaskel, Ordway, Hough, and Ritter.

Optimum Propellers and Ducted Fans

The optimum propeller and ducted fan are defined as configurations that operate with minimum induced- and profile-drag losses at any flight condition. The performance of the optimum propeller is a function of the blade number and diameter, and increases with both of these variables. When the profile drag is zero, the efficiency of the optimum propeller is equal to the ideal induced efficiency. This corresponds to the efficiency of a propeller operating with an optimum load distribution.

The profile drag losses of a propeller designed for a given condition are a minimum when the lift/drag ratio of the entire propeller is in the range of 60. This corresponds to a drag/lift angle γ of approximately 1° . By assuming that the optimum propeller is operating with an ideal induced load distribution and a drag/lift angle of 1° , the performance can easily be determined at any condition using the short method given in Volume I. Knowing the optimum performance, it is possible to find the change or loss in performance of a practical propeller in relationship to the best that can be obtained. This is very useful for finding the best compromise propeller design for a number of flight conditions.

The actual design of an optimum propeller of a given diameter and blade number operating at any condition of power, speed, rpm, and altitude requires finding the loading distribution for a combination of minimum profile and induced losses. This involves finding the blade solidity needed to operate at a lift coefficient and the corresponding blade camber for peak lift/drag ratio; it also involves determining the best distribution of loading, considering the induced losses for peak efficiency. This procedure involves the use of the theory of The Calculus of Variations and is detailed in Reference 1.

¹ Borst, et al.

PROPELLER SELECTION FOR RPV'S

The choice of the best propeller configuration for a remotely piloted vehicle depends on the flight conditions and their relative importance. For the advanced RPV aircraft, the flight speeds at the launch and landing conditions are equal and the performance requirements are such that one propeller will have peak performance for both conditions. The design cruise speed specified is that for maximum endurance where the power is minimum. At this condition the speed is close to that of the launch and landing speed conditions. Since the thrust required for cruise is lower than the launch thrust required, the optimum propeller chosen would be smaller than the optimum propeller designed for launch and landing conditions. It is therefore important that the propeller selected to give peak performance at the launch and landing conditions be as small as possible to obtain peak cruise performance. This means that the propeller designed for launch should develop only the minimum thrust needed for this condition.

The fourth operating condition to be considered is the dash case. Here, the specified requirement is to provide sufficient thrust so the airplane can exceed a speed of 100 knots. This is a full power or a maximum rpm condition, depending on the propeller used. At the dash condition, the efficiency becomes of secondary importance as long as the thrust available exceeds that required to obtain a flight speed of 100 knots. In Volume I it was shown that the optimum propeller designed for the dash condition is too small to meet the performance requirements of the launch and landing conditions. Thus, for peak performance at dash, the propellers chosen for the launch and landing conditions should also be the minimum size needed.

The thrust required for the launch condition must be met to obtain the needed rate of climb. When this level of thrust is obtained at launch, sufficient propeller thrust is available at the landing condition so the rate of climb is in the required 200 to 610 fpm range. Since propellers required for optimum performance at the cruise and dash operating flight conditions are smaller than those needed at the launch condition and will not develop the required performance at the launch condition at the lowest possible rpm necessary for peak performance and minimum noise, the launch condition has been selected as the primary design condition. At this condition, optimum minimum-size propellers and ducted fans are chosen to meet the aircraft performance requirements.

For RPV's equipped with fixed-pitch propellers operating at a certain engine power setting, the best configuration can be determined directly by finding the efficiency required at the

launch condition as a function of rpm and matching this performance with that obtained with the optimum propeller design. The required efficiency depends on the thrust horsepower requirements of the airplane and is a function of the aircraft drag, rate of climb, and the engine power/rpm characteristics of the specified throttle setting. It is apparent that the propeller efficiency required will increase as the power available decreases.

For the launch and landing conditions when the engines are operating at a maximum power setting, the efficiency requirements as a function of rpm are given in Figures 3 and 4 for both the direct-drive and geared engines. These efficiency requirements were calculated using the thrust horsepower characteristics of the airplane and the engine power output data given in Figures 1 and 2. The aircraft performance requirements used are given on page 10.

OPTIMUM PROPELLER DESIGN FOR 8000 RPM ENGINE

To find the peak propeller performance and corresponding power and rpm that will meet the efficiency requirements of the advanced RPV equipped with the 8000 rpm direct propeller drive engine as given in Figure 3, the performance of a series of optimum propellers must be found. At the design launch condition, the optimum efficiency of a series of optimum two-, three-, and four-bladed propellers was found for a range of diameters up to the maximum allowed of 2.5 feet. The variation of efficiency with propeller rpm of these optimum propeller configurations is shown, along with the efficiency required, in Figure 5. The lowest rpm where the efficiency of the optimum propellers crosses the efficiency-required curve is the best operating condition for any propeller. At this condition, the noise level of the propellers would also be a minimum, as the noise produced is a direct function of the tip speed and, therefore, rpm. Since the peak efficiency and minimum noise level occur at the lowest possible rotational speed, this criterion was used for selecting the best propeller. This, then, determines the optimum propeller diameter, blade number, operating rpm, and input power to meet the efficiency required to obtain the specified aircraft performance. For the direct-drive engine, the optimum two-bladed, 2.5-foot-diameter propeller meets the efficiency required at the launch condition when operating at a rpm of 5600 and with an efficiency of 73% (Figure 5). If a four-bladed, 2.5-foot-diameter propeller were used, the rpm would decrease to 5500 and the efficiency would increase to 75.5%. In determining whether a two- or a four-bladed propeller should be used, a detailed blade analysis is necessary to find the solidity for peak performance. If the loading is low, the solidity of the blades

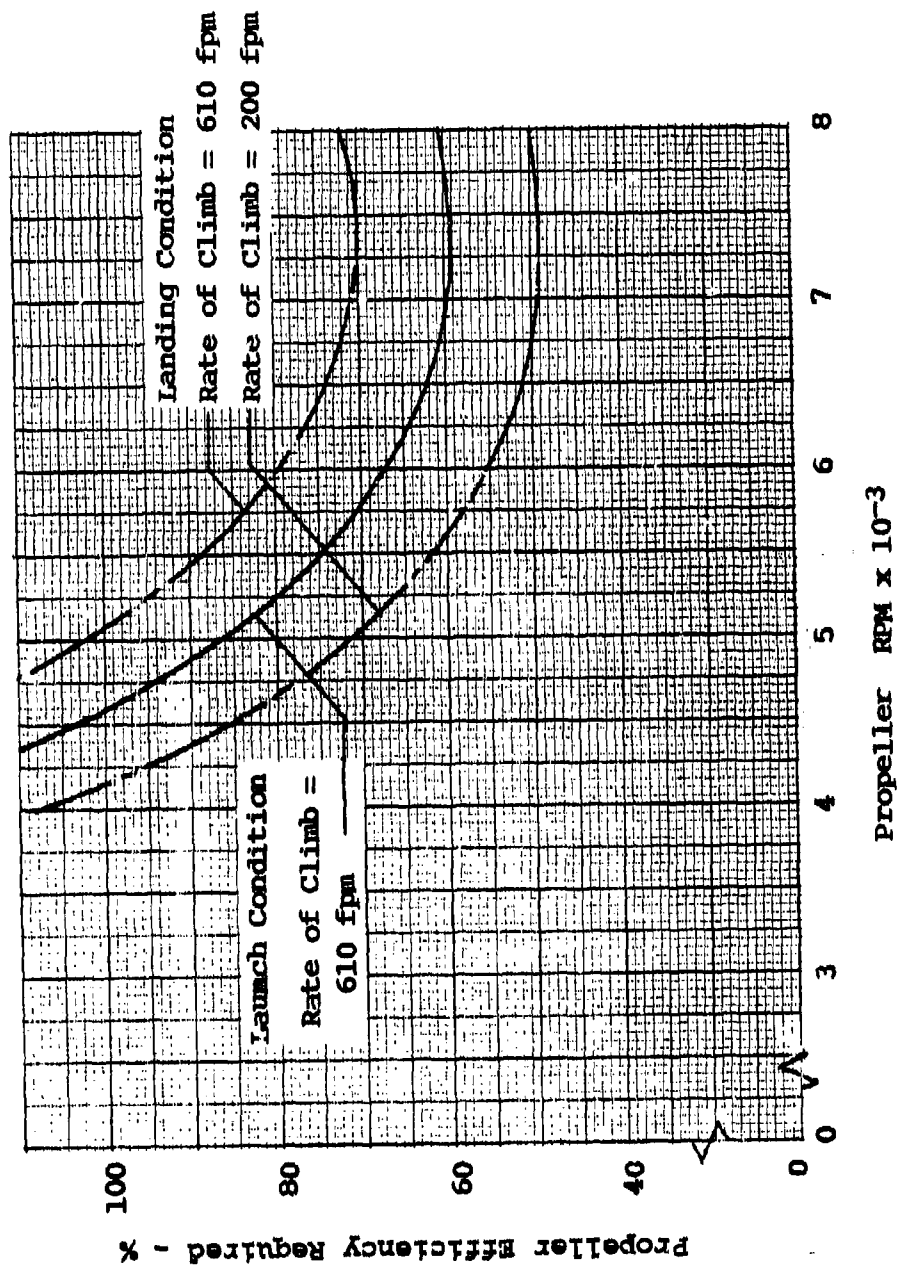


Figure 3. Efficiency Required for the Launch and Landing Conditions vs Propeller RPM — 8000 RPM Engine.

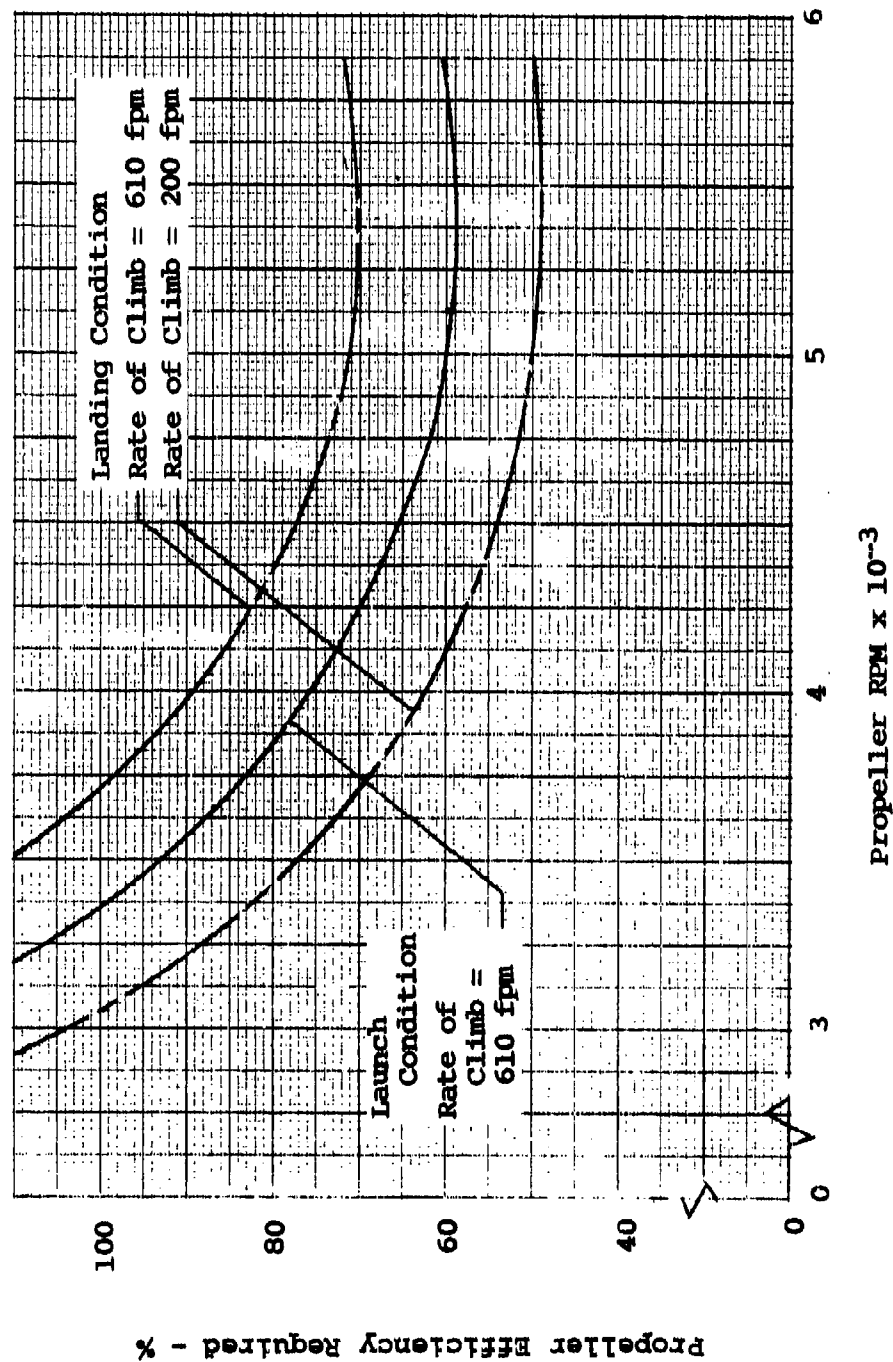


Figure 4. Efficiency Required for the Launch and Landing Conditions vs Propeller RPM — 5860 RPM Engine.

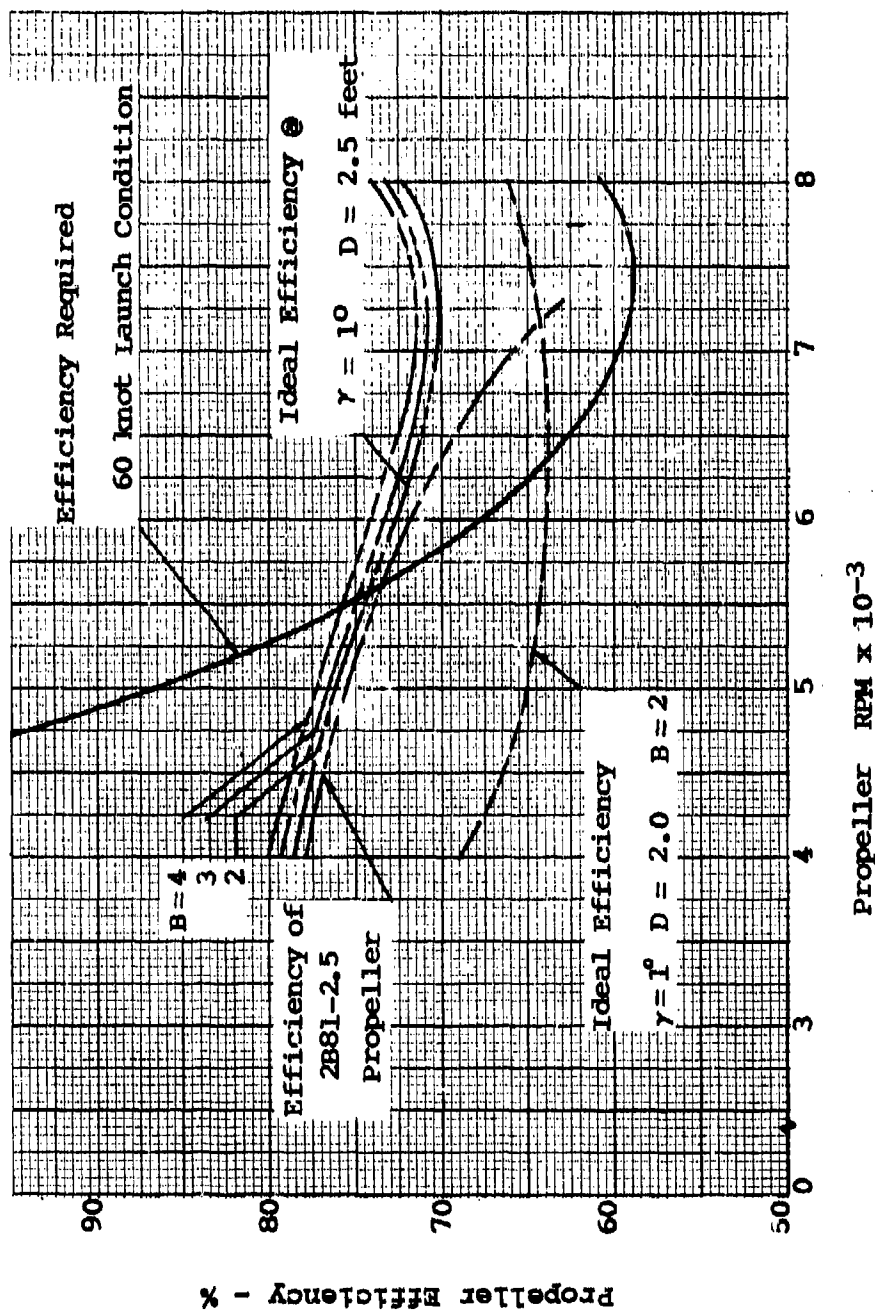


Figure 5. Efficiency Required and Available With Open Propellers at the Launch Condition vs Propeller RPM - 8000 RPM Engine.

for the four-bladed configuration might be too low, resulting in an impractical configuration.

In the above analysis only the operating rpm and power are established for each optimum propeller of the given blade number and diameter; the details of the blade, including solidity, camber, section type, and blade angle distribution, are not known. The design details of the blade required to achieve the stated performance were determined by the methods described previously and given in Reference 1. For the 8000 rpm engine, the analysis showed that the 2.5-foot-diameter propeller with two 81.1 activity factor blades is the optimum configuration. The blades with an activity factor of 81 are as small as are structurally practical, so that the propellers using three or four blades requiring activity factors below 50 were not considered. Since the improvement of performance using four blades is small, the loss in performance using the two-bladed configuration is minimal. The detailed characteristics of the propeller with two 81 activity factor blades designated 2B81-2.5 are given in Figure 6. Tables of the design data needed for fabrication of this blade are given in Appendix A. An efficiency map for determining the performance at any design condition of the 2B81-2.5 propeller is given in Figure 7. This map was calculated using the methods and data described in Volume I and includes all the necessary corrections for the low Reynolds numbers encountered.

Performance of 2B81-2.5 Propeller on the 8000 RPM Engine

The performance of the 2B81-2.5 propeller operating at a fixed blade angle using the 8000 rpm engine is given in Table 1 for the advanced RPV. The performance of the optimum propellers for each condition is also shown in the table for comparative purposes. This propeller meets all the aircraft performance requirements given on page 10. It operates at peak efficiency at the design launch condition and has an efficiency of 80.0% at the cruise condition, which is within 3.1% of the peak efficiency possible for an optimum propeller specifically designed for that condition. The performance of the propeller allows a dash speed of 123.5 knots. This is well above the minimum required.

OPTIMUM PROPELLER DESIGN FOR THE GEARED 5860 RPM ENGINE

With the geared 5860 rpm engine the operating propeller rpm needed to meet the performance requirements of the advanced RPV

¹ Borst, et al.

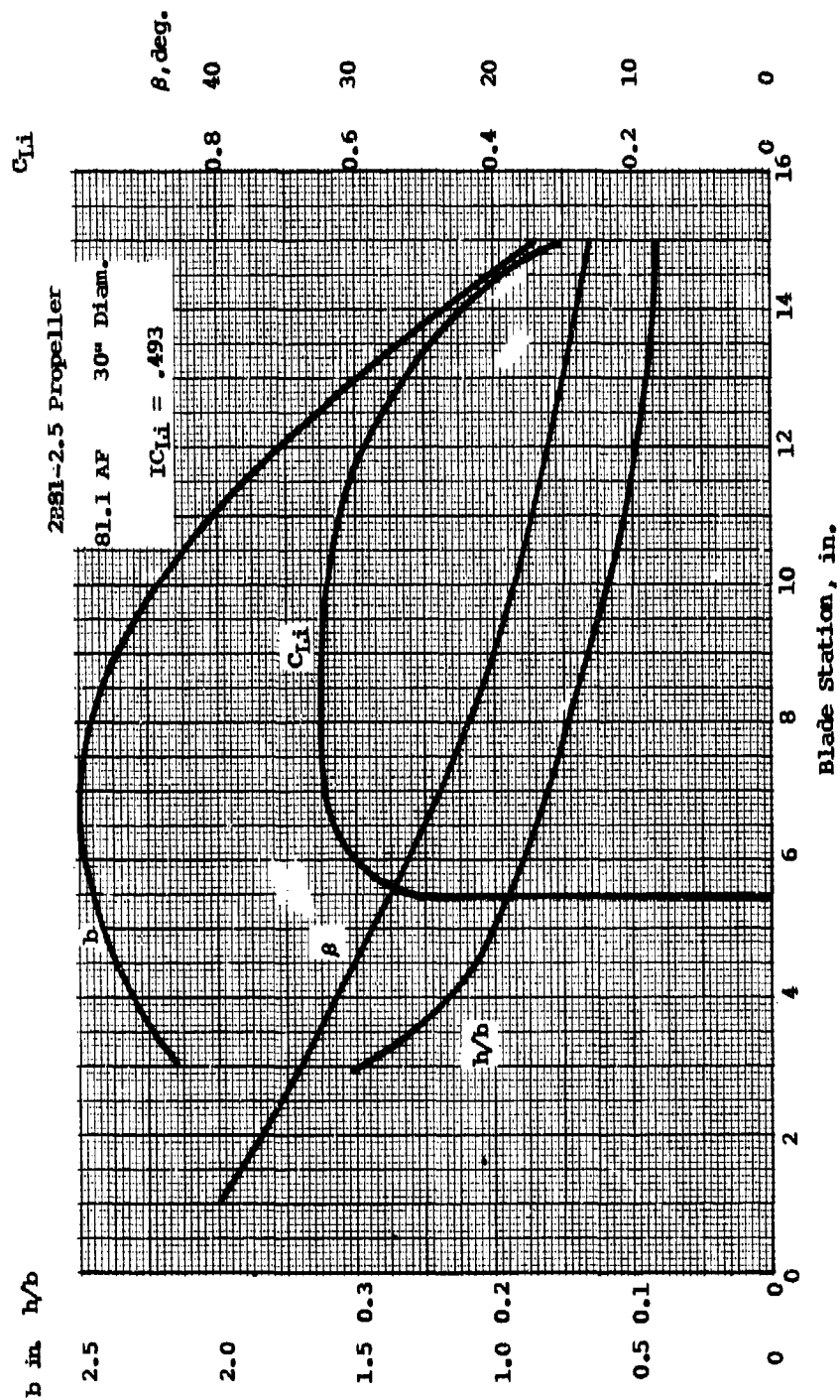


Figure 6. Blade Design Characteristics - 2B81-2.5 Propeller, 8000 RPM Engine.

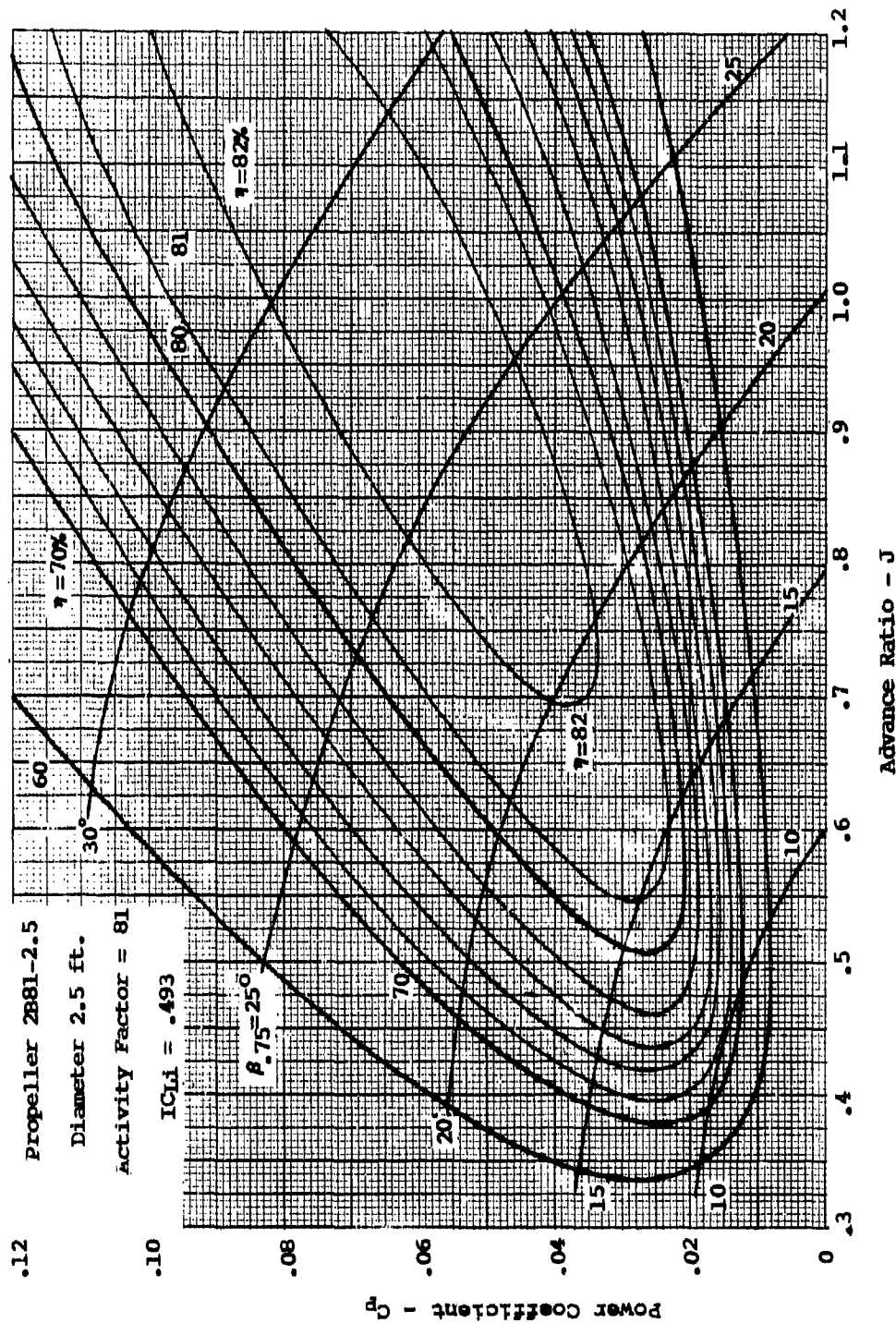


Figure 7. Performance Efficiency Map - 2B81-2.5 Propeller.

TABLE 1. CALCULATED PERFORMANCE

AIRPLANE: Advanced RPV												
ENGINE: 8000 RPM												
GEAR RATIO: 1.0												
PROPELLER:		MODEL					Ideal		2B81-2.5		5D130-1.75	
		NC. OF BLADES					2		2		5	
		DIAMETER					2.5		2.5		1.75	
		ACTIVITY FACTOR					81.0		81.0		130	
		INTEGRATED DESIGN C_L					.493		.493		.60	
		INDUCT LENGTH TO DIAMETER					None		None		.75	
		BLADE ANGLE AT $x = .75 = \beta, 75$					15.60		15.60		240	
CONDITION		SPEED		RATE OF AFT. TEMP.		PROP.		EFFICIENCY %				
	KT	CLIMB	FT	OF	HP	RPM						
LAUNCH	60	610	4000	95	10.3	5600	74.0	73.5				80.1
	60	610	4000	95	9.5	5250						
RECOVERY	60	200-610	4000	95	10.3	5600	74.0	73.5				80.1
	60	200-610	4000	95	9.5	5250						
CRUISE	75	0	4000	95	4.9	4960	83.1	81.5				83.0
	75	0	4000	95	4.82	4750						
DASH	123.5	0	4000	95	12.8	7500	85.2	77.0				76.6
	123	0	4000	95	12.8	7150						

are lower than the 8000 rpm engine. This occurs as the maximum output power is the same for both engines when the rpm times the gear ratio of the direct-drive engine is equal to that of the geared engine. Thus, the required efficiency of the optimum propeller occurs at a rpm of 4150 for the four-bladed, 2.5-foot-diameter configuration as compared with 5600 rpm for the direct-drive engine. As shown in Figure 8, the optimum efficiency of this propeller is approximately the same as that of the propeller used for the direct-drive engine. It would therefore appear that the main advantage of using the geared drive engine is the reduction of noise due to operation at reduced rpm.

As shown in Figure 8, the required efficiency can be obtained for the launch condition using propellers with lower diameters than the maximum allowable. These propellers will operate at increased rotational speeds and lower values of efficiency than the larger diameter propellers. Since the tip speeds of both configurations are approximately the same, there would be no noise advantage with the lower diameter configuration.

The details of the propeller for the geared engine needed to meet the performance requirement at the launch condition were found using the same optimum strip analysis procedure as was used previously. The results of this analysis indicated that a 2.5-foot-diameter propeller with four 81 activity factor blades, having an integrated design C_L of .465, would be required. The detailed characteristics of the blade are given in Figure 9. Tables of the blade section ordinates, from which the blade can be manufactured, are given in Appendix A. The blade is considered to be used with a 7.5-inch-diameter spinner, which gives an inboard cutoff of r/R of .25. This propeller configuration is designated 4B81-2.5.

Performance of 4B81-2.5 Propeller on the 5860 RPM Engine

A generalized efficiency map for the 4B81-2.5 propeller was calculated using the B-87 propeller computer program corrected to include the effects of low Reynolds number, and is given in Figure 10. From this map the performance of the propeller operating at the fixed-pitch blade angle needed for the launch condition is given in Table 2. The performance of this propeller meets or exceeds the requirements at all conditions.

Also shown in Table 2 is the optimum efficiency of propellers for each design condition. Comparison of the optimum propeller in each case to the 4B81-2.5 shows that this configuration has excellent performance at all the design operating conditions.

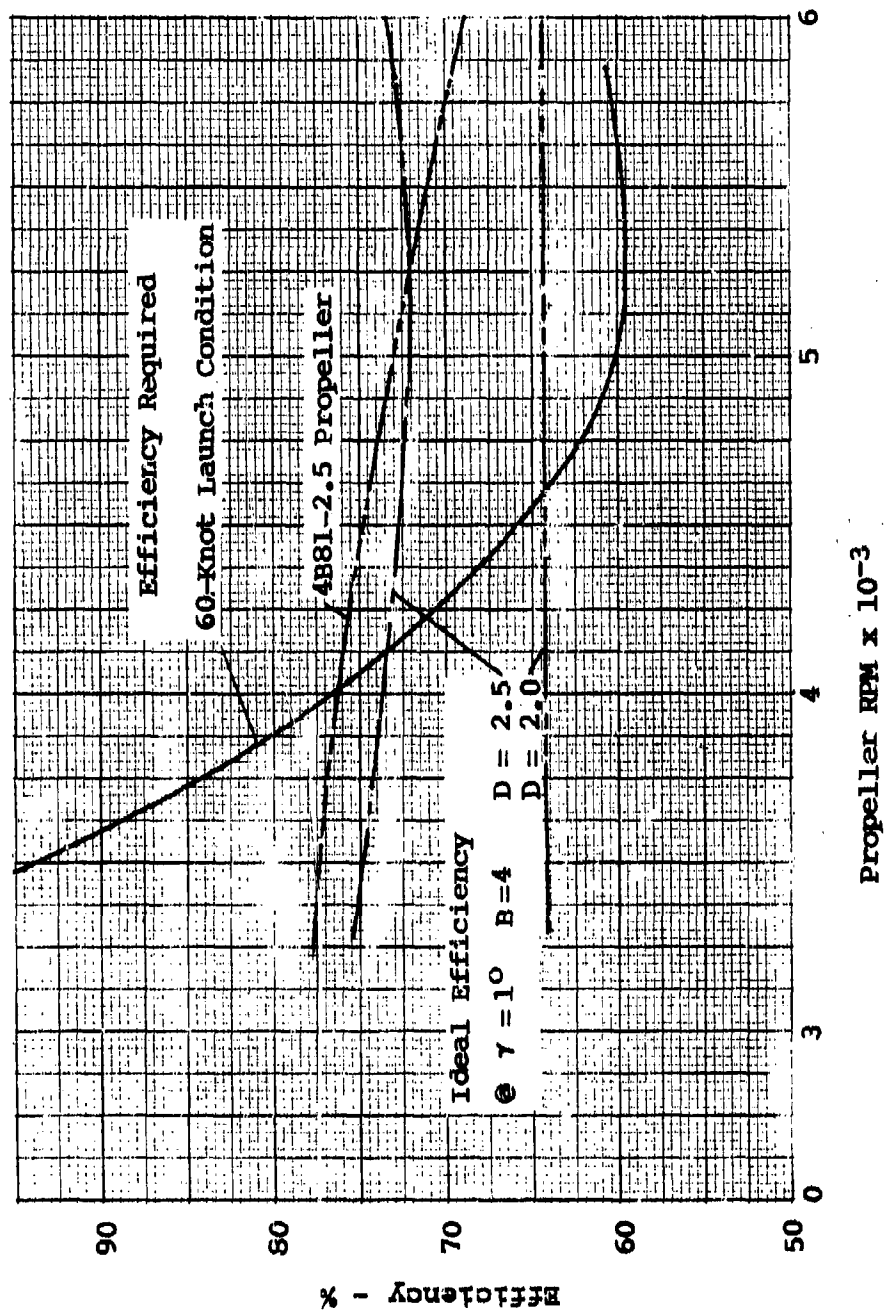


Figure 8. Efficiency Required and Available With Open Propellers at the Launch Condition vs Propeller RPM — 5860 RPM Engine.

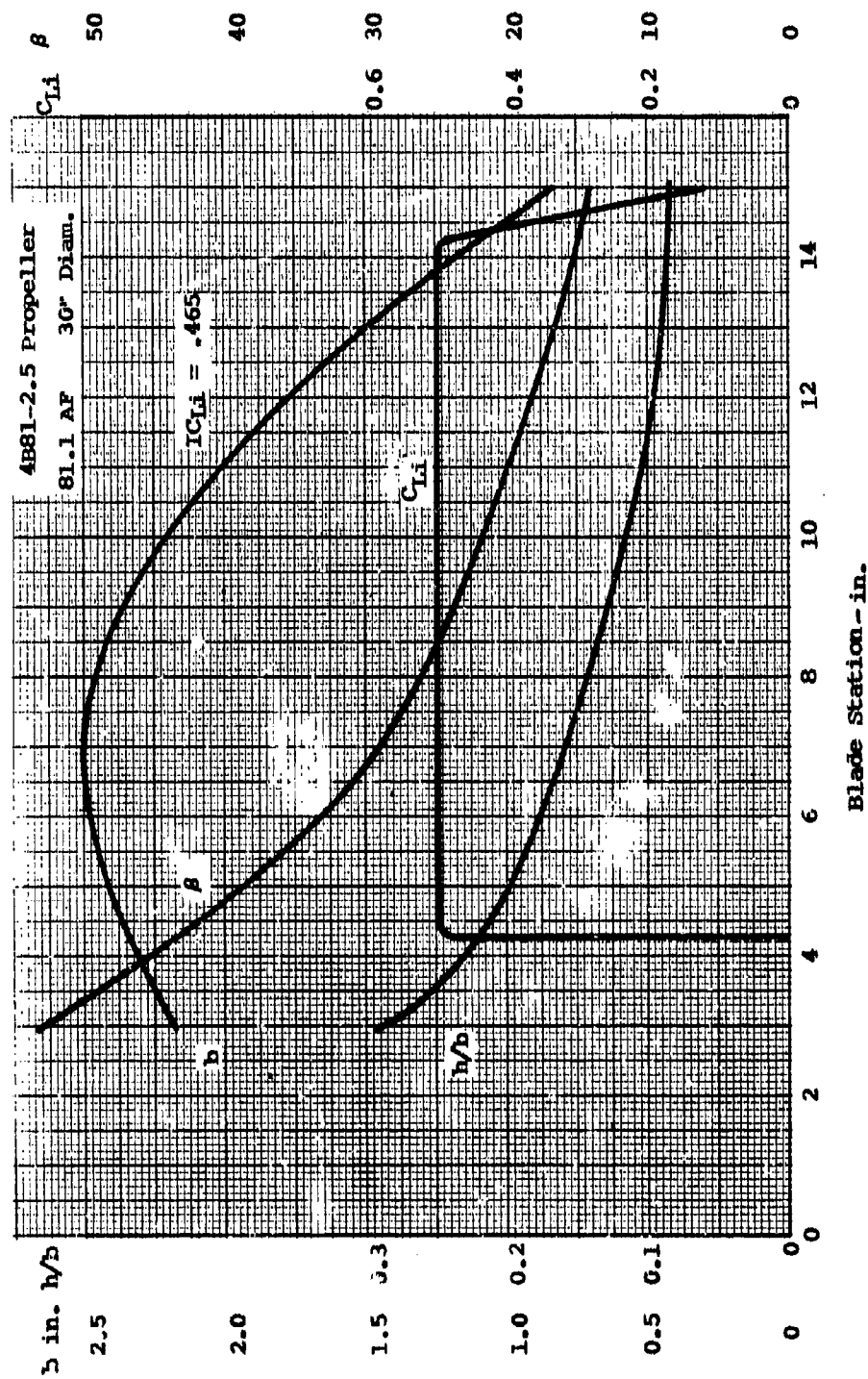


Figure 9. Blade Design Characteristics - 4B81-2.5 Propeller, 5860 RPM Engine.

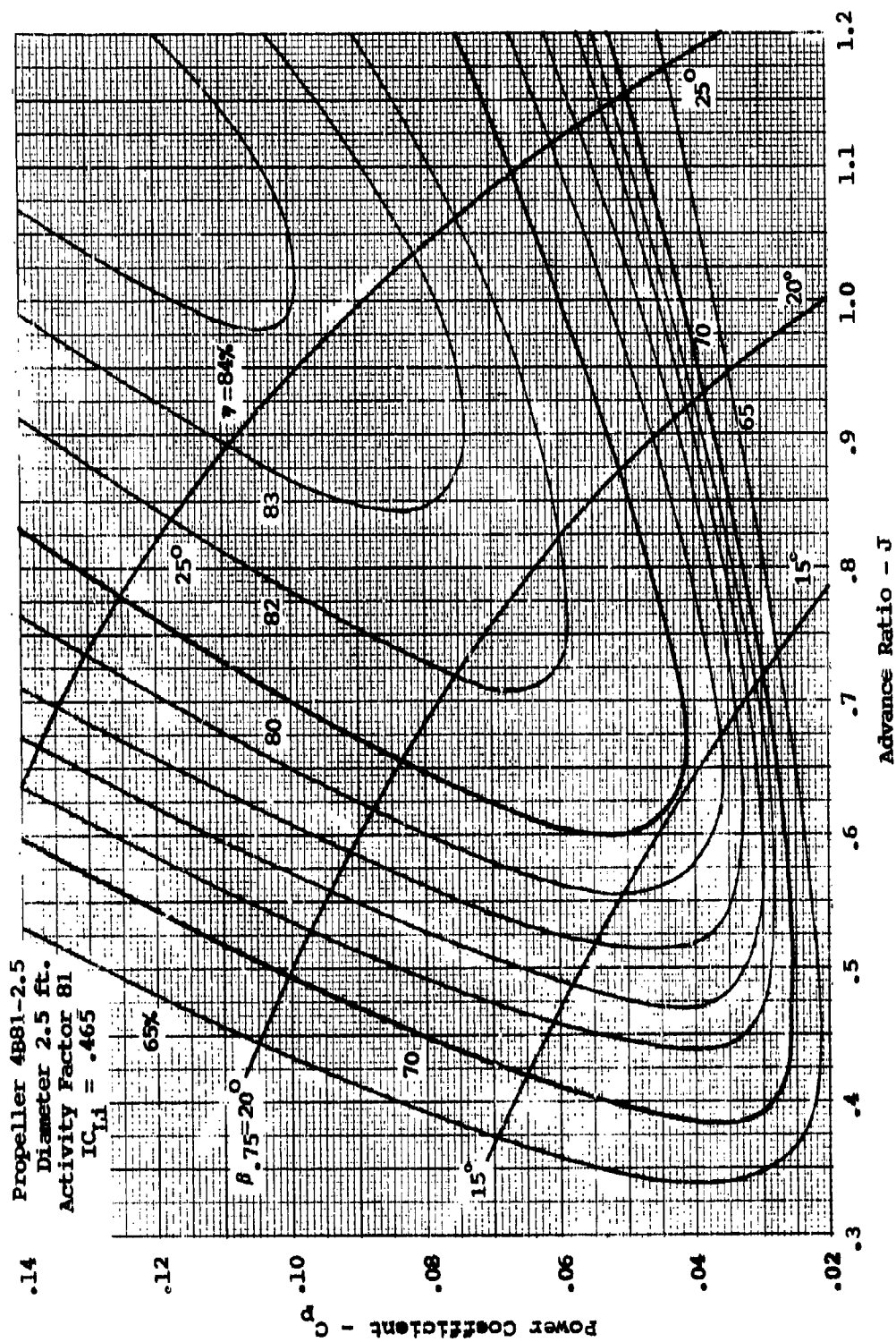


Figure 10. Performance Efficiency Map — 4B81-2.5 Propeller.

TABLE 2. CALCULATED PERFORMANCE

AIRPLANE: Advanced RPV ENGINE: 5860 RPM GEAR RATIO: .7325									
PROPELLER:		MODEL		Ideal		4B81-2.5		5D130-2	
		NO. OF BLADES		4		4		5	
		DIAMETER		2.5		2.5		2	
		ACTIVITY FACTOR		81.0		81.0		130	
		INTEGRATED DESIGN C_L		.464		.464		.7	
		DUCT LENGTH TO DIAMETER		None		None		.75	
		BLADE ANGLE AT $x=.75=\beta$		21.0°		21.0°		29.5°	
SPEED RATE OF ALT. TEMP. PROP.									
CONDITION		KT	CLIMB	FT	OF	HP	EFFICIENCY %		
LAUNCH		60	610	4000	95	10.0	4000	73.7	76.0
		60	610	4000	95	9.0	3750		84.0
RECOVERY		60	200-610	4000	95	10.0	4000	73.7	76.0
		60	200-610	4000	95	9.0	3750		84.0
CRUISE		75	0	4000	95	4.87	3600	84.8	82.2
		75	0	4000	95	4.99	3500		80.2
DASH		124	0	4000	95	12.8	5480	87.1	78.0
		120	0	4000	95	12.8	5100		72.0

DUCTED PROPELLERS

The existing RPV's incorporate shrouds to protect the propeller from damage and operating personnel from injury. The shrouds used are not effective for improving the rotor performance, as the tip clearance is large relative to the rotor diameter. Thus, the duct cannot control the tip losses by counteracting the radial flow velocity as in the case of a properly designed ducted propeller. In Volume I it was shown that with properly designed ducted propellers, important improvements in performance could be achieved compared with the open propeller. Further, it was shown that the performance increase would be obtained with rotors of lower diameter operating at reduced tip speeds. Because of the projected increase of performance shown, ducted propellers were designed for optimum performance on the advanced RPV's using both the direct-drive 8000 rpm and the geared 5860 rpm engines.

To determine the size requirements of the ducted propeller, an analysis was conducted in a manner similar to that performed for the open propellers. This is feasible, as variation of the efficiency required with rpm depends only on the engine characteristics and airplane requirements, and does not depend on the type of propeller or fan used. The efficiency needed to meet the performance requirements of the airplane for the launch condition is then compared to that produced with ducted propellers of various sizes. The efficiency of the various-size ducted fan configurations was determined from the characteristics of a known ducted propeller designed for operating in the same speed range as the RPV's. It was necessary to use the performance characteristics of an existing fan configuration for estimating the required fan size, as single-point methods such as those used for open propellers have not been developed as yet. Such methods would be useful for this purpose.

OPTIMUM DUCTED PROPELLER DESIGN FOR 8000 RPM ENGINE

Based on a five-bladed ducted propeller operating in a duct with a length to diameter ratio of 0.75, the efficiency at the launch condition was estimated for a series of diameters at a range of rotational speeds. Rotor diameters from 1.5 to 2.5 feet were considered. The results of these calculations are shown in Figure 11 in comparison with the efficiency required at the launch condition. The performance of rotors from 1.5 to 2.0 feet diameter will meet that required, with the 1.75-foot-diameter configuration having the best efficiency at the lowest rotational speed. This rotor diameter was therefore selected for further analysis and optimization.

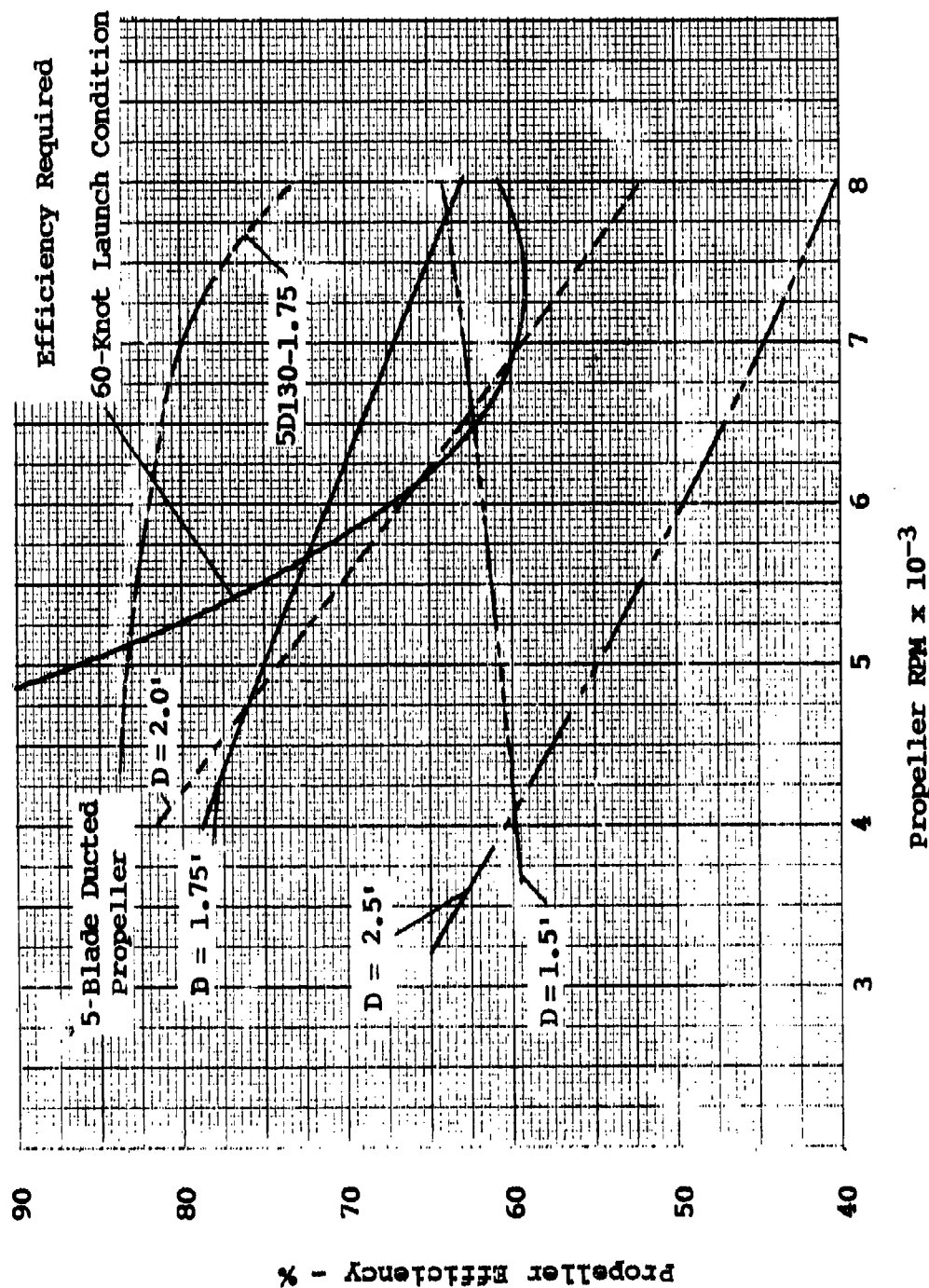


Figure 11. Efficiency Required and Available With Ducted Propellers at the Launch Condition vs Propeller RPM - 8000 RPM Engine.

Duct Design

As the duct and rotor interact, the design of each component depends on the other. The flow velocity at the rotor face is influenced by the shape of the duct and the thrust produced by the rotor. Since the design details and performance of the rotor are influenced by the upstream velocity, it is necessary to establish the duct configuration prior to designing the rotor blades.

The duct design parameters to be considered for a given rotor diameter are the duct length-to-diameter ratio, the airfoil cross section, the location of the rotor disk within the duct, and the angle of attack of the duct section with respect to the duct centerline. In choosing the above parameters for a given ducted propeller design, the main considerations are the avoidance of separation at the inlet and exit and controlling the flow in the propeller plane so that radial flow losses, such as those encountered with open propellers, are eliminated or reduced to a minimum.

The propeller or rotor of a pusher installation operates downstream of the engine; as a result, part of the rotor will be operating in the wake produced by the engine. This wake influences both the aerodynamic performance and the structural characteristics of the blades. Two opposing cylinders extend into the stream ahead of the rotor and thus will influence the flow into the rotor. To reduce the effect of the cylinder wake which can have a high velocity decrement, it appears possible to duct the airflow through the rotor hub in a manner illustrated in Figure 12. The exhaust flow can be mixed with the flow over the cylinders required for cooling and also passed through the rotor hub. The spokes of the rotor hub may be vanes or blades to force the flow through the spinner, and thus provide positive cooling airflow.

To provide minimum interference, the rotor hub has a conical shape to increase the axial distance between the struts and the rotor blades. This gives an axial distance to the propeller of approximately two stator chord lengths. This is the minimum distance for the rotor to be spaced from the stator vanes to minimize interference and noise. As a result of the above considerations, the engine/propeller was positioned in the duct with a length-to-diameter ratio of .75 at the mid-duct location as shown in Figure 12. The mid-duct location of the rotor also makes it possible to control the expansion of the flow prior to the exit of the duct. This is done by limiting the included angle of the duct to 70° . To maintain low skin friction drag, the duct length was kept relatively low at a length-to-diameter ratio of .75 where the diameter is the diameter of the rotor.

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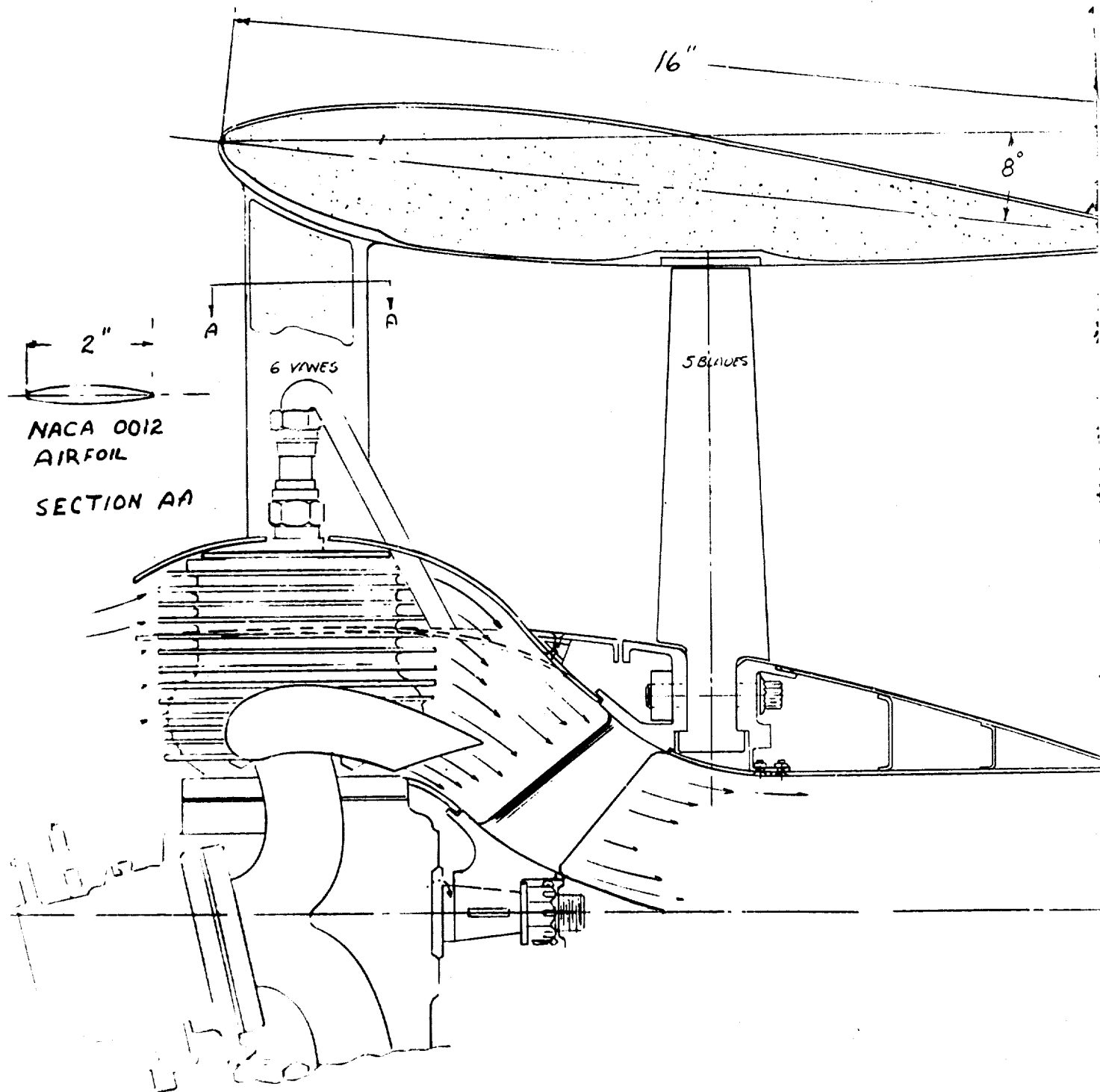
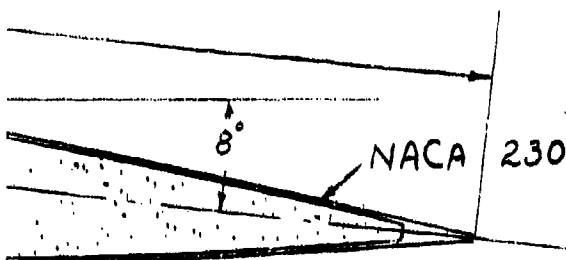
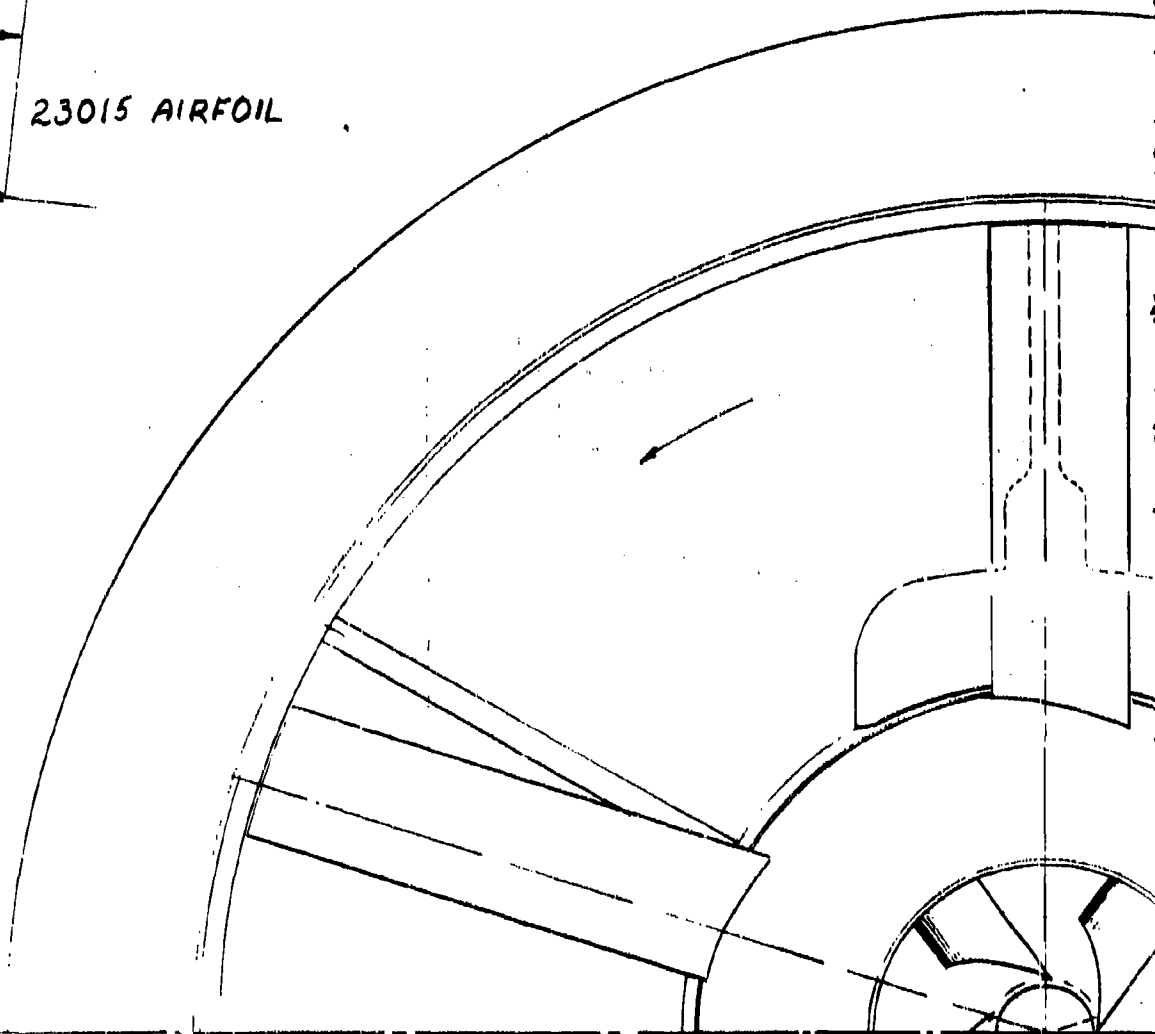
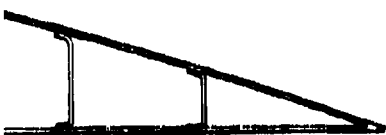
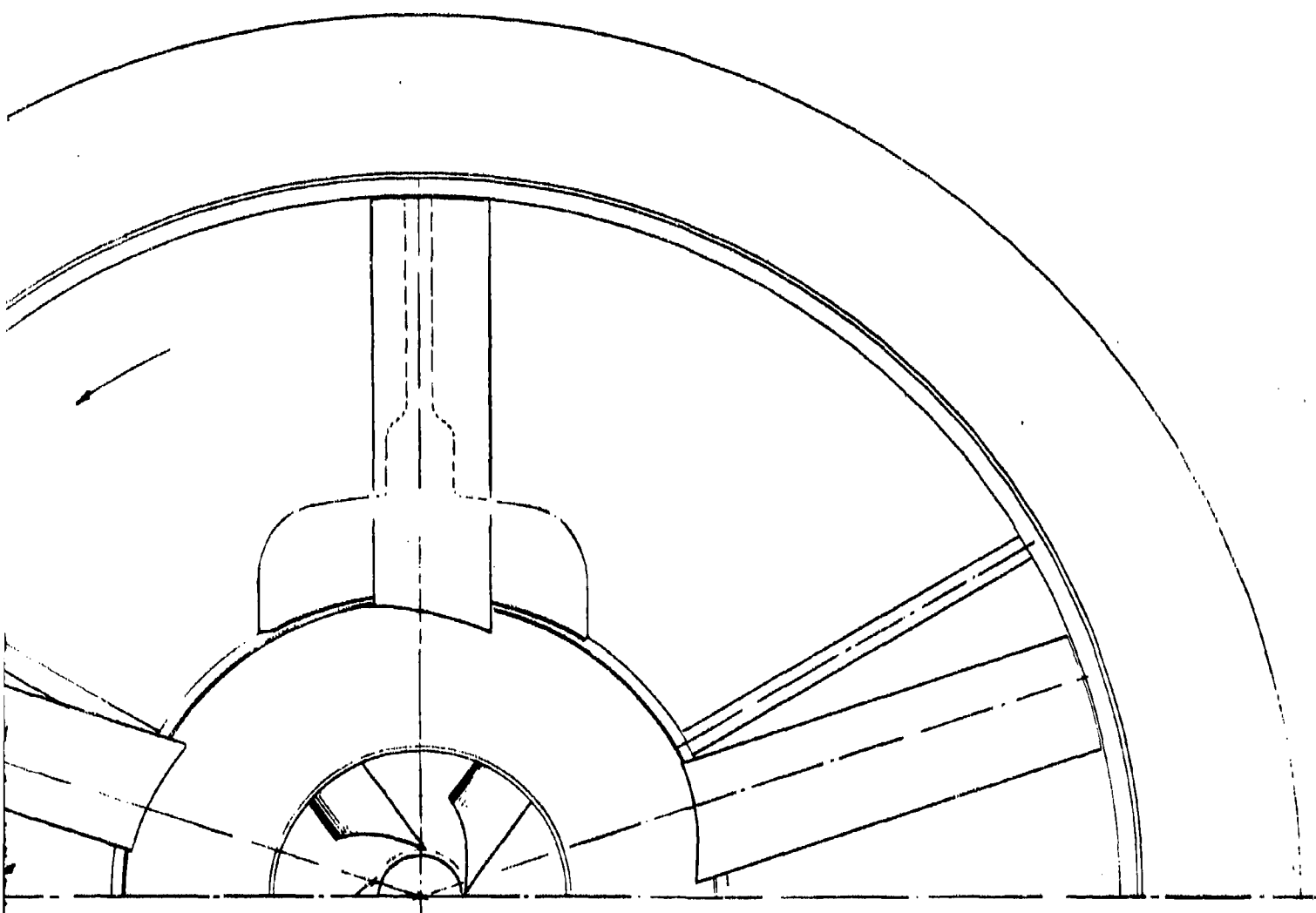


Figure 12. Conceptual Design of a Ducted Propeller Installation for Advanced RPV With 8000 RPM Engine.



NACA 23015 AIRFOIL





Calculations of the performance of several duct shapes indicated that ducts with a low thickness ratio will have small values of nose ratios, which can lead to separation at relatively low angles of attack. Further, ducts with low nose radii do not produce the thrust necessary for the development of the optimum ratio needed for peak performance. For these reasons, a 15% section using the ordinates of the NACA 23015 airfoil was chosen for the duct cross section. This duct section is oriented so that the normal upper surface is the inner surface of the duct. The above conclusions on the effect of leading-edge radius, duct length-to-diameter ratio, and propeller location are confirmed by the experimental data given in Reference 6.

Using the methods of Reference 5, the performance characteristics of the duct were calculated for a range of operating conditions that are encountered on the advanced RPV's. It was found that the thrust produced by the duct is only a function of the thrust loading of the rotor and is independent of the rotor rotational speed. The variation of duct thrust with rotor thrust is expressed in coefficient form, C'_T and C'_t , where

$$C'_T = T_R / qA = \text{rotor thrust coefficient}$$

$$C'_t = T_D / qA = \text{duct thrust coefficient}$$

$$T_R = \text{rotor thrust}$$

$$T_D = \text{duct thrust}$$

$$q = \text{free-stream dynamic pressure} = \frac{1}{2} \rho V_0^2$$

$$A = \text{disk area} = \pi D_R^2 / 4$$

The variation of the duct thrust coefficient with rotor thrust coefficient is given in Figure 12 for the duct configuration given in Figure 11.

The velocity induced by the duct and the propeller is dependent on the rotor thrust coefficient C'_T . This velocity is a function of the radial station and is expressed as the ratio of V_D to V_0 . It is calculated (knowing the duct geometry) using the method and data given in Reference 5.

⁵ Kaskel, Ordway, Hough, and Ritter.

⁶ Black, D.M., Wainauski, Harry S., and Rohrbach, C., SHROUDED PROPELLERS — A COMPREHENSIVE PERFORMANCE STUDY, AIAA Paper 68-994, Oct. 1968.

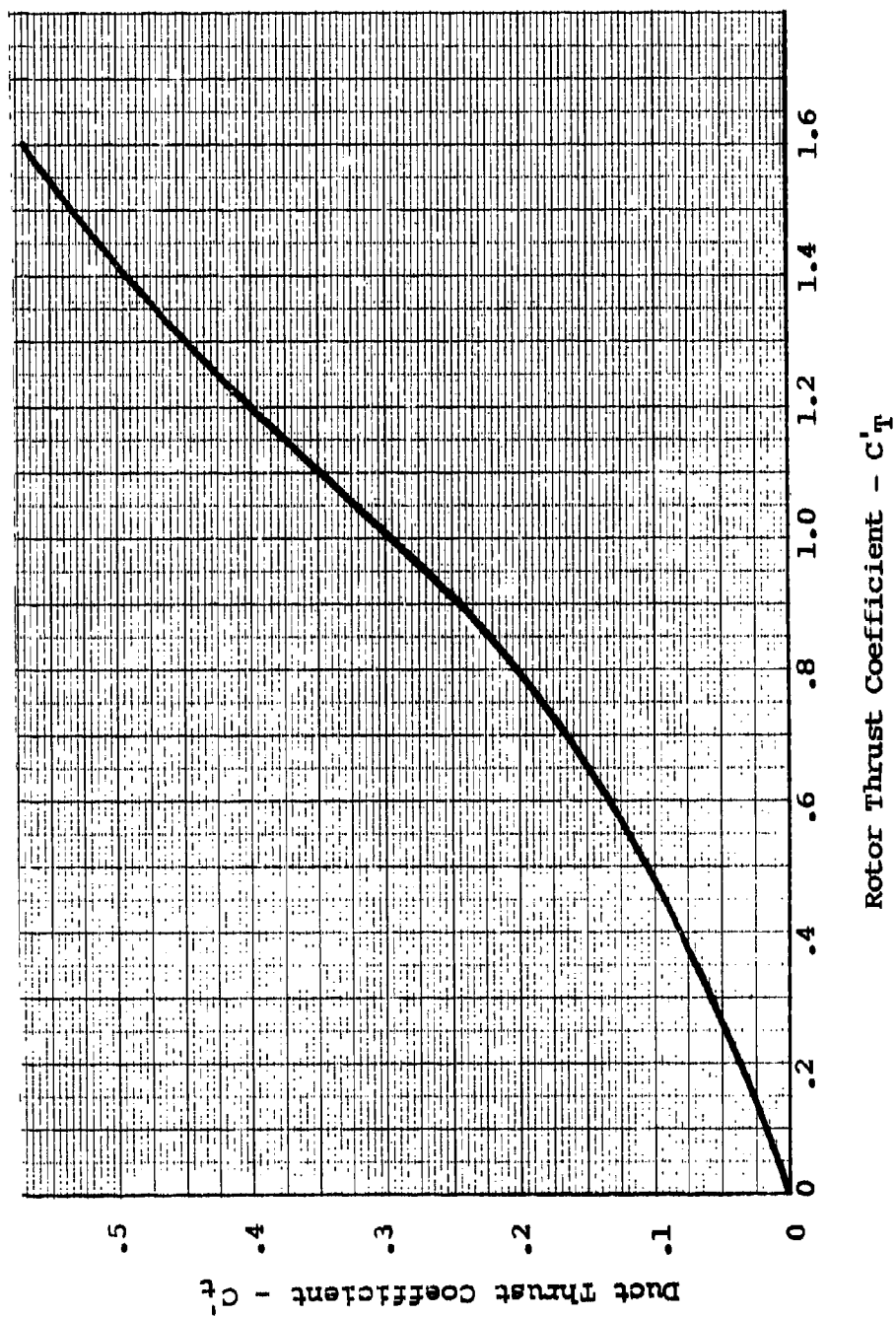


Figure 13. Duct Thrust Coefficient vs Rotor Thrust Coefficient.

Thus,

$$V_D/V_O = \mu = \text{duct velocity ratio}$$

where

V_D = velocity in the duct

V_O = free-stream velocity.

The duct velocity ratio μ is given as a function of the thrust coefficient C_T and x as shown in Figure 14. In the range of operation of the advanced RPV's, μ is independent of the rotational speed.

Ducted Propeller Design

The analysis of the efficiency requirements at the launch condition using the direct-drive 8000 rpm engine showed that a ducted propeller with a rotor diameter of 1.75 feet, operating at 5650 rpm, was the best configuration. At this condition, the details of the rotor were determined to give the optimum performance. The criterion used to establish the optimum blade configuration is basically the same as that used for optimum propeller designs. This blade loading condition corresponds to the vortex-free condition used in the design of compressors, which is the same as the rigid wake case used in propeller design. Based on this design criterion, a rotor with optimum blades was established. This rotor, operating in the duct given in Figure 12, has five 130 activity factor blades with an integrated design C_L of 0.6. The characteristics of the blade are given in Figure 15, with the detailed section ordinates from which the blade can be fabricated given in Appendix A.

PERFORMANCE OF 5D130-1.75 DUCTED PROPELLER ON 8000 RPM ENGINE

An efficiency map for the optimum ducted propeller described above and designated 5D130-1.75 was developed for the expected operating range and is given in Figure 16. The thrust determined from the efficiency given includes the thrust produced by both the rotor and the duct. The duct thrust includes the drag loss due to skin friction, which was calculated using the data given by Hoerner.⁷ Due to the location of the engine with respect to the duct and propeller, interference losses can be expected. These were estimated to be 1.5% of the calculated efficiency. It was assumed that a tip clearance of 0.1 inch would be maintained. Based on the test data given in Reference 7, the loss in performance was taken as 2.5% over the entire range.

⁷ Hoerner, S.F., FLUID-DYNAMIC DRAG, published by the author, 1965.

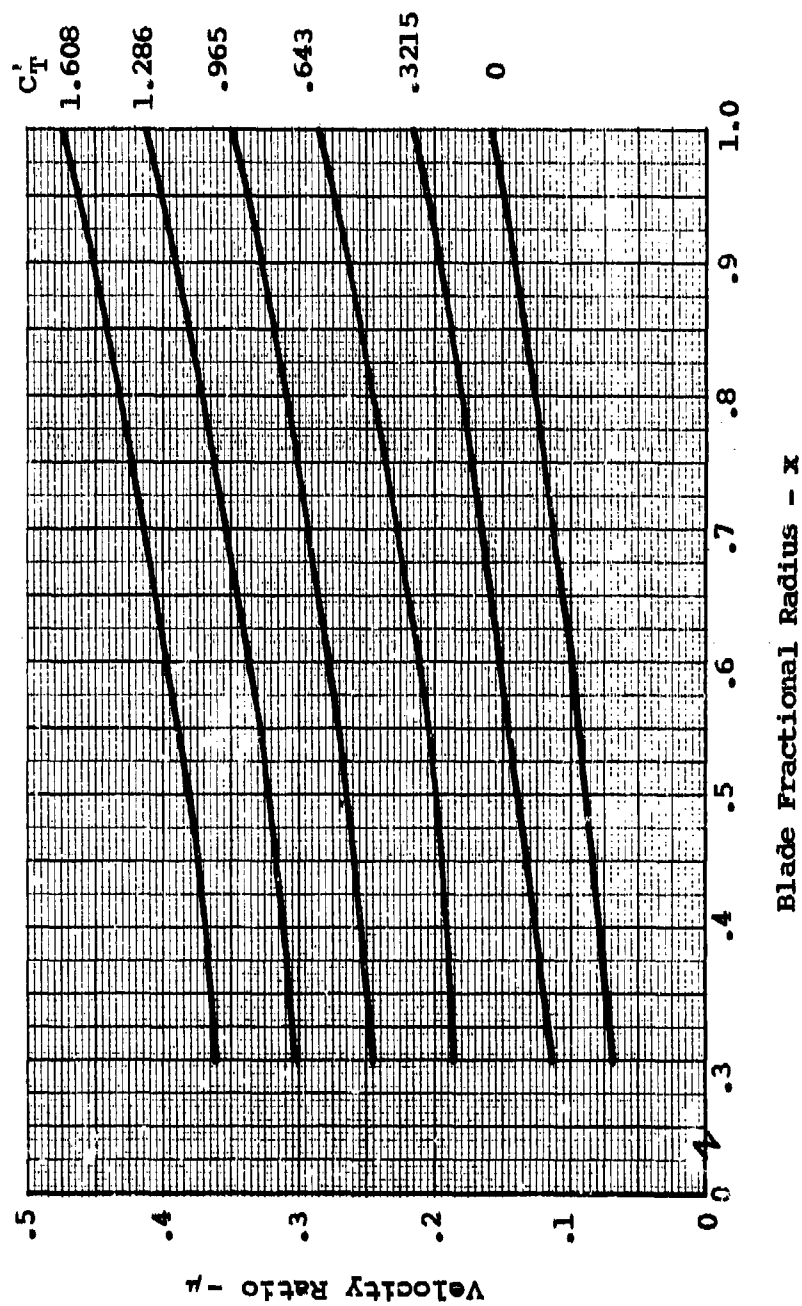


Figure 14. Duct Velocity at Rotor Face vs Rotor Thrust Coefficient.

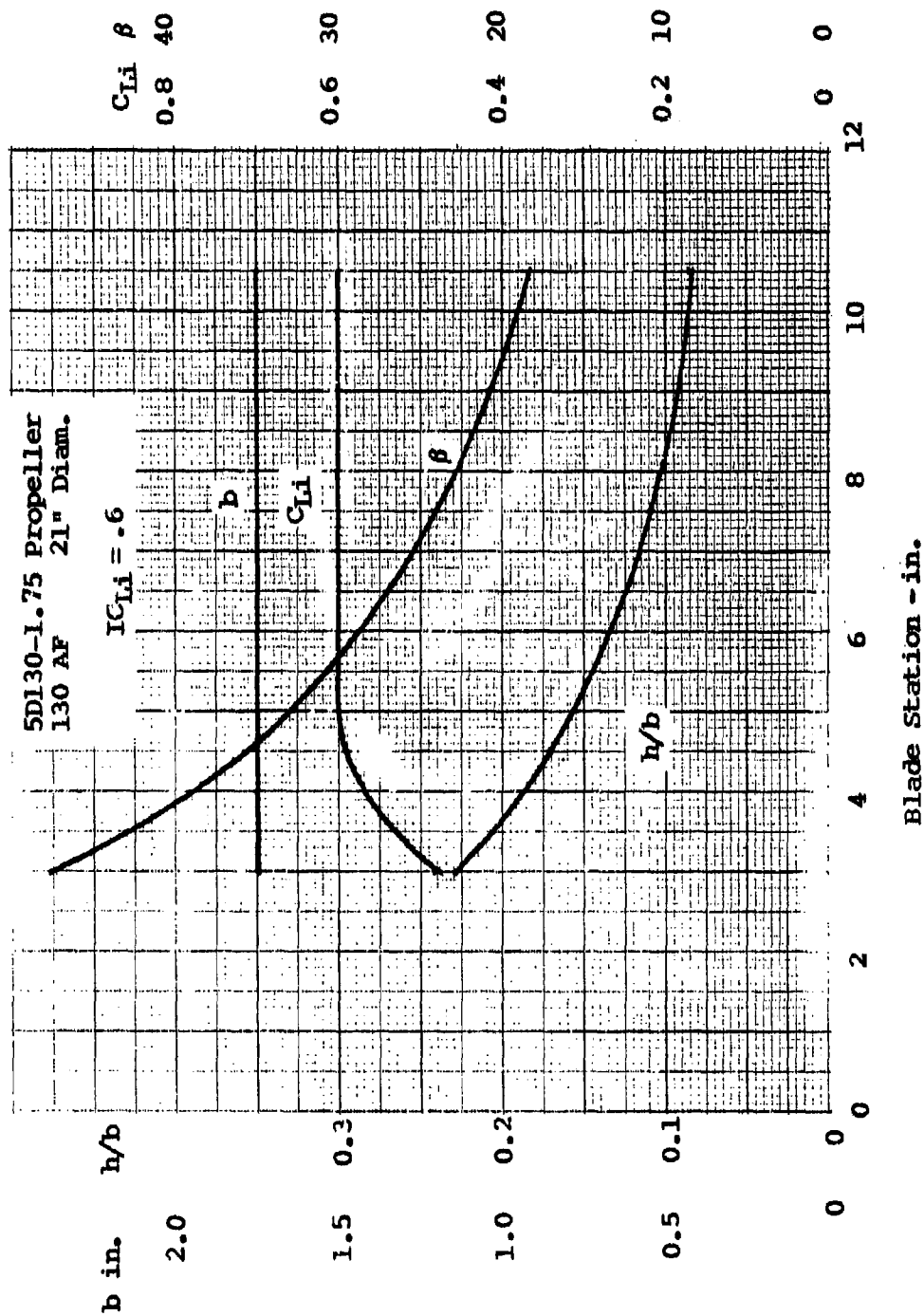


Figure 15. Blade Design Characteristics -
SD130-1.75 Ducted Propeller.

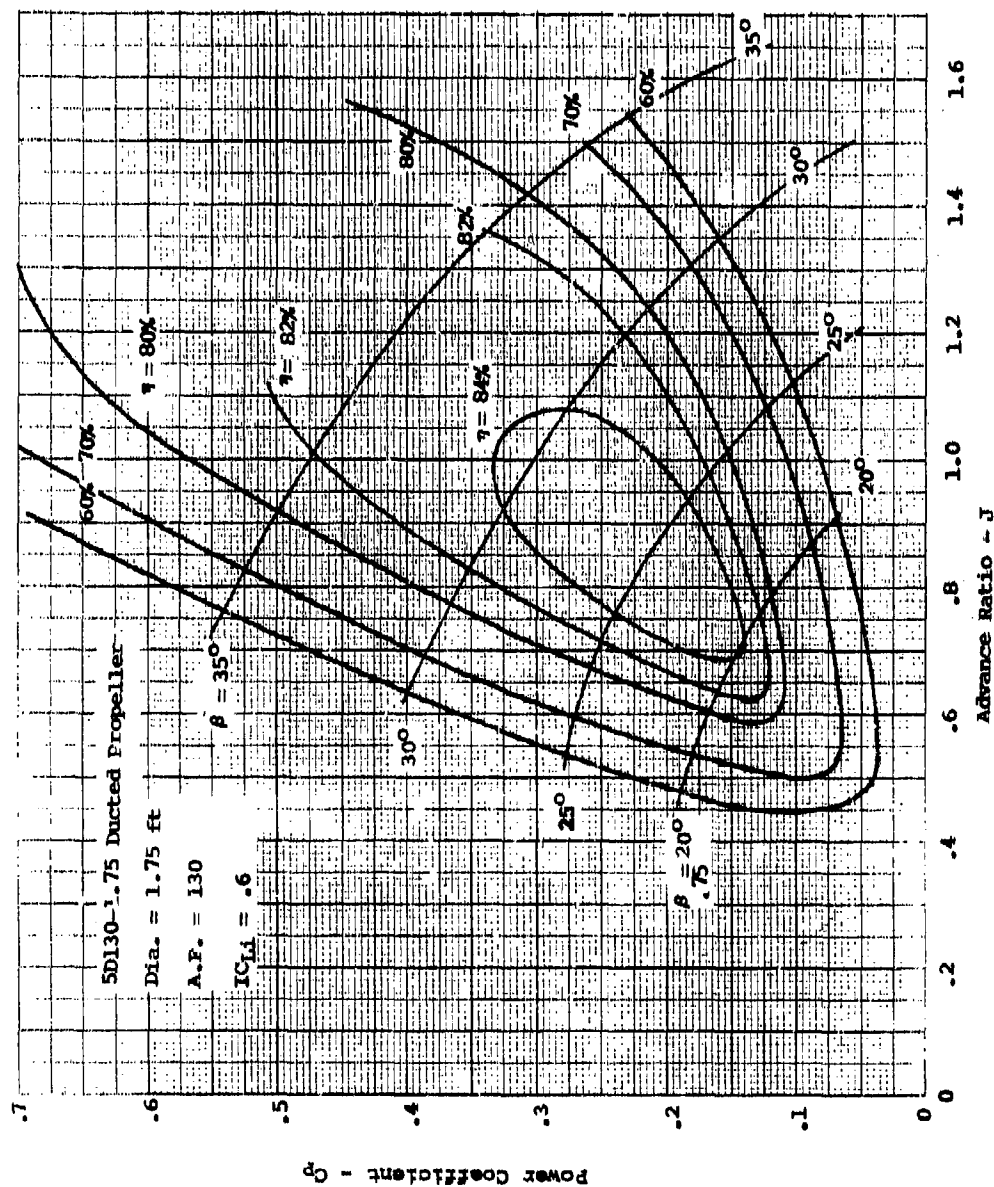


Figure 16. Performance Efficiency Map --
5D130-1.75 Ducted Propeller.

For the design operating conditions of the advanced RPV's, the performance of the 5D130-1.75 ducted fan was calculated and is given in Table 1. At all the flight conditions for the advanced RPV, the efficiency of the ducted fan is excellent and exceeds requirements. The high values of efficiency are maintained over a wide range of power with the ducted propeller configuration.

OPTIMUM DUCTED PROPELLER DESIGN FOR 5860 RPM ENGINE

The efficiency of a series of ducted propellers having a length-to-diameter ratio of 0.75 and installed on the 5860 rpm engine is shown in Figure 17. This efficiency, as a function of rpm, is compared with the efficiency required to obtain the thrust needed at the launch condition. Based on the criterion of operating at minimum tip speed, the ducted fan with 2-foot-diameter rotors appears to be optimum. This configuration, operating at 4000 rpm and full power, meets the efficiency needed at full power for obtaining the required rate of climb of 610 fpm at 60 knots. At this condition, the rotor tip speed is only 419 fps, which should result in low noise output.

Duct Design

The duct selected for the ducted fan used with the 5860 rpm engine has a length-to-diameter ratio of 0.75 and a cross section corresponding to a NACA 23015 airfoil section. This airfoil section was chosen as it has an essentially flat lower surface, which becomes the outer portion of the duct, and a large leading-edge radius, which tends to eliminate separation problems at the duct entrance. The airfoil is mounted in the duct with an expansion angle of 7° to provide a configuration with a high duct efficiency. The duct shape chosen is the same as that used with the 8000 rpm engine.

The cylinders on the 5860 rpm engine are large relative to the propeller, so that the wake produced will have a significant effect on performance. As the propeller operates aft of the engine, the blades go in and out of the wake produced by the cylinders with a corresponding periodic change of load. This is similar to a pusher propeller operating aft of a wing, and can lead to high alternating blade loads with corresponding high vibratory stresses. To minimize this effect, the location of the engine and fan relative to the duct has been arranged to reduce the wake size and the velocity relative to the cylinders. This was done by locating the cylinders just in front of the duct where the velocity is lower than that at the propeller face and using splitter plates aft of the cylinders. The splitter plates tend to reduce drag and so reduce

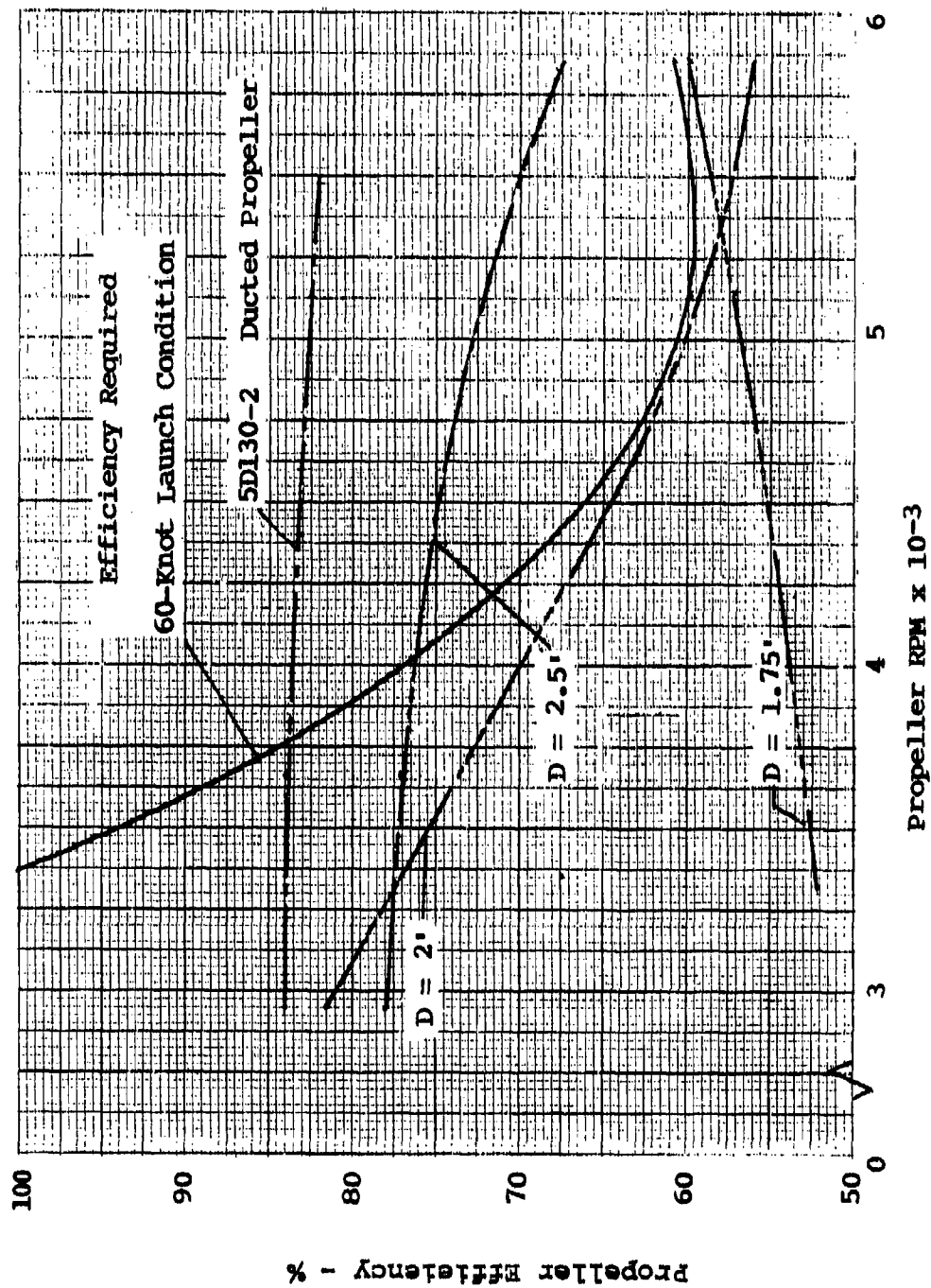


Figure 17. Efficiency Required and Available With 5-Bladed Ducted Propeller at Launch Condition vs Propeller RPM - 8000 RPM Engine.

the width and velocity decrement in the wake. The splitter plates are also used to support the outer portion of the duct. The general arrangement of the engine, duct, and rotor is given in Figure 18.

The arrangement of the duct, engine and propeller shown in Figure 18 results in a relatively large distance between the front supports of the duct and the duct trailing edge. To reduce the movement of the duct relative to the propeller so that minimum tip clearance can be maintained, the rear spinner is fixed so that it does not rotate. Six vanes, attached to the spinner, stabilize the duct. At the end of the fixed inner surface is a rotating shaft that can be used to start the engine.

Since the duct shape and length-to-diameter ratio of the ducted fans designed for both engines are the same, the thrust and velocity characteristics given in Figures 13 and 14 will apply to both designs.

Ducted Propeller Design

The propeller for the low rpm ducted fan was designed in the same manner as the one for the 8000 rpm engine. Based on operation at a minimum rpm of 4000 for the launch condition, the rotor using five 130 activity factor blades was optimized to find the best distributions of design C_L and blade angle. This resulted in a blade with an integrated design C_L of 0.7. This blade, installed in a five-blade hub operating in the above duct design given in Figure 18, is designated the 5D130-2 ducted propeller. The detailed blade characteristics are given in Figure 19. The section data needed for the blade fabrication is given in Appendix A.

PERFORMANCE OF 5D130-2 DUCTED PROPELLER ON THE 5860 RPM ENGINE

A generalized performance efficiency map was developed for the 5D130-2 ducted propeller configuration and is given in Figure 20. The efficiency shown includes the thrust of the propeller and duct as determined from Figure 20. The duct thrust includes a correction for skin friction based on the operating Reynolds number as determined from Hoerner.⁷ It is assumed that the tip clearance between the blade and the duct is 0.1 inch. This results in an efficiency loss of 2.5% based on the data given by Black.⁶ An additional efficiency of 2.5% was

⁶ Black, Wainauski, and Rohrbach.

⁷ Hoerner.

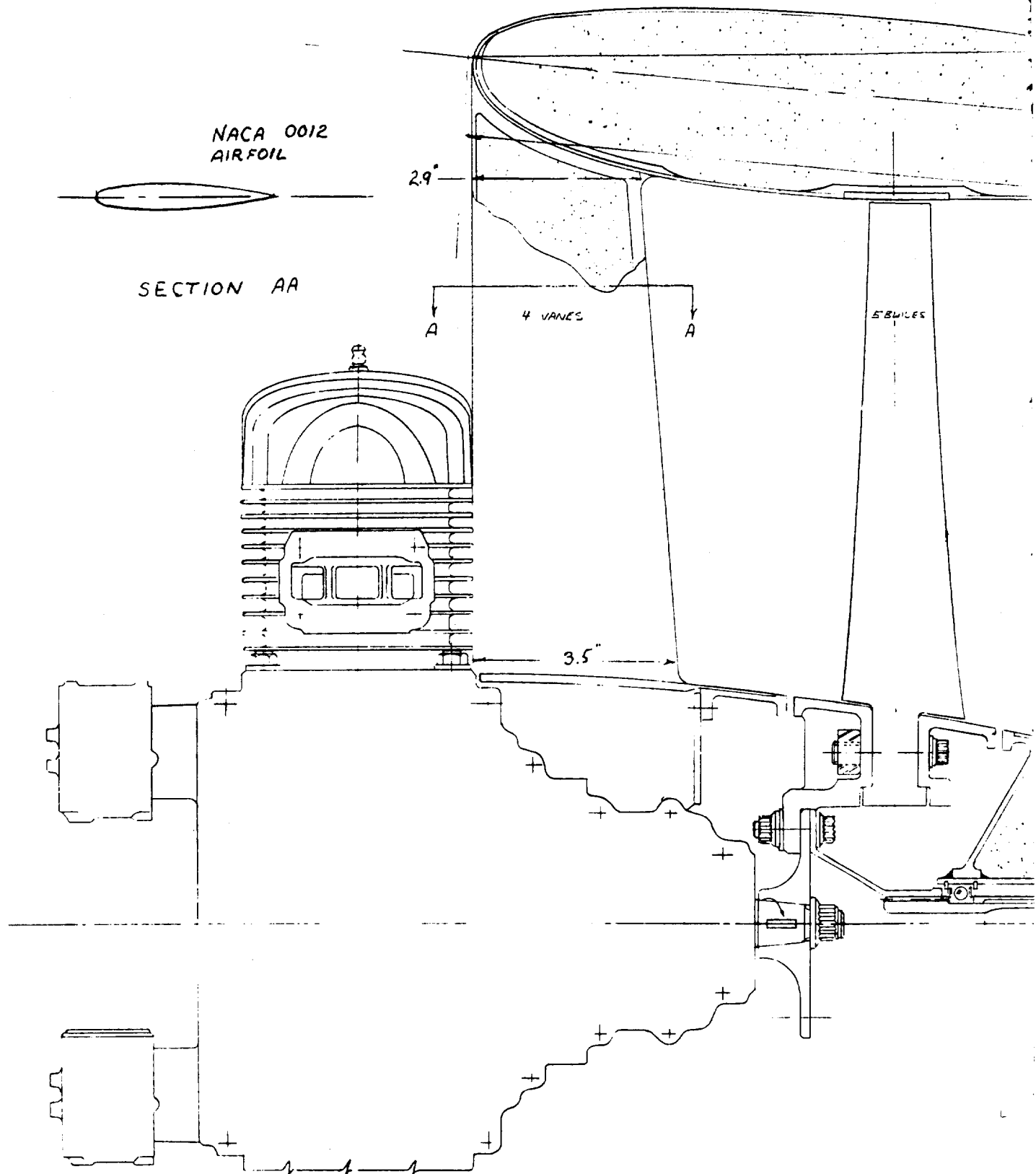
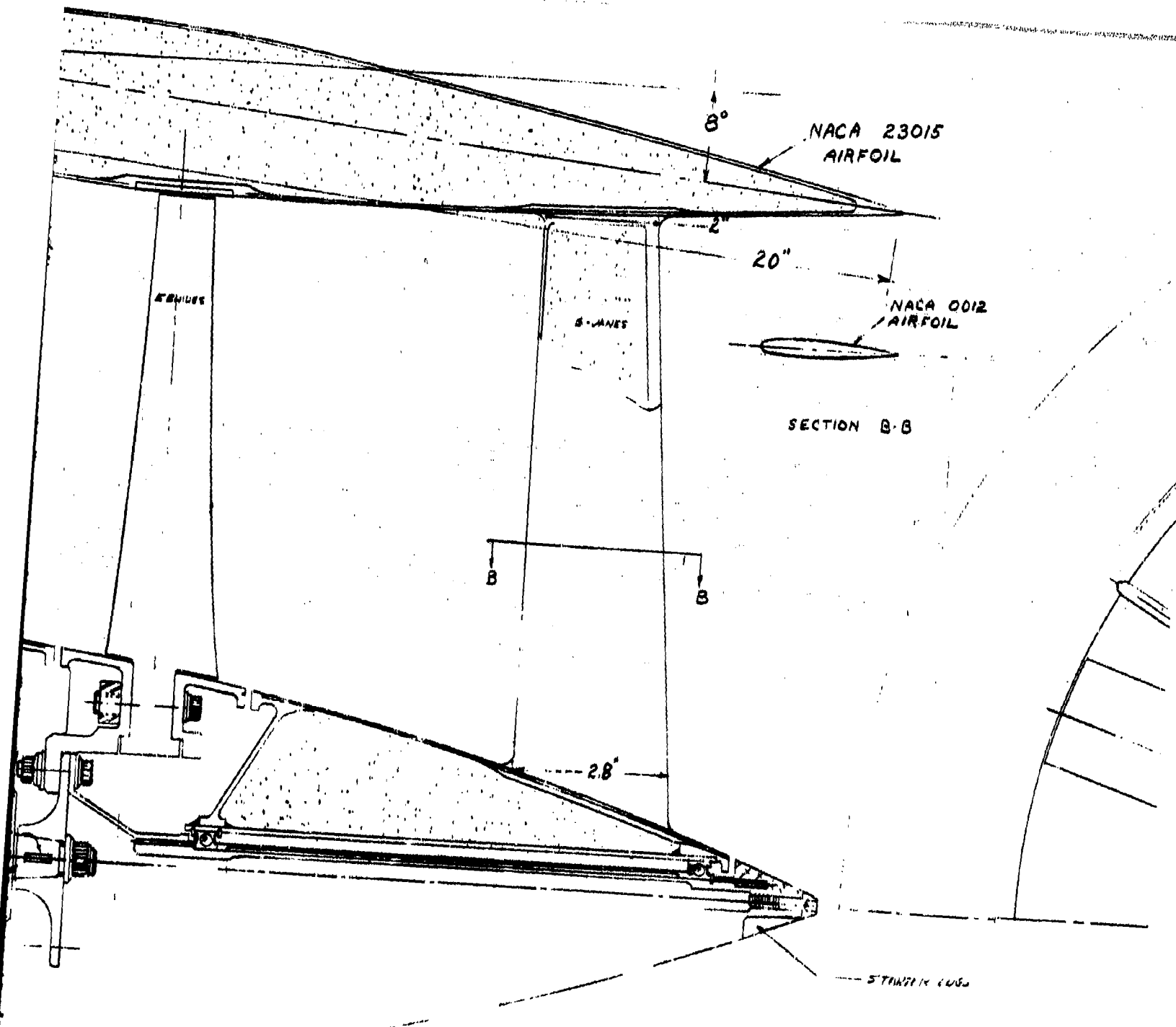


Figure 18. Conceptual Design of a Ducted Propeller Installation for Advanced RPV With 5860 RPM Engine.



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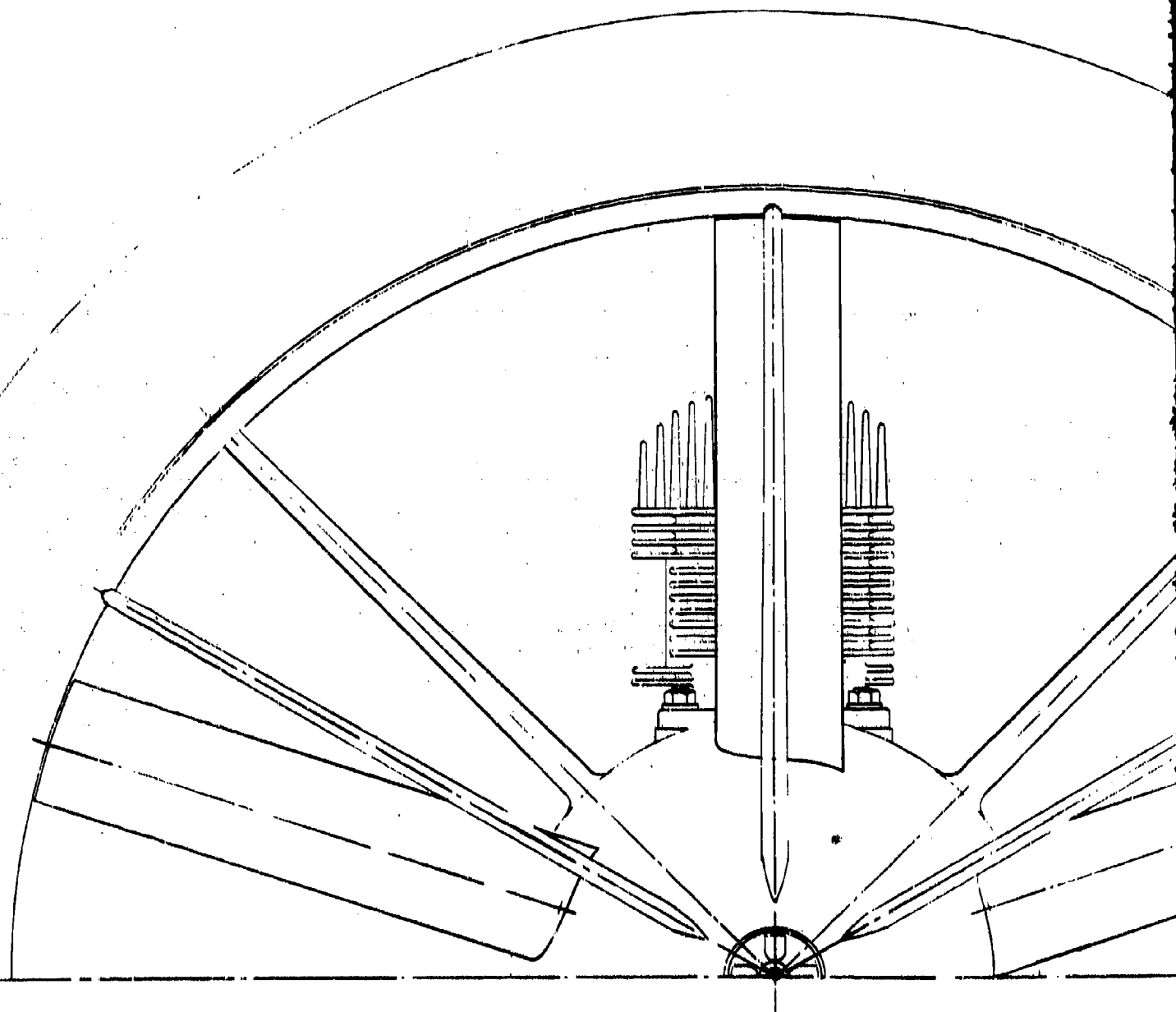
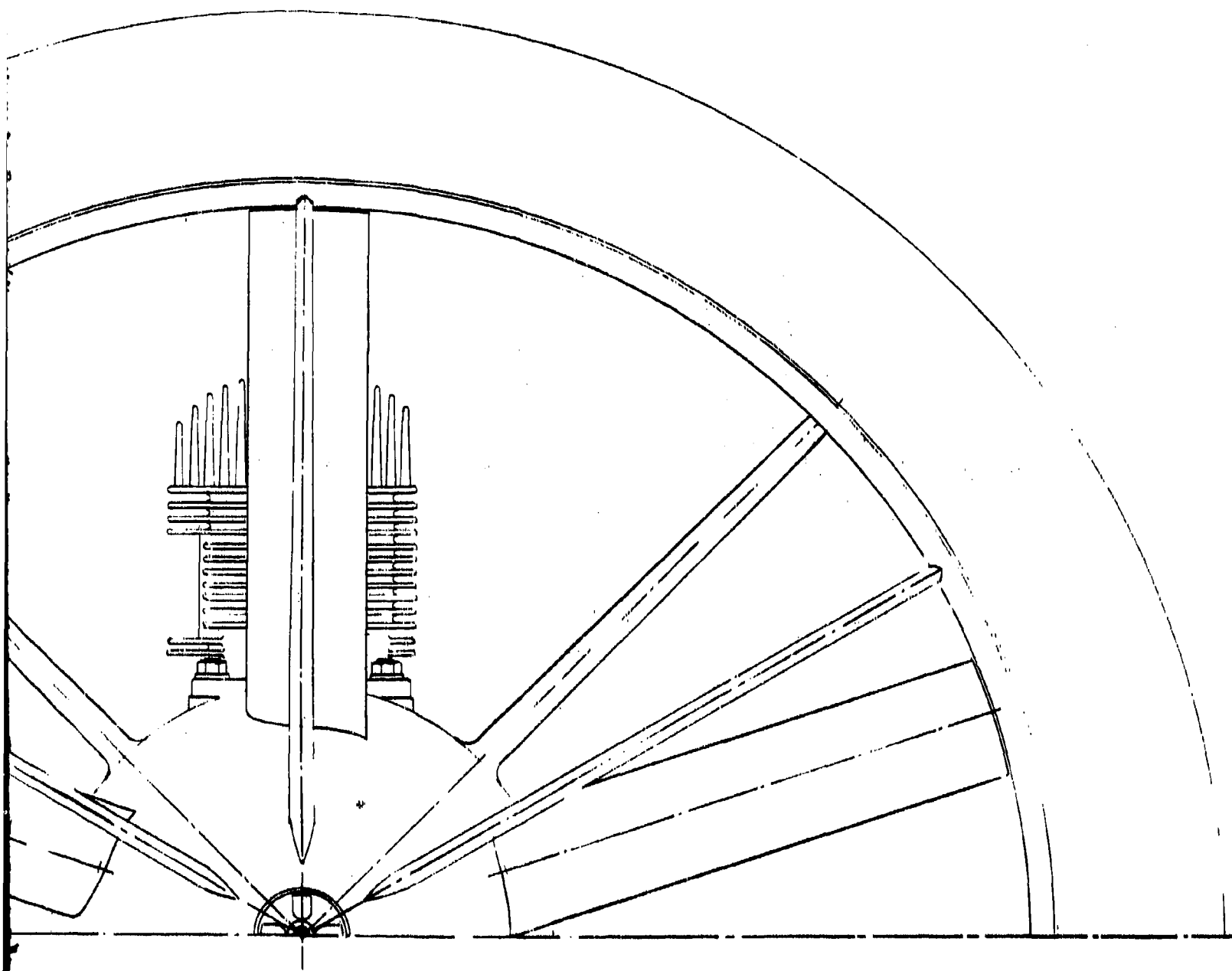


FIGURE 2080



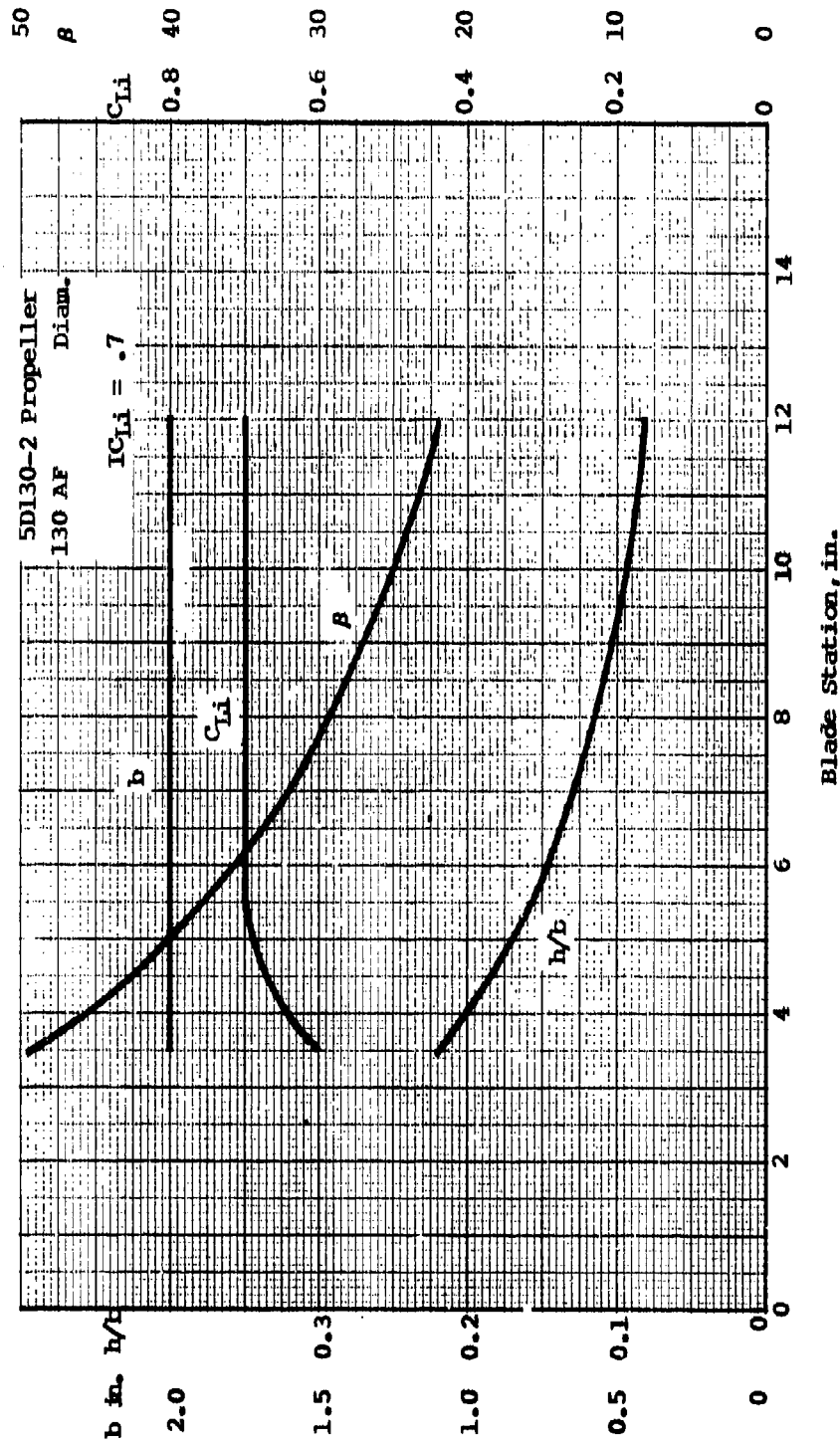


Figure 19. Blade Design Characteristics -- 5D130-2
Ducted Propeller.

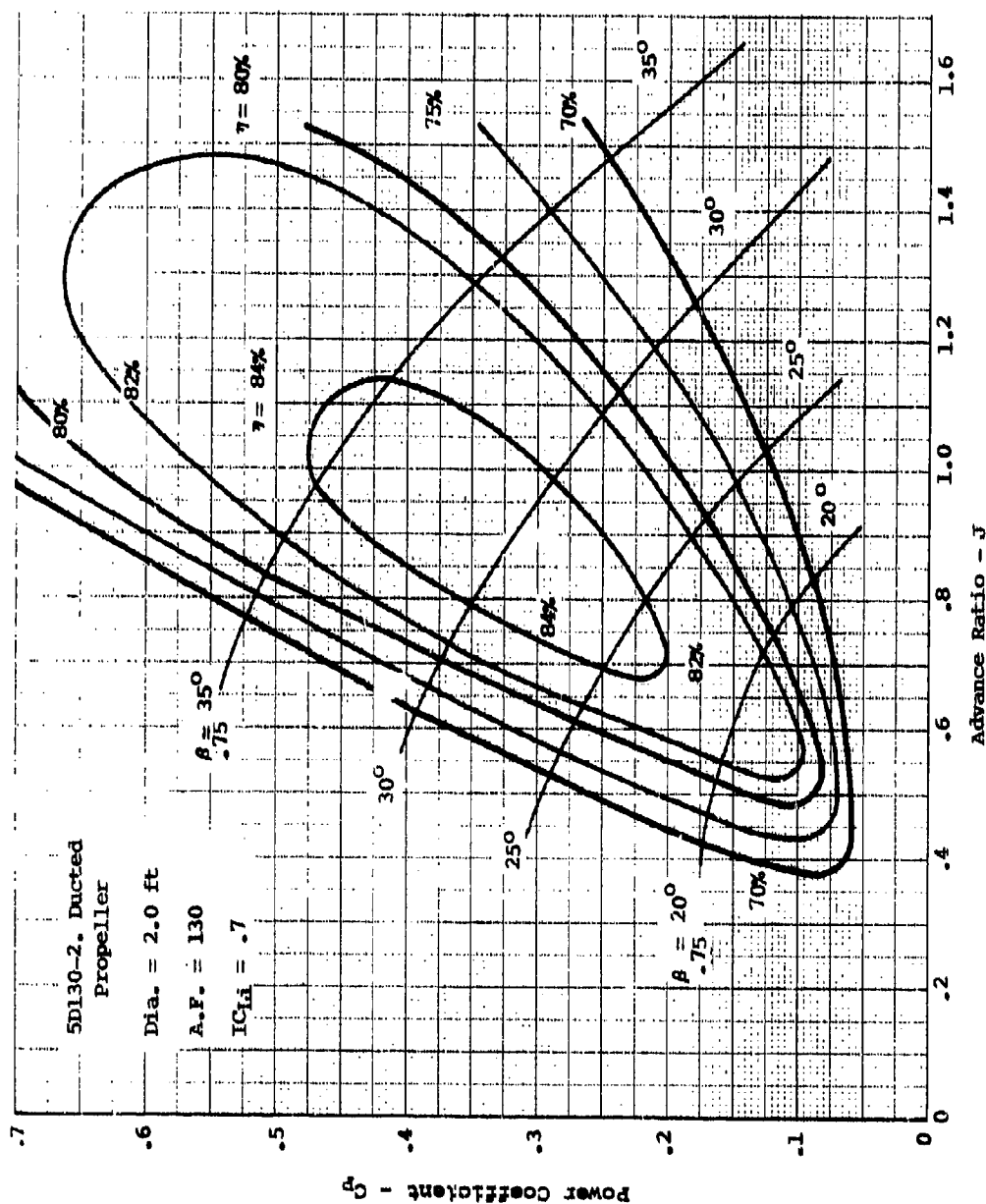


Figure 20. Performance Efficiency Map —
5D130-2 Ducted Propeller.

applied to account for the interference of the engine cylinders on the flow. Thus, the efficiency determined from the efficiency map (Figure 20) includes all the losses for skin friction and interference.

The performance of the 5D130-2 ducted propeller operating on the advanced RPV, using the 5860 rpm engine, was determined using the efficiency map (Figure 20) and is given in Table 2. Where the efficiency of the propeller intersects that required at the launch condition (Figure 17), the blade setting angle and rotational are established for the ducted propeller. For the 5D130-2 ducted propeller this occurs at an efficiency of 84% and 3750 rpm at the 60-knot launch condition. This higher performance compared to the original estimate is a result of the blade optimization, which made possible decreasing rpm and increasing efficiency.

With the propeller operating at a fixed blade angle, the performance of the 5D130-2 configuration was found at the design cruise and dash conditions using the efficiency map given in Figure 20. The performance is given in Table 2. Excellent performance, exceeding requirements, is obtained at all the operating conditions.

PERFORMANCE RESULTS

OPEN PROPELLERS

The performance of optimum open propellers is accurately determined using the single-point method given in Volume I and assuming a drag/lift angle of 1.0° . This is illustrated in Figure 5 by comparing the performance calculated at the launch condition, using the single-point method, with that found for an actual 2B81-2.5 optimum propeller. At the design rpm of the optimum propeller the efficiency determined by the two methods is nearly identical. Thus, the performance calculated by the single-point method at any rpm, assuming a drag/lift angle of 1.0° , is truly representative of what would be determined and expected for the optimum propeller. It should be noted that the actual blade configuration of the optimum propeller would be different for each rotational speed selected.

As shown in Table 1, the performance of the 2B81-2.5 propeller is excellent at all the design flight conditions of the advanced RPV. At the launch and landing conditions the performance is within 0.5% of the ideal configuration, while at the cruise condition the performance of the selected propeller is within 4% of the ideal. This performance is considered to be excellent for a fixed pitch propeller. The 2B81-2.5 propeller develops sufficient thrust so a dash speed of 123.5 knots is obtained, which is well above the minimum required of 100 knots. Even at this condition the efficiency is within 10% of the optimum for the engine operating at maximum power. Thus, it would appear that the fixed-pitch two-bladed propeller is the practical optimum configuration for the high rpm engine. Further, except for the possibility of reducing rpm at cruise to decrease the noise level, a variable blade angle propeller is not required for this installation.

As shown in Table 2, the performance of the 4B81-2.5 four-bladed open propeller, designed for peak efficiency at the launch condition, exceeds that of the propeller with the assumed drag/lift angle of 1° . This shows that the blade is operating with minimum profile losses at this condition. The efficiency of the four-bladed propeller exceeds that of the two-bladed configuration due to the better lift/drag ratios and higher induced efficiency. If higher rates of climb are required at the launch conditions, it would thus appear that the four-bladed configuration installed on the low rpm engine would provide the greater growth potential.

At the 75-knot cruise condition the performance of the four-bladed propeller on the low-speed engine is superior to that of the two-bladed configuration operating on the high-speed

engine. The four-bladed propeller also operates at an efficiency very close to the optimum. At the dash condition, good performance is obtained and the required dash speed of 100 knots is exceeded. With the lower operating rpm of the four-bladed propeller operating on the lower speed engine and its high performance, this configuration appears to be superior to the two-bladed propeller installed on the high-speed engine from both a noise and performance consideration.

DUCTED PROPELLERS

The performance of the 5D130-1.75 ducted propeller operating on the 8000 rpm engine is significantly better at the launch condition than the two-bladed open propeller. An increase in efficiency of almost 10% is obtained with a reduction of tip speed of over 35%. At the cruise condition the performance is better than the open propeller, and a similar reduction of tip speed is obtained which should be important in reducing the noise level of the configuration. At the dash flight condition the performance of the open propeller is slightly better than that of the ducted propeller, but the difference is insignificant as the dash speed difference is only one-half a knot.

The performance of the ducted propeller designed for the 5860 rpm engine is approximately 8% better than that of the open propeller at the launch condition. At the cruise and dash conditions, however, the open propeller has better efficiency than the ducted configuration. At these conditions there is a reduction of 28% in the tip speed of the ducted propeller compared to the open propeller, which may be important from noise considerations.

Although improved performance is obtained at the launch condition with the ducted propeller as compared with the four-bladed open propeller, the overall advantages do not appear to be as great as in the case of the installation with the 8000 rpm engine.

CONCLUSIONS

Based on the design and analysis of open and ducted propellers operating on advanced RPV's using engines with different output rotational speeds, it is concluded that:

1. At all the flight conditions analyzed the performance of the best open propeller for the high-speed engine will be below that of the best open propeller designed for the low-speed engine.
2. Ducted propellers can be designed with significant improvements in performance at the launch condition (and climb condition).
3. The ducted propeller operating on the high-speed engine has higher efficiency than either of the open propellers at the launch and cruise conditions, with nearly the same performance at the dash condition.
4. The ducted propeller on the low-speed engine has the best performance at the launch condition.
5. For either engine the ducted propellers operate at a lower tip speed than the open propellers, which should result in a corresponding noise reduction.
6. The tip speed of either the open or ducted propeller is less for the low-speed engine at corresponding conditions, which should result in a noise reduction.

RECOMMENDATIONS

Because the ducted propeller appears to have the greatest potential for peak performance and low noise, the following are recommended:

1. Fabricate and test in a wind tunnel the two ducted propeller configurations designed for this program. These tests should be run with the actual engines to duplicate the range of operating conditions to develop an efficiency map and cover the design operating conditions of the advanced RPV. Pressure distribution measurements on the duct should be made.
2. Conduct trade-off studies to determine the effect of changes in the performance at the launch and cruise conditions and their relative advantages.
3. Develop short methods for the design and analysis of ducted propellers, such as those given in Volume I for open propellers.

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APPENDIX A

FINALIZED BLADE GEOMETRY DETAILS

The necessary data for the fabrication of the blades for the two open propellers and two ducted propellers are given in this appendix. These data correspond to the blade design characteristic data given for the open propellers in Figures 6 and 9. The corresponding data for the blades of the ducted propellers are given in Figures 12 and 15. The developed planform for each of the blades is given in Figure A-1.

Both engines considered run clockwise when viewed from the anti-propeller end; thus, when used with pusher propellers, the blades are left-handed or rotate in the counterclockwise direction when viewed from the downstream or wake end.

To fabricate a blade, the detailed section ordinates are needed at a series radial station. Also needed is information on the stacking of these blade sections on the blade centerline and the section setting angle, defined as the blade angle. The detailed nomenclature used is shown in Figure A-2. The details of the blade shank will depend on the blade material and on the blade retention system used. The blade shank characteristics given in Figure A-1 are based on the preliminary retention designs given in Figures 12 and 18 and must be structurally analyzed prior to the blade fabrication. The outboard details of the blade are based on aerodynamic considerations as discussed in the body of this report. Small changes in the thickness ratio could be made without affecting the performance, if required for structural reasons.

The design characteristics necessary for fabrication of the four blades shown in Figure A-1 are given in the following tables and figures.

<u>Blade No.</u>	<u>Blade Characteristics</u>	<u>Blade Section Ordinates</u>	<u>Blade Plan-form Data</u>
	Figures:	Tables:	Tables:
2B81-2.5	6	A-1	A-5
4B81-2.5	9	A-2	A-5
5D130-1.75	12	A-3	A-6
5D130-2	15	A-4	A-6

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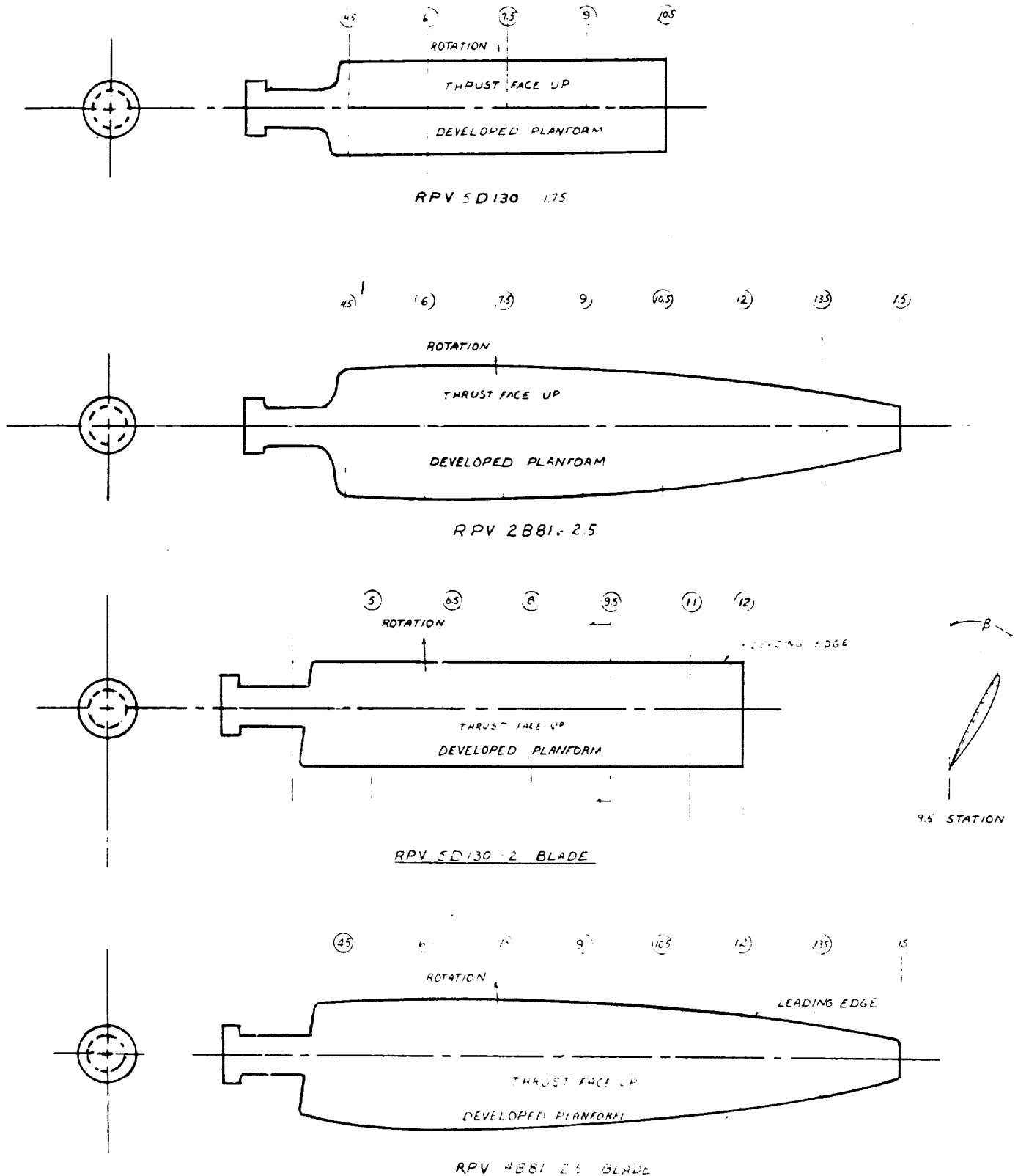


Figure A-1. Developed Blade Planforms.

Code for Blade Section

b	=	Blade Chord
CG	=	Center of Gravity
FA	=	Face Alignment
LER	=	Lead-Edge Radius
Y_L	=	Vertical Center of Lead-Edge Radius
LEA	=	Lead-Edge Alignment
h_u	=	Vertical Ordinate From Chord Line, Upper
h_L	=	Vertical Ordinate From Chord Line, Lower
x	=	Horizontal Location of Ordinates
x_{cg}	=	Horizontal Location, CG
Y_{cg}	=	Vertical Location, CG
TER	=	Trail-Edge Radius
β	=	Blade Angle
Y_T	=	Vertical Center of Trail-Edge Radius

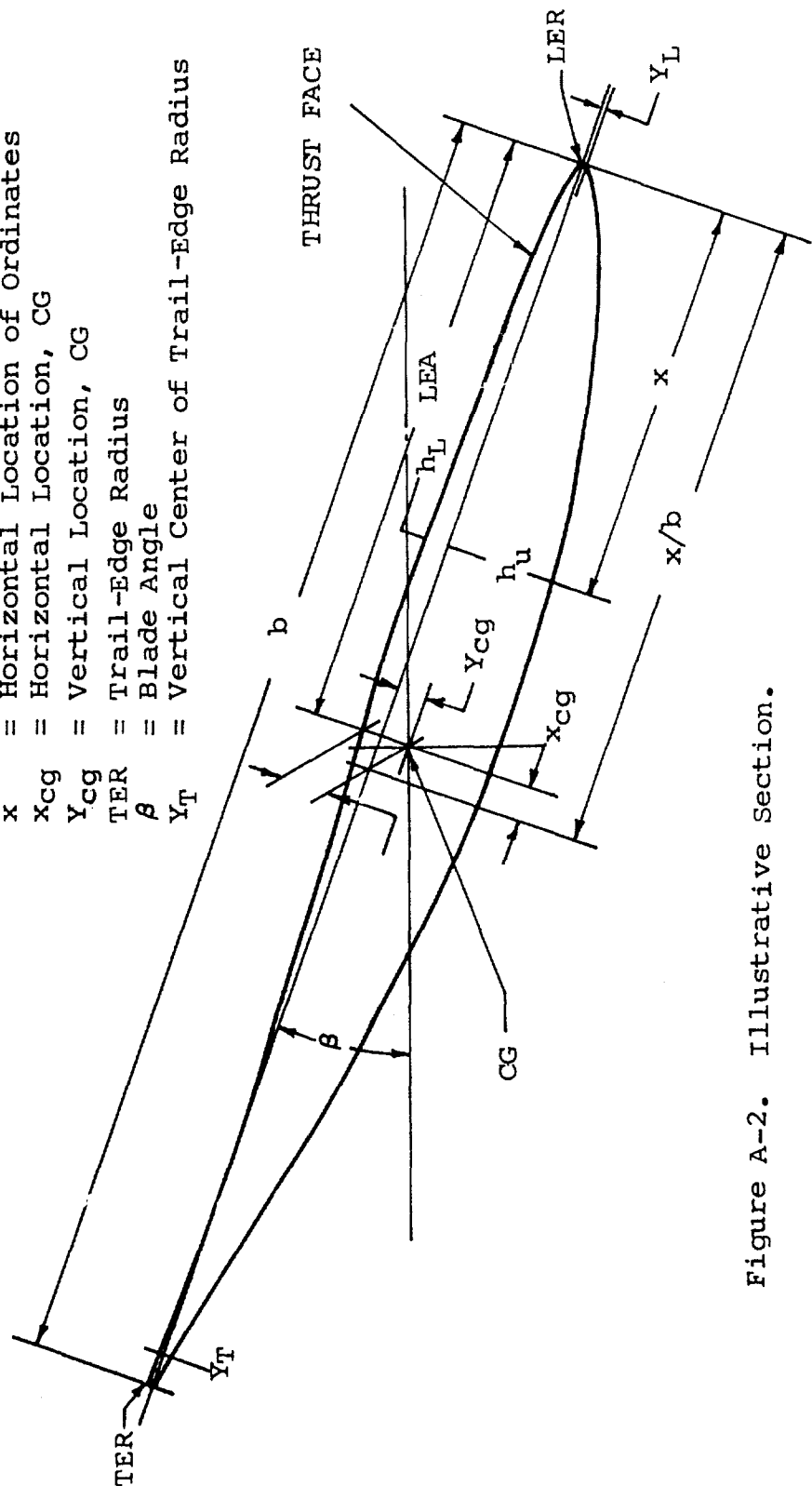


Figure A-2. Illustrative Section.

Table A-1
BLADE SECTION ORDINATES
RPV 2B81-2.5

7.5
.154
2.487
.65

6.0
.180
2.475
.6

4.5
.210
2.367
0

STATION:
h/b:
b:
C/L:

x	h _U	h _L
.062	.0807	.0393
.124	.1128	.0536
.249	.1615	.0719
.497	.2254	.0932
.746	.2621	.1035
.995	.2781	.1048
1.244	.2722	.0939
1.492	.2438	.0723
1.741	.1993	.0474
1.990	.1453	.0296
2.238	.0831	.0165
2.363	.0482	.0096
2.487	.0038	.0038

x	h _U	h _L
.0619	.0900	.0483
.124	.1250	.0678
.248	.1778	.0935
.495	.2467	.1241
.743	.2859	.1397
.990	.3023	.1431
1.238	.2940	.1303
1.485	.2607	.1023
1.733	.2099	.0691
1.980	.1505	.0440
2.228	.0844	.0238
2.351	.0486	.0134
2.475	.0045	.0045

x	h _U	h _L
.0592	.0750	.0750
.118	.1062	.1062
.237	.1504	.1504
.473	.2064	.2064
.710	.2374	.2374
.947	.2485	.2485
1.183	.2357	.2357
1.420	.1998	.1998
1.657	.1509	.1509
1.894	.1029	.1029
2.130	.0556	.0556
2.249	.0310	.0310
2.367	.0050	.0050

Table A-1 (continued)

BLADE SECTION ORDINATES

RPV 2B81-2.5

STATION: 9.0 10.5 12.0
 h/b : .130 .110 .098
 b : 2.361 2.127 1.785
 C_{Li} : .65 .63 .59

x	h_U	h_L	x	h_U	h_L	x	h_U	h_L
.059	.0665	.0303	.053	.0520	.0223	.045	.0391	.0165
.118	.0938	.0399	.106	.0740	.0282	.089	.0560	.0205
.236	.1353	.0518	.213	.1072	.0354	.179	.0813	.0253
.472	.1898	.0653	.425	.1512	.0430	.357	.1150	.0302
.708	.2214	.0713	.638	.1769	.0460	.536	.1347	.0319
.744	.2356	.0711	.851	.1887	.0451	.714	.1438	.0310
1.181	.2318	.0626	1.064	.1864	.0387	.893	.1423	.0262
1.417	.2094	.0469	1.276	.1695	.0278	1.071	.1298	.0185
1.653	.1731	.0295	1.489	.1415	.0162	1.250	.1089	.0104
1.889	.1280	.0178	1.702	.1058	.0089	1.428	.0820	.0054
2.125	.0746	.0101	1.914	.0625	.0051	1.607	.0489	.0030
2.243	.0435	.0061	2.021	.0367	.0032	1.696	.0288	.0020
2.361	.0031	.0031	2.127	.0023	.0023	1.785	.0017	.0017

Table A-1 (Continued)
BLADE SECTION ORDINATES
RPV 2B81-2.5

STATION: 13.5
h/b: .089
b: 1.344
C_{Li}: .50

x	h _u	h _L
.034	.0261	.0120
.067	.0373	.0150
.134	.0542	.0187
.269	.0766	.0226
.403	.0897	.0242
.538	.0958	.0238
.672	.0947	.0207
.806	.0865	.0154
.941	.0726	.0097
1.075	.0549	.0057
1.210	.0329	.0033
1.277	.0194	.0021
1.344	.0012	.0012

14.25
.085
1.095
.43

x	h _u	h _L
.027	.0197	.0099
.055	.0282	.0126
.110	.0408	.0159
.219	.0575	.0197
.329	.0672	.0213
.438	.0717	.0213
.548	.0709	.0190
.657	.0646	.0148
.767	.0542	.0101
.876	.0410	.0065
.986	.0246	.0038
1.040	.0146	.0023
1.095	.0009	.0009

14.625
.084
.96
.37

x	h _u	h _L
.024	.0165	.0092
.048	.0235	.0118
.096	.0339	.0152
.192	.0477	.0192
.288	.0557	.0211
.384	.0593	.0213
.480	.0585	.0193
.576	.0532	.0156
.672	.0445	.0113
.768	.0336	.0076
.864	.0202	.0044
.912	.0120	.0027
.960	.0008	.0008

Table A-2
BLADE SECTION ORDINATES
RPV 4B81-2.5

7.5
.154
2.487
.5

6
.180
2.475
0.5

STATION: 4.5
h/b: .210
b: 2.367
C_{Li}: .55

x	h _U	h _L	x	h _U	h _L	x	h _U	h _L
.0592	.0969	.0558	.0619	.0864	.0516	.062	.0757	.0439
.1184	.1336	.0804	.124	.1200	.0725	.124	.1058	.0603
.2365	.1894	.1130	.248	.1707	.1004	.249	.1510	.0821
.4734	.2615	.1526	.495	.2364	.1343	.497	.2100	.1083
.7101	.3020	.1733	.743	.2736	.1518	.746	.2437	.1217
.9468	.3183	.1787	.990	.2890	.1563	.995	.2581	.1248
1.184	.3075	.1639	1.238	.2804	.1439	1.244	.2516	.1145
1.420	.2695	.1296	1.485	.2475	.1155	1.492	.2241	.0921
1.657	.2133	.0879	1.733	.1982	.0809	1.741	.1818	.0649
1.894	.1499	.0556	1.980	.1416	.0529	1.990	.1319	.0429
2.130	.0820	.0290	2.228	.0794	.0289	2.238	.0754	.0242
2.249	.0467	.0158	2.351	.0456	.0163	2.363	.0437	.0140
2.367	.0050	.0050	2.475	.0045	.0045	2.487	.0038	.0038

Table A-2 (continued)
BLADE SECTION ORDINATES
RPV 4B81-2.5

STATION: 9			10.5			12		
h/b			.130			.098		
b:			2.361			1.785		
C _{Li} :			.5			.5		
x	h _u	h _L	x	h _u	h _L	x	h _u	h _L
.059	.0622	.0344	.053	.0488	.0252	.045	.0373	.0182
.118	.0875	.0460	.106	.0692	.0329	.089	.0532	.0232
.236	.1256	.0614	.213	.0998	.0427	.179	.0771	.0295
.472	.1754	.0796	.425	.1400	.0542	.357	.1085	.0366
.708	.2041	.0886	.638	.1634	.0595	.536	.1268	.0397
.744	.2167	.0901	.851	.1739	.0599	.714	.1352	.0396
1.181	.2123	.0821	1.064	.1711	.0539	.893	.1335	.0351
1.417	.1907	.0657	1.276	.1549	.0424	1.071	.1214	.0270
1.653	.1566	.0461	1.489	.1286	.0292	1.250	.1014	.0179
1.889	.1153	.0305	1.702	.0958	.0189	1.428	.0762	.0112
2.125	.0671	.0175	1.914	.0566	.0110	1.607	.0454	.0065
2.243	.0392	.0104	2.021	.0332	.0067	1.696	.0267	.0040
2.361	.0031	.0031	2.127	.0023	.0023	1.785	.0017	.0017

Table A-2 (continued)
BLADE SECTION ORDINATES
RPV 4B81-2.5

STATION: 13.5			14.25			14.625		
h/b: .089			.085			.084		
b: 1.344			1.095			0.96		
C/L: .5			.5			.3		
x	h _u	h _L	x	h _u	h _L	x	h _u	h _L
.034	.0261	.0120	.027	.0205	.0092	.024	.0158	.0098
.067	.0373	.0150	.055	.0294	.0113	.048	.0224	.0129
.134	.0542	.0186	.110	.0428	.0139	.096	.0322	.0169
.269	.0766	.0226	.219	.0606	.0156	.192	.0450	.0219
.403	.0897	.0242	.329	.0710	.0176	.288	.0524	.0243
.538	.0958	.0238	.438	.0758	.0172	.384	.0557	.0249
.672	.0947	.0207	.548	.0751	.0147	.480	.0548	.0230
.806	.0865	.0154	.657	.0687	.0108	.576	.0497	.0192
.941	.0726	.0097	.767	.0578	.0065	.672	.0414	.0144
1.075	.0549	.0057	.876	.0438	.0036	.768	.0311	.0100
1.210	.0329	.0033	.986	.0263	.0021	.864	.0187	.0059
1.277	.0194	.0021	1.040	.0156	.0013	.912	.0111	.0036
1.344	.0012	.0012	1.095	.0009	.0009	.960	.0008	.0008

Table A-3
BLADE SECTION ORDINATES
RPV 5D130-1.75

STATION: 3.0 4.5 6.0
 h/b: .230 .170 .135
 b: 1.75 1.75 1.75
 C_{Li}: .48 .590 .6

x	h _u	h _L	x	h _u	h _L	x	h _u	h _L
.0438	.0757	.0470	.0438	.0603	.0323	.0438	.0498	.0246
.0875	.1043	.0684	.0875	.0839	.0450	.0875	.0780	.0328
.175	.1475	.1256	.175	.0937	.0874	.175	.0797	.0642
.35	.2032	.1323	.35	.1662	.0813	.35	.1408	.0555
.525	.2342	.1509	.525	.1928	.0913	.525	.1641	.0613
.7	.2463	.1562	.7	.2041	.0933	.7	.1743	.0618
.875	.2366	.1440	.875	.1988	.0849	.875	.1711	.0553
1.05	.2054	.1145	1.05	.1768	.0669	1.05	.1540	.0428
1.225	.1602	.0783	1.225	.1429	.0454	1.225	.1268	.0285
1.4	.1109	.0494	1.4	.1030	.0291	1.4	.0933	.0180
1.575	.0595	.0251	1.575	.0582	.0160	1.575	.0541	.0102
1.663	.0336	.0133	1.663	.0336	.0091	1.663	.0316	.0061
1.75	.0040	.0040	1.75	.0030	.0030	1.75	.0024	.0024

Data for Fairing Only

Table A-3 (continued)
BLADE SECTION ORDINATES
 RPV 5D130-1.75

STATION:			9			10.5		
h/b:			.083			.085		
b:			1.75			1.75		
C _{LD} :			.6			.6		
x	h _U	h _L	x	h _U	h _L	x	h _U	h _L
.0438	.0421	.0189	.0438	.0370	.0148	.0438	.0346	.0129
.0875	.0600	.0241	.0875	.0532	.0180	.0875	.0509	.0152
.175	.0698	.0475	.175	.0631	.0361	.175	.0599	.0308
.35	.1223	.0376	.35	.1097	.0253	.35	.1038	.0195
.525	.1430	.0404	.525	.1287	.0263	.525	.1220	.0196
.7	.1525	.0399	.7	.1376	.0251	.7	.1306	.0181
.875	.1505	.0347	.875	.1363	.0206	.875	.1297	.0139
1.05	.1367	.0257	1.05	.1247	.0137	1.05	.1190	.0080
1.225	.1140	.0158	1.225	.1049	.0067	1.225	.1006	.0022
1.4	.0852	.0092	1.4	.0793	.0027	1.4	.0764	-.0006
1.575	.0503	.0053	1.575	.0474	.0014	1.575	.0460	-.0006
1.663	.0296	.0033	1.663	.0280	.0010	1.663	.0272	-.0001
1.75	.0019	.0019	1.75	.0016	.0016	1.75	.0015	.0015

Table A-4
BLADE SECTION ORDINATES
RPV 5D130-2

STATION:	5.0	6.5	8.0
$h/b:$	.172	.14	.117
$b:$	2	2	2
$C_{Li}:$	.69	.7	.7

x	h_u	h_L	x	h_u	h_L	x	h_u	h_L
.05	.0725	.0343	.05	.0611	.0270	.05	.0529	.0212
.1	.1097	.0485	.1	.0860	.0359	.1	.0752	.0269
.2	.1087	.1009	.2	.0948	.0759	.2	.0849	.0578
.4	.2000	.0864	.4	.1734	.0594	.4	.1538	.0406
.6	.2321	.0964	.6	.2021	.0650	.6	.1799	.0431
.8	.2459	.0980	.8	.2150	.0649	.8	.1920	.0419
1.0	.2401	.0879	1.0	.2113	.0569	1.0	.1896	.0353
1.2	.2142	.0672	1.2	.1905	.0422	1.2	.1723	.0243
1.4	.1739	.0434	1.4	.1571	.0259	1.4	.1437	.0129
1.6	.1255	.0267	1.6	.1156	.0152	1.6	.1072	.0062
1.8	.0709	.0145	1.8	.0668	.0085	1.8	.0631	.0035
1.9	.0410	.0082	1.9	.0389	.0051	1.9	.0370	.0023
2.00	.0034	.0034	2.00	.0028	.0028	2.00	.0023	.0023

Table A-4 (continued)
 BLADE SECTION ORDINATES
 RPV 5D130-2

STATION: 9.5
 h/b : .10
 b : 2
 C_{Li} : .7

11
 .088
 2
 .7

x	h_U	h_L
.05	.0469	.0168
.1	.0673	.0201
.2	.0776	.0443
.4	.1393	.0266
.6	.1636	.0269
.8	.1750	.0249
1.0	.1735	.0191
1.2	.1587	.0107
1.4	.1335	.0026
1.6	.1007	-.0011
1.8	.0599	-.0009
1.9	.0353	-.0002
2.00	.002	.002

x	h_U	h_L
.05	.0427	.0135
.1	.0618	.0153
.2	.0726	.0348
.4	.1293	.0168
.6	.1520	.0155
.8	.1629	.0129
1.0	.1621	.0077
1.2	.1490	.0010
1.4	.1262	-.0049
1.6	.0959	-.0066
1.8	.0576	-.0042
1.9	.0340	-.0022
2.00	.0618	.0018

x	h_U	h_L
.05	.0413	.0124
.1	.0600	.0137
.2	.0709	.0316
.4	.1259	.0135
.6	.1482	.0117
.8	.1589	.0089
1.0	.1583	.0039
1.2	.1457	-.0023
1.4	.1237	-.0075
1.6	.0942	-.0035
1.8	.0568	-.0054
1.9	.0336	-.0029
2.00	.0017	.0017

Table A-5
BLADE PLANFORM DATA
Blade 4B81-2.5

<u>STA</u> <u>in.</u>	<u>b</u> <u>in.</u>	<u>LEA</u> <u>in.</u>	<u>FA</u> <u>in.</u>	<u>LER</u> <u>in.</u>	<u>Y_L</u> <u>in.</u>	<u>TER</u> <u>in.</u>	<u>Y_T</u> <u>in.</u>	<u>β</u> <u>Deg.</u>
4.5	2.37	1.038	.249	.057	.014	.02	.005	42.3
6.0	2.48	1.095	.223	.047	.010		.004	33.3
7.5	2.49	1.107	.182	.038	.008		.004	27.6
9.0	2.36	1.056	.153	.028	.006		.004	23.9
10.5	2.13	.958	.117	.02	.004		.004	21.0
12.	1.79	.808	.088	.02	.003		.004	18.2
13.5	1.34	.606	.060	.02	.002		.004	15.9
14.25	1.10	.498	.047	.02	.001		.004	15.0
14.625	.96	.435	.040	.02	.001		.003	14.2

Blade 2B81-2.5

4.5	2.37	1.038	.249	.057	0	.02	0	30.2
6.0	2.48	1.095	.223	.047	.012		.005	26.4
7.5	2.49	1.107	.182	.038	.011		.006	22.8
9.0	2.36	1.056	.153	.028	.008		.006	19.4
10.5	2.13	.958	.117	.02	.005		.005	18.1
12.	1.79	.808	.088	.02	.003		.005	16.2
13.5	1.34	.606	.060	.02	.002		.004	14.5
14.25	1.10	.498	.047	.02	.001		.004	13.8
14.625	.96	.435	.040	.02	.001		.003	13.4

Table A-6
BLADE PLANFORM DATA
RPV 5D130-1.75 Blade

<u>STA</u> <u>in.</u>	<u>b</u> <u>in.</u>	<u>LEA</u> <u>in.</u>	<u>FA</u> <u>in.</u>	<u>LER</u> <u>in.</u>	<u>Y_L</u> <u>in.</u>	<u>TER</u> <u>in.</u>	<u>X_T</u> <u>in.</u>	<u>θ</u> <u>Deg.</u>	Data for Fairing Only
3	1.75	.762	.201	.048	.010	.02	.004	47.5	
4.5	1.75	.775	.149	.031	.008	↓	.005	35.7	
6.0	1.75	.782	.118	.022	.007	↓	.005	28.7	
7.5	1.75	.787	.096	.020	.006	↓	.005	24.2	
9.0	1.75	.791	.081	.020	.006	↓	.005	20.7	
10.5	1.75	.792	.074	.020	.006	↓	.005	18.4	

RPV 5D130-2.0

									Data for Fairing Only
3.5	2	.873	.22	.050	.013	.02	.005	49.6	
5.0	2	.885	.172	.031	.009	↓	.006	40.2	
6.5	2	.892	.14	.022	.007	↓	↓	33.8	
8	2	.898	.117	.02	.006	↓	↓	29.5	
9.5	2	.902	.10	↓	↓	↓	↓	26.1	
11	2	.905	.088	↓	↓	↓	↓	23.5	
12	2	.906	.084	↓	↓	↓	↓	22.0	

LIST OF SYMBOLS

A	disk area - sq ft
AF	blade activity factor
B	blade number
b	blade chord - ft
C _D	drag coefficient
C _L	lift coefficient
C _{Ld}	section design lift coefficient
C _P	power coefficient
C _Q	torque coefficient
C _T	thrust coefficient = $T/\rho n^2 D^4$
C' _T	propeller thrust coefficient = T/qA
C _t	duct thrust coefficient
CG	center of gravity
D	propeller diameter - ft
D _R	rotor diameter - ft
D _{SF}	duct skin friction drag - lb
FA	face alignment
h	maximum blade thickness - ft
h _L	vertical ordinate from chord line, lower
h _u	vertical ordinate from chord line, upper
hp	horsepower
J	advance ratio = V/nD
K(x)	circulation function - single rotation propellers
L	lift - lb

LIST OF SYMBOLS (continued)

LEA	lead-edge alignment
LER	lead-edge radius
mph	miles per hour
N	propeller rotational speed - rpm
n	propeller rotational speed - rps
P	power - ft-lb/sec
Q	torque - ft-lb
q	dynamic pressure - psf
R	propeller radius - ft
R.N.	Reynolds number
r	propeller radius at any station - ft
rpm	revolutions per minute
T	thrust - lb
T _D	duct thrust - lb
T _R	rotor thrust - lb
T _S	propeller shaft thrust
TER	trail-edge radius
Thp	thrust horsepower
V _D	velocity in duct - ft/sec
V _O	free-stream velocity - ft/sec
x	fractional radius at any station = r/R
x	horizontal location of ordinates
x _{CG}	horizontal location of center of gravity
y _{CG}	vertical location of center of gravity

LIST OF SYMBOLS (continued)

Y_L	vertical center of lead-edge radius
Y_T	vertical center of trail-edge radius
α	angle of attack - deg
α_i	induced angle of attack - deg
β	blade angle - deg
γ	drag/lift angle = $\tan^{-1} C_D/C_L$
η	propeller efficiency
μ	duct velocity ratio = V_D/V_0
ρ	mass density of air - slugs/cu ft
σ	propeller solidity

SUBSCRIPTS

ref	reference
.75	conditions at $x = .75$
i	induced