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APPLICATION OF PRONY'S METHOD TO TIME-DOMAIN REFLECTOMETER DATA--ETC(U)

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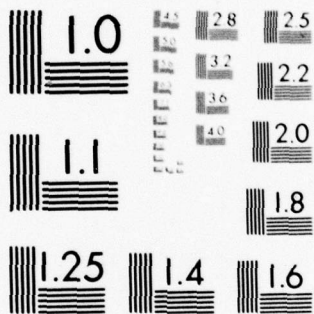
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requires much longer to calculate, the uniform error norm approximation is often orders of magnitude better than the least squared error approximation. The physical realizability of impedance functions obtained from experimental data has been investigated. Also, the procedures necessary to synthesize the lumped-element network have been evaluated. A simple partial fraction expansion of a realizable impedance function representing an antenna was found to yield nonrealizable element values. A general network synthesis procedure, such as Brune's method, is required.

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1. INTRODUCTION

The singularity expansion method (SEM) has been used to express the transient response of antennas and scatterers in a compact and useful form.* By using Prony's algorithm, Van Blaricum and Mittra¹ were able to extract the SEM poles and residues from analytical waveforms. The singularity expansions of experimental data have also been obtained,[†] and Van Blaricum and Schaubert² have demonstrated that the SEM can be used effectively to compute an antenna's impulse response from measured data.

The use of the SEM to calculate equivalent impedance circuits has also been discussed by many authors.[‡] However, there has been no previous application of the SEM to calculate equivalent lumped-element impedance circuits for antennas. Such circuits, when calculated by using the SEM, will be valid over a wide frequency range and will, therefore, be extremely useful for transient and out-of-band response analyses. Additional advantages of the SEM approach are that the goodness of fit to the data can be judged prior to calculating the circuit element values, and the impedance function can be readily synthesized by existing techniques.

Several problems relevant to the computation of equivalent circuits for antennas have been investigated, and the results are presented in sections 2, 3, and 4. Throughout the investigation, primary importance has been placed upon utilization of experimentally or numerically derived data. The SEM expansion of time-domain reflectometer (TDR) data for a dipole antenna with a quarter-wavelength ($\lambda/4$) balun has been obtained and used to calculate a lumped-element equivalent circuit.

2. RELATIONSHIP BETWEEN $Z(s)$ AND MEASURED QUANTITIES

The first step in calculating an equivalent circuit is deriving the relationship between the antenna impedance function $Z(s)$, where s is the Laplace transform frequency variable, and a measurable quantity. One of the most common and most easily used instruments for measuring impedance is the TDR, which excites the antenna with a step voltage and displays

¹M. L. Van Blaricum and R. Mittra, A Technique for Extracting the Poles and Residues of a System Directly from Its Transient Response, *IEEE Trans. Antennas Propag.*, AP-23 (November-1975), 777-782.

²M. L. Van Blaricum and D. H. Schaubert, An Experimental Transient Transfer Function via Prony's Method, *Proc. 1976 USNC/URSI Meeting*, Amherst, MA (October 1976), 97.

*See Selected Bibliography--Transient Response of Antennas and Scatterers.

†See Selected Bibliography--Experimental Data.

‡See Selected Bibliography--Equivalent Circuit Synthesis.

the resulting terminal voltage as a function of time (fig. 1). This terminal voltage, $v(t)$, is related to the antenna's impedance through the reflection coefficient and a convolution integral. Specifically, if the Laplace transform pair is defined by

$$V(s) = \int_0^{\infty} v(t) e^{-st} dt, \quad (1a)$$

$$v(t) = \frac{1}{2\pi j} \int_{\alpha-j\infty}^{\alpha+j\infty} V(s) e^{st} ds, \quad (1b)$$

where α is an appropriate constant, and $j = \sqrt{-1}$, then

$$V(s) = \frac{1}{2} V_0(s) [1 + R(s)]. \quad (2)$$

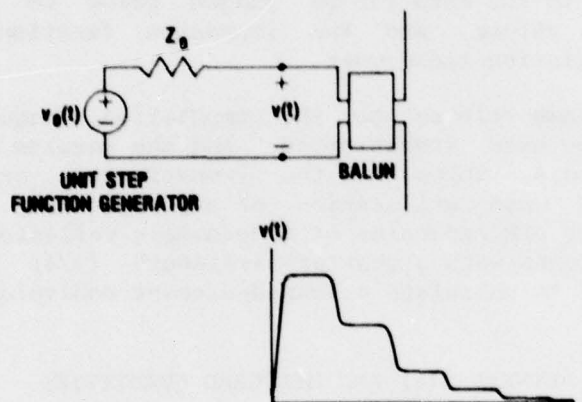


Figure 1. Time-domain reflectometer measurement of antenna impedance.

In equation (2), $V(s)$ and $V_0(s)$ are the Laplace transforms of the terminal voltage and excitation voltage, respectively, and $R(s)$ is the voltage reflection coefficient of the antenna,

$$R(s) = \frac{Z(s) - Z_0}{Z(s) + Z_0}, \quad (3)$$

where Z_0 is the source impedance of the TDR.

Applying the inverse Laplace transform (1b) to equation (2) yields

$$v(t) = \frac{1}{2} \int_0^t v_0(t - \tau) [\delta(\tau) + R(\tau)] d\tau. \quad (4)$$

Since $R(t)$ is the inverse Laplace transform of equation (3), the relationship between $v(t)$ and $Z(s)$ is not trivial. In the Laplace frequency domain, however, the relationship is quite simple. Substituting equation (3) into equation (2) yields

$$V(s) = V_0(s) \frac{Z(s)}{Z(s) + Z_0}. \quad (5)$$

That is,

$$Z(s) = Z_0 \frac{V(s)}{V_0(s) - V(s)}. \quad (6)$$

It is clear that an analytical expression for $Z(s)$ can be obtained if analytical expressions for $V_0(s)$ and $V(s)$, which are the Laplace transforms of measurable quantities, are available. The SEM, together with Prony's method, provides an efficient means of obtaining analytical expressions for $V_0(s)$ and $V(s)$ when $v_0(t)$ and $v(t)$ are known in sampled data form.

3. SINGULARITY EXPANSION AND PRONY'S METHOD

The singularity expansion of a transient waveform $v(t)$ is defined as the exponential approximation

$$v(t) = \sum_{n=1}^N A_n e^{s_n t}. \quad (7)$$

This expansion can be readily transformed to yield

$$V(s) = \sum_{n=1}^N \frac{A_n}{s - s_n}. \quad (8)$$

The s_n are the poles of the expansion and the A_n are the residues.

The problem of importance in this section is, given $v(t)$ at discrete points $t = t_i$, $i = 1, 2, \dots$, find the A_n and s_n that provide the best approximation in the form of equation (7). Because the s_n are not known, this is a nonlinear approximation problem. However, Prony has developed an algorithm that converts this problem into two linear problems plus one polynomial root finding problem, which can be readily solved on a digital computer.

An outline of Prony's method is presented below. Readers unfamiliar with the method may consult one of the references.^{1,*}

If the waveform $v(t)$ is sampled at uniform intervals Δt , then equation (7) becomes

$$v_k = v(k\Delta t) = \sum_{n=1}^N A_n e^{s_n k \Delta t}, \quad k = 0, 1, 2, \dots \quad (9)$$

Defining $Z_n \equiv e^{s_n \Delta t}$, equation (9) becomes

$$v_k = \sum_{n=1}^N A_n Z_n^k, \quad k = 0, 1, 2, \dots \quad (10)$$

It can be shown that Z_n satisfying equation (10) are the roots of the polynomial equation

$$\alpha_0 + \alpha_1 Z + \alpha_2 Z^2 + \dots + Z^N = 0, \quad (11)$$

¹M. L. Van Blaricum and R. Mittra, A Technique for Extracting the Poles and Residues of a System Directly from Its Transient Response, *IEEE Trans. Antennas Propag.*, **AP-23** (November 1975), 777-782.

*See Selected Bibliography--Transient Response of Antennas and Scatterers.

where the α_i are the solutions of the linear set of equations

$$\begin{bmatrix} v_0 & v_1 & v_2 & \cdot & \cdot & \cdot & v_{N-1} \\ v_1 & v_2 & v_3 & \cdot & \cdot & \cdot & v_N \\ v_2 & v_3 & v_4 & \cdot & \cdot & \cdot & v_{N+1} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ v_{N-1} & v_N & v_{N+1} & \cdot & \cdot & \cdot & v_{2N-1} \end{bmatrix} \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \cdot \\ \cdot \\ \cdot \\ \alpha_{N-1} \end{bmatrix} = \begin{bmatrix} -v_N \\ -v_{N+1} \\ -v_{N+2} \\ \cdot \\ \cdot \\ \cdot \\ -v_{2N} \end{bmatrix} \quad (12)$$

Prony's method, therefore, consists of three steps:

- Solve the linear set of equations (12) for the coefficients α_i .
- Obtain the N roots of the polynomial equation (11) and, therefore, the $s_n = (\ln Z_n)/\Delta t$.
- Solve the linear set of equations (10) for the residues A_n (Z_n are known from step b).

Since $v(t)$ is real, the poles s_n (and the roots Z_n) must either be real or occur in complex conjugate pairs.

4. UNIFORM NORM APPROXIMATION

Once the poles s_n are known, step c of Prony's method is a linear approximation problem. That is, one must find the A_n that provide the best approximation to $v(t)$ for a given set of s_n . Most workers in electromagnetic scattering have used the L_2^* (least squared error) norm when calculating the A_n . This norm has the advantage that it leads to a linear set of algebraic equations for the residues A_n . However, the L_2 norm has the disadvantage that it is not sensitive to the large deviations that often occur near $t = 0$. Figure 2 shows a singularity expansion obtained by the L_2 norm approximation method.

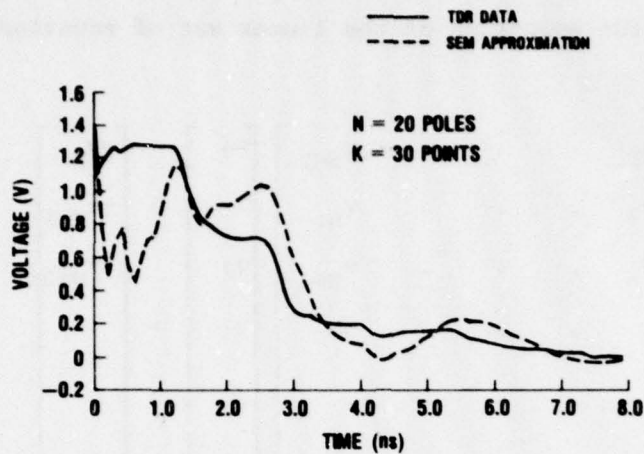


Figure 2. Typical L_2 -norm approximation of time-domain reflectometer (TDR) data.

In order to improve the accuracy of calculated singularity expansions, the third step of Prony's method has been replaced by the uniform norm approximation problem: Given the poles s_n , determine the residues A_n that minimize $\|e(t)\|$ where

$$\|e(t)\| = \max_t \left| v(t) - \sum_{n=1}^N A_n e^{s_n t} \right|. \quad (13)$$

The norm $\|\cdot\|$ defined by equation (13) is the uniform norm and is sensitive to any large deviations of the approximation. However, computation of the best uniform norm approximation is not a linear problem. Therefore, additional computation time is required. A comparison of the L_2 and L_∞ (uniform norm) approximation problems for step c of Prony's method is contained in table I. The two waveforms that were used for this comparison are shown in figure 3, and the parameters are defined by the discrete problem

$$\min_{A_n} \|e(k\Delta t)\|_\infty = \min_{A_n} \max_{0 \leq k \leq K} \left| v(k\Delta t) - \sum_{n=1}^N A_n e^{s_n k\Delta t} \right|. \quad (14)$$

TABLE 1. COMPARISON OF L_2 AND L_∞ NORM APPROXIMATIONS

Number of poles, N	Number of points, K	Δt (ns)	Computation time (s)		Squared error		Type of data
			L_2	L_∞	L_2	L_∞	
14	10	0.25	*	3.13	*	3.3	Impulse response
14	14	0.25	0.12	5.62	3.0	3.0	Impulse response
14	20	0.25	0.46	12.3	105.5	2.9	Impulse response
24	24	0.2	0.53	44.3	917.4	0.01	TDR
24	30	0.2	1.89	64.9	3.75	0.07	TDR
26	26	0.2	0.62	53.1	0.15	0.003	TDR
26	30	0.2	2.24	69.7	1076.1	0.0005	TDR

*Underspecified problem does not have unique solution.

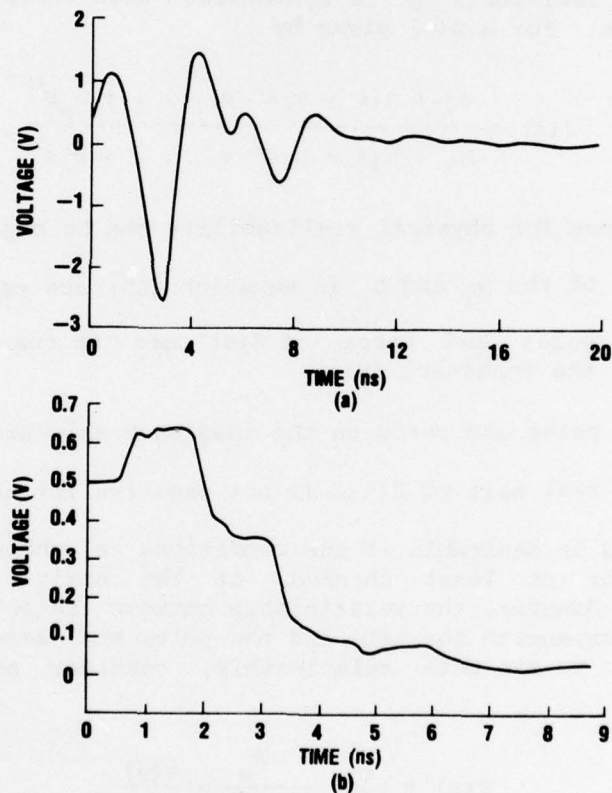


Figure 3. Test waveforms for comparison of L_2 - and L_∞ -norm approximations: (a) receive impulse response of dipole with corner reflector and (b) time-domain reflectometer data of dipole.

Although the computation time is significantly greater for the L_∞ -norm approximation than for the L_2 -norm approximation, the errors are sufficiently less to make the L_∞ -norm attractive. In fact, the uniform norm often yields a very good approximation when the L_2 norm totally fails due to ill-conditioning of the matrix equation.

Since the uniform norm is equivalent to the L_p norm when $p \rightarrow \infty$, the above approximation problem could also be solved by using the L_p norm with p equal to a large number. Doing so may result in some computational advantages.

5. PHYSICAL REALIZABILITY OF $Z(s)$

The impedance function $Z(s)$ derived from a TDR voltage $v(t)$ must be physically realizable to be synthesized with resistors, inductors, and capacitors. For a $Z(s)$ given by

$$Z(s) = \frac{a_0 + a_1s + a_2s^2 + \dots + a_ms^m}{b_0 + b_1s + b_2s^2 + \dots + b_ns^n}, \quad (15)$$

these conditions for physical realizability can be expressed:³

- a. All of the a_i and b_i in equation (15) are real and positive.
- b. The poles and zeros of $Z(s)$ are in the left half of the s plane or on the imaginary axis.
- c. The poles and zeros on the imaginary axis are simple.
- d. The real part of $Z(j\omega)$ is not negative for any value of ω .

It would be desirable if the conditions a through d could be guaranteed, or at least checked, at the early stages of the calculation. However, the relationship between the poles and residues of the SEM expansion for $v(t)$ and the poles and zeros of $Z(s)$ is complicated. To see this relationship, consider equation (8) for $V(s)$:

$$V(s) = \sum_{n=1}^N \frac{A_n}{s - s_n} = \frac{P(s)}{Q(s)}, \quad (16)$$

³W. H. Chen, *Linear Network Design and Synthesis*, McGraw-Hill Book Co., Inc., New York (1962).

where $P(s)$ and $Q(s)$ are polynomials of degree $N-1$ and N , respectively. Similarly, the SEM expansion of $v_0(t)$ leads to

$$V_0(s) = \frac{P_0(s)}{Q_0(s)} \quad (17)$$

Substituting equations (16) and (17) into equation (6) yields

$$Z(s) = \frac{Z_0 Q_0(s) P(s)}{P_0(s) Q(s) - Q_0(s) P(s)} \equiv Z_0 \frac{\tilde{P}(s)}{Q(s)} \quad (18)$$

The zeros of $Z(s)$ are the zeros of $V(s)$ plus the poles of $V_0(s)$. On the other hand, the poles of $Z(s)$ are different from the poles and zeros of either $V(s)$ or $V_0(s)$.

Usually, the generator voltage $v_0(t)$ is a unit step function so that $V_0(s) = 1/s$. Then equation (18) can be simplified to

$$Z(s) = Z_0 \frac{sP(s)}{Q(s) - sP(s)} \quad (19)$$

This equation provides an easily computed relationship between the SEM expansion of the measurable terminal voltage and the desired impedance function. Checks for the satisfaction of conditions a through d have been programmed for automatic testing with the digital computer.

A procedure for generating a physically realizable impedance function from a given $V(s)$ has been developed and tested. The procedure is based on a theorem in Weinberg⁴ and is described in appendix A. The results of numerical tests using the procedure, however, have not been encouraging. In general, when a nonrealizable impedance function is obtained, it is best to return to the original $v(t)$ and calculate a new SEM approximation with different values of N , K , or Δt .

6. SYNTHESIS OF LUMPED-ELEMENT EQUIVALENT CIRCUITS

When a physically realizable impedance function in the form of equation (15) has been obtained, a number of standard circuit synthesis procedures may be employed to determine an RLC network having the prescribed input impedance. The simplest synthesis

⁴L. Weinberg, *Network Analysis and Synthesis*, McGraw-Hill Book Co., Inc., New York (1962).

procedure is based on the partial fraction expansion of the impedance function in equation (15). This procedure has failed for the examples considered because the individual terms of the partial fraction expansion are not all physically realizable. Therefore, more general synthesis procedures such as those of Brune or Bott and Duffin must be used.^{3,4}

The Brune procedure has been selected for the examples that follow. This procedure leads to the least number of circuit elements for a given impedance function. A brief description of the Brune synthesis procedure is given in appendix B.

To illustrate the SEM method of equivalent circuit synthesis, a standard gain dipole with a $\lambda/4$ balun has been measured, and its equivalent circuit has been calculated. The original data obtained from the TDR experiment are shown in figure 4. An SEM approximation having 12 poles is shown in figure 5. The values of the poles and residues are given in table II.* The maximum deviation of this approximation is 20 dB below the 0.5-V signal level, and the total energy in the error signal is 20 dB below the energy in the actual signal. The impedance function calculated from equation (19) is

$$Z(s) = \frac{a_0 + a_1 \tilde{s} + \dots + a_{12} \tilde{s}^{12}}{b_0 + b_1 \tilde{s} + \dots + b_{12} \tilde{s}^{12}} \quad (20)$$

³W. H. Chen, *Linear Network Design and Synthesis*, McGraw-Hill Book Co., Inc., New York (1962).

⁴L. Weinberg, *Network Analysis and Synthesis*, McGraw-Hill Book Co., Inc., New York (1962).

*Since the SEM is an analytical representation of the experimental data, the poles and residues can be expressed to several significant figures. Maintaining these significant figures throughout the calculations insures that the resulting equivalent circuit will accurately model the SEM approximation to the data. Of course, the equivalent circuit model for the antenna cannot be more accurate than the original data.

where

$$\tilde{s} = s \times 10^{-9}$$

$$a_0 = 0.00 \quad b_0 = 0.765 \times 10^5$$

$$a_1 = 0.668 \times 10^7 \quad b_1 = 0.258 \times 10^6$$

$$a_2 = 0.197 \times 10^8 \quad b_2 = 0.286 \times 10^6$$

$$a_3 = 0.156 \times 10^8 \quad b_3 = 0.254 \times 10^6$$

$$a_4 = 0.112 \times 10^8 \quad b_4 = 0.177 \times 10^6$$

$$a_5 = 0.512 \times 10^7 \quad b_5 = 0.774 \times 10^5$$

$$a_6 = 0.206 \times 10^7 \quad b_6 = 0.353 \times 10^5$$

$$a_7 = 0.592 \times 10^6 \quad b_7 = 0.952 \times 10^4$$

$$a_8 = 0.157 \times 10^6 \quad b_8 = 0.287 \times 10^4$$

$$a_9 = 0.273 \times 10^5 \quad b_9 = 0.487 \times 10^3$$

$$a_{10} = 0.493 \times 10^4 \quad b_{10} = 0.957 \times 10^2$$

$$a_{11} = 0.428 \times 10^3 \quad b_{11} = 0.830 \times 10^1$$

$$a_{12} = 0.477 \times 10^2 \quad b_{12} = 0.105 \times 10^1$$

This impedance function is physically realizable as can be seen from the coefficients of equation (20), the locations of the poles and zeros (table III), and the value of the real part (fig. 6). By using Brune's synthesis procedure, the lumped-element equivalent circuit in figure 7 is obtained. This circuit can be substituted for the antenna in any network analysis code. For any frequency within the range of validity of the equivalent circuit (typically several decades), the computed voltages and currents within the network would be the same as if the antenna were attached. For receiver applications, a Thevenin representation of the antenna consisting of a source and the calculated equivalent impedance circuit can be used.

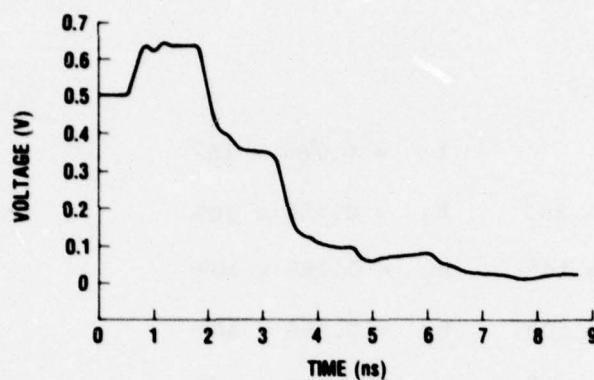


Figure 4. Time-domain reflectometer data of impedance of dipole with $\lambda/4$ balun.

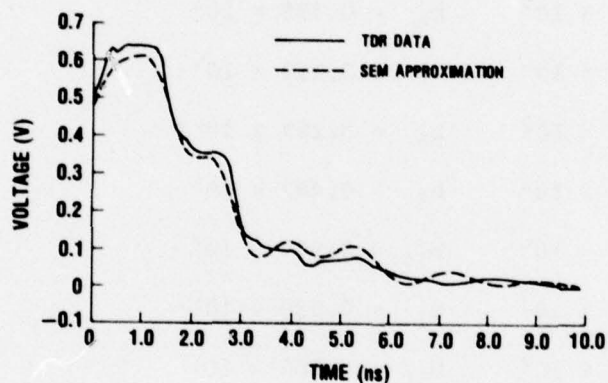


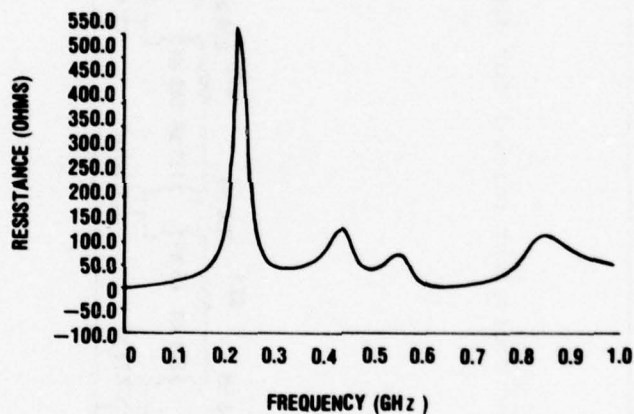
Figure 5. Twelve-pole singularity expansion method (SEM) approximation to time-domain reflectometer (TDR) data of dipole antenna.

TABLE II. POLES AND RESIDUES OF SINGULARITY EXPANSION METHOD APPROXIMATION TO DIPOLE DATA

Poles ($\times 10^{-3}$)	Residues
$-0.93124 \pm j5.05272$	$-0.04076 \pm j0.12436$
$-0.83504 \pm j3.97729$	$-0.19291 \pm j0.44911$
$-0.72280 \pm j1.82908$	$-0.31189 \pm j0.06252$
$-0.63500 \pm j2.92050$	$-0.10172 \pm j0.40237$
$-0.62839 \pm j3.38884$	$0.49460 \pm j0.00000$
$-0.46253 \pm j0.00000$	$0.39124 \pm j0.00000$

TABLE III. POLES AND ZEROS OF $Z(s)$ FOR DIPOLE

Poles ($\times 10^9$)	Zeros ($\times 10^9$)
$-2.3542 \pm j4.0616$	$-2.1230 \pm j1.8792$
$-0.3345 \pm j5.2596$	$-1.5150 \pm j5.6375$
$-0.1807 \pm j3.5347$	$-0.3259 \pm j2.3158$
$-0.1739 \pm j2.7866$	$-0.2395 \pm j3.2719$
$-0.0864 \pm j1.4718$	$-0.0504 \pm j4.3324$
$-0.4625 + j0.0000$	$-0.4625 + j0.0000$
$-1.2186 + j0.0000$	$0.0000 + j0.0000$

Figure 6. Real part of $Z(s)$ for dipole.

To check the synthesis procedure, a circuit that represents the TDR experiment has been analyzed by using the SPICE computer code. The circuit consisted of a 1-V step generator, a 50-ohm source resistor, and the lumped-element equivalent circuit (fig. 8). The calculated network response and the expected response (SEM approximation) are compared in figure 9. The comparison is excellent, and the equivalent circuit is a very good representation (errors at least 20 dB below signal) of the actual antenna.

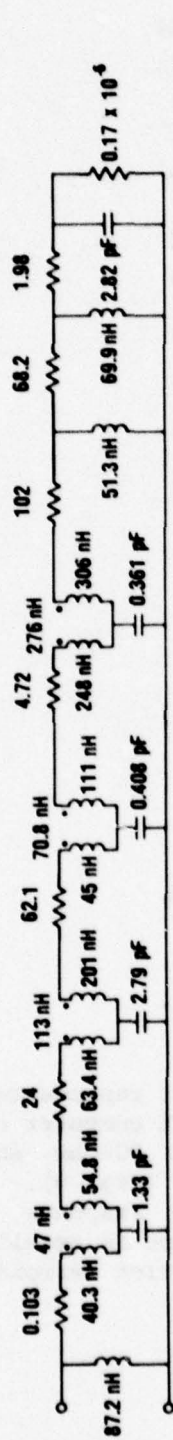


Figure 7. Lumped-element equivalent circuit for dipole with $\lambda/4$ balun.

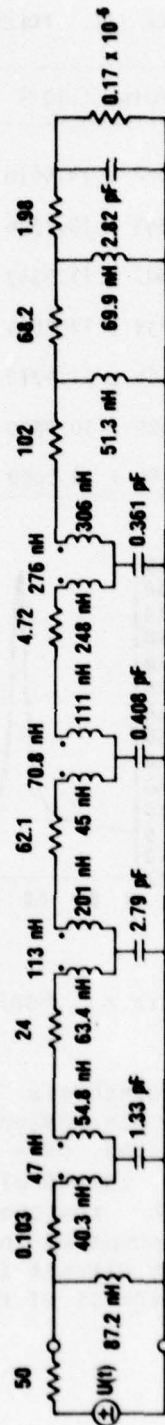


Figure 8. Lumped-element circuit representation of time-domain reflectometer measurement of dipole antenna.

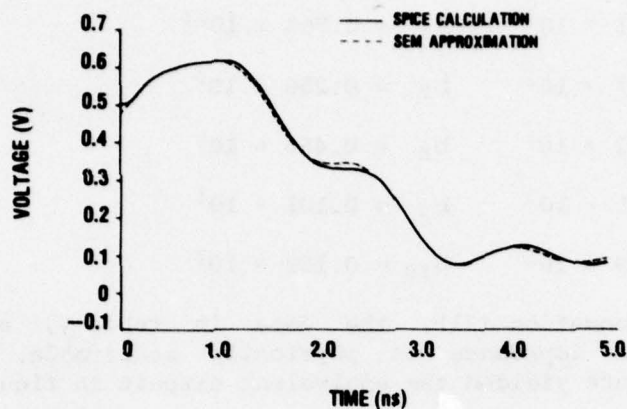


Figure 9. Comparison of desired (singularity expansion method--SEM--approximation) and calculated responses of lumped-element equivalent circuit for dipole.

The equivalent circuit for a yagi antenna has also been calculated. The TDR data and the 10-pole SEM approximation are shown in figure 10. The poles and residues of the approximation are given in table IV. The energy of the error signal is 26 dB below the energy of the actual signal. The impedance function for this voltage is

$$Z(s) = \frac{a_0 + a_1\tilde{s} + \dots + a_{10}\tilde{s}^{10}}{b_0 + b_1\tilde{s} + \dots + b_{10}\tilde{s}^{10}}, \quad (21)$$

where

$$\tilde{s} = s \times 10^{-10}$$

$$a_0 = 0.00 \quad b_0 = 0.151 \times 10^{-7}$$

$$a_1 = 0.194 \times 10^{-4} \quad b_1 = 0.918 \times 10^{-6}$$

$$a_2 = 0.112 \times 10^{-2} \quad b_2 = 0.200 \times 10^{-4}$$

$$a_3 = 0.218 \times 10^{-1} \quad b_3 = 0.237 \times 10^{-3}$$

$$a_4 = 0.184 \times 10^0 \quad b_4 = 0.225 \times 10^{-2}$$

$$a_5 = 0.887 \times 10^0 \quad b_5 = 0.161 \times 10^{-1}$$

$$\begin{aligned}
a_6 &= 0.451 \times 10^1 & b_6 &= 0.564 \times 10^{-1} \\
a_7 &= 0.107 \times 10^2 & b_7 &= 0.256 \times 10^0 \\
a_8 &= 0.312 \times 10^2 & b_8 &= 0.456 \times 10^0 \\
a_9 &= 0.315 \times 10^2 & b_9 &= 0.101 \times 10^1 \\
a_{10} &= 0.489 \times 10^2 & b_{10} &= 0.102 \times 10^1
\end{aligned}$$

Examination of equation (21), the data in table V, and figure 11 reveals that this impedance is physically realizable. The Brune synthesis procedure yielded the equivalent circuit in figure 12.

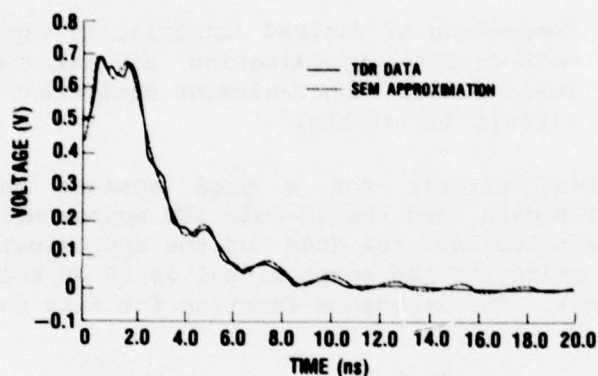


Figure 10. Ten-pole singularity expansion method (SEM) approximation to time-domain reflectometer (TDR) data of yagi antenna.

TABLE IV. POLES AND RESIDUES OF SINGULARITY EXPANSION METHOD APPROXIMATION TO YAGI DATA

Poles ($\times 10^{-3}$)	Residues
$-2.09150 \pm j1.36453$	$-1.05713 \pm j1.29353$
$-0.58429 \pm j4.24343$	$-0.03312 \pm j0.06859$
$-0.14017 \pm j2.47081$	$0.03728 \pm j0.00743$
$-0.34856 \pm j0.00000$	$0.40960 \pm j0.00000$
$-0.94222 \pm j0.00000$	$0.88773 \pm j0.00000$

TABLE V. POLES AND ZEROS OF $Z(s)$ FOR YAGI

Poles ($\times 10^{10}$)	Zeros ($\times 10^{10}$)
$-0.06556 \pm j0.01119$	$-0.14879 \pm j0.54307$
$-0.02358 \pm j0.40602$	$-0.08271 \pm j0.33522$
$-0.00976 \pm j0.26028$	$-0.07111 \pm j0.01615$
$-0.00260 \pm j0.10659$	$-0.00267 \pm j0.23826$
$-0.75164 + j0.00000$	$-0.03486 + j0.00000$
$-0.03486 + j0.00000$	$0.00000 + j0.00000$

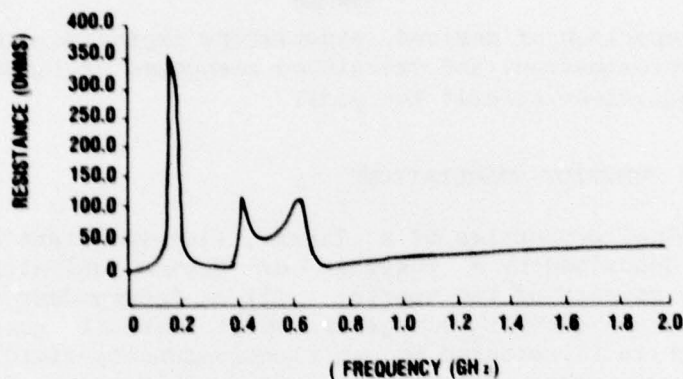


Figure 11. Real part of $Z(s)$ for yagi.

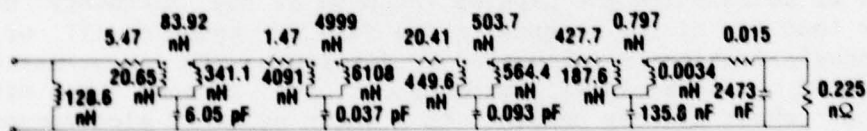


Figure 12. Lumped-element equivalent circuit for yagi antenna.

A SPICE analysis of the TDR response of this lumped-element equivalent circuit yielded the result shown in figure 13. There is excellent agreement between the expected and calculated waveforms.

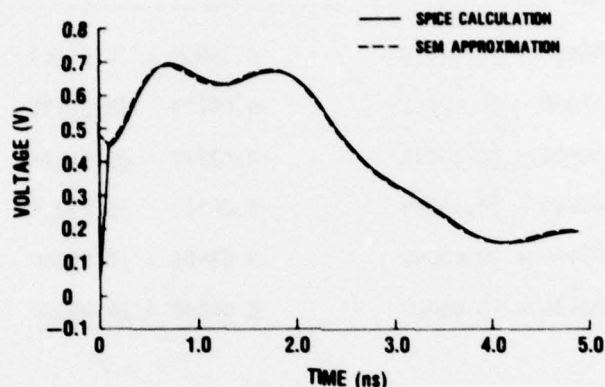


Figure 13. Comparison of desired (singularity expansion method--SEM--approximation) and calculated responses of lumped-element equivalent circuit for yagi.

7. TRANSFER FUNCTION CALCULATIONS

The terminal properties of a linear, time-invariant antenna can be completely described by a Thevenin or Norton equivalent circuit. These circuits consist of two parts: (1) an independent source that represents the open circuit voltage or short circuit current of the antenna when it is illuminated by an electromagnetic field and (2) an equivalent circuit that represents the antenna's input impedance or admittance. This equivalent impedance or admittance circuit can be synthesized by the techniques described above. Furthermore, the SEM can be used to characterize the independent source.

The SEM characterization of the antenna's voltage or current is obtained by calculating the impulse response or the frequency domain transfer function of the antenna in the form of equation (7) or (8). The characterization is calculated by finding an SEM approximation to the measured or calculated response of the antenna (open circuit voltage or short circuit current) to a short pulse of electromagnetic energy.^{2,5} This SEM approximation is an analytical estimate of the

²M. L. Van Blaricum and D. H. Schaubert, *An Experimental Transient Transfer Function via Prony's Method*, Proc. 1976 USNC/URSI Meeting, Amherst, MA (October 1976), 97.

⁵D. H. Schaubert, *Measurement of the Impulse Response of Communication Antennas*, Harry Diamond Laboratories TR-1832 (November 1977).

antenna's impulse response, and it may be convolved with any incident waveform to obtain the resulting terminal voltage or current. Therefore, the SEM can be used to generate equivalent circuits that completely characterize the terminal properties of an antenna.

8. CONCLUSIONS

The singularity expansion method (SEM) has been shown to be an effective tool for synthesizing lumped-element equivalent circuits from experimentally derived time-domain reflectometer (TDR) data. The poles of the SEM expansion are calculated by using the standard Prony algorithm. The residues, however, are calculated by an iterative scheme that yields a uniform norm approximation instead of a least squared error norm approximation. The uniform norm provides much better control of the early time errors and leads to a better overall approximation. The increased computation time required for the uniform norm approximation is readily justified because this time is not large compared to the total time for calculating the equivalent circuit. Furthermore, the time spent in calculating the SEM approximation is repaid by the final circuit, which produces exactly the voltage response prescribed by the SEM approximation.

Once the physical realizability of the calculated impedance function has been verified, a standard circuit synthesis procedure, such as that of Brune or Bott and Duffin, may be used to obtain the desired RLC equivalent circuit. In general, the partial fraction expansion of $Z(s)$ will not yield subcircuits that are physically realizable.

When combined with the previously developed technique for measuring the impulse response of antennas, SEM synthesis of equivalent impedance circuits is a powerful and generally applicable tool for converting experimentally obtained data into Thevenin and Norton equivalent circuits for use in wide-bandwidth analytical modelling of transmitter and receiver systems.

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APPENDIX A.--PHYSICAL REALIZABILITY VIA MULTIPLICATIVE FACTOR

Theorem 6.5 in Weinberg¹ states that $Z(s) = Z_0[1 + R(s)]/[1 - R(s)]$ is a physically realizable function if and only if

1. $R(s)$ is a real rational function with no poles in the right half plane or on the imaginary axis, and
2. $|R(j\omega)| \leq 1$ for all real ω .

This theorem and equation (18) in the main body of this report can be used to obtain an impedance function $Z_\alpha(s)$ that depends on the parameter α , $0 < \alpha \leq 1$. For $\alpha = 1$, $Z_\alpha(s)$ equals the impedance corresponding to $R(s)$. For $0 < \alpha < 1$, $Z_\alpha(s)$ is the impedance corresponding to a reflection coefficient $\alpha R(s)$.

The proposed scheme for calculating a physically realizable impedance function requires that the singularity expansion method (SEM) approximation to $V(s)$ and $V_0(s)$ be manipulated to obtain $R(s) = N(s)/D(s)$, where $N(s)$ and $D(s)$ are polynomials. Then

$$Z_\alpha(s) = Z_0 \frac{1 + \alpha R(s)}{1 - \alpha R(s)} \quad (A-1)$$

can be calculated for various α , and the largest $\alpha \leq 1$ for which $Z_\alpha(s)$ is physically realizable is the "best" impedance function. ("Best" is used in the sense that the poles and zeros of the reflection coefficient are those obtained from $V(s)$ and $V_0(s)$ and $|R_\alpha(j\omega)| = \alpha |R(j\omega)| \leq 1$.)

The impedance function $Z_\alpha(s)$ can be related directly to the SEM expressions for $V(s)$ and $V_0(s)$. Equation (2) in the main body of the report implies that

$$R(s) = 2 \frac{V(s)}{V_0(s)} - 1 = 2 \frac{P(s)Q_0(s)}{Q(s)P_0(s)} - 1. \quad (A-2)$$

Therefore,

$$R_\alpha(s) = \alpha R(s) = \frac{2\alpha P(s)Q_0(s) - Q(s)P_0(s)}{Q(s)P_0(s)}. \quad (A-3)$$

¹L. Weinberg, *Network Analysis and Synthesis*, McGraw-Hill Book Co., Inc., New York (1962).

Substituting equation (A-3) into equation (A-1) yields

$$Z_{\alpha}(s) = Z_0 \frac{(1 - \alpha)Q(s)P_0(s) + 2\alpha P(s)Q_0(s)}{(1 + \alpha)Q(s)P_0(s) - 2\alpha P(s)Q_0(s)} \quad (A-4)$$

For step function excitation, $V_0(s) = 1/s$, and equation (A-4) reduces to

$$Z_{\alpha}(s) = Z_0 \frac{(1 - \alpha)Q(s) + 2\alpha sP(s)}{(1 + \alpha)Q(s) - 2\alpha sP(s)}$$

APPENDIX B.--BRUNE SYNTHESIS OF RLC NETWORKS

A detailed description of the Brune synthesis procedure can be found in many texts.^{1,2} The following is a brief summary of the procedure.

The Brune synthesis procedure is based on the tee representation of a transformer (fig. B-1). In the tee representation, one of the inductances L_a or L_c is negative, but the network is still considered physically realizable.



$$\begin{aligned} L_p &= L_a + L_b \\ M &= L_b \\ L_s &= L_b + L_c \end{aligned}$$

Figure B-1. Equivalent representations of transformer.

Starting with the n th order impedance function,

$$Z(s) = \frac{a_n s^n + a_{n-1} s^{n-1} + \dots + a_0}{b_n s^n + b_{n-1} s^{n-1} + \dots + b_0}, \quad (B-1)$$

a single cycle of the synthesis procedure yields the network in figure B-2, where $Z_4(s)$ is of order $n-2$. The Brune cycle for reducing $Z(s)$ consists of four steps:

¹L. Weinberg, *Network Analysis and Synthesis*, McGraw-Hill Book Co., Inc., New York (1962).

²W. H. Chen, *Linear Network Design and Synthesis*, McGraw-Hill Book Co., Inc., New York (1962).

APPENDIX B

1. Remove $R_1 = \min_{\omega} \operatorname{Re} Z(j\omega)$ (fig. B-3).
2. Remove $L_1 = \operatorname{Im} Z(j\omega_1)/\omega_1$.
3. Remove the shunt arm $L_2 C_2$, which produces zeros in $Z_2(s)$ at $s = \pm j\omega_1$.
4. Remove L_3 , which causes $Z_3(s)$ to have a pole at infinity.

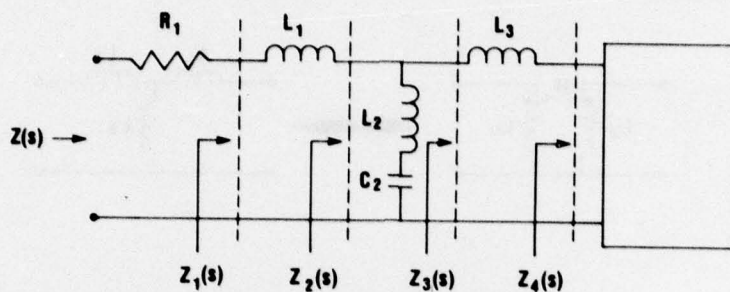


Figure B-2. Realization of single Brune cycle.

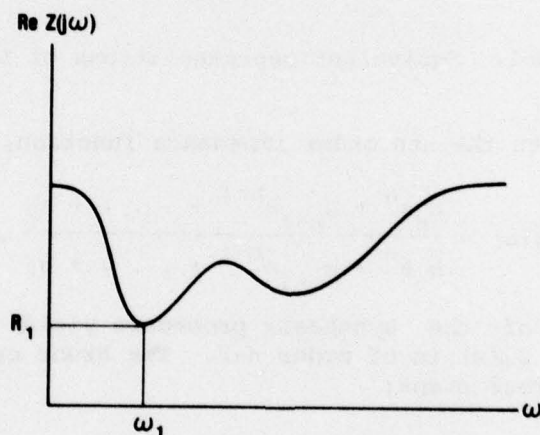


Figure B-3. Typical resistance function for Brune synthesis procedure.

APPENDIX B

These four steps require polynomial manipulations that can be readily programmed on the digital computer. The equivalence in figure B-1 may be used to convert a Brune circuit containing a negative inductance into a transformer as shown in figure B-4.

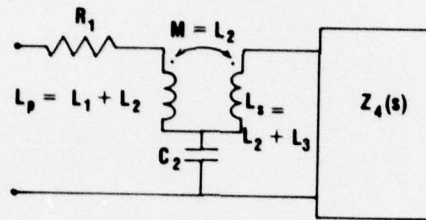


Figure B-4. Final Brune circuit with transformer.

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