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The response of an aircraft as a function of time on encountering an isolated ramp gust is derived from its response to a unit step gust. Two FORTRAN programs are described which treat separately straight ramp and smooth ramp (one-minus-cosine) gust profiles. The (critical) gust length causing maximum dynamic response is determined and responses to simple gust patterns (pairs) are investigated.

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THE RESPONSE OF AIRCRAFT TO DISCRETE RAMP GUSTS

by

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SUMMARY

The response of an aircraft as a function of time on encountering an isolated ramp gust is derived from its response to a unit step gust. Two FORTRAN programs are described which treat separately straight ramp and smooth ramp (one-minus-cosine) gust profiles. The (critical) gust length causing maximum dynamic response is determined and responses to simple gust patterns (pairs) are investigated.

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INTRODUCTION

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The problem discussed in this Report is the following:

Given the response F(t) of an aircraft flying at a certain speed to a unit step gust, predict the response to a family of ramp gusts and determine the critical gust length at which the extreme response occurs.

The investigation was treated, purely as a mathematical and computational exercise; the soundness and applicability of the physical theory (eg Jones¹) was taken for granted. Jones recalls in that Report that the earliest airworthiness requirements were based on response to discrete gusts but that recently more emphasis has been laid on irregular turbulence, the implication being that responses can be deduced adequately from stationary random process theory. He then argues that large aircraft loads cannot be satisfactorily predicted by spectral analysis and that the consideration of discrete ramp gusts is preferable if extreme responses are to be reliably estimated. Effectively, the argument is that the power spectrum (PSD) method estimates the root mean square response whereas one is really interested in the extreme behaviour and that the non-Gaussian character of turbulence can be so marked as to undermine the usefulness of the rms value. In particular, the PSD method is not to be trusted when the dominant aircraft mode is well damped.

The problem is the familiar one that the weight of interest falls in the tail of a distribution where probability levels are only poorly defined by empirical evidence. Any extrapolation from the relatively well defined body of the distribution is unsafe unless independent information on the shape of the distribution is available. In addition, Jones points out that dynamic response is produced by *changes* in wind velocity so that one must determine the probability distribution of velocity gradients. Many power spectrum investigations have only considered the velocity distribution.

To cope with situations in which non-Gaussian velocity gradients are significant, Jones has developed a statistical discrete gust model wherein the intensity, $w_{\rm H}$ of a ramp gust is related to its gradient distance (H) by

$$w_{\rm H} = w_0 {\rm H}^{\frac{1}{3}}$$
 (1)

where w_0 is derived directly from measurements and varies with altitude. This dependence of gust intensity on gradient length is valid over the range of wavelengths (λ) for which the Kolmogoroff (proportional to $\lambda^{\frac{5}{3}}$; $\lambda \approx 2$ H) power spectrum of atmospheric turbulence applies. Fortunately, the Kolmogoroff spectrum

appears to be valid far beyond the wavelengths associated with extreme aircraft response.

The ramp (see Fig 1) may be either straight or 'one-minus-cosine' (smooth).

$$w_1(x) = w_H(\frac{x}{H})$$
 $0 \le x \le H$ (2)

$$w_2(x) = \frac{1}{2}w_H(1 - \cos\frac{\pi x}{H}); \quad 0 \le x \le H.$$
 (3)

The latter has the advantage that $\frac{dw_2}{dx} = 0$ at x = 0, H (whereas $\frac{dw_1}{dx}$ is discontinuous at the end points) and is generally preferred.

In the next section we derive the ramp response from the unit step response and the gust profile. Later sections explain the computational details and the determination of that gust length, \overline{H} which causes the worst response, $\overline{\gamma}$. This is extended in section 7 to deduce the most adverse gust pair.

Although the problem appears straightforward at first sight, being basically one of numerical integration and interpolation there are complicating features. In addition, although the physical model is only approximate, we did not wish to further degrade it with rough calculations. Our efforts to ensure efficiency, accuracy and stability in the numerical operations are duly described.

Thus, in section 3 we describe the integration procedure based on cubic spline fitting of the step response, explaining the considerable economies which are possible with this formulation. The straight ramp generates a discontinuity in the slope of the ramp response which can be most troublesome (if, for example, the 'corner' is smoothed) if not properly handled as described in section 4. Extra trial gust lengths are needed to refine the estimate of the critical gust, \overline{H} , and more especially, the critical time, \overline{t} . These are kept to a minimum by an efficient interpolation procedure (see section 5).

The end result, we believe, are computer programs which can determine the critical ramp gust and gust pair with errors which are negligible compared to those arising from the fundamental physical assumptions and idealisations.

2 DERIVING THE RAMP RESPONSE FROM THE STEP RESPONSE

2.1 Assumptions

The important assertion which motivated the present work is that the response to an isolated ramp gust of a given length and profile shape is a meaningful quantity to calculate. Jones claims that such a representation is usually

reasonable at least in the vicinity of the critical gust length (*ie* that isolated gust which generates the worst behaviour). In other words, despite turbulence being comprised of gusts of all lengths, only a limited range is important and the magnitude of the worst response can be fairly well predicted by identifying the critical isolated gust (or, more generally, sequence of gusts).

A second major assumption is that the system is linear. A corollary is then that

 $\gamma(w_0, H) = w_0 \gamma(1, H)$

where y is the (extreme) response.

The linearity assumption also facilitates consideration of gust pairs whereby overswing tuning can result in a greater response than any single gust would cause. The effect, of course, is unimportant for well damped modes. Gust pairing is therefore an extension of the basic concept to moderately damped modes and involves finding the critical gust length, \overline{H}_0 , which results in the worst overswing as well as that (\overline{H}) which gives the extreme primary response. The most adverse situation is when the two gusts occur with opposite sign and appropriate spacing H_8 (see Fig 2 and section 7) so that the first overswing due to the first gust coincides with the primary peak due to the second gust. The spatial extent of the critical gust pattern is therefore

$$\bar{H}_2 = \bar{H} + \bar{H}_0 + H_s . \tag{4}$$

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More complex sequences of gusts are not considered. However, it is possible to extend the concept to include very lightly damped modes and a recent paper by Jones² envisages the superposition of responses to as many as eight successive gusts. Of course, the probability of the critical pattern occurring decreases as the number of component gusts is increased.

2.2 Theoretical considerations

Let the step response be F(t) and the ramp response be $\phi(H,t)$. Essentially, the calculation of ϕ involves the numerical integration of F(t). Note that strictly speaking we should write F(v,t) but it is customary to regard the aircraft velocity as constant during an encounter with turbulence. It is not a simple (*ie* linear) matter to determine ϕ (and hence $\gamma(H)$) at Jifferent speeds - F(t) must be recalculated.

A ramp gust can be thought of as a series of n step gusts (see Fig 3). Thus we have to combine together up to n step responses, each scaled and appropriately time shifted. The linearity assumption is essential if the means of combination is to be simple addition. The response at time t' will be given by

$$\phi(H,t') = \sum_{i=1}^{J} \delta w_i F(t' - \tau_i)$$
 (5)

where $n\delta t = H/v$; $\tau_i = i\delta t$; $j = min (n, t'/\delta t)$. Letting $\delta t \neq 0$,

$$\phi(H,t') = \int_{0}^{\min(t',H/v)} \frac{dw}{dt} \Big|_{\tau} F(t'-\tau)d\tau . \qquad (6)$$

Or, making the transformation $t = t' - \tau$,

$$\phi(H,t') = \int_{\max(0,t'-H/v)}^{t'} \frac{dw}{dt} \Big|_{t'-t} F(t)dt . \qquad (7)$$

The limits reflect the assumption that $\frac{dw}{dt} = 0$ when t < 0 or t > H/v. Expressed as functions of time and setting $w_0 = 1$, equations (2) and (3) become

$$w_1(t) = \frac{vt}{H^3}; w_2(t) = \frac{1}{2}H^3(1 - \cos\frac{\pi vt}{H}).$$
 (8)

Hence (7) gives, respectively,

$$\phi_{1}(H,t') = \frac{v}{H^{3}} \int_{\max(0,t'-H/v)}^{t'} F(t)dt \qquad (9)$$

$$\phi_{2}(H,t') = \frac{\pi}{2} \frac{v}{H^{3}} \int_{\max(0,t'-H/v)}^{t'} \sin \frac{\pi v}{H} (t'-t)F(t)dt \qquad (10)$$

Thus the straight ramp response at time t' is merely the area under the step response between t' and some earlier time, either zero or t' - H/v. Furthermore, differentiating (9) gives

$$\frac{\partial \Phi_1}{\partial t'} = \frac{v}{H^3} \left(F(t') - \left\{ \begin{matrix} 0 \\ F(t' - H/v) \end{matrix} \right\} \right)$$
(11)

$$\frac{\partial \phi_1}{\partial H} = -\frac{2}{3H} \phi_1(H,t') + \left\{ \frac{O}{F(t'-H/v)} \right\}.$$
(12)

From equation (11) we can envisage three possible situations concerning the time, t_{γ} , at which the maximum response, $\gamma(H)$ occurs. As depicted in Fig 4 t_{γ} can be greater than, less than, or equal to H/v. The first two present no computational difficulties because $\partial \phi_1/\partial t = 0$ but the third case could be troublesome because the discontinuity in $\partial \phi/\partial t'$ coincides with t_{γ} .

Now, in any numerical scheme integrals can only be computed at discrete (usually pre-determined) points. If one seeks maxima and minima of a function defined only as a set of points then some interpolation is necessary. Of necessity the interval between adjacent points will be represented by a fully continuous interpolating function and difficulties can be expected if the original function is less well behaved.

Trouble was indeed encountered with the straight ramp when the derivative discontinuity at t = H/v fell within the same integration interval as t_{γ} . The resulting interpolation errors superimposed a ripple on the $\gamma(H)$ curve making impossible the accurate determination of \overline{H} . The problem only arises if $F(0) \neq 0$ but unfortunately this is common because F(t) is the response to a step gust. The counter-measures adopted are described in section 4. There was no corresponding difficulty with the one-minus-cosine ramp because $\partial \phi_2/\partial t$ is everywhere smooth.

It can also be surmised that t_{γ} will increase with H until the first zero, t_0 , of F is reached. For $H > vt_0$ one would expect $t_{\gamma} = t_0$ (*ie* case 2 of Fig 4) and $\gamma(H)$ to decrease as H^{-1} . Equation (12) provides the condition to be satisfied at \overline{H} . First we observe that $\overline{t} < \overline{H}/v$ is impossible. For $\overline{t} > \overline{H}/v$ we have

 $\overline{\gamma} = \frac{3}{2} \overline{H}^{\frac{1}{2}} F(\overline{t} - \overline{H}/v) .$

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But from (11),

$$F(t) = F(t - H/v)$$
 (13)

Thus

$$\overline{\gamma} = \frac{3}{2} \overline{H}^{\frac{1}{3}} F(\overline{t}) . \qquad (14)$$

If t = H/v then neither derivative (11) or (12) is separately zero but we still have

$$\frac{\partial \phi_1}{\partial t^*} + v \frac{\partial \phi_1}{\partial H} = 0$$

giving $\overline{\gamma} = \frac{3}{2} \overline{H}^{\frac{1}{2}} F(\overline{H}/\nu)$.

Hence equation (14) remains valid when t = H/v. It is useful for checking purposes since the computer program does not use it directly but instead locates H by interpolation.

Equation (9) for the straight ramp gust encounter can be solved very economically because the integrand is independent of H. One merely has to store once and for all the accumulated area under F(t) at predetermined time points and then use appropriate bandwidths when considering different H. Equation (10) is far more complicated because the step response is weighted by an H-dependent factor. Furthermore, practical checking formulae analagous to (13) and (14) cannot be given.

EVALUATION OF INTEGRALS 3

The first operation in both the straight ramp and the smooth ramp programs is to fit a cubic spline to F(t). The two spare degrees of freedom are taken up by third derivative continuity at the second and penultimate points (knots). Subroutine TBO4A of the Harwell Subroutine Library is employed for this purpose. A particular virtue of the cubic spline is that the area between knots is easily calculated and will be accurate to third order even when the abscissae are unequally spaced (of Simpson's rule).

$$A_{i} = \frac{\Delta t}{2} (F_{i+1} + F_{i}) + \frac{\Delta t^{2}}{12} (F_{i}' - F_{i+1}')$$
(15)

where F' = dF/dt. The integral in (9) is therefore readily evaluated while that in (10) requires further manipulation as follows.

Using the cubic spline, the 'weighted' (in the sense of (10)) area between two adjacent knots is given by

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$$\int_{t_0}^{1} \sin \alpha(t' - t)F(t)dt = \left[\frac{F(t)}{\alpha} \cos \alpha(t' - t) + \frac{F'(t)}{\alpha^2} \sin \alpha(t' - t) - \frac{F''(t)}{\alpha^3} \cos \alpha(t' - t) - \frac{F''(t)}{\alpha^3} \cos \alpha(t' - t) - \frac{F'''(t)}{\alpha^4} \sin \alpha(t' - t)\right]_{t_0}^{t_1} .$$
 (16)

If we now consider the total integral over m intervals, the fact that F,F' and F" are continuous at the knots and that F''' is constant on each interval causes most terms to cancel leaving

$$\int_{t_0}^{t_m} \sin \alpha (t'-t) F(t) dt = \left[\frac{F(t)}{\alpha} \cos \alpha (t'-t) \dots \right]_{t_0}^{t_m} + \frac{1}{a^4} \sum_{i=1}^{m} F_i^{m} (\sin \alpha (t'-t_{i-1}))$$

Now, from equation (10) we have $\alpha = \pi v/H$, $t_m = t'$. Hence $\sin \alpha (t' - t_m) \equiv 0$, $\cos \alpha (t' - t_m) \equiv 1$ while at the lower integration limit we must distinguish the two cases (i) $t_0 = t' - H/v > 0$; (ii) $t_0 = 0$.

In the first case we have:

(i)
$$\sin \alpha (t' - t_0) = 0$$
, $\cos \alpha (t' - t_0) = -1$.

Therefore the first term on the right hand side of (17) becomes

$$\frac{F(t') + F(t' - H/v)}{\pi v/H} = \frac{F''(t') + F''(t' - H/v)}{(\pi v/H)^3} .$$
(18)

Regarding the second term on the right of (17), all intervals except possibly the first (depending whether or not H/v is an integral multiple of Δt) will be of the same size, Δt . Therefore,

$$\sum_{i=1}^{m} = \sum_{i=2}^{m} F_{i}^{m} (\sin \alpha(t' - t_{i-1}) - \sin \alpha(t' - t_{i})) + F_{1}^{m} (\sin \alpha(t' - t_{0}) - \sin \alpha(t' - t_{1}))$$

where $F_i^{"'}$ is the third derivative of F(t) in the ith interval following t_0 . But $t_0 = t' - H/v$, $t_1 - t_0 = \Delta t_1$ (say), $t' - t_1 = (m - i)\Delta t$ and $m = \frac{H}{v\Delta t}$ rounded up if necessary.

Therefore
$$\sum_{i=1}^{m} = \sum_{i=2}^{m} \left(F_{i}^{""} 2 \cos \alpha (t' - t_{i} + \frac{\Delta t}{2}) \sin \frac{\alpha \Delta^{t}}{2} \right) - F_{1}^{""} \sin \alpha \Delta t_{1}$$
$$= 2 \sin \frac{\pi v \Delta t}{2H} \sum_{i=2}^{m} \left(F_{i}^{""} \cos \frac{\pi v}{H} (m - i + \frac{1}{2}) \Delta t \right) - F_{1}^{""} \sin \frac{\pi v \Delta t_{1}}{H} .$$
$$\dots \dots (19)$$

In the second case we have:

(ii) All intervals are of length Δt and the integrand does not vanish at the lower limit. The first term on the right of (17) becomes

$$\frac{F(t')}{\alpha} - \frac{F''(t')}{\alpha^3} - \frac{F(0)}{\alpha} \cos \alpha t' + \frac{F''(0)}{\alpha^3} \cos \alpha t' - \frac{F'(0)}{\alpha^2} \sin \alpha t'$$
(20)

while, following the above analysis, the sum reduces to

$$2 \sin \frac{\pi v \Delta t}{2H} \sum_{i=1}^{m} F_{i}^{\prime \prime \prime} \cos \frac{\pi v}{H} (m - i + \frac{1}{2}) \Delta t \qquad (21)$$

where $m = t'/\Delta t$.

It is therefore necessary to evaluate F" at each knot and F" on each interval and to store them. In addition, in order to evaluate efficiently the integrals for all t' at given H it is helpful to first store

$$\cos (j - \frac{1}{2}) \frac{\pi v \Delta t}{H} \quad \text{for } j = 1, 2 \dots \text{ integer part of } \left(\frac{H}{v \Delta t}\right).$$

The only quantities which then have to be recalculated at each time point are cos $\alpha t'$ and sin $\alpha t'$ (for use in (20)). When t' > H/v even this becomes unnecessary although F(t' - H/v) and F''(t' - H/v) may then have to be specially interpolated from the cubic approximation over the relevant interval for use in (18).

Potential trouble spots in the above formulation are the heavy dependence on third derivatives and the presence of F'(0) and F''(0) in (20) because no restrictions are placed on the derivatives generated at the end points of the spline. Tests with analytic step responses have verified that the numerical scheme can be very accurate but ultimately of course, in any practical case much will depend on the nature of F(t) and the sampling interval, Δt (see also section 8). Perhaps the more important feature of the comparison with analytic expressions was the validation of the programming of equations (18) to (21).

4 OBTAINING $\gamma(H)$

The previous two sections explained how the ramp response is calculated at equally spaced time points for each trial gust length, H. We now have to detect its primary peak and, where applicable, the maximum overswing. No assumptions can be made about the shape of the curve which makes the location of global extrema all the more difficult. The basic procedure in both programs is to search the set of $\phi(t)$ for the largest positive and negative values and to then interpolate $\gamma(H)$ and t_{γ} using $d\phi/dt$. The situation depicted in Fig 5 will therefore be wrongly analysed as shown but it is hoped that Δt can be specified sufficiently small to render such a shortcoming unimportant.

As noted in section 2, when using the straight ramp model we have to be particularly careful to evaluate $\phi(H/v)$ explicitly because $d\phi/dt$ is there discontinuous. At other absicissae $d\phi/dt$ can be found directly from the tabulated step response using (11). We therefore have to consider the following possibilities.

- (1) $\phi(H/v)$ is the largest ordinate found initially,
- then either (a) $d\phi/dt$ changes sign at H/v in which case $t_{\gamma} = H/v$, or (b) $d\phi/dt$ does not change sign at H/v, *ie* t_{γ} lies between H/v and one or other of the adjacent abscissae.

(2) $\phi(H/v)$ is not the largest ordinate found initially but $j\Delta t$ is. Then t_{γ} lies somewhere between $(j - 1)\Delta t$ and $(j + 1)\Delta t$. Again, if H/v lies within this interval, care must be taken to locate the nearest point to $j\Delta t$ which

brackets the peak so that t and $\gamma(H)$ can be accurately interpolated, ie t lies in one of the intervals $((j - 1)\Delta t, j\Delta t), (j\Delta t, (j + 1)\Delta t), (H/v, j\Delta t), (j\Delta t, H/v).$

The process of interpolating the peak is merely that of finding a stationary point of a cubic given the function value and first derivative at two bracketing abscissae.

The smooth ramp model differs in that t = H/v does not present any special difficulties but on the other hand, obtaining $d\phi/dt$ is not so straightforward. In fact differentiating (10) gives

$$\frac{\partial \phi_2}{\partial t'} = \frac{\pi^2 v^2}{2H^3} \int_{\max(0,t'-H/v)}^{t'} \cos \frac{\pi v}{H} (t'-t)F(t)dt . \qquad (22)$$

Instead of evaluating the above integral we have elected to fit a cubic spline through $\phi_2(t)$ and to interpolate the required extrema from it having located the maximum and minimum tabulated points by inspection as in the straight ramp model.

5 FINDING \overline{H} , $\gamma(\overline{H})$ AND $t_{\gamma}(\overline{H})$

Both the primary peak of $\phi(t)$ and the largest overswing are determined for each trial value of H supplied by the user. Actually, we seek the extreme positive and negative responses $\gamma_{+}(H)$, $\gamma_{-}(H)$. It was originally suggested that we call the larger (in absolute value), γ_{+} and the smaller, γ_{-} but this seemed to be more confusing and could (if $\gamma_{+}(H)$ and $|\gamma_{-}(H)|$, as we have defined them, intersect) complicate the shape of $\gamma(H)$ causing discontinuities in $d\gamma/dH$ and corresponding difficulty in locating \overline{H} (see Fig 6). Using the revised convention, $\gamma(H)$ is more likely to be unimodal and only after $\overline{\gamma}_{+}$ and $\overline{\gamma}_{-}$ have been estimated do we decide which is the primary peak and which is the overswing. It is not necessarily the case that the sign of the primary is the same as that of the first peak in the ramp response nor that the worst overswing occurs after the primary.

In order to find \overline{H} it is essential that the user supplies trial values of H which bracket \overline{H} , or more precisely, that the extreme $\gamma(H_{trial})$ occurs neither at the smallest nor the largest gust length supplied. In principle the

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program could select its own trial values of H, choosing gust lengths anywhere between $v\Delta t$ and vt_{max} (where t_{max} is the last point of the given step response). In practice it was felt that the response to specific gusts could be of interest so the programs expect a minimum of three trial gust lengths. The user should avoid $H < v\Delta t$ or $H > vt_{max}$ and should be careful to supply enough time history, F(t). Strictly speaking, sufficient step response should be input to enable $\gamma(2H)$ to be calculated but this may not always be possible (see section 6).

Assuming these conditions have been fulfilled we will get a best estimate of H and lower and upper bounds H_{II} and H_{III} . We then use a safeguarded quadratic interpolation scheme outlined below to successively improve H until $max(H_U - H, H - H_L)$ is sufficiently small. Several further ramp responses will have to be calculated but we feel that the labour is justified. It may be thought possible to economise calculation by only evaluating \$ in the vicinity of t_{γ} but unfortunately t_{γ} can change discontinuously (and hence t cannot safely be interpolated) as the following example shows.

Consider the step response shown in Fig 7a in conjunction with the straight ramp model and let area A3 exceed area A1 and $t_{03} - t_{02} > t_{01}$. Then (see Fig 7b) $t_{\gamma} = H/v$ until $H = vt_{01}$ is reached after which $t_{\gamma} = t_{01}$ until H is sufficiently large that A3b = A1 when it switches discontinuously to t_{13} increasing to t_{03} when $H = v(t_{03} - t_{02})$. Eventually t_y will revert to t₀₁ at t₁₄

where $\int_{14}^{14} F(t)dt = AI$.

This is a powerful argument in favour of computing extra ramp responses rather than attempting to interpolate t directly from the responses at the given values of H. Although $\bar{\gamma}$ and H may perhaps be satisfactorily derived from the initial set it is apparent that t could be grossly in error and hence the most adverse gust pair predicted (see section 7) would be totally misleading.

We now describe the quadratic approximation procedure used to improve H. The simplest algorithm would be one which always maintained three points bracketing the peak so that interpolation was always possible. However, if the curve was very asymmetric such a process would be inefficient. For example, in Fig 8, point 3 would always be retained and slow convergence would be experienced because

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of its remoteness from \overline{H} . A preferable scheme is to maintain wherever possible the three highest points even if it means extrapolating \overline{H} . An artificial bound would then be needed to safeguard the extrapolation. Such algorithms are commonplace in one-dimensional search routines appearing in optimisation programs and are discussed by Brent³ and Gill and Murray⁴. If \overline{H} is the highest point so far found and H_L and H_U are the current bounds on \overline{H} then the artificial bound is defined by

$$\begin{aligned} H_{A} &= \widetilde{H} + \beta (H_{L} - \widetilde{H}) & \widetilde{H} \ge \mu \\ &= \widetilde{H} + \beta (H_{U} - \widetilde{H}) & \widetilde{H} < \mu \end{aligned}$$
 (23)

where $\mu = \frac{1}{2}(H_{L} + H_{U})$; $\beta = \frac{1}{2}(3 - \sqrt{5})$.

If the extrapolation predicts a point outside (H_L, H_U) then H_A is used instead. This strategy becomes comparable with the usual bracketing technique as the skewness of the curve decreases. In fact, the programs work with the curve $\gamma(\log H)$, making the determination of \overline{H} even easier. \overline{t} is not interpolated but is obtained directly from $\phi(\overline{H}, t)$ as of course is $\overline{\gamma}$.

A further refinement prevents the evaluation of ϕ at values of H which are too close together. The user supplies a parameter specifying the relative accuracy, $\Delta H/\bar{H}$, required and the program locates \bar{H} such that

$$\max(\log_{e}(\bar{H}/H_{L}), \log_{e}(H_{U}/\bar{H})) < \Delta H/\bar{H}$$
(24)

while keeping the minimum separation between any pair of (logarithmic) trial gust lengths $\Rightarrow \frac{\Delta H}{2R}$.

6 SENSITIVITY FACTOR

Jones¹ defines λ , the gust length sensitivity, a measure of the sharpness of the peak in $\gamma(H)$ by

$$\Lambda = \frac{1}{\log_{e}^{2}} \left(\frac{2\gamma(\bar{H}) - \gamma(2\bar{H}) - \gamma(\bar{H}/2)}{2\pi\gamma(\bar{H})} \right)^{4} .$$
 (25)

An alternative expression which ignores gusts longer than H is

$$\lambda_{w} = \frac{1}{\log_{e}^{2}} \left(\frac{\gamma(\bar{H}) - \gamma(\bar{H}/2)}{\pi \gamma(\bar{H})} \right)^{4} .$$
 (26)

These quantities are evaluated for both $\gamma_{1}(H)$ and $\gamma_{2}(H)$.

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Note that the calculation of $\gamma(2\overline{H})$ may require the provision of an uneconomic amount of step response. Likewise, the user should beware of suggesting too short a trial gust length or of occasions where \overline{H} is so short that $\overline{H} < 2v\Delta t$. The programs will ignore trial gust lengths $< v\Delta t$ but will attempt to determine $\gamma(\overline{H}/2)$ in similar circumstances so that λ may be evaluated. Note that for ramp gusts of very short gradient distance (straight or smooth) we have the asymptotic result

$$\phi(H,t) \sim H^3 F(t)$$
 as $H \to 0$. (27)

Thus a unit step gust can effectively be regarded as being equivalent to a ramp gust of length 1 ft. However, the programs do not exploit this feature, ie the maximum and minimum of F(t) are not determined.

7 GUST PAIRS

Having calculated $\overline{\gamma}_{+}$, \overline{t}_{+} , \overline{H}_{+} , $\overline{\gamma}_{-}$, \overline{t}_{-} and \overline{H}_{-} , we identify the primary peak with the larger of $\overline{\gamma}_{+}$ and $|\overline{\gamma}_{-}|$, the smaller becoming the worst overswing. We then deduce the most adverse gust pair situation as follows.

The maximum possible response is $(\bar{\gamma}_{+} + |\bar{\gamma}_{-}|)$ occurring at \bar{t}_{+} or \bar{t}_{-} whichever is the later. Let us assume for clarity that $\bar{t}_{-} > \bar{t}_{+}$. Then, as noted in section 2, the worst gust pair is a ramp gust of length \bar{H}_{-} followed by one of opposite sign of length \bar{H}_{+} , separated by H_{s} where (see Fig 2)

$$H_{g} = v(t_{-} - t_{+}) - H_{-}$$
 (28)

It is conceivable that $H_s < 0$ will result but again we leave aside the interpretation of such a situation.

8 ACCURACY

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As stated at the beginning of this paper, we have tackled a specific mathematical problem leaving it to others to assess the results and the validity of the models used. The interaction with our computer programs really only occurs through the specification of $\Delta H/\bar{H}$. The fixing of this parameter depends primarily on one's assessment of the accuracy of the discrete ramp model. Having made such an appraisal it is then up to the user to ensure that the step response is sufficiently well represented for the desired accuracy to be attained for it must be remembered that the numerical scheme has a finite accuracy and spline fitting, integrating and interpolating are all sources of error. The question then is - how close is the problem which the program has accurately solved to

that which one has tried to specify? In a practical situation the only insight available is by rerunning the program with fewer points defining the step response and comparing results. If they are too different then a finer tabulation of F(t) is probably necessary.

Both the straight ramp and smooth ramp programs have been tested on a two parameter family of analytic step responses suggested by Jones and which may be written as

$$F(t) = e^{-\xi t} \left(\cos \Omega t + \frac{(\alpha - 1)\xi}{\Omega} \sin \Omega t \right)$$
(29)

where $\Omega^2 = 1 - \xi^2$; $\alpha \ge 1$; $0 < \xi < 1$.

With $\Delta t = 0.5$ and v = 100, \overline{H} and \overline{t} were, with the smooth ramp program always obtained to within 5% while $\overline{\gamma}$ was even better (<1%). However, the dominant factor affecting numerical accuracy is the density of tabulated points relative to the scale on which F varies. The period of the above function exceeds 2 π by definition of Ω . $\Delta t = 0.5$ is thus not a large spacing by any means. Tests have suggested that the straight ramp program is relatively more accurate probably because of the need to spline fit $\phi(t)$ in the smooth ramp program.

Normally, F(t) would be obtained by solving a set of differential equations in which case Δt would automatically reflect the variability of F so that hopefully no appreciable loss of detail need occur in feeding F(t) to the ramp gust programs. A possible snag is that differential equation solvers tend to vary the step size thereby tabulating F(t) in a manner unacceptable to the smooth ramp program which requires equally spaced points.

In summary, we are confident that <1% error in H, t and γ is attainable on an ICL 1900 series computer for arbitrary F(t), providing F(t) is well tabulated (*ie* we expect the numerical methods to be able to handle many more than the 200 data points currently allowed by the dimensions of arrays). Obviously, precision will suffer when the programs are run on a machine possessing lower floating point accuracy and double precision working may then become compulsory.

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9 USING THE PROGRAMS

There should be little difficulty implementing the programs because they have been written in standard FORTRAN and the data required is minimal. The following quantities need to be input on channel 3.

- (a) TMAX the time of the last point in the step response. Set negative if analytic F(t) (programmed in FUNCTION FUNC(T)) is to be used.
- (b) TINT the time interval at which F(t) is to be calculated or supplied on channel 1. In the smooth ramp program, TINT is necessarily also the interval at which the ramp response is evaluated. If F(t) is supplied in tabular form to the straight ramp program then TINT is not read and the time points are expected on channel 1 and can be arbitrarily spaced. This facility will probably be little used (but see section 8) because the smooth ramp model is preferred and the same step response data cannot be used immediately.
- (c) VELY the speed of the aircraft in ft/s.
- (d) DT the time interval at which the straight ramp response is to be calculated. Not read in the smooth ramp program because DT = TINT necessarily.
- (e) NH the number of trial values of H supplied (NH ≥ 3)
- (f) H(I), I = 1,NH trial gust lengths. H to be bracketed; $t_{max} \ge H/v \ge \Delta t$ preferably (see section 5).
- (g) HLTOL accuracy required, $\Delta H/H$.

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- (h) IPR print control parameter giving different levels of output on channel 2 as follows
 - IPR = -1 data summary; γ and t_{γ} for trial H(I) and \overline{H} ; λ ; λ_{w} ; worst gust pair:
 - = 0 as IPR = 1 plus γ and t_{γ} for all extra gust lengths investigated including $\frac{1}{2}\overline{H}$ and $2\overline{H}$. Also the integrals in (9) or(10) are output at intervals of Δt for $H = \overline{H}$ and $\frac{1}{2}\overline{H}$.
 - = 1 as IPR = 0 plus the tabulated spline fit to the step response.
 - = 2 as IPR = 0 plus ramp responses for each H(I) supplied by the user. The multiplying factor $vH^{-2/3}$ or $\frac{\pi}{2}vH^{-2/3}$ is also output for converting the integrals to $\phi(H,t)$.
 - = 3 as IPR = 1 and IPR = 2 combined.
- (i) IANAL Set to zero if subroutine ANALYTIC to evaluate $\phi(H,t)$ analytically is either not provided or not being used. Otherwise, input any non-zero integer value.

This completes the channel 3 data. If TMAX > 0 then the step response must be tabulated on channel 1. Both programs currently include coding for F(t) given by (29) and the straight ramp version can determine $\phi(H,t)$ by integrating

analytically. They expect α and ξ to be input immediately after TMAX (< 0). Figs 9 to 12 typify the computer output (IPR = 3). The straight ramp program produces similar output except that the columns of the spline fit headed D2F/DT2 and D3F/DT3 are absent because they are not needed.

10 CONCLUSIONS

At the request of J.G. Jones of Flight Systems Department, RAE, Bedford, two FORTRAN programs have been written to facilitate the identification of the discrete gusts which produce the greatest aircraft response. The input to the programs is the calculated response as a function of time to a unit step gust. From this one can determine the behaviour on encountering a gust whose intensity profile is of ramp form. The process is in principle quite straightforward but converting the theory into an algorithm suitable for a computer required considerable care. Our declared aim was to reduce to negligible proportions the numerical errors accompanying the implementation so that consistent results can be obtained. This has been achieved at very moderate extra expense in terms of computational effort and thus seems well worthwhile.

The programs are very easy to use and have been applied at RAE, Bedford, to realistic step response profiles with satisfactory results although double precision working was needed because of the shorter wordlength of the SIGMA computer.

REFERENCES

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Fig 2 Critical gust pair

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Fig 3a&b Numerical approximation of a (smooth) ramp gust. The curves 1, 2, 3 are then added together to get $\phi(H,t)$. These diagrams are schematic; the actual computation is considerably more sophisticated (see text)

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Fig 4



Fig 6 Variation of extreme response with length of gust. Defining γ_{+} to be the envelope of the above curves could cause trouble, *eg* the worst gust pair would be wrongly predicted because the peak at \overline{H}_{-} would be ignored. Alternatively, because γ_{+} becomes double humped, \overline{H}_{+} might be missed

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Fig 7a&b



a Step response having the properties (i) $t_{03} - t_{02} > t_{01}$

(ii) area A3 > area A1 (A3b = A1)



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Fig 7a&b Showing possible discontinuous behaviour of $t_{\gamma}(H)$



AIRCRAFT RESPONSE TO SMOUTH RAMP GUSTS (A.G. PURCELL APRIL 1976) ANALYTIC STEP RESPONSE DEFINED BY ALPHA = 5.00 AND XI = 0.50 AIRCRAFT VELOCITY = 100. FT/SEC RAMP RESPONSE CALCULATED AT 0.20 SEC INTERVALS UP TO T = 10.00 SECS GUSTS OF LENGTH H < 20.0 FT ARE EFFECTIVELY STEPS SUCH H(I) IN THE DATA ARE IGNORED

Fig 9 Title page of output from the smooth ramp response program

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Fig 9

CUBIC SPLINE FIT TO STEP RESPONSE

$\begin{array}{cccccccccccccccccccccccccccccccccccc$	28E 00 28E 00 63E 00 59E 00 15E 00 15E 00 41E 00 84E 00 84E 00 87E-01 41E-01 55E-01
2.0000E-01 1.2514E U0 1.Uc15E 0U -2.2699E U0 1.30 4.0000E-01 1.4121E UU 5.9355E-01 -2.0093E UU 1.47 6.0000E-01 1.4926E UU 2.2122E-01 -1.7140E UU 1.50 8.0000E-01 1.5045E UU -9.1469E-U2 -1.4129E UU 1.49 1.0000E 00 1.460UE UU -3.4421E-01 -1.1146E UC 1.42 1.2000E 00 1.3707E UU -5.3869E-01 -8.3023E-U1 1.31 1.4000E 00 1.2481E UU -6.7845E-01 -5.6741E-U1 1.17 1.600UE 00 1.1027E UU -7.6837E-01 -3.3173E-U1 1.U2	28E 00 63E 00 59E 00 15E 00 16E 00 41E 00 84E 00 84E 00 87E-01 41E-01 55E-01
4.0000E-01 1.4121E UU 5.9355E-01 -2.0093E UU 1.47 6.0000E-01 1.4926E UU 2.4122E-01 -1.7140E UU 1.50 8.0000E-01 1.5045E UU -9.1469E-U2 -1.4129E UU 1.49 1.0000E 00 1.460UE UU -3.4421E-01 -1.1146E UC 1.42 1.2000E 00 1.3707E UU -5.3869E-01 -8.3023E-U1 1.31 1.4000E 00 1.2481E UU -6.7845E-01 -5.6741E-U1 1.17 1.600UE 00 1.1027E UU -7.6837E-01 -3.3173E-U1 1.U2	63E 00 59E 00 15E 00 16E 00 41E 00 84E 00 87E-01 41E-01 55E-01
6.0000E-01 1.4926E 00 2.4122E-01 -1.7140E 00 1.50 8.0000E-01 1.5045E 00 -9.1469E-02 -1.4129E 00 1.49 1.0000E 00 1.4600E 00 -3.4421E-01 -1.1146E 00 1.42 1.2000E 00 1.3707E 00 -5.3869E-01 -8.3023E-01 1.31 1.4000E 00 1.2481E 00 -6.7845E-01 -5.6741E-01 1.17 1.6000E 00 1.1027E 00 -7.6837E-01 -3.3173E-01 1.02	59E 00 15E 00 16E 00 41E 00 84E 00 47E 00 87E-01 41E-01 55E-01
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1.2000E 00 1.3707E 00 -5.3869E-01 -8.3023E-01 1.31 1.4000E 00 1.2481E 00 -6.7845E-01 -5.6741E-01 1.17 1.6000E 00 1.1027E 00 -7.6837E-01 -3.3173E-01 1.02	41E 00 84E 00 47E 00 87E-01 41E-01 55E-01
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1.6000E 00 1.1027E 00 -7.6837E-01 -5.3173E-01 1.02	47E 00 87E-01 41E-01 55E-01
	87E-01 41E-01 55E-01
1.8000E 00 9.4572E=01 -8.1422E=01 -1.2679E=01 8.61	41E-01
2 UDDDE 00 7. 1949E-U1 -8. 434E-01 4.5588E-U2 6.97	55E-01
2 2000F 00 6.1687F-01 -7 9927E-01 1.8507F-01 5 3/	
2 4000F 00 4 6143F-01 -7 5151E-01 2.9258E-01 5 87	256-01
2 6000F 00 3 1750F-01 -6 8525E-01 5.7003E-01 2 50	20F-01
2 8000F 00 1 8818F-01 -6 0624E-01 4 2007E-01 1 28	96F-01
3 0000F 00 7 5509F-02 -5 1964F-01 4 4586F-01 2 50	186-02
1 2000 00 - 1 96695-02 - 6 29975-01 - 6 0875-01 - 6 09	A56-02
3.6000000 = 0.0 = 0.000000000000000000000	175-01
3.40000000 = 1.5010000000000000000000000000000000000	5/ 5-01
3,000000000000000000000000000000000000	725-04
3.000000000000000000000000000000000000	*05-04
	075-01
	U76-01
4 ADDE DD -2,45012-01 6,40252 05 2,51122-01 2,45	775-04
2 + 3000 = 00 = 2 + 585 = 01 = 3 + 100 = 02 = 1 + 105 = -01 = 2 + 585 = -01 = 2 + 585 = -01 = 2 + 585 = -02 = -0	075-04
4,00000000 -2,200000000000000000000000000	SPE-01
5,00000000 -1,83/10-01 -1,077001 -5,00070-02 -1,95	6/5-01
5 4000E 00 -1 5751E-01 1 4/10E-01 2 4040E-02 -1 44	125-01
5 4000E 00 -1 50/9E=01 1 3424E=01 -5 9245E=08 -1 17	495-01
5 80006 00 -1 04185-01 1 51005-01 -2 73505-02 -9 11	50E-07
3.00000 = 0.0 = 7.86785 = 0.7 = 7.3805 = 0.1 = 6.5835 = 0.2 = 6.63	VOE-02
	205-02
4 40000 00 -3 34345002 4 007500 -5 8001000 -3 34345002 -1 35	846-02
4 40000 00 -3.34340 0 -3.34400 0 -3.344000 0 -3.344000 0 -3.344000 0 -3.344000 0 -3.344000 0 -3.344000 0 -3.344000 0 -3.344000 0 -3.344000 0 -3.344000 0 -3.344000 0 -3.344000 0 -3.344000 0 -3.344000 0 -3.344000 0 -3.344000 0 -3.3440000 0 -3.344000000 0 -3.3440000000000000000000000000000000000	55E-01
	ODE-DE
7 0000 00 1 41755-02 5 75715-02 -7 19385-02 1 96	43E-02
7 20005 00 2 42/25-02 4 45/85-02 -6 80005-02 2 84	UOF-02
7 40005 00 5 16715-02 3 05464-02 -6 23185-02 5 45	1×F=02
7 60005 00 3 65805-02 1 87735-02 -5 54145-02 5 82	125-02
7 8000F 00 3 92/7F-02 8 4554F-03 -4 7760F-02 3 99	63E-02
8 0000F 00 4 0066F-02 -2 9733E-04 -3 9767E-02 3 99	04E-02
8 2000F 00 3 9264F-02 -7 4527E-03 -3 1786F-02 3.84	13E-02
8 4000F 00 5 7189F-04 -1 5042E-02 -2 4104E-02 5.58	04E-02
8.6000F 00 3.414/F-02 -1.7146E-02 -1.6943E-02 3.23	17E-02
8.8000E 00 5.0444E-04 -1.9887E-02 -1.0468E-02 2.85	94E-02
9.0000E 00 2.6273E-06 -2.1413E-02 -4.7888E-03 2.41	31E-02
9.2000E 00 2.1947E-04 -2.1888E-02 3.7294E-05 1.96	81E-02
9.4000E 00 1./5/6E-UZ -2.1487E-02 3.9735E-U3 1.56	53E-02
9.6000E 00 1.33/9E-02 -2.0379E-02 7.1040E-03 1.05	126-02
9.8000E 00 9.4591E-U3 -1.8747E-02 9.2184E-US 1.05	126-02
1.0000E 01 5.9081E-03 -1.6692E-02 1.1333E-02 1.05	

Fig 10 Spline fit to F(t). Output only if IPR = 1 or 3

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Fig 10

C(T) [H4 687 10 18.372145 2 (MULTIPLY 5 0.07 LENGTH 5 GUST ANAS 1100MS 20 RESPONSE

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1.6613E-01 -4.7626E-02 -1.9219E-02 6.0091E-03 1.8912E-03 1.9019E-U1 -1.9137E-U2 -2.3656E-02 2.3656E-03 2,2753E-U1 6,1202E-U3 -5,1516E-U2 2,8947E-U2 3,9253E-U3 <.3049E-U1 2.39U6E-01 2.3744E-U1 0.408UE-U2 4.2612E-U2 2.3UU5E-U2 -88834E-02 4.3512E-U2 4.3UU5E-U2 -4.1059E-U2 1.10U3/E-U3 5.7478E-03 5.2193E-U5 4.398UE-U3 1.60385-U1 4.4V035-V2 0. 1.1.16406-U1 8.0/745-V2 0. 2 -3.72375-V4 -3.88485-V2 -3. 2 -1.10045-V4 -7.42135-V3 -4. 2 -1.10045-V3 0.15905-V3 -4. 0000E 00 4042E-01 3578E-02 5037E-02 3206E-02 3206E-03 -----

C(1) 1H4 667 10 8Y 11.573726 CMULTIPLY -0.05 LENGTH -6UST RAMP SMOOTH 20 RESPONSE

5.6234E-01 -4.4671E-02 -4.5760E-02 1.1215E-02 4.3975E-03 1 4.4285E-01 4.0606E-01 2 1.6711E-US -2.4292E-02 -2 0.130E-UC -5.2106E-UZ -3 0.8771E-03 9.4500E-03 5 7.5234E-03 9.4500E-03 4.07506-01 5.55086-02 -0.72046-02 5.43116-02 8.68986-03 8.68986-03 3.4748F-U1 4.3531E-U1 4.6751E-U1 4.7696E-U1 4.0943E-U1 1.5945E-U1 1.127E-U1 7.0480E-U2 -/.2754E-U2 1.7749E-U2 1.16593E-U2 -7.2879E-U2 -1.2592E-U2 -1.2392E-U2 -6.2140E-U3 9.9728E-U3 1.4540E-U2 1.1986E-U2 1.1100E-U2 9.9728E-03 1.20616-U1 2.61226-U1 -6.99226-U2 -2.70936-U2 0000E 00 1296E-01 9741E-02 5297E-02 2255E-02 3293E-03 -----

C(T) IH4 GET 10 7. 490991 ž CMULTIPLY -100.0 LENGTH 10 GUST RAMP SMOOTH 2 RESPONSE

6.4523E-01 -5.1675E-02 -1.0794E-01 1.7038E-02 1.2596E-02 8.8942E-01 2.3119E-02 -1.2310E-02 1.1413E-02 9. (8586-01 6.9046-02 -1.56196-02 4.05496-03 9.5541E-U1 1.6559E-U1 -1.4616E-U1 -5.1449E-U5 -2.0368E-U2 7.6835E-U1 9.U248E-U1 3.4599E-U1 2.5100E-01 1-1.5218E-U1 -1.51308E-01 1-2.8848E-02 -1.0136E-U2 2.2401E-U2 2.2401E-U2 5.2346-U1 4.4576E-01 1-1.4584E-01 -4.3104E-02 -4.3104E-04 2.58/36-01 5.4/956-01 -1.51016-01 -5.86096-02 2.4730E-UC 6.6446E-U2 6.4858E-U1 6.4858E-U1 -1.0846E-U1 -7.4903E-U2 2.3534E-U2 0.0000E 00 7.42865-01 -7.53926-02 -9.16295-02 2.10275-02 9.91665-03

((1) IH4 667 2 4. 593036 > CHULTIPLY -200.0 LENGTH -GUST RANP SHOOTH 2 RESPONSE

1.7172E 00 2.8750E-01 -2.7012E-01 -7.0001E-03 5.7383E-02 3.2410E-01 5.6609E-U1 8.4765E-U1 1.1362E U0 1.5987E 0U 1.6021E U0 1.3609E 0U 1.1866E UU 9.9468E-U1 8.0413E-U1 0.1799E-01 4.4576E-U1 -1.5014E-01 -2.0942E-01 -2.4942E-01 -2.4415E-U1 0.126202 -6.5204E-01 -1.66436E-01 -1.3595E-U1 -1.0505E-U1 -7.0445E-U2 -4.4307E-02 -6.7013E-02 5.9980E-02 4.4291E-U2 4.0220E-U2 4.0129E-U2 4.4307E-02 4.1398E-02 3.4024E-U2 1.4224E-U1 1.6428E UU 1.5177E UU 1.2516E-U2 -1.1567E-U2 -1.2.2559E-U1 -1.9645E-U1 -3.2.2911E-U2 5.2451E-U1 00006 00 72306 00 47546-01 50616-01 63786-03 OFFNOM

C(T) 1H4 2.893432 70 661 -(MULTIPLY 0.004 LENGTH 10 6457 ANAS SHOOTH 20 RESPONSE

1.4574E 00 4.1645E 00 -1.6496E-01 -4.7475E-01 5.3458E-02 1.191/E 00 6.3450E 00 15.5658E-02 13.1988E-01 9.4300E-01 2.4640E 00 1.3000E-01 45.6045E-01 7. 4416E-01 6. 5168E 00 5. 2655E-01 -5. 4454E-01 -2. 3883E-03 4.90866-01 2.51116 UU 5.52246-01 -4.1836-01 3,11136-01 4 2,44386 40 2 8,43516-01 5 8,43516-01 5 -4,29166-01 -4 1.74/16-01 2.32446 00 1.07446 00 -4.23466-01 7.28685-02 2.13365-02 1.55905-00 -1.55905-01 1.71765-02 1.94665 00 1.64615 00 -3.48915-01 0.0000E 00 1.7109E 00 1.9240E 00 -2.7379E-01 -2.2804E-01

Fig 11 Values of the integral in equation (10) at intervals of $\Delta t = 0.2$ Output if IPR = 2 or 3

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С-) Т	7.17546-01 4.49	9.0004E-01 4.64	1.11406 00 4.68	1.30746 00 5.46	1.2426E 00 6.76		petelocratel	nanelod lan lin	Output unle		0.0099	H+ 7+ E
T(+)	8.6373E-U1 -	9.91516-01 -	1.2555E UD -	1.9088E UU -	3.1257E 00 -	2.0425E 00	2.1483E 00	2.1764E 00	2.1355E 00	2.1604E 00	RED ACCURACY -	2.1483E UD
(+) ANNA (+)	4.40116 00	5.5207E 00	6.8271E UU	7.96116 00	7.3005E UD	8.0111E UD	8.0245E UO	8.U227E 00	8.U243E 00	8.U239E UD	ERION SATISFIED REQUI	8.0245E 00
IC LENGTH	0 25.00	0 50.00	0 100.00	00.002 0	00.00* 0	0 219.15	0 233.61	0 237.47	0 231.31	0 235.52	ERGENCE CRITI	5 233.61
-											ANOS	÷

4.141138 TO GET PHI(T)) CAULTIPLY BY RESPONSE TO SMOOTH RAMP GUST OF LENGTH 233.6 FT

1.78796 00	5.0645E-01	-5.18416-01	-3.09.485-02	4.67458-02	
1.5889E 00	1.0267E-01	-3.2034E-01	-5.7231E-02	5.01416-02	
1.2346E 00	9.1154E-01	-5. U9 UUE-U1	-8.6831E-02	>. c076E-UC	
1.US10E UD	1.12596 00	-4.0127E-U1	-1.19156-01	5.2141E-UL	
7.6498E-01	1.3384E 00	-2.34986-01	-1,5337E-01	4, 49196-02	
5.0179E-01	1.53886 00	-1.68U8E-01	-1.8840E-01	4.5003E-02	
2.8271E-01	1.715/E 00	-1.89448-02	-4.228/E-01	3.7011E-04	IT
1.42796-01	1.8560E UO	3.34096-02	-2.5513E-U1	2.561/E-UL	'¥'
2.91956-02	1.9340E 00	1.69166-01	-2.83486-01	1.05/06-04	
0.0000E 00	1.9087E 00	3.27516-01	-3.0515E-01	-8.2752E-03	4.2276E-02

6.573647 TO GET PHI(T)) CMULTIPLY BY RESPONSE TO SMOOTH RAMP GUST OF LENGTH 116.8 FT

0 1.0591E 00 9.9440E-01 1 6.05/7E-02 -8.694E-03 1 -1.4995E-01 -1.5337E-01 6 9.5156E-02 1.6955E-02 4 1.9170E-02 1.6046E-02 0.0000E 00 5.73735-U2 2.2915E-U1 4.8378E-U1 7.579UE-U1 9.7545E-U1 1.U718E 00 1.U84CE 00 9.0663E-01 8.0298E-U1 6.8V11E-U1 5.70Y6E-U1 4.5304E-U1 5.4117E-U1 2.5676E-U1 1.4272E-01 -6.5406E-02 -1.0934E-U1 1.4134E-U1 -1.6244E-U1 -1.73805E-U1 -1.7691E-01 -1.6594E-01 -1.1497E-01 -9.5757E-U2 -1.494E-U2 -5.8084E-02 -4.0836E-U2 -2.574E-02 -1.1603E-U2 8.9805E-06 2.2434E-02 2.6108E-U2 2.8173E-U2 2.8846E-02 2.8355E-U2 2.6928E-02 2.4787E-U2 2.4138E-02 1.2910E-02

1.3398E 00 3.4562E 00 1,1397E UU 6,8627E UU 116.81 ..

LAMBDA . U.291 0.210 . LAMBDA (V)

Results page Fig 12

from equations (26) and (25)

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lated estimates of \vec{H}_{-} . unless IPR = -1 .	, E	<pre>.6335E 00 1.52/4E 00 1.7679E 0 .6331E vv 9.9944E-01 7.7146E-0 .v744E-01 -5.3844E-01 -5.3119E-0 .50/8E-01 -9.5346E-02 -6.2892E-0 .1144E-04 5.7007E-02 5.4957E-0 (1)</pre>	.4244 UU 1.2095E UU 1.153E U .0725E-01 1.0842E-01 2.3583E-0 .8942E-01 -1.765UE-01 2.3583E-0 .6135E-U3 6.1775E-05 1.5603E-0 .0106E-02 2.3137E-02 1.9693E-0		t > ī,) (28)
Interpo	Н., Ţ., Т.	9.08/16-U1 1 1.46/86 UU 1 -4.57/06-U1 -3 -1.68488-U1 -1 -1.68488-U1 -1 3.40/46-U2 5 3.40/46-U2 5	1.16386 UD 1. 5.18626-U1 4. 1.97926-U1 4. 1.9705-U2 4. 2.91216-U2 4.		Max $(\overline{\gamma}_+, -\overline{\gamma})$ $\overline{\gamma}_+ - \overline{\gamma}$ \widetilde{H} (because $\overline{\gamma}_+$ \overline{H}_+ from equation
5.6856E UU 5.7975E UU 5.8970E UO 5.8657E 00 5.88196 00 5.8811E 00 5.8811E 00	5.6657E 00	6.94455-01 1.66/55 -01 -1.68/55 -01 -4.92075-01 4.92075-02	9.8607E-U1 4.414E-U1 -1.9896E-U1 -3.6182E-U2 5.1241E-U2	5.0718E 00 7.7568E 00	~ ~ ~ ~ ~
		4.5010E-01 1.8809E 00 -8.5854E-02 -2.4442E-01 4.0440E-02	7.2631E-U1 5.7081E-U1 -1.9146E-U1 -5.4654E-U2 3.2305E-U2	.03336 00	8.02455 6. 8.02455 9.36155 1. M1 = 265 1. M2 = 265 1. M2 = 233
FIED : 00//MBAR < 0		18455-V1 2.51245-01 V9846 VU 2.03445 UU 86325-V1 3.7425E-U2 10355-V1 -2.79425-U2 74765-U2 %Н	UJ63E-U1 4.458/E-U1 559/EE-U1 7.0414E-U1 4404E-U1 -1.7344E-U1 0445E-U2 -7.470UE-U2 U921E-U2 3.2398E-U2		TO AN ISOLATED GUST. Decurs wa to a pair of Gusts. RST GUST IS OF LENGTH OND GUST IS OF LENGTH PARATED BY A DISTANCE
57.69 55.06 70.85 65.91 68.29 68.35 68.35 68.35 68.15 68.15 68.15 68.15	65.91	0 2.56835-44 1. 0 4.06955 40 4. 1 5.59965-41 -1. 1 -3.54475-41 -3. 2 -9.20415-43 1. 2 9.20415-43 1.	0 5.06/5E-u2 4. 0 9.5999E-u1 4. 2 -1.0200E-u1 -1. 1 -1.1730E-u1 -9. 2 2.78442E-v2 3.	32.96 31.82 (u) • 0.258	ED VORST RESPONSE ED VORST RESPONSE CCURS VNEN THE F1 AND THE SEC THE THO BEING SE
C 000000000000000000000000000000000000	Te 6 2	0.0000E 0 5 5616E 0 5 5616E 0 5 1049E 0 5,1245E 0 5,1245E 0	0 00405 0 1 06926 0 4 67276-0 1 39176-0 1 39176-0 1 61466-0		ESTIMAT ESTIMAT MAICH 0

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Fig 12 (contd) Results page

Fig 12 (contd)