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DYNAMIC RESPONSE OF REINFORCED CONCRETE STRUCTURES.(U)
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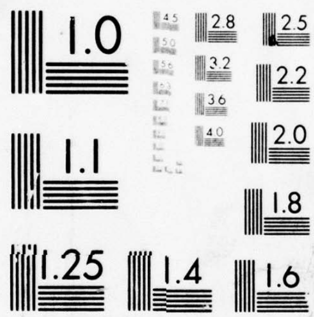

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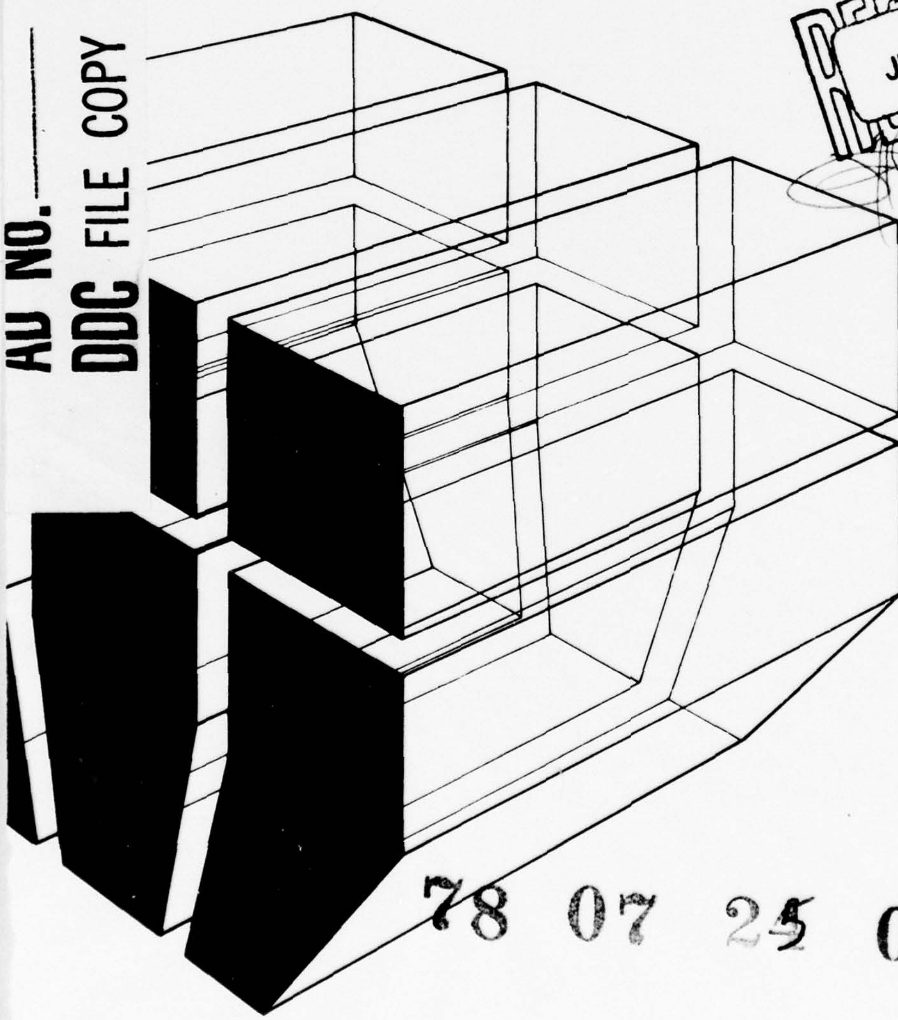
DYNAMIC RESPONSE OF REINFORCED CONCRETE STRUCTURES



by
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Beam-column and plane stress finite elements were described for an inelastic analysis of plane RC structures under earthquake-type ground motion. Material nonlinearities in the beam-column finite element were taken into account by considering cyclic inelastic deformations throughout the element. The plane stress finite element allowed for cracking of the element in tension. These elements were incorporated in the DRAIN-2D computer program, which was determined to be flexible and efficient. Preliminary results for simple structures showed that, with the addition of new finite elements described in this report,

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this program would be very useful for practical investigations of RC structures.

FOREWORD

This investigation was conducted for the Directorate of Military Construction, Office of the Chief of Engineers (OCE) as part of the RDT&E Army Program 6.27.19-A, Project 4A161102AT23, "Research in Military Engineering and Construction"; Task 02, "Analytical and Theoretical Studies of Complex Structural Systems"; Work Unit 004, "Energy Dissipation in Dynamically Loaded Structures." The OCE Technical Monitor was Mr. George Matsumura, DAEN-MCE-A.

The work was conducted by the Engineering and Materials Division (EM), U.S. Army Construction Engineering Research Laboratory (CERL). Dr. G. R. Williamson is Chief of EM. COL J. E. Hays is Commander and Director of CERL and Dr. L. R. Shaffer is Technical Director.

CONTENTS

DYNAMIC RESPONSE OF REINFORCED CONCRETE STRUCTURES

1 INTRODUCTION

Background

Reinforced concrete (RC) construction is one of the major types of construction used throughout the world. At present, the design and analysis of most reinforced concrete structures are based on linear elastic procedures, even though some of these structures will be subjected to loads in the inelastic range. Structures in severe earthquake zones, for example, may undergo many cycles of inelastic deformations during their service lives. Thus, there is a need for refined cyclic inelastic analyses of structures in order to develop a more economical, realistic approach and safer design procedures. Such analyses require knowledge of the behavior of structural materials under arbitrary complex cyclic loading.

Objective

The objective of this study is to develop criteria for determining damping coefficients and ductility factors in RC structures based on their energy dissipation characteristics.

The study is divided into three phases:

1. Development of a uniaxial model for the cyclic inelastic response of reinforced concrete.

2. Development of a beam-column finite element and a computer program for determining inelastic response for RC structures under seismic loads.

3. Extension of the computer program for estimating energy dissipation and equivalent damping coefficients and ductility factors.

The first phase of study has been completed and is described in CERL Technical Report M-180.[1] This report covers the work accomplished in the second phase.

[1]Sharma, S. K. and Bhattacharyya, R. K., *An Analytical Model for Uniaxial Cyclic Inelastic Behavior of Reinforced Concrete*, Technical Report M-180/ADA024910 (U.S. Army Construction Engineering Research Laboratory [CERL], 1976).

Approach

Beam-column and plane stress elements are developed using cubic Hermite polynomials. A constant acceleration method (a special case of Newmark's β method) is used in the solution procedure. The method assures a constant acceleration between time steps. The elements are incorporated in a commercially available computer program, DRAIN-2D, and a two-story, one-bay reinforced concrete structure is analyzed as an example problem.

Scope

The RC structures considered in this report are plane structures (two-dimensional) with inelastic beams and columns, and elastic shear walls. The shear walls may, however, fail in a brittle fracture mode. Structural elements which fail primarily by brittle fracture show a relatively linear elastic behavior up to the failure load. Hence there would be no significant increase in accuracy, but possibly a noticeable increase in computational time and cost, if the inelasticity of shear walls was also considered. The structures are idealized by beam-column (for beams and columns) and plane stress (for shear walls) finite elements.

Mode of Technology Transfer

The criteria developed in this study are to be incorporated into a seismic design manual, TM 5-809-10, *Seismic Designs for Buildings.*

2 BEAM-COLUMN AND PLANE STRESS FINITE ELEMENTS

The existing finite element computer programs for RC structures (e.g., SAKE, STRUDL, PRIEST, CONCRETE, etc.) use elastic beam-column elements with nonlinear moment-rotation springs at the ends. The properties of these nonlinear springs are chosen to simulate stiffness changes caused by cracking of the concrete, yielding of the reinforcement, and stress reversals.[2] Implicit is the assumption that inelastic deformations are concentrated only at the ends of beams and columns. However, this is too restrictive. In a real RC structure inelastic deformations can occur throughout the member. Thus, energy dissipation estimates based on these beam-column elements are not expected to be reliable. Consequently, a new beam-column finite element was developed which permits cyclic inelastic deformations at all points in the element.

The following assumptions are employed for the structure, prescribed forces, reinforcement, and beam-column and plane stress finite elements.

1. Only two-dimensional RC structures are considered.

2. Ground motion is parallel to the plane of the structure.

3. The structures are fixed at their bases on infinitely rigid foundations.

4. Mass of the floors is assumed to be concentrated at the member joints (nodal points).

5. Displacements and strains are assumed to be small.

6. Shear deformations in beams and columns are ignored.

7. There is no slip between the concrete and the reinforcing steel.

8. Kirchhoff's assumption, which states that normals to the centroidal axis remain straight and normal, is used.

9. Reinforcing steel is "smeared" in the concrete to obtain an equivalent tangent modulus.

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[2] Otani, S., *SAKE, A Computer Program for Inelastic Response of R/C Frames to Earthquakes*, Civil Engineers Studies, Structural Research Series No. 413 (University of Illinois at Urbana-Champaign, November 1974).

Beam-Column Element

Figure 1 shows a beam-column finite element. In the local coordinate system, the centroidal axis of the beam is on the x-axis from x = 0 to x = L. Displacements along x and y axes at any point x on the centroidal axis are denoted by u(x) and v(x), respectively. These displacement components are represented as

$$u = U_1 + U_2\psi$$
$$v = V_1\eta_1 + V_1'\eta_2 + V_2\eta_3 + V_2'\eta \qquad \text{[Eq 1]}$$

where

U_1, V_1 = displacement components at node 1

U_2, V_2 = displacement components at node 2

V_1', V_2' = derivatives with respect to x of v at nodes 1 and 2 respectively

ψ = a linear function of x defined in Appendix A

$\eta_1, \eta_2, \eta_3, \eta_4$ = cubic Hermite polynomials of x defined in Appendix A.

Axial strain at any point (x, y, z) of the beam is given by Sloane.[3]

$$\varepsilon_x(x,y,z) = \varepsilon_x(x,y,0) = u' - yv'' \qquad \text{[Eq 2]}$$

where

ε_x = strain at any point in the x direction

V',V" = derivatives with respect to x

therefore

$$\delta\varepsilon_x = \delta u' - y\delta v'' \qquad \text{[Eq 3]}$$

where δ denotes a virtual increment.

[3]Sloane, A., *Mechanics of Materials* (Dover Publications, Inc., 1952).

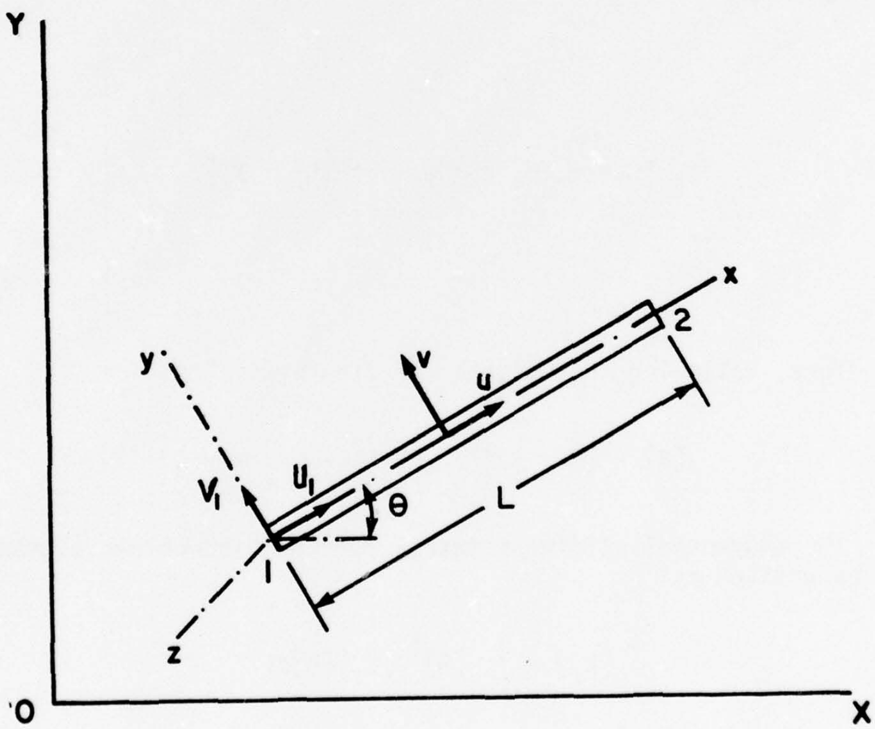

Figure 1. A beam-column finite element.

From Eq 3

$$\delta\varepsilon_x = \left[\frac{d}{dx'} \ - \frac{d^2}{dx^2} \right] \left[\frac{\delta u}{\delta v} \right] \qquad \text{[Eq 4]}$$

From Eqs 1 and 4

$$\delta\varepsilon_x = \left[\frac{d}{dx'} \ - \frac{d^2}{dx^2} \right] \begin{bmatrix} 1 & 0 & 0 & \psi & 0 & 0 \\ 0 & \eta_1 & \eta_2 & 0 & \eta_3 & \eta_4 \end{bmatrix} \begin{bmatrix} \delta U_1 \\ \delta V_1 \\ \delta V_1 \\ \delta U_2 \\ \delta V_2 \\ \delta V_2 \end{bmatrix}$$

which yields

$$\delta\varepsilon_x = [0 \ - y\eta_1'' \ - y\eta_2'' \ \psi' \ - y\eta_3'' \ - y\eta_4''] \begin{bmatrix} \delta U_1 \\ \delta V_1 \\ \delta V_1' \\ \delta U_2 \\ \delta V_2 \\ \delta V_2' \end{bmatrix} \qquad \text{[Eq 5]}$$

Therefore, following the notation of Zienkiewicz[4] yields

$$[B] = [0 \ - y\eta_1'' \ - y\eta_2'' \ \psi' \ - y\eta_3'' \ - y\eta_4''] \qquad \text{[Eq 6]}$$

The tangential stiffness matrix for the beam-column element may now be written as[5]

$$[k_T] = \int_V [B]^T \, [E_T][B]dv \qquad \text{[Eq 7]}$$

[4]Zienkiewicz, O. C., *The Finite Element Method in Engineering Science* (McGraw-Hill, 1971).
[5]Zienkiewicz.

where

$[k_T]$ = tangential stiffness matrix

$[B]^T$ = transpose of [B] defined by Eq 6

E^T = tangential modulus

V = volume of the beam-column element.

Assuming the breadth of the beam to be b_y at a distance y from the centroidal axis, Eq 7 may be written as

$$[k_T] = \int_0^L \int_{y_1}^{y_2} [B]^T [E_T][B]\, b_y dy dx \qquad \text{[Eq 8]}$$

The integral of Eq 8 is evaluated by numerical integration using Simpson's rule. A 5 x 5 matrix of integration points is chosen across the depth and length of the beam.

The tangent modulus at an integration point (x_i, y_i) is given by

$$E_T = \mu(x_i, y_i)\, E_s + \{1 - \mu(x_i, y_i)\}\, E_c \qquad \text{[Eq 9]}$$

where

$\mu(x_i, y_i)$ = reinforcement ratio (ratio of cross sections of reinforcement area to the total area) at (x_i, y_i)

E_s = tangent modulus of the reinforcing steel at (x_i, y_i)

E_c = tangent modulus of the concrete at (x_i, y_i).

The tangent moduli of the concrete and the reinforcing steel are obtained by their cyclic inelastic models as described in Sharma and Bhattacharyya.[6]

[6] Sharma, S. K. and Bhattacharyya, R. K., *An Analytical Model for Uniaxial Cyclic Inelastic Behavior of Reinforced Concrete*, Technical Report M-180/ADA024910 (CERL, 1976).

The tangential stiffness matrix in global coordinate system (X,Y,Z) is given as[7]

$$[K_T] = [\Gamma]^T[k_T][\Gamma] \qquad \text{[Eq 10]}$$

where

$[K_T]$ = tangential stiffness matrix in global coordinate system

$[\Gamma]$ = transformation matrix from local to global coordinate system

$[\Gamma]^T$ = transpose of $[\Gamma]$.

The transformation matrix $[\Gamma]$ is given in Appendix B.

Plane Stress Element

A plane stress finite element is used to discretize shear walls. Several highly accurate plane stress elements are available in the literature and one of these elements[8] has been selected. This plane stress element (Figure 2) is a triangular element with three nodes. Displacement components u and v within the element are written as

$$\begin{aligned} u &= \alpha_1 + \alpha_2 x + \alpha_3 y \\ v &= \alpha_4 + \alpha_5 x + \alpha_6 y \end{aligned} \qquad \text{[Eq 11]}$$

where (x,y) are local coordinates and $\alpha_1, \alpha_2, \ldots, \alpha_6$ are generalized nodal variables. Substituting nodal coordinates (X_i, y_i), i = 1, 2, 3 in Eq 11, $\alpha_1, \alpha_2, \ldots, \alpha_6$ can be expressed in terms of u_1, v_1, u_2, v_2, u_3 and v_3, where (u_i, v_i) denote the displacement components at node i, i = 1, 2, 3. This results in the following strain-displacement relation:[9]

[7]Zienkiewicz, O. C., *The Finite Element Method in Engineering Science* (McGraw-Hill, 1971).
[8]Zienkiewicz.
[9]Zienkiewicz.

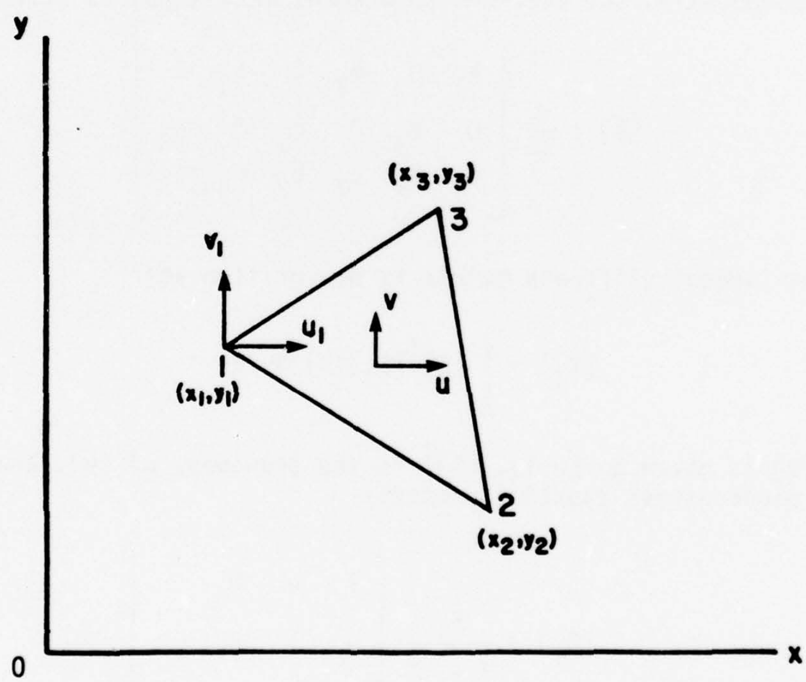

Figure 2. A plane stress finite element.

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \frac{1}{2\Delta} \begin{bmatrix} b_1 & 0 & b_2 & 0 & b_3 & 0 \\ 0 & c_1 & 0 & c_2 & 0 & c_3 \\ c_1 & b_1 & c_2 & b_2 & c_3 & b_3 \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \end{Bmatrix} \qquad \text{[Eq 12]}$$

where

$$b_1 = y_2 - y_3, b_2 = y_3 - y_1, b_3 = y_1 - y_2,$$

$$c_1 = x_3 - x_2, c_2 = x_1 - x_3, c_3 = x_2 - x_1$$

and

Δ = area of the triangle with nodes 1, 2, and 3.

Consequently, the strain-displacement matrix [B] is given by

$$[B] = \frac{1}{2\Delta} \begin{bmatrix} b_1 & 0 & b_2 & 0 & b_3 & 0 \\ 0 & c_1 & 0 & c_2 & 0 & c_3 \\ c_1 & b_1 & c_2 & b_2 & c_3 & b_3 \end{bmatrix} \qquad \text{[Eq 13]}$$

The tangent stiffness matrix is now written as[10]

$$[K_T] = \int_A [B]^T [D_T][B] \, dx \, dy \qquad \text{[Eq 14]}$$

where [B] is given by Eq 13, $[B]^T$ is the transpose of [B], and $[D_T]$ is the plane stress elasticity matrix:

$$[D_T] = \frac{E_T}{1 - \nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix}$$

[10]Zienkiewicz, O. C., *The Finite Element Method in Engineering Science* (McGraw-Hill, 1971).

where

E_T = Young's modulus of elasticity

ν = Poisson's ratio.

The plane stress element is assumed to be elastic, but may fail in tension (brittle failure). The failure of an element in tension is modeled by setting its elastic modulus E_T to zero once the principal tensile stress in the element reaches a prescribed value.

3 SOLUTION PROCEDURE

At any instant of time t, the incremental equation of equilibrium may be written as[11]

$$[\underline{M}]\{d\ddot{r}\} + [\underline{K}_t]\{dr\} = \{dP\} + \{C_r\} \qquad \text{[Eq 16]}$$

where

$[\underline{M}]$ = lumped mass matrix of the structure

$\{d\ddot{r}\}$ = increment of nodal acceleration matrix

$[\underline{K}_t]$ = structural tangential stiffness matrix at time t

$\{dP\}$ = increment of externally applied force matrix

$\{C_r\}$ = load correction matrix

$\{dr\}$ = increment of nodal displacement matrix

Eq 16 is solved by using a constant acceleration method (one of the special cases of the Newmark β method). This method assumes a constant acceleration between the time steps t and t+dt.

Therefore,

$$\{\ddot{r}\} = 1/2\ \{(\ddot{r}_t + \ddot{r}_{t+dt})\} \qquad \text{[Eq 17]}$$

where

$\{\ddot{r}\}$ = assumed constant acceleration between t and t+dt

$\{\ddot{r}_t\}$ = acceleration at time t

$\{\ddot{r}_{t+dt}\}$ = acceleration at time t+dt

[11] McNamara, J. F. and Sharma, S. K., *An Analytical Model for Determining Energy Dissipation in Dynamically Loaded Structures*, Technical Report M-165/ADA017040 (CERL, 1975).

Eq 17 is rewritten as

$$\{\ddot{r}\} = \left\{\ddot{r}_t + \frac{d\ddot{r}}{2}\right\} \qquad \text{[Eq 18]}$$

where

$$\{d\ddot{r}\} = \{\ddot{r}_{t+dt} - \ddot{r}_t\} \qquad \text{[Eq 19]}$$

Integration of Eq 18 yields

$$\{d\dot{r}\} = \left\{-2\dot{r}_t + \frac{2dr}{dt}\right\}$$

$$\{d\ddot{r}\} = \left\{-2\ddot{r}_t - \frac{4\dot{r}}{dt} + \frac{4dr}{(dt)^2}\right\} \qquad \text{[Eq 20]}$$

where

$\{\dot{r}_t\}$ = nodal velocity vector at time t

Substituting Eq 20 into Eq 16 yields

$$\left[\frac{4}{(dt)^2}\,[\underline{M}] + [\underline{K}_t]\right]\{dr\} = \{dP\} + \{C_r\} + [\underline{M}]\left\{2\ddot{r}_t + \frac{4\dot{r}_t}{dt}\right\} \qquad \text{[Eq 21]}$$

Eq 21 is solved for {dr}, which upon substitution in Eq 20 yields $\{d\dot{r}\}$ and $\{d\ddot{r}\}$. Thus, solving Eqs 21 and 20 successively for each time step yields time-history solutions for displacement, velocity, and acceleration of the structure.

4 RESULTS

DRAIN-2D

The beam-column and plane stress finite elements described in this report have been incorporated in a commercially available computer program, DRAIN-2D.[12] Other finite elements in the element library DRAIN-2D are (1) an elasto-plastic truss-element, (2) a beam element with plastic hinges at the ends, (3) a semi-rigid connection element, and (4) a shear panel element which has shear stiffness only. This program can be used for both static and dynamic (linear or nonlinear) analyses of plane frame structures. However, dynamic analyses are restricted to earthquake excitation in which all support points are assumed to move in phase. The earthquake excitation is defined by time histories of ground acceleration which may have both horizontal and vertical components. A two-story, one-bay reinforced concrete structure (Figure 3a) is analyzed by DRAIN-2D in the following example.

Example Problem

Figure 3b shows a typical cross-section of beams and columns of the structure. Hatched areas of the cross-section are steel reinforced. Table 1 gives the dimensions of these beams and columns and reinforcement ratios of the hatched areas. Dead weights on each story of the structure are 50 kips (22,680 kg).

The structure is subjected to a north-south component of the El Centro earthquake. The acceleration-time history of the earthquake (Figure 4) is used as the base acceleration for the structure.

Stress-strain response of the concrete and the reinforcing steel is predicted by their uniaxial cyclic models described in Sharma and Bhattacharyya. These models have been programmed into DRAIN-2D to obtain tangent moduli E_c and E_s (see Eq 9) at each time step of the numerical integration algorithm. A time step of 0.01 seconds is used for numerical integration, and results are printed out after every 10 steps.

The displacement-time response of node 5 (Figure 3a) is plotted in Figure 5. The maximum and minimum displacements for this node, which occur at 2.2 sec. and 2.7 sec., respectively, are 1.14 in. (29 mm) and 0.67 in. (17 mm). The maximum bending moment is 1414.48 kips-in. (16 297 000 kg-mm) which occurs at node 4 of member 5 (Figure 3a) at time 2.2 sec.

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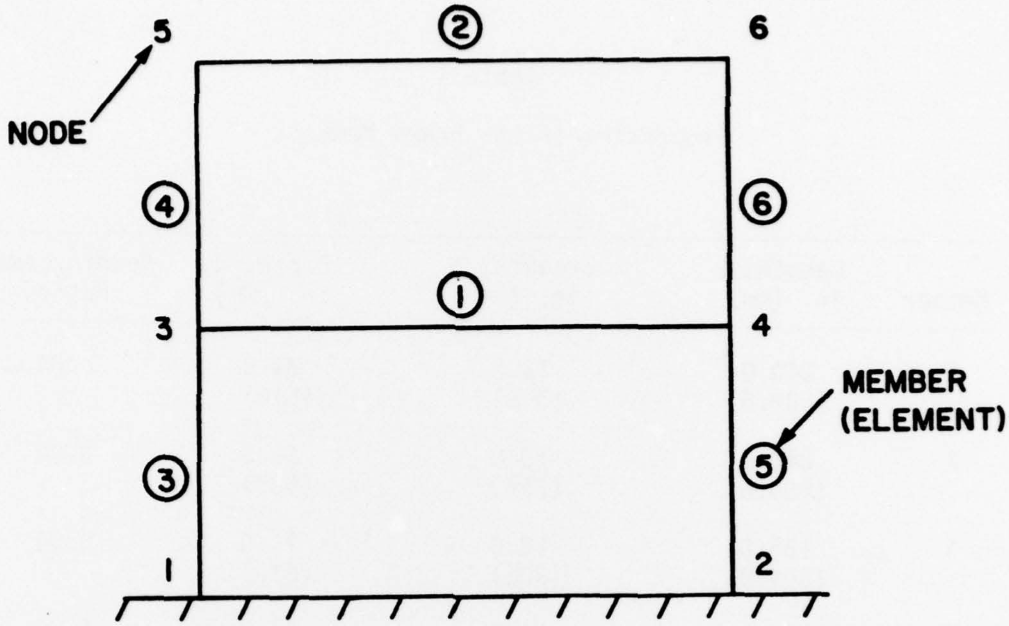
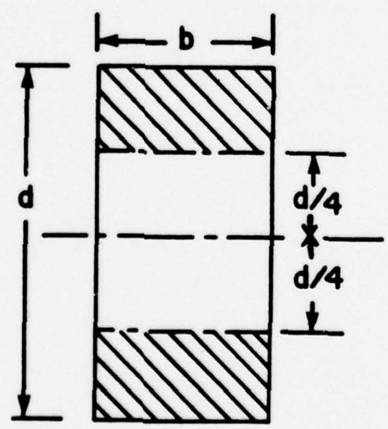
[12] Kanaan, A. E. and G. H. Powell, *DRAIN-2D, A General Purpose Computer Program for Dynamic Analysis Inelastic Plane Structures* (Earthquake Engineering Research Center, College of Engineering, University of California, Berkeley, April 1973).



(a) RC structure.



(b) Cross-section on the members.

Figure 3. Example problem.

Table 1

Properties of the Frame Members

Member	Length, L in. (mm)	Breadth, b in. (mm)	Depth, d in. (mm)	Reinforcement Ratio, μ
1	240.0 (609.6)	12.0 (305)	24.0 (610)	0.04
2	240.0 (609.6)	10.0 (254)	20.0 (508)	0.04
3	120.0 (304.8)	12.0 (305)	24.0 (610)	0.04
4	120.0 (304.8)	10.0 (254)	20.0 (508)	0.04
5	120.0 (304.8)	12.0 (305)	24.0 (610)	0.04
6	120.0 (304.8)	10.0 (254)	20.0 (508)	0.04

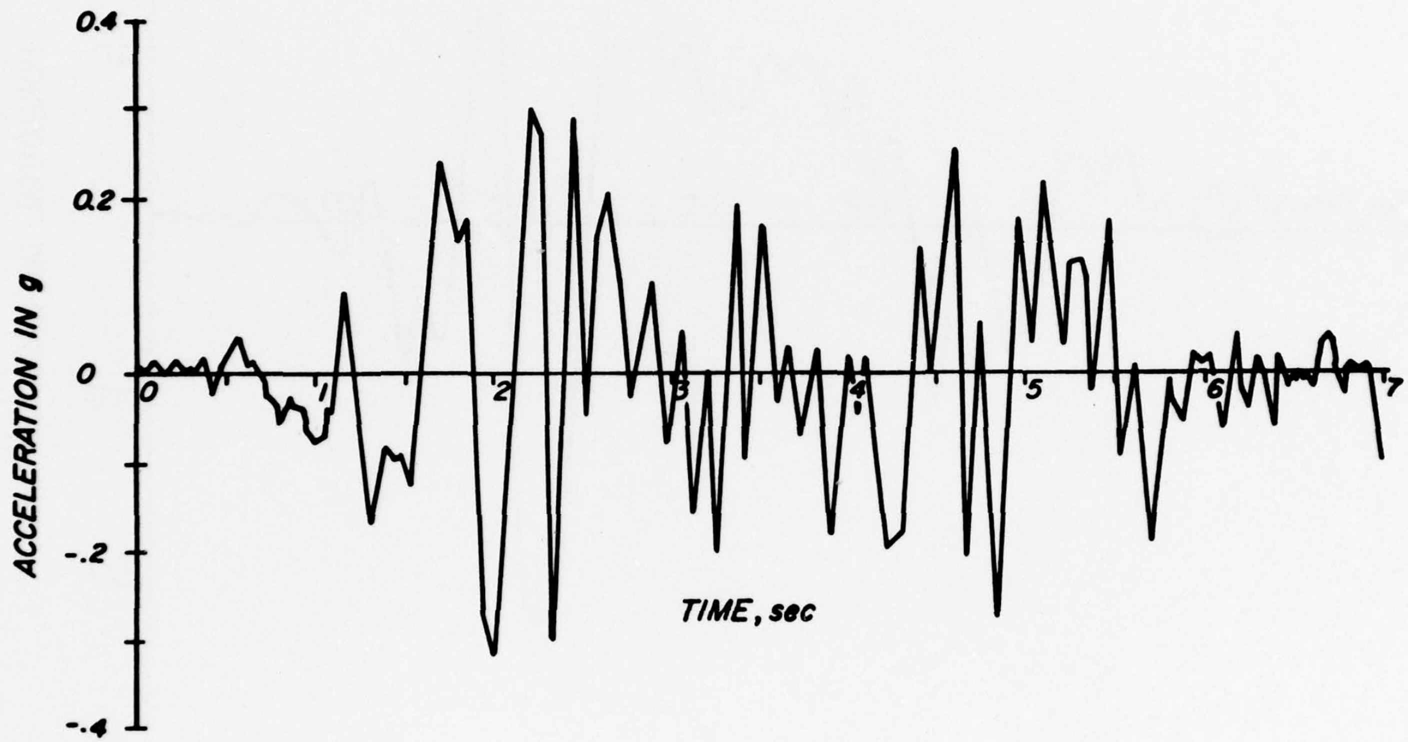


Figure 4. El Centro earthquake.

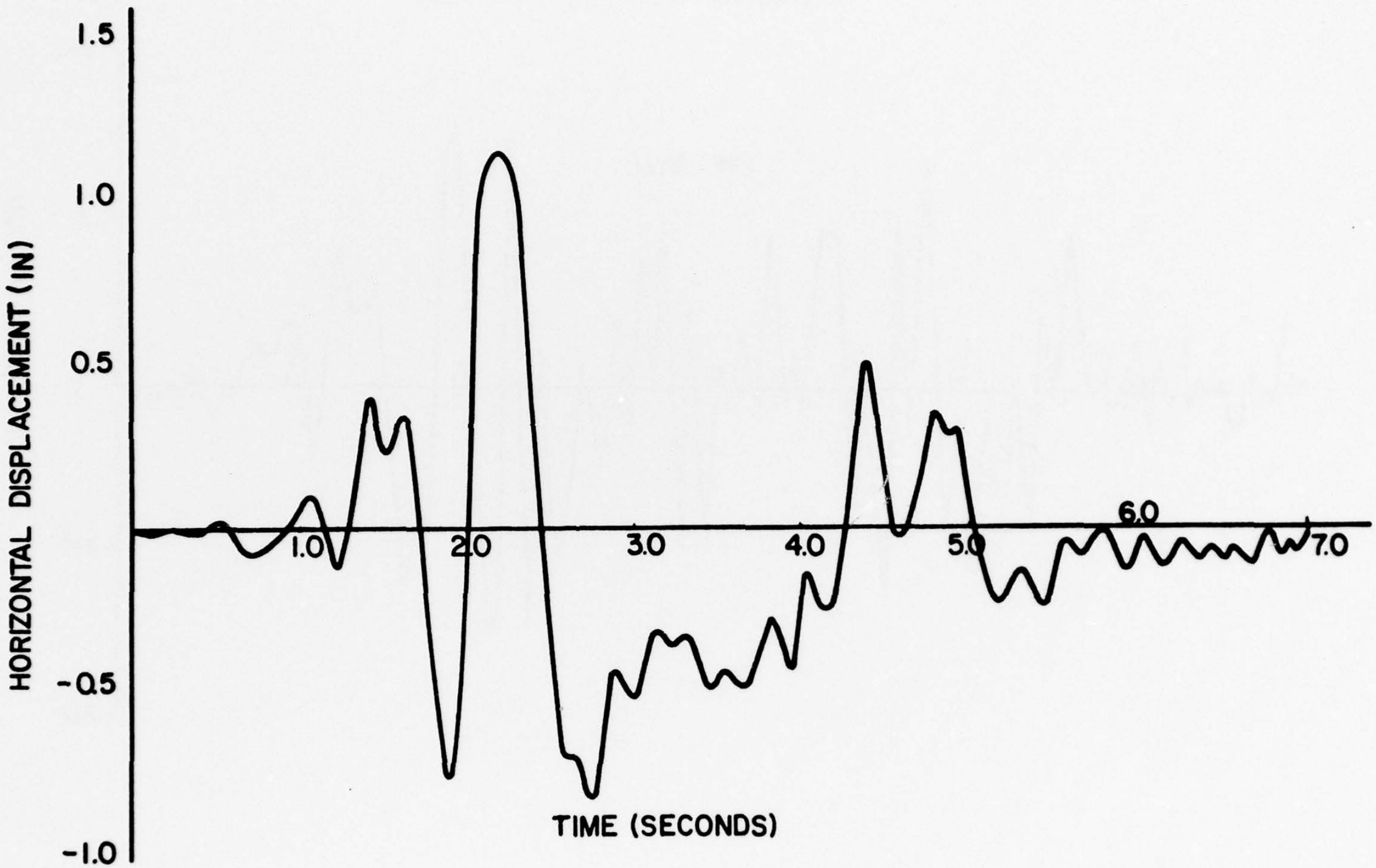


Figure 5. Horizontal displacement vs. time for node 5.

5 CONCLUSIONS

Beam-column and plane stress finite elements have been described for an inelastic analysis of plane RC structures under earthquake-type ground motion. Material nonlinearities in the beam-column finite element are taken into account by considering cyclic inelastic deformations throughout the element. The plane stress finite element allows for cracking of the element in tension. These elements have been incorporated in the DRAIN-2D computer program, which is both flexible and efficient. Preliminary results for simple structures show that, with the addition of new finite elements described in this report, this program will be very useful for practical investigations of RC structures.

In the third phase of this research project, this computer program is to be extended by adding new subroutines for the calculation of dissipated energy from increments of strains and stresses in the elements. Numerical estimates of dissipated energy will then be used in determining damping coefficients and ductility factors for RC structures.

The capability of RC structures to absorb energy through material damping and ductility contributes significantly to preventing catastrophic failure. This work will ultimately provide a scientific basis for estimating damping and/or ductility properties for the structural elements considered. Other elements may also be considered in the future, using similar analysis methods. Accounting for energy absorbtion with sufficient accuracy in this manner is recommended and will permit cost savings by minimizing design stress levels or load factors.

APPENDIX A:

FUNCTIONS ψ, η_1, η_2, η_3, and η_4

The functions ψ, η_1, η_2, η_3, and η_4 of Eq 1 are defined as follows

$$\psi = (U_2 - U_1)\, x/L$$

$$\eta_1 = c_1 + c_2 x + c_3 x^2 + c_4 x^3$$

$$\eta_2 = c_5 + c_6 x + c_7 x^2 + c_8 x^3$$

$$\eta_3 = c_9 + c_{10} x + c_{11} x^2 + c_{12} x^3$$

$$\eta_4 = c_{13} + c_{14} x + c_{15} x^2 + c_{16} x^3$$

where

$c_1 = 1$	$C_9 = 0$
$c_2 = 0$	$C_{10} = 0$
$c_3 = -3/L^2$	$C_{11} = 3/L^2$
$c_4 = 2/L^3$	$C_{12} = -2/L^3$
$c_5 = 0$	$C_{13} = 0$
$c_6 = 1$	$C_{14} = 0$
$c_7 = -2/L$	$C_{15} = -1/L$
$c_8 = 1/L^2$	$C_{16} = 1/L^2$

APPENDIX B:

TRANSFORMATION MATRIX [Γ]

The transformation matrix [Γ] is given by

$$[\Gamma] = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\theta & \sin\theta & 0 \\ 0 & 0 & 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

where θ is defined in Figure 1.

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Sharma, Sushil K
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