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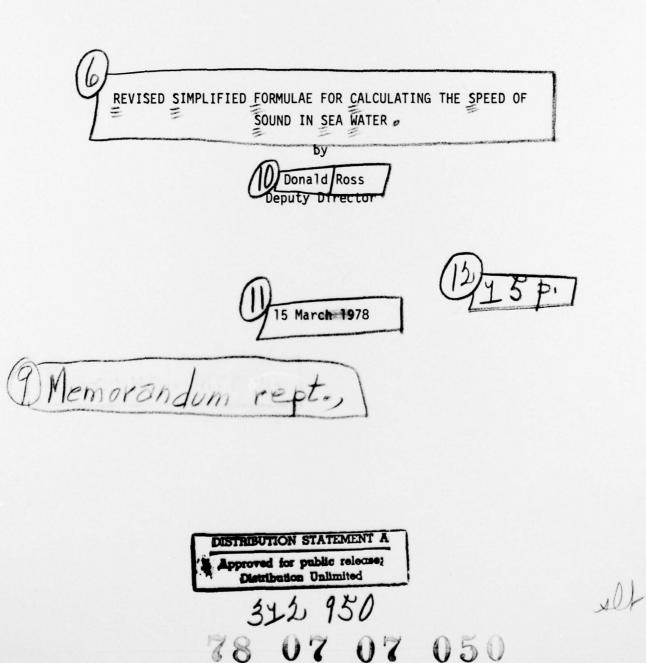


### SACLANTCEN MEMORANDUM SM-107

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## REVISED SIMPLIFIED FORMULAE FOR CALCULATING THE SPEED OF SOUND IN SEA WATER

### by

## Donald Ross

### ABSTRACT

Recently-published data on the speed of sound in water as a function of temperature, salinity, and pressure reveal significant differences from data used by Wilson in 1960 in the development of his sound speed formulae. Consequently the simple equations developed by Leroy using Wilson's formulae also require correction. In the present memorandum a number of simple equations are developed from the new data. Sound speeds calculated using these equations are shown to be in agreement with the new data to within 0.1 m/s over a wide range of temperatures and salinities at atmospheric pressure, and to within 0.5 m/s at great depths. It is also shown that the disagreement between experiments is of such magnitude as not to merit equations any more precise than those developed herein.

#### INTRODUCTION

For many purposes it is sufficient to assume that the speed of sound in sea water is given by its nominal value of 1500 metres per second. The true value varies from this value by as much as  $\pm 60$  or 70 m/s, which is less than 5%. However, the propagation of underwater sound is quite sensitive to small changes of the sound speed, and, for purposes of sound propagation calculations, sound speed should be known to an accuracy of about 0.1 m/s.

The speed of sound in water is a function of temperature, salinity, and pressure. Prior to 1950, when accurate direct measurements became possible, it had been the practice to calculate the speed of sound from the compressibility, B, and density,  $\rho$ , using the fundamental relationship:

$$c = \sqrt{\frac{B}{\rho}}$$
 [Eq. 1]

This was the method used by Kuwahara [1] shortly before World War II. His results were used universally for about twenty years. In the late 1950's consistent discrepancies were found with Kuwahara's tables, and in 1960 these were supplanted by formulae developed by Wilson [2 & 3] to represent his own measurements of sea-water samples. Wilson expressed the speed of sound (c) directly in terms of the three basic quantities by formulae of the form:

$$c(t,S,P) = c_0(0,35,0) + c_t(t) + c_s(S-35,t)$$
  
+  $c_p(P,t,S-35)$ 

[Eq. 2]

where t is temperature in °C,

S is salinity in parts per thousand,

P is absolute pressure in kg/cm<sup>2</sup>.

Each of the terms was expressed by a polynomial. In his first formula [2] Wilson fitted data in the narrow salinity range of 33% to 37%. His second formula [3] extended the salinity range to that of fresh water and is the one usually used.

Wilson's equations contain about twenty terms, each with coefficients to five significant figures. While readily implemented on a computer, they are not suitable for rapid hand calulation. Consequently a number of approximations have been developed, of which those by Kinsler and Frey [4] and Leroy [5] are the most widely used. Kinsler and Frey represented the first Wilson equation by:

$$c = 1449_{\circ}0 + 4_{\circ}60t - 0_{\circ}055t^{2} + 0_{\circ}00030t^{3} + 1_{\circ}39(S-35) - 0_{\circ}012t(S-35) + 0_{\circ}017z, \qquad [Eq. 3]$$

where z is depth in metres. Leroy developed two formulae, of which his second is best, both designed to fit Wilson's second formula. He also expressed the pressure dependencies in terms of depth, using a number of terms to cover this aspect. In deriving his formulae, Leroy recognized that for fresh water Wilson's formula does not agree well with the results of Greenspan and Tschiegg [6] and he adjusted his coefficients to obtain a better fit to their data.

Throughout the 1960's improvements were made in velocimeters and more precise laboratory measurements were also made, especially by Del Grosso at the US Naval Research Laboratory. Del Grosso [7] found that his, as well as other, laboratory data differed significantly, often by 0.5 to 1.5 m/s, from Wilson's. Mackenzie [8] also noted errors in Wilson's formulae, especially for fresh water. Since 1971, three papers have been published by Del Grosso [9, 10  $\leq$  11] and two by Millero and his students [12  $\leq$  13], which provide new data and new formulae. All of these are in close agreement with each other at atmospheric pressure and all differ significantly from those of Wilson for this condition.

In view of the large discrepancies between the Wilson formulae and recent data, it is proposed that the formulae given in this memorandum, which are based on the data in references 9 to 13, replace formulae based on Wilson's data in future sound-speed calculations.

1 SPEED OF SOUND IN WATER AT ATMOSPHERIC PRESSURE

As indicated by Eq. 2, the formula for sound speed can be divided into separate terms for the temperature, salinity, and pressure or depth dependencies. It is convenient to examine the temperature and salinity terms separately from those that are depth dependent. This separation approach is also useful since some users require formulae in terms of pressure and some prefer depth; thus two sets of equations can be developed for use below the sea surface.

Four equations for sound speed in water at atmospheric pressure are given in references 9 to 13, all of which agree with each other to within a maximum discrepancy of 0.1 m/s over the range of salinities from zero to 40%, and of temperatures from 0° to 40°C. All of these published equations express the salinity in parts per thousand relative to fresh water, rather than relative to 35%, as was done by Wilson and his followers.

The following simple equation agrees with the new equations to the same degree as they agree with each other; i.e., to within 0.1 m/s, and uses 35%, as the reference salinity.

$$c_a = 1449.10 \div 4.565t - 0.0517t^2$$
  
+ 2.21 x 10<sup>-4</sup>t<sup>3</sup> + 1.338(S-35) [Eq. 4]  
- 0.013t(S-35) + 1.0 x 10<sup>-4</sup>t<sup>2</sup>(S-35).

As compared with the published equations, which have as many as ten terms given to twelve significant figures, the present equation has only seven terms, none of which is given to more than five significant figures. Table 1 compares values calculated from Eq. 4 with those given by the other four equations for combinations of temperature and salinity covering the full range of practical importance. For comparison, values calculated using Wilson's second formula are also given. It is apparent that Eq. 4 is in far better agreement with the new data than is Wilson's formula\*, and also that Eq. 4 is in slightly better agreement with Del Grosso's results than with those of Millero.

Sound speeds are listed in Table 1 to the nearest 0.01 m/s. However, these are more precise by at least a factor of ten than can be expected from ordinary measurements. Thus, a precision of 0.05 m/s would require temperature measurements accurate to about 0.01°C and salinity to 0.03%.

<sup>&</sup>quot;Wilson's formula has been evaluated at P = 1.

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# SOUND SPEED AT ATMOSPHERIC PRESSURE

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which are the limits that can be achieved in field measurements. Normally, sound-speed errors of several tenths of a metre per second can be expected to be caused by errors in the measurement of temperature and salinity.

As demonstrated by the comparisons of Table 1, Eq. 4 is valid over the full range of salinities from fresh to very saline water. By using 35%, as the reference salinity, it is in a particularly useful form when dealing with the deep oceans, for which the salinity is close to that value. For the Mediterranean Sea, however, it is useful to use 38%, as the reference salinity and 15°C as the reference temperature. With these values, Eq. 4 transforms into:

 $c_a = 1510.18 + 3.133(t-15) - 0.0414(t-15)^2$ + 2.2 x 10<sup>-4</sup>(t-15)<sup>3</sup> + 1.166(S-38) [Eq. 5] - 0.010(t-15)(S-38).

It is apparent that sound speeds in the Mediterranean generally exceed 1500 m/s.

### 2 DEPENDENCE of SOUND SPEED ON PRESSURE

As expressed by Eq. 1, the speed of sound is directly a function of the compressibility and density of the medium, both of which quantities are complicated functions of the pressure, temperature, and salinity. Equations 4 and 5 have been developed for sea water at atmospheric pressure and are based on several sets of recently-measured data that agree closely. The situation is less satisfactory for variations with pressure. Wilson made measurements for fresh water [14] and sea water [2] and included pressure terms in both his equations [2 & 3]. More recently, Del Grosso and Mader [9] and Chen and Millero [13] have made comprehensive measurements. The latter find that their results are in reasonably good agreement with those of Wilson once his data have been corrected by about 0.5 m/s [15]. All in all, discrepancies between the various data of several tenths of a metre per second are common and exceed a half a metre per second for depths of over 3000 m.

The equations published by these various investigators include as many as twenty pressure-dependent terms, sometimes given to as many as twelve significant figures. In view of the large discrepancies between the experimental data, wherein results are certainly not valid to better than 0.1 m/s, the burdening of computational facilities with any of these equations is clearly not warranted. Using a weighted average of the results of references 3, 10, 11 and 13, it is found that the pressure-dependence of the sound speed can be represented by an equation having only six terms:

$$c_p = 0.1592P + 1.25 \times 10^{-7}P^2$$
  
+ 2.0 x 10<sup>-4</sup>tP - 7.5 x 10<sup>-7</sup>tP<sup>2</sup> [Eq. 6]  
+ 2.0 x 10<sup>-4</sup>(S-35)P - 2.4 x 10<sup>-7</sup>(S-35)P<sup>2</sup>,

where P is gauge pressure in kg/cm<sup>2</sup>. This equation gives sufficiently accurate results for the full range of temperatures, salinities, and pressures found in actual seas and oceans.

Table 2 compares the values for the pressure term calculated using Eq. 6 with those published for comparable conditions in references 3, 10 and 13. It is apparent that Eq. 6 agrees quite well with the results of Chen and Millero, being only slightly lower to account for the lower values found by Wilson and Del Grosso. The maximum discrepancy between the various values increases approximately linearly with depth to about 3000 m, being about 0.03 m/s at 100 m, 0.1 m/s at 300 m, and 0.3 m/s at 1000 m. For greater depths, the various investigators generally agree to within about 0.6 m/s.

Equation 6, as it stands, is especially simple to use in the Atlantic and Pacific Oceans, for which the salinity is close to 35% and the temperature at great depths is under 5°C. The Mediterranean Sea, on the other hand, is characterized by salinities of close to 38% and temperatures above 12°C. For use with Mediterranean data, a more suitable form is:

$$c_{P} = 0.1630P + 2.0 \times 10^{-6} (t-15)P$$
  
- 7.5 x  $10^{-7} (t-15)P^{2}$ 

[Eq. 7]

of which, generally, only the first term is needed.

### 3 DEPENDENCE OF SOUND SPEED ON DEPTH

Acoustic propagation calculations require the sound-speed profile, which is the sound speed as a function of depth. For this reason it is necessary to express the pressure in terms of depth and then solve for the depth as a function of pressure. Expressions for depth can then be substituted into Eqs. 6 and 7 to obtain formulae for sound speed as a function of depth.

The actual relationship of pressure to depth is very complex, requiring the solution of an integral equation. Leroy [5] developed an approximate formula that has been adopted by many other investigators. His

TABL	.E	2
	-	-

# DEPENDENCE OF SOUND SPEED ON PRESSURE

(S = 35%)

Р	t (°C)					
(kg/cm <sup>2</sup> )	0	5	10	15	20	25
10	1.593 1.605 1.57 1.605	1.60 1.61 1.58 1.60	1.61 1.615 1.59 1.595	1.62 1.62 1.605 1.595	1.63 1.63 1.60 1.60	1.64 1.64 1.58 1.61
20	3.19 3.21 3.14 3.21	3.21 3.22 3.155 3.20	3.23 3.23 3.19 3.19 3.19	3.25 3.24 3.21 3.19	3.26 3.26 3.19 3.20	3.28 3.28 3.15 3.22
30	4.79 4.81 4.71 4.82	4.82 4.83 4.73 4.80	4.84 4.85 4.79 4.79	4.87 4.87 4.82 4.79	4.90 4.90 4.80 4.81	4.92 4.93 4.8 4.84
50	7.99 8.02 7.88 8.04	8.03 8.05 7.91 8.00	8.07 8.07 7.98 7.98	8.11 8.10 8.03 7.98	8.15 8.15 7.98 8.01	8.20 8.21 8.06
100	16.05 16.07 15.85 16.13	16.11 15.15 15.89 16.05	16.17 16.20 16.02 16.02	16.23 16.26 16.09 16.00	16.30 16.34 16.0 16.05	16.37 16.43 16.14
200	32.34 32.46 32.085 32.49	32.39 32.49 32.07 32.30	32.44 32.50 32.23 32.20	32.49 32.54 32.26 32.15	32.54 32.64 32.2 32.23	
300	48.88 49.13 48.65 49.07	48.84 49.0 48.5 48.75	48.8 48.9 48.6 48.5	48.77 48.85 48.5 48.4		
400	65.68 66.03 65.50 65.89	65.48 65.7 65.15 65.4	65.28 65.4 65.1 65.0	65.08 65.2 64.85 64.8	- - -	
500	82.72 83.15 82.58 82.93	82.3 82.5 81.95 82.25	81.85 82.0 81.7 81.7	81.4 81.6 81.2 81.4	ROSS	- - Eq. 6
600	100.02 100.04 99.85 100.2	99.3 99.5 98.85 99.3	98.5 98.65 98.3 98.5	97.8 98.0 97.9	CHEN & MIL DEL GROSSO WILSON II - -	
800	135.35 135.35 134.75 135.2	133.75 133.65 132.9 133.6	132.15 132.25 131.7 132.1	130.6 130.9 130.8	-	-

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equation is:

 $P = 0.102506(1 + 0.00528sin^2\phi) + 0.2524\zeta^2, \quad [Eq. 8]$ 

where z is depth in metres,

ζ is depth in kilometres,

For mid-latitudes, Eq. 8 can be closely represented by:

$$P = 0.10268z + 0.2524\zeta^2 = 0.10268(1 + \frac{\zeta}{400})z. \qquad [Eq. 9]$$

Inverting Eq. 9, one can then express the depth in terms of the pressure by:

$$z = 9.74P - 2.2 \times 10^{-1}P^2$$
. [Eq. 10]

A more accurate formula, involving the latitude, is given by Bisset and Berman [16] and may be written:

$$z = \frac{9.7512P}{1 + 0.0053 \sin^2 \phi} - 2.07 \times 10^{-8} P^2, \qquad [Eq. 11]$$

where P is in kg/cm<sup>2</sup> gauge.

In view of the state of our knowledge of the dependence of sound speed on pressure it is adequate to use Eqs. 9 and 10 to convert between pressure and depth. Substituting Eq. 9 into Eq. 6, the resultant expression for the depth dependence of sound speed can be represented by:

$$c_{z} = 0.01635z + 1.75 \times 10^{-7} z^{2}$$
  
+ 2.05 x 10<sup>-5</sup>tz - 8.1 x 10<sup>-9</sup>tz<sup>2</sup> [Eq. 12]  
+ 2.05 x 10<sup>-5</sup>(S-35)z - 2.55 x 10<sup>-9</sup>(S-35)z<sup>2</sup>.

Similarly, for the Mediterranean, Eq. 7 becomes:

$$c_z = 0.01673z + 4.5 \times 10^{-8}z^2$$
  
+ 2.05 x 10<sup>-5</sup>(t-15)z - 8.0 x 10<sup>-9</sup>(t-15)z^2, [Eq. 13]

which for most purposes can be represented with sufficient accuracy simply by:

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#### CONCLUSIONS

Recently published data on the speed of sound in sea water by Del Grosso and Millero and their co-workers confirm that equations based on data published by Wilson around 1960 contain significant errors. However, while these recent investigators obtain results in close agreement for values at sea level, their data diverge for high pressures. Each of the groups has used computer curve fitting to obtain formulae involving upwards of twenty terms, calculated to many significant figures, to represent their data. The present investigation finds that the sound speed can be calculated from the temperature, salinity, and pressure by formulae having no more than thirteen terms, none of which is specified to more than five significant figures.

The approach taken has been to fit the published data for sea-level sound speeds with an equation that represents variations with temperature and salinity to within 0.1 m/s over the entire range from 0° to 40°C and from 0%, to 40%. It is shown in Table 1 that Eq. 4 satisfies this requirement. A second equation was developed to account for the dependence of sound speed on pressure. This equation, Eq. 6, has only six terms and gives results that agree with the mean of the various data sets better than they agree with each other. For investigators requiring sound speed as a function of depth, rather than pressure, a formula given by Leroy has been used to convert Eq. 6 to an equivalent one in terms of depth, namely Eq. 12. Thus, the complete expression for the sound speed as a function of all of the variables is given by Eq. 4 together with either Eq. 6 or Eq. 12.

Equations 4, 6 and 12 have been developed using 0°C and 35%, as reference values of temperature and salinity, as was done by Wilson. This is a particularly good choice for work in the deep oceans, for which the salinity is close to 35%, and the temperature is under 5°C at depth. The Mediterranean Sea, on the other hand, is better represented by formulae using 15°C and 38%, as the reference values. A separate set of equations (Eqs. 5, 7 and 13) have been derived for use when working in the Mediterranean.

The equations developed in the present report are for calculating sound speed from independent measurements of temperature, salinity, and pressure or depth. In view of the accuracies of the sound-speed formulae and of the data on which they are based, it is clear that the oceanographic measurements need to be only sufficiently accurate as to not cause additional errors. Thus, near the surface it is worthwhile to measure temperature to 0.01°C and salinity to 0.02%. However, for depths in excess of a few hundred metres, errors of twice this amount are tolerable, and for depths in excess of 1000 m, temperature need be known only to 0.05°C and salinity only to 0.1%.

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