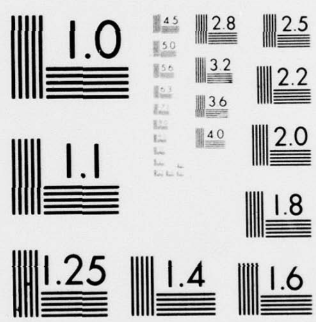
AD-A056 459 ARMY AVIATION RESEARCH AND DEVELOPMENT COMMAND ST LO--ETC F/G 14/4
A MARKOV MODEL FOR RELIABILITY, AVAILABILITY AND MAINTAINABILIT--ETC(U)
JUN 78 H Y LAW, T D EVANS

UNCLASSIFIED NL

| OF |

AD
A056459

END
DATE
FILMED
8-78
DDC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

*LAW & EVANS

LEVEL II

(11) Jun '78 (12) 13p.

AD A056459

(6) A MARKOV MODEL FOR RELIABILITY, AVAILABILITY AND MAINTAINABILITY POLICY GUIDELINE OF AN AIRMOBILE COMBAT SYSTEM.

JUN 1978

(10) HAROLD Y. H. LAW
TIMOTHY D. EVANS
US ARMY AVIATION RESEARCH AND DEVELOPMENT COMMAND
ST. LOUIS, MISSOURI 63166

Introduction

The simulation of aircraft reliability, availability, and maintainability (RAM) is an extremely complex task which deals with details of aircraft missions, scheduling, maintenance, supply, ground equipment, manpower, etc. Implementation of such simulation often requires complex models with laborious input preparation and tedious output digestion. From the top-level decision makers' point of view, it is helpful to gain an insight into the overall trend of significant interactions among the aircraft RAM characteristics so as to formulate overall policy guidelines in anticipation of future actions.

The objective of this paper is to describe a projection model which will facilitate analysis of such interactions and to project the aircraft availability at various stages of operation in a combat scenario. This availability can be expressed in terms of aircraft population at various stages and is a crucial piece of information for decision and policy guideline. The scheduling, maintenance, manpower, supply, etc., are governed to a large extent by availability. The reverse is also true in order to improve availability and therefore mission effectiveness.

The model presented here is based on the mathematical concept of Markov Chain Processes, supported by the real world RAM operational sequences of an airmobile combat system. This model can be used as a management tool which permits observations of the impact of proposed actions prior to their implementation. Although the operation of an airmobile combat system is addressed in this paper, this model is flexible and can be adapted to other operating systems as well.

Modeling Philosophy

A model is a way of abstracting the real world so that the static and dynamic interrelationships are represented (Reference 1). With an appropriate model of a real world situation, we should then be able to predict certain outcomes or determine how the real world would behave if we implemented a particular alternative decision. One objective of model building is to identify the important variables and relationships and then translate a perception of the real world into these essential relationships and variables, and thus into a model which is tractable and, hopefully, computationally manageable. Along this modeling philosophy, we try to build a model that is simple which can be used as a management tool, and that will capture the essence of the RAM characteristics of an airmobile combat system to approximate the operations of a fleet of aircraft in a combat scenario. This model can then be used to project the movement of aircraft at various stages in the scenario, to study the interactions among the RAM characteristics, and to assess the impacts of decisions (policies) on the overall aircraft availability and RAM characteristics, so that guideline for a workable policy can be formulated.

Scenario of an Airmobile Combat System

In order to identify the essential RAM variables and relationships of an airmobile combat system, it is necessary to understand and define the scenario under which this system operates. Based on the flow of operation, the scenario can be aggregated into seven stages through which a fleet of aircraft will normally proceed. Let us assume that a fleet of aircraft starts at the ready pool stage (R) where refueling, arming, preflight preparation, etc., will be performed. This fleet of aircraft will be replenished from the reserve stage (S). From ready pool the aircraft will either remain in ready pool or go to combat mission stage (C). We define that attrition can occur only during combat due to component failure or combat damage. From the mission stage an aircraft can either remain in mission, go to attrition stage (A), come back to ready pool after the mission is accomplished, go to scheduled maintenance stage (SM), or to unscheduled maintenance stage (UM). Scheduled maintenance requirements are specified by the user according to flight time or calendar time. Unscheduled maintenance is based on component failure. From SM stage, an aircraft can either remain in that stage due to maintenance delay or other factors, or it can go to UM stage if component failure is discovered. It will return to ready pool when scheduled maintenance is completed. From the UM stage, an aircraft can either remain in that stage, or go to the not operationally ready supply (NORS) stage due to waiting for parts. Again it will go back to ready pool after maintenance is performed. From NORS, an aircraft will either remain in NORS or return to UM stage.

The above seven stages characterize in general terms the RAM dynamics of a fleet of aircraft in operation.

Model Development

This chain of transition from stage to stage can be illustrated by a directed graph (Figure 1). This graph specified the transitional

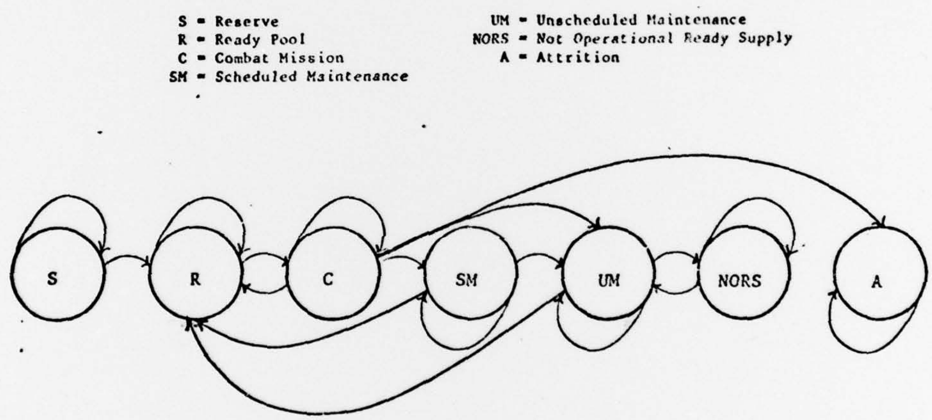


FIGURE 1: DIRECTED GRAPH

directions of aircraft and therefore defines the movement of aircraft and the interrelationships of the RAM characteristics. One can look upon the stages as a set of outcomes of an experiment. The probabilities of an aircraft to move from one stage to the next reflect the RAM characteristics such as the reliability of components, the availability of manpower, equipment and parts, and the scheduling of the resources and facilities, etc. If one can assign values of these transitional probabilities from one stage to the other, one should then be able to predict the outcomes. These values can be obtained from historical data recorded in terms of the amount of time that the system is in various stages of readiness or operability. Since we specify a finite number of stages (seven in this case) where an aircraft can be tracted, this set of outcomes is finite.

When considering a fleet of aircraft, we cannot say that the present condition of this fleet is independent of the past. We can say, however, that the future condition of the fleet is dependent at most on the present, i.e., it does not matter how the fleet arrives at its present condition. For example, an aircraft in mission stage depends only on its condition at the ready pool stage where preflight checking has been performed to certify the readiness of the aircraft to fly a mission. It does not depend on how it arrived at the ready pool stage. A simple model on logistic guideline of an airmobile combat system based on the concept of Regular Markov Processes was suggested by Law (reference 2). The application of such concept can be extended to the scenario described here. This scenario fits well into the concept of Markov Processes, in particular the Absorbing Markov Chain Processes.

Absorbing Markov Chain Processes

Before constructing a Markov model, one needs to make sure that the characteristics of the real world situation to be modeled satisfy the basic assumptions of a Markov process. Assuming a sequence of experiments, the outcome of each experiment is one of a finite number of possible outcomes. It is assumed that the probability of an outcome of any given experiment is not necessarily independent of the outcomes of previous experiments but depends at most upon the outcome of the immediately preceding experiment. Finally, we assume that the probability p_{ij} of an outcome on any experiment is known, given that the outcome of the preceding experiment occurred. The outcomes are called "states", and the numbers p_{ij} are called "transition probabilities". A matrix of these probabilities is called the "transition matrix". If we assume that the process begins in some particular state, then we have enough information to determine the tree measure for the process and can calculate probabilities of statements relating to the overall sequence of experiments. When these assumptions are satisfied, one can then translate the Markov processes into the operations of an airmobile combat system.

The seven stages are the states of a Markov chain. A state is called an absorbing state if it is impossible to leave it. In the scenario of this study, the attrition stage is an absorbing state. We define that when an aircraft is attrited due to component failure or combat damage, it is lost and not salvagable and remains in the attrition stage. A Markov chain is absorbing if (1) it has at least one absorbing state, and (2) from every state it is possible to go to an absorbing state (not necessarily in one step). According to the directed graph in Figure 1, an aircraft starting at any stage will be able to go to the attrition stage.

Model Description

The scenario under study fits well into the framework of absorbing Markov Chain Processes, and all the assumptions for a Markov chain are satisfied in the real world situation. Since the main objective of this model is to tract the dynamic distribution of aircraft population at different stages, we want the model to predict and project the available aircraft population in any stage at any time, and to facilitate a sensitivity analysis of the population dynamics with respect to the RAM characteristics. This sensitivity analysis provides information for an impact study of decision on a proposed action. The impact will lead to formulation of guideline for a policy that may be optimal under a particular situation.

Now consider some of the questions one would like to ask and obtain clues and solutions from a model. A policy guideline on RAM includes considerations such as scheduling of manpower and equipment, projected availability of aircraft, reliability of components, ordering of parts and supplies, anticipated attrition in combat, and shipment of aircraft from reserve, etc. Some of the questions of interest are: (1) What is the time history of aircraft population distribution in every stage under a prescribed policy? (2) How is this population distribution affected by a change in policy (a change in one or more transition probabilities)? (3) On the average, how many stages will an aircraft go through, starting at any stages, before it is attrited, and how sensitive is this flow towards a change in policy? (4) On the average, how many times will an aircraft be in each stage, starting from any stage, before it is attrited, and again how sensitive is this towards a change of policy? (5) What is the probability that an aircraft starting at any stage will end up in attrition?

The aircraft population at any stage is a basic piece of information based on which the RAM policy is formulated. In a combat scenario, one is concerned with mission effectiveness. The aircraft availability is an important factor that contributes to mission effectiveness. Therefore one would like to have an idea of available quantity of aircraft so that adequate preparation can be made to keep a continuous flow of aircraft. Since each transition probability is a description of the decision and policy in RAM, one can conduct sensitivity analysis and investigate how the policies affect the aircraft availability and how one can improve decision policies to provide timely maintenance and to maintain mission effectiveness. This sets the tone of an overall policy guideline. On a more detailed level of management, question (3) leads to some insight on how one should prepare for the availability and scheduling of manpower, equipment, supplies, etc., at various stages to anticipate and service the flow of aircraft. Question (4) is similiar to question (3) except it is more concerned with the local scheduling and supply of a particular stage.

According to the structure of this absorbing Markov model, an aircraft will ultimately end up at the attrition stage. This is true in reality when an aircraft has accumulated sufficient number of flight hours. In the scenario under study, there is only one attrition stage. This means the probability of attrition for any aircraft will be 1. This is true but not very interesting. However, if there is more than one attrition stage in the model, i.e., attrition due to combat damage, component failure, accidents, and other causes, then the model can provide information on the probability of attrition due to various causes. This information may be of interest to the decision maker, and the Markov model is flexible to accommodate this feature. However this feature is not included in this study. The questions above are by no means all one wants to ask in formulating policy guideline but are questions of major interest and concern.

The condition of a system can be expressed as a state vector containing w states, $S(s_1, s_2, \ldots, s_w)$. If the system is in state s_i, the probability that it will be in state s_j is p_{ij} such that $s_j = p_{ij} s_i$. This transition probability can be considered as a description of the decision that takes the system from state s_i to state s_j. This decision is the result of actions implemented according to a policy. Thus the transition of a system at any state can be described by the transition matrix

$$p = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1w} \\ p_{21} & p_{22} & \cdots & p_{2w} \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ p_{w1} & p_{w2} & \cdots & p_{ww} \end{bmatrix} \qquad \text{such that } S_{k+1} = PS_k \qquad (1)$$

where k indicates the time period and

$$\sum_{j=1}^{w} p_{ij} = 1, \; i = 1,2,\ldots,w \text{ and } p_{ij} \leq 1.$$

For an arbitrary absorbing Markov chain, the absorbing states can be grouped together and the transition matrix can be rearranged in the following canonical form

$$P = \begin{array}{c c} & \begin{array}{cc} r & t \end{array} \\ \begin{array}{c} r \\ t \end{array} & \left[\begin{array}{c|c} I & 0 \\ \hline R & Q \end{array}\right] \end{array} \qquad (2)$$

where I is an rxr identity matrix, 0 is an rxt zero matrix, R and Q are the partitions of the remaining elements in matrix P, and the elements in R and Q are less than unity by definition.

A theorem (reference 3) shows that the inverse matrix, $(I-Q)^{-1}$, exists and is called the fundamental matrix.

Let $N = (I-Q)^{-1}$ and $N = (n_{ij})$, $i,j = 1,2,\ldots,t$
Futhermore, a theorem (reference 3) indicates that the matrix of probability of absorption is a txr matrix B such that $B = NR$. These matrices contain important information and interpretation which are of interest to policy guideline in this study.

The elements n_{ij} of the fundamental matrix N is the expected number of times before attrition that an aircraft will be in state j if it starts in state i. This information provides an answer to question (4). The sum of n_{ij} along the row elements represents the expected number of times that an aircraft will be in a nonabsorbing state (nonattrition stage) if it starts at state i. This provides an answer to question (3). To answer question (5), we notice that the interpretation of element b_{ij} of matrix B is precisely the probability that an aircraft will be attrited at the absorbing state j if it starts at state i. We shall demonstrate below how this model can be applied to the scenario of an airmobile combat system. According to the scenario illustrated in Figure 1, the canonical form of the transition matrix can be presented as

$$P = \begin{array}{c c} & \begin{array}{ccccccc} A & S & R & C & SM & UM & NORS \end{array} \\ \begin{array}{c} A \\ S \\ R \\ C \\ SM \\ UM \\ NORS \end{array} & \left[\begin{array}{c|cccccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & p_{22} & p_{23} & 0 & 0 & 0 & 0 \\ 0 & 0 & p_{33} & p_{34} & 0 & 0 & 0 \\ p_{41} & 0 & p_{43} & p_{44} & p_{45} & p_{46} & 0 \\ 0 & 0 & p_{53} & 0 & p_{55} & p_{56} & 0 \\ 0 & 0 & p_{63} & 0 & 0 & p_{66} & p_{67} \\ 0 & 0 & 0 & 0 & 0 & p_{76} & p_{77} \end{array}\right] \end{array} = \left[\begin{array}{c|c} I & 0 \\ \hline R & Q \end{array}\right] \qquad (3)$$

where $0 \leq p_{ij} \leq 1$
Let $A = (I-Q)$, then $N = A^{-1}$.
The time history of aircraft population distribution and the effect of change of policy on available population can be obtained by the recurrsive equation (1), $S_{k+1} = PS_k$. The sensitivity mentioned in questions (3) and (4) can be investigated through the partial derivatives $\frac{\delta b_{ik}}{\delta p_{pq}}$ or $\frac{\delta b_{ik}}{\delta a_{pq}}$ and $\frac{\delta n_{ij}}{\delta p_{pq}}$ or $\frac{\delta n_{ij}}{\delta a_{pq}}$. A moment of reflection on Cramer's rule indicates that a_{pq} can be isolated in the expressions of b_{ik} and n_{ij}. Therefore analytical expressions of the partial derivatives as a function of a_{pq} can be obtained. Let the minor of matrix A be M_{ij} and the cofactor be C_{ij} such that $C_{ij} = (-1)^{i+j}M_{ij}$. The elements of N can be expressed as

$$n_{ij} = \frac{(-1)^{j+i}|M_{ji}|}{|A|} = \frac{(-1)^{j+i}(a_{pq}C'_{pq} + K_1)}{a_{pq}C_{pq} + K_2} \qquad (4)$$

$$q \neq i, \quad p \neq j$$

where $|M_{ji}| = \sum_{\substack{h=1 \\ h\neq i}}^{t} a_{mh}C'_{mh}$, $m \neq j$ and C'_{mh} is the cofactor of M_{ji} and

K_1, K_2 are constant terms not involving a_{pq}. Therefore

$$\frac{\delta n_{ij}}{\delta a_{pq}} = \frac{(-1)^{j+i}(C'_{pq}K_2 - C_{pq}K_1)}{(a_{pq}C_{pq} + K_2)^2}, \quad p\neq j,\ q\neq i\ . \qquad (5)$$

Similarly,

$$\frac{\delta \sum_{j=1}^{t} n_{ij}}{\delta a_{pq}} = \sum_{j=1}^{t} \frac{(-1)^{j+i}(C'_{pq}K_2 - C_{pq}K_1)}{(a_{pq}C_{pq} + K_2)^2}\ . \qquad (6)$$

Since B = NR, where $R = (r_{jk})$, then

$$b_{ik} = \sum_{j=1}^{t} n_{ij} r_{jk}, \quad i,j = 1,\ldots,t, \text{ and}$$

$$\frac{\delta b_{ik}}{\delta a_{pq}} = \sum_{j=1}^{t} \frac{n_{ij}}{a_{pq}} r_{jk}, \quad k = 1,\ldots,r \, . \tag{7}$$

Hence the sensitivity of the flow of this system towards a policy at any stage can be obtained in equations (5), (6), and (7). The answers to the above five questions can also be obtained through a simple algorithm (Figure 2). This algorithm can be easily adapted to accommodate as many stages as one wishes to have. A listing of the algorithm is available upon request.

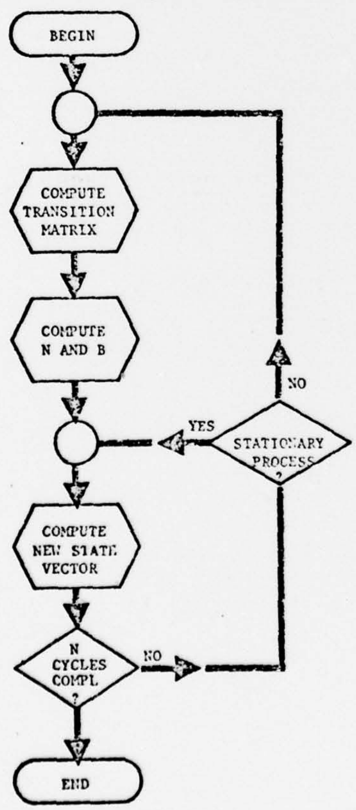


FIGURE 2: FLOW DIAGRAM OF ALGORITHM

Model Application and Discussion

Now let us apply the model to a real system. The data in Table 1 gives the required parameters and the average condition of a helicopter fleet in 1971. Some of the results of this application are presented in Figures 3 to 7. These figures show the trend of population dynamic which is the primary information for policy making. The aircraft population distributions at all seven stages are presented in Figure 3. The aircraft availability at the stages of ready pool, mission, scheduled and unscheduled maintenance fluctuates at the early time period, and gradually stablizes. For a limited population of aircraft at the reserve stage, it is seen that the aircraft at that stage diminishes as timé goes on, and the attrition accumulates at a fairly constant rate. If a policy of skipping scheduled maintenance is in effect, it is observed in Figure 3 that more aircraft are available for mission, and yet more aircraft are also attrited. However, the SM, UM, and NORS stages are not affected significantly by this policy. When the rate from reserve to ready pool is increased as in Figure 4, the available aircraft at ready and mission stages increases sharply in the early time period, and attrition also increases sharply. Therefore more manpower, equipment and facilities are needed at the maintenance stages. It is interesting to notice that at a later time period, aircraft is actually less available for mission because of limited reserve and high attrition. Therefore the reserve pool would need to be built up in order to maintain a certain level of aircraft availability. When the rate of attrition is increased, the drastic effects are observed in Figure 5, and the level of availability becomes very low. It was observed that shortening the waiting for supply at the NORS stage does not noticeably affect the availability. The graph of which is not shown due to lack of space. One example of the sensitivity of the flow of aircraft towards a change of policy is presented in Figures 6 and 7. In this example, the sensitivity increases as the transition probability increases. In other situations, the reverse trend may be true.

Conclusion

In this paper we have demonstrated the application of absorbing Markov Chain Processes to analysis of reliability, availability, and maintainability policies. A model was presented and applied to a real world situation. This model is simple and flexible. Because of the simplicity of the model structure, the algorithm of which can be easily programmed and be made interactive. The Markov Chain Processes presented here show great promise in analyzing RAM policies. The model presented here is not intended to replace simulation models, but can be used to gain insight into trends which would result from overall policy changes.

References

1. Gass, Saul, Roger L. Sission, A Guide to Models in Governmental Planning and Operations, PB-245-269, August, 1974.

2. Law, Harold Y. H., "A Macroscopic Modeling Concept for Logistic Policy Guideline of an Airmobile Combat System", Proceedings of

the 15th Army Operations Research Symposium, 26-29 Oct. 1976, Ft. Lee, Virginia.

3. Feller, William, An Introduction to Probability Theory and Its Applications, Volume I, 3rd edition, John Wiley, New York, 1970.

Table 1

Condition and Parameters of a Helicopter Fleet

Average inventory: 2,526 aircraft
Total flight hours: 1,096,510
Total flight: 2,917,955
Mean time between scheduled maintenance: 2.749 hours
Mean time between unscheduled maintenance: 2.652 hours
Mean time between all maintenance: 1.35 hours
Mean elapsed scheduled maintenance time: 5.40 hours
Mean elapsed unscheduled maintenance time: 5.195 hours
Mean elapsed maintenance time: 5.30 hours
Clock hours: 8,760

$P_{23} = 1.0 - \exp(-r_2 t_c)$
$P_{33} = \exp(-r_3 t_c)$
$P_{41} = 1.0 - \exp(-r_1 t_c)$
$P_{44} = \exp(-r_4 t_c)$
$P_{45} = 1.0 - \exp(-r_5 t_c)$

Rate of attrition, $r_1 = 0.1$ per clock hour
Rate from reserve to ready pool, $r_2 = 0.1$
Rate of launch, $r_3 = 0.1319$
Rate of leaving mission, $r_4 = 2.6611$
Rate from mission to scheduled maintenance, $r_5 = 0.0180$
Rate from mission to unscheduled maint., $r_6 = 0.0187$
Rate from scheduled to unsched. maint., $r_7 = 0.0$
Average length of mission, $t_c = 1.0$ hour

$P_{46} = 1.0 - \exp(-r_6 t_c)$
$P_{56} = 1.0 - \exp(-r_7 t_c)$
$P_{55} = r_8 = 0.1852$
$P_{66} = 0.2003$
$P_{67} = 0.1$
$P_{77} = 0.3165$

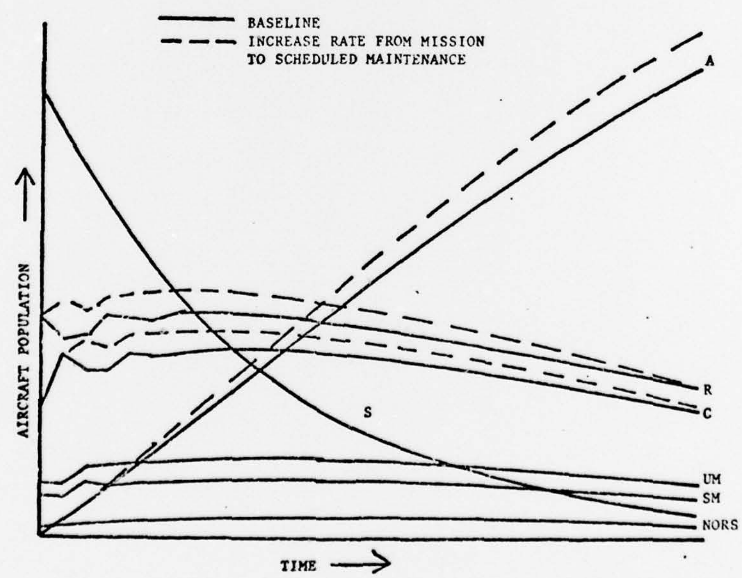


FIGURE 3: DYNAMICS OF AIRCRAFT POPULATION DISTRIBUTION

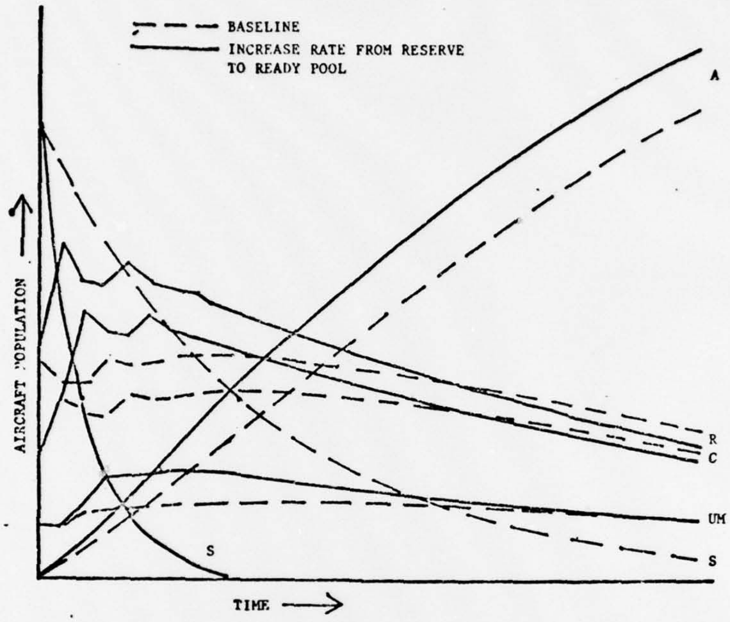


FIGURE 4: DYNAMICS OF AIRCRAFT POPULATION DISTRIBUTION

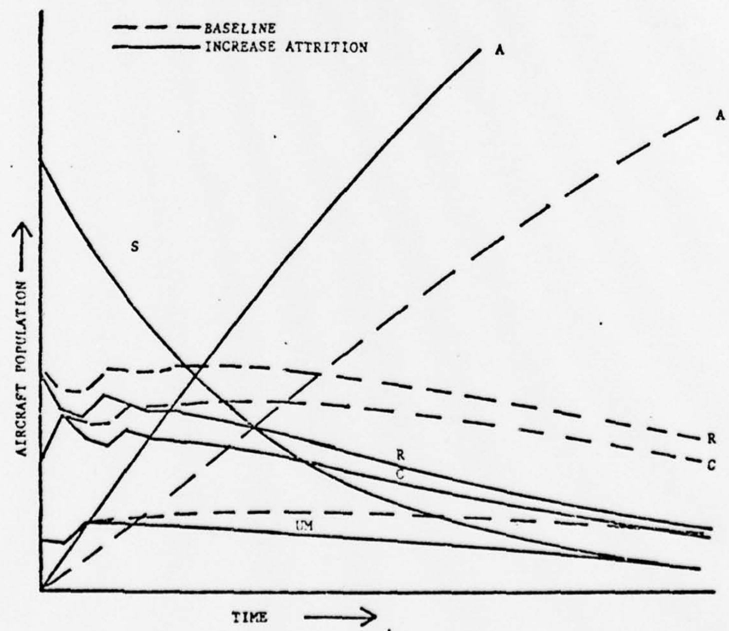


FIGURE 5: DYNAMICS OF AIRCRAFT POPULATION DISTRIBUTION

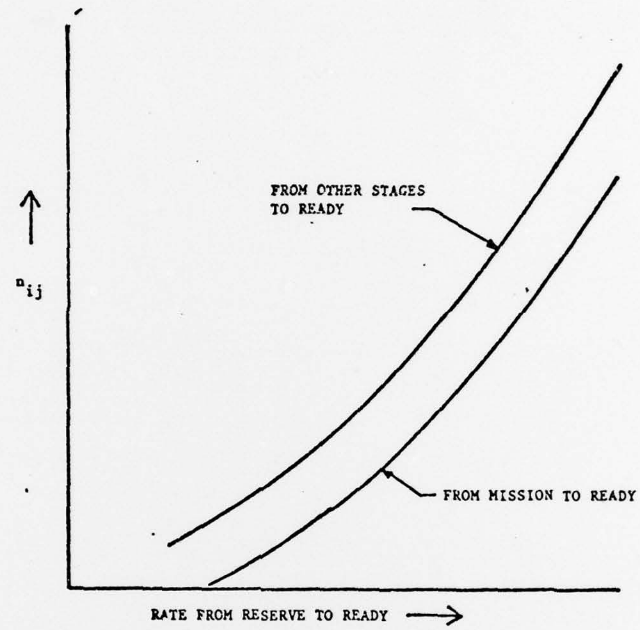


FIGURE 6: AIRCRAFT FLOW SENSITIVITY

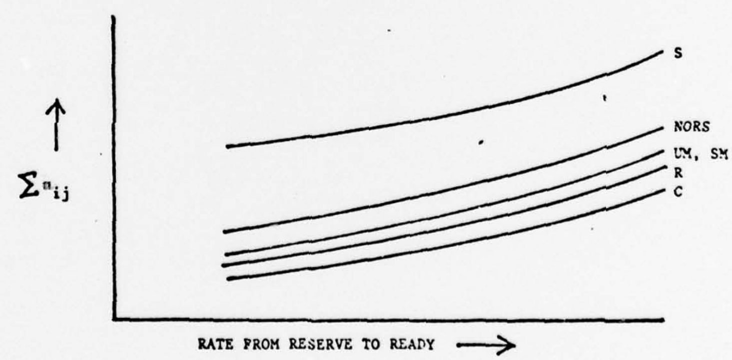


FIGURE 7: AIRCRAFT FLOW SENSITIVITY