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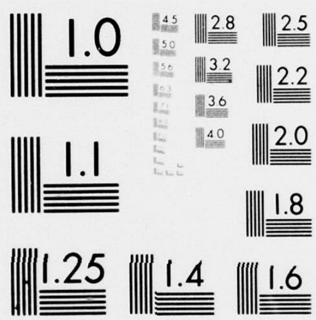

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ANALYSIS OF A NONLINEAR ELECTROMAGNETIC FIELD PENETRATION PROBLEM

JUN 1978

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INTRODUCTION

Typical electromagnetic shielding problems involve the calculation of the time-dependent electric or magnetic field penetration of electrically conducting media. Two material properties which enter into such calculations are the electrical conductivity σ and the permeability μ. In most cases the electrical conductivity does not significantly vary with electromagnetic field level and can be regarded as a constant for a given material. In nonmagnetic materials the permeability is also independent of field level and has the value for free space (vacuum) $\mu_o = 4\pi \times 10^{-7}$H/m. For media whose material properties are independent of the applied electromagnetic field level, the partial differential equations which govern electromagnetic field propagation are linear. The analytical theory for electromagnetic field calculations in media with constant permeability has been well developed and extensive analyses exist. In fact, quite often the exact analytical solution can be achieved by any one of a number of standard linear mathematical techniques.

On the other hand, ferromagnetic materials, which are commonly used in electromagnetic shield fabrication, not only have a permeability that is dependent on the magnetic field strength H but also exhibit the phenomenon of saturation. Although ferromagnetic materials usually have relatively large (in some cases relatively enormous) permeabilities compared to nonmagnetic materials, the permeability can be greatly reduced if the material is driven into saturation. For these materials the partial differential equations which govern electromagnetic field propagation are nonlinear and considerably less amenable to analytic solution. Due to the difficulties associated with the nonlinear partial differential equations which arise in this case, relatively few analyses are available so that it is difficult to assess the effect of a variable permeability on the time-dependent electromagnetic field penetration.

This study examines the time-dependent penetration of a step increase in magnetic field strength H into a semi-infinite conducting medium having a permeability which varies with magnetic field strength. The medium is considered to be isotropic, homogeneous, and initially demagnetized. The medium at any point is presumed to follow its initial magnetization curve for which a simple approximation is assumed. The analytical approach consists of mathematical analysis supplemented with numerical calculations. The problem can be simplified from one involving a second order nonlinear partial differential equation to one involving a second order nonlinear ordinary differential equation utilizing a simple transformation of variables. Although a formal parametric representation of the solution of the second order differential equation is considered, a simple closed form solution in terms of elementary functions does not seem to exist. Hence several analytical techniques, as well as numerical calculations, have been employed to deduce properties of the solution.

MATHEMATICAL FORMULATION OF THE PROBLEM

The propagation of electromagnetic fields in a conducting medium (where the displacement current can be neglected) is governed by the equations

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad (1)$$

and

$$\nabla \times \vec{H} = \sigma \vec{E} , \qquad (2)$$

where $\vec{E}$ is the electric field, $\vec{H}$ is the magnetic field strength, and $\vec{B}$ is the magnetic flux density. In a homogeneous and isotropic medium, it can be assumed that $\vec{B} = B(\vec{H})$, i.e., that the magnetic flux density is a function of the applied magnetic field strength and is in the same direction. With the differential permeability defined as $\mu_d(H) = \partial B/\partial H$, (1) can be written as:

$$\nabla \times \vec{E} = -\mu_d(H)\frac{\partial \vec{H}}{\partial t} . \qquad (3)$$

Consider a homogeneous, isotropic, conducting medium of semi-infinite extent ($x \geq 0$). In the present problem it is presumed that the medium is initially demagnetized, and at time $t = 0$, a step increase in magnetic field strength in the $\hat{y}$ direction occurs at the surface $x = 0$, e.g., as the result of an imposed surface current in the $\hat{z}$ direction.

Since it is presumed that $\vec{H} = H_y(x,t)\hat{y}$, the propagation of the electromagnetic fields following the onset of the step increase in $H_y(x,t)$ is governed by the equations

$$\frac{\partial E_z(x,t)}{\partial x} = \mu_d(H_y)\frac{\partial H_y(x,t)}{\partial t} \quad , \tag{4}$$

$$\frac{\partial H_y(x,t)}{\partial x} = \sigma E_z(x,t) \ . \tag{5}$$

By eliminating $E_z(x,t)$ from (4) and (5), it is found that the partial differential equation governing the penetration of $H_y(x,t)$ is

$$\frac{\partial^2 H_y(x,t)}{\partial x^2} = \sigma\mu_d(H)\frac{\partial H_y(x,t)}{\partial t} \ . \tag{6}$$

In general, $B(H) = \mu_o[H + M(H)]$, where $M(H)$ is the magnetization. As H is increased from zero, $\mu_d(H)$ usually starts from an initial value μ_i (μ_i is usually larger than μ_o), increases to some maximum value, and then decreases to μ_o as the material undergoes saturation. In many cases before saturation, the variation of B with H [hence the variation of $\mu_d(H)$] is primarily due to $M(H)$. In this study it is assumed that over the range of interest $B(H)$ can be represented by the simple approximation

$$B(H) = B_s[1 - \exp(-H/H_m)],$$

where B_s represents a saturation value for B and H_m is a parameter which indicates the shape of the magnetization curve for the medium. With this representation of the magnetization curve,

$$\mu_d(H) = \mu_i \exp(-H/H_m) \ ,$$

where $\mu_i \equiv B_s/H_m$ is the initial slope of the B-H curve. The variation of $B(H)$ with H is shown in Figure 1a and the corresponding variation of $\mu_d(H)$ with H is shown in Figure 1b. For large H, $\mu_d(H)$ actually approaches μ_o as a lower limit; hence this simple representation is appropriate as long as $\mu_i \exp(-H/H_m) > \mu_o$ over the range of H under consideration.

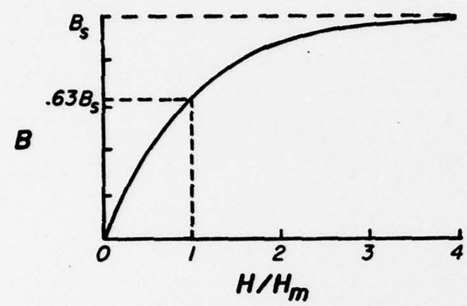

FIGURE 1a B(H) vs. H

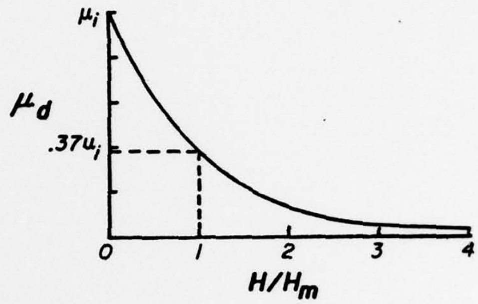

FIGURE 1b μ_d(H) vs. H

The present problem, therefore, requires the solution of the nonlinear partial differential equation

$$\frac{\partial^2 H_y(x,t)}{\partial x^2} = \sigma\mu_i \exp(-H/H_m) \frac{\partial H_y(x,t)}{\partial t} \tag{7}$$

subject to the initial condition

$$H_y(x,0) = 0 \tag{8}$$

and the auxiliary conditions

$$H_y(0,t) = H_o \tag{9}$$

and

$$H_y(\infty,t) = 0 \; . \tag{10}$$

The electric field can be determined from the solution to the above problem by noting from (5) that

$$E_z(x,t) = \frac{1}{\sigma} \frac{\partial H_y(x,t)}{\partial x} \; . \tag{11}$$

REDUCTION IN THE NUMBER OF INDEPENDENT VARIABLES

Using an approach similar to that used for nonlinear diffusion phenomena analysis, the number of independent variables appearing in the problem can be reduced by the transformation

$$H_y(x,t) = H(\zeta); \qquad \zeta = \frac{\sqrt{\sigma\mu_i}}{2} \frac{x}{\sqrt{t}} .$$

The new variables are introduced by evaluating the partial derivatives in terms of the new variables and substituting these quantities into the original partial differential equation. On calculating the partial derivatives it is found that

$$\frac{\partial H_y(x,t)}{\partial x} = \frac{\partial \zeta}{\partial x} \frac{dH(\zeta)}{d\zeta} = \frac{\sqrt{\sigma\mu_i}}{2\sqrt{t}} \frac{dH(\zeta)}{d\zeta} ,$$

$$\frac{\partial^2 H_y(x,t)}{\partial x^2} = \frac{\partial^2 \zeta}{\partial x^2} \frac{dH(\zeta)}{d\zeta} + \left(\frac{\partial \zeta}{\partial x}\right)^2 \frac{d^2 H(\zeta)}{d\zeta^2} = \frac{\sigma\mu_i}{4t} \frac{d^2 H(\zeta)}{d\zeta^2} ,$$

and

$$\frac{\partial H_y(x,t)}{\partial t} = \frac{\partial \zeta}{\partial t} \frac{dH}{d\zeta} = -\frac{1}{2} \frac{\sqrt{\sigma\mu_i}}{2} \frac{x}{t^{3/2}} \frac{dH(\zeta)}{d\zeta} = -\frac{1}{2} \frac{\zeta}{t} \frac{dH(\zeta)}{d\zeta} .$$

Upon substituting these quantities into (7) and cancelling a factor of $\sigma\mu_i/4t$, it is found that the magnetic field strength satisfies the equation

$$\frac{d^2 H(\zeta)}{d\zeta^2} = -2\zeta \exp(-H/H_m) \frac{dH(\zeta)}{d\zeta} , \tag{12}$$

in which the variables x and t no longer appear explicitly. The electric field is related to $H(\zeta)$ by

$$E_z(x,t) = \frac{1}{2} \sqrt{\frac{\mu_i}{\sigma}} \frac{1}{\sqrt{t}} \frac{dH(\zeta)}{d\zeta} \tag{13}$$

in which a factor of $\sqrt{t}$ appears. Noting that both $x = 0$ and $t = \infty$ imply that $\zeta = 0$ and that both $x = \infty$ and $t = 0$ imply that $\zeta = \infty$, it follows from (9) that

$$H(0) = H_o \tag{14}$$

and from (8) and (10) that

$$H_y(\infty) = 0 . \tag{15}$$

For computational purposes, it is convenient to normalize the problem in terms of H_o. With

$$F(\zeta) = H(\zeta)/H_o = H_y(x,t)/H_o \ , \tag{16}$$

the problem has been reduced to solving the nonlinear second order ordinary differential equation

$$\frac{d^2F}{d\zeta^2} = -\,2\zeta\,\exp(-\alpha F)\,\frac{dF}{d\zeta} \tag{17}$$

subject to the conditions

$$F(0) = 1 \tag{18}$$

and

$$F(\infty) = 0 \ , \tag{19}$$

where $\alpha = H_o/H_m$. Having solved for $F(\zeta)$, the electric field can then be evaluated from the relation

$$\frac{dF(\zeta)}{d\zeta} = \frac{2}{H_o}\sqrt{\frac{\sigma}{\mu_i}}\,\sqrt{t}\,E_z(x,t) \ . \tag{20}$$

ANALYTICAL ASPECTS OF THE ORDINARY DIFFERENTIAL EQUATION

Classical Linear Solution

In classical analyses of electromagnetic field penetration, it is assumed that the permeability is constant, which corresponds to $\alpha = 0$ in the present investigation. For $\alpha = 0$ the problem reduces to

$$\frac{d^2F}{d\zeta^2} = -\,2\zeta\,\frac{dF}{d\zeta} \tag{21}$$

subject to conditions (18) and (19). This equation can be integrated in a straightforward manner, to obtain the classic solution

$$F(\zeta) = 1 - \mathrm{erf}(\zeta) \ , \tag{22}$$

where the error function is defined as

$$\mathrm{erf}(\zeta) = \frac{2}{\sqrt{\pi}}\int_0^{\zeta} \exp(-\zeta_1^2)\,d\zeta_1 \ .$$

It follows immediately that

$$\frac{dF(\zeta)}{d\zeta} = -\,\frac{2}{\sqrt{\pi}}\exp(-\zeta^2). \tag{23}$$

Formal Solution of the Nonlinear Problem

Most of the standard linear techniques are of relatively little use in achieving an exact solution in nonlinear cases ($\alpha \neq 0$); however, in the course of an investigation of a related problem in nonlinear diffusion phenomena, Fujita [1] found a formal solution which can be directly adapted to the problem presently under investigation. The problem is satisfied with F and ζ being given by the parametric representations

$$F = \frac{2}{\alpha} \int_0^{\xi} \left[\xi_1^2 - \beta \ln(\xi_1^2)\right]^{-1/2} d\xi_1 , \tag{24}$$

$$\zeta = \frac{1}{(2\beta)^{1/2}} \left\{ \left[\xi^2 - \beta \ln(\xi^2)\right]^{1/2} - \xi \right\} \times \exp\left\{ \int_0^{\xi} \left[\xi_1^2 - \beta \ln(\xi_1)^2\right]^{-1/2} d\xi_1 \right\} , \tag{25}$$

where the derived quantity β is related to the given quantity α by

$$\alpha = 2 \int_0^1 \left[\xi_1^2 - \beta \ln(\xi_1^2)\right]^{-1/2} d\xi_1 . \tag{26}$$

The parametric representation of $dF/d\zeta$ can be evaluated from

$$\frac{dF}{d\zeta} = \left(\frac{dF}{d\xi}\right)/\left(\frac{d\zeta}{d\xi}\right) = -\frac{4}{\alpha\beta^{1/2}}\, \xi \exp\left\{ - \int_0^{\xi} \left[\xi_1^2 - \beta \ln(\xi_1^2)\right]^{-1/2} d\xi_1 \right\}. \tag{27}$$

The parameter ξ ($0 \leq \xi \leq 1$) relates a value of F to the corresponding value of ζ. (Note that $\xi = 0$ implies $\zeta = \infty$ and $F = 0$, while $\xi = 1$ implies $\zeta = 0$ and $F = 1$.) To evaluate F versus ζ, a value of ξ is selected and the value of F and the corresponding value of ζ are evaluated. The integrals appearing in the formal solution do not appear to be evaluated in closed form, thus numerical integration seems to be required; however, the above representation offers the advantage of allowing the computational effort to be directed to the very accurate calculation of the solution at only a few points, or one point for that matter, since the knowledge of intermediate points is not required. Of special interest is $dF/d\zeta$ evaluated at $\zeta = 0$ where

$$\frac{dF(0)}{d\zeta} = - \frac{4}{\alpha\beta^{1/2}} \exp(-\alpha/2) \ . \tag{28}$$

In addition to its relation to the electric field at $x = 0$, the calculated value of $dF(0)/d\zeta$ can be used to transform the problem from a two-point problem (with a condition at $x = \infty$ as well as one at $x = 0$) to an initial value problem (with both boundary conditions specified at $x = 0$). Such a transformation is often useful from a computational standpoint.

Taylor Series Expansion of the Solution

If $F(\zeta)$ was a known function of ζ and if $F(\zeta)$ and its derivatives were continuous, then $F(\zeta)$ could be expanded in a Taylor (Maclaurin) series about $\zeta = 0$

$$F(\zeta) = \sum_{m=0}^{\infty} \frac{d^m F(0)}{d\zeta^m} \frac{\zeta^m}{m!} = F(0) + \frac{dF(0)}{d\zeta} \zeta + \frac{d^2F(0)}{d\zeta^2} \frac{\zeta^2}{2!} + \dots ,$$

where F and its derivatives are evaluated $\zeta = 0$. For example, the solution (22) to the linear case ($\alpha = 0$) has the series representation

$$F(\zeta) = 1 - \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n \zeta^{2n+1}}{(2n+1)n!} = 1 - \frac{2}{\sqrt{\pi}} (\zeta - \frac{\zeta^3}{3} + \frac{\zeta^5}{5 \cdot 2!} - \dots) \ .$$

A series expansion approach can also be used to develop the solution $F(\zeta)$ from the second order differential equation. If $F(0)$ and $dF(0)/d\zeta$ are given as boundary conditions, then the approach is straight-forward since the second and higher order derivatives can, in principle, be determined from successive differentiation of the second order ordinary differential equation. Although $F(0)$ is one of the auxiliary conditions in the problem presently under investigation, it is evident that a difficulty arises because the second condition $F(\infty) = 0$ is specified at $\zeta = \infty$. For this reason we are led to consider the associated initial value problem

$$\frac{d^2F}{d\zeta^2} = - 2\zeta \exp(-\alpha F) \frac{dF}{d\zeta} \tag{29}$$

subject to

$$F(0) = 1 \tag{30}$$

and

$$\frac{dF(0)}{d\zeta} = \gamma \ , \tag{31}$$

where γ is to be chosen such that $F(\infty) = 0$. This value for γ can be computed from Eq. (27).

The second derivative can be determined directly from (29) from which it follows that

$$\frac{d^2F(0)}{d\zeta^2} = 0 \ .$$

The third derivative can be found by differentiation of Eq. (29)

$$\frac{d^3F}{d\zeta^3} = -\ 2 \left\{ \exp(-\alpha F)\frac{dF}{d\zeta} + \zeta\frac{d}{d\zeta}\left[\exp(-\alpha F)\ \frac{dF}{d\zeta}\right]\right\} , \tag{32}$$

from which it follows that

$$\frac{d^3F(0)}{d\zeta^3} = -\ 2\ \gamma\ \exp(-\alpha) \ .$$

Differentiating (32) and evaluating the result at $\zeta = 0$ yields

$$\frac{d^4F(0)}{d\zeta^4} = -2\alpha\gamma^2\ \exp(-\alpha) \ .$$

Similarly, it is found that

$$\frac{d^5F(0)}{d\zeta^5} = -6\alpha^2\gamma^3\ \exp(-\alpha) + 12\gamma\ \exp(-2\alpha) \ .$$

The approach could, in principle, be extended indefinitely; however, in practice it becomes rather tedious after the first few terms.

To fifth order terms the solution $F(\zeta)$ is given by

$$F(\zeta) = 1 + \gamma\zeta - \frac{\gamma\ \exp(-\alpha)}{3}\ \zeta^3 + \frac{\alpha\gamma^2\ \exp(-\alpha)}{6}\ \zeta^4$$

$$+ \frac{2\gamma\ \exp(-2\alpha) - \alpha^2\gamma^3\ \exp(-\alpha)}{20}\ \zeta^5 + \ldots \ . \tag{33}$$

The series representation (33) can be used to calculate $F(\zeta)$ for small values of ζ.

In the study of pulse penetration it is frequently of interest to know the time that it takes for the fields at a certain point to rise to a given value. For this purpose a convenient manipulation is the reversion of series. Given a series for the dependent variable y in terms of the independent variable x,

$$y = ax + bx^2 + cx^3 + dx^4 + ex^5 + \ldots .$$

Using a reversion of series one can write x as a function of y:

$$x = Ay + By^2 + Cy^3 + Dy^4 + Ey^5 + \ldots ,$$

where

$$A = \frac{1}{a} \; ; \; B = -\frac{b}{a^3} \; ; \; C = \frac{2b^2 - ac}{a^5} \; ; \; D = \frac{5abc - a^2d - 5b^3}{a^7} \; ;$$

$$E = \frac{6a^2bd + 3a^2c^2 + 14b^4 - 21ab^2c - a^3e}{a^9} \; ; \ldots .$$

For example, the classic linear solution (22) can be rewritten as

$$(1-F) = \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)n!} \zeta^{2n+1} = \frac{2}{\sqrt{\pi}} (\zeta - \frac{\zeta^3}{3} + \frac{\zeta^5}{10} - \ldots) .$$

A reversion of series yields

$$\zeta = \frac{\sqrt{\pi}}{2} \left[(1 - F) + \frac{\pi}{12} (1 - F)^3 + \frac{7\pi^2}{480} (1 - F)^5 + \ldots \right]$$

which is suitable for calculating the value of ζ at which the solution $F(\zeta)$ has a specified value for small values of $(1 - F)$ (which implies small ζ).

Equation (33) can be rewritten as

$$(1 - F) = -\gamma\zeta + \frac{\gamma \exp(-\alpha)}{3} \zeta^3 - \frac{\alpha\gamma^2}{6} \exp(-\alpha)\zeta^4$$

$$- \frac{2\gamma \exp(-2\alpha) - \alpha^2\gamma^3 \exp(-\alpha)}{20} \zeta^5 + \ldots .$$

A reversion of series yields

$$\zeta = \frac{1}{\gamma}\left[(1 - F) + \frac{\exp(-\alpha)}{3\gamma^2}(1 - F)^3 + \frac{\alpha \exp(-\alpha)}{6\gamma^2}(1 - F)^4 + \left(\frac{1}{20}\frac{\alpha^2\exp(-\alpha)}{\gamma^2} - \frac{7}{30}\frac{\exp(-2\alpha)}{\gamma^4}\right)(1 - F)^5 + \ldots\right]$$

which can be used [for small values of $(1 - F)$] to examine the variation with α of the value of ζ at which $F(\zeta)$ has a specified value.

NUMERICAL CALCULATIONS

For a specified value of β the corresponding value of α can be computed by numerical integration of (26) and the value of $dF(0)/d\zeta = \gamma$ can be calculated from (28). Some numerical calculations were performed and the results are shown in Table 1. An examination of Table 1 reveals that rather large changes in β correspond to relatively small changes in α. This situation can make it difficult to determine the value of β which corresponds to a specified value of α. On the other hand, it is evident that $dF(0)/d\zeta = \gamma$ exhibits a relatively moderate variation with α. While α varies in the range $0 \le \alpha \le \infty$, γ varies in the range $-2/\sqrt{\pi} \le \gamma \le 0$. This moderate variation makes it possible to accurately represent γ as a function of α using the regression equation

$$\frac{dF(0)}{d\zeta} = \gamma = -1.1284 + 2.0490 \times 10^{-1}\alpha - 3.4426 \times 10^{-2}\alpha^2 + 4.4139 \times 10^{-3}\alpha^3 - 4.1562 \times 10^{-4}\alpha^4 + 2.7025 \times 10^{-5}\alpha^5 - 1.0736 \times 10^{-6}\alpha^6 + 0.9395 \times 10^{-7}\alpha^7 - \ldots ,$$

which provides a convenient means of evaluating γ for $0 \le \alpha < 10$.

The variations of F and $dF/d\zeta$ with ζ were calculated for $\alpha = 0 (\beta = \infty)$, $\alpha = 2 (\beta = 3.959 \times 10^{-1})$, and $\alpha = 5 (\beta = 5.922 \times 10^{-3})$ and are shown in Figures 2a and 2b. The magnetic field strength H can be determined from (16) and the electric field from (20). In shielding applications, the time variation is often of interest; therefore the variations of F and $dF/d\zeta$ with $1/\zeta^2$ are shown in Figures 3a and 3b.

Table 1

Numerical Calculations of α and $dF(0)/d\zeta$.*

β	α	$\gamma = \frac{dF(0)}{d\zeta}$
∞	0	$-2/\sqrt{\pi}$
5×10^{4}	0.0112	-1.1260
1×10^{4}	0.0249	-1.1233
5×10^{3}	0.0351	-1.1212
1×10^{3}	0.0773	-1.1127
5×10^{2}	0.1083	-1.1066
1×10^{2}	0.2326	-1.0825
5×10^{1}	0.3197	-1.0662
1×10^{1}	0.6424	-1.0098
5×10^{0}	0.8475	-0.9769
1×10^{0}	1.5058	-0.8847
5×10^{-1}	1.8681	-0.8414
1×10^{-1}	2.8683	-0.7432
5×10^{-2}	3.3552	-0.7043
1×10^{-2}	4.5797	-0.6255
5×10^{-3}	5.1378	-0.5965
1×10^{-3}	6.4838	-0.5392
5×10^{-4}	7.0801	-0.5183
1×10^{-4}	8.4931	-0.4767
5×10^{-5}	9.1115	-0.4612
1×10^{-5}	10.5651	-0.4300

*The calculations were performed on a CDC 6600 computer using simple trapezoidal numerical integration.

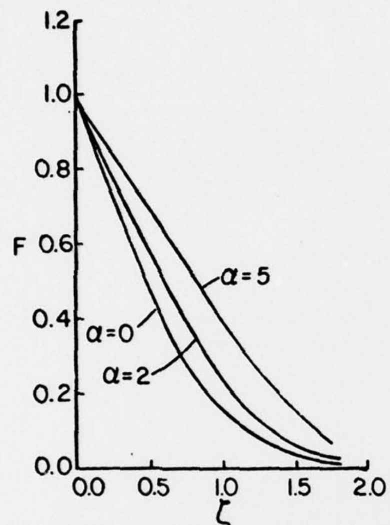

FIGURE 2a. F(ζ) VERSUS ζ

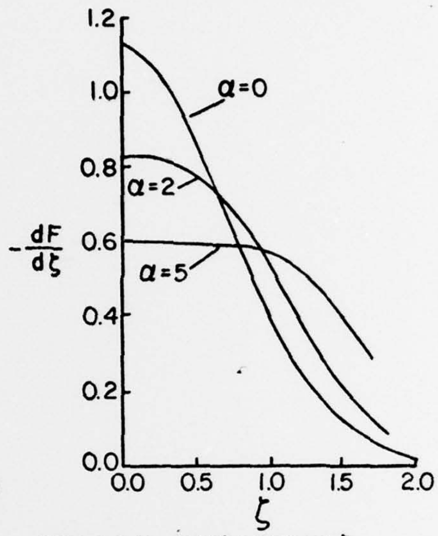

FIGURE 2b. dF/dζ VERSUS ζ

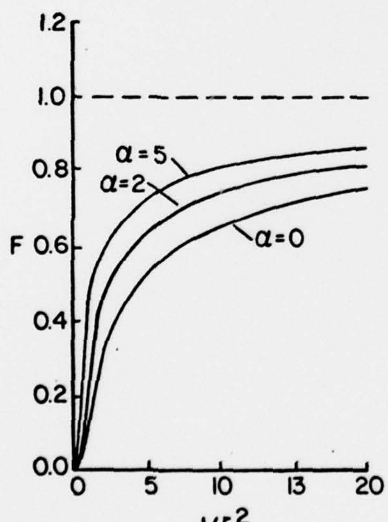

FIGURE 3a. F(ζ) VERSUS $1/\zeta^2$

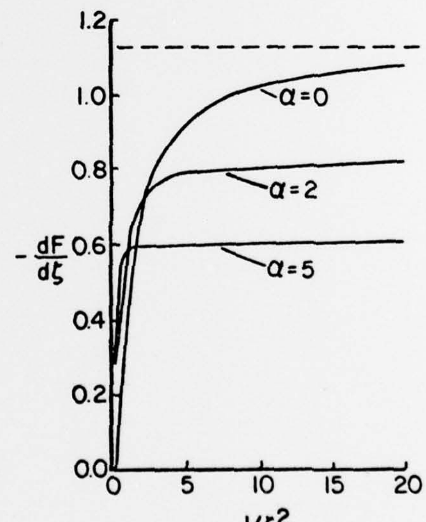

FIGURE 3b. dF(ζ)/dζ VERSUS $1/\zeta^2$

CONCLUSIONS

In the linear case ($\alpha = 0$), the solution $F(\zeta)$ is linearly scaled by H_o and the shape of the solution $F(\zeta)$ remains unchanged. Examination of the preceding analyses and numerical calculations reveals that the nonlinear case is more complicated. As might be expected, the results in the nonlinear case may not be simply scaled in the usual linear fashion. Although by the nature of the problem the final value H_o is the same, the manner in which this value is approached depends on the value of H_o. The variation in permeability with applied magnetic field strength in the present problem results in a more rapid penetration of the medium by the magnetic field. In other words, at a given position in the medium, the magnetic field strength H will reach a given percentage of the applied field H_o in a shorter time. Alternatively, at a given time the point at which the magnetic field strength has a prescribed value will have penetrated to a greater distance. In addition it is evident that for a given value of the parameter α the solution $F(\zeta)$ at some $\zeta = c$ will have the same value for all combinations of x and t which are related by $\zeta = c$. This indicates that if a certain value for the magnetic field strength H has reached location x at some time t, then the same value of H will occur at some location $x' = ax$ at some time $t' = a^2t$. It might be pointed out that for a given α, the solution $F(\zeta)$ will have reached a prescribed value F_p at some ζ_p. Thus, one can consider a "penetration thickness"

$$\delta_p \equiv \zeta_p(2/\sqrt{\sigma\mu_i})\sqrt{t}$$

beyond which the magnetic field strength has changed by less than 'p' percent of H_o. For example, the linear solution (22) has a value less than 0.01 when the error function argument is about 2. If it is necessary to calculate the electromagnetic fields in a shield of finite thickness, then the solution $F(\zeta)$ will be a good approximation if the "penetration thickness" is small with respect to the shield thickness.

REFERENCE

[1] J. Crank, *Mathematics of Diffusion*, Oxford University Press, London, 1964, p. 166-170.