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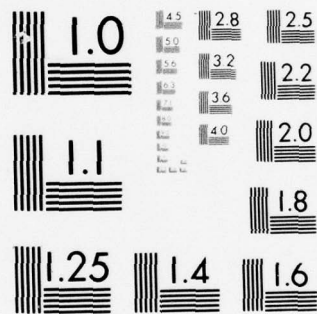
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THESIS

(6) COMPARISON OF FOUR SEQUENTIAL
PROBABILITY RATIO TESTS

by

(10) Sung Hwan/Wie

(11) March 1978

Thesis Advisor:

Donald R. Barr

Approved for public release; distribution
unlimited.

(12) 51p.

251 454

78 07 10 006

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Comparison of Four Sequential Probability Ratio Tests		5. TYPE OF REPORT & PERIOD COVERED Master's Thesis March 1978
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Sung Hwan Wie		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		12. REPORT DATE March 1978
		13. NUMBER OF PAGES 52
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Naval Postgraduate School Monterey, California 93940		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) An investigation of expected sample size ($E[N]$), variance of sample size ($V[N]$) and robustness of four sequential tests applicable to testing bombing system accuracy is made using computer simulation. Operating characteristics, $E[N]$, $V[N]$ and error rates for these tests are presented.		

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COMPARISON OF FOUR SEQUENTIAL
PROBABILITY RATIO TESTS

by

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Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
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ABSTRACT

An investigation of expected sample size ($E[N]$), variance of sample size ($V[N]$) and robustness of four sequential tests applicable to testing bombing system accuracy is made using computer simulation.

Operating characteristics, $E[N]$, $V[N]$ and error rates for these tests are presented.

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I. OBJECTIVE

In this thesis the Sequential Rayleigh Test and three sequential binomial tests, which are applicable to testing bombing system accuracy, are compared by computer simulation.

The objective of this thesis is to investigate expected sample size, variance of sample size, error rates of these tests, and to investigate their robustness.

II. DESCRIPTION OF THE TESTS

A. INTRODUCTION

There are two types of tests of a system (which is taken to be a bombing system in what follows). The sample size is determined as a direct result of the experiment in one case (and is therefore random), and the sample size is selected prior to commencing the test in the other case.

The former procedure is called a sequential test [1]. The test procedures being considered here are sequential, and are based on assumed circular normal distribution (that is, bivariate normal) with the same variance in each coordinate. In the coordinate system of the target plane it is assumed that:

$$\begin{aligned} X &\sim \text{Normal } (0, \sigma^2) \\ Y &\sim \text{Normal } (0, \sigma^2). \end{aligned}$$

The origin of coordinate system is the target, and X and Y are distances from the weapon impact to the target along the X and Y axes.

If it is further assumed that X and Y are independent, then $\frac{X}{\sigma}$ is normal with mean 0, variance 1. $\frac{Y}{\sigma}$ is normal with mean 0, variance 1. Thus $(\frac{X}{\sigma})^2 + (\frac{Y}{\sigma})^2$ has Chi-square distribution with two degrees of freedom.

The density function of Chi-square with two degrees of freedom is [2].

$$\begin{aligned} f_T(t) &= \frac{1}{2 \cdot \Gamma(1)} \cdot e^{-t/2}, \quad t > 0 \\ &= \frac{1}{2} e^{-t/2} \end{aligned}$$

This is the exponential density function with parameter $\lambda = \frac{1}{2}$. Here,

$\frac{X^2}{\sigma^2} + \frac{Y^2}{\sigma^2} = \frac{X^2 + Y^2}{\sigma^2}$, where $X^2 + Y^2$ is squared miss distance of the impact from the target. Let this be another random variable Z . Then,

$$\frac{Z}{\sigma^2} \sim \text{Exp}\left(\frac{1}{2}\right)$$

The density function of Z is derived as follows:

$$F_Z(z) = P [Z \leq z]$$

$$= P \left[\frac{Z}{\sigma^2} \leq \frac{z}{\sigma^2} \right].$$

Let $\frac{Z}{\sigma^2}$ be Z' ;

then $F_Z(z) = F_{Z'}\left(\frac{z}{\sigma^2}\right)$

Where Z' is exponentially distributed with parameter $\lambda = \frac{1}{2}$.

Then

$$F_{Z'}\left(\frac{z}{\sigma^2}\right) = 1 - \exp[-z/2 \cdot \sigma^2] = F_Z(z).$$

From the relation that $\frac{Z}{\sigma^2} = Z'$ the density function of Z is [5]

$$\frac{1}{2\sigma^2} \exp[-z/2 \cdot \sigma^2] = f_Z(z)$$

which is the exponential density function with parameter $\lambda = \frac{1}{2\sigma^2}$.

Assume C is the median of Z (C represents CEP^2 which is defined to be the median of the squared Circular Error Probable [6]).

Then by definition:

$$F_Z(C) = \frac{1}{2}$$

$$1 - \exp[-C/2 \cdot \sigma^2] = \frac{1}{2}$$

$$-\frac{C}{2\sigma^2} = \ln\left(\frac{1}{2}\right)$$

$$\sigma^2 = \frac{C}{2\ln 2}$$

Thus $Z \sim \text{Exp}\left(\frac{\ln 2}{C}\right)$

If a bombing system has been specified to have median radial miss distance γ_0 and if a system with radial miss distance γ_1 is unacceptable, this can be tested with $C_0 = \gamma_0^2$ as the null hypothesized median and $C_1 = \gamma_1^2$ as the alternative hypothesized median. Here, $\frac{\ln 2}{C_0}$, $\frac{\ln 2}{C_1}$ are the parameters under null and alternative exponential distributions, respectively.

In what follows C_0 will be treated as median under null hypothesis and C_1 as median under alternative, where these quantities relate to the squared radial miss distribution.

B. TEST CASE A: SEQUENTIAL RAYLEIGH TEST

If a system is to be tested with null hypothesis $H_0: CEP^2 = C_0$ and alternative hypothesis $H_1: CEP^2 = C_1$ with type I and type II error rate α and β , then a sequential test for H_0 against H_1 can be defined as follows. We shall call this the "Sequential Rayleigh Test" in what follows; it is an application of Wald's Sequential probability ratio test to the exponential situation described above.

Two positive constants are chosen, B and A , where $B < A$. At each stage (n -th experiment or observation), the probability

ratio $\prod_{i=1}^n \frac{f(z_i; C_1)}{f(z_i; C_0)}$, called the likelihood

ratio [3], is computed, where $f(z_i; C_j)$ is the exponential density function with parameter $\frac{\ln 2}{C_j}$, $j = 1, 2$

If $B < \prod_{i=1}^n \frac{f(z_i, C_1)}{f(z_i, C_0)} < A$, then another observation is made (this means the test enters the $(n+1)^{st}$ stage).

If the likelihood ratio does not fall in the interval (B, A) , called the continuation region, the test terminates.

In termination, the conclusion is to:

$$\text{Accept } H_0 \text{ if } \prod_{i=1}^n \frac{f(z_i, C_1)}{f(z_i, C_0)} \leq B;$$

$$\text{Reject } H_0 \text{ if } \prod_{i=1}^n \frac{f(z_i, C_1)}{f(z_i, C_0)} \geq A.$$

For a test with approximate level of significance α and power $1-\beta$, one may define [1]

$$A = \frac{1-\beta}{\alpha}, \text{ and } B = \frac{\beta}{1-\alpha}.$$

Thus, an approximate bound for each stage can be obtained as follows:

The explicit form of the likelihood ratio is

$$\begin{aligned} & \prod_{i=1}^n \frac{f(z_i, C_1)}{f(z_i, C_0)} \\ &= \frac{(\ln 2/C_1) \cdot \exp[-z_1 \cdot \ln 2/C_1] \cdot \dots \cdot (\ln 2/C_1) \cdot \exp[-z_n \cdot \ln 2/C_1]}{(\ln 2/C_0) \cdot \exp[-z_1 \cdot \ln 2/C_0] \cdot \dots \cdot (\ln 2/C_0) \cdot \exp[-z_n \cdot \ln 2/C_0]} \\ &= \frac{(\ln 2/C_1)^n \cdot \left(\prod_{i=1}^n z_i \right) \cdot \exp[-\ln 2/C_1]}{(\ln 2/C_0)^n \cdot \left(\prod_{i=1}^n z_i \right) \cdot \exp[-\ln 2/C_0]} \end{aligned}$$

$$= \left(\frac{C_o}{C_1} \right)^n \cdot e^{-\left(\sum_{i=1}^n z_i \right) \cdot \left[\frac{\ell n 2}{C_1} - \frac{\ell n 2}{C_o} \right]}$$

This is equated to A and the logarithm is taken to obtain rejection bound R_n for the n-th stage;

$$\ell n \left(\frac{1-\beta}{\alpha} \right) = n \cdot \ell n \left(\frac{C_o}{C_1} \right) - \left(\sum_{i=1}^n z_i \right) \cdot \left[\frac{\ell n 2}{C_1} - \frac{\ell n 2}{C_o} \right]$$

The test rejects H_o if

$$\sum_{i=1}^n z_i \geq \frac{-\ell n \left(\frac{1-\beta}{\alpha} \right) + n \cdot \ell n \left(\frac{C_o}{C_1} \right)}{(\ell n 2) \cdot \left(\frac{1}{C_1} - \frac{1}{C_o} \right)} = R_n$$

where $\sum_{i=1}^n z_i$ is sum of squared radial miss distances.

The same procedure can be applied to B:

$$\ell n \left(\frac{\beta}{1-\alpha} \right) = n \cdot \ell n \left(\frac{C_o}{C_1} \right) - \left(\sum_{i=1}^n z_i \right) \cdot \left(\frac{\ell n 2}{C_1} - \frac{\ell n 2}{C_o} \right)$$

The test accepts H_o if

$$\sum_{i=1}^n z_i \leq \frac{-\ell n \left(\frac{\beta}{1-\alpha} \right) + n \cdot \ell n \left(\frac{C_o}{C_1} \right)}{(\ell n 2) \cdot \left(\frac{1}{C_1} - \frac{1}{C_o} \right)} = A_n$$

Here, it is assumed $C_1 > C_o$, which is, we envision, true in the bombing system test.

C. CASE B: SEQUENTIAL BINOMIAL TEST WITH $P_0 = \frac{1}{2}$

In the study of bombing system the target is defined to be a point on the impact plane and impact of a bomb within (over) some distance $\sqrt{r}$ from the target is defined to be a hit (miss).

A null hypothesis that $CEP^2 = C_0$ is to be tested against an alternative hypothesis that $CEP^2 = C_1$, with Type I and Type II error rates α and β , where $C_0 < C_1$.

Also this system can be tested with $CEP^2 \leq C_0$ as null hypothesis and $CEP^2 \geq C_1$ as alternative hypothesis without altering the test procedure.

Let P_0 be defined as the probability of hit under the null hypothesis, and P_1 to be the hit probability under the alternative hypothesis. Then under H_0 , $CEP^2 \leq C_0$, which says the true median of squared radial miss distance is less than or equal to C_0 , implies the probability of hit is greater than or equal to 0.5.

Similarly, $CEP^2 \geq C_1$ implies the probability of hit is less than or equal to 0.5 under the alternative hypothesis. So the new hypotheses are defined as $H_0: P_0 \geq 0.5$ and $H_1: P_1 \leq 0.5$. From this hit or miss criterion r , a value of squared radial miss distance may be obtained which gives P_0 value of 0.5; i.e., if

$$F_z(r) = 1 - e^{-\frac{\ln 2}{C_0} \cdot r} = 0.5$$

$$\text{then } \ln(0.5) = (\ln 2) \cdot \left(-\frac{r}{C_0}\right), \text{ so } r = C_0.$$

The alternate hit probability P_1 is found as follows:

$$\begin{aligned} P_1 &= P_r [\text{hit} \mid \text{CEP}^2 = C_1] \\ &= P_r [\text{observed squared radial miss distance} \leq r \mid \text{CEP}^2 = C_1] \\ &= 1 - e^{-\frac{\ln 2}{C_1} \cdot r} \\ &= 1 - e^{-\frac{\ln 2}{C_1} \cdot C_0} = 1 - 2^{-\frac{C_0}{C_1}} \end{aligned}$$

Let the random variable Z_i be defined as:

$$Z_i = \begin{cases} 0 & \text{if miss (squared miss distance} > r) \\ 1 & \text{if hit (squared miss distance} \leq r) \end{cases}$$

Then the likelihood ratio becomes

$$\begin{aligned} \prod_{i=1}^n \frac{f(z_i, P_1)}{f(z_i, P_0)} &= \frac{(P_1)^Z (1-P_1)^{n-Z}}{(1/2)^Z (1/2)^{n-Z}} \\ &= \left(\frac{P_1}{1-P_1}\right)^Z \cdot (1-P_1)^n \cdot 2^n \end{aligned}$$

$$\text{Where } Z = \sum_{i=1}^n z_i$$

For a sequential probability ratio test for the binomial situation, the experiment is continued as long as this value remains between B and A.

An approximate acceptance boundary is found by substituting $\frac{\beta}{1-\alpha}$ for B [1].

$$\text{Then } \ln \left(\frac{\beta}{1-\alpha} \right) \geq \left(\sum_{i=1}^n z_i \right) \cdot \ln \left(\frac{P_1}{1-P_1} \right) + n \cdot \ln(2(1-P_1)).$$

Substituting $(1-2^{-C_0/C_1})$ for P_1 gives

$$\ln \left(\frac{\beta}{1-\alpha} \right) \geq \left(\sum_{i=1}^n z_i \right) \ln \left(\frac{1-2^{-C_0/C_1}}{2^{-C_0/C_1}} \right) + n \cdot \ln 2^{1-C_0/C_1}$$

Solving for $\sum_{i=1}^n z_i$, which is a convenient test statistic:

$$\sum_{i=1}^n z_i \geq \frac{\ln \left(\frac{\beta}{1-\alpha} \right) - n \cdot (1 - C_0/C_1) \cdot \ln 2}{\ln (2^{C_0/C_1} - 1)} = A_n$$

(The inequality changes because $C_0 < C_1$ implies $2^{C_0/C_1} - 1 \leq 0$.)

Where $\sum_{i=1}^n z_i$ is the number of bombs which hit the target, out of the total fire, n. And substituting $\frac{1-\beta}{\alpha}$ for A, an approximate rejection boundary is found to be.

$$\sum_{i=1}^n z_i \leq \frac{\ln \left(\frac{1-\beta}{\alpha} \right) - n \cdot \left(1 - \frac{C_0}{C_1} \right) \cdot \ln 2}{\ln \left(2^{C_0/C_1} - 1 \right)} = R_n$$

The test now operates as follows:

in stage n ,

Accept H_0 if $\sum_{i=1}^n z_i \geq A_n$;

Reject H_0 if $\sum_{i=1}^n z_i \leq R_n$;

Continue to stage $n+1$ otherwise.

D. CASE C: A SEQUENTIAL BINOMIAL TEST WITH NULL PARAMETER WHICH MINIMIZES $E[N]$ UNDER H_0 .

This case also has null hypothesis $CEP^2 \leq C_0$ and alternative $CEP^2 \geq C_1$. Let $P_0(r)$ be the probability of hitting a target of radius $\sqrt{r}$, and let $P_1(r)$ denote that probability under H_1 . In mathematical form:

$$P_0(r) = P_0(Z \leq r | CEP^2 = C_0) = 1 - e^{-\frac{\ln 2}{C_0} \cdot r}$$

$$P_1(r) = P_1(Z \leq r | CEP^2 = C_1) = 1 - e^{-\frac{\ln 2}{C_1} \cdot r}$$

Thus: $P_1(r) = 1 - [1 - P_0(r)]^{C_0/C_1}$

It is desired to determine r so as to minimize expected sample size n , required to test H_0 vs H_1 with Type I, II error rates α, β respectively, using the Binomial Sequential Probability Ratio test.

The average sample size function is

$$E(N) \cong \frac{(1-L(P)) \cdot \ln A + L(P) \cdot \ln B}{P \ln \left(\frac{P_1(r)}{P_0(r)} \right) + (1-P) \ln \left(\frac{1-P_1(r)}{1-P_0(r)} \right)}, \quad [1], [4]$$

where P is the true probability of hit and $L(P) = P_r[\text{accept } H_0 | P]$.

If $P_0(r)$ is chosen to be the underlying hit probability, then

$$L(P_0(r)) = 1 - \alpha.$$

By substituting $L(P_0(r)) = 1 - \alpha$,

$$A = \frac{1-\beta}{\alpha}, \quad P_1(r) = 1 - (1-P_0(r))^{C_0/C_1}, \quad B = \frac{\beta}{1-\alpha}$$

in $E[N]$ function, the expression above becomes

$$E(N) \cong \frac{\alpha \ln \left(\frac{1-\beta}{\alpha} \right) + (1-\alpha) \ln \left(\frac{\beta}{1-\alpha} \right)}{P_0(r) \left[\ln \left(\frac{1 - (1-P_0(r))^{C_0/C_1}}{P_0(r)} \right) \right] + (1-P_0(r)) \ln [1 - P_0(r)]^{-1+C_0/C_1}}$$

If a value of $P_0(r)$ which minimizes $E(N)$ is obtained, r is also obtained from $P_0(r)$. The numerator is a negative constant as long as $\beta < 0.5$ and $\alpha < 0.5$ and the denominator is a negative variable depending on r .

Therefore, in order to minimize $E(N)$ it is necessary only to maximize the absolute value of the denominator.

$$\text{Let } \frac{C_0}{C_1} = \frac{1}{k}$$

Then for various values of K , $P_0(r)$ may be found.

Figure II shows a plot of $P_0(r)$ vs k resulting from this minimization. As k approaches infinity $P_0(r)$ approaches 1.0; as k decreases to zero $P_0(r)$ decreases to around 0.63. Specifically, for $k = 2$

$$P_0(r) = 0.8416, P_1(r) = 0.6020.$$

From this r is found to be

$$-C_0 \cdot \frac{\ln(0.1584)}{\ln 2} = 2.65836 \cdot C_0$$

Let Z be a random variable such that

$$Z_i = \begin{cases} 0 & \text{if squared miss distance} > r \\ 1 & \text{if squared miss distance} \leq r \end{cases}$$

where $r = 2.65836$.

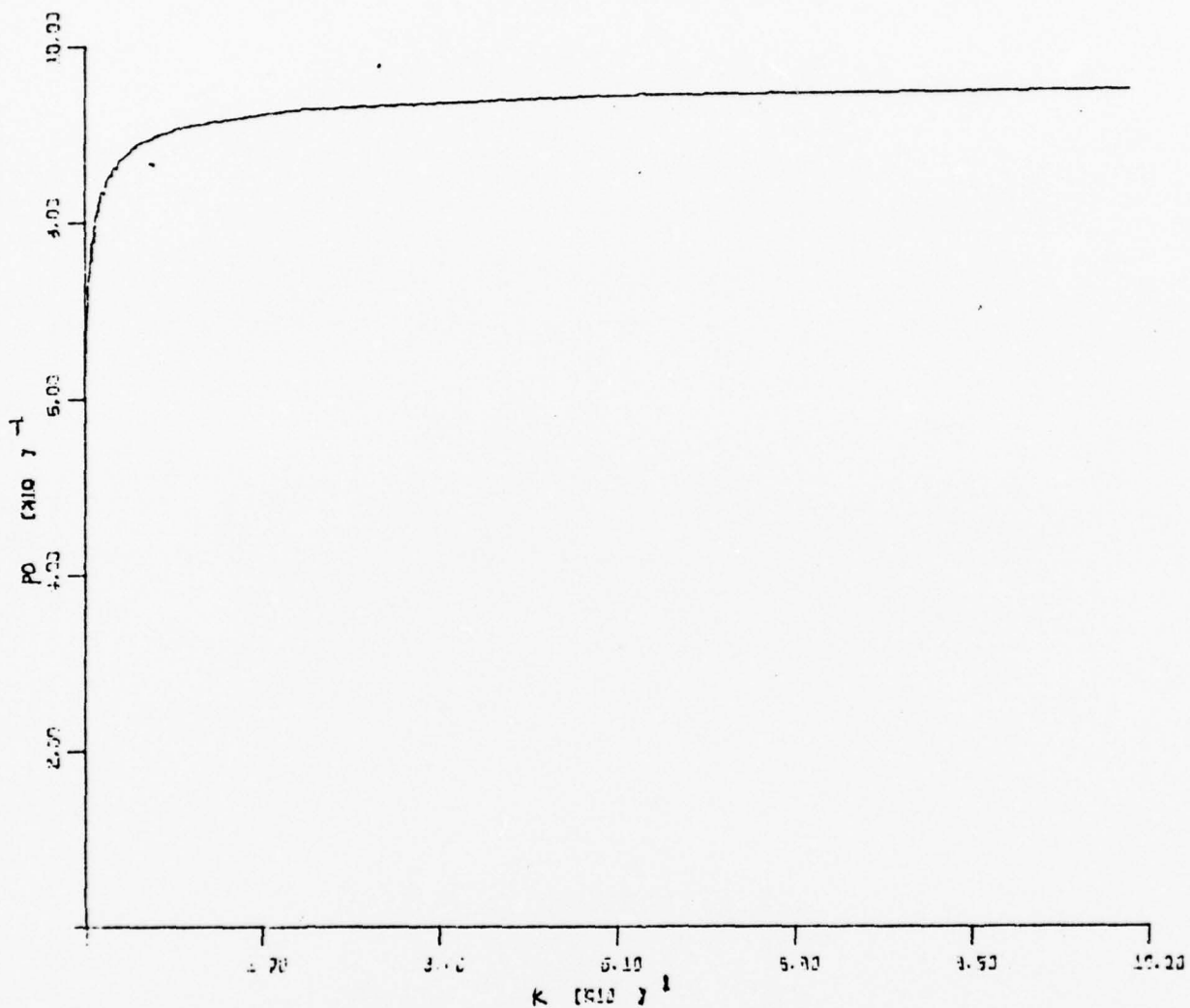
Then the hypotheses in this Binomial test becomes:

$$H_0: P_0 = 0.8416, H_1: P_1 = 0.602.$$

If $P_0 = 0.8416$, $P_1 = 0.602$ are substituted in the likelihood ratio (similar to Case B above), acceptance and rejection bounds at stage n are:

Accept H_0 if

$$\sum_{i=1}^n Z_i \geq -1.2564 \cdot \ln\left(\frac{\beta}{1-\alpha}\right) + n \cdot 0.7333 = A_n$$



K	0.001	0.1	0.3	0.5	0.90	1.01	1.414	1.5
P_O	0.632	0.655	0.707	0.745	0.789	0.800	0.820	0.824
K	2.0	3.0	5.0	7.0	10.0	20.0	50.0	100.0
P_O	0.842	0.863	0.886	0.899	0.910	0.928	0.944	0.953

FIGURE II-1

Reject H_0 if

$$\sum_{i=1}^n Z_i \leq -1.2564 \cdot \ln \left(\frac{1-\beta}{\alpha} \right) + n \cdot 0.7333 = R_n$$

Continue otherwise.

E. CASE D: A Sequential Binomial test with null parameter
which minimizes maximum value of $E(N)$

In the binomial situation described above, the hypotheses
about the parameter are

$$H_0: P = P_0$$

$$H_1: P = P_1.$$

The maximum average value of $E(N)$ occurs very nearly at

$$P^* = \frac{\ln \left(\frac{1-P_0(r)}{1-P_1(r)} \right)}{\ln \left(\frac{P_1(r)}{P_0(r)} \right) - \ln \left(\frac{1-P_1(r)}{1-P_0(r)} \right)} \quad [4]$$

from which

$$\begin{aligned} E(N) &\approx \frac{\ln A \cdot \ln B}{\ln \left(\frac{P_1(r)}{P_0(r)} \right) - \ln \left(\frac{1-P_1(r)}{1-P_0(r)} \right)} \\ &= \frac{\ln A \cdot \ln B}{\ln \left[\frac{1-(1-P_0(r))^{C_0/C_1}}{P_0(r)} \right] \cdot \ln \left[\frac{(1-P_0(r))^{C_0/C_1}}{1-P_0(r)} \right]} \end{aligned}$$

$$(P_1(r) = 1 - [1-P_0(r)]^{C_0/C_1}, \text{ See Case B})$$

The numerator is a negative constant for given α and β .

Similarly as in Case C by maximizing the absolute value of the denominator, the maximum value of $E[N]$ is minimized (or at least nearly so).

Figure II-2 shows the relation between $P_0(r)$ and $k = \frac{C_1}{C_0}$. Specifically for $k = 2$
 $P_0(r) = 0.89867$, $P_1(r) = 0.68167$ with $r = - \frac{\ln(1-0.89867)}{\ln 2} = 3.30286$ is obtained.

Hence the hypotheses for this case are $H_0: P=0.89867$, $H_1: P=0.68167$, and "hit" is defined by squared radial miss distance less than or equal to 3.30286.

Let Z be a random variable such that

$$Z_i = \begin{cases} 0 & \text{if squared miss distance} > 3.30286 \\ 1 & \text{if squared miss distance} \leq 3.30286. \end{cases}$$

Proceeding similarly as in Case B, the decision at stage n is

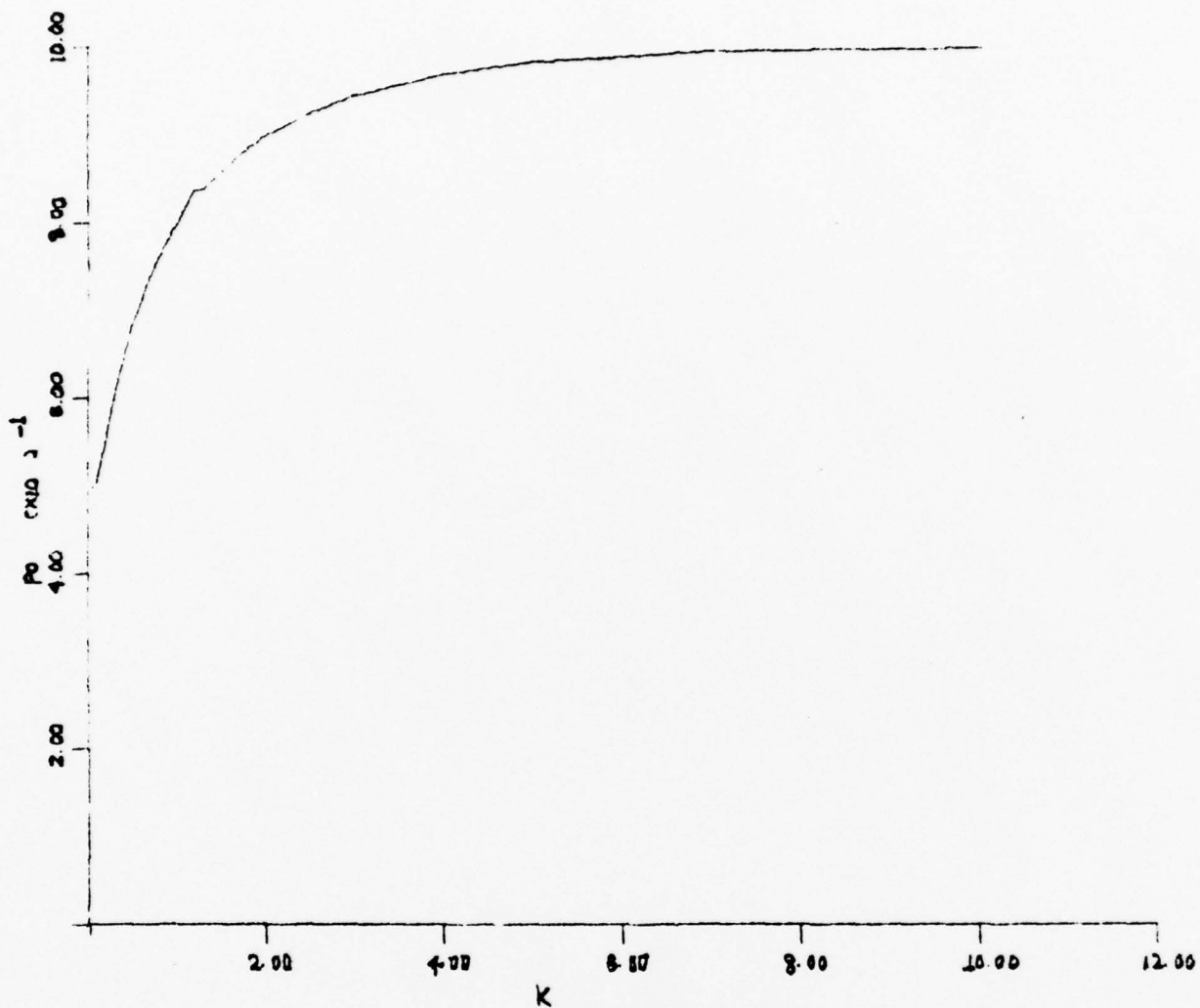
Reject H_0 if

$$\sum_{i=1}^n Z_i \leq -1.42167 \cdot \ln\left(\frac{1-\beta}{\alpha}\right) + n \cdot 0.80552 = R_n$$

Accept H_0 if

$$\sum_{i=1}^n Z_i \geq -1.42167 \cdot \ln\left(\frac{\beta}{1-\alpha}\right) + n \cdot 0.80552 = A_n$$

Continue to state $n+1$ otherwise.



K	0.05	0.1	0.3	0.5	0.7	0.9	1.01	1.414
P_O	0.5	0.504	0.602	0.681	0.738	0.780	0.794	0.851
K	1.5	2.0	2.5	3.0	4.0	5.0	7.0	10.0
P_O	0.859	0.899	0.925	0.944	0.968	0.982	0.994	0.999

FIGURE II-2

III. DETERMINATION OF SIMULATION FACTORS

For convenience the hypotheses

$$H_0: CEP^2 = 1$$

$$H_1: CEP^2 = 2 \text{ were selected.}$$

This means $k = 2$ in Case C and D.

Type I and Type II error rates were selected to be 0.05. But actual simulation gives Type I error rates .0258, .0468, .0396, .0292 and Type II error rates .0396, .0402, .0420, .0400 for Case A, B, C, and D, respectively. This point is explained in Chapter 3.3 of [1]. Approximate error rates of 0.05, 0.05 (Type I, II) were obtained by adjusting the bounds A and B, which is possible by changing α, β in $A = \frac{1-\beta}{\alpha}$, $B = \frac{\beta}{1-\alpha}$. The adjustment factors used here are:

$$\text{CASE A } \alpha = 2.0 \times 0.05 \quad \beta = 1.2 \times 0.05$$

$$\text{CASE B } \alpha = 1.2 \times 0.05 \quad \beta = 1.2 \times 0.05$$

$$\text{CASE C } \alpha = 1.5 \times 0.05 \quad \beta = 1.15 \times 0.05$$

$$\text{CASE D } \alpha = 1.5 \times 0.05 \quad \beta = 1.1 \times 0.05$$

How many replications are enough? Assuming N_1, N_2 have Binomial distribution with probability of success 0.05, a sample size n is found such that $P_r(|\hat{P}_1 - \hat{P}_2| < c) = 1 - \alpha$, where α is the significance level, $\hat{P}_i, i=1,2$ is the estimation of P_i . The above equation implies

$$P_r(-c < \hat{P}_1 - \hat{P}_2 < c) = 1 - \alpha$$

$$P_r(-c < \frac{N_1}{n} - \frac{N_2}{n} < c) = 1 - \alpha$$

$$P_r(-nc < \frac{N_1}{1} - \frac{N_2}{2} < nc) = 1 - \alpha$$

By the Normal approximation , $(N_1 - N_2) \sim$ Normal with mean 0 and
Variance $2np(1-p)$.

Thus

$$P_r \left[- \frac{nc}{\sqrt{2np(1-p)}} < \frac{N_1 - N_2}{\sqrt{2np(1-p)}} < \frac{nc}{\sqrt{2np(1-p)}} \right] = 1-\alpha$$

gives

$$\frac{nc}{\sqrt{2np(1-p)}} = z_{1-\frac{\alpha}{2}}$$

The following table shows n for various values of α and c.

α $z_{1-\frac{\alpha}{2}}$	0.2	0.1	0.05	0.025	0.02
c	1.285	1.645	1.96	2.24	2.33
0.2	3.9	6.4	9.1	11.9	12.8
0.1	15.7	25.7	36.1	47.6	51.5
0.05	62.7	102.8	145.9	190.7	206.3
0.025	250.9	411.3	583.9	762.7	825.2
0.01	1568.7	2570.7	3649.5	4766.7	5157.5
0.001	156866.4	257072.3	361237.5	476672.0	515745.5

Assuming $c = 0.01$ and $\alpha = 0.02$, an approximate value of 5000 is obtained through the table. By this number of replications, obtaining a difference in estimated error rates greater than 0.01 is significant at level 0.02. Exponential random samples were generated by the Monte Carlo method. For an exponential variate T to have median m , it is necessary to use the scale parameter $\lambda = \frac{\ln 2}{m}$

$$\text{so } F_T(t) = 1 - e^{-\frac{\ln 2}{m} \cdot t}$$

But $U = F_T(T)$ is uniformly distributed on $(0,1)$ [8].

Thus $2^{-\frac{T}{m}} = 1 - F(T) = 1 - U$ is also uniformly distributed.

Finally $\frac{T}{m} \cdot \ln 2 = -\ln U$, or

$$T = -m \cdot \frac{\ln U}{\ln 2}.$$

By changing the median of the population sampled, the operating characteristic function, expected sample size at termination, and its variance can be estimated for each test, and those can be compared.

The medians to be generated are those which result in a 0.1 difference of operating characteristic function values in Case C, which is based on minimizing the maximum $E(N)$, those which yield Type I, II error rates, three points which yield approximate maximum $E(N)$ value (for Cases B, C, D), and five more points in both tails.

By changing the skewness of the sample distribution, comparison of the robustness of the four tests of hypotheses about the medians under null and alternative hypothesis was performed.

This is based on the assumption that the underlying distribution is WEIBULL with shape parameter α and scale parameter λ .

If $T \sim \text{WEIBULL}\{\alpha, \lambda\}$,
the distribution function of T is

$$F_T(T) = 1 - e^{-(\lambda T)^\alpha}$$

Let m be the median. Then

$$P_r[T \geq m] = P_r[T \leq m];$$

$$1 - e^{-(\lambda m)^\alpha} = 0.5,$$

$$\ln(0.5) = -(\lambda m)^\alpha.$$

Solving for λ ,

$$\lambda = \frac{1}{m} [\ln 2]^{1/\alpha}$$

Now $1 - e^{-(\lambda T)^\alpha} \sim U(0,1)$, so

$$\ln(1-U) = -(\lambda T)^\alpha,$$

$$T = \frac{1}{\lambda} [\ln(U)]^{\frac{1}{\alpha}}, \text{ where } U \sim U(0,1).$$

Substituting $\lambda = \frac{1}{m} (\ln 2)^{1/\alpha}$

$$T = m \cdot \left(\frac{\ln U}{\ln 2}\right)^{\frac{1}{\alpha}} \sim \text{WEIBULL}\left\{\alpha, \frac{1}{m} (\ln 2)^{1/\alpha}\right\}.$$

Following histograms in Figure III-1 show the effect of changing α in WEIBULL. As α decreases the distribution is widely spread and it is said that the distribution has heavy tail (Figure III-1, B). In the opposite case; i.e. α increases, it has light tail (Figure III-1, A).

Figure III-2 is observed keeping α fixed at 1 and median is 0.5, 1.0, 2.0 in WEIBULL.

This is exponential distribution with median 0.5, 1.0, 2.0.

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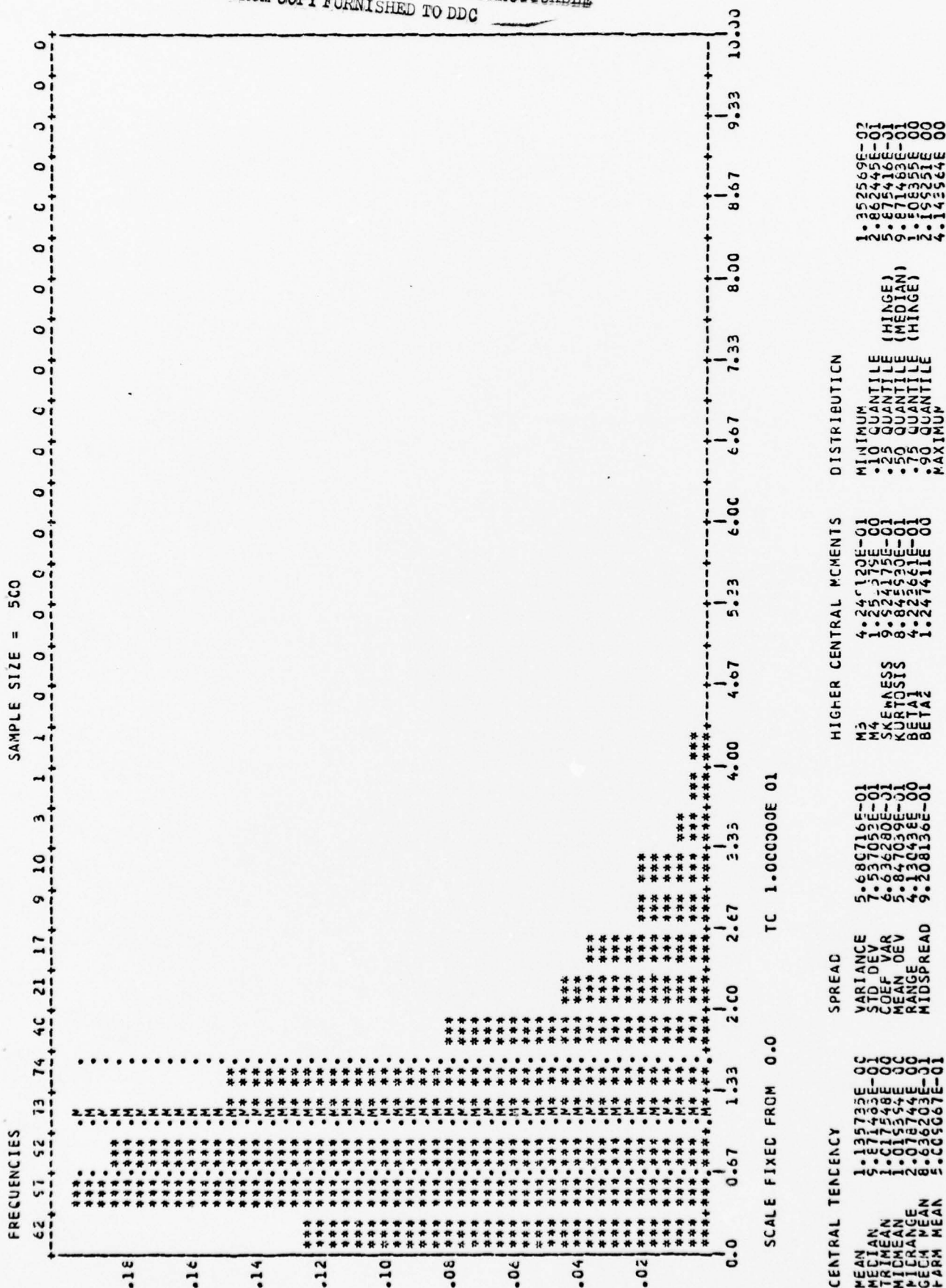


FIGURE III-1A, WEIBULL{1.5, 0.7832}

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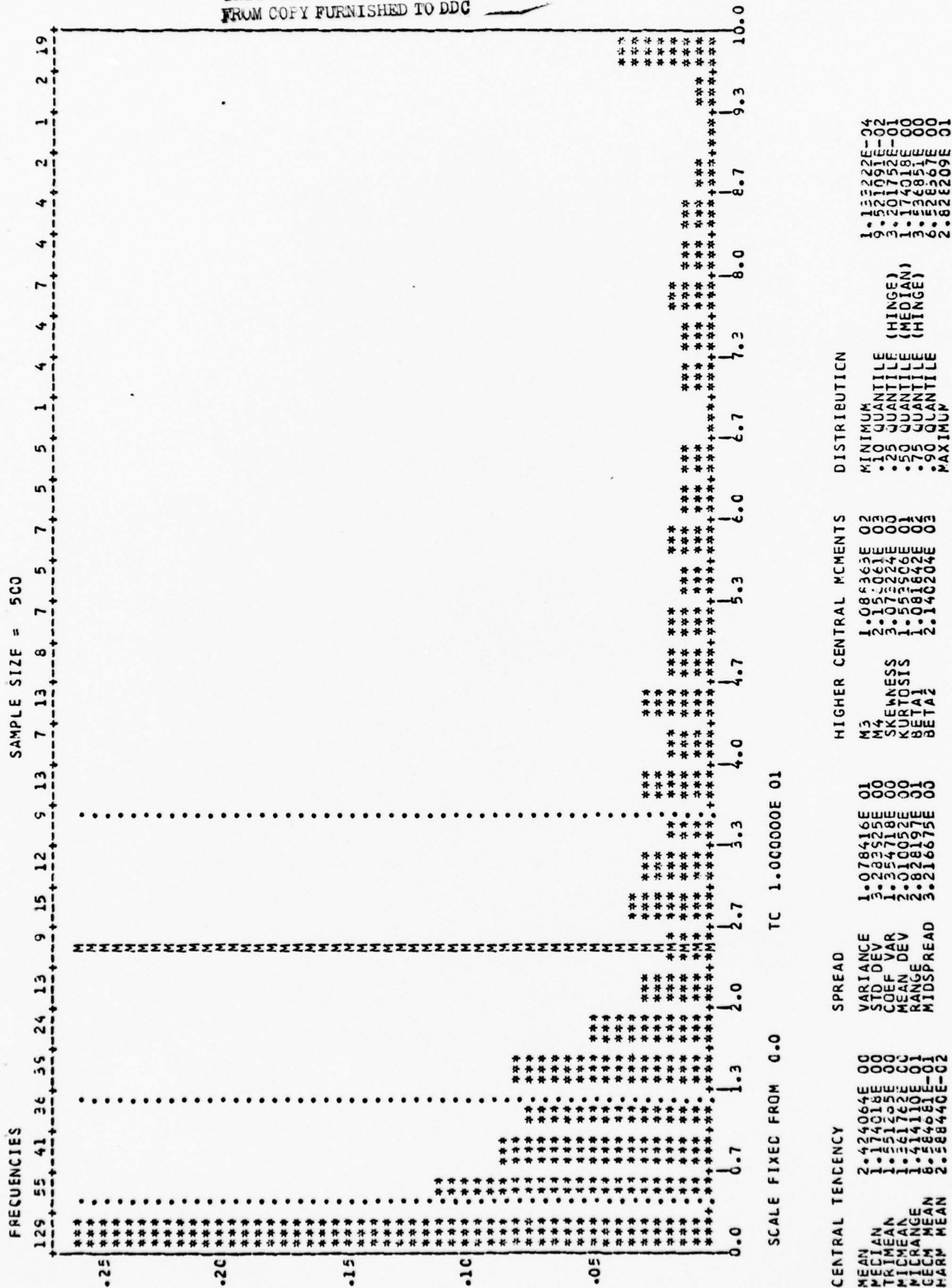


FIGURE III-1B, {WEIBULL 0.7, 0.5923}

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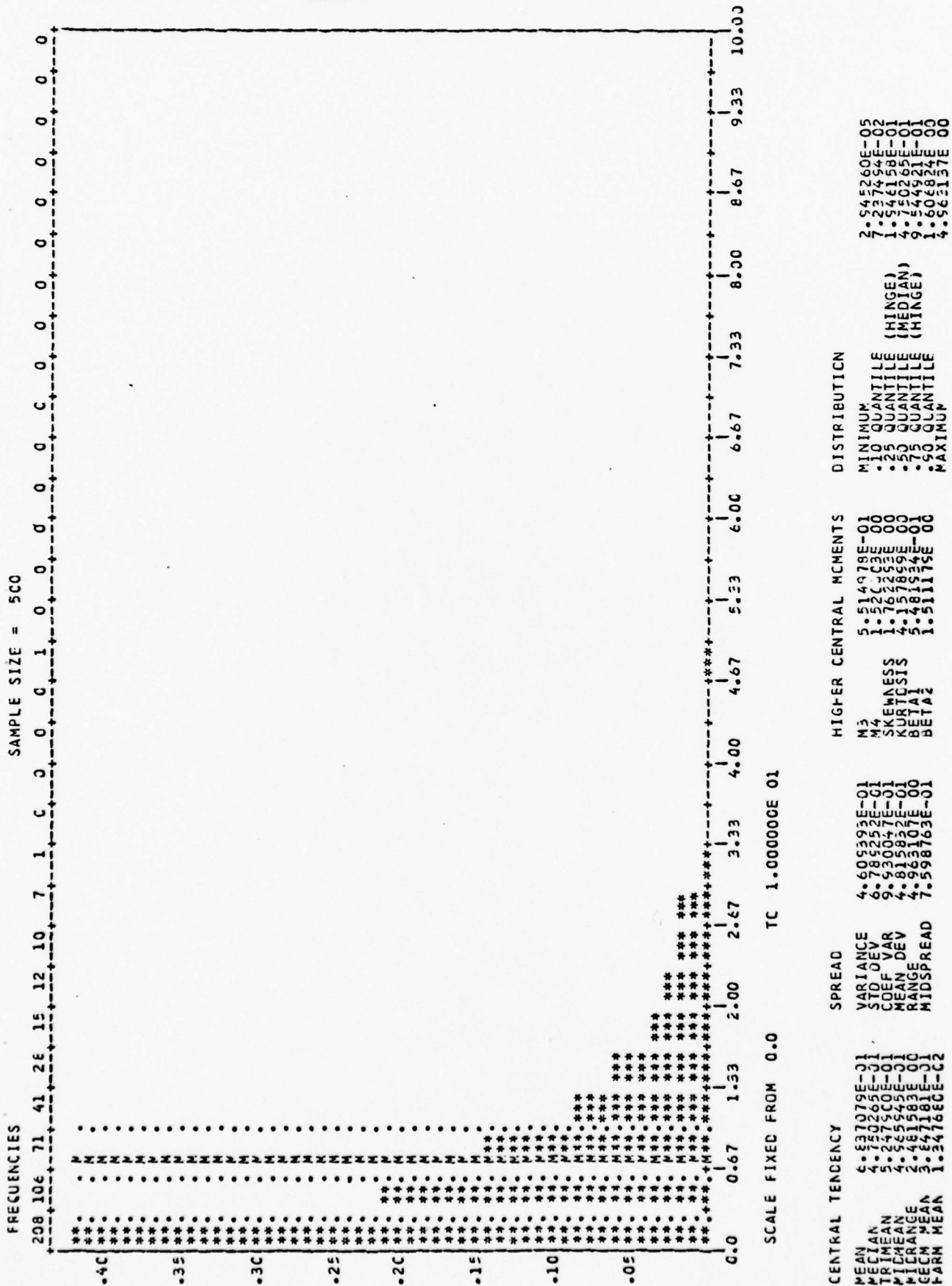
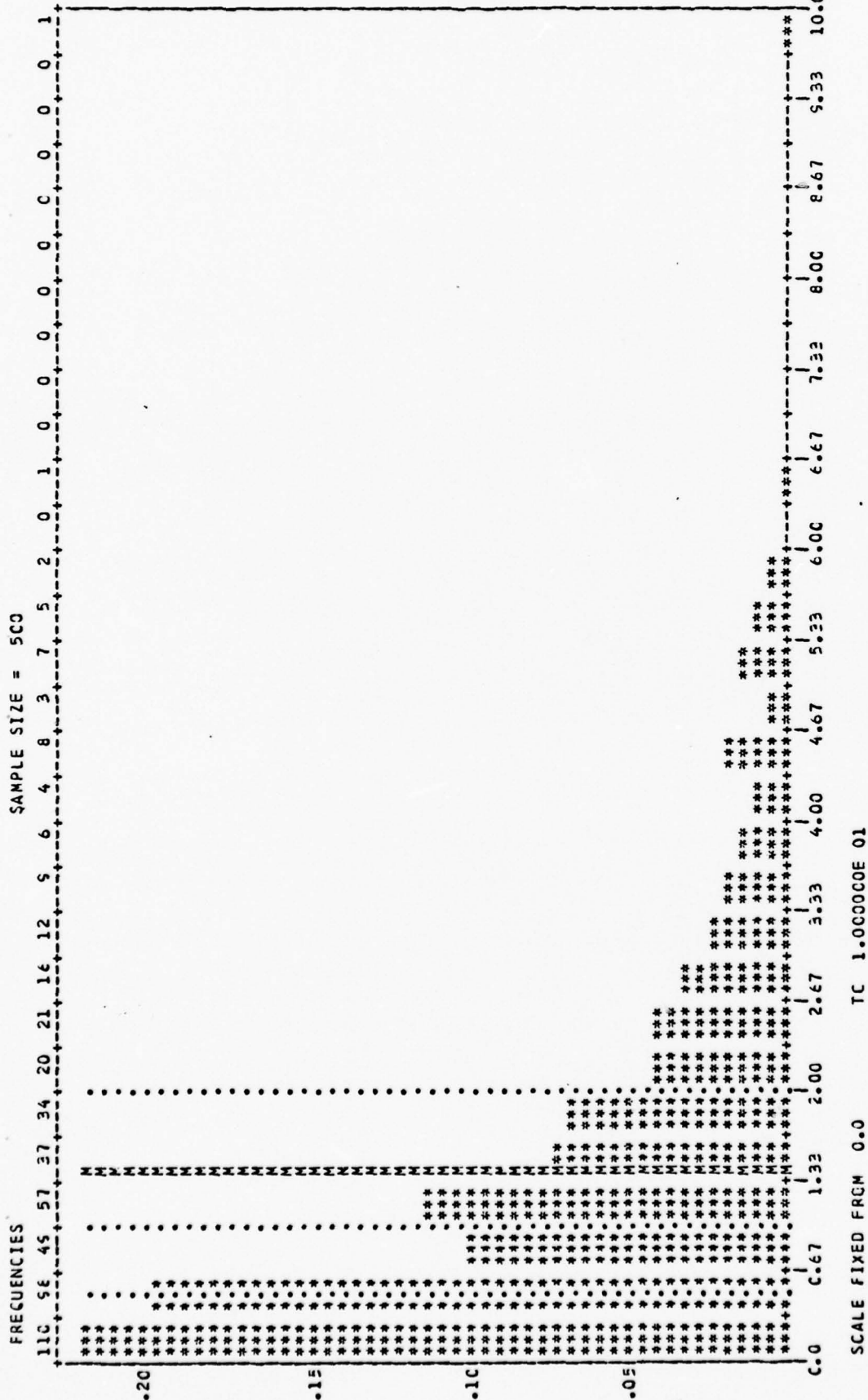


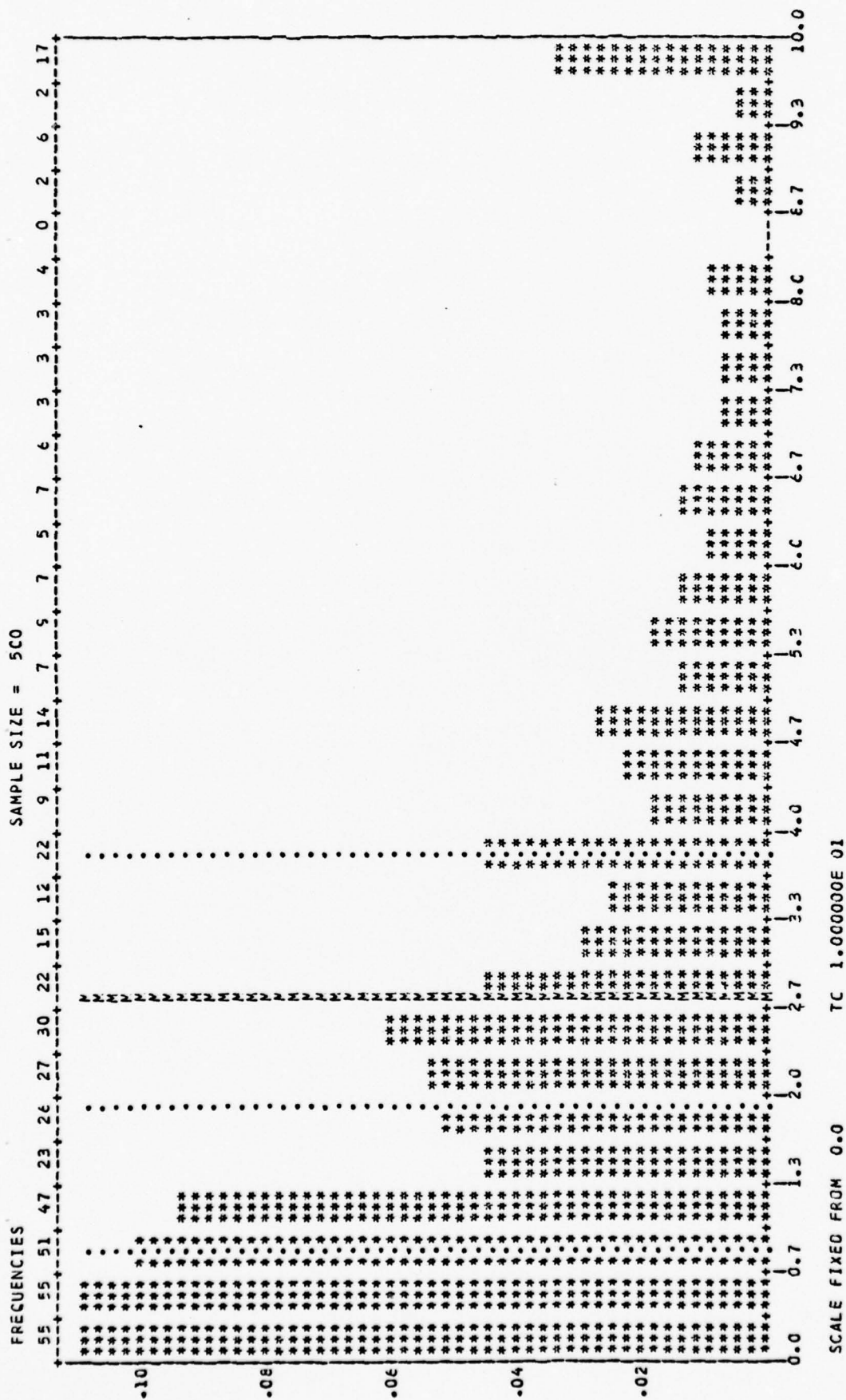
FIGURE III-2A, WEIBULL{1,1.3863}

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CENTRAL TENDENCY		SPREAD		HIGHER CENTRAL MOMENTS		DISTRIBUTION	
MEAN	1.367415E-01	VARIANCE	1.843758E-00	M3	4.411988E-00	MINIMUM	5.890521E-05
MEAN	1.367415E-01	STD DEV	1.357850E-01	M4	2.431387E-01	.10 QUANTILE	1.447379E-01
TRIMEAN	1.604558E-01	COEF VAR	9.930047E-01	SKEWNESS	1.762229E-00	.25 QUANTILE	1.592337E-01
MEAN	1.367415E-01	MEAN DEV	9.930047E-01	KURTOSIS	1.192730E-00	.50 QUANTILE	1.592337E-01
MEAN	1.367415E-01	RANGE	9.930047E-01	BETA1	2.332533E-01	.75 QUANTILE	1.592337E-01
MEAN	1.367415E-01	MIDSPREAD	1.515151E-00	BETA2	2.417888E-01	.90 QUANTILE	1.592337E-01
MEAN	1.367415E-01					MAXIMUM	1.592337E-01

FIGURE III-2B, WEIBULL(1,0.6931)



CENTRAL TENDENCY		SPREAD		HIGHER CENTRAL MOMENTS		DISTRIBUTION	
MEAN	2.74832E-04	VARIANCE	7.37503E-00	M3	3.52323E-01	MINIMUM	1.17810E-04
MECIAN	2.74832E-04	STD DEV	2.71107E-01	M4	3.52323E-02	.10 QUANTILE	2.85455E-01
MEAN	2.74832E-04	COEFF VAR	9.93004E-01	SKENESS	1.76222E-00	.25 QUANTILE	7.78463E-01
MECRANGE	2.74832E-04	MEAN DEV	1.92263E-01	KURTOSIS	4.15785E-01	.50 QUANTILE	1.90010E-00
MECUM MEAN	2.74832E-04	RANGE	1.92263E-01	BETA1	3.50443E-01	.75 QUANTILE	3.61756E-00
MECUM MEAN	2.74832E-04	MIDSPREAD	3.03550E-01	BETA2	3.86661E-02	.90 QUANTILE	6.42725E-00
MECUM MEAN	2.74832E-04					MAXIMUM	1.98525E-01

FIGURE III-2C, WEIBULL{1,0.3466}

IV. RESULTS AND COMMENTS

Simulation of the four tests were performed 5,000 times (5,000 replications). In each replication $H_0: CEP^2=1$; $H_1: CEP^2=2$; and Type I, II error rates are 0.05.

The following tables IV-1,2,3 show the mean number of terminations, their variances, and number of acceptances of H_0 in 5,000 replications.

Table IV-1 shows the effects of changing median values, where the shape parameter of the WEIBULL variate was fixed at 1.0 (the exponential variate with changing median). Table IV-2 and 3 shows the effects of changing shape parameter, the median being fixed at 1.0 in table IV-2 and at 2.0 in table IV-3.

For ease of comparison these were graphed. Each figure contains four curves and each curve represents the case A, B, C or D.

In testing the difference of $E[N]$, null hypothesis would be $E[N_1] = E[N_2]$, where N_1, N_2 are random variables representing the number of termination by test cases which one wishes to test the difference of the $E[N]$.

By the central limit theorem [2] the difference in $E[N]$ greater than $\frac{1.96 \times \hat{\sigma}}{n}$ is significant at level 0.05. $\hat{\sigma}$ is estimated standard deviation of N . But it is different between the test cases being considered.

The larger one might be selected so that we may not reject H_0 erroneously.

THIS CASE IS OBSERVED CHANGING MEDIAN AND SHAPE PARAMETER IS FIXED AT 1.

MEDIAN	CASE A			CASE B			CASE C			CASE D		
	E(N)	V(N)	# ACCEPT	E(N)	V(N)	# ACCEPT	E(N)	V(N)	# ACCEPT	E(N)	V(N)	# ACCEPT
0.54	7.035	2.530	5000.	10.382	17.428	5000.	10.294	6.230	4999.	11.874	4.756	5000.
0.67	8.350	7.245	4996.	13.605	47.889	4997.	11.830	15.527	4998.	13.151	14.061	4997.
0.84	10.646	23.490	4950.	19.569	156.983	4953.	15.145	58.313	4941.	16.064	45.647	4936.
1.00	13.728	63.682	4745.	28.178	408.287	4771.	19.803	142.636	4752.	20.176	131.629	4744.
1.08	15.470	94.006	4487.	33.284	620.832	4484.	22.603	235.429	4489.	22.914	214.950	4538.
1.19	17.843	174.842	3944.	39.552	967.506	3935.	25.780	344.258	3957.	26.317	345.588	4030.
1.27	18.444	189.203	3426.	43.717	1232.651	3404.	27.081	419.085	3474.	27.707	428.855	3503.
1.33	18.805	201.792	2898.	45.522	1328.232	2917.	28.445	504.042	2987.	28.615	484.615	3037.
1.39	18.892	220.836	2530.	46.051	1360.952	2468.	28.027	507.703	2545.	28.248	487.113	2575.
1.40	18.395	214.427	2418.	47.216	1362.500	2433.	27.763	473.886	2399.	27.808	493.729	2506.
1.40	18.159	203.286	2284.	46.197	1558.806	2365.	28.192	510.770	2381.	27.746	456.406	2448.
1.47	17.468	195.246	1971.	45.525	1275.995	1938.	27.178	487.229	1945.	27.320	456.467	2026.
1.55	16.244	186.480	1423.	43.445	1168.529	1423.	25.548	440.333	1485.	25.638	447.553	1524.
1.65	14.138	133.647	951.	39.535	908.235	909.	22.901	349.643	966.	22.891	351.952	955.
1.82	11.882	96.718	501.	34.765	603.266	475.	19.117	215.990	470.	18.667	226.866	503.
2.00	9.432	63.624	257.	30.126	428.890	236.	15.676	153.003	234.	15.529	157.854	226.
2.45	6.453	26.128	58.	21.725	150.938	70.	11.052	54.749	70.	10.482	56.675	42.
2.67	5.476	17.995	37.	20.003	124.732	33.	9.810	39.096	32.	9.135	41.058	23.
3.25	4.128	9.233	6.	16.328	62.552	7.	7.792	20.556	5.	7.055	17.663	4.
3.45	3.843	7.617	3.	15.635	53.751	4.	7.282	15.668	1.	6.647	14.994	5.

TABLE IV-1

THIS CASE IS OBSERVED CHANGING SHAPE PARAMETER AND MEDIAN IS FIXED AT 1.

MEDIAN	CASE A			CASE B			CASE C			CASE D		
	E(N)	V(N)	# ACCEPT	E(N)	V(N)	# ACCEPT	E(N)	V(N)	# ACCEPT	E(N)	V(N)	# ACCEPT
0.50	6.139	23.478	1001.	27.915	395.861	4742.	23.350	374.262	1057.	18.571	227.114	492.
0.60	8.654	46.590	1569.	27.818	418.116	4738.	27.007	442.582	1828.	24.335	382.756	1161.
0.70	11.147	73.146	2389.	27.631	398.115	4715.	28.510	525.863	2868.	27.450	485.572	2274.
0.80	13.616	95.577	3405.	27.516	381.577	4722.	26.742	390.584	3794.	28.004	474.156	3488.
0.90	14.415	90.572	4277.	28.033	400.513	4726.	22.555	250.668	4420.	24.262	268.240	4341.
1.00	13.728	63.682	4745.	28.178	408.287	4771.	15.803	142.636	4752.	20.176	131.635	4744.
1.10	12.755	40.668	4948.	28.351	435.550	4766.	16.602	81.464	4891.	17.214	70.285	4898.
1.20	11.679	24.907	4982.	28.020	396.977	4731.	14.586	47.508	4559.	15.006	34.312	4977.
1.30	10.507	15.607	5000.	28.154	421.004	4771.	13.094	26.650	4982.	13.421	16.557	4992.
1.40	10.420	11.415	5000.	28.234	412.444	4746.	11.892	17.057	4998.	12.621	10.211	4999.
1.50	10.056	8.504	5000.	27.924	391.072	4712.	11.043	10.840	4998.	11.950	5.687	5000.

TABLE IV-2

THIS CASE IS OBSERVED CHANGING SHAPE PARAMETER AND MEDIAN IS FIXED AT 2.

MEDIAN	CASE A			CASE B			CASE C			CASE D		
	E(N)	V(N)	# ACCEPT	E(N)	V(N)	# ACCEPT	E(N)	V(N)	# ACCEPT	E(N)	V(N)	# ACCEPT
0.50	3.573	10.354	206.	46.124	1307.159	2250.	12.237	77.664	86.	9.791	44.278	30.
0.60	4.650	15.043	193.	44.425	1221.383	1603.	13.193	93.457	94.	10.442	55.562	41.
0.70	5.585	21.765	230.	41.048	952.357	1056.	13.433	97.599	132.	11.550	71.607	66.
0.80	6.741	32.653	225.	37.358	766.110	685.	14.326	110.328	150.	12.567	88.580	103.
0.90	8.083	45.046	220.	33.644	558.666	376.	15.105	130.406	181.	13.781	108.805	178.
1.00	9.432	63.624	257.	30.126	428.890	236.	15.978	153.003	234.	15.529	157.854	226.
1.10	11.371	83.627	321.	26.460	297.943	171.	17.154	176.051	267.	17.012	188.225	355.
1.20	13.104	112.655	302.	24.215	210.740	101.	17.995	169.367	348.	19.005	225.854	504.
1.30	15.630	159.630	304.	21.774	161.597	58.	19.049	222.104	407.	20.867	285.435	731.
1.40	18.170	222.495	352.	20.191	122.917	28.	20.203	256.703	620.	23.113	357.848	1035.
1.50	21.008	290.551	394.	18.723	100.249	22.	21.274	282.634	691.	25.514	415.375	1367.

TABLE IV-3

REFERENCE FOR THE FIGURES IV-1 - IV-9

_____	CASE A
+ _____ + _____ +	CASE B
* _____ * _____ *	CASE C
x _____ x _____ x	CASE D

FIGURE IV-1, 4, 7; Expected Sample Sizes
 FIGURE IV-2, 5, 8; Variances of the Sample Sizes
 FIGURE IV-3, 6, 9; Operating Characteristic Curves

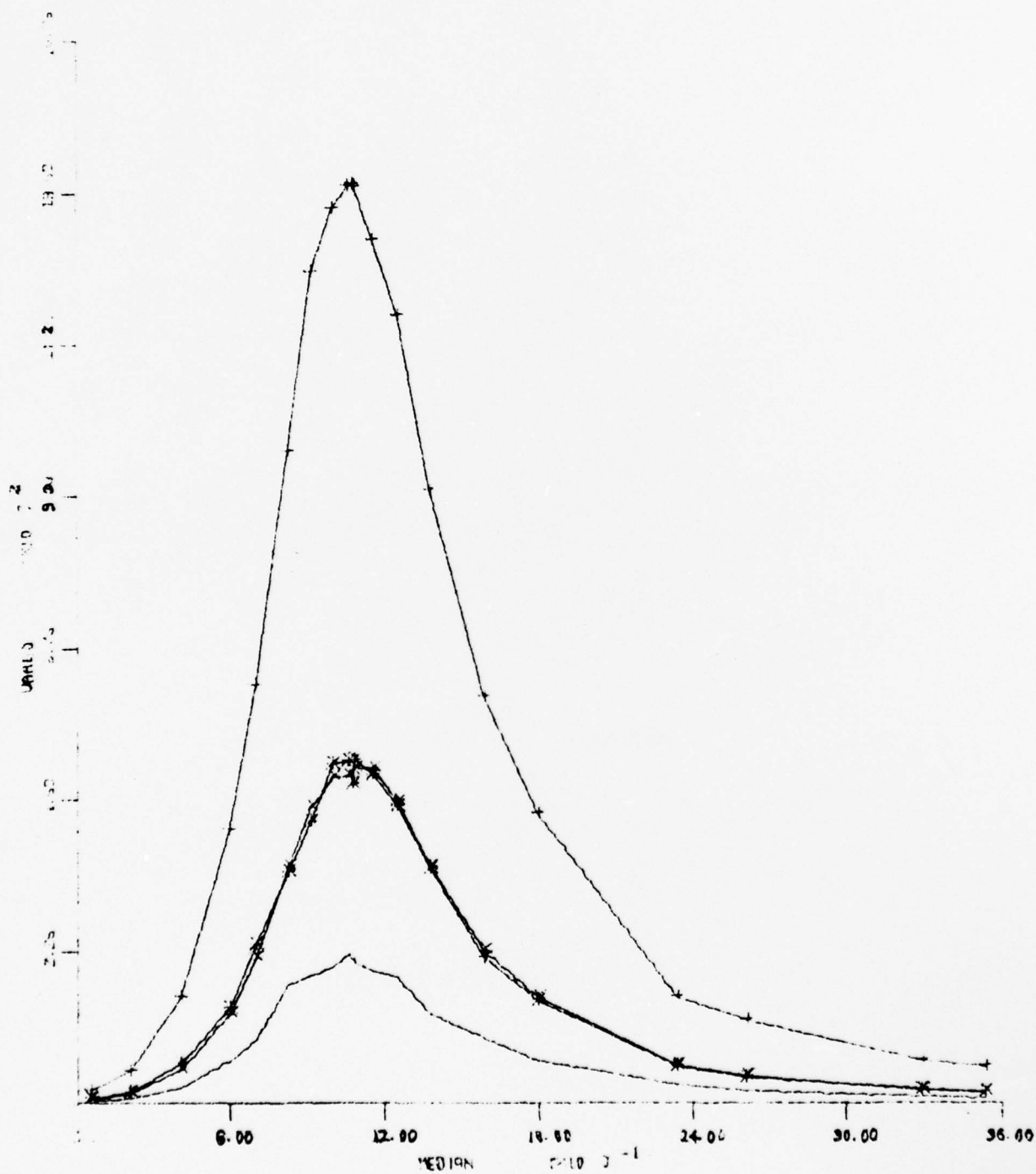


FIGURE IV-1

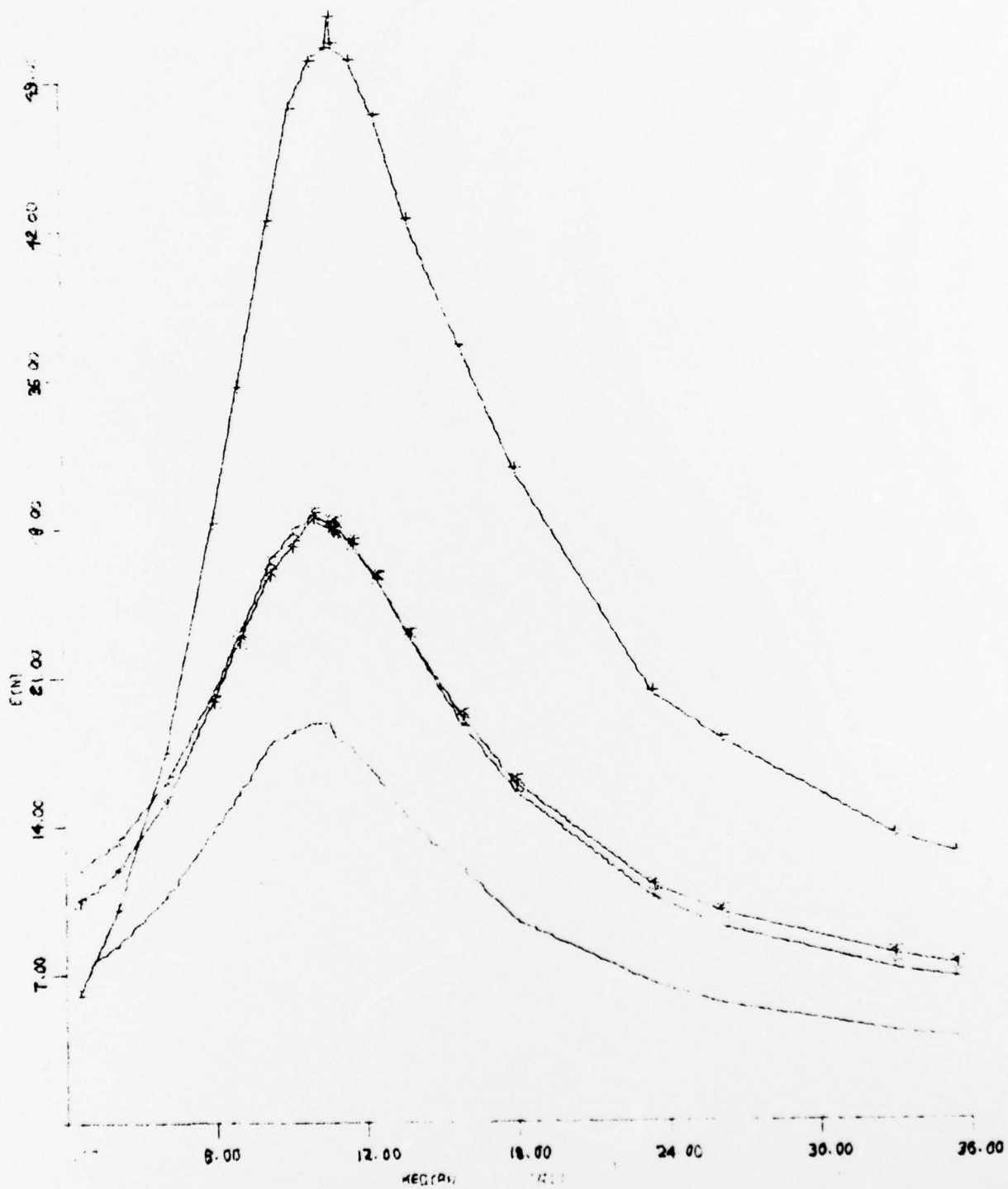


FIGURE IV-2

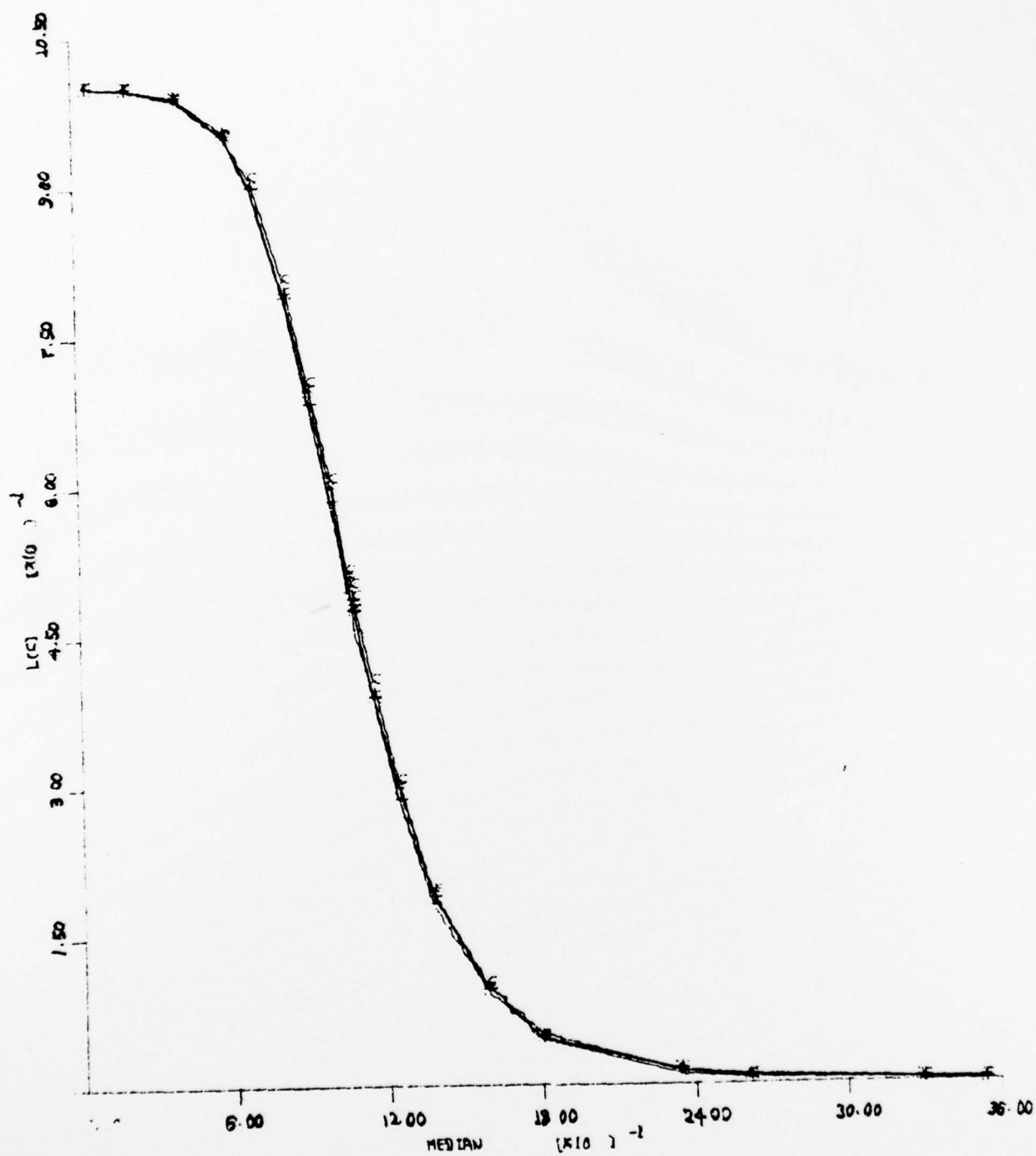


FIGURE IV-3

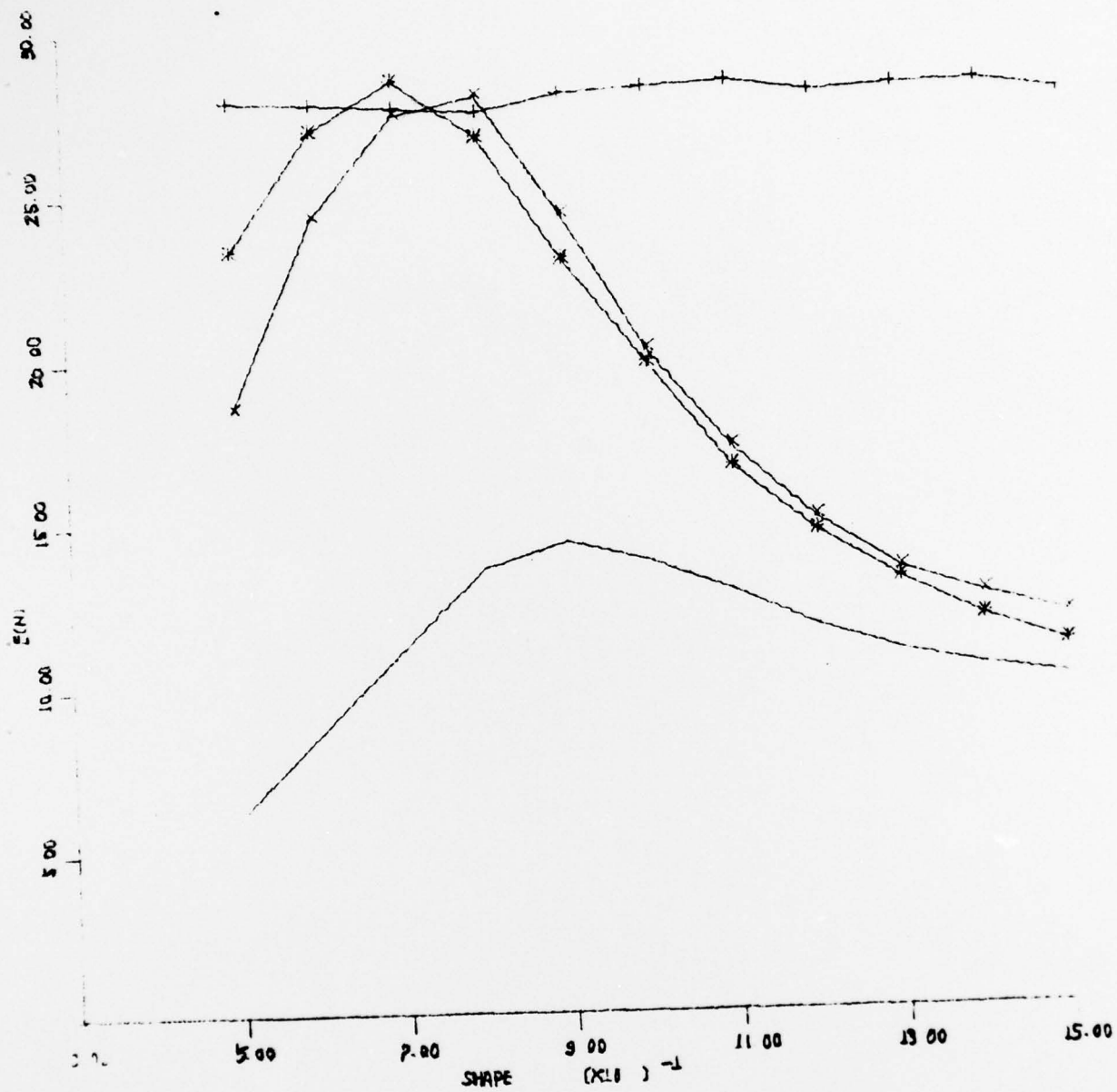


FIGURE IV-4

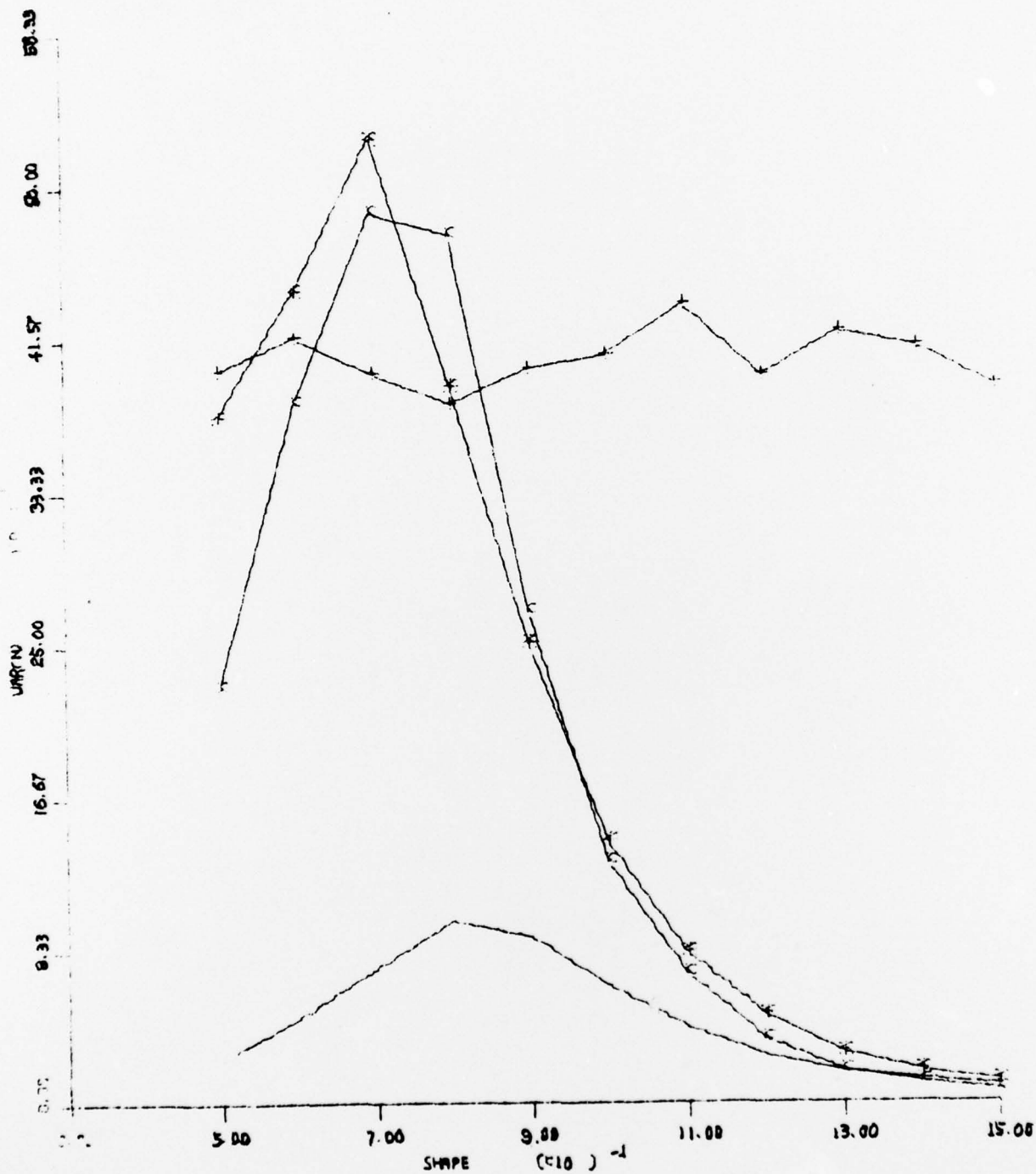


FIGURE IV-5

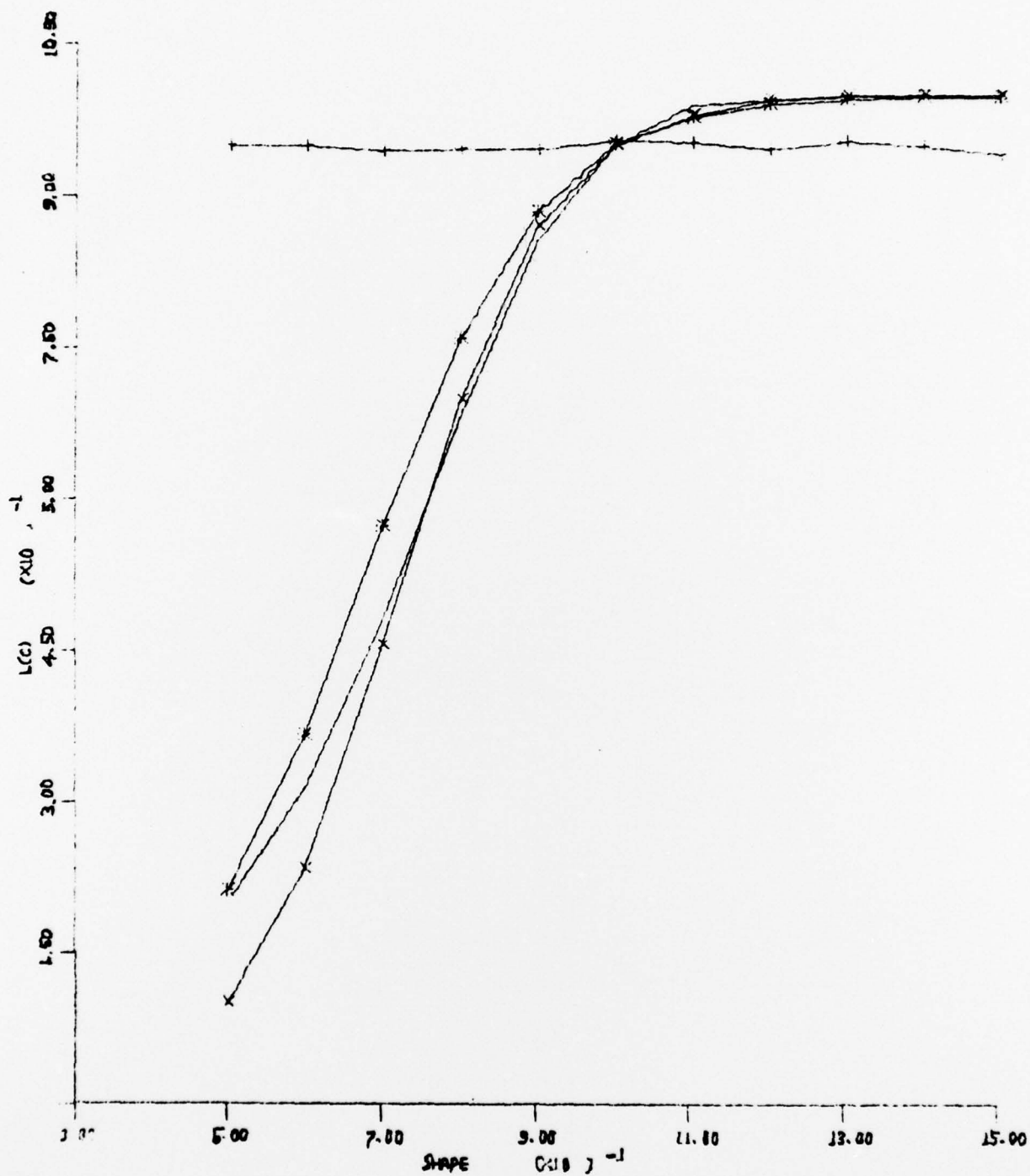


FIGURE IV-6

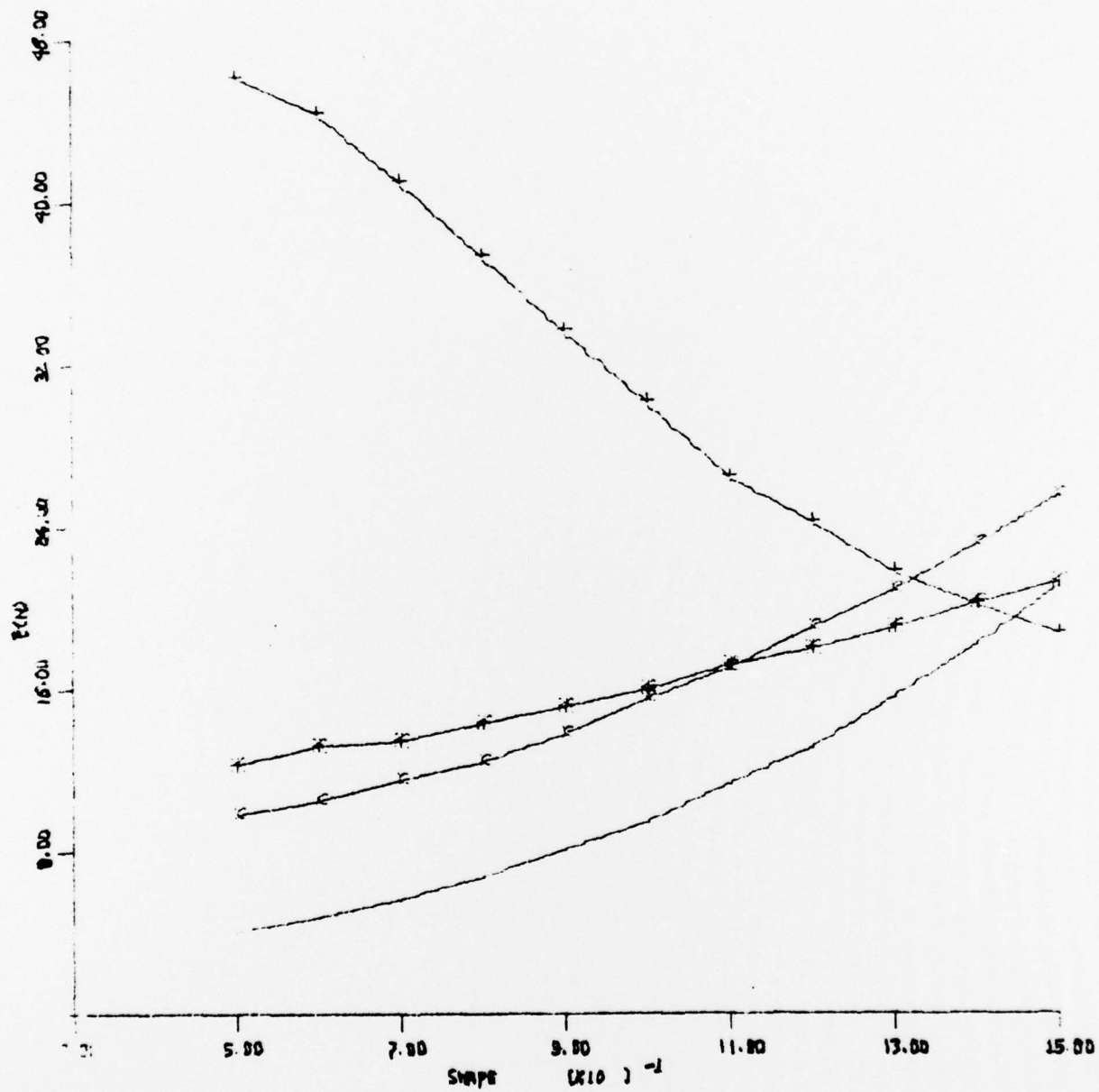


FIGURE IV-7

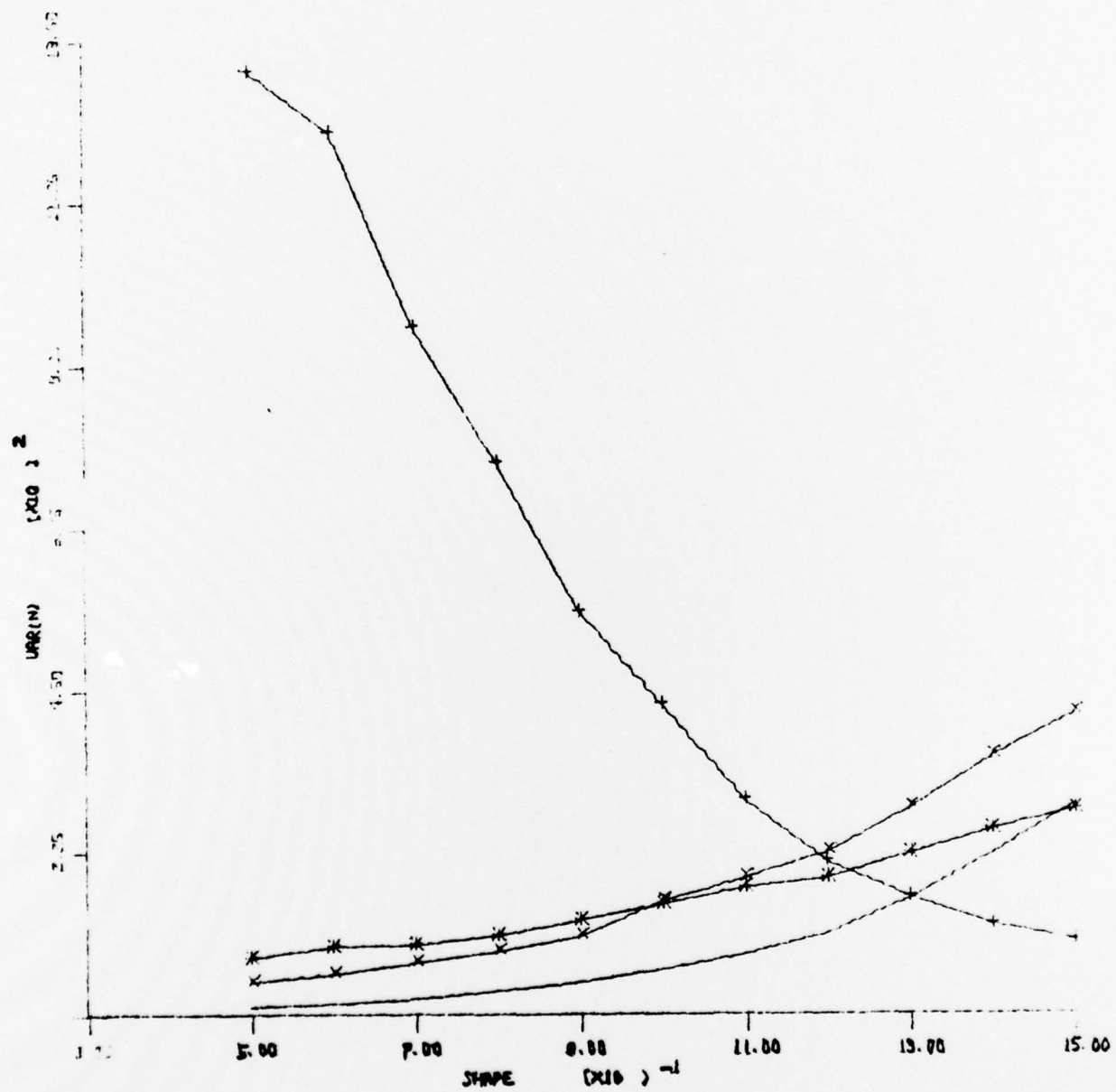


FIGURE IV-8

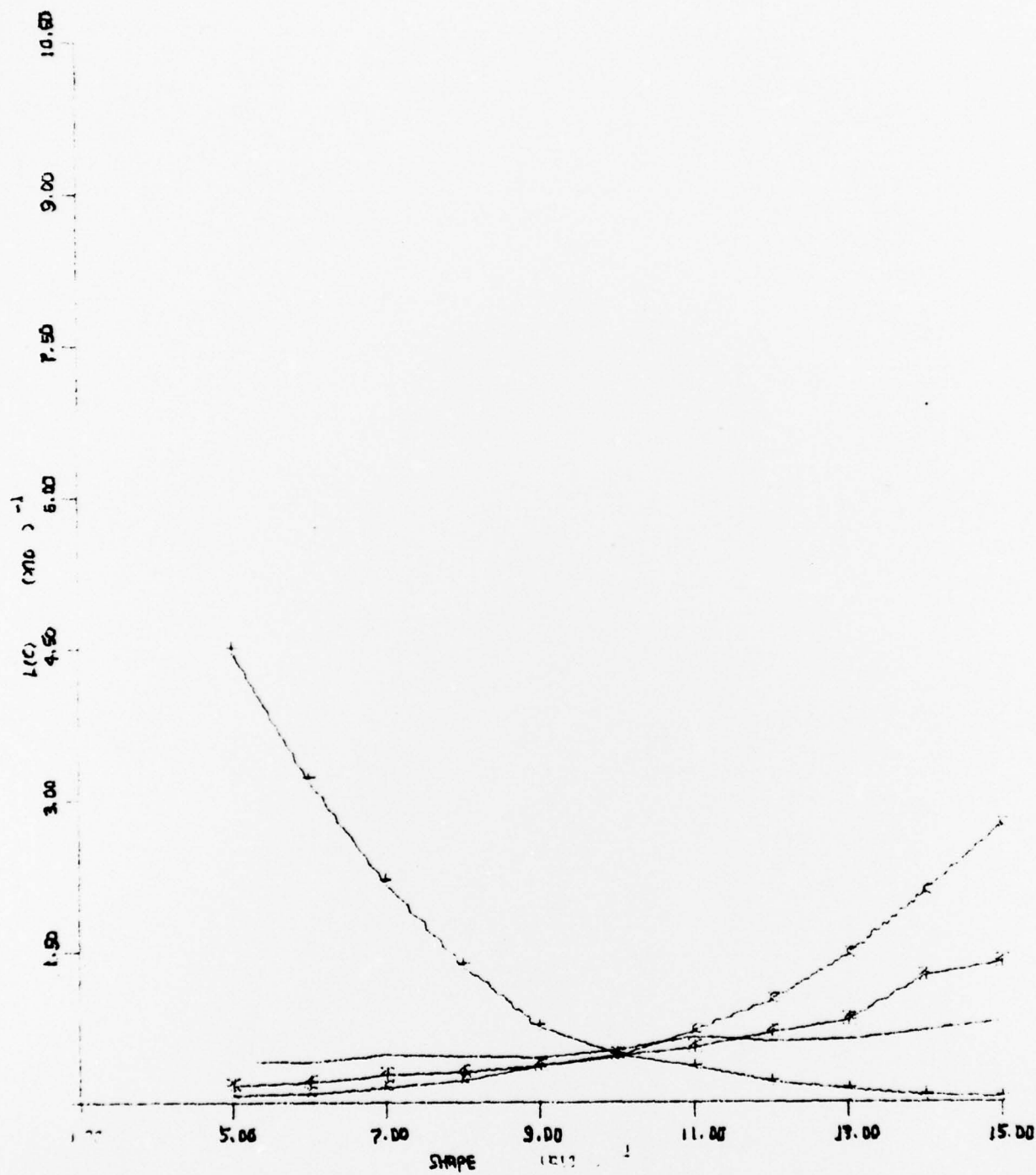


FIGURE IV-9

The interpretation of the figures is as follows:

- a. Variances or standard deviations are approximately proportional to expected sample sizes.
- b. Test Case A contains the minimum expected sample size and Test Case B includes the maximum expected sample size in almost all instances.
- c. Case C and Case D yield similar results when median is changing and shape parameter is fixed at 1.0.
- d. Test Case B is invariant about the change of skewness when the underlying median is 1.0 (See Figures IV-4,5,6).
- e. Case B also gives the lowest error rates if the bomb impact has a heavy tail distribution under the alternative hypothesized median, and has light tail distribution under null hypothesized median (Figure IV-6.9).
- f. Case C gives the lowest error rate when the true median is 2.0 and bomb impacts are clustered around the median.

NOTES:

- a. The adjusting factors of the error rates table were developed iteratively and there is no guarantee that these adjusting factors are the best ones.
- b. There could be further investigation of effects under error rates other than 0.05, or of changing null and alternative hypothesized median values.
- c. The Fortran program for this simulation is attached as an Appendix.

V. CONCLUSION

If it is certain that the bomb impact is Rayleigh distributed, then Sequential Rayleigh Test is appropriate to test the system.

In case one has doubt about the bomb impact distribution, the Sequential Binomial Test gives better tests than the Sequential Rayleigh Test.

Further, if there is some reason to believe that the median is likely to be 1 (so the test is likely to accept H_0), Case B (Null parameter $P_0=0.5$) is better than any of the other three tests.

If the median is not likely to be 1, test case A (Sequential Rayleigh Test) or test case C (which has null parameter that minimized $E[N]$ under H_0) are the best tests.

APPENDIX

```

C      DIMENSION BOUND(300,2,4),DATA(4,4),TEST(5000,2,4)
C      BOUND----CONTAINS DECISION BOUNDARIES FOR EACH STAGES
C      TEST----FIRST ELEMENT REPRESENTS NUMBER OF REPLICATION
C      SECOND ELEMENT REPRESENTS DECISION
C      THIRD ELEMENT REPRESENTS TEST CASE(A,B,C OR D)
C      DATA----TEST RESULT,FIRST ROW---MEAN NUMBER OF TERMINA
C      TION SECCND ROW---VARIANCE,THIRD ROW---NUMBER ACCEP
C      TANCE FOURTH ROW---NUMBER REJECTION
C      ISEED= 688777
C      NCOUNT=5000
C      READ(5,2)ALPHA,BETA,CO,C1
C      CO----MEDIAN UNDER THE NULL HYPOTHESIS
C      C1----MEDIAN UNDER THE ALTERNATIVE HYPOTHESIS
C      WRITE(6,140)ALPHA,BETA,CO,C1
140  FORMAT(5X,'ALPHA=',F6.3,' BETA=',F6.3,' CO=',F6.3
C      *,', C1=',F6.3)
C      R=-CO*ALOG(.1584)/(ALOG(2.))
C      R----HIT MISS CRITERION IN CASE C
C      R2----HIT MISS CRITERION IN CASE C
C      P40=.898657
C      R2=(ALOG(1-P40))/(ALOG(.5))
C      P41=1.-1./2**((R2/2.))
C      2 FORMAT(4F6.3)
C      COMPUTE BOUND OF REJECTION AND ACCEPTANCE
C      B1=ALOG(1.2*BETA/(1-2.*ALPHA))
C      A1=ALOG((1-1.2*BETA)/(2.*ALPHA))
C      B2=ALOG(1.2*BETA/(1-1.2*ALPHA))
C      A2=ALOG((1.-1.2*BETA)/(1.2*ALPHA))
C      B3=ALOG(1.15*BETA/(1-1.5*ALPHA))
C      A3=ALOG((1-1.15*BETA)/(1.5*ALPHA))
C      B4=ALOG(1.1*BETA/(1-1.5*ALPHA))
C      A4=ALOG((1-1.1*BETA)/(1.5*ALPHA))
C      AL=(ALOG(2.))*(1./CO-1./C1)
C      AL1=ALOG(2**((CO/C1)-1.))
C      F1=1-SQRT(.1584)
C      AL2=ALOG(.1584*P1/(.8416*(1-P1)))
C      AL3=(ALOG(2.))*(1-(CO/C1))
C      AL4=ALOG((1-P1)/.1584)
C      AL5=ALOG((1-P41)/(1-P40))
C      AL6=ALOG(P41*(1-P40)/(P40*(1-P41)))
C      DO 5 I=1,300
C      BOUND(I,1,1)=(I*ALOG(C1/CO)+B1)/AL
C      BOUND(I,2,1)=(A1-I*ALOG(CO/C1))/AL
C      BOUND(I,1,2)=(A2-I*AL3)/AL1
C      BOUND(I,2,2)=(B2-I*AL3)/AL1
C      BOUND(I,1,3)=(A3-I*AL4)/AL2
C      BOUND(I,2,3)=(B3-I*AL4)/AL2
C      BOUND(I,1,4)=(A4-I*AL5)/AL6
C      BOUND(I,2,4)=(B4-I*AL5)/AL6
C      5 CONTINUE
C      300 READ(5,6,END=1000) FMED,SHAPE
C      FMED----MEDIAN VALUE TO BE GENERATED
C      SHAPE----SHAPE PARAMETER IN WEIBULL DISTRIBUTION
C      6 FORMAT(2F6.3)
C      WRITE(6,102)FMED,SHAPE
C      KCUNT=1.
C      CO 7 I=1,4
C      DO 7 J=1,4
C      DATA(I,J)=0.
C      T1(2,3,4)----TEST STATISTIC FOR TEST CASE A(B,C,D)
C      DCZN1(2,3,4)---TAKES VALUE 1. IF TEST TERMINATED,OTHE
C      RWISE ).
C      10 T1=0.
C      T2=0.
C      T3=0.
C      T4=0.
C      N=0.
C      DCZN1=0

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        DCZN2=0
        DCZN3=0.
        DCZN4=0
20  CALL GSUB(1SEED,1,U)
    X=FMED*(ALOG(U)/ALOG(.5))**((1./SHAPE)
    N=N+1
    IF(DCZN1.EQ.1.)GO TO 40
    T1=T1+X
    IF(T1.GT.BOUND(N,1,1).AND.T1.LT.BCUND(N,2,1)) GO TO 40
    TEST(KOUNT,1,1)=N
    DATA(1,1)=DATA(1,1)+N
    IF(T1.LE.BOUND(N,1,1))GO TO 30
    TEST(KOUNT,2,1)=-1.
    GO TO 31
30  TEST(KOUNT,2,1)=1.
31  DCZN1=1.
40  IF(DCZN2.EQ.1.)GO TO 60
    IF(X.LE.CO)T2=T2+1.
    IF(T2.GT.BOUND(N,1,2).AND.T2.LT.BCUND(N,2,2))GO TO 60
    TEST(KOUNT,1,2)=N
    DATA(2,1)=DATA(2,1)+N
    IF(T2.GE.BOUND(N,2,2))GO TO 50
    TEST(KOUNT,2,2)=-1.
    GO TO 51
50  TEST(KOUNT,2,2)=1.
51  DCZN2=1.
60  IF(DCZN3.EQ.1.)GO TO 80
    IF(X.LE.R)T3=T3+1.
    IF(T3.GT.BOUND(N,1,3).AND.T3.LT.BOUND(N,2,3))GO TO 80
    TEST(KOUNT,1,3)=N
    DATA(3,1)=DATA(3,1)+N
    IF(T3.GE.BOUND(N,2,3))GO TO 70
    TEST(KOUNT,2,3)=-1.
    GO TO 71
70  TEST(KOUNT,2,3)=1.
71  DCZN3=1.
80  IF(DCZN4.EQ.1) GO TO 90
    IF(X.LE.R2) T4=T4+1.
    IF(T4.GT.BOUND(N,1,4).AND.T4.LT.BCUND(N,2,4)) GO TO 20
    TEST(KOUNT,1,4)=N
    DATA(4,1)=DATA(4,1)+N
    IF(T4.GE.BOUND(N,2,4)) GO TO 78
    TEST(KOUNT,2,4)=-1.
    GO TO 79
78  TEST(KOUNT,2,4)=1.
79  DCZN4=1.
90  DDD=DCZN1*DCZN2*DCZN3
    IF(DDD.NE.1.)GO TO 20
    KCUNT=KOUNT+1
    IF(KOUNT.LE.NCOUNT)GO TO 10
    DC 100 I=1,4
100  DATA(I,1)=DATA(I,1)/NCOUNT
    CO 110 I=1,NCOUNT
    DC 110 II=1,4
    DATA(II,2)=(TEST(I,1,II)-DATA(II,1))**2./FLCAT(NCUNT-
    *1)+DATA(II,2)
    IF(TEST(I,2,II).EQ.1.) DATA(II,3)=DATA(II,3)+1.
110  CONTINUE
    DC 120 J=1,4
120  DATA(J,4)=NCOUNT-DATA(J,3)
    WRITE(6,130) ((DATA(I,J),J=1,4),I=1,3)
130  FORMAT(/,4X,' MEAN',F10.5,' VAR',F10.5,' NO ACCEPT '
    *,F5.0,' NO REJECT ',F5.0)
101  FORMAT(1JF10.5)
102  FORMAT(5X,' MEDIAN IS',F7.3,' SHAPE PARAMETER IS',
    *F7.3)
    GO TO 300
1000 STOP
END

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