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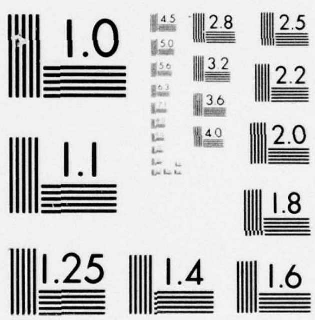

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TECHNICAL REPORT ARBRL-TR-02068

USER'S MANUAL FOR THE BRL SUBROUTINE TO CALCULATE BESSEL FUNCTIONS OF INTEGRAL ORDER AND COMPLEX ARGUMENT

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May 1978

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The formulas used in the BRL Bessel Function subroutine are discussed in enough detail to enable the user to correctly implement the computer code. Accuracy limits for the Hankel Asymptotic Series are discussed in Appendix A. An annotated listing of the computer code and sample results are found in Appendix B and C.

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NOTATION

This manual has been written as an aid to the mathematician or programmer using the BRL Bessel Function Subroutine. FORTRAN symbols for variables and arithmetic operations are used in the body of the report for consistency with excerpts from the coding.

As an aid to the reader who is unfamiliar with standard FORTRAN, the following symbols are defined:

	Symbol	Operation	Algebraic notation		FORTRAN notation
1.	+	add	$a + b$	=	A + B
2.	-	subtract	$a - b$	=	A - B
3.	*	multiply	$a \times b$	=	A * B
4.	/	divide	$a \div b$	=	A / B
5.	**	raise to the power of	a^2	=	A ** 2

Numbers are written in specific ways to define their type:

1. Integer: 2
2. Real: 2. or 2.0
3. Standard notation 2.78×10^5: 2.78 E+05

USER'S MANUAL FOR THE BRL SUBROUTINE TO CALCULATE BESSEL FUNCTIONS OF INTEGRAL ORDER AND COMPLEX ARGUMENT

I. INTRODUCTION

It became apparent in the late 1960's that a subroutine to compute Bessel functions of integral order, first and second kind, ordinary and modified, for wide ranges of integral order and complex argument was necessary to solve a general class of problems involving the Laplace and biharmonic equations in cylindrical coordinates. While there were computer codes to calculate Bessel functions of real arguments, none was available for complex arguments. Although tables of Bessel functions for complex arguments had been published, the tables were limited in scope and accuracy; interpolation between given values resulted in a further loss of accuracy.

The three basic methods of calculation used to compute the ordinary and modified Bessel functions of the first and second kind include:

1. A Weber-Schlafli series,

2. Gauss continued fractions, and

3. Hankel asymptotic series.

Each method will be discussed in enough detail to enable the programmer to understand the subroutine. The rationale for using Gauss continued fractions to compute Bessel functions of the second kind is described by A. S. Elder in a previous report.[1]

The subroutine* is written in FORTRAN IV and has been code-checked on BRLESC** I and BRLESC II. Both computers have a 72 binary bit or 17 decimal digit word length. Examples run on BRLESC II have agreed to sixteen decimal places with double precision runs on the UNIVAC 1108 and the CDC 7600. More thorough checks using independent methods of calculation and multiple precision arithmetic are in progress. The results of these studies will be reported at a later date.

Complex arithmetic is not available on either BRLESC I or BRLESC II. It was necessary, therefore, to code each complex arithmetic operation as a binomial operation. Given two complex numbers $a+bi$ and $c+di$,

---

1 *A. S. Elder, "Formulas for Calculating Bessel Functions of Integral Order and Complex Argument." Ballistic Research Laboratory Report No. 1423, November 1968. (AD680209)*

* *An annotated listing of the subroutine is given in Appendix B.*

** *Ballistic Research Laboratory Electronic Scientific Computer.*

$$(a+bi) * (c+di) = (ac-bd) + (ad+bc)i \text{ and}$$

$$\frac{a+bi}{c+di} = \frac{(a+bi)}{(c+di)}\frac{(c-di)}{(c-di)} = \frac{ac+bd}{c^2+d^2} + \frac{bc-ad}{c^2+d^2}\,i \quad .$$

With some exceptions, the real and imaginary parts of each complex number will either be prefixed by R or C respectively or will contain an R or an I within the respective variable name. For example, the complex number $z = a+bi$ could be called $RZ + CZi$ where RZ = a and CZ=b or $ZR + ZI\ i$ where ZR = a and ZI = b.

II. INPUT AND OUTPUT VARIABLES

The first five variables in the argument list of the subroutine statement

```
SUBROUTINE BESSEL (PHI, CHI, ORD, OPT1, OPT2, FJI, SYK, JPR, LERR)
```

are input variables. Each variable must be real, not integer.

PHI	Real part of complex argument of Bessel function
CHI	Imaginary part of complex argument of Bessel function
ORD	Order of Bessel function. The subroutine will always compute orders m = ORD and n = ORD + 1.
OPT1=1.	Compute ordinary Bessel functions of first and second kind.
=2.	Compute modified Bessel functions of first and second kind.
OPT2=1.	Argument is given in rectangular coordinates, i.e., $z = a+bi$ where PHI=a and CHI=b;
=2.	Argument is given in polar coordinates; i.e., $z = \rho\cos\theta + i\rho\sin\theta$ where PHI = ρ and CHI = θ, in degrees. If θ is not in the interval $-180° < \theta \leq 180°$, the subroutine adjusts the angle by adding or subtracting 360° until it lies within the interval.

Error messages are printed if the input is not in the correct format. The output contains two real arrays of four elements each and two integer variables.

	Ordinary	Modified		Ordinary	Modified
FJI(1)	$Re\ J_m(z)$	$Re\ I_m(z)$	SYK(1)	$Re\ Y_m(z)$	$Re\ K_m(z)$
FJI(2)	$Im\ J_m(z)$	$Im\ I_m(z)$	SYK(2)	$Im\ Y_m(z)$	$Im\ K_m(z)$
FJI(3)	$Re\ J_n(z)$	$Re\ I_n(z)$	SYK(3)	$Re\ Y_n(z)$	$Re\ K_n(z)$
FJI(4)	$Im\ J_n(z)$	$Im\ I_n(z)$	SYK(4)	$Im\ Y_n(z)$	$Im\ K_n(z)$

JPR	= 1	Indicates Weber-Schlafli series used
	= 2	Indicates Gauss continued fraction used
	= 3	Indicates Hankel asymptotic series used
LERR	= 0	No error detected in subroutine
	= 1	Error occurred in subroutine. A printed error message from the subroutine will indicate the section of code where error made.

It is necessary to declare the arrays corresponding to FJI(4) and SYK(4) in a DIMENSION statement in the calling program.

III. METHODS OF CALCULATION

A. The Differential Equation

The complete solution of the differential equation

$$\frac{d^2y}{dz^2} + \frac{1}{z}\frac{dy}{dz} + \left(k^2 - \frac{n^2}{z^2}\right) y = 0$$

is a linear combination of the ordinary Bessel functions of the first, $J_n(kz)$, and the second, $Y_n(kz)$, kinds

$$y = a\, J_n(kz) + b\, Y_n(kz)$$

where a, b, k are constants and n is the integer order.

A change of sign in the differential equation

$$\frac{d^2y}{dz^2} + \frac{1}{z}\frac{dy}{dz} - \left(k^2 + \frac{n^2}{z^2}\right) y = 0$$

yields a complete solution which is a linear combination of the modified Bessel functions of the first, $I_n(kz)$, and the second, $K_n(kz)$, kinds

$$y = a\, I_n(kz) + b\, K_n(kz)$$

where a, b, k are constants and n is the integral order.

B. Preliminary Calculations

The following values are computed at the beginning of the code up to location 140 from the input parameters PHI and CHI:

		Rectangular Coordinates	Polar Coordinates
1.	z	PHI + i CHI	PHI (cos CHI+ $i$sin CHI)
2.	ρ	$\sqrt{PHI^2 + CHI^2}$	PHI
3.	θ	$2*ARCTAN\left(\frac{\sin Z}{1+\cos Z}\right)$	CHI (π/180)
4.	ZR	PHI	$\rho * \cos\theta$
5.	ZI	CHI	$\rho * \sin\theta$
6.	sinZ	CHI/ρ	
7.	cosZ	PHI/ρ	

Because the principal values of the ARCTAN lie between $-\pi/2$ and $\pi/2$, the half-angle formula was used to compute ARCTAN z. This should make the calculation independent of machine limits imposed on any predefined ARCTAN function.

To calculate the series for Bessel functions of the first and second kinds, several quantities needed later, can be computed before generating the term of the series:

1. $\log z = \log\rho + i\theta$

$\log z + \psi = \log\rho + \psi + i\theta = ZRL + i\ ZIL$

where $\psi = \gamma - \ln 2$ = a constant
γ = Euler's constant = .57721...
$ZRL = \log\rho + \psi$
$ZIL = i\theta$

An error test prevents trying to compute log 0.

2. $\left(\frac{z}{2}\right)^2 = \frac{1}{4}(ZR + iZI)^2 = XR + iXI$

where $XR = \frac{1}{4}(ZR^2 - ZI^2)$

$XI = \frac{1}{2}(ZR * ZI)$

3. $T1 = \rho^2 = ZR^2 + ZI^2$

4. $Y = \frac{z}{\rho^2} = \frac{ZR}{ZR^2+ZI^2} - \frac{i\ ZI}{ZR^2+ZI^2} = YR + i\ YI$

The signs of alternating terms differ in the infinite series used to calculate ordinary and modified Bessel functions. Thus,

SGN = - 1.0 for ordinary Bessel functions and
SGN = 1.0 for modified Bessel functions.

Tests are performed beginning three lines before location 180 and ending at 180 to determine the method of calculation:

1.	$\rho \leq 2.5$	Use Weber-Schlafli series
2.	$2.5 < \rho < 21.0$	Use recurrence formulas for Bessel functions of the first kind and Gauss continued fractions for Bessel functions of the second kind.
3.	$\frac{(\text{order} + 1)^2}{\rho} < 1.95$ and $\rho \geq 21.0$	Use Hankel asymptotic series.

The limits given here are suitable for BRLESC I and BRLESC II; these constants must be changed for computers with higher accuracy. Further discussion of these limits is in Appendix A.

C. Weber-Schlafli Series: $|z| \leq 2.5$

The subroutine always calculates Bessel functions in pairs of order m and n = m+1. Because orders zero through three are used so frequently, the initial conditions for the appropriate series are calculated directly. Initial conditions for orders n>3 are calculated from general formulas. The series calculations for the Bessel functions of both the first and second kind for orders m and n are done simultaneously and are accurate for small values of $|z|$ where $0 < |z| \leq 2.5$.

Given the equations for the ordinary Bessel function of the first (J_m) kind and the modified Bessel function of the first (I_m) kind,[2] one can immediately observe that the difference between the equations is in the sign of alternating terms of the series.

$$J_o(z) = 1 - \frac{\left(\frac{z}{2}\right)^2}{(1!)^2} + \frac{\left(\frac{z}{2}\right)^4}{(2!)^2} - \frac{\left(\frac{z}{2}\right)^6}{(3!)^2} + \ldots$$

$$I_o(z) = 1 + \frac{\left(\frac{z}{2}\right)^2}{(1!)^2} + \frac{\left(\frac{z}{2}\right)^4}{(2!)^2} + \frac{\left(\frac{z}{2}\right)^6}{(3!)^2} + \ldots$$

2 *British Association for the Advancement of Science, Bessel Functions, Part I, Mathematical Tables Vol VI, University Press, Cambridge England, 1937.*

$$J_1(z) = \frac{z}{2} - \frac{\left(\frac{z}{2}\right)^3}{1!2!} + \frac{\left(\frac{z}{2}\right)^5}{2!3!} - \frac{\left(\frac{z}{2}\right)^7}{3!4!} + \ldots$$

$$I_1(z) = \frac{z}{2} + \frac{\left(\frac{z}{2}\right)^3}{1!2!} + \frac{\left(\frac{z}{2}\right)^5}{2!3!} + \frac{\left(\frac{z}{2}\right)^7}{3!4!} + \ldots$$

$$J_n(z) = \frac{\left(\frac{z}{2}\right)^n}{n!}\left[1 - \frac{\left(\frac{z}{2}\right)^2}{1!(n+1)} + \frac{\left(\frac{z}{2}\right)^4}{2!(n-1)(n+2)} - \frac{\left(\frac{z}{2}\right)^6}{3!(n+1)(n+2)(n+3)} + \ldots\right]$$

$$= \frac{\left(\frac{z}{2}\right)^n}{n!} \sum_{k=o}^{\infty} AA_k \left(\frac{z}{2}\right)^{2k} \quad \text{where } AA_k = (-1)^k \frac{n!}{k!(n+k)!}$$

$$I_n(z) = \frac{\left(\frac{z}{2}\right)^n}{n!}\left[1 + \frac{\left(\frac{z}{2}\right)^2}{1!(n+1)} + \frac{\left(\frac{z}{2}\right)^4}{2!(n+1)(n+2)} + \frac{\left(\frac{z}{2}\right)^6}{3!(n+1)(n+2)(n+3)} + \ldots\right]$$

$$= \frac{\left(\frac{z}{2}\right)^n}{n!} \sum_{k=o}^{\infty} BB_k \left(\frac{z}{2}\right)^{2k} \quad \text{where } BB_k = \frac{n!}{k!(n+k)!}$$

Because of the similarities of these equations, only the coding for orders 0 and 1 of ordinary Bessel functions will be detailed here. It is immediately seen that the common factor of the terms of $J_0(z)$ is $1+0i$ and of $J_1(z)$ is $\frac{z}{2} = \frac{ZR}{2} + \frac{ZI}{2} i$. The common factors for orders 0 and 1 are specified at location 201 in the code:

```
CMR1  = 1.          }    1 + 0i
CMI1  = 0.          }

CNR1  = OR = ZR/2.  }    z/2 = ZR/2. + ZI/2. i
CNI1  = OI = ZI/2.  }
```

The terms of the series are computed beginning at location 500. The series for ordinary and modified Bessel functions are identical in powers of $\frac{z}{2}$ but remain different in the sign of alternating coefficients.

The real coefficient for order m=0 can be written as

$$AA_k = \frac{SGN}{FR(BM+FR)} * AA_{k-1} = AA * AA_{k-1}$$

and for order n = m+1 = 1 by

$$BB_k = \frac{SGN}{FR(BN+FR)} * BB_{k-1} = BB*BB_{k-1}$$

where

SGN = - 1 for ordinary Bessel functions
= 1 for modified Bessel functions
BM = order m
BN = order n
FR = k.

For $k \geq 2$, each term of the factored series is the product of the previous term and $\left(\frac{z}{2}\right)^2 \left[\frac{\pm 1}{k(n+k)}\right]$. In the code $\left(\frac{z}{2}\right)^2$ = XR + iXI and $\frac{\pm 1}{k(n+k)}$ = BB for order n. The terms of the factored series are stored in the arrays TRM and TIM for $J_m(z)$ and in TRN and TIN for $J_n(z)$. Each term is computed and stored (omitting the complex notation) as

$$TM_k = TM_{k-1} * AA * \left(\frac{z}{2}\right)^2 \quad \text{for order m.}$$

The terms of the factored series are summed from the smallest term to the largest term in order to avoid cancellation error. This sum, a complex number, is then multiplied by the common factor, also complex, to compute the sum of the infinite series:

$$STR + i\ STI = \sum_k TRM(K) + i \sum_k TIM(K)\ , \text{order m}$$

$$SUR + i\ SUI = \sum_k TRN(K) + i \sum_k TIN(K)\ , \text{order n}$$

$$J_m(z) = (CMR1 + iCMI1)(STR + iSTI) = RSM + i\ CJM$$

where RJM = CMR1 * STR - CMI1 * STI and
CJM = CMR1 * STI + CMI1 * STR.

Similarly

$$J_n(z) = RJN + i\, CJN \text{ for order n.}$$

Calculation of the ordinary and modified Bessel functions of the second kind is more complex. The Weber-Schlafli series contains a logarithmic term, the sum of an infinite series, and the sum of a finite series. As before, the difference between the ordinary and modified Bessel functions of the second kind can be observed in the signs of corresponding terms:

$$Y_o(z) = \frac{2}{\pi}\left[J_o(z)\left\{\gamma - \ln 2 + \ln z\right\} - \frac{1}{2}\left\{\frac{\left(\frac{z}{2}\right)^0}{0!0!}(0+0) - \frac{\left(\frac{z}{2}\right)^2}{1!1!}(1+1) + \frac{\left(\frac{z}{2}\right)^4 (1+\frac{1}{2}+1+\frac{1}{2})}{2!2!} - \ldots\right\} + \left\{0\right\}\right]$$

$$K_o(z) = -\, I_o(z)\left\{\gamma - \ln 2 + \ln z\right\} + \frac{1}{2}\left\{\frac{\left(\frac{z}{2}\right)^0}{0!0!}(0+0) + \frac{\left(\frac{z}{2}\right)^2}{1!1!}(1+1) + \frac{\left(\frac{z}{2}\right)^4 (1+\frac{1}{2}+1+\frac{1}{2})}{2!2!} + \ldots\right\} + \left\{0\right\}$$

where $\gamma = .5772156649$, Euler's constant

$$Y_n(z) = \frac{2}{\pi}\left[J_n(z)\left\{\gamma - \ln 2 + \ln z\right\} - \frac{1}{2}\left\{\sum_{r=0}^{\infty}\frac{(-1)^r}{r!(n+r)!}\left(\frac{z}{2}\right)^{n+2r}\left(1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{r}+1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n+r}\right)\right\} - \frac{1}{2}\sum_{n=o}^{n-1}\frac{(n-r-1)!}{r}\left(\frac{z}{2}\right)^{2r-n}\right]$$

$$K_n(z) = F_1(n,z) + F_2(n,z) + F_3(n,z)$$

where $F_1(n,z) = (-1)^{n+1}\, I_n(z)\left\{\gamma - \ln z + \ln z\right\}$

$$F_2(n,z) = (-1)^n \frac{1}{2} \sum_{r=o} \frac{1}{r!(n+r)!} \left(\frac{z}{2}\right)^{n+2r} \left(1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{r}+1+\frac{1}{2}+\frac{1}{3}+\ldots\frac{1}{n+r}\right)$$

$$F_3(n,z) = \frac{1}{2} \sum_{r=o}^{n-1} \frac{(-1)^r\ (n-r-1)!}{r!} \left(\frac{z}{2}\right)^{2r-n}$$

The common factor of the infinite series for both the ordinary and the modified Bessel function of the second kind is $(\pm 1)^n \left(\frac{1}{2}\right)\left(\frac{z}{2}\right)^n$. The finite series is factored

$$\sum_{r=0}^{m-1} \frac{(m-r-1)!}{r!} \left(\frac{z}{2}\right)^{2r-m} (SGN)^r =$$

$$(m-1)! \left(\frac{z}{2}\right)^{-m} \left[1 + \sum_{r=1}^{m-1} \frac{(SGN)^r}{r!(m-r)(m-r+1)\ldots(m-1)} \left(\frac{z}{2}\right)^{2r}\right]$$

where SGN = 1.0 for the ordinary Bessel functions and
SGN =-1.0 for the modified Bessel functions, and
$(m-1)! \left(\frac{z}{2}\right)^{-m}$ is the common factor of the finite series.

These common factors, defined for orders 0 and 1 at location 201, are given by

$$\left.\begin{array}{l} CMR2 = SGN * .5 = \pm\ .5 \\ CMI2 = 0.0 \end{array}\right\} \text{for order 0, infinite series}$$

$$\left.\begin{array}{l} CNR2 = SGN * .5 * OR = \pm \frac{1}{2}\left(\frac{ZR}{2}\right) \\ CNI2 = SGN * .5 * OI = \pm \frac{1}{2}\left(\frac{ZI}{2}\right) \end{array}\right\} \text{for order 1, infinite series}$$

$$\left.\begin{array}{l} CMR3 = 0. \\ CMI3 = 0. \end{array}\right\} \text{for order 0, finite series}$$

$$\left.\begin{array}{l} CNR3 = SGN * YR = \pm \dfrac{ZR}{ZR^2 + ZI^2} \\ CNI3 = SGN * YI = \pm \dfrac{-ZI}{ZR^2 + ZI^2} \end{array}\right\} \text{for order 1, finite series}$$

3 *National Bureau of Standards, Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, U.S. Government Printing Office, Washington, D.C. 1964.*

When the common factor for the finite series is calculated for the n^{th} order beginning at location 300 in the code, a check is made to prevent machine overflow.

The terms of the factored finite series and their sum are computed at location 400. This sum is stored in CMR4 + i CMI4 for order m and CNR4 + i CNI4 for order n=m+1. The sum and common factor are not multiplied together to get the value of the finite series until the logarithmic term and the infinite series term of the equation for the Bessel function of the second kind are assembled at location 560.

The Weber-Schlafli or infinite series term of the equation for Bessel functions of the second kind is calculated in the same manner and in the same section of coding (location 500) as previously described for Bessel functions of the first kind. After factoring the common factor of the terms of the infinite series, the powers of $\frac{z}{2}$ are identical to those in the infinite series for Bessel functions of the first kind. The coefficients of the terms differ in their numerators, however, and these real numerators are calculated and stored in P for order m and Q for order n=m+1. These numerators, P and Q, are calculated as the remaining portion of each term of the factored infinite series, TRM(K) + i TIM(K) for order m and TRN(K)+ i TIN(K) for order n, is computed. The products

$$P*[TRM(K)+iTIM(K)]=TRV(K)+iTIV(K) \quad \text{for order m and}$$
$$Q*[TRN(K)+iTIN(K)]=TRW(K)+iTIW(K) \quad \text{for order n}$$

contain the entire k^{th} term of the factored infinite series for Bessel functions of the second kind. The sums of the factored infinite series are given by

$$SVR+iSVI= \sum_k [TRV(K)+iTIV(K)] \quad \text{for order m}$$

$$SWR+iSWI= \sum_k [TRW(K)+iTIW(K)] \quad \text{for order n=m+1.}$$

As mentioned above, the sum and common factor are multiplied at location 560.

Each of the factors of the logarithmic term has already been computed. Therefore it is now possible to assemble each of the three terms of the equations for Bessel functions of the second kind as follows:

a. Logarithmic term: $J_m(z) * (\log z - \log 2 + \gamma)$, ordinary
$I_m(z) * (\log z - \log 2 + \gamma)$, modified

$$RLV + i\ CLV = T1 * (RJM + i\ CJM) * (ZRL + i\ ZIL)$$

where T1 is the appropriate sign and
RJM + i CJM contains either the ordinary or the modified Bessel function of the first kind.

b. Finite or power series term = FV1R + i FV1I
= (CMR3+iCMI3) * (CMR4+iCMI4), for order m

where CMR3+iCMI3 is the common factor and
CMR4+iCMI4 is the sum of the factored finite series

c. Weber-Schlafli or infinite series term = FV2R+iFV2I
= T2 * (SVR + i SVI) * (CMR2+iCMI2), for order m

where CMR2 + i CMI2 is the common factor
and SVR + i SVI is the sum of the factored infinite series
and T2 is the appropriate sign.

Finally, these three terms are added and then multiplied by their common factor so that

$$Y_m(z) \text{ or } K_m(z) = \text{SGPI} * [\,(\text{RLV}+i\text{CLV}) + (\text{FV1R} + i\ \text{FV1I}) + (\text{FV2R} + i\ \text{FV2I})\,]$$

where $\text{SGPI} = \frac{2}{\pi}$ for ordinary Bessel functions

$= 1$ for modified Bessel functions.

D. Gauss Continued Fraction and Recurrence: $2.5<|z| \leq 21.0$

Bessel functions of the first kind, $J_n(z)$ and $I_n(z)$, for medium values of $|z|$ where $2.5 < |z| \leq 21.0$ can be calculated using the recurrence relations:[4]

$$J_{m-1}(z) = \frac{2m}{z} J_m(z) - J_n(z)\ , \quad n = m+1$$

$$I_{m-1}(z) = \frac{2m}{z} I_m(z) + I_n(z).$$

A starting value can be obtained using the series approximations described in the previous section if the order is larger than $|z|$. At location 610 in the code, r is incremented until

$$T1_r = \left(T1_{r-1} * \frac{\rho^2}{4}\right) / [r(n+r)] < 10^{-10}$$

where initially $T1_0$=1., r = 1, and n is the order m + 1.
When the order FN=n+r is sufficiently large so that $T1 < 10^{-10}$, these

4 *N.W. McLachlan,* *Bessel Functions for Engineers*, *Clarendon Press, Oxford, 1955.*

new high orders, BM = FN and BN = FN+1, are used to calculate the starting values of J or I. A downward recurrence using the above recurrence relations is performed at location 630 until the high order BM has decreased to the original order ORD.

The corresponding recurrence relation for modified Bessel functions of the second kind is [4]

$$K_{m-1}(z) = K_n(z) - \frac{2m}{z} K_m(z).$$

Although the Weber-Schlafli series is useful to start the recurrence when $n > |z|$, functions of lower order cannot be calculated accurately from the recurrence relation as the difference of two nearly like numbers occurs in the course of the calculations.

The equation[5]

$$K_n(z) = \left(\frac{\pi}{2z}\right)^{\frac{1}{2}} \frac{e^{-z}}{\Gamma(n+\frac{1}{2})} \int_0^\infty e^{-u} u^{n-\frac{1}{2}} \left(1+\frac{u}{2z}\right)^{n-\frac{1}{2}} du$$

was chosen because it does not separate the analytic and the logarithmic parts of the function. It is valid provided z does not lie on the negative half of the real axis. This equation is related to the confluent hypergeometric function discussed by Wall[6]:

$$f(a,b;v) = \frac{1}{\Gamma(a)} \int_0^\infty \frac{e^{-u} u^{a-1}}{(1+vu)^b} du. \qquad (1)$$

It is seen that

$$K_n(z) = \left(\frac{\pi}{2z}\right)^{\frac{1}{2}} e^{-z} f(a,b;v) \qquad (2)$$

where $v = \frac{1}{2z}$, $a = \frac{1}{2} + n$, and $b = \frac{1}{2} - n$.

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5 G.N. Watson, *A Treatise on the Theory of Bessel Functions*, The MacMillan Co, New York, 1948.

6 H.S. Wall, *Analytic Theory of Continued Fractions*, D. Van Nostrand Co., Inc., New York, 1948.

Similarly,

$$K_{n-1}(z) = \left(\frac{\pi}{2z}\right)^{\frac{1}{2}} e^{-z} f(a-1,\ b+1;\ v).$$

If a quotient function is defined as

$$Q_n(z) = \frac{K_{n-1}(z)}{K_n(z)}$$

then

$$Q_n(z) = \frac{f(a-1,\ b+1;\ v)}{f(a,b;v)}. \tag{3}$$

The expression on the right-hand side of equation (3) can be reduced to a form that is expressible in terms of Gauss continued fractions. The procedure is outlined below:

1. Integrate equation (1) by parts to derive the recurrence relation

   $$f(a,b;v) = f(a+1,b;v) + bv\ f(a+1,b+1;v). \tag{4}$$

2. Let a = a-1 and b= b+1, then substitute in equation (4) to get

   $$f(a-1,b+1;v) = f(a,b+1;v) + (b+1)v\ f(a,b+2;v) \tag{5}$$

3. Substitute equation (5) into the numerator of equation (3):

$$Q_n(z) = \frac{f(a,b+1;v)}{f(a,b;v)} + \frac{(b+1)v\ f(a,b+2;v)}{f(a,b;v)}$$

$$= \frac{f(a,b+1;v)}{f(a,b;v)} \left[1 + \frac{(b+1)v\ f(a,b+2;v)}{f(a,b+1;v)}\right]$$

$$= F_1(a,b;v) \left[1 + v(b+1)G_1(a,b;v)\right]. \tag{6}$$

$F_1(a,b;v)$ and $G_1(a,b;v)$ are forms of the Gauss continued fraction

$$\frac{f(A,B;v)}{f(A,B-1;v)} = \cfrac{1}{1 + \cfrac{Av}{1 + \cfrac{Bv}{1 + \cfrac{(A+1)v}{1 + \cfrac{(B+1)v}{1 + \cfrac{(A+2)v}{1 + \cdots}}}}}} \tag{7}$$

Let A = a and B = b+1. The left hand side of equation (7) is now equivalent to $F_1(a,b;v)$ from equation (6):

$$\frac{f(A,B;v)}{f(A,B-1;v)} = \frac{f(a,\ b+1;v)}{f(a,b;v)} = F_1(a,b;v).$$

If A=a, B=b+2, then similarly

$$\frac{f(a,\ b+2;\ v)}{f(a,\ b+1;\ v)} = G_1\ (a,b;v).$$

These continued fractions are computed at location 760 in the code by an iterative procedure. If it is assumed that

$$F_\ell(a,b;v) = \frac{f(a+\ell-1,\ b+\ell\ ;\ v)}{f(a+\ell-1,\ b+\ell-1\ ;\ v)}$$

and

$$G_\ell(a,b;v) = \frac{f(a+\ell-1,\ b+\ell+1;\ v)}{f(a+\ell-1,\ b+\ell\ ;\ v)}$$

then it can be shown that

$$F_\ell(a,b;v) = \frac{1 + (b+\ell)v\ \ F_{\ell+1}\ (a,b;v)}{1 + (b+\ell)v\ \ F_{\ell+1}\ (a,b;v) + (a+\ell-1)v}$$

and

$$G_\ell(a,b;v) = \frac{1 + (b+\ell+1)v\ \ G_{\ell+1}(a,b;v)}{1 + (b+\ell+1)v\ \ G_{\ell+1}(a,b;v) + (a+\ell-1)v}.$$

To start the iterative process

1. Choose $\ell = \ell_{max} \simeq 50$ or any number sufficiently large to keep truncation error to a minimum
2. Let $F_{\ell-1}(a,b;v) = 0$ and $G_{\ell+1}\ (a,b;v) = 0$
3. Iterate backwards until $\ell = 0$.

The quotient Q_n is calculated at location 770. This quotient is used to compute both the ordinary and the modified Bessel functions of the second kind.

The modified Bessel functions of the second kind are obtained from the Wronskian relation[3]

$$I_n(z)\,K_{n-1}(z) + I_{n-1}(z)\,K_n(z) = \frac{1}{z}\,.$$

By definition of Q_n,

$$K_{n-1}(z) = K_n(z)\,Q_n(z)$$

and, therefore,

$$K_n(z) = \frac{1}{z[I_{n-1}(z) + I_n(z)Q_n(z)]} \tag{8}$$

If z lies in the right half-plane, equation (8) can be used directly to calculate $K_n(z)$. The values for I_n and I_{n-1} have already been obtained using the recurrence described in the beginning of this section, and the value of Q_n has just been calculated using the Gauss continued fraction. These values are assembled in location 780 through 790 and finally stored as $K_n(z)$.

If z lies in the left half-plane, analytic continuation formulas are used to obtain the correct functional values:[3]

1. For z in quadrant II, then

$$K_n(z) = (-1)^n\,[K_n(t) - i\pi I_n(z)]\,,\ t=-z$$

2. For z in quadrant III, then

$$K_n(z) = (-1)^n\,[K_n(t) + i\pi I_n(z)]\,,\ t=-z$$

The ordinary Bessel functions of the second kind are calculated in terms of Hankel functions which are linear combinations of ordinary Bessel functions:[3]

$$H_n^{(1)}(z) = J_n(z) + i\,Y_n(z) \tag{9}$$

$$H_n^{(2)}(z) = J_n(z) - i\,Y_n(z). \tag{10}$$

Up to this point in the code, the quotient function Q_n and the ordinary Bessel functions of the first kind $J_n(z)$ have been calculated for medium values of $|z|$. So that equations (9) and (10) can be solved for

$Y_n(z)$, a method for calculating $H_n(z)$ in terms of Q_n or $J_n(z)$ must be found. This method starts at location 820 in the code.

Using the integral representation of $H_n^{(1)}(z)$

$$H_n^{(1)}(z) = \left(\frac{2}{\pi z}\right)^{\frac{1}{2}} \frac{e^{i(z - \frac{n\pi}{2} - \frac{\pi}{4})}}{\Gamma(n+\frac{1}{2})} \int_0^\infty e^{-u} u^{n-\frac{1}{2}} \left(1 + \frac{iu}{2z}\right)^{n-\frac{1}{2}} du$$

and substituting equation (1) yields

$$H_n^{(1)}(z) = \left(\frac{2}{\pi z}\right)^{\frac{1}{2}} e^{i(z-\frac{1}{2}n\pi-\frac{1}{4}\pi)} \quad f(a,b;v)$$

where
$$a = n+\tfrac{1}{2}$$
$$b = \tfrac{1}{2}-n$$
$$v = \frac{i}{2z}$$
$$-\frac{\pi}{2} < \arg z < \frac{3\pi}{2} .$$

Substituting $v=\frac{i}{2z}$ into the definition of $K_n(z)$ given in equation (2) gives

$$K_n(-iz) = \left(\frac{i\pi}{2z}\right)^{\frac{1}{2}} e^{iz} f(a,b;v).$$

Therefore $H_n^{(1)}(z)$ can be written as

$$H_n^{(1)}(z) = \frac{-2i}{\pi} e^{-\frac{1}{2}n\pi i} \quad K_n(-iz)$$

and similarly

$$H_{n-1}^{(1)}(z) = \frac{-2i}{\pi} e^{-\frac{1}{2}(n-1)\pi i} \quad K_{n-1}(-iz) .$$

If another quotient function $P_n^{(1)}(z)$ is defined, then

$$P_n^{(1)}(z) = \frac{H_{n-1}^{(1)}(z)}{H_n^{(1)}(z)} = \frac{i\, K_{n-1}(-iz)}{K_n(-iz)}$$

$$P_n^{(1)}(z) = i\, Q_n(-iz)$$

Likewise if[5]

$$H_n^{(2)}(z) = \left(\frac{2}{\pi z}\right)^{\frac{1}{2}} \frac{e^{-i(z-\frac{1}{2}n\pi-\frac{1}{4}\pi}}{\Gamma\ (n+\frac{1}{2})} \int_0^\infty e^{-u}\, u^{n-\frac{1}{2}} \left(1-\frac{iu}{2z}\right)^{n-\frac{1}{2}} du$$

and $-\frac{3\pi}{2} < \arg z < \frac{\pi}{2}$, then

$$P_n^{(2)}(z) = -i\, Q_n(iz)$$

where $v = -\frac{i}{2z}$. Using the Wronskian relation

$$J_{n-1}(z)\, Y_n(z) - J_n(z)\, Y_{n-1}(z) = \frac{-2}{\pi z}$$

and substituting from equation (9)

$$Y_n(z) = i\, [J_n(z) - H_n^{(1)}(z)] \tag{11}$$

in the Wronskian yields

$$J_{n-1}(z)\, H_n^{(1)}(z) - J_n(z)\, H_{n-1}^{(1)}(z) = -\frac{2i}{\pi z}. \tag{12}$$

Likewise

$$J_{n-1}(z)\, H_n^{(2)}(z) - J_n(z)\, H_{n-1}^{(2)}(z) = \frac{2i}{\pi z} \tag{13}$$

when

$$Y_n(z) = -i\ [J_n(z) - H_n^{(2)}(z)\ ]\ . \tag{14}$$

Dividing both sides of equation (12) by $H_n^{(1)}(z)$, substituting $P_n^{(1)}(z)$ and then solving for $H_n^{(1)}(z)$ finally yields

$$H_n^{(1)}(z) = -\frac{4}{\pi}\ \frac{i}{2z}\left[\frac{1}{J_{n-1}(z) - iJ_n(z)\ Q_n(iz)}\right]. \tag{15}$$

Similarly

$$H_n^{(2)}(z) = \frac{4}{\pi}\ \frac{i}{2z}\left[\frac{1}{J_{n-1}(z) + iJ_n(z)\ Q_n(iz)}\right] \tag{16}$$

Finally, using the Hankel function calculated by equations (15) and (16) and the ordinary Bessel functions of the first kind calculated by recurrence, the ordinary Bessel function of the second kind $Y_n(z)$ is calculated from equation (11) if Im $z \leq 0$ or from equation (14) if Im $z \geq 0$.

E. Hankel Asymptotic Series: $|z| > 21.0$

Ordinary and modified Bessel functions of the first and second kind for large argument ($|z| > 21.0$) are calculated using the Hankel asymptotic series beginning at location 1000 in the code.

If z does not lie in the right half-plane, it is rotated through the positive real axis $\pm 180^{\circ}$ to either quadrant I or quadrant IV so that the argument of z lies within the range of all the formulas used. After the functions are calculated for z in the right half-plane, analytic continuation is used to compute the functional value for the original z lying in the left hand plane.

Calculation of the Hankel asymptotic series begins at location 1040 using the formulas given by McLachlan[4]:

$$H_m^{(1)}(z) = \frac{e^{-\rho \sin\theta}}{\sqrt{\frac{\pi\rho}{2}}} \quad e^{i(\rho\cos-\frac{1}{2}\theta - \frac{1}{2}m\pi - \frac{1}{4}\pi)}(P_m+iQ_m) \qquad (16)$$

$$H_m^{(2)}(z) = \frac{e^{\rho\sin\theta}}{\sqrt{\frac{\pi\rho}{2}}} \quad e^{-i(\rho\cos\theta + \frac{1}{2}\theta - \frac{1}{2}m\pi - \frac{1}{4}\pi)}(P_m+iQ_m) \qquad (17)$$

where

$$P_m = \sum_{k=0}^{\infty}(-1)^k \frac{(m,2k)}{(2z)^{2k}} = 1- \frac{(4m^2-1^2)(4m^2-3^2)}{2!(8z)^2} + \frac{(4m^2-1^2)(4m^2-3^2)(4m^2-5^2)(4m^2-7^2)}{4!(8z)^4} - \ldots \qquad (18)$$

$$Q_m = \sum_{k=0}^{\infty}(-1)^k \frac{(m,2k+1)}{(2z)^{2k+1}} = \frac{4m^2-1^2}{1!(8z)} - \frac{(4m^2-1^2)(4m^2-3^2)(4m^2-5^2)}{3!(8z)^3} + \ldots \qquad (19)$$

and the notation $(\nu, m)^5$ following Hankel is defined as

$$(\nu, m) = \frac{(-1)^m(\frac{1}{2}-\nu)_m \; (\frac{1}{2}+\nu)_m}{m!} = \frac{\Gamma(\nu+m+\frac{1}{2})}{m!\,\Gamma(\nu-m+\frac{1}{2})}$$

$$= \frac{[4\nu^2-1^2][4\nu^2-3^2]\ldots[4\nu^2-(2m-1)^2]}{m!(2)^{2m}}, \quad (\nu,0) = 1$$

The terms of the P and Q series are evaluated for order m and n=m+1. The coding for this process begins after location 1040 and ends just before 1090. The terms of P_m are the even numbered terms of a series $(P+Q)_m$; the terms of Q_m are the odd numbered terms of $(P+Q)_m$. Using $\frac{i}{8z}$ = RW + i CW as the argument of each series, instead of $\frac{1}{8z}$, will not only affect the signs of alternate terms in P_m but will in effect multiply Q_m by i as well as change the signs of alternate terms. The coding for the terms of the series $P_m \pm i\, Q_m$ can be summarized as follows:

$$P_m + i\, Q_m : \quad RS1+iCS1=\sum_k TRM(K)+i\sum_k TIM(K)$$

$$P_m - i\, Q_m : \quad RS2+iCS2=\sum_k TRN(K)+i\sum_k TIN(K)$$

$$P_n + i\, Q_n : \quad RS3+iCS3=\sum_k TRV(K)+i\sum_k TIV(K)$$

$$P_n - i\, Q_n : \quad RS4+iCS4=\sum_k TRW(K)+i\sum_k TIW(K)$$

Successive terms of the combined series are computed until either

$$|\, (P_m + i\, Q_m)_k \,| < 10^{-36} \quad \text{or}$$

$|\, (P_n + i\, Q_n)_1 \,| < \; |\, (P_n + i\, Q_n)_k \,|$. After completing

the calculation of $P \pm iQ$, the program divides into two distinct sections to compute either the ordinary or the modified Bessel functions.

At location 1100 the Hankel asymptotic functions for the ordinary Bessel functions are computed. Equation (16) can be rewritten in terms of the computer code for order m as

$$H_m^{(1)}(z) = \frac{C6}{\frac{1}{(C2*C1)}} * e^{i(RED-ZETA-C3-C4)} * (RS1+iCS1)$$

$$FM1 \quad = C8 * [COS(ALPHA)+ i\, sin(ALPHA)] * (RS1 + i\, CS1)$$

and equation (17) for order m as

$$H_m^{(2)}(z) = \frac{C7}{\frac{1}{C2*C1}} * e^{i(RED+ZETA-C3-C4)} * (RS2 + i\, CS2)$$

$$FM2 \quad = C9 * [COS(ALPH1) + iSIN(ALPH1)] * (RS2 + i\, CS2)$$

where

$C1 = \sqrt{\frac{2}{\pi}}$

$C2 = \sqrt{\frac{1}{\rho}}$

$C3 = .25\pi$

$C4 = .5*m*\pi$

$C6 = e^{\rho\sin\theta}$

$C7 = e^{\rho\sin\theta}$

$C8 = C1 * C2 * C6$

$C9 = C1 * C2 * C7$

$ALPHA = \rho\cos\theta - \frac{1}{2}\theta - \frac{1}{2}m\pi - \frac{1}{4}\pi$

$ALPH1 = \rho\cos\theta + \frac{1}{2}\theta - \frac{1}{2}m\pi - \frac{1}{4}\pi$

For order n = m+1, the exponential term changes to

$$e^{i(RED-ZETA-C3-C5)} = e^{iBETA} \quad \text{for } H_n^{(1)}(z) = FN1$$

and to

$$e^{i(RED+ZETA-C3-C5)} = e^{iBETA1} \quad \text{for } H_n^{(2)}(z) = FN2$$

where

$$C5 = .5 * n * \pi.$$

If the real part, ZR, of the given argument z lies in the right halfplane, then the Hankel functions will be used directly to calculate the ordinary Bessel functions and are stored in

$$HM1 = FM1 = H_m^{(1)}(z)$$

$$HM2 = FM2 = H_m^{(2)}(z)$$

$$HN1 = FN1 = H_n^{(1)}(z)$$

$$HN2 = FN2 = H_n^{(2)}(z)$$

until the Bessel functions are assembled at location 1140.

If the given z was rotated to the right half plane, then analytic continuation must be used to obtain the correct Hankel asymptotic functions

$$HM1 = H_m^{(1)}(-z) \qquad HN1 = H_n^{(1)}(-z)$$

$$HM2 = H_m^{(2)}(-z) \qquad HN2 = H_n^{(2)}(-z)$$

from the values FM1, FM2, FN1, and FN2 just calculated.

If the original z is in quadrant II, then the following formulas are used at location 1120:

$$H_m^{(1)}(-z) \quad =-e^{-im\pi}\ H_m^{(2)}(z) =-e^{-im\pi}*FM2$$

$$H_m^{(2)}(-z) = e^{im\pi} * H_m^{(1)}(z) + 2\cos(m\pi) * H_m^{(2)}(z)$$

$$= e^{im\pi} * FM1 + 2\cos(m\pi) * FM2$$

where z is in the right half-plane and -z is in the left half-plane[4]. Since $e^{i\pi} = -1$, then

```
HM1 = -FM2                if the order m is even
HN1 =  FN2                if the order n is odd
HM2 =  FM1 + 2. * FM2     if the order m is even
HM2 = -FN1 - 2. * FN2     if the order n is odd.
```

Each sign will change if m is odd and n is even.

At location 1130, the formulas to calculate the correct Hankel functions for an argument originally given in quadrant III, $H_m(-z)$, from Hankel functions of the corresponding argument lying in quadrant I, $H_m(z)$, were coded as follows:

$$H_m^{(1)}(-z) = (e^{im\pi} + e^{-im\pi})\ H_m^{(1)}(z) + e^{-im\pi}\ H_m^{(2)}(z)$$

```
HM1 = 2.*FM1 + FM2        If order m is even
HN1 = 2.*FN1 - FN2        if order n=m+1 is odd
```

$$H_m^{(2)}(-z) = -\ e^{im\pi}\ H_m^{(1)}(z)$$

HM2 = -FM1 if order m is even
HN2 = FN1 if order n=m+1 is odd

Each sign will change if m is odd and n is even.

Since the appropriate values of the Hankel function for a given z are stored in HM1, HM2, HN1, and HN2, their linear combinations will yield the correct values of the ordinary Bessel functions for large values of z^4:

$$J_m(z) = \tfrac{1}{2}\,[H_m^{(1)}(z) + H_m^{(2)}(z)] = .5\ (HM1 + HM2)$$

$$J_n(z) = \tfrac{1}{2}\,[H_n^{(1)}(z) + H_n^{(2)}(z)] = .5\ (HN1 + HN2)$$

$$Y_m(z) = -\frac{i}{2}\,[H_m^{(1)}(z) - H_m^{(2)}(z)] = -.5i(HM1 - HM2)$$

$$Y_n(z) = -\frac{i}{2}\,[H_n^{(1)}(z) - H_n^{(2)}(z)] = -.5i(HN1 - HN2).$$

The formulas given by Watson[5] for modified Bessel functions of the first kind were rewritten to separate the real and imaginary parts of the factor $\frac{1}{\sqrt{z}}$:

$$I_m(z) = \frac{e^z}{\sqrt{2\pi z}} \sum_{k=0}^{\infty} \frac{(-1)^k (m,k)}{(2z)^k} + \frac{e^{-z}}{\sqrt{2\pi z}}\, e^{(m+\frac{1}{2})\pi i} \sum_{k=0}^{\infty} \frac{(m,k)}{(2z)^k}\ , \quad -\frac{\pi}{2} < \arg z < \frac{3\pi}{2}$$

where $\sqrt{z} = \sqrt{\rho e^{i\theta}} = \sqrt{\rho}\cdot e^{\frac{1}{2}i\theta}$

Since the magnitude of each term of $P_m \pm Q_m$ is numerically the same as described in equation (18) and (19) and the appropriate sign changes are made when the terms are summed so that $P_m+Q_m = RS1 + i\ CS1$ and $P_m-Q_m = RS2 - i\ CS2$ are equivalent to the summations required for $I_m(z)$, we can write

$$I_m(z) = \frac{1}{\sqrt{2\pi}\,\sqrt{\rho}}\, e^{ZR}\, e^{i(ZI-\frac{1}{2}\theta)} (P_m+Q_m) + \frac{1}{\sqrt{2\pi}\,\sqrt{\rho}}\, e^{-ZR} e^{i(-ZI-\frac{1}{2}\theta+\frac{1}{2}\pi+m\pi)} (P_m-Q_m)$$

Using the notation of the coding, this equation can be rewritten as

$$FM1 = C1*C2*C4*\ e^{i ALPH1}(RS1+iCS1) + C1*C2*C5\ e^{i ALPH2}(RS2+iCS2)$$

$$= C8[C4*(COSA1+iSINA1)(RS1+iCS1) + C5(COSA2+iSINA2)(RS1+iCS2)]$$

where

$$C1 = \frac{1}{\sqrt{2\pi}} \qquad C5=e^{-ZR}$$

$$C2 = \frac{1}{\sqrt{\rho}} \qquad C8= C1 * C2$$

$$C4 = e^{ZR} \qquad ALPH1 = ZI - \tfrac{1}{2}\theta$$

$$e^{i ALPH1}=COSA1 + i\ SINA1 \qquad ALPH2 = - ZI - \tfrac{1}{2}\theta + \tfrac{1}{2}\pi + m\pi$$

The mechanics of computing $P_m \pm Q_m$ are identical to those used for the Hankel functions just discussed. The argument used is $\frac{1}{8z} = RW + i\ CW$, and the series $P_m \pm Q_m$ is calculated and stored in the same section of coding. Thus when computation of the modified Bessel functions is begun at location 1200, the factors are stored as

$$P_m + Q_m = RS1 + i\ CS1$$

$$P_m - Q_m = RS2 + i\ CS2$$

$$P_n + Q_n = RS3 + i\ CS3$$

$$P_n - Q_n = RS4 + i\ CS4.$$

It is noted that the order affects two factors of the equation for $I_m(z)$: ALPH2 and $P_m \pm Q_m$. For order n = m+1, these factors become

$$ALPH3 = - ZI - \tfrac{1}{2}\theta + \tfrac{1}{2}\pi + n\pi \text{ and } P_n \pm Q_n.$$

Thus

$$I_n(z) = C8[C4*e^{i\text{ALPH1}}(RS3+iCS3) + C5 * e^{i\text{ALPH3}}(RS4 + i\ CS4)]$$

$$FN1 = C8[C4(COSA1+iSINA1)(RS3+iCS3) + C5(COSA2+iSINA2)(RS4+iCS4)]$$

Modified Bessel functions of the second kind were obtained from the equation[5]

$$K_m(z) = \sqrt{\frac{\pi}{2z}}\ e^{-z} \sum_{k=o}^{\infty} \frac{(m,k)}{(2z)^k}$$

$$= \sqrt{\frac{\pi}{2}}\ \sqrt{\frac{1}{\rho}}\ e^{-ZR}\ e^{i\ (-ZI\ -\ \frac{1}{2}\theta)}(P_m - Q_m)$$

and coded as

$$FM2 = C3 * C2 * C5 * e^{i\text{ALPH4}}\ (RS2 + i\ CS2)$$

$$= C9 * (COSA4 + i\ SINA4)(RS2 + i\ CS2)$$

where

$$C3 = \sqrt{\frac{\pi}{2}}$$

$$C9 = C2 * C3 * C5$$

$$ALPH4 = -\ ZI - \tfrac{1}{2}\theta.$$

For order n=m+1, the only factor which requires change is $P_m - Q_m$ to $P_n - Q_n$:

$$K_n(z) = \sqrt{\frac{\pi}{2}}\ \sqrt{\frac{1}{\rho}}\ e^{-ZR}\ e^{i(-ZI-\frac{1}{2}\theta)}(P_n - Q_n)$$

FN2 = C9 * (COSA4 + iSINA4)(RS4 + iCS4).

If the original z was not rotated to the right half plane at location 1000, then the values of the function have been calculated for the original input value and are stored in the output arrays of the subroutine as follows:

FJI(1) + i FJI(2) = RFM1 + i CFM1
FJI(3) + i FJI(4) = RFN1 + i CFN1
SYK(1) + i SYK(2) = RFM2 + i CFM2
SYK(3) + i SYK(4) = RFN2 + i CFN2.

If z was rotated however, analytic continuation must be used to find the corresponding functional values of the original input value. The formulas,[4] coded at location 1230, for an input value from quadrant II are

$I_m(-z) = I_m(z)$ when m is an even order

$I_n(-z) = -I_n(z)$ when n is an odd order

$K_m(-z) = K_m(z) - \pi i\, I_m(z)$ when m is even

$K_n(-z) = -K_n(z) - \pi i\, I_n(z)$ when n is odd.

The corresponding equations[4] for an input value from quadrant III are coded at location 1240:

$I_m(-z) = I_m(z)$, m even

$I_n(-z) = -I_n(z)$, n odd

$K_m(-z) = K_m(z) + \pi i\, I_m(z)$, m even

$K_n(-z) = -K_n(z) + \pi i\, I_n(z)$, n odd.

IV. CONCLUSIONS

Both the derivation and the coding of the formulas used in the subroutine were carefully checked during preparation of this report. The subroutine is accurate and efficient.

Because sufficiently accurate tables are not available, it is difficult to check the accuracy of the calculations when the argument is complex. Partial verification has been accomplished in the region of overlap between methods of calculation used within the subroutine. Further verification by independent methods of calculation is in progress.

Chebyshev approximations are generally more efficient than continued fraction approximations for real variables but are not appropriate for complex variable.

One of the reviewers has suggested that extension of the program to include fractional and arbitrary real orders would be useful in various physical applications. Only integral orders were considered in this report as they are generally sufficient for our applications in elasticity. Fractional orders involve rather complicated formulas for analytic continuation and should properly be the subject of a separate subroutine.

V. ACKNOWLEDGMENTS

The authors wish to thank Dr. J.B. Campbell, National Research Council of Canada, for disclosure of his method of calculating Bessel functions of complex argument by means of Gauss continued fractions. His derivation, which differs from ours, is contained in a letter to the principal investigator, Mr. A.S. Elder, dated 22 December 1969.

Dr. F. Olver also made helpful suggestions during the early stages of this work.

APPENDIX A

ATTAINABLE ACCURACY OF THE HANKEL ASYMPTOTIC SERIES

The logic of our subroutine required a set of simple inequalities, depending only on argument and order, for determining the method of calculating the Bessel functions. The inequalities used to select the Hankel asymptotic series were originally obtained empirically, after considerable numerical experimentation. In this Appendix we show these empirical inequalities are conservative and may be derived by asymptotic methods of analysis.

We assume the argument is real, positive, and large compared with the order. The terms of a Hankel asymptotic series decreases numerically until a minimum is reached, then increases without bound. The error is generally less than the first term neglected. It is common practice therefore to terminate the series just before the minimum term. We calculate the approximate value of the smallest term in terms of order and argument. The required inequalities are obtained by equating the smallest term and the allowable error.

The Hankel asymptotic series for the modified Bessel function of the second kind is

$$K_n(x) \simeq \sqrt{\frac{\pi}{2x}}\, e^{-x}\; {}_2F_0\left(n+\tfrac{1}{2}, -n+\tfrac{1}{2}, -\frac{1}{2x}\right)$$

where

$${}_2F_0\left(n+\tfrac{1}{2}, -n+\tfrac{1}{2}, -\frac{1}{2x}\right) = \frac{1}{\Gamma(n+\frac{1}{2})\,\Gamma(-n+\frac{1}{2})} \sum_{k=o}^{\infty} (-1)^k\, T_k$$

and

$$T_k = \frac{\Gamma(k+\frac{1}{2}+n)\,\Gamma(k+\frac{1}{2}-n)}{\Gamma(k+1)\,(2x)^k} \quad ,k \text{ integral.}$$

Now assume $k > n$ and regard k as a variable, not merely an index.

$$T(k) = \frac{\Gamma(k+\frac{1}{2}+n)\,\Gamma(k+\frac{1}{2}-n)}{\Gamma(k+1)\,(2x)^k} \quad , k \text{ variable}$$

Let

$$u = \log T(k)$$

or

$$u = \log \Gamma(k+\tfrac{1}{2}+n) + \log \Gamma(k+\tfrac{1}{2}-n) - \log \Gamma(k+1) - k \log 2x$$

To find the minimum, differentiate u with respect to k and set this derivative equal to zero. We have

$$\frac{d \log \Gamma(z)}{dz} = \psi(z), \text{ the psi function}$$

Hence

$$\frac{du}{dk} = \psi(k+\tfrac{1}{2}+n) + \psi(k+\tfrac{1}{2}-n) - \psi(k+1) - \log 2x$$

The leading terms of the asymptotic formula for $\psi(z)$ are sufficient for the required approximation:

$$\psi(z) \simeq \log z - \frac{1}{2z} - \frac{1}{12z^2} .$$

The formula

$$\psi(z) \simeq \log(z - \frac{1}{2} + \frac{1}{24z}),$$

obtained by using the series expansion for log (1+h) where $h = -\frac{1}{2z} + \frac{1}{24z^2}$ is more convenient. Let

$$a = \frac{1}{24(k+\frac{1}{2}+n)}$$

$$b = \frac{1}{24(k+\frac{1}{2}-n)}$$

$$c = \frac{1}{24(k+1)}$$

then

$$\frac{du}{dk} \simeq \log(k+n+a) + \log(k-n+b) - \log(k+\tfrac{1}{2}+c) - \log 2x$$

or

$$\frac{du}{dk} \simeq \frac{(k+n+a)(k-n+b)}{(k+\frac{1}{2}+c)(2x)} .$$

We find

$$\frac{du}{dk} \simeq 0$$

if

$$2x \simeq \frac{(k+n+a)(k-n+b)}{(k+\frac{1}{2}+c)} .$$

We now assume that

$$n^2 < dx$$

where d is a constant in the range

$$1 < d < 10 .$$

Then

$$k \simeq 2x$$

when u is a minimum.

A more accurate value of k is obtained by an iterative procedure. Under the order conditions assumed above we obtain the following cubic equation of k:

$$k^3 - \frac{1}{2}k^2 - n^2k + \frac{1}{2}n^2 + \frac{7}{24}k - 2xk^2 = 0$$

or

$$k = \frac{1}{2} + \frac{2xk^2 - \frac{7}{24}k}{k^2 - n^2}$$

To solve by iteration, we assume

$$k_{i+1} = \frac{1}{2} + \frac{2xk_i^2 - \frac{7}{24} k_i}{k_i^2 - n^2} ,$$

$$k_i = 2x$$

We finally obtain

$$k = 2x + \frac{1}{2} + \frac{n^2}{2x} + \delta$$

where

$$\delta = - \frac{7}{48x} - \frac{n^2}{4x^2} - \frac{n^4}{8x^3} + O(1/x^2).$$

The fractions given above are of order $1/x$ since we have assumed that n^2 is of the same order as x, and consequently n^2 is of the same order as k. If we solve for x in terms of k, we find

$$2x = k - \frac{1}{2} - \frac{n^2}{k} + \frac{7}{24k} + \frac{n^2}{24^2} + O(1/k^2).$$

We now approximate u for large values of x by using Stirling's formula for the gamma function and the logarithmic series:

$$\log \Gamma(z) \approx (z - \frac{1}{2}) \log z - z + \frac{1}{2} \log 2\pi + \frac{1}{12z} - \frac{1}{360z^2} + O(1/z^5)$$

$$\log (1+z) = z - \frac{1}{2} z^2 + \frac{1}{3} z^3 - \frac{1}{4} z^4 + O(1/z^5)$$

We approximate $\log \Gamma(k+\frac{1}{2}+n)$, $\log (k+\frac{1}{2}-n)$, and $\log \Gamma(k+1)$ by Stirling's formula, then represent the logarithms which occur as log k + a series. We also express 2x in terms of k. We finally obtain, after considerable algebra, the formula

$$u = -\frac{1}{2}\log k - k + \frac{1}{2} - \frac{1}{12k} + \frac{2n^2}{k} + \frac{n^2}{k^2} - \frac{n^4}{6k^3} + O(1/k^2)$$

In terms of x this becomes

$$u = -\frac{1}{2}\log 2x - 2x + \frac{n^2}{2x} + \varepsilon$$

where

$$\varepsilon = -\frac{1}{48x} + \frac{n^2}{8x^2} - \frac{n^4}{48x^3} + O(1/x^2) .$$

Since $u = \log \Gamma(k)$, $\Gamma(k) = \exp(u)$.

Hence

$$T_k \simeq \frac{\pi}{x} e^{-2x+n^2/2x} [1 + \varepsilon + O(1/2^2)]$$

In our program we used the Hankel asymptotic expansion provided

$$2x < 42$$

and

$$\frac{n^2}{2x} < 1.$$

These requirements are evidently conservative.

APPENDIX B

LISTING OF SUBROUTINE BESSEL

```
      SUBROUTINE BESSEL (PHI,CHI,ORD,OPT1,OPT2,FJI,SYK,JPR,LERR)
C
C     ORDINARY AND MODIFIED BESSEL FUNCTIONS OF INTEGRAL ORDER
C     FOR REAL AND COMPLEX ARGUMENT
C
C     COMPLEX VARIABLE NOTATION     (I = SQUARE ROOT OF -1)
C
C
C (I) RECTANGULAR COORDINATES  -  Z = X+I*Y, PHI = X, CHI = Y
C (I) POLAR COORDINATES     -  Z = RHO*COS(ANGLE)+I*RHO*SIN(ANGLE)
C (I)                       -  PHI = RHO,  THE RADIUS VECTOR
C (I)                       -  CHI = ANGLE IN DEGREES
C (I)                       -  180 < CHI <= 180  (PROGRAM CORRECTS)
C (I) ORD  = ORDER TO BE COMPUTED - IF ORD = M THEN PROGRAM WILL
C (I)        COMPUTE ORDERS M AND M+1. EX.  IF M+1 = N THEN WILL
C (I)        GIVE JM(Z) AND JN(Z)
C (I) OPT1 = 1  COMPUTE ORDINARY BESSEL FUNCTIONS
C (I) OPT1 = 2  COMPUTE MODIFIED BESSEL FUNCTIONS
C (I) OPT2 = 1  ARGUMENT IS IN RECTANGULAR COORDINATES
C (I) OPT2 = 2  ARGUMENT IS IN POLAR COORDINATES - ANGLE IN DEGREES
C
C
C (R) FJI AND SYK ARE ONE DIMENSIONAL ARRAYS OF 4 VALUES
C (R) FJI - BESSEL FUNCTIONS OF THE FIRST KIND
C (R)     FJI(1) = REAL JM(Z) OR IM(Z)
C (R)     FJI(2) = IMAGINARY JM(Z) OR IM(Z)
C (R)     FJI(3) = REAL JN(Z) OR IN(Z)
C (R)     FJI(4) = IMAGINARY JN(Z) OR IN(Z)
C (R) SYK - BESSEL FUNCTIONS OF THE SECOND KIND
C (R)     SYK(1) = REAL YM(Z) OR KM(Z)
C (R)     SYK(2) = IMAGINARY YM(Z) OR KM(Z)
C (R)     SYK(3) = REAL YN(Z) OR KN(Z)
C (R)     SYK(4) = IMAGINARY YN(Z) OR KN(Z)
```

```
C
C (R) JPR IS AN INTEGER INDICATING WHICH METHOD OF COMPUTATION WAS USED
C (R)      JPR = 1   SERIES
C (R)      JPR = 2   CONTINUED FRACTION OF GAUSS
C (R)      JPR = 3   HANKEL ASYMPTOTIC SERIES
C
C (R) LERR IS AN INTEGER INDICATING IF THERE WAS A RUN ERROR IN THE
C (R)       SUBROUTINE.  LERR = 0, NO ERROR.  LERR = 1, THERE WAS AN
C (R)       ERROR.  USER CAN TEST THIS NUMBER AND THEN DECIDE WHETHER
C (R)       TO CONTINUE OR NOT.
C
C
C
      DIMENSION FJI(4),SYK(4),EPSI(9),TRM(100),TIM(100),TRN(100),
     1          TIN(100),TRV(100),TIV(100),TRW(100),TIW(100)
C
C
      DATA PI/3.1415926535897932/,PSI/-.11593151565841244881O7/
      DATA EPSI/ 1.E-9, 1.E-36, 1.E-48, 1.E-75, 1.E+75, 1.E-10, 1.E-14,
     1      1.E+10, 350.0/
C
      LERR=0
      OM=ORD
      IF(PHI .EQ. 0.0 .AND. CHI .EQ. 0.0) GOTO 5000  JERR=1  z=0
      IF(AMOD(ORD,1.) .NE. 0.) GOTO 5060    JERR=7;  Integral orders only
      IF(OPT1 .LT. 1.0  .OR.  OPT1 .GT. 2.0) GOTO 5070  JERR=8; type B.F. wrong
      N=OPT2
      IF(N .EQ. 0 .OR. N .GT. 2) GOTO 5010   JERR=2; coordinate system wrong
      GOTO  (100,130),  N
  100 ZR=PHI                                                Rectangular
      ZI=CHI                                                Coordinates
      RHO=SQRT(PHI**2+CHI**2)
      SINZ=CHI/RHO
      COSZ=PHI/RHO
      IF(COSZ) 102,101,101
```

```
101 Z=SINZ/(1.0+COSZ)
    THETA=2.0*ATAN(Z)
    GOTO  140
102 T1=1.0
    IF(SINZ)  103,104,104
103 T1=-1.0
104 Z=-SINZ/(1.0-COSZ)
    THETA=2.0*ATAN(Z)+T1*PI
    GOTO 140
130 IF(CHI .GE. 0.0 .AND. CHI .LE. 180.0)  GOTO  136     Polar
    T1=-1.0*CHI                                          Coordinates
    IF(T1 .GT. 0.0  .AND.  T1 .LT. 180.0)  GOTO  136
    T1=-1.0
    IF(CHI)  132,134,134
132 T1=1.0
134 CHI=T1*360.0+CHI
    GOTO  130
136 RHO=PHI
    THETA=PI*CHI/180.
    ZR=RHO*COS(THETA)
    ZI=RHO*SIN(THETA)
140 BM=OM
    BN=OM+1.
    RN=BN
    MI=OM
    NI=MI+1        1.E-48
    IF(RHO .LT. EPSI(3))  GOTO 5020  JERR=3; prevents getting log 0
    ZRL=ALOG(RHO)+PSI
    ZIL=THETA
    N=OPT1
    GOTO (150,160), N
150 SGN=-1.0
    SGPI=2./PI      signs for ordinary B.F.
    GOTO 170
160 SGN=1.0         signs for modified B.F.
```

```
      SGPI=1.0          signs for modified B.F.
  170 OR=.5*ZR
      OI=.5*ZI
      XR=.25*(ZR*ZR-ZI*ZI)   (z/2)^2 = ((RZ+iZI)/2)^2 = (ZR^2-ZI^2)/4 + (iZR•ZI)/2 = XR+iXI
      XI=.5*ZR*ZI
      T1=ZR*ZR+ZI*ZI
      YR=ZR/T1               1/z = 1/(ZR+iZI) = ZR/(ZR^2+ZI^2) - iZI/(ZR^2+ZI^2) = YR + i YI
      YI=-ZI/T1
      IF(RHO .LT. 21.0)  GOTO  180        Weber-Schlafli
      T1=(BN*BN)/RHO
      IF(T1 .LE. 1.95)  GOTO  1000        Hankel Asymptotic Series
  180 IF(RHO .GT. 2.5)  GOTO  600         Gauss Continued Fraction and Recurrence
C
C     SET UP INITIAL CONDITIONS FOR  M=0  N=1
C
      JPR=1
      ASSIGN 2000 TO JS1                  Return for small |z|<2.5
  200 IF(BM-1.) 201,210,220
  201 CMR1=1.              common factor of J0 and I0
      CMI1=0.0
      CNR1=OR              common factor of J1 and I1
      CNI1=OI
      CMR2=SGN*.5          common factor of infinite series for Y0 and K0
      CMI2=0.0
      CNR2=SGN*.5*OR       common factor of infinite series for Y1 and K1
      CNI2=SGN*.5*OI
      SRV=0.0
      SRW=1.0
      CMR3=0.0             common factor of finite series for Y0 and K0
      CMI3=0.0
      CNR3=SGN*YR          common factor of finite series for Y1 and K1
      CNI3=SGN*YI
      CMR4=0.0             sum of terms of finite series for Y0 and K0
      CMI4=0.0
      CNR4=1.0             sum of terms of finite series for Y1 and K1
```

sum of terms of finite series for Y_1 and K_1

```
      CNI4=0.0
      GOTO 500
C
C     SET UP INITIAL CONDITIONS FOR  M=1  N=2
C
  210 CMR1=OR
      CMI1=OI
      CNR1=XR*.5
      CNI1=XI*.5
      CMR2=SGN*.5*OR
      CMI2=SGN*.5*OI
      CNR2=SGN*.5*CNR1
      CNI2=SGN*.5*CNI1
      SRV=1.0
      SRW=1.5
      CMR3=SGN*YR
      CMI3=SGN*YI
      CNR3=SGN*2.*(YR*YR-YI*YI)
      CNI3=SGN*4.*YI*YR
      CMR4=1.0
      CMI4=0.0
      CNR4=1.-SGN*XR
      CNI4=-SGN*XI
      GOTO 500
C
C     SET UP INITIAL CONDITIONS FOR  M=2  N=3
C
  220 IF(BM .GT. 2.)  GOTO 300
      CMR1=.5*XR
      CMI1=.5*XI
      CNR1=(XR*OR-XI*OI)/6.
      CNI1=(XR*OI+XI*OR)/6.
      CMR2=SGN*.5*CMR1
      CMI2=SGN*.5*CMI1
      CNR2=SGN*.5*CNR1
```

```
      CNI2=SGN*.5*CNI1
      SRV=1.5
      SRW=11./6.
      CMR3=SGN*2.*(YR*YR-YI*YI)
      CMI3=SGN*4.*YR*YI
      T1=SGN*6.*(CNR1**2+CNI1**2)
      CNR3=CNR1/T1
      CNI3=-CNI1/T1
      CMR4=1.-SGN*XR
      CMI4=-SGN*XI
      CNR4=1.-SGN*.5*XR+(XR**2-XI**2)/4.
      CNI4=-SGN*.5*XI+.5*XR*XI
      GOTO 500
C
C     CODING FOR THE I TH ORDER FOR RHO < 2.5
C
  300 T1=XR/2.
      T2=XI/2.
      T3=3.
      T4=1./3.
      SRV=1.5
  310 CMR1=(OR*T1-OI*T2)*T4
      CMI1=(OR*T2+OI*T1)*T4
      T5=ABS(CMR1)+ABS(CMI1)                                  JERR=4; constant
      IF(T5 .GT. EPSI(5) .OR. T5 .LT. EPSI(4))  GOTO 5030     term of finite
      T1=CMR1      1.E+75                  1.E-75             series too large
      T2=CMI1
      SRV=SRV+T4         1.E-09
      IF(ABS(T3-BM) .LE. EPSI(1))  GOTO 320
      T3=T3+1.
      T4=1./T3
      GOTO 310
  320 T4=1./(T3+1.)
      SRW=SRV+T4
      CNR1=(OR*CMR1-OI*CMI1)*T4
```

```
      CNI1=(OR*CMI1+OI*CMR1)*T4
      T1=(CMR1**2+CMI1**2)*2.*BM*SGN
      T2=(CNR1**2+CNI1**2)*2.*BN*SGN
      CMR3=CMR1/T1
      CMI3=-CMI1/T1
      CNR3=CNR1/T2
      CNI3=-CNI1/T2
      CMR2=SGN*.5*CMR1
      CMI2=SGN*.5*CMI1
      CNR2=SGN*.5*CNR1
      CNI2=SGN*.5*CNI1
C
C     COMPUTE THE FINITE SERIES
C
      T3=0.0
      FR=1.0
      TRM(1)=1.0
      TIM(1)=0.0
      TRN(1)=1.0
      TIN(1)=0.0
      K=1               1.E-09
  400 IF(ABS(FR-BM) .LE. EPSI(1))  GOTO 410
      T1=-SGN/(FR*(BM-FR))
      TRM(K+1)=T1*(TRM(K)*XR-TIM(K)*XI)
      TIM(K+1)=T1*(TRM(K)*XI+TIM(K)*XR)
  410 T1=-SGN/(FR*(BN-FR))
      TRN(K+1)=T1*(TRN(K)*XR-TIN(K)*XI)
      TIN(K+1)=T1*(TRN(K)*XI+TIN(K)*XR)
      IF(ABS(FR-BM) .LE. EPSI(1))  GOTO 430
      T2=TRN(K+1)**2+TIN(K+1)**2
      IF(T2 .LT. T3)  GOTO 420
      T3=T2
      T4=TRN(K+1)
      T5=TIN(K+1)       1.E-36
  420 IF(T2 .LT. EPSI(2))  GOTO 430
```

```
      FR=FR+1.0
      K=K+1
      GOTO 400
  430 CNR4=TRN(K+1)+TRN(K)
      CNI4=TIN(K+1)+TIN(K)
      CMR4=TRM(K)
      CMI4=TIM(K)
  440 K=K-1
      CNR4=CNR4+TRN(K)
      CNI4=CNI4+TIN(K)
      CMR4=CMR4+TRM(K)
      CMI4=CMI4+TIM(K)
      IF(K .GT. 1)  GOTO 440
C
C     COMPUTE THE INFINITE SERIES
C
  500 TRM(1)=1.0
      TRN(1)=1.0
      TRV(1)=SRV
      TRW(1)=SRW
      TIM(1)=0.0
      TIN(1)=0.0
      TIV(1)=0.0
      TIW(1)=0.0
      K=1
      FR=1.0
      T1=BN+1.0
      T3=SRW
      T4=0.0
      T6=0.0
  510 AA=SGN/(FR*(BM+FR))
      BB=SGN/(FR*(BN+FR))
      T2=T3
      T3=T2+1.0/T1
      T4=T4+1.0/FR
```

```
      P=T4+T2
      Q=T4+T3
      TRM(K+1)=AA*(TRM(K)*XR-TIM(K)*XI)        Jm or Im
      TRV(K+1)=P*TRM(K+1)      Ym or Km
      TRN(K+1)=BB*(TRN(K)*XR-TIN(K)*XI)        Jn or In
      TRW(K+1)=Q*TRN(K+1)      Yn or Kn
      TIM(K+1)=AA*(TRM(K)*XI+TIM(K)*XR)
      TIV(K+1)=P*TIM(K+1)
      TIN(K+1)=BB*(TRN(K)*XI+TIN(K)*XR)
      TIW(K+1)=Q*TIN(K+1)
      T5=TRW(K+1)**2+TIW(K+1)**2
      T9=TRN(K+1)**2+TIN(K+1)**2
      IF(T9 .LT. T6)  GOTO 520
      T6=T9
  520 IF(T5 .LT. EPSI(2))  GOTO 530  ----> out of loop
      K=K+1
      FR=FR+1.0
      T1=T1+1.0
      GOTO 510
  530 STR=TRM(K+1)    Jm or Im
      STI=TIM(K+1)
      SUR=TRN(K+1)    Jn or In
      SUI=TIN(K+1)
      SVR=TRV(K+1)    Ym or Km
      SVI=TIV(K+1)
      SWR=TRW(K+1)    Yn or Kn
      SWI=TIW(K+1)
      FR=FR+1.0
  540 STR=STR+TRM(K)
      STI=STI+TIM(K)
      SUR=SUR+TRN(K)
      SUI=SUI+TIN(K)
      SVR=SVR+TRV(K)
      SVI=SVI+TIV(K)
      SWR=SWR+TRW(K)
```

(Margin annotation with arrow spanning lines P=T4+T2 through GOTO 510: Calculate terms in series until it converges)

```
      SWI=SWI+TIW(K)
      K=K-1
      IF(K .GT. 0)  GOTO 540
      RJM=CMR1*STR-CMI1*STI
      CJM=CMR1*STI+CMI1*STR
      RJN=CNR1*SUR-CNI1*SUI
      CJN=CNR1*SUI+CNI1*SUR
      T1=1.0
      T2=1.0
      IF(OPT1 .EQ. 1.0)  GOTO 560        ordinary B.F.
      IF(AMOD(BM,2.) .EQ. 0.0)  GOTO 550
      T1=1.0
      T2=-1.0
      GOTO 560                 modified B.F. differ in signs of even terms
  550 T1=-1.0
      T2=1.0
  560 RLV=T1*(RJM*ZRL-CJM*ZIL)        logarithmic term for Y_m or K_m
      CLV=T1*(RJM*ZIL+CJM*ZRL)
      RLW=T2*(RJN*ZRL-CJN*ZIL)        logarithmic term for Y_n or K_n
      CLW=T2*(RJN*ZIL+CJN*ZRL)
      FV1R=CMR3*CMR4-CMI3*CMI4        finite term for Y_m or K_m
      FV1I=CMR3*CMI4+CMI3*CMR4
      FW1R=CNR3*CNR4-CNI3*CNI4        finite term for Y_n or K_n
      FW1I=CNR3*CNI4+CNI3*CNR4
      FV2R=T2*(CMR2*SVR-CMI2*SVI)     infinite series term for Y_m or K_m
      FV2I=T2*(CMR2*SVI+CMI2*SVR)
      FW2R=T1*(CNR2*SWR-CNI2*SWI)     infinite series term for Y_n or K_n
      FW2I=T1*(CNR2*SWI+CNI2*SWR)
      RGM=-(RLV+FV1R+FV2R)
      RYM=-SGPI*RGM                   three terms summed and multiplied by
      CGM=-(CLV+FV1I+FV2I)            their common factor - order m
      CYM=-SGPI*CGM
      RGN=-(RLW+FW1R+FW2R)
      RYN=-SGPI*RGN                   - order n
      CGN=-(CLW+FW1I+FW2I)
```

```
      CYN=-SGPI*CGN
      FJI(1)=RJM                                  Results stored in output arrays.
      FJI(2)=CJM
      FJI(3)=RJN
      FJI(4)=CJN
      SYK(1)=RYM
      SYK(2)=CYM
      SYK(3)=RYN
      SYK(4)=CYN
      GOTO  JS1,(2000,630)
C
C     COMPUTATION FOR Z > 2.5 BY USE OF RECURSION FORMULAS
C
  600 ASSIGN 630 TO JS1          Return to recursion after using series to calculate
      TX=RHO*RHO/4.0             starting value for recurrence procedures.
      T1=1.0
      FR=1.0
      FN=RN+1.0                                   Increase order until
  610 T1=T1*TX/(FR*FN)                              T1<1.E-10
      IF(T1 .LT. EPSI(6))  GOTO 620
      FR=FR+1.0   1.E-10
      FN=FN+1.0
      GOTO 610
  620 BM=FN
      BN=FN+1.0
      MI=BM
      NI=BN
      GOTO 200    Go compute B.F. of order FN and FN+1 using series.
  630 RJHN1=RJN   Return from series.  Use B.F. of high order to start downward
      CJHN1=CJN           recursion.
      RJHM1=RJM
      CJHM1=CJM
  640 RJHM=2.*BM*(YR*RJHM1-YI*CJHM1)+SGN*RJHN1
      CJHM=2.*BM*(YR*CJHM1+YI*RJHM1)+SGN*CJHN1
      MI=MI-1
```

$J_{m-1} = \frac{2m}{z} \cdot J_m - J_n$

```
      NI=NI-1       Decrease order to compute J(m-2) = 2(m-1)/z . J(m-1) - J(n-1) until BM = ORD.
      BM=BM-1.0
      IF(ABS(BM-ORD) .LT. .001)  GOTO 650     finished recursion
      RJHN1=RJHM1
      CJHN1=CJHM1          J(n) = J(m) = J(n-1)
      RJHM1=RJHM
      CJHM1=CJHM           J(m) = J(m-1)
      GOTO 640
  650 SRJM1=RJHM           Save values of J(m) and J(n) computed by downward recursion.
      SCJM1=CJHM
      SRJN1=RJHM1
      SCJN1=CJHM1
C
C     COMPUTE Y AND K BY USING CONTINUED FRACTIONS OF GAUSS
C
      T1=1.0
      IF(OPT1 .EQ. 1.0)  GOTO 720        ordinary B.F.
      IF(ZR .GE. 0.0)  GOTO 710
      T1=-1.0              modified B.F.; left half-plane
  710 RW=T1*.5*YR          1/2z ; rotated by T1 to right half-plane if necessary.
      CW=T1*.5*YI
      GOTO 740
  720 T3=1.0               ordinary B.F.
      IF(ZI .GE. 0.0)  GOTO 730
      T1=-1.0
      T3=-1.0
  730 RW=-.5*T1*YI         1/2z ; rotated by T1 to right half-plane if necessary.
      CW=.5*T1*YR
  740 AA=RN+.5
      BB=-RN+.5
      CLL=50.0
      FJI(1)=SRJM1
      FJI(2)=SCJM1         J or I computed by recursion are stored in output arrays.
      FJI(3)=SRJN1
      FJI(4)=SCJN1
```

```
      T1=1.0
      T2=-1.0
      IF(AMOD(RN,2.) .EQ. 0.0)  GOTO 750          n even
      T1=-1.0     n odd
      T2=1.0
  750 RFL1=0.0
      CFL1=0.0
      RGL1=0.0
      CGL1=0.0
  760 BL=BB+CLL       Begin using continued fraction for Y or K
      BL1=BL+1.0
      ALL=AA+CLL-1.0
      RNF=1.0+BL*(RW*RFL1-CW*CFL1)
      CNF=BL*(RW*CFL1+CW*RFL1)
      RDF=RNF+ALL*RW                F_l(a,b;v)
      CDF=CNF+ALL*CW
      T4=RDF**2+CDF**2
      RFL=(RNF*RDF+CNF*CDF)/T4
      CFL=(CNF*RDF-RNF*CDF)/T4
      RNG=1.0+BL1*(RW*RGL1-CW*CGL1)
      CNG=BL1*(RW*CGL1+CW*RGL1)
      RDG=RNG+ALL*RW                G_l(a,b;v)
      CDG=CNG+ALL*CW
      T4=RDG**2+CDG**2
      RGL=(RNG*RDG+CNG*CDG)/T4
      CGL=(CNG*RDG-RNG*CDG)/T4
      CLL=CLL-1.0
      IF(CLL .LT. .5)  GOTO 770
      RFL1=RFL
      CFL1=CFL
      RGL1=RGL
      CGL1=CGL
      GOTO 760
  770 RH=(BB+1.0)*(RW*RGL-CW*CGL)
      CH=(BB+1.0)*(RW*CGL+CW*RGL)      Q_n
```

```
      RQN=RFL*(1.0+RH)-CFL*CH
      CQN=CFL*(1.0+RH)+RFL*CH
      IF(OPT1 .EQ. 1.0)  GOTO 820          ordinary B.F.
      J=1          z in right-half plane
      IF(ZR .GE. 0.0)  GOTO 790          right half-plane
      J=2       z rotated to right half-plane; analytic continuation to be used.
      T3=-1.0
      IF(ZI .GE. 0.0)  GOTO 780          Quad II
      T3=1.0                 Quad III
  780 SRJN1=T1*SRJN1
      SCJN1=T1*SCJN1            -I_n   when n is odd
      SRJM1=T2*SRJM1
      SCJM1=T2*SCJM1            I_m    when m is even
  790 RNKN=2.0*RW
      CNKN=2.0*CW               1/z
      RDKN=RQN*SRJN1-CQN*SCJN1+SRJM1
      CDKN=RQN*SCJN1+CQN*SRJN1+SCJM1       Q_n I_n + I_m = Q_{m+1} I_{m+1} + I_m
      T4=RDKN**2+CDKN**2
      RKN=(RNKN*RDKN+CNKN*CDKN)/T4
      CKN=(CNKN*RDKN-RNKN*CDKN)/T4
      RKM=RQN*RKN-CQN*CKN
      CKM=RQN*CKN+CQN*RKN                  K_m = K_n Q_n = K_{n-1}
      GOTO (810,800), J
  800 T4=T3*PI              Analytic continuation used to find correct value
      RKN=T1*RKN-T4*SCJN1
      CKN=T1*CKN+T4*SRJN1              for K
      RKM=T2*RKM-T4*SCJM1
      CKM=T2*CKM+T4*SRJM1
  810 SYK(1)=RKM            Results stored in output arrays.
      SYK(2)=CKM
      SYK(3)=RKN
      SYK(4)=CKN
  820 IF(OPT1 .EQ. 2.0)  GOTO  830
      T4=4.0/PI             Analytic continuation to find correct value
      RPN=T3*(-CQN)                    for Y
```

```
      CPN=T3*RQN
      RNH=RW
      CNH=CW
      RDH=RPN*FJI(3)-CPN*FJI(4)-FJI(1)
      CDH=RPN*FJI(4)+CPN*FJI(3)-FJI(2)
      T5=RDH**2+CDH**2
      RHN=T4*((RNH*RDH+CNH*CDH)/T5)
      CHN=T4*((CNH*RDH-RNH*CDH)/T5)
      RHM=RHN*RPN-CHN*CPN
      CHM=RHN*CPN+CHN*RPN
      SYK(1)=T3*(CHM-FJI(2))
      SYK(2)=T3*(FJI(1)-RHM)        Y  stored in output arrays
      SYK(3)=T3*(CHN-FJI(4))
      SYK(4)=T3*(FJI(3)-RHN)
  830 JPR=2
      GOTO  2000            Return from subroutine.
C
C     COMPUTE BESSEL FUNCTIONS USING HANKEL ASYMPTOTIC SERIES
C
 1000 IF(OPTI .EQ. 2.0)  GOTO  1020 ——> modified B.F.
      T1=1.0         z in right half-plane
      T2=-1.0
      ZETA=.5*THETA
      IF(ZR .GE. 0.0)  GOTO  1010          right half-plane
      T1=-1.0        rotate z to right half-plane.
      T2=1.0
      ZETA=.5*(THETA-PI)      rotate - 180° from Quad II to Quad IV
      IF(ZI .GE. 0.0)  GOTO  1010
      ZETA=.5*(THETA+PI)      rotate 180° from Quad III to Quad I
 1010 RED=T1*ZR      Puts z in right half-plane
      CMD=T1*ZI
      RW=T1*YI/8.0   -i
      CW=T2*YR/8.0   8z
      J=1
      GOTO  1040
```

Ordinary

Ordinary

Modified

```
1020 T1=1.0
     ZETA=.5*THETA
     IF(ZR .GE. 0.0)  GOTO  1030
     T1=-1.0
     ZETA=.5*(THETA+PI)          rotate from Quad III to Quad I
     IF(ZI .LT. 0.0)  GOTO  1030
     ZETA=.5*(THETA-PI)          rotate from Quad II to Quad IV
1030 RED=T1*ZR           Puts z in right half-plane
     CMD=T1*ZI
     RW=T1*YR/8.         1
     CW=T1*YI/8.         8z
     J=2                 1.E+10
1040 IF(ABS(ZETA) .GT. EPSI(8))  GOTO 5050
     K=3
     TC1=1.0
     TC2=1.0
     FM2=4.0*ORD*ORD     4m^2
     FN2=4.0*RN*RN           4n^2
     T1=FM2-1.0
     T2=FN2-1.0
     T4=-1.0
     TRM(1)=1.0
     TRN(1)=1.0
     TRV(1)=1.0
     TRW(1)=1.0
     TIM(1)=0.0
     TIN(1)=0.0
     TIV(1)=0.0
     TIW(1)=0.0
     RT3=T1*RW           (4m^2 -1) (-i/8z)
     CT3=T1*CW
     RT4=T2*RW
     CT4=T2*CW           (4n^2-1) (i/8z)
     TRM(2)=-RT3
     TRN(2)=RT3
```

```
      TRV(2)=-RT4
      TRW(2)=RT4
      TIM(2)=-CT3
      TIN(2)=CT3
      TIV(2)=-CT4
      TIW(2)=CT4
      T6=RT4**2+CT4**2
 1050 TC1=TC1+1.0
      TC2=TC2+2.0
      T4=-1.0*T4
      T1=FM2-TC2*TC2
      T2=FN2-TC2*TC2
      T3=T1/TC1
      T5=T2/TC1
      TRT3=T3*(RT3*RW-CT3*CW)        m th term of series   Pm ± Qm
      TCT3=T3*(RT3*CW+CT3*RW)
      TRT4=T5*(RT4*RW-CT4*CW)        Pn ± Qn
      TCT4=T5*(RT4*CW+CT4*RW)
      RT3=TRT3
      CT3=TCT3
      RT4=TRT4
      CT4=TCT4
      TRM(K)=T4*RT3
      TRN(K)=RT3
      TRV(K)=T4*RT4
      TRW(K)=RT4
      TIM(K)=T4*CT3
      TIN(K)=CT3
      TIV(K)=T4*CT4
      TIW(K)=CT4                            1.E-36
      IF((RT3*RT3+CT3*CT3) .LT. EPSI(2))  GOTO  1070
      T7=RT4**2+CT4**2
      IF(T6 .LT. T7) GOTO 1060
      T7=T6
      K=K+1
```

```
      GOTO  1050
1060  K=K-1
1070  RS1=0.0
      RS2=0.0
      RS3=0.0
      RS4=0.0
      CS1=0.0
      CS2=0.0
      CS3=0.0
      CS4=0.0
1080  RS1=RS1+TRM(K)        Pm+(i)Qm   Jm or Im
      CS1=CS1+TIM(K)
      RS2=RS2+TRN(K)        Pm-(i)Qm   Ym or Km
      CS2=CS2+TIN(K)
      RS3=RS3+TRV(K)        Pn+(i)Qn   Jn or In
      CS3=CS3+TIV(K)
      RS4=RS4+TRW(K)
      CS4=CS4+TIW(K)        Pn-(i)Qn   Yn or Kn
      IF(K .EQ. 1)  GOTO  1090
      K=K-1
      GOTO  1080
1090  HPI=.5*PI
      THPI=1.5*PI
      GOTO  (1100,1200), J
1100  C1=SQRT(2.0/PI)
      C2=1.0/SQRT(RHO)                                     Ordinary B.F.
      C3=.25*PI
      C4=.5*ORD*PI
      C5=.5*RN*PI
                         350.0
      IF(ABS(CMD) .GT. EPSI(9))  GOTO 5040
      C6=EXP(-CMD)
      C7=EXP(CMD)
      ALPHA=RED-ZETA-C3-C4    ±ZR - θ/2 - π/4 - mπ/2
      SINA=SIN(ALPHA)
      COSA=COS(ALPHA)
```

```
      ALPH1=RED+ZETA-C3-C4
      SINA1=SIN(ALPH1)
      COSA1=COS(ALPH1)
      BETA=RED-ZETA-C3-C5
      SINB=SIN(BETA)
      COSB=COS(BETA)
      BETA1=RED+ZETA-C3-C5
      SINB1=SIN(BETA1)
      COSB1=COS(BETA1)
 1112 C8=C1*C2*C6
      C9=C1*C2*C7
      RFM1=C8*(COSA*RS1-SINA*CS1) ________ H_n^(1)(z)
      CFM1=C8*(COSA*CS1+SINA*RS1)
      RFM2=C9*(COSA1*RS2+SINA1*CS2)_______ H_n^(2)(z)
      CFM2=C9*(COSA1*CS2-SINA1*RS2)
      RFN1=C8*(COSB*RS3-SINB*CS3) ________ H_n+1^(1)(z)
      CFN1=C8*(COSB*CS3+SINB*RS3)
      RFN2=C9*(COSB1*RS4+SINB1*CS4)_______ H_n+1^(2)(z)
      CFN2=C9*(COSB1*CS4-SINB1*RS4)
      IF(ZR .LT. 0.0)  GOTO  1115
      RHM1=RFM1          J_m
      RHM2=RFM2              Y_m
      RHN1=RFN1          J_n
      RHN2=RFN2              Y_n
      CHM1=CFM1
      CHM2=CFM2
      CHN1=CFN1          Correct Hankel functions if z was given in the
      CHN2=CFN2          right half-plane.
      GOTO  1140
 1115 T1=1.0                                                z lies in left
      IF(AMOD(BM,2.) .EQ. 0.0)  GOTO  1120                  half-plane.
      T1=-1.0
 1120 T2=-1.0*T1
      T3=-1.0*T2
      IF(ZI .LT. 0.0)  GOTO  1130
```

```
      RHM1=T2*RFM2
      CHM1=T2*CFM2
      RHM2=T1*(RFM1+2.0*RFM2)                      IV → II
      CHM2=T1*(CFM1+2.0*CFM2)
      RHN1=T3*RFN2
      CHN1=T3*CFN2
      RHN2=T2*(RFN1+2.0*RFN2)
      CHN2=T2*(CFN1+2.0*CFN2)
      GOTO  1140
1130  RHM1=T1*(2.0*RFM1+RFM2)
      CHM1=T1*(2.0*CFM1+CFM2)                      I → III
      RHM2=T2*RFM1
      CHM2=T2*CFM1
      RHN1=T2*(2.0*RFN1+RFN2)
      CHN1=T2*(2.0*CFN1+CFN2)
      RHN2=T3*RFN1
      CHN2=T3*CFN1
1140  FJI(1)=.5*(RHM1+RHM2)             Output arrays for ordinary B.F.
      FJI(2)=.5*(CHM1+CHM2)
      FJI(3)=.5*(RHN1+RHN2)
      FJI(4)=.5*(CHN1+CHN2)
      SYK(1)=.5*(CHM1-CHM2)
      SYK(2)=.5*(RHM2-RHM1)
      SYK(3)=.5*(CHN1-CHN2)
      SYK(4)=.5*(RHN2-RHN1)
      JPR=3
      GOTO  2000                        Return
1200  C1=1.0/SQRT(2.0*PI)               Modified B.F.
      C2=1.0/SQRT(RHO)
      C3=SQRT(.5*PI)
      IF(ABS(RED) .GT. EPSI(9))  GOTO 5040
      C4=EXP(RED)
      C5=EXP(-RED)
      ALPH1=CMD-ZETA
      SINA1=SIN(ALPH1)
```

```
      COSA1=COS(ALPH1)
      T1=1.0
      IF(ZI .GT. 0.0) GOTO 1210
      T1=-1.0
 1210 ALPH2=-CMD-ZETA+T1*(ORD+.5)*PI
      SINA2=SIN(ALPH2)
      COSA2=COS(ALPH2)
      ALPH3=-CMD-ZETA+T1*(RN+.5)*PI
      SINA3=SIN(ALPH3)
      COSA3=COS(ALPH3)
      ALPH4=-ZETA-CMD
      SINA4=SIN(ALPH4)
      COSA4=COS(ALPH4)
 1219 C8=C1*C2
      C9=C3*C2*C5
      RFM1=C8*(C4*(COSA1*RS1-SINA1*CS1)+C5*(COSA2*RS2-SINA2*CS2))     I_m
      CFM1=C8*(C4*(COSA1*CS1+SINA1*RS1)+C5*(COSA2*CS2+SINA2*RS2))
      RFN1=C8*(C4*(COSA1*RS3-SINA1*CS3)+C5*(COSA3*RS4-SINA3*CS4))     I_n
      CFN1=C8*(C4*(COSA1*CS3+SINA1*RS3)+C5*(COSA3*CS4+SINA3*RS4))
      RFM2=C9*(COSA4*RS2-SINA4*CS2)     K_m
      CFM2=C9*(COSA4*CS2+SINA4*RS2)
      RFN2=C9*(COSA4*RS4-SINA4*CS4)     K_n
      CFN2=C9*(COSA4*CS4+SINA4*RS4)
      IF(ZR .LT. 0.0)  GOTO  1220
      FJI(1)=RFM1          Output arrays for z in right half-plane
      FJI(2)=CFM1
      FJI(3)=RFN1
      FJI(4)=CFN1
      SYK(1)=RFM2
      SYK(2)=CFM2
      SYK(3)=RFN2
      SYK(4)=CFN2
      GOTO  1250
 1220 T1=1.0
      IF(AMOD(BM,2.) .EQ. 0.0)  GOTO  1230
```

```
      T1=-1.0
 1230 T2=-1.0*T1
      IF(ZI .LT. 0.0)  GOTO  1240
      FJI(1)=T1*RFM1          Output arrays for z in Quadrant II.
      FJI(2)=T1*CFM1
      FJI(3)=T2*RFN1
      FJI(4)=T2*CFN1
      SYK(1)=T1*(RFM2+PI*FJI(2))
      SYK(2)=T1*(CFM2-PI*FJI(1))
      SYK(3)=T2*(RFN2+PI*FJI(4))
      SYK(4)=T2*(CFN2-PI*FJI(3))
      GOTO  1250
 1240 FJI(1)=T1*RFM1          Output arrays for z in Quadrant III.
      FJI(2)=T1*CFM1
      FJI(3)=T2*RFN1
      FJI(4)=T2*CFN1
      SYK(1)=T1*(RFM2-PI*FJI(2))
      SYK(2)=T1*(CFM2+PI*FJI(1))
      SYK(3)=T2*(RFN2-PI*FJI(4))
      SYK(4)=T2*(CFN2+PI*FJI(3))
 1250 JPR=3
      GOTO  2000             Return
C     ______________________________________________________________
C     ERROR PRINT OUTS
C
 5000 JERR=1
      GOTO 5090
 5001 WRITE(6,5002)
 5002 FORMAT(// 49H CAN NOT COMPUTE THE BESSEL FUNCTION IF Z IS ZERO)
      WRITE(6,5003)
 5003 FORMAT(// 47H J 0 EQUAL 1  -  ALL OTHER ORDERS OF J ARE ZERO)
      WRITE(6,5004)
 5004 FORMAT(38H0ALL ORDERS OF Y ARE EQUAL TO INFINITY)
      GOTO 5100
 5010 JERR=2
```

```
      GOTO 5090
 5011 WRITE(6,5012) OPT2
 5012 FORMAT(// 30H OPT2 IS NOT 1 OR 2     OPT2 =, E12.4)
      GOTO 5100
 5020 JERR=3
      GOTO 5090
 5021 WRITE(6,5022) RHO
 5022 FORMAT(// 29H RHO IS OUTSIDE RANGE   RHO =, E12.4)
      GOTO 5100
 5030 JERR=4
      GOTO 5090
 5031 WRITE(6,5032)
 5032 FORMAT(// 43H CONSTANT TERM OF FINITE SERIES WILL EXCEED,
     123H NUMBER SIZE OF MACHINE)
      WRITE(6,5033) OM,RN,CMR1,CMI1
 5033 FORMAT(4HOM =, E12.4, 5H  N =, E12.4, 8H  CMR1 =, E12.4,
     1 8H  CMI1 =, E12.4)
      WRITE(6,5034) PHI,CHI
 5034 FORMAT(6HOPHI =, E15.8, 7H  CHI =, E15.8)
      GOTO 5100
 5040 JERR=5
      GOTO 5090
 5041 WRITE(6,5042)
 5042 FORMAT(/ 52H ERROR IN ASYMPTOTIC SECTION - ARGUMENT FOR EXP TOO ,
     1 5HLARGE)
      WRITE(6,5043) RED,CMD
 5043 FORMAT(1HO,5HRED =,E15.8,4X,5HCMD =,E15.8)
      GOTO 5100
 5050 JERR=6
      GOTO 5090
 5051 WRITE(6,5052)
 5052 FORMAT(/ 52H ARGUMENT FOR SIN AND COS IN ASYMPTOTIC SECTION TOO ,
     1 5HLARGE)
      WRITE(6,5053) ZETA
 5053 FORMAT(1HO,6HZETA =,E15.8)
```

```
      GOTO 5100
 5060 JERR=7
      GOTO 5090
 5061 WRITE(6,5062)
 5062 FORMAT(55H SUBROUTINE CALCULATES INTEGRAL ORDER ONLY- CHECK INPUT)
      GOTO 5100
 5070 JERR=8
      GOTO 5090
 5071 WRITE(6,5072)
 5072 FORMAT(33H OPT1 IS NOT 1 OR 2 - CHECK INPUT)
      GOTO 5100
      GOTO (5001,5011,5021,5031,5041,5051,5061,5071), JERR
 5090 WRITE(6,5091)
 5091 FORMAT(// 40H RUN ERROR IN BESSEL FUNCTION SUBROUTINE)
      WRITE(6,5092) PHI,CHI,ORD,OPT1,OPT2
 5092 FORMAT(1H0,5HPHI =,F15.8,4X,5HCHI =,F15.8,4X,5HORD =,F6.1,4X,
     1 6HOPT1 =,F6.1,4X,6HOPT2 =,F6.1)
      GOTO (5001,5011,5021,5031,5041,5051), JERR
 5100 WRITE(6,5101)
 5101 FORMAT(1H1)
      LERR=1
C
 2000 RETURN
      END
```

APPENDIX C

SAMPLE OUTPUTS FROM SUBROUTINE BESSEL

The results tabulated in this appendix were selected to show calculations in either rectangular or polar coordinates, in each of the four quadrants, and of various orders. The results are presented in pairs as calculated by the subroutine. For any given RHO, the first page of the tables gives order m and n=m+1 for Bessel functions of the first kind; page two contains orders m and n for Bessel functions of the second kind. Note that each table has two sets of duplicate arguments corresponding to RHO = 2.5 and RHO = 21.0. These are the cut-off points between various means of calculation; the results for each method are shown for comparison.

Samples for polar coordinates with ANG = 0°, 30°, 60°, 90°, 120°, 150°, 180°, -120°, and -60° for ordinary Bessel function of order zero and one begin on page 70. Corresponding modified Bessel functions begin on page 88. This sample set shows calculations in each of the four quadrants.

Calculations for rectangular coordinates as input begin on page 106 for values corresponding to ANG = 45°. Samples for orders m=0 and 1, for m=10 and 11, and for n=50 and 51 are displayed. Note that there are errors detected by the subroutine in the calculations for the first three arguments of all the tables for m=50. The error prints (found on page 114 and 117) have been included for the user's information.

OPT1= 1.0 ORDINARY BESSEL FUNCTIONS, FIRST KIND,
OPT2= 2.0 OF COMPLEX ARGUMENT IN POLAR COORDINATES
ORD = 0.0 OF ORDER M = 0

RHO	ANG	RE J	IM J	METHOD
0.1	0.0	0.997501562066040E 00	0.000000000000000E 00	1
0.5	0.0	0.938469807240813E 00	0.000000000000000E 00	1
1.0	0.0	0.765197686557967E 00	0.000000000000000E 00	1
1.5	0.0	0.511827671735918E 00	0.000000000000000E 00	1
2.0	0.0	0.223890779141236E 00	0.000000000000000E 00	1
2.5	0.0	-0.483837764681980E-01	0.000000000000000E 00	1
2.5	0.0	-0.483837764681980E-01	0.000000000000000E 00	2
3.0	0.0	-0.260051954901933E 00	0.000000000000000E 00	2
5.0	0.0	-0.177596771314338E 00	0.000000000000000E 00	2
10.0	0.0	-0.245935764451349E 00	0.000000000000000E 00	2
15.0	0.0	-0.142244728267808E-01	0.000000000000000E 00	2
20.0	0.0	0.167024664340582E 00	0.000000000000000E 00	2
21.0	0.0	0.365790710008631E-01	0.000000000000000E 00	2
21.0	0.0	0.365790710008630E-01	0.000000000000000E 00	3
25.0	0.0	0.962667832759579E-01	0.000000000000000E 00	3
30.0	0.0	-0.863679835810404E-01	0.000000000000000E 00	3
40.0	0.0	0.736689058423751E-02	0.000000000000000E 00	3
50.0	0.0	0.558123276692516E-01	0.000000000000000E 00	3
75.0	0.0	0.346439138050968E-01	0.000000000000000E 00	3
100.0	0.0	0.199858503042229E-01	0.000000000000000E 00	3

AND ORDER N = M+1 = 1

RHO	ANG	RE J	IM J	METHOD
0.1	0.0	0.499375260362420E-01	0.000000000000000E 00	1
0.5	0.0	0.242268457674874E 00	0.000000000000000E 00	1
1.0	0.0	0.440050585744934E 00	0.000000000000000E 00	1
1.5	0.0	0.557936507910100E 00	0.000000000000000E 00	1
2.0	0.0	0.576724807756873E 00	0.000000000000000E 00	1
2.5	0.0	0.497094102464274E 00	0.000000000000000E 00	1
2.5	0.0	0.497094102464274E 00	0.000000000000000E 00	2
3.0	0.0	0.339058958525936E 00	0.000000000000000E 00	2
5.0	0.0	-0.327579137591465E 00	0.000000000000000E 00	2
10.0	0.0	0.434727461688616E-01	0.000000000000000E 00	2
15.0	0.0	0.205104038613522E 00	0.000000000000000E 00	2
20.0	0.0	0.668331241758496E-01	0.000000000000000E 00	2
21.0	0.0	0.171120272763902E 00	0.000000000000000E 00	2
21.0	0.0	0.171120272763900E 00	0.000000000000000E 00	3
25.0	0.0	-0.125350249580290E 00	0.000000000000000E 00	3
30.0	0.0	-0.118751062616623E 00	0.000000000000000E 00	3
40.0	0.0	0.126038318037585E 00	0.000000000000000E 00	3
50.0	0.0	-0.975118281251752E-01	0.000000000000000E 00	3
75.0	0.0	-0.851399950448292E-01	0.000000000000000E 00	3
100.0	0.0	-0.771453520141122E-01	0.000000000000000E 00	3

OPT1= 1.0 ORDINARY BESSEL FUNCTIONS, SECOND KIND,
OPT2= 2.0 OF COMPLEX ARGUMENT IN POLAR COORDINATES
ORD = 0.0 OF ORDER M = 0

RHO	ANG	RE Y	IM Y	METHOD
0.1	0.0	-0.153423865135037E 01	0.000000000000000E 00	1
0.5	0.0	-0.444518733506707E 00	0.000000000000000E 00	1
1.0	0.0	0.882569642156771E-01	0.000000000000000E 00	1
1.5	0.0	0.382448923797759E 00	0.000000000000000E 00	1
2.0	0.0	0.510375672649745E 00	0.000000000000000E 00	1
2.5	0.0	0.498070359615232E 00	0.000000000000000E 00	1
2.5	0.0	0.498070359615232E 00	0.780625564189563E-17	2
3.0	0.0	0.376850010012790E 00	0.832667268468867E-16	2
5.0	0.0	-0.308517625249034E 00	-0.555111512312578E-16	2
10.0	0.0	0.556711672835995E-01	-0.374700270810990E-15	2
15.0	0.0	0.205464296038919E 00	0.442354486374086E-16	2
20.0	0.0	0.626405968093843E-01	-0.240085729075190E-14	2
21.0	0.0	0.170201758422154E 00	0.640980324373430E-15	2
21.0	0.0	0.170201758422156E 00	0.000000000000000E 00	3
25.0	0.0	-0.127249432268006E 00	0.000000000000000E 00	3
30.0	0.0	-0.117295731686664E 00	0.000000000000000E 00	3
40.0	0.0	0.125936417058261E 00	0.000000000000000E 00	3
50.0	0.0	-0.980649954700772E-01	0.000000000000000E 00	3
75.0	0.0	-0.853690476477757E-01	0.000000000000000E 00	3
100.0	0.0	-0.772443133650832E-01	0.000000000000000E 00	3

AND ORDER N = M+1 = 1

RHO	ANG	RE Y	IM Y	METHOD
0.1	0.0	-0.645895109470203E 01	0.000000000000000E 00	1
0.5	0.0	-0.147147239267024E 01	0.000000000000000E 00	1
1.0	0.0	-0.781212821300289E 00	0.000000000000000E 00	1
1.5	0.0	-0.412308626973911E 00	0.000000000000000E 00	1
2.0	0.0	-0.107032431540938E 00	0.000000000000000E 00	1
2.5	0.0	0.145918137966786E 00	0.000000000000000E 00	1
2.5	0.0	0.145918137966786E 00	-0.166533453693773E-15	2
3.0	0.0	0.324674424791800E 00	-0.124900090270330E-15	2
5.0	0.0	0.147863143391227E 00	-0.111022302462516E-15	2
10.0	0.0	0.249015424206954E 00	0.650521303491303E-16	2
15.0	0.0	0.210736280368736E-01	-0.624500451351651E-15	2
20.0	0.0	-0.165511614362523E 00	-0.957567358739198E-15	2
21.0	0.0	-0.325392607558651E-01	0.301147995429574E-14	2
21.0	0.0	-0.325392607558653E-01	0.000000000000000E 00	3
25.0	0.0	-0.988299647832375E-01	0.000000000000000E 00	3
30.0	0.0	0.844255706617472E-01	0.000000000000000E 00	3
40.0	0.0	-0.579350582154967E-02	0.000000000000000E 00	3
50.0	0.0	-0.567956685620147E-01	0.000000000000000E 00	3
75.0	0.0	-0.352137851605804E-01	0.000000000000000E 00	3
100.0	0.0	-0.203723120027597E-01	0.000000000000000E 00	3

OPT1= 1.0
OPT2= 2.0
ORD = 0.0

ORDINARY BESSEL FUNCTIONS, FIRST KIND,
OF COMPLEX ARGUMENT IN POLAR COORDINATES
OF ORDER M = 0

RHO	ANG	RE J	IM J	METHOD
0.1	30.0	0.998749219183994E 00	-0.216371034482641E-02	1
0.5	30.0	0.968268487155552E 00	-0.532808826876282E-01	1
1.0	30.0	0.867618103497088E 00	-0.202980518393593E 00	1
1.5	30.0	0.684054268254105E 00	-0.418782462297495E 00	1
2.0	30.0	0.401876909028543E 00	-0.650962462072000E 00	1
2.5	30.0	0.140682542445071E-01	-0.832987043495826E 00	1
2.5	30.0	0.140682542445068E-01	-0.832987043495826E 00	2
3.0	30.0	-0.465411477666209E 00	-0.887568825861950E 00	2
5.0	30.0	-0.177477694131959E 01	0.131251655129563E 01	2
10.0	30.0	-0.504718923288674E 01	-0.181437389325676E 02	2
15.0	30.0	0.185917703645046E 03	0.199776629495281E 02	2
20.0	30.0	-0.923017123951714E 03	0.174137723919345E 04	2
21.0	30.0	0.117282087083194E 04	0.294578373110085E 04	2
21.0	30.0	0.117282087083194E 04	0.294578373110085E 04	3
25.0	30.0	-0.138638036945787E 05	-0.163841413058516E 05	3
30.0	30.0	0.226423045537541E 06	-0.752236282122239E 05	3
40.0	30.0	-0.276939913381966E 08	-0.131340690983884E 08	3
50.0	30.0	0.144801894531719E 10	0.380099335560008E 10	3
75.0	30.0	-0.215590057027113E 14	-0.890542624880618E 15	3
100.0	30.0	-0.643080421887415E 20	0.196724241684646E 21	3

AND ORDER N = M+1 = 1

RHO	ANG	RE J	IM J	METHOD
0.1	30.0	0.433012476411755E-01	0.249375130235453E-01	1
0.5	30.0	0.216436240685357E 00	0.117228400708627E 00	1
1.0	30.0	0.430804407423258E 00	0.188828534743413E 00	1
1.5	30.0	0.633195425326950E 00	0.174387857582875E 00	1
2.0	30.0	0.799860900592473E 00	0.447975866038960E-01	1
2.5	30.0	0.890855053772562E 00	-0.210366347877049E 00	1
2.5	30.0	0.890855053772563E 00	-0.210366347877049E 00	2
3.0	30.0	0.852941735217775E 00	-0.574590706638979E 00	2
5.0	30.0	-0.141571113811487E 01	-0.154462726355877E 01	2
10.0	30.0	0.174815980619617E 02	-0.573017405009032E 01	2
15.0	30.0	-0.141953614488187E 02	0.183464045550592E 03	2
20.0	30.0	-0.174014288024260E 04	-0.873467366158393E 03	2
21.0	30.0	-0.288670348898402E 04	0.122050261384379E 04	2
21.0	30.0	-0.288670348898402E 04	0.122050261384378E 04	3
25.0	30.0	0.159795501524473E 05	-0.140132778554756E 05	3
30.0	30.0	0.778976641808558E 05	0.223458193208723E 06	3
40.0	30.0	0.127508522875685E 08	-0.276651089568002E 08	3
50.0	30.0	-0.376948439914233E 10	0.147389856943119E 10	3
75.0	30.0	0.887459425151825E 15	-0.266460721975677E 14	3
100.0	30.0	-0.196512843420341E 21	-0.632937118915498E 20	3

OPT1= 1.0 ORDINARY BESSEL FUNCTIONS, SECOND KIND,
OPT2= 2.0 OF COMPLEX ARGUMENT IN POLAR COORDINATES
ORD = 0.0 OF ORDER M = 0

RHO	ANG	RE Y	IM Y	METHOD
0.1	30.0	-0.153623193926885E 01	0.337624848247792E 00	1
0.5	30.0	-0.460618331886311E 00	0.383850315053005E 00	1
1.0	30.0	0.901620659975732E-01	0.429105025534784E 00	1
1.5	30.0	0.476846347044623E 00	0.395727680561784E 00	1
2.0	30.0	0.771118384999964E 00	0.241239102429762E 00	1
2.5	30.0	0.957226150662270E 00	-0.519703336133048E-01	1
2.5	30.0	0.957226150662271E 00	-0.519703336133051E-01	2
3.0	30.0	0.987933482284138E 00	-0.470487054202657E 00	2
5.0	30.0	-0.131602701590907E 01	-0.174608854068330E 01	2
10.0	30.0	0.181453749679991E 02	-0.504760910158662E 01	2
15.0	30.0	-0.199777298060961E 02	0.185917611981269E 03	2
20.0	30.0	-0.174137724348276E 04	-0.923017117111215E 03	2
21.0	30.0	-0.294578373583096E 04	0.117282087152003E 04	2
21.0	30.0	-0.294578373583096E 04	0.117282087152003E 04	3
25.0	30.0	0.163841413064353E 05	-0.138638036944732E 05	3
30.0	30.0	0.752236282122149E 05	0.226423045537497E 06	3
40.0	30.0	0.131340690983884E 08	-0.276939913381966E 08	3
50.0	30.0	-0.380099335560008E 10	0.144801894531719E 10	3
75.0	30.0	0.890542624880818E 15	-0.215590057027113E 14	3
100.0	30.0	-0.196724241684646E 21	-0.643080421887415E 20	3

AND ORDER N = M+1 = 1

RHO	ANG	RE Y	IM Y	METHOD
0.1	30.0	-0.560205452682501E 01	0.315122890834242E 01	1
0.5	30.0	-0.132205671909700E 01	0.614768527633812E 00	1
1.0	30.0	-0.781564828632038E 00	0.416719012330624E 00	1
1.5	30.0	-0.498546827364022E 00	0.492874773780581E 00	1
2.0	30.0	-0.203248655590212E 00	0.632904405511599E 00	1
2.5	30.0	0.157022589135603E 00	0.743200458638169E 00	1
2.5	30.0	0.157022589135604E 00	0.743200458638169E 00	2
3.0	30.0	0.582456784835582E 00	0.743173898106327E 00	2
5.0	30.0	0.157450875053365E 01	-0.140965398371385E 01	2
10.0	30.0	0.572981256372800E 01	0.174799025577091E 02	2
15.0	30.0	-0.183464140662370E 03	-0.141952960640065E 02	2
20.0	30.0	0.873467372993656E 03	-0.174014287575281E 04	2
21.0	30.0	-0.122050261324380E 04	-0.288670348418297E 04	2
21.0	30.0	-0.122050261324379E 04	-0.288670348418297E 04	3
25.0	30.0	0.140132778555922E 05	0.159795501518595E 05	3
30.0	30.0	-0.223458193208767E 06	0.778976641808642E 05	3
40.0	30.0	0.276651089568002E 08	0.127508522875685E 08	3
50.0	30.0	-0.147389856943119E 10	-0.376948439914233E 10	3
75.0	30.0	0.266460721975677E 14	0.887459425151825E 15	3
100.0	30.0	0.632937118915498E 20	-0.196512843420341E 21	3

OPT1= 1.0 ORDINARY BESSEL FUNCTIONS, FIRST KIND,
OPT2= 2.0 OF COMPLEX ARGUMENT IN POLAR COORDINATES
ORD = 0.0 OF ORDER M = 0

RHO	ANG	RE J	IM J	METHOD
0.1	60.0	0.100124921831594E 01	-0.216641667409577E-02	1
0.5	60.0	0.103075492385372E 01	-0.549722926707177E-01	1
1.0	60.0	0.111675011576078E 01	-0.230032066040576E 00	1
1.5	60.0	0.123667048430371E 01	-0.555489344711846E 00	1
2.0	60.0	0.134639083726984E 01	-0.108096813237748E 01	1
2.5	60.0	0.136528827443692E 01	-0.187222368675946E 01	1
2.5	60.0	0.136528827443692E 01	-0.187222368675946E 01	2
3.0	60.0	0.115579200904768E 01	-0.300262937063371E 01	2
5.0	60.0	-0.841142625147852E 01	-0.110161430764326E 02	2
10.0	60.0	0.139420238990127E 02	0.735811296869062E 03	2
15.0	60.0	0.264264623150749E 05	-0.370001823305540E 05	2
20.0	60.0	-0.284281315817822E 07	0.911112779480107E 06	2
21.0	60.0	-0.477276355792684E 07	0.501674968032690E 07	2
21.0	60.0	-0.477276355792685E 07	0.501674968032691E 07	3
25.0	60.0	0.191597942931977E 09	0.657900165249202E 08	3
30.0	60.0	-0.791224672881744E 10	-0.115929487827113E 11	3
40.0	60.0	0.442567473084090E 14	-0.543106951188833E 14	3
50.0	60.0	0.333369769107119E 18	0.139303970580571E 18	3
75.0	60.0	0.667141394754871E 27	0.331993344232796E 27	3
100.0	60.0	0.140873927196622E 37	0.821458496603586E 36	3

AND ORDER N = M+1 = 1

RHO	ANG	RE J	IM J	METHOD
0.1	60.0	0.250625130181200E-01	0.433012476317786E-01	1
0.5	60.0	0.132852976850616E 00	0.216435506551306E 00	1
1.0	60.0	0.313774275620088E 00	0.430710447441400E 00	1
1.5	60.0	0.595335397446254E 00	0.631590710386300E 00	1
2.0	60.0	0.103784157792652E 01	0.787852816867388E 00	1
2.5	60.0	0.170951050029616E 01	0.833733833953888E 00	1
2.5	60.0	0.170951050029616E 01	0.833733833953889E 00	2
3.0	60.0	0.268076012848432E 01	0.649163851044207E 00	2
5.0	60.0	0.957501107181050E 01	-0.827919220327030E 01	2
10.0	60.0	-0.703135000007194E 03	0.326273435112711E 02	2
15.0	60.0	0.363761230388357E 05	0.250203467325038E 05	2
20.0	60.0	-0.927597044231115E 06	-0.276917161684861E 07	2
21.0	60.0	-0.497065707239388E 07	-0.461265029179396E 07	2
21.0	60.0	-0.497065707239389E 07	-0.461265029179396E 07	3
25.0	60.0	-0.626931553093386E 08	0.188930116622977E 09	3
30.0	60.0	0.113578900648696E 11	-0.789555341300883E 10	3
40.0	60.0	0.540003338141960E 14	0.434327008570482E 14	3
50.0	60.0	-0.136412455763170E 18	0.331177068637690E 18	3
75.0	60.0	-0.327836046940875E 27	0.664395375463492E 27	3
100.0	60.0	-0.814359066955208E 36	0.140469309166744E 37	3

OPT1 = 1.0 ORDINARY BESSEL FUNCTIONS, SECOND KIND,
OPT2 = 2.0 OF COMPLEX ARGUMENT IN POLAR COORDINATES
ORD = 0.0 OF ORDER M = 0

RHO	ANG	RE Y	IM Y	METHOD
0.1	60.0	-0.154094963408115E 01	0.672214671953312E 00	1
0.5	60.0	-0.513688568911397E 00	0.750750466256815E 00	1
1.0	60.0	-0.672311230821477E-03	0.912223551678364E 00	1
1.5	60.0	0.462874176302913E 00	0.109739204484137E 01	1
2.0	60.0	0.104997404927546E 01	0.125636964387044E 01	1
2.5	60.0	0.186788105112190E 01	0.130975779196637E 01	1
2.5	60.0	0.186788105112189E 01	0.130975779196637E 01	2
3.0	60.0	0.300838927511044E 01	0.112312378570383E 01	2
5.0	60.0	0.110203993371652E 02	-0.841317848451188E 01	2
10.0	60.0	-0.735811319255232E 03	0.139420609380260E 02	2
15.0	60.0	0.370001823305092E 05	0.264264623146102E 05	2
20.0	60.0	-0.911112779480103E 06	-0.284281315817822E 07	2
21.0	60.0	-0.501674968032690E 07	-0.477276355792683E 07	2
21.0	60.0	-0.501674968032691E 07	-0.477276355792685E 07	3
25.0	60.0	-0.657900165249202E 08	0.191597942931977E 09	3
30.0	60.0	0.115929487827113E 11	-0.791224672881744E 10	3
40.0	60.0	0.543106951188833E 14	0.442567473084090E 14	3
50.0	60.0	-0.139303970580571E 18	0.333369769107119E 18	3
75.0	60.0	-0.331993344232796E 27	0.667141394754871E 27	3
100.0	60.0	-0.821458496603586E 36	0.140873927196622E 37	3

AND ORDER N = M+1 = 1

RHO	ANG	RE Y	IM Y	METHOD
0.1	60.0	-0.325856199422923E 01	0.544954422799028E 01	1
0.5	60.0	-0.895388056159714E 00	0.101090494078167E 01	1
1.0	60.0	-0.759268252174481E 00	0.593342730843681E 00	1
1.5	60.0	-0.820634843664161E 00	0.693282531826976E 00	1
2.0	60.0	-0.899449749548632E 00	0.106546001211328E 01	1
2.5	60.0	-0.898759057909209E 00	0.170968573692400E 01	1
2.5	60.0	-0.898759057909209E 00	0.170968573692400E 01	2
3.0	60.0	-0.685907596070777E 00	0.267177226709970E 01	2
5.0	60.0	0.827749018104930E 01	0.957031507778632E 01	2
10.0	60.0	-0.326273054291926E 02	-0.703134975775553E 03	2
15.0	60.0	-0.250203467329825E 05	0.363761230388742E 05	2
20.0	60.0	0.276917161684861E 07	-0.927597044231119E 06	2
21.0	60.0	0.461265029179396E 07	-0.497065707239388E 07	2
21.0	60.0	0.461265029179396E 07	-0.497065707239389E 07	3
25.0	60.0	-0.188930116622977E 09	-0.626931553093386E 08	3
30.0	60.0	0.789555341300883E 10	0.113578900648696E 11	3
40.0	60.0	-0.434327008570482E 14	0.540003338141960E 14	3
50.0	60.0	-0.331177068637690E 18	-0.136412455763170E 18	3
75.0	60.0	-0.664395375463492E 27	-0.327836046940875E 27	3
100.0	60.0	-0.140469309166744E 37	-0.814359066955208E 36	3

```
OPT1=   1.0          ORDINARY BESSEL FUNCTIONS,  FIRST KIND,
OPT2=   2.0        OF COMPLEX ARGUMENT IN    POLAR   COORDINATES
ORD  =  0.0                     OF ORDER M =    0
```

RHO	ANG	RE J	IM J	METHOD
0.1	90.0	0.100250156293410E 01	-0.436528768825978E-18	1
0.5	90.0	0.106348337074132E 01	-0.112437677948591E-16	1
1.0	90.0	0.126606587775201E 01	-0.492800158794543E-16	1
1.5	90.0	0.164672318977289E 01	-0.128397127942107E-15	1
2.0	90.0	0.227958530233607E 01	-0.277396608853054E-15	1
2.5	90.0	0.328983914405012E 01	-0.548623456898064E-15	1
2.5	90.0	0.328983914405012E 01	-0.548623456898063E-15	2
3.0	90.0	0.488079258586502E 01	-0.103416265816831E-14	2
5.0	90.0	0.272398718236044E 02	-0.106099399509331E-13	2
10.0	90.0	0.281571662846626E 04	-0.232901399075728E-11	2
15.0	90.0	0.339649373297914E 06	-0.429171217031784E-09	2
20.0	90.0	0.435582825595536E 08	-0.740386821268910E-07	2
21.0	90.0	0.115513961922158E 09	-0.206421926801154E-06	2
21.0	90.0	0.115513961922158E 09	-0.258161885294257E-11	3
25.0	90.0	0.577456060646632E 10	0.262409486519982E-08	3
30.0	90.0	0.781672297823978E 12	0.293993966939457E-06	3
40.0	90.0	0.148947747934199E 17	0.416432400070393E-02	3
50.0	90.0	0.293255378384934E 21	0.652475155705402E 02	3
75.0	90.0	0.172263907803581E 32	0.253756945183767E 13	3
100.0	90.0	0.107375170713108E 43	0.118223545927515E 24	3

AND ORDER N = M+1 =   1

RHO	ANG	RE J	IM J	METHOD
0.1	90.0	0.437619636314585E-17	0.500625260470927E-01	1
0.5	90.0	0.238785912473406E-16	0.257894305390896E 00	1
1.0	90.0	0.611167664059176E-16	0.565159103992485E 00	1
1.5	90.0	0.129785187400855E-15	0.981666428577908E 00	1
2.0	90.0	0.258846383800546E-15	0.159063685463733E 01	1
2.5	90.0	0.497708511868781E-15	0.251671624528870E 01	1
2.5	90.0	0.497708511868781E-15	0.251671624528870E 01	2
3.0	90.0	0.932046316860350E-15	0.395337021740261E 01	2
5.0	90.0	0.975414836273125E-14	0.243356421424505E 02	2
10.0	90.0	0.222231093286090E-11	0.267098830370126E 04	2
15.0	90.0	0.415633216591527E-09	0.328124921970206E 06	2
20.0	90.0	0.722608466770682E-07	0.424549733851278E 08	2
21.0	90.0	0.201691576593984E-06	0.112729199137776E 09	2
21.0	90.0	0.117284663250688E-09	0.112729199137776E 09	3
25.0	90.0	0.771180398201133E-08	0.565786512987871E 10	3
30.0	90.0	0.866887654586501E-06	0.768532038938957E 12	3
40.0	90.0	0.123337242674162E-01	0.147073961632594E 17	3
50.0	90.0	0.193754515243797E 03	0.290307859010356E 21	3
75.0	90.0	0.756143539294969E 13	0.171111601529653E 32	3
100.0	90.0	0.352883710781244E 24	0.106836939033816E 43	3

OPT1= 1.0 ORDINARY BESSEL FUNCTIONS, SECOND KIND,
OPT2= 2.0 OF COMPLEX ARGUMENT IN POLAR COORDINATES
ORD = 0.0 OF ORDER M = 0

RHO	ANG	RE Y	IM Y	METHOD
0.1	90.0	-0.154512013002621E 01	0.100250156293410E 01	1
0.5	90.0	-0.588503458697208E 00	0.106348337074132E 01	1
1.0	90.0	-0.268032482033988E 00	0.126606587775201E 01	1
1.5	90.0	-0.136112848623590E 00	0.164672318977289E 01	1
2.0	90.0	-0.725070913438698E-01	0.227958530233607E 01	1
2.5	90.0	-0.396916851260922E-01	0.328983914405012E 01	1
2.5	90.0	-0.396916851260922E-01	0.328983914405012E 01	2
3.0	90.0	-0.221158553745547E-01	0.488079258586502E 01	2
5.0	90.0	-0.234982618119395E-02	0.272398718236044E 02	2
10.0	90.0	-0.113191368953861E-04	0.281571662846626E 04	2
15.0	90.0	-0.620839395847520E-07	0.339649373297914E 06	2
20.0	90.0	0.736731835757803E-07	0.435582825595536E 08	2
21.0	90.0	0.206290670575582E-06	0.115513961922158E 09	2
21.0	90.0	-0.128674606719740E-09	0.115513961922158E 09	3
25.0	90.0	-0.262630021894500E-08	0.577456060646632E 10	3
30.0	90.0	-0.293993980515230E-06	0.781672297823978E 12	3
40.0	90.0	-0.416432400070393E-02	0.148947747934199E 17	3
50.0	90.0	-0.652475155705402E 02	0.293255378384934E 21	3
75.0	90.0	-0.253756945183767E 13	0.172263907803581E 32	3
100.0	90.0	-0.118223545927515E 24	0.107375170713108E 43	3

AND ORDER N = M+1 = 1

RHO	ANG	RE Y	IM Y	METHOD
0.1	90.0	-0.500625260470933E-01	0.627315242134332E 01	1
0.5	90.0	-0.257894305390896E 00	0.105452316875680E 01	1
1.0	90.0	-0.565159103992485E 00	0.383186043874565E 00	1
1.5	90.0	-0.981666428577908E 00	0.176590558384380E 00	1
2.0	90.0	-0.159063685463733E 01	0.890413858440260E-01	1
2.5	90.0	-0.251671624528870E 01	0.470403546833581E-01	1
2.5	90.0	-0.251671624528870E 01	0.470403546833580E-01	2
3.0	90.0	-0.395337021740261E 01	0.255643780439264E-01	2
5.0	90.0	-0.243356421424505E 02	0.257488089096837E-02	2
10.0	90.0	-0.267098830370126E 04	0.118721801334200E-04	2
15.0	90.0	-0.328124921970206E 06	0.649798876445071E-07	2
20.0	90.0	-0.424549733851278E 08	0.726353737796087E-07	2
21.0	90.0	-0.112729199137776E 09	0.201825922415101E-06	2
21.0	90.0	-0.112729199137776E 09	0.251630484367976E-09	3
25.0	90.0	-0.565786512987871E 10	0.771405301838412E-08	3
30.0	90.0	-0.768532038938957E 12	0.866887668386711E-06	3
40.0	90.0	-0.147073961632594E 17	0.123337242674162E-01	3
50.0	90.0	-0.290307859010356E 21	0.193754515243797E 03	3
75.0	90.0	-0.171111601529653E 32	0.756143539294969E 13	3
100.0	90.0	-0.106836939033816E 43	0.352883710781244E 24	3

OPT1= 1.0 ORDINARY BESSEL FUNCTIONS, FIRST KIND,
OPT2= 2.0 OF COMPLEX ARGUMENT IN POLAR COORDINATES
ORD = 0.0 OF ORDER M = 0

RHO	ANG	RE J	IM J	METHOD
0.1	120.0	0.100124921831594E 01	0.216641667409577E-02	1
0.5	120.0	0.103075492385372E 01	0.549722926707177E-01	1
1.0	120.0	0.111675011576078E 01	0.230032066040576E 00	1
1.5	120.0	0.123667048430371E 01	0.555489344711846E 00	1
2.0	120.0	0.134639083726984E 01	0.108096813237748E 01	1
2.5	120.0	0.136528827443692E 01	0.187222368675946E 01	1
2.5	120.0	0.136528827443692E 01	0.187222368675946E 01	2
3.0	120.0	0.115579200904768E 01	0.300262937063371E 01	2
5.0	120.0	-0.841142625147850E 01	0.110161430764326E 02	2
10.0	120.0	0.139420238990115E 02	-0.735811296869062E 03	2
15.0	120.0	0.264264623150750E 05	0.370001823305540E 05	2
20.0	120.0	-0.284281315817823E 07	-0.911112779480102E 06	2
21.0	120.0	-0.477276355792687E 07	-0.501674968032690E 07	2
21.0	120.0	-0.477276355792687E 07	-0.501674968032691E 07	3
25.0	120.0	0.191597942931977E 09	-0.657900165249208E 08	3
30.0	120.0	-0.791224672881743E 10	0.115929487827114E 11	3
40.0	120.0	0.442567473084093E 14	0.543106951188833E 14	3
50.0	120.0	0.333369769107120E 18	-0.139303970580575E 18	3
75.0	120.0	0.667141394754872E 27	-0.331993344232806E 27	3
100.0	120.0	0.140873927196622E 37	-0.821458496603613E 36	3

AND ORDER N = M+1 = 1

RHO	ANG	RE J	IM J	METHOD
0.1	120.0	-0.250625130181200E-01	0.433012476317786E-01	1
0.5	120.0	-0.132852976850615E 00	0.216435506551306E 00	1
1.0	120.0	-0.313774275620088E 00	0.430710447441400E 00	1
1.5	120.0	-0.595335397446254E 00	0.631590710386300E 00	1
2.0	120.0	-0.103784157792652E 01	0.787852816867388E 00	1
2.5	120.0	-0.170951050029616E 01	0.833733833953889E 00	1
2.5	120.0	-0.170951050029616E 01	0.833733833953890E 00	2
3.0	120.0	-0.268076012848432E 01	0.649163851044207E 00	2
5.0	120.0	-0.957501107181051E 01	-0.827919220327028E 01	2
10.0	120.0	0.703135000007194E 03	0.326273435112699E 02	2
15.0	120.0	-0.363761230388357E 05	0.250203467325039E 05	2
20.0	120.0	0.927597044231110E 06	-0.276917161684862E 07	2
21.0	120.0	0.497065707239388E 07	-0.461265029179399E 07	2
21.0	120.0	0.497065707239388E 07	-0.461265029179399E 07	3
25.0	120.0	0.626931553093391E 08	0.188930116622977E 09	3
30.0	120.0	-0.113578900648697E 11	-0.789555341300881E 10	3
40.0	120.0	-0.540003338141960E 14	0.434327008570484E 14	3
50.0	120.0	0.136412455763175E 18	0.331177068637691E 18	3
75.0	120.0	0.327836046940884E 27	0.664395375463494E 27	3
100.0	120.0	0.814359066955233E 36	0.140469309166744E 37	3

OPT1= 1.0 ORDINARY BESSEL FUNCTIONS, SECOND KIND,
OPT2= 2.0 OF COMPLEX ARGUMENT IN POLAR COORDINATES
ORD = 0.0 OF ORDER M = 0

RHO	ANG	RE Y	IM Y	METHOD
0.1	120.0	-0.154528246742934E 01	0.133028376467856E 01	1
0.5	120.0	-0.623633154252833E 00	0.131075938145063E 01	1
1.0	120.0	-0.460736443311973E 00	0.132127667984319E 01	1
1.5	120.0	-0.648104513120779E 00	0.137594892376606E 01	1
2.0	120.0	-0.111196221547949E 01	0.143641203066924E 01	1
2.5	120.0	-0.187656632239702E 01	0.142081875690747E 01	1
2.5	120.0	-0.187656632239702E 01	0.142081875690747E 01	2
3.0	120.0	-0.299686946615698E 01	0.118846023239153E 01	2
5.0	120.0	-0.110118868157001E 02	-0.840967401844514E 01	2
10.0	120.0	0.735811274482892E 03	0.139419868599982E 02	2
15.0	120.0	-0.370001823305988E 05	0.264264623155397E 05	2
20.0	120.0	0.911112779480105E 06	-0.284281315817824E 07	2
21.0	120.0	0.501674968032691E 07	-0.477276355792687E 07	2
21.0	120.0	0.501674968032691E 07	-0.477276355792688E 07	3
25.0	120.0	0.657900165249208E 08	0.191597942931977E 09	3
30.0	120.0	-0.115929487827114E 11	-0.791224672881743E 10	3
40.0	120.0	-0.543106951188833E 14	0.442567473084093E 14	3
50.0	120.0	0.139303970580575E 18	0.333369769107120E 18	3
75.0	120.0	0.331993344232806E 27	0.667141394754872E 27	3
100.0	120.0	0.821458496603613E 36	0.140873927196622E 37	3

AND ORDER N = M+1 = 1

RHO	ANG	RE Y	IM Y	METHOD
0.1	120.0	0.317195549896567E 01	0.539941920195404E 01	1
0.5	120.0	0.462517043057102E 00	0.745198987080434E 00	1
1.0	120.0	-0.102152642708320E 00	-0.342058203964951E-01	1
1.5	120.0	-0.442546577108438E 00	-0.497388263065532E 00	1
2.0	120.0	-0.676255884186144E 00	-0.101022314373976E 01	1
2.5	120.0	-0.768708609998568E 00	-0.170933526366832E 01	1
2.5	120.0	-0.768708609998569E 00	-0.170933526366832E 01	2
3.0	120.0	-0.612420106017637E 00	-0.268974798986895E 01	2
5.0	120.0	0.828089422549127E 01	-0.957970706583469E 01	2
10.0	120.0	-0.326273815933484E 02	0.703135024238835E 03	2
15.0	120.0	-0.250203467320252E 05	-0.363761230387971E 05	2
20.0	120.0	0.276917161684862E 07	0.927597044231107E 06	2
21.0	120.0	0.461265029179399E 07	0.497065707239388E 07	2
21.0	120.0	0.461265029179399E 07	0.497065707239388E 07	3
25.0	120.0	-0.188930116622977E 09	0.626931553093391E 08	3
30.0	120.0	0.789555341300881E 10	-0.113578900648697E 11	3
40.0	120.0	-0.434327008570484E 14	-0.540003338141960E 14	3
50.0	120.0	-0.331177068637691E 18	0.136412455763175E 18	3
75.0	120.0	-0.664395375463494E 27	0.327836046940884E 27	3
100.0	120.0	-0.140469309166744E 37	0.814359066955233E 36	3

OPT1 = 1.0
OPT2 = 2.0
ORD = 0.0

ORDINARY BESSEL FUNCTIONS, FIRST KIND,
OF COMPLEX ARGUMENT IN POLAR COORDINATES
OF ORDER M = 0

RHO	ANG	RE J	IM J	METHOD
0.1	150.0	0.998749219183994E 00	0.216371034482641E-02	1
0.5	150.0	0.968268487155552E 00	0.532808826876282E-01	1
1.0	150.0	0.867618103497088E 00	0.202980518393593E 00	1
1.5	150.0	0.684054268254105E 00	0.418782462297495E 00	1
2.0	150.0	0.401876909028543E 00	0.650962462072001E 00	1
2.5	150.0	0.140682542445072E-01	0.832987043495826E 00	1
2.5	150.0	0.140682542445074E-01	0.832987043495826E 00	2
3.0	150.0	-0.465411477666209E 00	0.887568825861952E 00	2
5.0	150.0	-0.177477694131959E 01	-0.131251655129563E 01	2
10.0	150.0	-0.504718923288672E 01	0.181437389325677E 02	2
15.0	150.0	0.185917703645047E 03	-0.199776629495285E 02	2
20.0	150.0	-0.923017123951726E 03	-0.174137723919346E 04	2
21.0	150.0	0.117282087083194E 04	-0.294578373110088E 04	2
21.0	150.0	0.117282087083194E 04	-0.294578373110087E 04	3
25.0	150.0	-0.138638036945787E 05	0.163841413058518E 05	3
30.0	150.0	0.226423045537542E 06	0.752236282122233E 05	3
40.0	150.0	-0.276939913381967E 08	0.131340690983886E 08	3
50.0	150.0	0.144801894531719E 10	-0.380099335560015E 10	3
75.0	150.0	-0.215590057027017E 14	0.890542624880833E 15	3
100.0	150.0	-0.643080421887448E 20	-0.196724241684650E 21	3

AND ORDER N = M+1 = 1

RHO	ANG	RE J	IM J	METHOD
0.1	150.0	-0.433012476411755E-01	0.249375130235453E-01	1
0.5	150.0	-0.216436240685357E 00	0.117228400708627E 00	1
1.0	150.0	-0.430804407423258E 00	0.188828534743413E 00	1
1.5	150.0	-0.633195425326950E 00	0.174387857582875E 00	1
2.0	150.0	-0.799860900592474E 00	0.447975866038962E-01	1
2.5	150.0	-0.890855053772563E 00	-0.210366347877049E 00	1
2.5	150.0	-0.890855053772563E 00	-0.210366347877048E 00	2
3.0	150.0	-0.852941735217776E 00	-0.574590706638979E 00	2
5.0	150.0	0.141571113811486E 01	-0.154462726355877E 01	2
10.0	150.0	-0.174815980619618E 02	-0.573017405009030E 01	2
15.0	150.0	0.141953614488191E 02	0.183464045550593E 03	2
20.0	150.0	0.174014288024260E 04	-0.873467366158404E 03	2
21.0	150.0	0.288670348898404E 04	0.122050261384378E 04	2
21.0	150.0	0.288670348898403E 04	0.122050261384378E 04	3
25.0	150.0	-0.159795501524474E 05	-0.140132778554756E 05	3
30.0	150.0	-0.778976641808553E 05	0.223458193208725E 06	3
40.0	150.0	-0.127508522875686E 08	-0.276651089568003E 08	3
50.0	150.0	0.376948439914240E 10	0.147389856943119E 10	3
75.0	150.0	-0.887459425151841E 15	-0.266460721975595E 14	3
100.0	150.0	0.196512843420345E 21	-0.632937118915534E 20	3

OPT1= 1.0 ORDINARY BESSEL FUNCTIONS, SECOND KIND,
OPT2= 2.0 OF COMPLEX ARGUMENT IN POLAR COORDINATES
ORD = 0.0 OF ORDER M = 0

RHO	ANG	RE Y	IM Y	METHOD
0.1	150.0	-0.154055935995850E 01	0.165987359012020E 01	1
0.5	150.0	-0.567180097261568E 00	0.155268665925810E 01	1
1.0	150.0	-0.315798970789612E 00	0.130613118145939E 01	1
1.5	150.0	-0.360718577550368E 00	0.972380855946426E 00	1
2.0	150.0	-0.530806539144037E 00	0.562514715627323E 00	1
2.5	150.0	-0.708747936329382E 00	0.801068421023191E-01	1
2.5	150.0	-0.708747936329382E 00	0.801068421023193E-01	2
3.0	150.0	-0.787204169439765E 00	-0.460335901129762E 00	2
5.0	150.0	0.130900608668220E 01	-0.180346534195588E 01	2
10.0	150.0	-0.181421028971362E 02	-0.504676936418683E 01	2
15.0	150.0	0.199775960929605E 02	0.185917795308825E 03	2
20.0	150.0	0.174137723490415E 04	-0.923017130792225E 03	2
21.0	150.0	0.294578372637077E 04	0.117282087014385E 04	2
21.0	150.0	0.294578372637076E 04	0.117282087014385E 04	3
25.0	150.0	-0.163841413052680E 05	-0.138638036946842E 05	3
30.0	150.0	-0.752236282122323E 05	0.226423045537586E 06	3
40.0	150.0	-0.131340690983886E 08	-0.276939913381967E 08	3
50.0	150.0	0.380099335560015E 10	0.144801894531719E 10	3
75.0	150.0	-0.890542624880833E 15	-0.215590057027017E 14	3
100.0	150.0	0.196724241684650E 21	-0.643080421887448E 20	3

AND ORDER N = M+1 = 1

RHO	ANG	RE Y	IM Y	METHOD
0.1	150.0	0.555217950077791E 01	0.306462641306007E 01	1
0.5	150.0	0.108759991767975E 01	0.181896046263099E 00	1
1.0	150.0	0.403907759145211E 00	-0.444889802515893E 00	1
1.5	150.0	0.149771112198271E 00	-0.773516076873319E 00	1
2.0	150.0	0.113653482382419E 00	-0.966817395673348E 00	1
2.5	150.0	0.263710106618494E 00	-0.103850964890696E 01	1
2.5	150.0	0.263710106618494E 00	-0.103850964890696E 01	2
3.0	150.0	0.566724628442376E 00	-0.962709572329223E 00	2
5.0	150.0	0.151474577658389E 01	0.142176829251588E 01	2
10.0	150.0	0.573053553645263E 01	-0.174832935662144E 02	2
15.0	150.0	-0.183463950438815E 03	0.141954268336313E 02	2
20.0	150.0	0.873467359323141E 03	0.174014288473239E 04	2
21.0	150.0	-0.122050261444377E 04	0.288670349378508E 04	2
21.0	150.0	-0.122050261444377E 04	0.288670349378508E 04	3
25.0	150.0	0.140132778553590E 05	-0.159795501530352E 05	3
30.0	150.0	-0.223458193208681E 06	-0.778976641808468E 05	3
40.0	150.0	0.276651089568003E 08	-0.127508522875686E 08	3
50.0	150.0	-0.147389856943119E 10	0.376948439914240E 10	3
75.0	150.0	0.266460721975595E 14	-0.887459425151841E 15	3
100.0	150.0	0.632937118915534E 20	0.196512843420345E 21	3

OPT1= 1.0 ORDINARY BESSEL FUNCTIONS, FIRST KIND,
OPT2= 2.0 OF COMPLEX ARGUMENT IN POLAR COORDINATES
ORD = 0.0 OF ORDER M = 0

RHO	ANG	RE J	IM J	METHOD
0.1	180.0	0.997501562066040E 00	0.870877619651472E-18	1
0.5	180.0	0.938469807240813E 00	0.211250130396333E-16	1
1.0	180.0	0.765197686557967E 00	0.767419287775000E-16	1
1.5	180.0	0.511827671735918E 00	0.145950687737144E-15	1
2.0	180.0	0.223890779141236E 00	0.201154028899528E-15	1
2.5	180.0	-0.483837764681980E-01	0.216724857566377E-15	1
2.5	180.0	-0.483837764681980E-01	0.216724857566377E-15	2
3.0	180.0	-0.260051954901933E 00	0.177388959061534E-15	2
5.0	180.0	-0.177596771314338E 00	-0.285638238652527E-15	2
10.0	180.0	-0.245935764451348E 00	0.758136109421493E-16	2
15.0	180.0	-0.142244728267808E-01	0.536531936320838E-15	2
20.0	180.0	0.167024664340582E 00	0.233105148436447E-15	2
21.0	180.0	0.365790710008630E-01	0.626687259181050E-15	2
21.0	180.0	0.365790710008630E-01	0.617561557447743E-15	3
25.0	180.0	0.962667832759579E-01	-0.562050406216485E-15	3
30.0	180.0	-0.863679835810404E-01	-0.617561557447743E-15	3
40.0	180.0	0.736689058423750E-02	0.874300631892311E-15	3
50.0	180.0	0.558123276692516E-01	-0.853483950180589E-15	3
75.0	180.0	0.346439138050968E-01	-0.111716191852906E-14	3
100.0	180.0	0.199858503042229E-01	-0.135308431126191E-14	3

AND ORDER N = M+1 = 1

RHO	ANG	RE J	IM J	METHOD
0.1	180.0	-0.499375260362420E-01	0.868699517870800E-17	1
0.5	180.0	-0.242268457674874E 00	0.395814558459201E-16	1
1.0	180.0	-0.440050585744934E 00	0.567035165072307E-16	1
1.5	180.0	-0.557936507910100E 00	0.365886124578319E-16	1
2.0	180.0	-0.576724807756873E 00	-0.224868548936317E-16	1
2.5	180.0	-0.497094102464274E 00	-0.107784474246316E-15	1
2.5	180.0	-0.497094102464274E 00	-0.107784474246316E-15	2
3.0	180.0	-0.339058958525936E 00	-0.195183706222714E-15	2
5.0	180.0	0.327579137591465E 00	-0.977308982733256E-16	2
10.0	180.0	-0.434727461688616E-01	-0.436477163775979E-15	2
15.0	180.0	-0.205104038613522E 00	-0.729786137597045E-16	2
20.0	180.0	-0.668331241758496E-01	0.570904807722158E-15	2
21.0	180.0	-0.171120272763902E 00	0.104119888464657E-15	2
21.0	180.0	-0.171120272763900E 00	0.117961196366423E-15	3
25.0	180.0	0.125350249580290E 00	0.437150315946155E-15	3
30.0	180.0	0.118751062616623E 00	-0.444089209850063E-15	3
40.0	180.0	-0.126038318037585E 00	0.403323208164608E-16	3
50.0	180.0	0.975118281251752E-01	0.492661467177413E-15	3
75.0	180.0	0.851399950448292E-01	0.461436444609831E-15	3
100.0	180.0	0.771453520141122E-01	0.356485674313234E-15	3

OPT1= 1.0 ORDINARY BESSEL FUNCTIONS, SECOND KIND,
OPT2= 2.0 OF COMPLEX ARGUMENT IN POLAR COORDINATES
ORD = 0.0 OF ORDER M = 0

RHO	ANG	RE Y	IM Y	METHOD
0.1	180.0	-0.153423865135037E 01	0.199500312413208E 01	1
0.5	180.0	-0.444518733506707E 00	0.187693961448163E 01	1
1.0	180.0	0.882569642156769E-01	0.153039537311593E 01	1
1.5	180.0	0.382448923797759E 00	0.102365534347184E 01	1
2.0	180.0	0.510375672649745E 00	0.447781558282471E 00	1
2.5	180.0	0.498070359615231E 00	-0.967675529363960E-01	1
2.5	180.0	0.498070359615232E 00	-0.967675529363960E-01	2
3.0	180.0	0.376850010012790E 00	-0.520103909803867E 00	2
5.0	180.0	-0.308517625249033E 00	-0.355193542628677E 00	2
10.0	180.0	0.556711672835994E-01	-0.491871528902696E 00	2
15.0	180.0	0.205464296038918E 00	-0.284489456535616E-01	2
20.0	180.0	0.626405968093838E-01	0.334049328681166E 00	2
21.0	180.0	0.170201758422153E 00	0.731581420017253E-01	2
21.0	180.0	0.170201758422154E 00	0.731581420017258E-01	3
25.0	180.0	-0.127249432268005E 00	0.192533566551915E 00	3
30.0	180.0	-0.117295731686663E 00	-0.172735967162080E 00	3
40.0	180.0	0.125936417058259E 00	0.147337811684750E-01	3
50.0	180.0	-0.980649954700755E-01	0.111624655338503E 00	3
75.0	180.0	-0.853690476477735E-01	0.692878276101932E-01	3
100.0	180.0	-0.772443133650805E-01	0.399717006084455E-01	3

AND ORDER N = M+1 = 1

RHO	ANG	RE Y	IM Y	METHOD
0.1	180.0	0.645895109470203E 01	-0.998750520724829E-01	1
0.5	180.0	0.147147239267024E 01	-0.484536915349748E 00	1
1.0	180.0	0.781212821300289E 00	-0.880101171489867E 00	1
1.5	180.0	0.412308626973911E 00	-0.111587301582020E 01	1
2.0	180.0	0.107032431540938E 00	-0.115344961551375E 01	1
2.5	180.0	-0.145918137966786E 00	-0.994188204928548E 00	1
2.5	180.0	-0.145918137966786E 00	-0.994188204928548E 00	2
3.0	180.0	-0.324674424791800E 00	-0.678117917051873E 00	2
5.0	180.0	-0.147863143391227E 00	0.655158275182930E 00	2
10.0	180.0	-0.249015424206953E 00	-0.869454923377231E-01	2
15.0	180.0	-0.210736280368734E-01	-0.410208077227045E 00	2
20.0	180.0	0.165511614362521E 00	-0.133666248351700E 00	2
21.0	180.0	0.325392607558648E-01	-0.342240545527800E 00	2
21.0	180.0	0.325392607558651E-01	-0.342240545527800E 00	3
25.0	180.0	0.988299647832367E-01	0.250700499160579E 00	3
30.0	180.0	-0.844255706617463E-01	0.237502125233245E 00	3
40.0	180.0	0.579350582154959E-02	-0.252076636075169E 00	3
50.0	180.0	0.567956685620137E-01	0.195023656250350E 00	3
75.0	180.0	0.352137851605795E-01	0.170279990089657E 00	3
100.0	180.0	0.203723120027590E-01	0.154290704028223E 00	3

OPT1= 1.0 ORDINARY BESSEL FUNCTIONS, FIRST KIND,
OPT2= 2.0 OF COMPLEX ARGUMENT IN POLAR COORDINATES
ORD = 0.0 OF ORDER M = 0

RHO	ANG	RE J	IM J	METHOD
0.1	-120.	0.100124921831594E 01	-0.216641667409577E-02	1
0.5	-120.	0.103075492385372E 01	-0.549722926707177E-01	1
1.0	-120.	0.111675011576078E 01	-0.230032066040576E 00	1
1.5	-120.	0.123667048430371E 01	-0.555489344711846E 00	1
2.0	-120.	0.134639083726984E 01	-0.108096813237748E 01	1
2.5	-120.	0.136528827443692E 01	-0.187222368675946E 01	1
2.5	-120.	0.136528827443692E 01	-0.187222368675946E 01	2
3.0	-120.	0.115579200904768E 01	-0.300262937063371E 01	2
5.0	-120.	-0.841142625147850E 01	-0.110161430764326E 02	2
10.0	-120.	0.139420238990119E 02	0.735811296869061E 03	2
15.0	-120.	0.264264623150750E 05	-0.370001823305540E 05	2
20.0	-120.	-0.284281315817823E 07	0.911112779480102E 06	2
21.0	-120.	-0.477276355792687E 07	0.501674968032690E 07	2
21.0	-120.	-0.477276355792687E 07	0.501674968032691E 07	3
25.0	-120.	0.191597942931977E 09	0.657900165249208E 08	3
30.0	-120.	-0.791224672881743E 10	-0.115929487827114E 11	3
40.0	-120.	0.442567473084093E 14	-0.543106951188833E 14	3
50.0	-120.	0.333369769107120E 18	0.139303970580575E 18	3
75.0	-120.	0.667141394754872E 27	0.331993344232806E 27	3
100.0	-120.	0.140873927196622E 37	0.821458496603613E 36	3

AND ORDER N = M+1 = 1

RHO	ANG	RE J	IM J	METHOD
0.1	-120.	-0.250625130181200E-01	-0.433012476317786E-01	1
0.5	-120.	-0.132852976850615E 00	-0.216435506551306E 00	1
1.0	-120.	-0.313774275620088E 00	-0.430710447441400E 00	1
1.5	-120.	-0.595335397446254E 00	-0.631590710386300E 00	1
2.0	-120.	-0.103784157792652E 01	-0.787852816867388E 00	1
2.5	-120.	-0.170951050029616E 01	-0.833733833953889E 00	1
2.5	-120.	-0.170951050029616E 01	-0.833733833953887E 00	2
3.0	-120.	-0.268076012848432E 01	-0.649163851044207E 00	2
5.0	-120.	-0.957501107181050E 01	0.827919220327027E 01	2
10.0	-120.	0.703135000007194E 03	-0.326273435112702E 02	2
15.0	-120.	-0.363761230388357E 05	-0.250203467325039E 05	2
20.0	-120.	0.927597044231110E 06	0.276917161684862E 07	2
21.0	-120.	0.497065707239388E 07	0.461265029179399E 07	2
21.0	-120.	0.497065707239388E 07	0.461265029179399E 07	3
25.0	-120.	0.626931553093391E 08	-0.188930116622977E 09	3
30.0	-120.	-0.113578900648697E 11	0.789555341300881E 10	3
40.0	-120.	-0.540003338141960E 14	-0.434327008570484E 14	3
50.0	-120.	0.136412455763175E 18	-0.331177068637691E 18	3
75.0	-120.	0.327836046940884E 27	-0.664395375463494E 27	3
100.0	-120.	0.814359066955233E 36	-0.140469309166744E 37	3

OPT1= 1.0
OPT2= 2.0
ORD = 0.0

ORDINARY BESSEL FUNCTIONS, SECOND KIND,
OF COMPLEX ARGUMENT IN POLAR COORDINATES
OF ORDER M = 0

RHO	ANG	RE Y	IM Y	METHOD
0.1	-120.	-0.154528246742934E 01	-0.133028376467856E 01	1
0.5	-120.	-0.623633154252833E 00	-0.131075938145063E 01	1
1.0	-120.	-0.460736443311973E 00	-0.132127667984319E 01	1
1.5	-120.	-0.648104513120779E 00	-0.137594892376606E 01	1
2.0	-120.	-0.111196221547949E 01	-0.143641203066924E 01	1
2.5	-120.	-0.187656632239702E 01	-0.142081875690747E 01	1
2.5	-120.	-0.187656632239702E 01	-0.142081875690747E 01	2
3.0	-120.	-0.299686946615698E 01	-0.118846023239153E 01	2
5.0	-120.	-0.110118868157001E 02	0.840967401844514E 01	2
10.0	-120.	0.735811274482892E 03	-0.139419868599986E 02	2
15.0	-120.	-0.370001823305988E 05	-0.264264623155397E 05	2
20.0	-120.	0.911112779480106E 06	0.284281315817824E 07	2
21.0	-120.	0.501674968032690E 07	0.477276355792687E 07	2
21.0	-120.	0.501674968032691E 07	0.477276355792688E 07	3
25.0	-120.	0.657900165249208E 08	-0.191597942931977E 09	3
30.0	-120.	-0.115929487827114E 11	0.791224672881743E 10	3
40.0	-120.	-0.543106951188833E 14	-0.442567473084093E 14	3
50.0	-120.	0.139303970580575E 18	-0.333369769107120E 18	3
75.0	-120.	0.331993344232806E 27	-0.667141394754872E 27	3
100.0	-120.	0.821458496603613E 36	-0.140873927196622E 37	3

AND ORDER N = M+1 = 1

RHO	ANG	RE Y	IM Y	METHOD
0.1	-120.	0.317195949896567E 01	-0.539941920195404E 01	1
0.5	-120.	0.462517043057102E 00	-0.745198987080435E 00	1
1.0	-120.	-0.102152642708320E 00	0.342058203964951E-01	1
1.5	-120.	-0.442546577108438E 00	0.497388263065532E 00	1
2.0	-120.	-0.676255884186144E 00	0.101022314373976E 01	1
2.5	-120.	-0.768708609998568E 00	0.170933526366832E 01	1
2.5	-120.	-0.768708609998566E 00	0.170933526366832E 01	2
3.0	-120.	-0.612420106017637E 00	0.268974798986895E 01	2
5.0	-120.	0.828089422549127E 01	0.957970706583468E 01	2
10.0	-120.	-0.326273815933488E 02	-0.703135024238834E 03	2
15.0	-120.	-0.250203467320252E 05	0.363761230387971E 05	2
20.0	-120.	0.276917161684862E 07	-0.927597044231107E 06	2
21.0	-120.	0.461265029179399E 07	-0.497065707239388E 07	2
21.0	-120.	0.461265029179399E 07	-0.497065707239388E 07	3
25.0	-120.	-0.188930116622977E 09	-0.626931553093391E 08	3
30.0	-120.	0.789555341300881E 10	0.113578900648697E 11	3
40.0	-120.	-0.434327008570484E 14	0.540003338141960E 14	3
50.0	-120.	-0.331177068637691E 18	-0.136412455763175E 18	3
75.0	-120.	-0.664395375463494E 27	-0.327836046940884E 27	3
100.0	-120.	-0.140469309166744E 37	-0.814359066955233E 36	3

OPT1= 1.0 ORDINARY BESSEL FUNCTIONS, FIRST KIND,
OPT2= 2.0 OF COMPLEX ARGUMENT IN POLAR COORDINATES
ORD = 0.0 OF ORDER M = 0

RHO	ANG	RE J	IM J	METHOD
0.1	-60.0	0.100124921831594E 01	0.216641667409577E-02	1
0.5	-60.0	0.103075492385372E 01	0.549722926707177E-01	1
1.0	-60.0	0.111675011576078E 01	0.230032066040576E 00	1
1.5	-60.0	0.123667048430371E 01	0.555489344711846E 00	1
2.0	-60.0	0.134639083726984E 01	0.108096813237748E 01	1
2.5	-60.0	0.136528827443692E 01	0.187222368675946E 01	1
2.5	-60.0	0.136528827443692E 01	0.187222368675946E 01	2
3.0	-60.0	0.115579200904768E 01	0.300262937063371E 01	2
5.0	-60.0	-0.841142625147852E 01	0.110161430764326E 02	2
10.0	-60.0	0.139420238990131E 02	-0.735811296869062E 03	2
15.0	-60.0	0.264264623150749E 05	0.370001823305540E 05	2
20.0	-60.0	-0.284281315817822E 07	-0.911112779480108E 06	2
21.0	-60.0	-0.477276355792684E 07	-0.501674968032690E 07	2
21.0	-60.0	-0.477276355792685E 07	-0.501674968032691E 07	3
25.0	-60.0	0.191597942931977E 09	-0.657900165249202E 08	3
30.0	-60.0	-0.791224672881744E 10	0.115929487827113E 11	3
40.0	-60.0	0.442567473084090E 14	0.543106951188833E 14	3
50.0	-60.0	0.333369769107119E 18	-0.139303970580571E 18	3
75.0	-60.0	0.667141394754871E 27	-0.331993344232796E 27	3
100.0	-60.0	0.140873927196622E 37	-0.821458496603586E 36	3

AND ORDER N = M+1 = 1

RHO	ANG	RE J	IM J	METHOD
0.1	-60.0	0.250625130181200E-01	-0.433012476317786E-01	1
0.5	-60.0	0.132852976850616E 00	-0.216435506551306E 00	1
1.0	-60.0	0.313774275620088E 00	-0.430710447441400E 00	1
1.5	-60.0	0.595335397446254E 00	-0.631590710386300E 00	1
2.0	-60.0	0.103784157792652E 01	-0.787852816867388E 00	1
2.5	-60.0	0.170951050029616E 01	-0.833733833953888E 00	1
2.5	-60.0	0.170951050029616E 01	-0.833733833953889E 00	2
3.0	-60.0	0.268076012848432E 01	-0.649163851044205E 00	2
5.0	-60.0	0.957501107181049E 01	0.827919220327029E 01	2
10.0	-60.0	-0.703135000007194E 03	-0.326273435112714E 02	2
15.0	-60.0	0.363761230388357E 05	-0.250203467325038E 05	2
20.0	-60.0	-0.927597044231116E 06	0.276917161684861E 07	2
21.0	-60.0	-0.497065707239388E 07	0.461265029179396E 07	2
21.0	-60.0	-0.497065707239389E 07	0.461265029179396E 07	3
25.0	-60.0	-0.626931553093386E 08	-0.188930116622977E 09	3
30.0	-60.0	0.113578900648696E 11	0.789555341300883E 10	3
40.0	-60.0	0.540003338141960E 14	-0.434327008570482E 14	3
50.0	-60.0	-0.136412455763170E 18	-0.331177068637690E 18	3
75.0	-60.0	-0.327836046940875E 27	-0.664395375463492E 27	3
100.0	-60.0	-0.814359066955208E 36	-0.140469309166744E 37	3

OPT1= 1.0 ORDINARY BESSEL FUNCTIONS, SECOND KIND,
OPT2= 2.0 OF COMPLEX ARGUMENT IN POLAR COORDINATES
ORD = 0.0 OF ORDER M = 0

RHO	ANG	RE Y	IM Y	METHOD
0.1	-60.0	-0.154094963408115E 01	-0.672214671953312E 00	1
0.5	-60.0	-0.513688568911397E 00	-0.750750466256815E 00	1
1.0	-60.0	-0.672311230821477E-03	-0.912223551678364E 00	1
1.5	-60.0	0.462874176302913E 00	-0.109739204484137E 01	1
2.0	-60.0	0.104997404927546E 01	-0.125636964387044E 01	1
2.5	-60.0	0.186788105112190E 01	-0.130975779196637E 01	1
2.5	-60.0	0.186788105112189E 01	-0.130975779196637E 01	2
3.0	-60.0	0.300838927511043E 01	-0.112312378570383E 01	2
5.0	-60.0	0.110203993371652E 02	0.841317848451188E 01	2
10.0	-60.0	-0.735811319255231E 03	-0.139420609380263E 02	2
15.0	-60.0	0.370001823305092E 05	-0.264264623146102E 05	2
20.0	-60.0	-0.911112779480104E 06	0.284281315817822E 07	2
21.0	-60.0	-0.501674968032690E 07	0.477276355792683E 07	2
21.0	-60.0	-0.501674968032691E 07	0.477276355792685E 07	3
25.0	-60.0	-0.657900165249202E 08	-0.191597942931977E 09	3
30.0	-60.0	0.115929487827113E 11	0.791224672881744E 10	3
40.0	-60.0	0.543106951188833E 14	-0.442567473084090E 14	3
50.0	-60.0	-0.139303970580571E 18	-0.333369769107119E 18	3
75.0	-60.0	-0.331993344232796E 27	-0.667141394754871E 27	3
100.0	-60.0	-0.821458496603586E 36	-0.140873927196622E 37	3

AND ORDER N = M+1 = 1

RHO	ANG	RE Y	IM Y	METHOD
0.1	-60.0	-0.325856199422923E 01	-0.544954422799028E 01	1
0.5	-60.0	-0.895388056159714E 00	-0.101090494078167E 01	1
1.0	-60.0	-0.759268252174481E 00	-0.593342730843681E 00	1
1.5	-60.0	-0.820634843664161E 00	-0.693282531826976E 00	1
2.0	-60.0	-0.899449749548632E 00	-0.106546001211328E 01	1
2.5	-60.0	-0.898759057909209E 00	-0.170968573692400E 01	1
2.5	-60.0	-0.898759057909209E 00	-0.170968573692400E 01	2
3.0	-60.0	-0.685907596070775E 00	-0.267177226709970E 01	2
5.0	-60.0	0.827749018104930E 01	-0.957031507778631E 01	2
10.0	-60.0	-0.326273054291929E 02	0.703134975775553E 03	2
15.0	-60.0	-0.250203467329825E 05	-0.363761230388742E 05	2
20.0	-60.0	0.276917161684861E 07	0.927597044231119E 06	2
21.0	-60.0	0.461265029179396E 07	0.497065707239388E 07	2
21.0	-60.0	0.461265029179396E 07	0.497065707239389E 07	3
25.0	-60.0	-0.188930116622977E 09	0.626931553093386E 08	3
30.0	-60.0	0.789555341300883E 10	-0.113578900648696E 11	3
40.0	-60.0	-0.434327008570482E 14	-0.540003338141960E 14	3
50.0	-60.0	-0.331177068637690E 18	0.136412455763170E 18	3
75.0	-60.0	-0.664395375463492E 27	0.327836046940875E 27	3
100.0	-60.0	-0.140469309166744E 37	0.814359066955208E 36	3

OPT1= 2.0 MODIFIED BESSEL FUNCTIONS, FIRST KIND,
OPT2= 2.0 OF COMPLEX ARGUMENT IN POLAR COORDINATES
ORD = 0.0 OF ORDER M = 0

RHO	ANG	RE I	IM I	METHOD
0.1	0.0	0.100250156293410E 01	0.000000000000000E 00	1
0.5	0.0	0.106348337074132E 01	0.000000000000000E 00	1
1.0	0.0	0.126606587775201E 01	0.000000000000000E 00	1
1.5	0.0	0.164672318977289E 01	0.000000000000000E 00	1
2.0	0.0	0.227958530233607E 01	0.000000000000000E 00	1
2.5	0.0	0.328983914405012E 01	0.000000000000000E 00	1
2.5	0.0	0.328983914405012E 01	0.000000000000000E 00	2
3.0	0.0	0.488079258586502E 01	0.000000000000000E 00	2
5.0	0.0	0.272398718236044E 02	0.000000000000000E 00	2
10.0	0.0	0.281571662846626E 04	0.000000000000000E 00	2
15.0	0.0	0.339649373297914E 06	0.000000000000000E 00	2
20.0	0.0	0.435582825595536E 08	0.000000000000000E 00	2
21.0	0.0	0.115513961922158E 09	0.000000000000000E 00	2
21.0	0.0	0.115513961922158E 09	-0.656281127863413E-10	3
25.0	0.0	0.577456060646632E 10	-0.110267687259032E-11	3
30.0	0.0	0.781672297823978E 12	-0.678788669188652E-14	3
40.0	0.0	0.148947747934199E 17	-0.267153066152906E-18	3
50.0	0.0	0.293255378384934E 21	-0.108549010830313E-22	3
75.0	0.0	0.172263907803581E 32	-0.123189551629872E-33	3
100.0	0.0	0.107375170713108E 43	-0.148225080162921E-44	3

AND ORDER N = M+1 = 1

RHO	ANG	RE I	IM I	METHOD
0.1	0.0	0.500625260470927E-01	0.000000000000000E 00	1
0.5	0.0	0.257894305390896E 00	0.000000000000000E 00	1
1.0	0.0	0.565159103992485E 00	0.000000000000000E 00	1
1.5	0.0	0.981666428577908E 00	0.000000000000000E 00	1
2.0	0.0	0.159063685463733E 01	0.000000000000000E 00	1
2.5	0.0	0.251671624528870E 01	0.000000000000000E 00	1
2.5	0.0	0.251671624528870E 01	0.000000000000000E 00	2
3.0	0.0	0.395337021740261E 01	0.000000000000000E 00	2
5.0	0.0	0.243356421424505E 02	0.000000000000000E 00	2
10.0	0.0	0.267098830370126E 04	0.000000000000000E 00	2
15.0	0.0	0.328124921970207E 06	0.000000000000000E 00	2
20.0	0.0	0.424549733851278E 08	0.000000000000000E 00	2
21.0	0.0	0.112729199137776E 09	0.000000000000000E 00	2
21.0	0.0	0.112729199137776E 09	0.671729105586440E-10	3
25.0	0.0	0.565786512987871E 10	0.112451818539236E-11	3
30.0	0.0	0.768532038938957E 12	0.690010526799061E-14	3
40.0	0.0	0.147073961632594E 17	0.270472110544047E-18	3
50.0	0.0	0.290307859010356E 21	0.109629178779181E-22	3
75.0	0.0	0.171111601529653E 32	0.124008214186857E-33	3
100.0	0.0	0.106836939033816E 43	0.148964370994738E-44	3

```
OPT1=   2.0          MODIFIED BESSEL FUNCTIONS, SECOND KIND,
OPT2=   2.0        OF COMPLEX ARGUMENT IN     POLAR   COORDINATES
ORD =   0.0                          OF ORDER M =    0
```

RHO	ANG	RE K	IM K	METHOD
0.1	0.0	0.242706902470202E 01	0.000000000000000E 00	1
0.5	0.0	0.924419071227666E 00	0.000000000000000E 00	1
1.0	0.0	0.421024438240708E 00	0.000000000000000E 00	1
1.5	0.0	0.213805562647526E 00	0.000000000000000E 00	1
2.0	0.0	0.113893872749533E 00	0.000000000000000E 00	1
2.5	0.0	0.623475532003661E-01	0.000000000000000E 00	1
2.5	0.0	0.623475532003662E-01	0.000000000000000E 00	2
3.0	0.0	0.347395043862792E-01	0.000000000000000E 00	2
5.0	0.0	0.369109833404259E-02	0.000000000000000E 00	2
10.0	0.0	0.177800623161676E-04	0.000000000000000E 00	2
15.0	0.0	0.981953648239643E-07	0.000000000000000E 00	2
20.0	0.0	0.574123781533652E-09	0.000000000000000E 00	2
21.0	0.0	0.206176796998532E-09	0.000000000000000E 00	2
21.0	0.0	0.206176796998532E-09	0.000000000000000E 00	3
25.0	0.0	0.346416156221312E-11	0.000000000000000E 00	3
30.0	0.0	0.213247749646306E-13	0.000000000000000E 00	3
40.0	0.0	0.839286110009958E-18	0.000000000000000E 00	3
50.0	0.0	0.341016774978950E-22	0.000000000000000E 00	3
75.0	0.0	0.387011704558692E-33	0.000000000000000E 00	3
100.0	0.0	0.465662822917592E-44	0.000000000000000E 00	3

AND ORDER N = M+1 =   1

RHO	ANG	RE K	IM K	METHOD
0.1	0.0	0.985384478087061E 01	0.000000000000000E 00	1
0.5	0.0	0.165644112000330E 01	0.000000000000000E 00	1
1.0	0.0	0.601907230197235E 00	0.000000000000000E 00	1
1.5	0.0	0.277387800456844E 00	0.000000000000000E 00	1
2.0	0.0	0.139865881816522E 00	0.000000000000000E 00	1
2.5	0.0	0.738908163477471E-01	0.000000000000000E 00	1
2.5	0.0	0.738908163477471E-01	0.000000000000000E 00	2
3.0	0.0	0.401564311281942E-01	0.000000000000000E 00	2
5.0	0.0	0.404461344545216E-02	0.000000000000000E 00	2
10.0	0.0	0.186487734538256E-04	0.000000000000000E 00	2
15.0	0.0	0.101417293697621E-06	0.000000000000000E 00	2
20.0	0.0	0.588305796955704E-09	0.000000000000000E 00	2
21.0	0.0	0.211029922331280E-09	0.000000000000000E 00	2
21.0	0.0	0.211029922331280E-09	0.000000000000000E 00	3
25.0	0.0	0.353277807319994E-11	0.000000000000000E 00	3
30.0	0.0	0.216773200189155E-13	0.000000000000000E 00	3
40.0	0.0	0.849713195486105E-18	0.000000000000000E 00	3
50.0	0.0	0.344410222671756E-22	0.000000000000000E 00	3
75.0	0.0	0.389583294674220E-33	0.000000000000000E 00	3
100.0	0.0	0.467985373563693E-44	0.000000000000000E 00	3

OPT1= 2.0 MODIFIED BESSEL FUNCTIONS, FIRST KIND,
OPT2= 2.0 OF COMPLEX ARGUMENT IN POLAR COORDINATES
ORD = 0.0 OF ORDER M = 0

RHO	ANG	RE I	IM I	METHOD
0.1	30.0	0.100124921831594E 01	0.216641657409577E-02	1
0.5	30.0	0.103075492385372E 01	0.549722926707177E-01	1
1.0	30.0	0.111675011576078E 01	0.230032066040576E 00	1
1.5	30.0	0.123667048430371E 01	0.555489344711846E 00	1
2.0	30.0	0.134639083726984E 01	0.108096813237748E 01	1
2.5	30.0	0.136528827443692E 01	0.187222368675946E 01	1
2.5	30.0	0.136528827443692E 01	0.187222368675946E 01	2
3.0	30.0	0.115579200904768E 01	0.300262937063371E 01	2
5.0	30.0	-0.841142625147851E 01	0.110161430764326E 02	2
10.0	30.0	0.139420238990112E 02	-0.735811296869062E 03	2
15.0	30.0	0.264264623150750E 05	0.370001823305540E 05	2
20.0	30.0	-0.284281315817823E 07	-0.911112779480102E 06	2
21.0	30.0	-0.477276355792687E 07	-0.501674968032690E 07	2
21.0	30.0	-0.477276355792687E 07	-0.501674968032691E 07	3
25.0	30.0	0.191597942931977E 09	-0.657900165249208E 08	3
30.0	30.0	-0.791224672881743E 10	0.115929487827114E 11	3
40.0	30.0	0.442567473084093E 14	0.543106951188833E 14	3
50.0	30.0	0.333369769107120E 18	-0.139303970580575E 18	3
75.0	30.0	0.667141394754872E 27	-0.331993344232806E 27	3
100.0	30.0	0.140873927196622E 37	-0.821458496603613E 36	3

AND ORDER N = M+1 = 1

RHO	ANG	RE I	IM I	METHOD
0.1	30.0	0.433012476317786E-01	0.250625130181200E-01	1
0.5	30.0	0.216435506551306E 00	0.132852976850615E 00	1
1.0	30.0	0.430710447441400E 00	0.313774275620088E 00	1
1.5	30.0	0.631590710386300E 00	0.595335397446254E 00	1
2.0	30.0	0.787852816867388E 00	0.103784157792652E 01	1
2.5	30.0	0.833733833953889E 00	0.170951050029616E 01	1
2.5	30.0	0.833733833953890E 00	0.170951050029616E 01	2
3.0	30.0	0.649163851044207E 00	0.268076012648432E 01	2
5.0	30.0	-0.827919220327029E 01	0.957501107181050E 01	2
10.0	30.0	0.326273435112697E 02	-0.703135000007194E 03	2
15.0	30.0	0.250203467325039E 05	0.363761230388357E 05	2
20.0	30.0	-0.276917161684862E 07	-0.927597044231110E 06	2
21.0	30.0	-0.461265029179399E 07	-0.497065707239388E 07	2
21.0	30.0	-0.461265029179399E 07	-0.497065707239388E 07	3
25.0	30.0	0.188930116622977E 09	-0.626931553093391E 08	3
30.0	30.0	-0.789555341300881E 10	0.113578900648697E 11	3
40.0	30.0	0.434327008570485E 14	0.540003338141959E 14	3
50.0	30.0	0.331177068637691E 18	-0.136412455763175E 18	3
75.0	30.0	0.664395375463493E 27	-0.327836046940886E 27	3
100.0	30.0	0.140469309166744E 37	-0.814359066955237E 36	3

OPT1= 2.0 MODIFIED BESSEL FUNCTIONS, SECOND KIND,
OPT2= 2.0 OF COMPLEX ARGUMENT IN POLAR COORDINATES
ORD = 0.0 OF ORDER M = 0

RHO	ANG	RE K	IM K	METHOD
0.1	30.0	0.242392102434459E 01	-0.516846256815038E 00	1
0.5	30.0	0.893250392565207E 00	-0.439829973479423E 00	1
1.0	30.0	0.362389588393415E 00	-0.321269575592633E 00	1
1.5	30.0	0.145479566342242E 00	-0.218778061109181E 00	1
2.0	30.0	0.486853918890205E-01	-0.141404959925468E 00	1
2.5	30.0	0.682139610809449E-02	-0.872270778898894E-01	1
2.5	30.0	0.682139610809434E-02	-0.872270778898893E-01	2
3.0	30.0	-0.904763679472972E-02	-0.513151252314320E-01	2
5.0	30.0	-0.668571872457182E-02	-0.275240121248720E-02	2
10.0	30.0	0.351641132100538E-04	0.581807459710908E-04	2
15.0	30.0	0.703819382856184E-07	-0.729927342651241E-06	2
20.0	30.0	-0.562771542024696E-08	0.620347212798921E-08	2
21.0	30.0	-0.806212004251352E-09	0.334315475908424E-08	2
21.0	30.0	-0.806212004251356E-09	0.334315475908425E-08	3
25.0	30.0	0.969035062838646E-10	-0.189392538258369E-10	3
30.0	30.0	-0.107024398035999E-11	-0.514598884923899E-12	3
40.0	30.0	0.284412265869036E-16	-0.176145793260146E-15	3
50.0	30.0	0.274523432667463E-19	-0.352845356298280E-20	3
75.0	30.0	0.892933747143352E-29	-0.553107478653961E-30	3
100.0	30.0	0.306606764758007E-38	0.131925157232660E-40	3

AND ORDER N = M+1 = 1

RHO	ANG	RE K	IM K	METHOD
0.1	30.0	0.852075595264435E 01	-0.505051977044309E 01	1
0.5	30.0	0.137924079967794E 01	-0.106649617099292E 01	1
1.0	30.0	0.429145102552943E 00	-0.516097392814518E 00	1
1.5	30.0	0.153854998905324E 00	-0.296949830154990E 00	1
2.0	30.0	0.433829349723874E-01	-0.175296051937276E 00	1
2.5	30.0	0.275261051325484E-03	-0.102141382938034E 00	1
2.5	30.0	0.275261051325509E-03	-0.102141382938034E 00	2
3.0	30.0	-0.141180996487075E-01	-0.577169397204243E-01	2
5.0	30.0	-0.737645016383444E-02	-0.267353025286437E-02	2
10.0	30.0	0.380629722935504E-04	0.598191890727621E-04	2
15.0	30.0	0.605551926192099E-07	-0.751939458826199E-06	2
20.0	30.0	-0.567272730313496E-08	0.640572087355649E-08	2
21.0	30.0	-0.783700709800426E-09	0.342102715177404E-08	2
21.0	30.0	-0.783700709800430E-09	0.342102715177405E-08	3
25.0	30.0	0.983859991400325E-10	-0.202183876712622E-10	3
30.0	30.0	-0.108984612069155E-11	-0.513196089561275E-12	3
40.0	30.0	0.276586756660357E-16	-0.178221639977460E-15	3
50.0	30.0	0.276719087080291E-19	-0.369502242943097E-20	3
75.0	30.0	0.897895862877622E-29	-0.585889913977018E-30	3
100.0	30.0	0.307935779001252E-38	0.561720245364178E-41	3

OPT1= 2.0 MODIFIED BESSEL FUNCTIONS, FIRST KIND,
OPT2= 2.0 OF COMPLEX ARGUMENT IN POLAR COORDINATES
ORD = 0.0 OF ORDER M = 0

RHO	ANG	RE I	IM I	METHOD
0.1	60.0	0.998749219183994E 00	0.216371034482641E-02	1
0.5	60.0	0.968268487155552E 00	0.532808826876282E-01	1
1.0	60.0	0.867618103497088E 00	0.202980518393593E 00	1
1.5	60.0	0.684054268254105E 00	0.418782462297495E 00	1
2.0	60.0	0.401876909028543E 00	0.650962462072001E 00	1
2.5	60.0	0.140682542445072E-01	0.832987043495826E 00	1
2.5	60.0	0.140682542445073E-01	0.832987043495826E 00	2
3.0	60.0	-0.465411477666210E 00	0.887568825861952E 00	2
5.0	60.0	-0.177477694131959E 01	-0.131251655129564E 01	2
10.0	60.0	-0.504718923288672E 01	0.181437389325677E 02	2
15.0	60.0	0.185917703645047E 03	-0.199776629495284E 02	2
20.0	60.0	-0.923017123951721E 03	-0.174137723919345E 04	2
21.0	60.0	0.117282087083194E 04	-0.294578373110087E 04	2
21.0	60.0	0.117282087083194E 04	-0.294578373110087E 04	3
25.0	60.0	-0.138638036945788E 05	0.163841413058517E 05	3
30.0	60.0	0.226423045537542E 06	0.752236282122231E 05	3
40.0	60.0	-0.276939913381966E 08	0.131340690983886E 08	3
50.0	60.0	0.144801894531719E 10	-0.380099335560014E 10	3
75.0	60.0	-0.215590057027040E 14	0.890542624880827E 15	3
100.0	60.0	-0.643080421887438E 20	-0.196724241684648E 21	3

AND ORDER N = M+1 = 1

RHO	ANG	RE I	IM I	METHOD
0.1	60.0	0.249375130235453E-01	0.433012476411755E-01	1
0.5	60.0	0.117228400708627E 00	0.216436240685357E 00	1
1.0	60.0	0.188828534743413E 00	0.430804407423258E 00	1
1.5	60.0	0.174387857582875E 00	0.633195425326950E 00	1
2.0	60.0	0.447975866038961E-01	0.799860900592474E 00	1
2.5	60.0	-0.210366347877049E 00	0.890855053772563E 00	1
2.5	60.0	-0.210366347877049E 00	0.890855053772563E 00	2
3.0	60.0	-0.574590706638980E 00	0.852941735217776E 00	2
5.0	60.0	-0.154462726355877E 01	-0.141571113811487E 01	2
10.0	60.0	-0.573017405009030E 01	0.174815980619618E 02	2
15.0	60.0	0.183464045550592E 03	-0.141953614488190E 02	2
20.0	60.0	-0.873467366158400E 03	-0.174014288024260E 04	2
21.0	60.0	0.122050261384378E 04	-0.288670348898403E 04	2
21.0	60.0	0.122050261384378E 04	-0.288670348898403E 04	3
25.0	60.0	-0.140132778554757E 05	0.159795501524473E 05	3
30.0	60.0	0.223458193208724E 06	0.778976641808554E 05	3
40.0	60.0	-0.276651089568002E 08	0.127508522875685E 08	3
50.0	60.0	0.147389856943119E 10	-0.376948439914239E 10	3
75.0	60.0	-0.266460721975630E 14	0.887459425151834E 15	3
100.0	60.0	-0.632937118915518E 20	-0.196512843420344E 21	3

OPT1= 2.0 MODIFIED BESSEL FUNCTIONS, SECOND KIND,
OPT2= 2.0 OF COMPLEX ARGUMENT IN POLAR COORDINATES
ORD = 0.0 OF ORDER M = 0

RHO	ANG	RE K	IM K	METHOD
0.1	60.0	0.241650623557041E 01	-0.103849173342117E 01	1
0.5	60.0	0.807230998595527E 00	-0.918001918050869E 00	1
1.0	60.0	0.177214810618352E 00	-0.688814732114711E 00	1
1.5	60.0	-0.912065368802299E-01	-0.452902344864404E 00	1
2.0	60.0	-0.188740482377896E 00	-0.252329276549754E 00	1
2.5	60.0	-0.195154333181329E 00	-0.103733171233773E 00	1
2.5	60.0	-0.195154333181329E 00	-0.103733171233773E 00	2
3.0	60.0	-0.157652433648003E 00	-0.797269697981867E-02	2
5.0	60.0	0.551422492012628E-02	0.450636343411047E-01	2
10.0	60.0	-0.256987844620590E-02	-0.659528211515843E-03	2
15.0	60.0	0.105018051508056E-03	-0.143985125031484E-03	2
20.0	60.0	0.673763180530214E-05	0.107450317178827E-04	2
21.0	60.0	0.743003145931799E-05	0.108084318129907E-05	2
21.0	60.0	0.743003145931799E-05	0.108084318129907E-05	3
25.0	60.0	-0.916897023495287E-06	0.165750212208903E-06	3
30.0	60.0	0.140634499924458E-07	-0.684190811475629E-07	3
40.0	60.0	-0.335555064709345E-09	0.231751110688451E-09	3
50.0	60.0	0.242723567642160E-11	0.390639179282618E-12	3
75.0	60.0	-0.656988907321024E-17	-0.358382051825935E-17	3
100.0	60.0	0.161330574139091E-22	0.179816316067128E-22	3

AND ORDER N = M+1 = 1

RHO	ANG	RE K	IM K	METHOD
0.1	60.0	0.488192135337377E 01	-0.876051491938466E 01	1
0.5	60.0	0.625698893182445E 00	-0.189253989694293E 01	1
1.0	60.0	-0.221252868729644E-01	-0.931067993198272E 00	1
1.5	60.0	-0.220415164022503E 00	-0.509187719029843E 00	1
2.0	60.0	-0.262254649207587E 00	-0.248894357140429E 00	1
2.5	60.0	-0.231935295671493E 00	-0.837921802884959E-01	1
2.5	60.0	-0.231935295671493E 00	-0.837921802884959E-01	2
3.0	60.0	-0.172422915334882E 00	0.123560067375052E-01	2
5.0	60.0	0.951455588394162E-02	0.469377299793186E-01	2
10.0	60.0	-0.266329185211349E-02	-0.567821450125988E-03	2
15.0	60.0	0.102706222772161E-03	-0.149401231490252E-03	2
20.0	60.0	0.705254312001200E-05	0.107368057372259E-04	2
21.0	60.0	0.754146309656039E-05	0.942462905344266E-06	2
21.0	60.0	0.754146309656040E-05	0.942462905344274E-06	3
25.0	60.0	-0.923308357907071E-06	0.183145683005242E-06	3
30.0	60.0	0.132022305819469E-07	-0.691949830356485E-07	3
40.0	60.0	-0.335171635472393E-09	0.236817972602643E-09	3
50.0	60.0	0.244279627729629E-11	0.371686318707572E-12	3
75.0	60.0	-0.661248202176921E-17	-0.355800048753178E-17	3
100.0	60.0	0.162511570102619E-22	0.179570122720726E-22	3

OPT1= 2.0 MODIFIED BESSEL FUNCTIONS, FIRST KIND,
OPT2= 2.0 OF COMPLEX ARGUMENT IN POLAR COORDINATES
ORD = 0.0 OF ORDER M = 0

RHO	ANG	RE I	IM I	METHOD
0.1	90.0	0.997501562066040E 00	0.435438809825736E-18	1
0.5	90.0	0.938469807240813E 00	0.105625065198166E-16	1
1.0	90.0	0.765197686557967E 00	0.383709643887500E-16	1
1.5	90.0	0.511827671735918E 00	0.729753438685718E-16	1
2.0	90.0	0.223890779141236E 00	0.100577014449764E-15	1
2.5	90.0	-0.483837764681980E-01	0.108362428783189E-15	1
2.5	90.0	-0.483837764681980E-01	0.108362428783189E-15	2
3.0	90.0	-0.260051954901933E 00	0.886944795307671E-16	2
5.0	90.0	-0.177596771314338E 00	-0.142819119326263E-15	2
10.0	90.0	-0.245935764451348E 00	0.379068054710744E-16	2
15.0	90.0	-0.142244728267808E-01	0.268265968160419E-15	2
20.0	90.0	0.167024664340582E 00	0.116552574218223E-15	2
21.0	90.0	0.365790710008630E-01	0.313343629590525E-15	2
21.0	90.0	0.365790710008630E-01	0.307582580231733E-15	3
25.0	90.0	0.962667832759579E-01	-0.281108228552071E-15	3
30.0	90.0	-0.863679835810404E-01	-0.310077921104454E-15	3
40.0	90.0	0.736689058423750E-02	0.443551294907440E-15	3
50.0	90.0	0.558123276692516E-01	-0.431184208826982E-15	3
75.0	90.0	0.346439138050968E-01	-0.554149313519979E-15	3
100.0	90.0	0.199858503042229E-01	-0.671256137793521E-15	3

AND ORDER N = M+1 = 1

RHO	ANG	RE I	IM I	METHOD
0.1	90.0	0.434349758935400E-17	0.499375260362420E-01	1
0.5	90.0	0.197907279229600E-16	0.242268457674874E 00	1
1.0	90.0	0.283517582536154E-16	0.440050585744934E 00	1
1.5	90.0	0.182943062289159E-16	0.557936507910100E 00	1
2.0	90.0	-0.112434274468159E-16	0.576724807756873E 00	1
2.5	90.0	-0.538922371231579E-16	0.497094102464274E 00	1
2.5	90.0	-0.538922371231579E-16	0.497094102464274E 00	2
3.0	90.0	-0.975918531113569E-16	0.339058958525936E 00	2
5.0	90.0	-0.488654491366627E-16	-0.327579137591465E 00	2
10.0	90.0	-0.218238581887990E-15	0.434727461688616E-01	2
15.0	90.0	-0.364893068798521E-16	0.205104038613522E 00	2
20.0	90.0	0.285452403861079E-15	0.668331241758496E-01	2
21.0	90.0	0.520599442323284E-16	0.171120272763902E 00	2
21.0	90.0	0.592134811533188E-16	0.171120272763900E 00	3
25.0	90.0	0.218550823533705E-15	-0.125350249580290E 00	3
30.0	90.0	-0.223073209697027E-15	-0.118751062616623E 00	3
40.0	90.0	0.203501934564608E-16	0.126038318037585E 00	3
50.0	90.0	0.249837881021774E-15	-0.975118281251753E-01	3
75.0	90.0	0.229001601353774E-15	-0.851399950448292E-01	3
100.0	90.0	0.177155148727077E-15	-0.771453520141122E-01	3

OPT1= 2.0 MODIFIED BESSEL FUNCTIONS, SECOND KIND,
OPT2= 2.0 OF COMPLEX ARGUMENT IN POLAR COORDINATES
ORD = 0.0 OF ORDER M = 0

RHO	ANG	RE K	IM K	METHOD
0.1	90.0	0.240997643796791E 01	-0.156687178966551E 01	1
0.5	90.0	0.698248393783854E 00	-0.147414492602178E 01	1
1.0	90.0	-0.138633715204054E 00	-0.120196971531721E 01	1
1.5	90.0	-0.600749364688181E 00	-0.803977026714764E 00	1
2.0	90.0	-0.801696231883694E 00	-0.351686813478300E 00	1
2.5	90.0	-0.782367091369019E 00	0.760010583527108E-01	1
2.5	90.0	-0.782367091369019E 00	0.760010583527108E-01	2
3.0	90.0	-0.591954611480711E 00	0.408488655535789E 00	2
5.0	90.0	0.484618352492666E 00	0.278968356031196E 00	2
10.0	90.0	-0.874480650774623E-01	0.386314995427672E 00	2
15.0	90.0	-0.322742561505432E 00	0.223437496669011E-01	2
20.0	90.0	-0.983956193764207E-01	-0.262361729230340E 00	2
21.0	90.0	-0.267352296943552E 00	-0.574582703657243E-01	2
21.0	90.0	-0.267352296943554E 00	-0.574582703657248E-01	3
25.0	90.0	0.199882940793320E 00	-0.151215509562235E 00	3
30.0	90.0	0.184247704482131E 00	0.135666511361780E 00	3
40.0	90.0	-0.197820461324826E 00	-0.115718846696199E-01	3
50.0	90.0	0.154040134671555E 00	-0.876697992927334E-01	3
75.0	90.0	0.134097386467104E 00	-0.544185325508450E-01	3
100.0	90.0	0.121335083699666E 00	-0.313937002457459E-01	3

AND ORDER N = M+1 = 1

RHO	ANG	RE K	IM K	METHOD
0.1	90.0	-0.784416824669526E-01	-0.101456966545058E 02	1
0.5	90.0	-0.380554403413957E 00	-0.231138342938652E 01	1
1.0	90.0	-0.691229843692084E 00	-0.122712623014357E 01	1
1.5	90.0	-0.876404617209956E 00	-0.647652876756467E 00	1
2.0	90.0	-0.905917209595990E 00	-0.168126150312431E 00	1
2.5	90.0	-0.780833590222288E 00	0.229207675130978E 00	1
2.5	90.0	-0.780833590222287E 00	0.229207675130978E 00	2
3.0	90.0	-0.532592566619444E 00	0.509997393867205E 00	2
5.0	90.0	0.514560106063313E 00	0.232262882507286E 00	2
10.0	90.0	-0.682868299977346E-01	0.391152513659556E 00	2
15.0	90.0	-0.322176670464920E 00	0.331023775125629E-01	2
20.0	90.0	-0.104981225963653E 00	-0.259985035882543E 00	2
21.0	90.0	-0.268795095897672E 00	-0.511125512719340E-01	2
21.0	90.0	-0.268795095897675E 00	-0.511125512719345E-01	3
25.0	90.0	0.196899711603543E 00	-0.155241745658778E 00	3
30.0	90.0	0.186533732961182E 00	0.132615376283035E 00	3
40.0	90.0	-0.197980527008845E 00	-0.910041766375504E-02	3
50.0	90.0	0.153171221438080E 00	-0.892144275550727E-01	3
75.0	90.0	0.133737591479752E 00	-0.553136843827839E-01	3
100.0	90.0	0.121179635573066E 00	-0.320007528622542E-01	3

OPT1 = 2.0 MODIFIED BESSEL FUNCTIONS, FIRST KIND,
OPT2 = 2.0 OF COMPLEX ARGUMENT IN POLAR COORDINATES
ORD = 0.0 OF ORDER M = 0

RHO	ANG	RE I	IM I	METHOD
0.1	120.0	0.998749219183994E 00	-0.216371034482641E-02	1
0.5	120.0	0.968268487155552E 00	-0.532808826876282E-01	1
1.0	120.0	0.867618103497088E 00	-0.202980518393593E 00	1
1.5	120.0	0.684054268254105E 00	-0.418782462297495E 00	1
2.0	120.0	0.401876909028543E 00	-0.650962462072001E 00	1
2.5	120.0	0.140682542445071E-01	-0.832987043495826E 00	1
2.5	120.0	0.140682542445070E-01	-0.832987043495826E 00	2
3.0	120.0	-0.465411477666209E 00	-0.887568825861951E 00	2
5.0	120.0	-0.177477694131959E 01	0.131251655129563E 01	2
10.0	120.0	-0.504718923288674E 01	-0.181437389325676E 02	2
15.0	120.0	0.185917703645046E 03	0.199776629495281E 02	2
20.0	120.0	-0.923017123951714E 03	0.174137723919345E 04	2
21.0	120.0	0.117282087083194E 04	0.294578373110086E 04	2
21.0	120.0	0.117282087152003E 04	0.294578373583096E 04	3
25.0	120.0	-0.138638036944732E 05	-0.163841413064353E 05	3
30.0	120.0	0.226423045537497E 06	-0.752236282122149E 05	3
40.0	120.0	-0.276939913381966E 08	-0.131340690983884E 08	3
50.0	120.0	0.144801894531719E 10	0.380099335560008E 10	3
75.0	120.0	-0.215590057027113E 14	-0.890542624880818E 15	3
100.0	120.0	-0.643080421887415E 20	0.196724241684646E 21	3

AND ORDER N = M+1 = 1

RHO	ANG	RE I	IM I	METHOD
0.1	120.0	-0.249375130235453E-01	0.433012476411755E-01	1
0.5	120.0	-0.117228400708627E 00	0.216436240685357E 00	1
1.0	120.0	-0.188828534743413E 00	0.430804407423258E 00	1
1.5	120.0	-0.174387857582875E 00	0.633195425326950E 00	1
2.0	120.0	-0.447975866038960E-01	0.799860900592473E 00	1
2.5	120.0	0.210366347877049E 00	0.890855053772562E 00	1
2.5	120.0	0.210366347877049E 00	0.890855053772563E 00	2
3.0	120.0	0.574590706638979E 00	0.852941735217775E 00	2
5.0	120.0	0.154462726355877E 01	-0.141571113811487E 01	2
10.0	120.0	0.573017405009032E 01	0.174815980619617E 02	2
15.0	120.0	-0.183464045590592E 03	-0.141953614488187E 02	2
20.0	120.0	0.873467366158393E 03	-0.174014288024260E 04	2
21.0	120.0	-0.122050261384378E 04	-0.288670348898402E 04	2
21.0	120.0	-0.122050261324380E 04	-0.288670348418297E 04	3
25.0	120.0	0.140132778555923E 05	0.159795501518594E 05	3
30.0	120.0	-0.223458193208767E 06	0.778976641808645E 05	3
40.0	120.0	0.276651089568002E 08	0.127508522875684E 08	3
50.0	120.0	-0.147389856943120E 10	-0.376948439914233E 10	3
75.0	120.0	0.266460721975702E 14	0.887459425151825E 15	3
100.0	120.0	0.632937118915495E 20	-0.196512843420342E 21	3

OPT1= 2.0
OPT2= 2.0
ORD = 0.0

MODIFIED BESSEL FUNCTIONS, SECOND KIND,
OF COMPLEX ARGUMENT IN POLAR COORDINATES
OF ORDER M = 0

RHO	ANG	RE K	IM K	METHOD
0.1	120.0	0.240970873904661E 01	-0.209917147634580E 01	1
0.5	120.0	0.639844168967294E 00	-0.212390324789952E 01	1
1.0	120.0	-0.460467294788806E 00	-0.203688792795325E 01	1
1.5	120.0	-0.140685044388629E 01	-0.169611751893943E 01	1
2.0	120.0	-0.223379937098602E 01	-0.101020426850169E 01	1
2.5	120.0	-0.281206030956330E 01	0.595364470503959E-01	1
2.5	120.0	-0.281206030956330E 01	0.595364470503964E-01	2
3.0	120.0	-0.294603213653123E 01	0.147010597611235E 01	2
5.0	120.0	0.412890658018550E 01	0.553056256626908E 01	2
10.0	120.0	-0.570028068176518E 02	0.158568721435260E 02	2
15.0	120.0	0.627617841761819E 02	-0.584077547958437E 03	2
20.0	120.0	0.547069794851625E 04	0.289974380499925E 04	2
21.0	120.0	0.925445253612081E 04	-0.368452543286325E 04	2
21.0	120.0	0.925445255098086E 04	-0.368452543502495E 04	3
25.0	120.0	-0.514722979645912E 05	0.435544238372023E 05	3
30.0	120.0	-0.236321997767850E 06	-0.711328976463960E 06	3
40.0	120.0	-0.412618949912378E 08	0.870032397366579E 08	3
50.0	120.0	0.119411728022968E 11	-0.454908568086734E 10	3
75.0	120.0	-0.279772216803415E 16	0.677296139343382E 14	3
100.0	120.0	0.618027432459506E 21	0.202029672906893E 21	3

AND ORDER N = M+1 = 1

RHO	ANG	RE K	IM K	METHOD
0.1	120.0	-0.501795623485456E 01	-0.883885842709823E 01	1
0.5	120.0	-0.130565339689015E 01	-0.226082377940123E 01	1
1.0	120.0	-0.133128667462205E 01	-0.152429033073630E 01	1
1.5	120.0	-0.176882693247131E 01	-0.105704333128747E 01	1
2.0	120.0	-0.225058247998744E 01	-0.389630126113781E 00	1
2.5	120.0	-0.256676839667373E 01	0.577093192764555E 00	1
2.5	120.0	-0.256676839667373E 01	0.577093192764556E 00	2
3.0	120.0	-0.250717257396541E 01	0.181748594953549E 01	2
5.0	120.0	0.443807315522297E 01	0.489952739371004E 01	2
10.0	120.0	-0.549173967526165E 02	0.180013048781045E 02	2
15.0	120.0	0.445959405364378E 02	-0.576369447100834E 03	2
20.0	120.0	0.546682008171418E 04	0.274407867141044E 04	2
21.0	120.0	0.906884646654275E 04	-0.383432204439631E 04	2
21.0	120.0	0.906884645145981E 04	-0.383432204251139E 04	3
25.0	120.0	-0.502012373638280E 05	0.440240107640243E 05	3
30.0	120.0	-0.244722729522422E 06	-0.702014618169181E 06	3
40.0	120.0	-0.400579838736334E 08	0.869125030594447E 08	3
50.0	120.0	0.118421844961669E 11	-0.463038891786156E 10	3
75.0	120.0	-0.278803601041600E 16	0.837111046629098E 14	3
100.0	120.0	0.617363305225386E 21	0.198843060296921E 21	3

OPT1= 2.0 MODIFIED BESSEL FUNCTIONS, FIRST KIND,
OPT2= 2.0 OF COMPLEX ARGUMENT IN POLAR COORDINATES
ORD = 0.0 OF ORDER M = 0

RHO	ANG	RE I	IM I	METHOD
0.1	150.0	0.100124921831594E 01	-0.216641667409577E-02	1
0.5	150.0	0.103075492385372E 01	-0.549722926707177E-01	1
1.0	150.0	0.111675011576078E 01	-0.230032066040576E 00	1
1.5	150.0	0.123667048430371E 01	-0.555489344711846E 00	1
2.0	150.0	0.134639083726984E 01	-0.108096813237748E 01	1
2.5	150.0	0.136528827443692E 01	-0.187222368675946E 01	1
2.5	150.0	0.136528827443692E 01	-0.187222368675946E 01	2
3.0	150.0	0.115579200904768E 01	-0.300262937063371E 01	2
5.0	150.0	-0.841142625147852E 01	-0.110161430764326E 02	2
10.0	150.0	0.139420238990131E 02	0.735811296869060E 03	2
15.0	150.0	0.264264623150748E 05	-0.370001823305540E 05	2
20.0	150.0	-0.284281315817823E 07	0.911112779480112E 06	2
21.0	150.0	-0.477276355792684E 07	0.501674968032691E 07	2
21.0	150.0	-0.477276355792684E 07	0.501674968032691E 07	3
25.0	150.0	0.191597942931976E 09	0.657900165249196E 08	3
30.0	150.0	-0.791224672881747E 10	-0.115929487827113E 11	3
40.0	150.0	0.442567473084088E 14	-0.543106951188835E 14	3
50.0	150.0	0.333369769107119E 18	0.139303970580570E 18	3
75.0	150.0	0.667141394754872E 27	0.331993344232791E 27	3
100.0	150.0	0.140873927196622E 37	0.821458496603572E 36	3

AND ORDER N = M+1 = 1

RHO	ANG	RE I	IM I	METHOD
0.1	150.0	-0.433012476317786E-01	0.250625130181200E-01	1
0.5	150.0	-0.216435506551306E 00	0.132852976850616E 00	1
1.0	150.0	-0.430710447441400E 00	0.313774275620088E 00	1
1.5	150.0	-0.631590710386300E 00	0.595335397446254E 00	1
2.0	150.0	-0.787852816867387E 00	0.103784157792652E 01	1
2.5	150.0	-0.833733833953888E 00	0.170951050029616E 01	1
2.5	150.0	-0.833733833953888E 00	0.170951050029616E 01	2
3.0	150.0	-0.649163851044205E 00	0.268076012848432E 01	2
5.0	150.0	0.827919220327029E 01	0.957501107181049E 01	2
10.0	150.0	-0.326273435112714E 02	-0.703135000007192E 03	2
15.0	150.0	-0.250203467325037E 05	0.363761230388357E 05	2
20.0	150.0	0.276917161684861E 07	-0.927597044231120E 06	2
21.0	150.0	0.461265029179396E 07	-0.497065707239389E 07	2
21.0	150.0	0.461265029179397E 07	-0.497065707239389E 07	3
25.0	150.0	-0.188930116622977E 09	-0.626931553093380E 08	3
30.0	150.0	0.789555341300885E 10	0.113578900648696E 11	3
40.0	150.0	-0.434327008570480E 14	0.540003338141961E 14	3
50.0	150.0	-0.331177068637690E 18	-0.136412455763170E 18	3
75.0	150.0	-0.664395375463492E 27	-0.327836046940871E 27	3
100.0	150.0	-0.140469309166744E 37	-0.814359066955196E 36	3

OPT1= 2.0 MODIFIED BESSEL FUNCTIONS, SECOND KIND,
OPT2= 2.0 OF COMPLEX ARGUMENT IN POLAR COORDINATES
ORD = 0.0 OF ORDER M = 0

RHO	ANG	RE K	IM K	METHOD
0.1	150.0	0.241711502563664E 01	-0.262867093185884E 01	1
0.5	150.0	0.720549841759892E 00	-0.279838212295094E 01	1
1.0	150.0	-0.360277460369740E 00	-0.318710438397697E 01	1
1.5	150.0	-0.159964167815190E 01	-0.366633684729069E 01	1
2.0	150.0	-0.334727615155274E 01	-0.408840660330208E 01	1
2.5	150.0	-0.587494278409222E 01	-0.420195253511342E 01	1
2.5	150.0	-0.587494278409222E 01	-0.420195253511342E 01	2
3.0	150.0	-0.944208600903053E 01	-0.357971255947053E 01	2
5.0	150.0	-0.346149198785393E 02	0.264280273190697E 02	2
10.0	150.0	0.231161939983633E 04	-0.438002180380589E 02	2
15.0	150.0	-0.116239500991081E 06	-0.830211798686766E 05	2
20.0	150.0	0.286234521460649E 07	0.893096093326110E 07	2
21.0	150.0	0.157605839406140E 08	0.149940789309040E 08	2
21.0	150.0	0.157605839406140E 08	0.149940789309040E 08	3
25.0	150.0	0.206685432594239E 09	-0.601922689958013E 09	3
30.0	150.0	-0.364203227292086E 11	0.248570561966428E 11	3
40.0	150.0	-0.170622080796839E 15	-0.139036672215877E 15	3
50.0	150.0	0.437636330591806E 18	-0.104731201755585E 19	3
75.0	150.0	0.104298785128244E 28	-0.209588650466755E 28	3
100.0	150.0	0.258068797815870E 37	-0.442568494763250E 37	3

AND ORDER N = M+1 = 1

RHO	ANG	RE K	IM K	METHOD
0.1	150.0	-0.859949215942257E 01	-0.518655465189436E 01	1
0.5	150.0	-0.179661073575937E 01	-0.174644836835048E 01	1
1.0	150.0	-0.142489606172647E 01	-0.186921417032079E 01	1
1.5	150.0	-0.202415630994444E 01	-0.228115056598015E 01	1
2.0	150.0	-0.330385841177638E 01	-0.265040867351788E 01	1
2.5	150.0	-0.537086089001635E 01	-0.272139347073682E 01	1
2.5	150.0	-0.537086089001636E 01	-0.272139347073682E 01	2
3.0	150.0	-0.840773822603407E 01	-0.209712532513696E 01	2
5.0	150.0	-0.300734079910769E 02	0.260071758731990E 02	2
10.0	150.0	0.220896371244148E 04	-0.102501762861972E 03	2
15.0	150.0	-0.114278960904945E 06	-0.786037374858550E 05	2
20.0	150.0	0.291413205964810E 07	0.869960920802097E 07	2
21.0	150.0	0.156157797421468E 08	0.144910682702787E 08	2
21.0	150.0	0.156157797421468E 08	0.144910682702787E 08	3
25.0	150.0	0.196956356150180E 09	-0.593541466424606E 09	3
30.0	150.0	-0.356818639880748E 11	0.248046125983344E 11	3
40.0	150.0	-0.169647052002075E 15	-0.136447853938065E 15	3
50.0	150.0	0.428552368883717E 18	-0.104042344586957E 19	3
75.0	150.0	0.102992731665136E 28	-0.208725963063514E 28	3
100.0	150.0	0.255838446213068E 37	-0.441297349733077E 37	3

OPT1= 2.0 MODIFIED BESSEL FUNCTIONS, FIRST KIND,
OPT2= 2.0 OF COMPLEX ARGUMENT IN POLAR COORDINATES
ORD = 0.0 OF ORDER M = 0

RHO	ANG	RE I	IM I	METHOD
0.1	180.0	0.100250156293410E 01	-0.873057537651956E-18	1
0.5	180.0	0.106348337074132E 01	-0.224875355897181E-16	1
1.0	180.0	0.126606587775201E 01	-0.985600317589086E-16	1
1.5	180.0	0.164672318977289E 01	-0.256794255884214E-15	1
2.0	180.0	0.227958530233607E 01	-0.554793217706108E-15	1
2.5	180.0	0.328983914405012E 01	-0.109724691379613E-14	1
2.5	180.0	0.328983914405012E 01	-0.109724691379613E-14	2
3.0	180.0	0.488079258586502E 01	-0.206832531633662E-14	2
5.0	180.0	0.272398718236044E 02	-0.212198799018661E-13	2
10.0	180.0	0.281571662846626E 04	-0.465802798151456E-11	2
15.0	180.0	0.339649373297914E 06	-0.858342434063567E-09	2
20.0	180.0	0.435582825595535E 08	-0.148077364253782E-06	2
21.0	180.0	0.115513961922158E 09	-0.412843853602309E-06	2
21.0	180.0	0.115513961922158E 09	0.191721100653139E-09	3
25.0	180.0	0.577456060646632E 10	0.525149776101740E-08	3
30.0	180.0	0.781672297823978E 12	0.587987954242574E-06	3
40.0	180.0	0.148947747934199E 17	0.832864800140786E-02	3
50.0	180.0	0.293255378384934E 21	0.130495031141080E 03	3
75.0	180.0	0.172263907803581E 32	-0.225307621375586E 18	3
100.0	180.0	0.107375170713108E 43	-0.187252873228355E 29	3

AND ORDER N = M+1 = 1

RHO	ANG	RE I	IM I	METHOD
0.1	180.0	-0.500625260470927E-01	0.875239272629169E-17	1
0.5	180.0	-0.257894305390896E 00	0.477571824946812E-16	1
1.0	180.0	-0.565159103992485E 00	0.122233532811835E-15	1
1.5	180.0	-0.981666428577908E 00	0.259570374801710E-15	1
2.0	180.0	-0.159063685463733E 01	0.517692767601091E-15	1
2.5	180.0	-0.251671624528870E 01	0.995417023737562E-15	1
2.5	180.0	-0.251671624528870E 01	0.995417023737562E-15	2
3.0	180.0	-0.395337021740261E 01	0.186409263372070E-14	2
5.0	180.0	-0.243356421424505E 02	0.195082967254625E-13	2
10.0	180.0	-0.267098830370126E 04	0.444462186572181E-11	2
15.0	180.0	-0.328124921970206E 06	0.831266433183054E-09	2
20.0	180.0	-0.424549733851278E 08	0.144521693354136E-06	2
21.0	180.0	-0.112729199137776E 09	0.403383153187968E-06	2
21.0	180.0	-0.112729199137776E 09	0.436088058177308E-09	3
25.0	180.0	-0.565786512987871E 10	0.154269815185818E-07	3
30.0	180.0	-0.768532038938957E 12	0.173377532987332E-05	3
40.0	180.0	-0.147073961632594E 17	0.246674485348324E-01	3
50.0	180.0	-0.290307859010356E 21	0.387509030487594E 03	3
75.0	180.0	-0.171111601529653E 32	0.223820659602942E 18	3
100.0	180.0	-0.106836939033816E 43	0.186323654714074E 29	3

AD-A056 369 ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND ABERD--ETC F/G 12/1
USER'S MANUAL FOR THE BRL SUBROUTINE TO CALCULATE BESSEL FUNCTI--ETC(U)
MAY 78 K L ZIMMERMAN, A S ELDER, A K DEPUE
UNCLASSIFIED ARBRL-TR-02068 SBIE-AD-E430 057 NL

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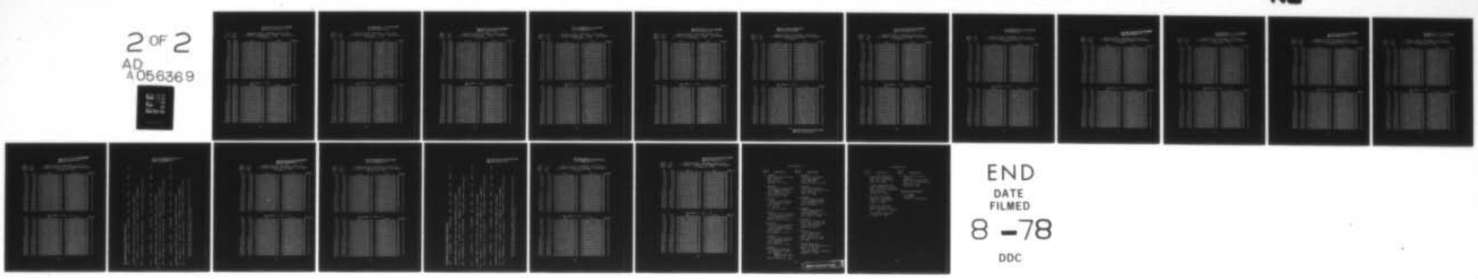

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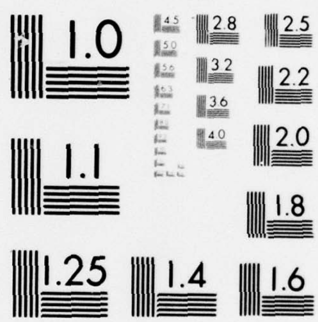

MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

OPT1= 2.0 MODIFIED BESSEL FUNCTIONS, SECOND KIND,
OPT2= 2.0 OF COMPLEX ARGUMENT IN POLAR COORDINATES
ORD = 0.0 OF ORDER M = 0

RHO	ANG	RE K	IM K	METHOD
0.1	180.0	0.242706902470202E 01	-0.314945154532604E 01	1
0.5	180.0	0.924419071227666E 00	-0.334103154473585E 01	1
1.0	180.0	0.421024438240708E 00	-0.397746326050642E 01	1
1.5	180.0	0.213805562647525E 00	-0.517333347548647E 01	1
2.0	180.0	0.113893872749531E 00	-0.716152843905026E 01	1
2.5	180.0	0.623475532003626E-01	-0.103353344864400E 02	1
2.5	180.0	0.623475532003628E-01	-0.103353344864400E 02	2
3.0	180.0	0.347395043862728E-01	-0.153334621314491E 02	2
5.0	180.0	0.369109833397593E-02	-0.855765812057633E 02	2
10.0	180.0	0.177804768254 12E-04	-0.884583467458021E 04	2
15.0	180.0	0.954988025388458E-07	-0.106703997594910E 07	2
20.0	180.0	-0.464624635921088E-06	-0.136842380492082E 09	2
21.0	180.0	-0.129678104075971E-05	-0.362897814161703E 09	2
21.0	180.0	0.808486398348583E-09	-0.362897814161703E 09	3
25.0	180.0	0.165015309479177E-07	-0.181413171789836E 11	3
30.0	180.0	0.184721865877254E-05	-0.245569594835846E 13	3
40.0	180.0	0.261652193755583E-01	-0.467933150678824E 17	3
50.0	180.0	0.409962231162789E 03	-0.921288942359803E 21	3
75.0	180.0	-0.707824768111331E 18	-0.541183027234398E 32	3
100.0	180.0	-0.588272250897780E 29	-0.337329047490248E 43	3

AND ORDER N = M+1 = 1

RHO	ANG	RE K	IM K	METHOD
0.1	180.0	-0.985384478087061E 01	-0.157276064049696E 00	1
0.5	180.0	-0.165644112000330E 01	-0.810198855218683E 00	1
1.0	180.0	-0.601907230197235E 00	-0.177549968921218E 01	1
1.5	180.0	-0.277387804568 45E 00	-0.308399604029608E 01	1
2.0	180.0	-0.139865881816524E 00	-0.499713305705781E 01	1
2.5	180.0	-0.738908163477502E-01	-0.790649726736906E 01	1
2.5	180.0	-0.738908163477502E-01	-0.790649726736906E 01	2
3.0	180.0	-0.401564311282000E-01	-0.124198788319127E 02	2
5.0	180.0	-0.404461344551345E-02	-0.764526745751127E 02	2
10.0	180.0	-0.186487874170170E-04	-0.839115723273213E 04	2
15.0	180.0	-0.104028794217285E-06	-0.103083484432132E 07	2
20.0	180.0	-0.454616595922667E-06	-0.133376232495068E 09	2
21.0	180.0	-0.126747658055954E-05	-0.354149223856296E 09	2
21.0	180.0	-0.158104096221935E-08	-0.354149223856296E 09	3
25.0	180.0	-0.484688245839154E-07	-0.177747075270288E 11	3
30.0	180.0	-0.544681586098255E-05	-0.241441460757901E 13	3
40.0	180.0	-0.774950750998339E-01	-0.462046477399303E 17	3
50.0	180.0	-0.121739552337953E 04	-0.912029037146316E 21	3
75.0	180.0	-0.703153339930224E 18	-0.537562950309542E 32	3
100.0	180.0	-0.585353024839736E 29	-0.335638142800658E 43	3

OPT1= 2.0 MODIFIED BESSEL FUNCTIONS, FIRST KIND,
OPT2= 2.0 OF COMPLEX ARGUMENT IN POLAR COORDINATES
ORD = 0.0 OF ORDER M = 0

RHO	ANG	RE I	IM I	METHOD
0.1	-120.	0.998749219183994E 00	0.216371034482641E-02	1
0.5	-120.	0.968268487155552E 00	0.532808826876282E-01	1
1.0	-120.	0.867618103497088E 00	0.202980518393593E 00	1
1.5	-120.	0.684054268254105E 00	0.418782462297495E 00	1
2.0	-120.	0.401876909028543E 00	0.650962462072001E 00	1
2.5	-120.	0.140682542445071E-01	0.832987043495826E 00	1
2.5	-120.	0.140682542445075E-01	0.832987043495825E 00	2
3.0	-120.	-0.465411477666209E 00	0.887568825861951E 00	2
5.0	-120.	-0.177477694131959E 01	-0.131251655129563E 01	2
10.0	-120.	-0.504718923288673E 01	0.181437389325676E 02	2
15.0	-120.	0.185917703645047E 03	-0.199776629495281E 02	2
20.0	-120.	-0.923017123951714E 03	-0.174137723919345E 04	2
21.0	-120.	0.117282087083194E 04	-0.294578373110086E 04	2
21.0	-120.	0.117282087152003E 04	-0.294578373583096E 04	3
25.0	-120.	-0.138638036944732E 05	0.163841413064353E 05	3
30.0	-120.	0.226423045537497E 06	0.752236282122149E 05	3
40.0	-120.	-0.276939913381966E 08	0.131340690983884E 08	3
50.0	-120.	0.144801894531719E 10	-0.380099335560008E 10	3
75.0	-120.	-0.215590057027113E 14	0.890542624980818E 15	3
100.0	-120.	-0.643080421887415E 20	-0.196724241684646E 21	3

AND ORDER N = M+1 = 1

RHO	ANG	RE I	IM I	METHOD
0.1	-120.	-0.249375130235453E-01	-0.433012475411755E-01	1
0.5	-120.	-0.117228400708627E 00	-0.216436240685357E 00	1
1.0	-120.	-0.188828534743413E 00	-0.430804407423258E 00	1
1.5	-120.	-0.174387857582875E 00	-0.633195425326950E 00	1
2.0	-120.	-0.447975866038960E-01	-0.799860900592473E 00	1
2.5	-120.	0.210366347877049E 00	-0.890855053772562E 00	1
2.5	-120.	0.210366347877048E 00	-0.890855053772562E 00	2
3.0	-120.	0.574590706638979E 00	-0.852941735217775E 00	2
5.0	-120.	0.154462726355876E 01	0.141571113811486E 01	2
10.0	-120.	0.573017405009031E 01	-0.174815980619617E 02	2
15.0	-120.	-0.183464045550592E 03	0.141953614488187E 02	2
20.0	-120.	0.873467366158393E 03	0.174014288024260E 04	2
21.0	-120.	-0.122050261384378E 04	0.288670348898402E 04	2
21.0	-120.	-0.122050261324380E 04	0.288670348418297E 04	3
25.0	-120.	0.140132778555923E 05	-0.159795501518594E 05	3
30.0	-120.	-0.223458193208767E 06	-0.778976641808645E 05	3
40.0	-120.	0.276651089568002E 08	-0.127508522875684E 08	3
50.0	-120.	-0.147389856943120E 10	0.376948439914233E 10	3
75.0	-120.	0.266460721975702E 14	-0.887459425151825E 15	3
100.0	-120.	0.632937118915495E 20	0.196512843420342E 21	3

OPT1= 2.0 MODIFIED BESSEL FUNCTIONS, SECOND KIND,
OPT2= 2.0 OF COMPLEX ARGUMENT IN POLAR COORDINATES
ORD = 0.0 OF ORDER M = 0

RHO	ANG	RE K	IM K	METHOD
0.1	-120.	0.240970873904661E 01	0.209917147634580E 01	1
0.5	-120.	0.639844168967294E 00	0.212390324789952E 01	1
1.0	-120.	-0.460467294788806E 00	0.203688792795325E 01	1
1.5	-120.	-0.140685044388629E 01	0.169611751893943E 01	1
2.0	-120.	-0.223379937098602E 01	0.101020426850169E 01	1
2.5	-120.	-0.281206030956330E 01	-0.595364470503959E-01	1
2.5	-120.	-0.281206030956329E 01	-0.595364470503951E-01	2
3.0	-120.	-0.294603213653123E 01	-0.147010597611235E 01	2
5.0	-120.	0.412890658018549E 01	-0.553056256626907E 01	2
10.0	-120.	-0.570028068176518E 02	-0.158568721435260E 02	2
15.0	-120.	0.627617841761820E 02	0.584077547958437E 03	2
20.0	-120.	0.547069794851625E 04	-0.289974380499925E 04	2
21.0	-120.	0.925445253612081E 04	0.368452543286325E 04	2
21.0	-120.	0.925445255098086E 04	0.368452543502495E 04	3
25.0	-120.	-0.514722979645912E 05	-0.435544238372023E 05	3
30.0	-120.	-0.236321997767850E 06	0.711328976463960E 06	3
40.0	-120.	-0.412618949912378E 08	-0.870032397366579E 08	3
50.0	-120.	0.119411728022968E 11	0.454908568086734E 10	3
75.0	-120.	-0.279772216803415E 16	-0.677296139343382E 14	3
100.0	-120.	0.618027432459506E 21	-0.202029672906893E 21	3

AND ORDER N = M+1 = 1

RHO	ANG	RE K	IM K	METHOD
0.1	-120.	-0.501795623485456E 01	0.883885842709823E 01	1
0.5	-120.	-0.130565339689015E 01	0.226082377940123E 01	1
1.0	-120.	-0.133128667462205E 01	0.152429033073630E 01	1
1.5	-120.	-0.176882693247131E 01	0.105704333128747E 01	1
2.0	-120.	-0.225058247998744E 01	0.389630126113781E 00	1
2.5	-120.	-0.256676839667373E 01	-0.577093192764555E 00	1
2.5	-120.	-0.256676839667373E 01	-0.577093192764553E 00	2
3.0	-120.	-0.250717257396541E 01	-0.181748594953549E 01	2
5.0	-120.	0.443807315522296E 01	-0.489952739371004E 01	2
10.0	-120.	-0.549173967526165E 02	-0.180013048781045E 02	2
15.0	-120.	0.445959405364379E 02	0.576369447100835E 03	2
20.0	-120.	0.546682008171418E 04	-0.274407867141044E 04	2
21.0	-120.	0.906884646654275E 04	0.383432204439631E 04	2
21.0	-120.	0.906884645145981E 04	0.383432204251139E 04	3
25.0	-120.	-0.502012373638280E 05	-0.440240107640243E 05	3
30.0	-120.	-0.244722729522422E 06	0.702014618169181E 06	3
40.0	-120.	-0.400579838736334E 08	-0.869125030594447E 08	3
50.0	-120.	0.118421844961669E 11	0.463038891786156E 10	3
75.0	-120.	-0.278803601041600E 16	-0.837111046629098E 14	3
100.0	-120.	0.617363305225386E 21	-0.198843060296921E 21	3

OPT1= 2.0 MODIFIED BESSEL FUNCTIONS, FIRST KIND,
OPT2= 2.0 OF COMPLEX ARGUMENT IN POLAR COORDINATES
ORD = 0.0 OF ORDER M = 0

RHO	ANG	RE I	IM I	METHOD
0.1	-60.0	0.998749219183994E 00	-0.216371034482641E-02	1
0.5	-60.0	0.968268487155552E 00	-0.532808826876282E-01	1
1.0	-60.0	0.867618103497088E 00	-0.202980518393593E 00	1
1.5	-60.0	0.684054268254105E 00	-0.418782462297495E 00	1
2.0	-60.0	0.401876909028543E 00	-0.650962462072001E 00	1
2.5	-60.0	0.140682542445072E-01	-0.832987043495826E 00	1
2.5	-60.0	0.140682542445072E-01	-0.832987043495826E 00	2
3.0	-60.0	-0.465411477666210E 00	-0.887568825861951E 00	2
5.0	-60.0	-0.177477694131959E 01	0.131251655129563E 01	2
10.0	-60.0	-0.504718923288670E 01	-0.181437389325677E 02	2
15.0	-60.0	0.185917703645047E 03	0.199776629495284E 02	2
20.0	-60.0	-0.923017123951721E 03	0.174137723919345E 04	2
21.0	-60.0	0.117282087083194E 04	0.294578373110087E 04	2
21.0	-60.0	0.117282087083194E 04	0.294578373110087E 04	3
25.0	-60.0	-0.138638036945788E 05	-0.163841413058517E 05	3
30.0	-60.0	0.226423045537542E 06	-0.752236282122231E 05	3
40.0	-60.0	-0.276939913381966E 08	-0.131340690983886E 08	3
50.0	-60.0	0.144801894531719E 10	0.380099335560014E 10	3
75.0	-60.0	-0.215590057027040E 14	-0.890542624880827E 15	3
100.0	-60.0	-0.643080421887438E 20	0.196724241684648E 21	3

AND ORDER N = M+1 = 1

RHO	ANG	RE I	IM I	METHOD
0.1	-60.0	0.249375130235453E-01	-0.433012476411755E-01	1
0.5	-60.0	0.117228400708627E 00	-0.216436240685357E 00	1
1.0	-60.0	0.188828534743413E 00	-0.430804407423258E 00	1
1.5	-60.0	0.174387857582875E 00	-0.633195425326950E 00	1
2.0	-60.0	0.447975866038961E-01	-0.799860900592474E 00	1
2.5	-60.0	-0.210366347877049E 00	-0.890855053772563E 00	1
2.5	-60.0	-0.210366347877049E 00	-0.890855053772563E 00	2
3.0	-60.0	-0.574590706638979E 00	-0.852941735217775E 00	2
5.0	-60.0	-0.154462726385877E 01	0.141571113811486E 01	2
10.0	-60.0	-0.573017405009029E 01	-0.174815980619618E 02	2
15.0	-60.0	0.183464045580592E 03	0.141953614488190E 02	2
20.0	-60.0	-0.873467366158400E 03	0.174014288024260E 04	2
21.0	-60.0	0.122050261384378E 04	0.288670348898403E 04	2
21.0	-60.0	0.122050261384378E 04	0.288670348898403E 04	3
25.0	-60.0	-0.140132778554757E 05	-0.159795501524473E 05	3
30.0	-60.0	0.223458193208724E 06	-0.778976641808554E 05	3
40.0	-60.0	-0.276651089568002E 08	-0.127508522875685E 08	3
50.0	-60.0	0.147389856943119E 10	0.376948439914239E 10	3
75.0	-60.0	-0.266460721975630E 14	-0.887459425151834E 15	3
100.0	-60.0	-0.632937118915518E 20	0.196512843420344E 21	3

OPT1= 2.0 MODIFIED BESSEL FUNCTIONS, SECOND KIND,
OPT2= 2.0 OF COMPLEX ARGUMENT IN POLAR COORDINATES
ORD = 0.0 OF ORDER M = 0

RHO	ANG	RE K	IM K	METHOD
0.1	-60.0	0.241650623557041E 01	0.103849173342117E 01	1
0.5	-60.0	0.807230998595527E 00	0.918001918050869E 00	1
1.0	-60.0	0.177214810618352E 00	0.688814732114711E 00	1
1.5	-60.0	-0.912065368802299E-01	0.452902344864404E 00	1
2.0	-60.0	-0.188740482377896E 00	0.252329276549754E 00	1
2.5	-60.0	-0.195154333181329E 00	0.103733171233773E 00	1
2.5	-60.0	-0.195154333181329E 00	0.103733171233773E 00	2
3.0	-60.0	-0.157652433648003E 00	0.797269697981873E-02	2
5.0	-60.0	0.551422492012627E-02	-0.450636343411048E-01	2
10.0	-60.0	-0.256987844620590E-02	0.659528211515844E-03	2
15.0	-60.0	0.105018051508056E-03	0.143985125031484E-03	2
20.0	-60.0	0.673763180530214E-05	-0.107450317178827E-04	2
21.0	-60.0	0.743003145931799E-05	-0.108084318129907E-05	2
21.0	-60.0	0.743003145931799E-05	-0.108084318129907E-05	3
25.0	-60.0	-0.916897023495287E-06	-0.165750212208903E-06	3
30.0	-60.0	0.140634499924458E-07	0.684190811475629E-07	3
40.0	-60.0	-0.335555064709345E-09	-0.231751110688451E-09	3
50.0	-60.0	0.242723567642160E-11	-0.390639179282618E-12	3
75.0	-60.0	-0.656988907321024E-17	0.358382051825935E-17	3
100.0	-60.0	0.161330574139091E-22	-0.179816316067128E-22	3

AND ORDER N = M+1 = 1

RHO	ANG	RE K	IM K	METHOD
0.1	-60.0	0.488192135337377E 01	0.876051491938466E 01	1
0.5	-60.0	0.625698893182445E 00	0.189253989694293E 01	1
1.0	-60.0	-0.221252868729644E-01	0.931067993198272E 00	1
1.5	-60.0	-0.220415164022503E 00	0.509187719029843E 00	1
2.0	-60.0	-0.262254649207587E 00	0.248894357140429E 00	1
2.5	-60.0	-0.231935295671493E 00	0.837921802884959E-01	1
2.5	-60.0	-0.231935295671493E 00	0.837921802884959E-01	2
3.0	-60.0	-0.172422915334883E 00	-0.123560067375051E-01	2
5.0	-60.0	0.951455588394161E-02	-0.469377299793186E-01	2
10.0	-60.0	-0.266329185211349E-02	0.567821450125989E-03	2
15.0	-60.0	0.102706222772161E-03	0.149401231490252E-03	2
20.0	-60.0	0.705254312001200E-05	-0.107368057372259E-04	2
21.0	-60.0	0.754146309656039E-05	-0.942462905344265E-06	2
21.0	-60.0	0.754146309656040E-05	-0.942462905344274E-06	3
25.0	-60.0	-0.923308357907071E-06	-0.183145683005242E-06	3
30.0	-60.0	0.132022305819469E-07	0.691949830356485E-07	3
40.0	-60.0	-0.335171635472393E-09	-0.236817972602643E-09	3
50.0	-60.0	0.244279627729629E-11	-0.371686318707572E-12	3
75.0	-60.0	-0.661248202176921E-17	0.355800048753178E-17	3
100.0	-60.0	0.162511570102619E-22	-0.179570122720726E-22	3

OPT1= 1.0 ORDINARY BESSEL FUNCTIONS, FIRST KIND,
OPT2= 1.0 OF COMPLEX ARGUMENT IN RECTANGULR COORDINATES
ORD = 0.0 OF ORDER M = 0

RZ	CZ	RE J	IM J	METHOD
0.1	0.1	0.999998437500068E 00	-0.249999956597223E-02	1
0.4	0.4	0.999023463990838E 00	-0.624932183821995E-01	1
0.7	0.7	0.984381781213087E 00	-0.249566040036660E 00	1
1.1	1.1	0.921072183546256E 00	-0.557560062303087E 00	1
1.4	1.4	0.751734182713808E 00	-0.972291627306661E 00	1
1.8	1.8	0.399968417129531E 00	-0.145718204415980E 01	1
1.8	1.8	0.399968417129532E 00	-0.145718204415980E 01	2
2.1	2.1	-0.221380249598694E 00	-0.193758678526605E 01	2
3.5	3.5	-0.623008247866636E 01	-0.116034381550200E 00	2
7.1	7.1	0.138840465941633E 03	-0.563704585539067E 02	2
10.6	10.6	-0.296725453463513E 04	0.295270788734412E 04	2
14.1	14.1	0.474893702650619E 05	-0.114775197360066E 06	2
14.8	14.8	-0.761556554206425E 05	-0.233697769382106E 06	2
14.8	14.8	-0.761556554206424E 05	-0.233697769382106E 06	3
17.7	17.7	0.979771694974212E 04	0.380878991144405E 07	3
21.2	21.2	-0.461176025779852E 08	-0.109955713182506E 09	3
28.3	28.3	-0.112596696872074E 12	-0.456281302856412E 11	3
35.4	35.4	-0.117623968512357E 15	0.501926462544625E 14	3
53.0	53.0	-0.357078660147134E 22	-0.344834075060094E 22	3
70.7	70.7	0.736870687809498E 29	-0.190691140936238E 30	3

AND ORDER N = M+1 = 1

RZ	CZ	RE J	IM J	METHOD
0.1	0.1	0.353995148150766E-01	0.353111264751009E-01	1
0.4	0.4	0.182243123755112E 00	0.171195179717015E 00	1
0.7	0.7	0.395868261019711E 00	0.307556631375537E 00	1
1.1	1.1	0.664865417959769E 00	0.367864989002090E 00	1
1.4	1.4	0.997077651926429E 00	0.299775437002033E 00	1
1.8	1.8	0.137309689764511E 01	0.386684439659503E-01	1
1.8	1.8	0.137309689764511E 01	0.386684439659508E-01	2
2.1	2.1	0.173264422112848E 01	-0.487454177016071E 00	2
3.5	3.5	-0.359776667766673E 00	-0.579790790179263E 01	2
7.1	7.1	0.594776104262634E 02	0.131878639175687E 03	2
10.6	10.6	-0.295486529135246E 04	-0.282609360425738E 04	2
14.1	14.1	0.113602518986651E 06	0.445843747040039E 05	2
14.8	14.8	0.228461290297083E 06	-0.788772751671598E 05	2
14.8	14.8	0.228461290297083E 06	-0.788772751671597E 05	3
17.7	17.7	-0.375480847314302E 07	0.643068143930479E 05	3
21.2	21.2	0.108110203693180E 09	-0.468857346720725E 08	3
28.3	28.3	0.442207220322677E 11	-0.112008564758730E 12	3
35.4	35.4	-0.506754590676279E 14	-0.116434867731361E 15	3
53.0	53.0	0.341517291263255E 22	-0.357028750862372E 22	3
70.7	70.7	0.190278413904035E 30	0.727499568271638E 29	3

OPT1= 1.0 ORDINARY BESSEL FUNCTIONS, SECOND KIND,
OPT2= 1.0 OF COMPLEX ARGUMENT IN RECTANGULR COORDINATES
ORD = 0.0 OF ORDER M = 0

RZ	CZ	RE Y	IM Y	METHOD
0.1	0.1	-0.153842159526420E 01	0.505439955738290E 00	1
0.4	0.4	-0.482393383094042E 00	0.571481278133628E 00	1
0.7	0.7	0.670431986996671E-01	0.669258408389366E 00	1
1.1	1.1	0.523860648455003E 00	0.710099216086656E 00	1
1.4	1.4	0.998816080719742E 00	0.622882297646259E 00	1
1.8	1.8	0.150154678540620E 01	0.329497091683066E 00	1
1.8	1.8	0.150154678540620E 01	0.329497091683067E 00	2
2.1	2.1	0.198025892051388E 01	-0.253925451783052E 00	2
3.5	3.5	0.123362974699498E 00	-0.622296023988910E 01	2
7.1	7.1	0.563703761330810E 02	0.138840270165412E 03	2
10.6	10.6	-0.295270788733448E 04	-0.296725452956579E 04	2
14.1	14.1	0.114775197360115E 06	0.474893702649435E 05	2
14.8	14.8	0.233697769382161E 06	-0.761556554206704E 05	2
14.8	14.8	0.233697769382161E 06	-0.761556554206703E 05	3
17.7	17.7	-0.380878991144405E 07	0.979771694974448E 04	3
21.2	21.2	0.109955713182506E 09	-0.461176025779852E 08	3
28.3	28.3	0.456281302856412E 11	-0.112596696872074E 12	3
35.4	35.4	-0.501926462544625E 14	-0.117623968512357E 15	3
53.0	53.0	0.344834075060094E 22	-0.357078660147134E 22	3
70.7	70.7	0.190691140936238E 30	0.736870687809498E 29	3

AND ORDER N = M+1 = 1

RZ	CZ	RE Y	IM Y	METHOD
0.1	0.1	-0.458503001245842E 01	0.445369489632128E 01	1
0.4	0.4	-0.114038728983511E 01	0.851446423687312E 00	1
0.7	0.7	-0.778860430737329E 00	0.549927678082084E 00	1
1.1	1.1	-0.633363626891833E 00	0.664223271700701E 00	1
1.4	1.4	-0.446711055312949E 00	0.946116622516332E 00	1
1.8	1.8	-0.113316018485763E 00	0.131369149508362E 01	1
1.8	1.8	-0.113316018485764E 00	0.131369149508362E 01	2
2.1	2.1	0.455687927670856E 00	0.168154261033741E 01	2
3.5	3.5	0.580601677642325E 01	-0.367147294143033E 00	2
7.1	7.1	-0.131878844677735E 03	0.594776890612845E 02	2
10.6	10.6	0.282609360944631E 04	-0.295486529124553E 04	2
14.1	14.1	-0.445843747041235E 05	0.113602518986598E 06	2
14.8	14.8	0.788772751671323E 05	0.228461290297026E 06	2
14.8	14.8	0.788772751671322E 05	0.228461290297026E 06	3
17.7	17.7	-0.643068143930455E 05	-0.375480847314301E 07	3
21.2	21.2	0.468857346720725E 08	0.108110203693180E 09	3
28.3	28.3	0.112008564758730E 12	0.442207220322677E 11	3
35.4	35.4	0.116434867731361E 15	-0.506754590676279E 14	3
53.0	53.0	0.357028750862372E 22	0.341517291263255E 22	3
70.7	70.7	-0.727499568271638E 29	0.190278413904035E 30	3

OPT1= 2.0
OPT2= 1.0
ORD = 0.0

MODIFIED BESSEL FUNCTIONS, FIRST KIND,
OF COMPLEX ARGUMENT IN RECTANGULR COORDINATES
OF ORDER M = 0

RZ	CZ	RE I	IM I	METHOD
0.1	0.1	0.999998437500068E 00	0.249999956597223E-02	1
0.4	0.4	0.999023463990838E 00	0.624932183821995E-01	1
0.7	0.7	0.984381781213087E 00	0.249566040036660E 00	1
1.1	1.1	0.921072183546256E 00	0.557560062303087E 00	1
1.4	1.4	0.751734182713808E 00	0.972291627306661E 00	1
1.8	1.8	0.399968417129531E 00	0.145718204415980E 01	1
1.8	1.8	0.399968417129531E 00	0.145718204415980E 01	2
2.1	2.1	-0.221380249598694E 00	0.193758678526605E 01	2
3.5	3.5	-0.623008247866636E 01	0.116034381550200E 00	2
7.1	7.1	0.138840465941633E 03	0.563704585539067E 02	2
10.6	10.6	-0.296725453463513E 04	-0.295270788734412E 04	2
14.1	14.1	0.474893702650618E 05	0.114775197360066E 06	2
14.8	14.8	-0.761556554206425E 05	0.233697769382106E 06	2
14.8	14.8	-0.761556554206424E 05	0.233697769382106E 06	3
17.7	17.7	0.979771694974212E 04	-0.380878991144405E 07	3
21.2	21.2	-0.461176025779852E 08	0.109955713182506E 09	3
28.3	28.3	-0.112596696872074E 12	0.456281302856411E 11	3
35.4	35.4	-0.117623968512357E 15	-0.501926462544625E 14	3
53.0	53.0	-0.357078660147134E 22	0.344834075060094E 22	3
70.7	70.7	0.736870687809498E 29	0.190691140936238E 30	3

AND ORDER N = M+1 = 1

RZ	CZ	RE I	IM I	METHOD
0.1	0.1	0.353111264751009E-01	0.353995148150766E-01	1
0.4	0.4	0.171195179717015E 00	0.182243123755112E 00	1
0.7	0.7	0.307556631375537E 00	0.395868261019711E 00	1
1.1	1.1	0.367864989002090E 00	0.664865417959769E 00	1
1.4	1.4	0.299775437002033E 00	0.997077651926429E 00	1
1.8	1.8	0.386684439659503E-01	0.137309689764511E 01	1
1.8	1.8	0.386684439659503E-01	0.137309689764511E 01	2
2.1	2.1	-0.487454177016071E 00	0.173264422112848E 01	2
3.5	3.5	-0.579790790179263E 01	-0.359776666776673E 00	2
7.1	7.1	0.131878639175687E 03	0.594776104262634E 02	2
10.6	10.6	-0.282609360425738E 04	-0.295486529135246E 04	2
14.1	14.1	0.445843747040038E 05	0.113602518986650E 06	2
14.8	14.8	-0.788772751671599E 05	0.228461290297083E 06	2
14.8	14.8	-0.788772751671597E 05	0.228461290297083E 06	3
17.7	17.7	0.643068143930531E 05	-0.375480847314302E 07	3
21.2	21.2	-0.468857346720726E 08	0.108110203693180E 09	3
28.3	28.3	-0.112008564758731E 12	0.442207220322675E 11	3
35.4	35.4	-0.116434867731361E 15	-0.506754590676281E 14	3
53.0	53.0	-0.357028750862373E 22	0.341517291263255E 22	3
70.7	70.7	0.727499568271636E 29	0.190278413904035E 30	3

OPT1= 2.0 MODIFIED BESSEL FUNCTIONS, SECOND KIND,
OPT2= 1.0 OF COMPLEX ARGUMENT IN RECTANGULR COORDINATES
ORD = 0.0 OF ORDER M = 0

RZ	CZ	RE K	IM K	METHOD
0.1	0.1	0.242047398103817E 01	-0.776850646536661E 00	1
0.4	0.4	0.855905872118634E 00	-0.671581695094368E 00	1
0.7	0.7	0.286706208728316E 00	-0.494994636518720E 00	1
1.1	1.1	0.529349154877105E-01	-0.331395562338559E 00	1
1.4	1.4	-0.416645139915095E-01	-0.202400067764704E 00	1
1.8	1.8	-0.696879725890451E-01	-0.110696099155675E 00	1
1.8	1.8	-0.696879725890454E-01	-0.110696099155675E 00	2
2.1	2.1	-0.670292333037986E-01	-0.511218840459867E-01	2
3.5	3.5	-0.115117271994907E-01	0.111875865098696E-01	2
7.1	7.1	0.129466330214806E-03	-0.307524569088144E-03	2
10.6	10.6	-0.151434720734704E-07	0.796289439837763E-05	2
14.1	14.1	-0.771523310986096E-07	-0.185894151111944E-06	2
14.8	14.8	-0.863603016230482E-07	-0.438782975284030E-07	2
14.8	14.8	-0.863603016230483E-07	-0.438782975284031E-07	3
17.7	17.7	0.372329131364329E-08	0.370270353525263E-08	3
21.2	21.2	-0.129382693760208E-09	-0.528999660662834E-10	3
28.3	28.3	-0.947481164909944E-13	0.401108139940073E-13	3
35.4	35.4	-0.291507708939681E-16	0.725581322036560E-16	3
53.0	53.0	-0.134279063473278E-23	0.234542909132694E-25	3
70.7	70.7	-0.989841799673071E-32	-0.223653552604146E-31	3

AND ORDER N = M+1 = 1

RZ	CZ	RE K	IM K	METHOD
0.1	0.1	0.694024215596480E 01	-0.714668171405197E 01	1
0.4	0.4	0.105118208541252E 01	-0.152240340653209E 01	1
0.7	0.7	0.241995966429738E 00	-0.740322276841983E 00	1
1.1	1.1	-0.100868098500986E-02	-0.417044285166257E 00	1
1.4	1.4	-0.800493978070668E-01	-0.230805929518123E 00	1
1.8	1.8	-0.933137881353576E-01	-0.117256135859871E 00	1
1.8	1.8	-0.933137881353575E-01	-0.117256135859871E 00	2
2.1	2.1	-0.802702225239221E-01	-0.498983077875148E-01	2
3.5	3.5	-0.115777543932525E-01	0.127373904842186E-01	2
7.1	7.1	0.123519602311802E-03	-0.322801862589603E-03	2
10.6	10.6	0.167969487368452E-06	0.815074977761545E-05	2
14.1	14.1	-0.817455627943058E-07	-0.187837873182164E-06	2
14.8	14.8	-0.885402180189412E-07	-0.431863574625504E-07	2
14.8	14.8	-0.885402180189413E-07	-0.431863574625506E-07	3
17.7	17.7	0.382757174164956E-08	0.370311686906463E-08	3
21.2	21.2	-0.131523347037960E-09	-0.520159950583746E-10	3
28.3	28.3	-0.952340010546482E-13	0.412954857636697E-13	3
35.4	35.4	-0.288473914324528E-16	0.732758366983542E-16	3
53.0	53.0	-0.134901028899142E-23	0.298652678713081E-25	3
70.7	70.7	-0.100122092144351E-31	-0.224095534616114E-31	3

OPT1= 1.0
OPT2= 1.0
ORD = 10.0

ORDINARY BESSEL FUNCTIONS, FIRST KIND,
OF COMPLEX ARGUMENT IN RECTANGULR COORDINATES
OF ORDER M = 10

RZ	CZ	RE J	IM J	METHOD
0.1	0.1	0.611623735795193E-23	0.269114439175657E-19	1
0.4	0.4	0.149321578946547E-14	0.262803187137177E-12	1
0.7	0.7	0.611582900226281E-11	0.269050736563710E-09	1
1.1	1.1	0.793291168863320E-09	0.154998986606129E-07	1
1.4	1.4	0.250253496844590E-07	0.274529832271821E-06	1
1.8	1.8	0.363605823725003E-06	0.254276774651378E-05	1
1.8	1.8	0.363605823725002E-06	0.254276774651378E-05	2
2.1	2.1	0.323286010775992E-05	0.155869179424557E-04	2
3.5	3.5	0.143148233854888E-02	0.224612826316871E-02	2
7.1	7.1	0.260753673517600E 01	-0.200930838256087E 01	2
10.6	10.6	-0.360629894321824E 03	0.512097391695860E 01	2
14.1	14.1	0.200843431588253E 05	0.345354688032786E 04	2
14.8	14.8	0.396725751352578E 05	-0.192829979288223E 05	2
14.8	14.8	0.396725751352578E 05	-0.192829979288223E 05	2
17.7	17.7	-0.899667994150609E 06	-0.119971051178568E 06	2
21.2	21.2	0.362676922167992E 08	-0.800522552376349E 06	2
28.3	28.3	0.436003211045719E 11	-0.243947378381352E 11	2
35.4	35.4	0.275088840093355E 14	-0.565730622375762E 14	2
53.0	53.0	0.296430630646120E 22	0.893512183384775E 21	3
70.7	70.7	-0.186106987046289E 28	0.143497330272958E 30	3

AND ORDER N = M+1 = 11

RZ	CZ	RE J	IM J	METHOD
0.1	0.1	-0.864786369307638E-22	0.865146772050578E-22	1
0.4	0.4	-0.420141963093843E-14	0.424541394927196E-14	1
0.7	0.7	-0.846774215861799E-11	0.882812427644266E-11	1
1.1	1.1	-0.712355794179662E-09	0.782476876590248E-09	1
1.4	1.4	-0.161829028714864E-07	0.191326190411046E-07	1
1.8	1.8	-0.177819866294338E-06	0.231404053791075E-06	1
1.8	1.8	-0.177819866294338E-06	0.231404053791075E-06	2
2.1	2.1	-0.122148116626401E-05	0.179341852848689E-05	2
3.5	3.5	-0.158117262649281E-03	0.582452210326785E-03	2
7.1	7.1	0.136841020207753E 01	0.428692355897700E 00	2
10.6	10.6	-0.981018993460347E 02	-0.187113550842524E 03	2
14.1	14.1	0.267592047931165E 04	0.135467130244025E 05	2
14.8	14.8	0.219101213849011E 05	0.212096147729540E 05	2
14.8	14.8	0.219101213849011E 05	0.212096147729540E 05	2
17.7	17.7	-0.108790250971704E 06	-0.659260162652377E 06	2
21.2	21.2	0.751892386195086E 07	0.271496911372131E 08	2
28.3	28.3	0.265741231004139E 11	0.317511568408991E 11	2
35.4	35.4	0.516957356649574E 14	0.161660714817601E 14	2
53.0	53.0	-0.538102020742943E 21	0.275117597780780E 22	3
70.7	70.7	-0.132968692261500E 30	0.821048825501880E 28	3

OPT1= 1.0 ORDINARY BESSEL FUNCTIONS, SECOND KIND,
OPT2= 1.0 OF COMPLEX ARGUMENT IN RECTANGULR COORDINATES
ORD = 10.0 OF ORDER M = 10

RZ	CZ	RE Y	IM Y	METHOD
0.1	0.1	-0.328556911928389E 15	0.118280485360633E 19	1
0.4	0.4	-0.841095931765907E 09	0.121115936722483E 12	1
0.7	0.7	-0.328495808907082E 07	0.118229159841426E 09	1
1.1	1.1	-0.128077133173387E 06	0.204666053868917E 07	1
1.4	1.4	-0.127961209612894E 05	0.114707739489792E 06	1
1.8	1.8	-0.213764841091431E 04	0.121933464087239E 05	1
1.8	1.8	-0.213764841091430E 04	0.121933464087239E 05	2
2.1	2.1	-0.493290554806983E 03	0.193337081633117E 04	2
3.5	3.5	-0.749484973061489E 01	0.906730888914586E 01	2
7.1	7.1	0.200525054963510E 01	0.260049969281913E 01	2
10.6	10.6	-0.512092714723398E 01	-0.360629863015010E 03	2
14.1	14.1	-0.345354688100589E 04	0.200843431584614E 05	2
14.8	14.8	0.192829979286764E 05	0.396725751349513E 05	2
14.8	14.8	0.192829979286764E 05	0.396725751349513E 05	2
17.7	17.7	0.119971051178579E 06	-0.899667994150602E 06	2
21.2	21.2	0.800522552376349E 06	0.362676922167992E 08	2
28.3	28.3	0.243947378381352E 11	0.436003211045719E 11	2
35.4	35.4	0.565730622375762E 14	0.275088840093355E 14	2
53.0	53.0	-0.893512183384775E 21	0.296430630646120E 22	3
70.7	70.7	-0.143497330272958E 30	-0.186106987046289E 28	3

AND ORDER N = M+1 = 11

RZ	CZ	RE Y	IM Y	METHOD
0.1	0.1	0.167232049545260E 21	0.167315686480960E 21	1
0.4	0.4	0.340428373010103E 13	0.344710545459303E 13	1
0.7	0.7	0.163034556138586E 10	0.171397039894544E 10	1
1.1	1.1	0.182175904322358E 08	0.203915862487474E 08	1
1.4	1.4	0.730748258078004E 06	0.893723967252240E 06	1
1.8	1.8	0.583109063431023E 05	0.801122800449983E 05	1
1.8	1.8	0.583109063431023E 05	0.801122800449983E 05	2
2.1	2.1	0.707922405142849E 04	0.112789237077249E 05	2
3.5	3.5	0.767591714986101E 01	0.467986544456886E 02	2
7.1	7.1	-0.446373214768201E 00	0.136768936673261E 01	2
10.6	10.6	0.187113631238629E 03	-0.981019486384826E 02	2
14.1	14.1	-0.135467130252376E 05	0.267592048007010E 04	2
14.8	14.8	-0.212096147734389E 05	0.219101213849584E 05	2
14.8	14.8	-0.212096147734389E 05	0.219101213849584E 05	2
17.7	17.7	0.659260162652391E 06	-0.108790250971716E 06	2
21.2	21.2	-0.271496911372131E 08	0.751892386195086E 07	2
28.3	28.3	-0.317511568408991E 11	0.265741231004139E 11	2
35.4	35.4	-0.161660714817601E 14	0.516957356649574E 14	2
53.0	53.0	-0.275117597780780E 22	-0.538102020742943E 21	3
70.7	70.7	-0.821048825501880E 28	-0.132968692261500E 30	3

OPT1= 2.0
OPT2= 1.0
ORD = 10.0

MODIFIED BESSEL FUNCTIONS, FIRST KIND,
OF COMPLEX ARGUMENT IN RECTANGULR COORDINATES
OF ORDER M = 10

RZ	CZ	RE I	IM I	METHOD
0.1	0.1	-0.611623735794651E-23	0.269114439175657E-19	1
0.4	0.4	-0.149321578946542E-14	0.262803187137177E-12	1
0.7	0.7	-0.611582900226275E-11	0.269050736563710E-09	1
1.1	1.1	-0.793291168863321E-09	0.154998986606129E-07	1
1.4	1.4	-0.250253496844590E-07	0.274529832271821E-06	1
1.8	1.8	-0.363605823725003E-06	0.254276774651378E-05	1
1.8	1.8	-0.363605823725004E-06	0.254276774651378E-05	2
2.1	2.1	-0.323286010775993E-05	0.155869179424557E-04	2
3.5	3.5	-0.143148233854888E-02	0.224612826316871E-02	2
7.1	7.1	-0.260753673517600E 01	-0.200930838256087E 01	2
10.6	10.6	0.360629894321825E 03	0.512097391695863E 01	2
14.1	14.1	-0.200843431588253E 05	0.345354688032786E 04	2
14.8	14.8	-0.396725751352578E 05	-0.192829979288223E 05	2
14.8	14.8	-0.396725751352578E 05	-0.192829979288223E 05	2
17.7	17.7	0.899667994150609E 06	-0.119971051178568E 06	2
21.2	21.2	-0.362676922167992E 08	-0.800522552376370E 06	2
28.3	28.3	-0.436003211045722E 11	-0.243947378381354E 11	2
35.4	35.4	-0.275088840093355E 14	-0.565730622375763E 14	2
53.0	53.0	-0.296430630646120E 22	0.893512183384779E 21	3
70.7	70.7	0.186106987046308E 28	0.143497330272958E 30	3

AND ORDER N = M+1 = 11

RZ	CZ	RE I	IM I	METHOD
0.1	0.1	-0.865146772050577E-22	0.864786369307638E-22	1
0.4	0.4	-0.424541394927196E-14	0.420141963093844E-14	1
0.7	0.7	-0.882812427644266E-11	0.846774215861799E-11	1
1.1	1.1	-0.782476876590248E-09	0.712355794179662E-09	1
1.4	1.4	-0.191326190411046E-07	0.161829028714864E-07	1
1.8	1.8	-0.231404053791075E-06	0.177819866294338E-06	1
1.8	1.8	-0.231404053791075E-06	0.177819866294338E-06	2
2.1	2.1	-0.179341852848689E-05	0.122148116626401E-05	2
3.5	3.5	-0.582452210326785E-03	0.158117262649281E-03	2
7.1	7.1	-0.428692355897700E 00	-0.136841020207753E 01	2
10.6	10.6	0.187113550842525E 03	0.981018993460347E 02	2
14.1	14.1	-0.135467130244029E 05	-0.267592047931165E 04	2
14.8	14.8	-0.212096147729540E 05	-0.219101213849011E 05	2
14.8	14.8	-0.212096147729540E 05	-0.219101213849011E 05	2
17.7	17.7	0.659260162652377E 06	0.108790250971703E 06	2
21.2	21.2	-0.271496911372131E 08	-0.751892386195088E 07	2
28.3	28.3	-0.317511568408993E 11	-0.265741231004141E 11	2
35.4	35.4	-0.161660714817602E 14	-0.516957356649575E 14	2
53.0	53.0	-0.275117597780780E 22	0.538102020742943E 21	3
70.7	70.7	-0.821048825501880E 28	0.132968692261500E 30	3

OPT1= 2.0
OPT2= 1.0
ORD = 10.0

MODIFIED BESSEL FUNCTIONS, SECOND KIND,
OF COMPLEX ARGUMENT IN RECTANGULR COORDINATES
OF ORDER N = 10

RZ	CZ	RE K	IM K	METHOD
0.1	0.1	-0.516095990399813E 15	-0.185794551936000E 19	1
0.4	0.4	-0.132119040009998E 10	-0.190248468520000E 12	1
0.7	0.7	-0.516000009998758E 07	-0.185713929998958E 09	1
1.1	1.1	-0.201183090335154E 06	-0.321488685636900E 07	1
1.4	1.4	-0.201000998027854E 05	-0.180182495845472E 06	1
1.8	1.8	-0.335781026784897E 04	-0.191532637495901E 05	1
1.8	1.8	-0.335781026784898E 04	-0.191532637495901E 05	2
2.1	2.1	-0.774858967049554E 03	-0.303693177154728E 04	2
3.5	3.5	-0.117693542167043E 02	-0.142406469297858E 02	2
7.1	7.1	-0.637402905454530E-02	0.110537602856658E-01	2
10.6	10.6	0.734657116508621E-04	-0.491766297663483E-04	2
14.1	14.1	-0.106504575261069E-05	0.571702783836276E-06	2
14.8	14.8	-0.229247513385171E-06	0.481438718693554E-06	2
14.8	14.8	-0.229247513385171E-06	0.481438718693554E-06	2
17.7	17.7	0.183955157448295E-07	-0.118850629481139E-07	2
21.2	21.2	-0.334464394276640E-09	0.312993346829909E-09	2
28.3	28.3	-0.754000741451816E-13	0.238316979671642E-12	2
35.4	35.4	0.489087772242371E-16	0.151190321838016E-15	2
53.0	53.0	-0.190604711334409E-23	0.100145990940450E-23	3
70.7	70.7	-0.241893536263197E-31	-0.250740217542298E-31	3

AND ORDER N = N+1 = 11

RZ	CZ	RE K	IM K	METHOD
0.1	0.1	-0.262818865739459E 21	-0.262687489148076E 21	1
0.4	0.4	-0.541470058614938E 13	-0.534743637861033E 13	1
0.7	0.7	-0.269229840689868E 10	-0.256094081923128E 10	1
1.1	1.1	-0.320310287770538E 08	-0.286161241340099E 08	1
1.4	1.4	-0.140385832492841E 07	-0.114785667960073E 07	1
1.8	1.8	-0.125840075226127E 06	-0.915945574961898E 05	1
1.8	1.8	-0.125840075226127E 06	-0.915945574961897E 05	2
2.1	2.1	-0.177168919322129E 05	-0.111290191393590E 05	2
3.5	3.5	-0.735114028722467E 02	-0.120582173775761E 02	2
7.1	7.1	0.113228551202426E-02	0.277730281683614E-01	2
10.6	10.6	0.774283960722704E-04	-0.126285905934596E-03	2
14.1	14.1	-0.119137177633636E-05	0.131177628077950E-05	2
14.8	14.8	-0.900523327059640E-07	0.761733802564592E-06	2
14.8	14.8	-0.900523327059640E-07	0.761733802564592E-06	2
17.7	17.7	0.195023044452420E-07	-0.223532054667289E-07	2
21.2	21.2	-0.323768271062867E-09	0.492078441502813E-09	2
28.3	28.3	-0.379461141532566E-13	0.299085092924960E-12	2
35.4	35.4	0.816159354734579E-16	0.165486300927510E-15	2
53.0	53.0	-0.198702942340726E-23	0.130614544118130E-23	3
70.7	70.7	-0.279722629479767E-31	-0.250210917135198E-31	3

```
RUN ERROR IN BESSEL FUNCTION SUBRCUTINE

PHI =     0.07C71C68    CHI =     0.07071068    ORD =  50.0    OPT1 =   1.0    CPT2 =   1.0

CONSTANT TERM CF FINITE SERIES WILL EXCEED NUMBER SIZE OF MACHINE

M =  0.50COE 02  N =  0.51COE 02  CMR1 =  0.8848E-77  CMI1 = -0.2135E-93

PHI = 0.70710678E-01  CHI = 0.7C710678E-01
```

(*First argument in following two tables.*)

```
PHI =     0.70710678    CHI =     C.7C710678    ORD =  50.0    OPT1 =   1.0    CPT2 =   1.0

CONSTANT TERM OF FINITE SERIES WILL EXCEED NUMBER SIZE OF MACHINE

M =  0.5CCOE C2  N =  0.51COE 02  CMR1 =  0.2862E-75  CMI1 = -0.5553E-91

PHI = 0.70710678E 00  CHI = 0.7C71C678E 00
```

(*Second argument in following two tables.*)

```
PHI =     0.35355339    CHI =     0.35355339    ORD =  50.0    OPT1 =   1.0    CPT2 =   1.0

CONSTANT TERM OF FINITE SERIES WILL EXCEED NUMBER SIZE OF MACHINE

M =  0.5CCOE 02  N =  0.51COE 02  CMR1 =  0.6209E-92  CMI1 =  0.3680E-76

PHI = C.35355339E C0  CHI = 0.35355339E 00
```

(*Third argument in following two tables.*)

* *Note that first three lines of tables for ordinary Bessel functions of first and second kinds for both order fifty and fifty-one are incorrect.*

OPT1= 1.0 ORDINARY BESSEL FUNCTIONS, FIRST KIND,
OPT2= 1.0 OF COMPLEX ARGUMENT IN RECTANGULR COORDINATES
ORD = 50.0 OF ORDER M = 50

RZ	CZ	RE J	IM J	METHOD
0.1	0.1	-0.429252413337410E 29	-0.245786600037392E 29	1
0.4	0.4	-0.429252413337410E 29	-0.245786600037392E 29	1
0.7	0.7	-0.429252413337410E 29	-0.245786600037392E 29	1
1.1	1.1	0.205367793874257E-72	0.186192588244278E-70	1
1.4	1.4	0.644656977282852E-66	0.328732953471727E-64	1
1.8	1.8	0.705686006626560E-61	0.230263892760918E-59	1
1.8	1.8	0.705686006626555E-61	0.230263892760918E-59	2
2.1	2.1	0.924628879614436E-57	0.209446652302951E-55	2
3.5	3.5	0.317109274379212E-45	0.257465432802150E-44	2
7.1	7.1	0.137797334215050E-29	0.258239215245509E-29	2
10.6	10.6	0.168119542102589E-20	0.850142387367874E-21	2
14.1	14.1	0.315755184582353E-14	-0.129119382400413E-14	2
14.8	14.8	0.327958898809374E-13	-0.218894332496436E-13	2
14.8	14.8	0.327958898809374E-13	-0.218894332496436E-13	2
17.7	17.7	0.213221956077210E-10	-0.251040981006069E-09	2
21.2	21.2	-0.239155530076743E-05	-0.794765583796044E-06	2
28.3	28.3	0.645328508960991E 01	0.768054357860994E 00	2
35.4	35.4	-0.597129414019722E 06	0.732890427471766E 06	2
53.0	53.0	-0.884780393070384E 16	0.239624979589022E 17	2
70.7	70.7	0.220221917163971E 26	-0.115126332010885E 26	2

AND ORDER N = M+1 = 51

RZ	CZ	RE J	IM J	METHOD
0.1	0.1	0.156591141253776E 29	-0.397736223240534E 29	1
0.4	0.4	0.156591141253776E 29	-0.397736223240534E 29	1
0.7	0.7	0.156591141253776E 29	-0.397736223240534E 29	1
1.1	1.1	-0.191520726286526E-72	0.195709680015350E-72	1
1.4	1.4	-0.447019986626318E-66	0.464552384364386E-66	1
1.8	1.8	-0.387083258256211E-61	0.411073551019486E-61	1
1.8	1.8	-0.387083258256211E-61	0.411073551019486E-61	2
2.1	2.1	-0.416747155004580E-57	0.454467468393452E-57	2
3.5	3.5	-0.784866109304612E-46	0.100049087662271E-45	2
7.1	7.1	-0.860678552299070E-31	0.273714532454698E-30	2
10.6	10.6	0.807708282446699E-22	0.264821799259687E-21	2
14.1	14.1	0.605413755092633E-15	0.281155310376800E-15	2
14.8	14.8	0.786895858472873E-14	0.191073133908328E-14	2
14.8	14.8	0.786895858472873E-14	0.191073133908328E-14	2
17.7	17.7	0.491969186858778E-10	-0.368108898310817E-10	2
21.2	21.2	-0.273202649735711E-06	-0.680980779321034E-06	2
28.3	28.3	0.124071168374831E 01	0.213995744670417E 01	2
35.4	35.4	-0.432246897728040E 06	-0.449719518074126E 05	2
53.0	53.0	-0.153566473930436E 17	0.131172197791362E 16	2
70.7	70.7	0.125916766525331E 26	0.116420408670867E 26	2

OPT1= 1.0
OPT2= 1.0
ORD = 50.0

ORDINARY BESSEL FUNCTIONS, SECOND KIND,
OF COMPLEX ARGUMENT IN RECTANGULR COORDINATES
OF ORDER M = 50

RZ	CZ	RE Y	IM Y	METHOD
0.1	0.1	C.2457866C0037392E 29	-0.429252413337410E 29	1
0.4	0.4	C.2457866C0037392E 29	-0.429252413337410E 29	1
0.7	0.7	C.2457866C0037392E 29	-0.429252413337410E 29	1
1.1	1.1	-C.392471496102547E 67	0.341871276189906E 69	1
1.4	1.4	-C.395118C14323426E 61	0.193580971327639E 63	1
1.8	1.8	-C.881048786827628E 56	0.276203327357115E 58	1
1.8	1.8	-C.881048786827623E 56	0.276203327357115E 58	2
2.1	2.1	-C.139385C01129492E 53	0.303336372898177E 54	2
3.5	3.5	-C.312165264059983E 42	0.243410074977057E 43	2
7.1	7.1	-C.106165406799218E 28	0.189725013220189E 28	2
10.6	10.6	-C.307477080428418E 19	0.138523368119331E 19	2
14.1	14.1	-C.165539145232948E 13	-0.835742580043550E 12	2
14.8	14.8	-C.124998394780933E 12	-0.100230612660254F 12	2
14.8	14.8	-C.124998394780933E 12	-0.100230612660254E 12	2
17.7	17.7	C.940255462936916E 06	-0.248695543948999E 08	2
21.2	21.2	C.242338C63679547E 04	-0.361391704647179E 03	2
28.3	28.3	-C.768940931406652E 0C	0.645313639315310E 01	2
35.4	35.4	-C.732890427471765E 06	-0.597129414019717E 06	2
53.0	53.0	-C.239624979589022E 17	-0.884780393070384E 16	2
70.7	70.7	C.115126332C10885E 26	0.220221917163971E 26	2

AND ORDER N = M+1 = 51

RZ	CZ	RE Y	IM Y	METHOD
0.1	0.1	C.397736223240534E 29	0.156591141253776E 29	1
0.4	0.4	C.397736223240534E 29	0.156591141253776E 29	1
0.7	0.7	C.397736223240534E 29	0.156591141253776E 29	1
1.1	1.1	C.159346971210087E 71	0.162973226285750E 71	1
1.4	1.4	C.670727726271899E 64	0.698108090311364E 64	1
1.8	1.8	C.756815600549577E 59	0.805658812966017E 59	1
1.8	1.8	C.756815600549577E 59	0.805658812966017E 59	2
2.1	2.1	C.682804702858506E 55	0.747198271234929E 55	2
3.5	3.5	C.301079781120135E 44	0.387617779937833E 44	2
7.1	7.1	C.612264371956562E 28	0.208646027017757E 29	2
10.6	10.6	-C.748676679649721E 19	0.212188192751660E 20	2
14.1	14.1	-C.870477840366334E 13	0.326105797824442E 13	2
14.8	14.8	-C.756230643295086E 12	0.117548073911384E 12	2
14.8	14.8	-C.756230643295086E 12	0.117548073911384E 12	2
17.7	17.7	-C.725776602160101E 08	-0.690246825091243E 08	2
21.2	21.2	C.430869344110093E 04	-0.705738676639261E 04	2
28.3	28.3	-C.214162965165847E 01	0.124233203354175E 01	2
35.4	35.4	C.449719518074244E 05	-0.432246897728036E 06	2
53.0	53.0	-C.131172197791362E 16	-0.153566473930436E 17	2
70.7	70.7	-C.116420408670867E 26	0.125916766525331E 26	2

```
RUN ERROR IN BESSEL FUNCTION SUBROUTINE

PHI =     0.07071068    CHI =     0.07071068    ORD =  50.0    OPT1 =   2.0    OPT2 =   1.0

CONSTANT TERM OF FINITE SERIES WILL EXCEED NUMBER SIZE OF MACHINE

M =  0.5000E 02  N =  0.5100E 02  CMR1 =  0.8848E-77  CMI1 = -0.2135E-93

PHI = 0.70710678E-01  CHI = 0.70710678E-01
```
(First argument in tables on next two pages.)

```
PHI =     0.35355339    CHI =     0.35355339    ORD =  50.0    OPT1 =   2.0    OPT2 =   1.0

CONSTANT TERM OF FINITE SERIES WILL EXCEED NUMBER SIZE OF MACHINE

M =  0.5000E 02  N =  0.5100E 02  CMR1 =  0.6209E-92  CMI1 =  0.3680E-76

PHI = 0.35355339E 00  CHI = 0.35355339E 00
```
(Second argument in tables on next two pages.)

```
PHI =     0.70710678    CHI =     0.70710678    ORD =  50.0    OPT1 =   2.0    OPT2 =   1.0

CONSTANT TERM OF FINITE SERIES WILL EXCEED NUMBER SIZE OF MACHINE

M =  0.5000E 02  N =  0.5100E 02  CMR1 =  0.2862E-75  CMI1 = -0.5553E-91

PHI = 0.70710678E 00  CHI = 0.70710678E 00
```
(Third argument in tables on next two pages.)

* *Note that first three lines of tables for modified Bessel functions of first and second kind for both order fifty and fifty-one are incorrect.*

OPT1= 2.0
OPT2= 1.0
ORD = 50.0

MODIFIED BESSEL FUNCTIONS, FIRST KIND,
OF COMPLEX ARGUMENT IN RECTANGULR COORDINATES
OF ORDER M = 50

RZ	CZ	RE I	IM I	METHOD
0.1	0.1	-0.429252413306806E 29	0.245786600059863E 29	1
0.4	0.4	-0.429252413306806E 29	0.245786600059863E 29	1
0.7	0.7	-0.429252413306806E 29	0.245786600059863E 29	1
1.1	1.1	-0.205367793874271E-72	0.186192588244278E-70	1
1.4	1.4	-0.644656977282865E-66	0.328732953471727E-64	1
1.8	1.8	-0.705686006626564E-61	0.230263892760918E-59	1
1.8	1.8	-0.705686006626571E-61	0.230263892760918E-59	2
2.1	2.1	-0.924628879614447E-57	0.209446652302951E-55	2
3.5	3.5	-0.317109274379212E-45	0.257465432802150E-44	2
7.1	7.1	-0.137797334215050E-29	0.258239215245509E-29	2
10.6	10.6	-0.168119542102589E-20	0.850142387367875E-21	2
14.1	14.1	-0.315755184582353E-14	-0.129119382400413E-14	2
14.8	14.8	-0.327958898809374E-13	-0.218894332496435E-13	2
14.8	14.8	-0.327958898809374E-13	-0.218894332496435E-13	2
17.7	17.7	-0.213221956072711E-10	-0.251040981006069E-09	2
21.2	21.2	0.239155530076743E-05	-0.794765583796045E-06	2
28.3	28.3	-0.645328508960991E 01	0.768054357860982E 00	2
35.4	35.4	0.597129414019719E 06	0.732890427471764E 06	2
53.0	53.0	0.884780393070385E 16	0.239624979589022E 17	2
70.7	70.7	-0.220221917163974E 26	-0.115126332010917E 26	2

AND ORDER N = M+1 = 51

RZ	CZ	RE I	IM I	METHOD
0.1	0.1	-0.397736223217170E 29	0.156591141276811E 29	1
0.4	0.4	-0.397736223217170E 29	0.156591141276811E 29	1
0.7	0.7	-0.397736223217170E 29	0.156591141276811E 29	1
1.1	1.1	-0.195709680015350E-72	0.191520726286526E-72	1
1.4	1.4	-0.464552384364386E-66	0.447019986626318E-66	1
1.8	1.8	-0.411073551019486E-61	0.387083258256210E-61	1
1.8	1.8	-0.411073551019486E-61	0.387083258256210E-61	2
2.1	2.1	-0.454467468393452E-57	0.416747155004579E-57	2
3.5	3.5	-0.100049087662271E-45	0.784866109304612E-46	2
7.1	7.1	-0.273714532454698E-30	0.860678552299069E-31	2
10.6	10.6	-0.264821799259687E-21	-0.807708282446698E-22	2
14.1	14.1	-0.281155310376800E-15	-0.605413755092633E-15	2
14.8	14.8	-0.191073133908328E-14	-0.786895858472873E-14	2
14.8	14.8	-0.191073133908328E-14	-0.786895858472873E-14	2
17.7	17.7	0.368108898310817E-10	-0.491969186858778E-10	2
21.2	21.2	0.680980779321034E-06	0.273202649735711E-06	2
28.3	28.3	-0.213995744670417E 01	-0.124071168374831E 01	2
35.4	35.4	0.449719518074120E 05	0.432246897728038E 06	2
53.0	53.0	-0.131172197791361E 16	0.153566473930437E 17	2
70.7	70.7	-0.116420408670861E 26	-0.125916766525353E 26	2

OPT1= 2.0 MODIFIED BESSEL FUNCTIONS, SECOND KIND,
OPT2= 1.0 OF COMPLEX ARGUMENT IN RECTANGULR COORDINATES
ORD = 50.0 OF ORDER N = 50

RZ	CZ	RE K	IM K	METHOD
0.1	0.1	-0.980163873578377E-31	0.245477787644502E-31	1
0.4	0.4	-0.980163873578377E-31	0.245477787644502E-31	1
0.7	0.7	-0.980163873578377E-31	0.245477787644502E-31	1
1.1	1.1	-0.616492784449619E 67	-0.537010144875788E 69	1
1.4	1.4	-0.620649925549742E 61	-0.304076278698843E 63	1
1.8	1.8	-0.138394819807594E 57	-0.433859172061085E 58	1
1.8	1.8	-0.138394819807596E 57	-0.433859172061085E 58	2
2.1	2.1	-0.218945447784511E 53	-0.476479660331743E 54	2
3.5	3.5	-0.490348050138380E 42	-0.382347651678832E 43	2
7.1	7.1	-0.166764231032898E 28	-0.298019353867387E 28	2
10.6	10.6	-0.482983868510577E 19	-0.217591997817103E 19	2
14.1	14.1	-0.260028281272681E 13	0.131278137487850E 13	2
14.8	14.8	-0.196347019377148E 12	0.157441878199128E 12	2
14.8	14.8	-0.196347019377148E 12	0.157441878199128E 12	2
17.7	17.7	0.147694982743014E 07	0.390650046925346E 08	2
21.2	21.2	0.380663740145578E 04	0.567672758437288E 03	2
28.3	28.3	-0.139262646895438E-02	0.233571848163099E-03	2
35.4	35.4	0.255232250889884E-08	-0.852066258074482E-08	2
53.0	53.0	-0.550693251547524E-19	-0.243342193077792E-18	2
70.7	70.7	-0.819297723063889E-28	0.180456867180876E-27	2

AND ORDER N = N+1 = 51

RZ	CZ	RE K	IM K	METHOD
0.1	0.1	-0.108211771986173E-30	0.442668118625159E-31	1
0.4	0.4	-0.108211771986173E-30	0.442668118625159E-31	1
0.7	0.7	-0.108211771986173E-30	0.442668118625159E-31	1
1.1	1.1	-0.255997745215569E 71	-0.250301637062697E 71	1
1.4	1.4	-0.109658562396689E 65	-0.105357664870739E 65	1
1.8	1.8	-0.126552590405696E 60	-0.118880316540435E 60	1
1.8	1.8	-0.126552590405696E 60	-0.118880316540435E 60	2
2.1	2.1	-0.117369629984332E 56	-0.107254711916842E 56	2
3.5	3.5	-0.608868584926740E 44	-0.472935014255719E 44	2
7.1	7.1	-0.327740412839842E 29	-0.961742626496752E 28	2
10.6	10.6	-0.333304433763555E 20	0.117601857835078E 20	2
14.1	14.1	-0.512245789369152E 13	0.136734339420379E 14	2
14.8	14.8	-0.184644082721818E 12	0.118788431669766E 13	2
14.8	14.8	-0.184644082721818E 12	0.118788431669766E 13	2
17.7	17.7	0.108423717743516E 09	0.114004722074677E 09	2
21.2	21.2	0.110857172089913E 05	-0.676807982949692E 04	2
28.3	28.3	-0.254523950366308E-02	0.262669339987454E-02	2
35.4	35.4	-0.527636956398760E-08	-0.184133912680484E-07	2
53.0	53.0	-0.250717694998951E-18	-0.327062543208598E-18	2
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