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LITERATURE REVIEW-ELASTIC CONSTANTS FOR AIRPORT PAVEMENT MATERIALS
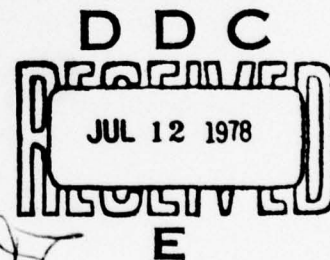
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FINAL REPORT

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16. Abstract

A literature review was made to support an ongoing study to develop a method for evaluating airport pavements based on the layered elastic theory and using constants as determined from vibratory test results. The review covered the definitions and relations between elastic constants, methods used by various researchers for measuring elastic constants, and values of elastic constants found (or used) by various researchers. The review also included a study to determine the sensitivity of pavement responses to arbitrarily assigned values of elastic constants and an examination of the relationships between vibrator test results and elastic constants. The latter subject included a special preliminary examination of the relationship between the results from tests with the WES 16-kip vibrator and elastic constants. A summary discussion was given of the findings from the literature review and the special preliminary examination. Finally recommendations were made to facilitate further planning and implementation of the ongoing study mentioned.

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PREFACE

This study was conducted by the Soils and Pavements Laboratory
(S&PL), U. S. Army Engineer Waterways Experiment Station (WES), for the
U. S. Department of Transportation, Federal Aviation Administration,
under Inter-Agency Agreement No. FA73WAI-377, "New Pavement Design
Methodology."

The literature review reported herein was made during June 1975-
January 1976 under the general supervision of Mr. James P. Sale, Chief,
S&PL, and Mr. Alfred H. Joseph, Chief, Pavement Investigation Division
(PID). This report was prepared by Mr. James L. Green, PID.

Director of WES during the conduct of the investigation and the
preparation of this report was COL G. H. Hilt, CE. Technical Director
was Mr. F. R. Brown.

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METRIC CONVERSION FACTORS

Approximate Conversions to Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
	LENGTH			
in	inches	*2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
	AREA			
in²	square inches	6.5	square centimeters	cm²
ft²	square feet	0.09	square meters	m²
yd²	square yards	0.8	square meters	m²
mi²	square miles	2.6	square kilometers	km²
	acres	0.4	hectares	ha
	MASS (weight)			
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
	VOLUME			
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft³	cubic feet	0.03	cubic meters	m³
yd³	cubic yards	0.76	cubic meters	m³
	TEMPERATURE (exact)			
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
	LENGTH			
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
	AREA			
cm²	square centimeters	0.16	square inches	in²
m²	square meters	1.2	square yards	yd²
km²	square kilometers	0.4	square miles	mi²
ha	hectares (10,000 m²)	2.5	acres	
	MASS (weight)			
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	
	VOLUME			
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m³	cubic meters	35	cubic feet	ft³
m³	cubic meters	1.3	cubic yards	yd³
	TEMPERATURE (exact)			
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F

°F -40   0   32   40   80   98.6   120   160   200   212
°C -40   -20   0   20   37   40   60   80   100
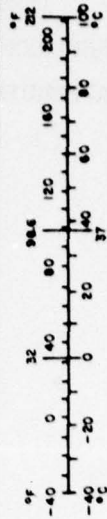
*1 in = 2.54 (exactly). For other exact conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures. Price $2.25, SD Catalog No. C13.10:286.

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INTRODUCTION

BACKGROUND

The nondestructive nature, speed of testing, ease of operation, accuracy and repeatability of results, and the fact that moving-wheel loads can be simulated have made vibratory testing devices popular for airport pavement evaluation. At least four types of portable vibratory testing devices are in use today. These devices are the Dynaflect, the Road Rater, a vibrator developed by the Civil Engineering Research Facility of the Air Force, and a vibrator developed by the U. S. Army Engineer Waterways Experiment Station (WES*). The first two of these devices are available commercially and the other two are research devices. The WES testing device is made available to any public agency that requests it for evaluation purposes.

The vibrators listed above (and others not mentioned) are associated with different procedures that permit pavement evaluation, but none of the vibratory procedure combinations is considered to be fully satisfactory. In a WES study[1] for the Federal Aviation Administration (FAA), a procedure was developed using the WES 16-kip** vibrator that is considered to be a significant improvement in nondestructive methods of evaluating airport pavements. This procedure was based on an experimental correlation of vibratory test results with allowable aircraft loads as computed using the conventional direct sampling evaluation procedures. Improvement and upgrading of this procedure is continuing.

It is probable that the procedure referenced above will serve airport evaluation requirements for several years to come. However, being empirical, it can be applied confidently only to the same kinds of pavements and aircraft loading from which it was developed. New kinds of pavements and aircraft of the future could severely test the efficacy

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* For convenience, necessary abbreviations and symbols are listed on page A1.

** A table of factors for converting units of measurement is presented on page 4.

of the evaluation procedure. Clearly, there is a need for more analytical means of designing and evaluating airport pavements than now exist. That need has long been recognized and a considerable effort has been made, and is being made, to fill it.

Currently, WES is conducting a study for FAA to satisfy commitments under FAA-ER-430-002b, Amendment 1, dated 10 September 1974, paragraph 3.4, which states, in part:

> Phase B of the nondestructive pavement evaluation is concerned with development of the nondestructive evaluation method compatible with the selected rational design procedure for flexible and rigid pavements described in paragraph 3.3* A theoretical analysis will be made to determine if the vibratory load-deflection data can be converted to appropriate modulus or other parameters for use with the rational procedure. The end product is the load-carrying capacity of airport pavements based on the nondestructive test and rational procedure. Results of the analysis will be verified by experimental results using all available data at the Waterways Experiment Station and any additional data obtained by the associated experiments described in the previous paragraphs on the climatological effects. A tentative nondestructive evaluation methodology developed in Phase B for both flexible and rigid pavement is to be made available by the end of fiscal year 1976.

The WES study in progress is based on layered elastic theory and results of tests with the WES 16-kip vibrator. The evaluation procedure, which is a reverse of the design procedure, will consist of first determining the values of the required elastic constants from vibrator results and then using those values as inputs to one of the existing layered elastic computer programs for pavement design, such as CHEVRON, BISTRO, or AFPAV, to predict the pavement performance.

The vibrator test results will be in the form of deflections measured at the pavement surface under known loads. The success of the whole endeavor rests heavily on the accuracy and ease with which it will be possible to extract the elastic constants from the deflection

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* Paragraph 3.3: "These procedures will be selected or adopted from various rational design methods such as finite element, layered elastic theory, finite difference, and layered viscoelastic theory."

measurements. A reasonable degree of confidence exists that an acceptable degree of accuracy will be obtained. Several models that relate elastic constants to vibrator test results are available. However, one of these, developed by Weiss,[2,3] is unique in that it is the only one based on a nonlinear theory.

Because of the importance of elastic constants per se to the ongoing study, a corollary literature search was considered essential regarding elastic constants (how measured, what values obtained, what causes variation in values, etc.) to support and enhance the larger study. Accordingly, the study reported herein was devoted to the development of empirical equations for deriving elastic constants from measurements made with the WES 16-kip vibrator. This report also includes a brief summary of that effort.

PURPOSE

The purpose of this study was to conduct a review of literature in support of an ongoing study to develop a method for evaluating airport pavements based on the layered elastic theory and using elastic constants as determined from vibratory test results. This report is expected to provide a convenient reference in facilitating efforts to develop such a method. The specific objectives of the study were: (a) review the elastic constants known to be of significance in applying the layered elastic theory to the evaluation of airport pavements; (b) determine the sensitivity of pavement responses to changes in assigned values of elastic constants; and (c) make a preliminary examination of the relationships between vibratory test results and elastic constants.

SCOPE

The review of elastic constants consisted of defining the pertinent elastic constants and stating their relationships, examining methods for measuring them, reporting specific values and variations or values of the constants, and indicating their relationships with other parameters. The sensitivity study consisted of a review of work by

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Packard[4] and Pichumani[5] in which they, independently, used the layered elastic theory to study the magnitudes of changes that occurred in predicted pavement responses with arbitrary changes in elastic constants. The examination of the relationships between vibratory test results and material parameters mainly comprised a review of existing models, but also included a preliminary relationship between the results from tests with the WES 16-kip vibrator and elastic moduli developed especially for this study. The report includes a summary discussion and recommendations to be considered in planning studies to develop airport evaluation methodology based on the layered elastic theory and vibratory test devices.

REVIEW OF ELASTIC CONSTANTS

The review of those elastic constants known to be of significance
in applying the layered elastic theory to the evaluation of airport
pavements consists of (a) defining the elastic constants and stating
relationships between them, (b) examining test methods for measuring the
constants, (c) indicating values and variations of the constants found
by several researchers, and (d) showing the relationships of the
constants to other parameters. The names and symbols for the elastic
constants and the units in which they are expressed are given as they
were used by the references cited.

DEFINITIONS AND RELATION-
SHIPS OF ELASTIC CONSTANTS

QUASI-STATIC ELASTIC CONSTANTS

Elastic Modulus, E . In the study of materials, it is common to
plot the relationships between stress, σ , and strain, ε . Figure 1
illustrates a material under uniaxial stress, and Figure 2 shows a
generalized stress-strain graph for the material. Each material has a
unique curve. For a certain distance from the origin, the experimental
values of stress versus strain lie essentially on a straight line,
except that for some materials the straight part of the curve hardly
exists. Nevertheless, up to some point, such as A (Figure 2), the re-
lationship between stress and strain may be said to be linear for all
materials. This generalization is known as Hooke's law and for uniaxial
loading is expressed in equation form as

$$\sigma = E\varepsilon \quad \text{or} \quad \varepsilon = \frac{\sigma}{E}$$

which means that stress is directly proportional to strain where the
constant of proportionality is E . For the case of a three-dimensional
state of stress, the generalized Hooke's law, valid for an isotropic
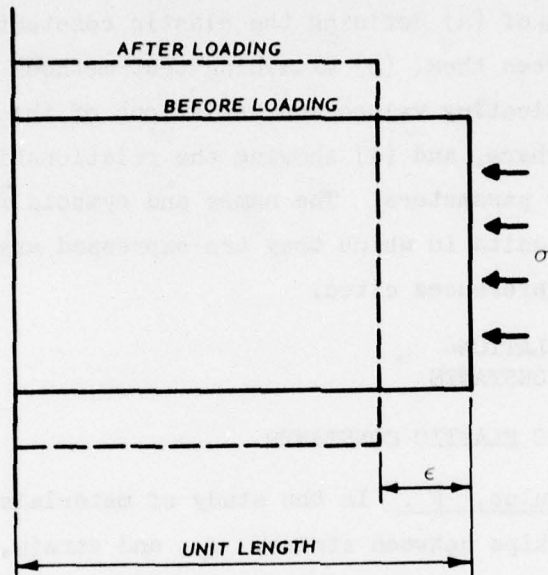homogeneous material, leads to the equations:

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Figure 1. Material under uniaxial stress
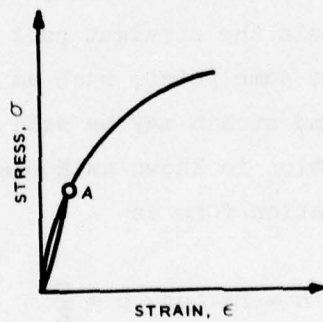
Figure 2. Stress versus strain

$$\varepsilon_x = \frac{1}{E} [\sigma_x - \nu(\sigma_y + \sigma_z)]$$

$$\varepsilon_y = \frac{1}{E} [\sigma_y - \nu(\sigma_x + \sigma_z)]$$

$$\varepsilon_z = \frac{1}{E} [\sigma_z - \nu(\sigma_x + \sigma_y)]$$

where

ε = normal strain in the x, y, or z direction

E = elastic modulus

σ = normal stress in the x, y, or z direction

ν = Poisson's ratio

The constant E, the elastic modulus, is a definite property of a material. It also is called the coefficient of elasticity and Young's modulus.

Unconfined clay is one material for which a stress-strain curve is far from linear. For such a material, it is often expedient to employ variations of E to characterize the material. The most commonly used are E_i, the initial tangent modulus; E_t, the tangent modulus; E_s, the secant modulus; and E_h, the hysteresis modulus.

Poisson's Ratio, ν. The elastic modulus reflects a property of a material in the direction of an applied force. At right angles to a force applied to an element, Figure 3 shows the lateral expansion or contraction that takes place. The lateral deformations per unit of length are termed lateral strains. Lateral strains bear a constant relationship to the longitudinal or axial strains caused by an axial force, provided a material remains elastic and is homogeneous and isotropic. For the two-dimensional element (Figure 3), the positive normal stress is σ_x/E. However, the positive normal stress, σ_y, also produces a strain normal to the y-z plane that is opposite in direction to the strain produced by σ_x. This lateral strain is equal to
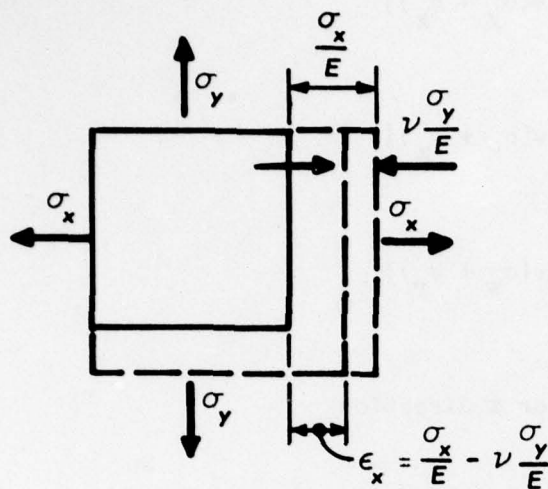
Figure 3. Two-dimensional homogeneous isotropic rectangular element of unit thickness subjected to a biaxial state of stress

and the total lateral deformation in the x direction, ϵ_x , is equal to

$$\frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E}$$

For the three-dimensional state of stress, the lateral strain produced by σ_z is equal to

$$\nu \frac{\sigma_z}{E}$$

Poisson's ratio is a definite property of a material, just as is the elastic modulus.

Shearing Modulus of Elasticity, G . For small deformations in the elastic range of a material, Figure 4 illustrates the linear relationship between the shearing stress, τ , and the angle, γ . If γ is defined as the shearing strain, mathematically the extension of Hooke's law for shearing stress and strain is

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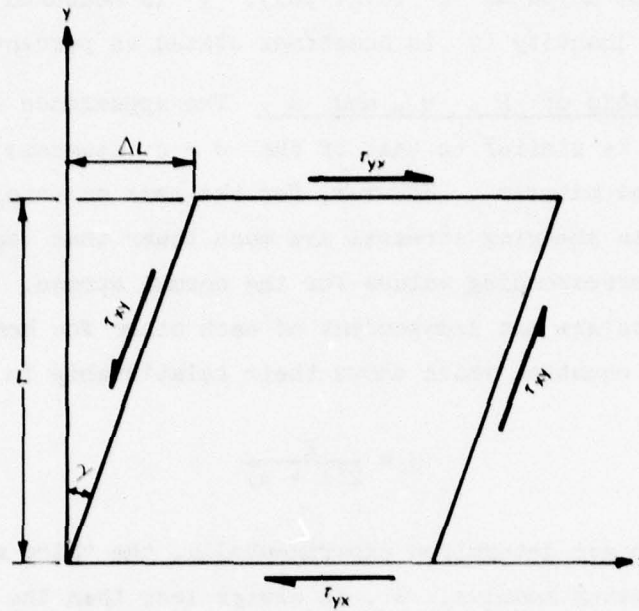
$$\frac{\Delta L}{L} = \text{TAN } \gamma = \gamma \text{ (RADIANS), FOR SMALL STRAINS}$$

Figure 4. Material under shear stress

$$\tau = G\gamma \quad \text{or} \quad \gamma = \frac{\tau}{G}$$

where G is a constant of proportionality called the shearing modulus
of elasticity or the modulus of rigidity. For a three-dimensional
state of stress, the generalized Hooke's law leads to three equations
for shearing strain:

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

$$\gamma_{yz} = \frac{\tau_{yz}}{G}$$

$$\gamma_{zx} = \frac{\tau_{zx}}{G}$$

Like E , G is a constant for a given elastic material. It is mea-
sured in the same units as E (e.g. psi); γ is measured in radians,
a dimensionless quantity (γ is sometimes stated as percent).

Relationship of E , ν , and G . The appearance of the
$\tau - \gamma$ diagrams is similar to that of the $\sigma - \epsilon$ diagrams for a tension
test for the same material. However, for the same material, the numeri-
cal values of the shearing stresses are much lower than (approximately
one half) the corresponding values for the normal stress. The three
elastic constants are not independent of each other for homogeneous
materials. The equation which shows their relationship is

$$G = \frac{E}{2(1 + \nu)}$$

Thus, if any two are determined experimentally, the third may be com-
puted. The shearing modulus, G , is always less than the elastic
modulus, E , since the Poisson's ratio, ν , is a positive quantity.

DYNAMIC ELASTIC CONSTANTS

The airport pavement evaluation method under consideration is
concerned with in-place states of stress in pavements subjected to mov-
ing wheel loads; therefore, the material properties under dynamic load-
ing must be considered. The stress-strain relationships indicated
earlier for quasi-static loading apply also for dynamic loading in the
case of linear elastic materials; however, it should be noted that
quantitative values of the dynamic response are frequency dependent.
Poisson's ratio and shearing modulus of elasticity usually have the
same basic definitions, and the relationship, $G = E/2(1 + \nu)$, for
quasi-static conditions is also used for dynamic conditions. However,
for repetitive loading conditions, G is commonly determined for a
given strain amplitude and a given cycle of loading. The dynamic
elastic modulus, however, takes a number of forms. Those forms referred
to in this study are defined in the following paragraphs.

Resilient Modulus, M_R . The resilient modulus of elasticity is

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the ratio of the repeatedly applied deviator stress, σ_o ($\sigma_o = \sigma_1 - \sigma_3$ where σ_1 equals axial stress and σ_3 equals confining stress), to the recoverable axial strain. M_R is usually determined at a given stress level and cycle of loading.

Complex Modulus, E^* . The complex modulus, E^* , is a complex number, $E^* = E_1 + iE_2$, whose magnitude, $|E^*| = \sqrt{E_1^2 + E_2^2}$, gives the ratio of the axial stress to axial strain during a uniaxial compression test. The laboratory test to determine $|E^*|$ is similar to that used to determine M_R , except that sinusoidal loading is used for $|E^*|$ and pulse loading is used for M_R . The values of $|E^*|$ are used inter-changeably for elastic moduli values by some researchers. If values for E_1 , the real component of the complex modulus where strain is in phase with stress, and E_2 , the imaginary component of the complex modulus where the modulus of strain is 90 deg out of phase with the stress, are desired, the "loss angle," θ_L , must be measured in the laboratory. The strain vector lags behind the stress vector by θ_L , the tangent of which is known as the "loss tangent," defined by $\tan \theta_L = E_2/E_1$. Know-ing θ_L and $|E^*|$ allows the solution of the two simultaneous equa-tions for E_1 and E_2 .

Stiffness Modulus. The stiffness modulus is a modulus of elas-ticity computed from the results of repetitive load tests on thin cir-cular slabs.

Dynamic Stiffness Modulus, DSM . The dynamic stiffness modulus is the ratio of load to deflection when the load is a specific steady-state vibratory load.

METHODS FOR MEASUR-
ING ELASTIC CONSTANTS

The following paragraphs reference and/or briefly describe several test methods for measuring elastic constants for portland cement concrete (PCC) and asphaltic concrete (AC) wearing surfaces, treated and untreated base materials, and subgrade materials. A detailed descrip-tion of test methods is beyond the scope of this report. The tests

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discussed usually are performed in the laboratory, with one exception, the wave velocity test, which normally is a field test.

Each paragraph is headed by a test reference (ASTM,[6] CRD-C,[7] etc.) or name reference (Massachusetts Institute of Technology (MIT), Glynn and Kirwan, etc.) followed by a symbol for the constant measured (E, ν, etc.) by the reference, in parentheses.

PCC WEARING SURFACES

ASTM C469-65[6] and CRD-C 21[7] (E and ν). Quasi-static methods for determining ν and E of PCC are ASTM Designation C469-65 for compression testing and CRD-C 21 for flexure testing. There apparently is no established procedure for directly determining the quasi-static shear modulus of PCC.

ASTM C597-71[6] (E). This test method is sometimes used to determine the modulus of elasticity of concrete. It involves the pulse velocity of compressional waves. However, the procedure is not recommended for establishing the compliance of the modulus of elasticity of field concrete with that assumed in design because the pulse velocity is not influenced by minute cracks in a concrete pavement, and the test procedure may assign a higher value of E to a cracked pavement than actually justified.

MIT and WES (Dynamic E). A dynamic modulus of elasticity for PCC has been determined under a single concentrated load by a machine developed by MIT and modified by Lundeen at WES.[8] The device yielded satisfactory results for compressive testing, but improvements to the apparatus were recommended for flexural testing. The impact load is produced by driving a piston with nitrogen under 1,000-psi pressure. A constant-slope loading pulse is produced that peaks in approximately 1 msec. Instrumentation consists of a 30,000-lb capacity load cell and an oscilloscope modified to provide only one trigger sweep or to be used with a d-c amplifier and an x-y recorder. The strains are measured with SR-4 strain gages and are recorded on a dual-trace oscilloscope equipped with a camera. For the test, a cylindrical specimen encased in a

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flexible membrane is placed in a triaxial compression chamber, subjected
to a constant lateral fluid pressure, and then loaded axially to failure.
The size of the test specimen is limited to 1-1/2 in. in diameter by
3 in. in height.

Murillo Engineering[9] (E). Values of E at rupture were observed
by Murillo Engineering from results of tests on concrete beams loaded
at three points. The test is similar to that described in CRD-C 21-58.[7]

Barkan[10] (Dynamic E). Barkan used a sound generator to excite
longitudinal or transverse waves of various frequencies in concrete
samples to determine dynamic values of E .

CRD-C 18-59[7] (Dynamic E , G , and ν). Procedures for measur-
ing frequencies of concrete prisms and cylinders of PCC for the purpose
of calculating dynamic E , G , and ν are described in CRD-C 18-59.
As with the pulse velocity test, the procedure is not recommended for
establishing the compliance of the modulus of elasticity of field con-
crete with that assumed in design. Also, specimens of the same concrete
that vary in size and shape may yield different computed modulus values.
In this procedure, E and G are determined independently and ν is
calculated using the formula $\nu = E/(2G - 1)$.

AC WEARING SURFACES

No references were found to quasi-static E , ν , or G or
dynamic ν or G testing procedures for AC.

Washington State University[11] (WSU) and the Asphalt Institute[12]
(AI) ($|E^*|$). Both WSU and AI used $|E^*|$ for the elastic modulus of
asphalt concrete. Neither of the above references describes the test
apparatus or test procedure in detail; however, WSU reveals that test
specimens are compacted to average densities (density of test track
under study) in molds 4 in. in diameter and 8 in. high. The complex
modulus is the axially applied sinusoidal stress function divided by the
resulting sinusoidal strain. In the series of tests described in
Reference 11, three frequencies of stress application (1, 4, and 16 cps)
and three temperatures (40, 70, and 100°F) were used in combination with

a single stress level (35 psi) to describe the response of asphalt concrete under dynamic loading.

Glynn and Kirwan[13] (Stiffness Modulus). In their study of environmental factors and flexible pavements, Glynn and Kirwan initially tried to determine resilient moduli of bituminous mixtures using beam specimens with third-point loading. They found that the concentrated loads induced localized displacements and that testing of short beams was complicated by shear deflection. They developed a device for testing thin circular slabs, the results from which allow computation of stiffness modulus. The device consists of a loading frame that incorporates a flexible pressure pad and pulse generator. The pressure pad area was made approximately equal to a tire imprint. Pulsed loading is produced by a rotor valve fitted to a compressed air line. The slab diameter was three times the pad diameter. A testing rate of 25 cpm was used along with a maximum applied pressure of 5 psi. Higher pressures produced excessive creep. The authors noted that the Texas Transportation Institute earlier developed a similar device of which they were not aware at the time of their study. They found the stiffness modulus of dense AC to be between 75,000 and 125,000 psi at 20°C. In their calculations, Poisson's ratio was estimated to be 0.35 or 0.45 for several independent tests.

TREATED BASE MATERIALS

The method for testing treated base materials depends on the amount and type of treating agent used in the base material. For example, if a granular base course material is treated with enough cement, it will behave similar to PCC and beam-shaped specimens could be tested in flexure. However, materials stabilized with small amounts of lime or salt may require small cylindrical test specimens. Therefore, tests to determine E , ν , and G of treated materials may either be similar to those described earlier for PCC and AC or to those described later for untreated bases and subgrade materials, i.e. soils, depending on the nature of the material. Most treated bases probably resemble wearing surfaces more closely than they do subgrades, and their elastic

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constants are probably influenced by mix proportions, temperature, stress level, and frequency of loading to the same extent that those of comparable wearing surface materials are influenced.

UNTREATED BASE MATERIALS

Untreated base materials usually are crushed rock or granular soil. Methods for measuring quasi-static values of elastic constants in untreated base materials are the same as those for subgrade materials. Three types of tests used to determine dynamic values of elastic moduli are discussed in the following paragraphs.

Repetitive Triaxial Compression Test (M_R). This is the most common test for dynamic elastic modulus. It is described later under test methods for subgrade materials.

Dorman and Klomp[14] (Dynamic E). These researchers determined dynamic values of E by measuring the velocity of transverse waves that were radiated from a vibratory load on the untreated base material. In this type of testing, electronic sensing devices are placed on the surface at various distances from the vibrator to determine a wavelength. E is then calculated from the relationship

$$E = q \times d \times f(\lambda)^2$$

where

q = factor dependent on Poisson's ratio

d = density of the material

f = frequency of wavelength

λ = wavelength

Hardin[15] (G). Hardin has done extensive work with the shear modulus of soils and base course materials. For shear testing of base course materials, he used a torsional resonant column device at the University of Kentucky. A torsionally vibrating table produces torsional vibration of the sample about its axis. The frequency of vibration is measured, and the resonant frequency is used to compute the

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shear modulus. Theory for the resonant column test results is found in "The Nature of Damping in Sands."[16]

SUBGRADE MATERIALS

Quasi-static tests from which the elastic constants of E, ν, and G can be determined for soils are probably routine for most soils laboratories; however, there are some difficulties in determining ν because of the problem of measuring the small magnitude of lateral strains involved. Standard triaxial tests generally are used for E and ν determinations.

Dynamic testing of soils is less well established then quasi-static testing but is becoming more common in the fields of road and airport design and evaluation because test results more nearly simulate the behavior of soils under moving wheel loads. At least seven laboratories are presently engaged in research of dynamic testing of soils: the Asphalt Institute Laboratory in College Park, Maryland; Construction Engineering Research Laboratory in Champaign, Illinois; University of Kentucky in Lexington; University of California in Berkeley; Cold Regions Research and Engineering Laboratory in Hanover, New Hampshire; WES; and University of Illinois in Urbana-Champaign, Illinois.

WES (M_R). The test for resilient modulus is similar to a standard triaxial test. Definitions and procedures suggested by WES are found in Technical Report S-75-10.[17] The deviator stress is applied repetitively and at several stress levels. A constant lateral stress (chamber pressure) is maintained. The test equipment is similar to a standard triaxial cell, the major difference being that there must be some external loading source capable of providing a variable load of fixed cycle and load duration. The expression for resilient modulus is

$$M_R = \frac{\sigma_d}{\epsilon_R}$$

where

σ_d = deviator stress = $\sigma_1 - \sigma_3$, or the repeated axial stress
σ_1 = total axial stress

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σ_3 = total radial stress (continuing pressure in the triaxial test)

ε_R = recoverable axial strain

Resilient Poisson's ratio is

$$\frac{\varepsilon_{L_r}}{\varepsilon_{R_r}}$$

where

ε_{L_r} = resilient lateral strain

ε_{R_r} = resilient axial strain

No published results of laboratory determination of resilient Poisson's ratios were found. Load versus time curves for the resilient modulus tests can take a variety of shapes, that is, haversine, sawtooth, haver square, etc.

Dorman and Klomp[14] (Dynamic E). Dorman and Klomp used their wave velocity method to determine an in-place dynamic elastic modulus for subgrade materials.

Hardin[18] (G). In his work with the shear modulus of soils, Hardin obtained dynamic values by applying a torsional load about the axis of a hollow cylinder of soil that was confined in a pressure chamber. The system could apply a maximum torque of 60 kg-cm, and the rate of loading for the tests varied from approximately 0.2 to 450 kg-cm per hour. Tests were conducted for strain amplitudes as small as about 0.00001 to as large as 0.005.

VALUES AND VARIATIONS
OF ELASTIC CONSTANTS

The following paragraphs provide values of elastic constants and/or describe their variation with several factors, for PCC and AC wearing surfaces, treated and untreated base materials and subgrade materials, as found by several researchers. The elastic constant(s) discussed in a paragraph is (are) shown in parentheses at the end of the paragraph heading. Quantitative data are given in the same units as used by the referenced authors.

PCC WEARING SURFACES

Popov[19] (E). Popov indicated that the elastic modulus of concrete varies with water-cement ratio. He gave a value of E = 2,000,000 psi for 7-1/2-gal/sack concrete and E = 3,000,000 psi for 6-gal/sack concrete.

Barkan[10] (E). In electroacoustic testing of concrete, Barkan showed how Young's modulus (E) varies with the age (7 and 28 days) and composition of concrete. Table 1 shows that in five of six cases aging increased E from 6.7 to 26.2 percent. In the sixth case, E was reduced 8.2 percent.

Yoder[20] and the Portland Cement Association[4] (PCA) (E and ν). Both Yoder and the PCA suggested that the elastic modulus and Poisson's ratio of concrete can be assumed to be 4,000,000 psi and 0.15, respectively, for most cases.

Materials Research and Development[21] (MRD) (E and ν). MRD used ASTM Test Procedure C469-65 in evaluating three PCC test sites at San Francisco International Airport and determined the elastic moduli of the concrete to be 3,440,000, 3,560,000, and 3,600,000 psi. They assumed Poisson's ratio to be 0.15.

Murillo Engineering[9] (E). Murillo Engineering used the results of tests on concrete beams loaded at three points to determine the modulus of elasticity at rupture for six test sites at Houston Intercontinental Airport. Table 2 presents the results, and E values computed used the equation

$$E = \frac{P\ell^3}{48I\Delta}\left[1 + (2.4 + 1.5\nu)\left(\frac{h}{\ell}\right)^2 - 0.84\left(\frac{h}{\ell}\right)^2\right]$$

where

P = maximum load, lb

ℓ = span, in.

I = moment of inertia of beam, in.^{4}

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Δ = deflection, in.

ν = Poisson's ratio

h = depth of beam, in.

E values range from 362,663 to 1,289,642 psi.

Table 1

Variation of Young's Modulus with Age and Composition
of Concrete (After Barkan[10])

Composition of Concrete*	Age of Concrete days	Young's Modulus	
		tons/m²	psi
1:2.55:2.55	7	3,600,000	4,600,000
	28	3,810,000	4,910,000
1:3.0:3.0	7	3,020,000	3,890,000
	28	3,810,000	4,910,000
1:1.93:3.23	7	3,530,000	4,550,000
	28	4,110,000	5,300,000
1:2.64:4.05	7	4,320,000	5,570,000
	28	3,960,000	5,110,000
1:3.76:3.0	7	3,100,000	4,000,000
	28	3,670,000	4,730,000
1:4.65:6.18	7	2,950,000	3,800,000
	28	3,310,000	4,270,000

* Cement:sand:coarse aggregate (by volume).

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Table 2

Summary of Flexural Strength of Concrete Beams (After
Murillo[9]) at Houston Intercontinental Airport

Beam No.	Location	Modulus of Elasticity at Rupture, psi
1	Sta 15+12.5, Taxiway A	362,663
2	Sta 60+37.5, Taxiway B	642,534
3	Sta 83+12.5, Taxiway B	601,780
4	Sta 89+37.5, Runway 14-32	762,248
5	Sta 88+37.5, Taxiway K	1,289,642
6	Sta 70+12.5, Taxiway K	683,308

Neville[22] (E, ν, Dynamic E, Dynamic ν). Neville suggested that the higher the E of the aggregate is, the higher will be the modulus of the resulting concrete. Figure 5 shows the relationship between E and compressive strength for four gravel aggregate mixes and a similar (single) relationship for several expanded clay (lightweight) aggregate mixes of different proportions. The E of lightweight aggregate concrete is usually between 40 and 80 percent of the E of ordinary concrete of the same compressive strength. Since the E of lightweight aggregate differs little from the E of the cement paste, mix proportions do not affect the E of lightweight aggregate concrete.

Neville further stated that the quasi-static ν of both ordinary and lightweight concrete varies from 0.11 to 0.21. Dynamic determination yields average values of about 0.24.

In the determination of the natural frequency of concrete beams for the purpose of calculating E, Neville suggested that since during the vibration of the specimen a negligible stress is applied, the

24
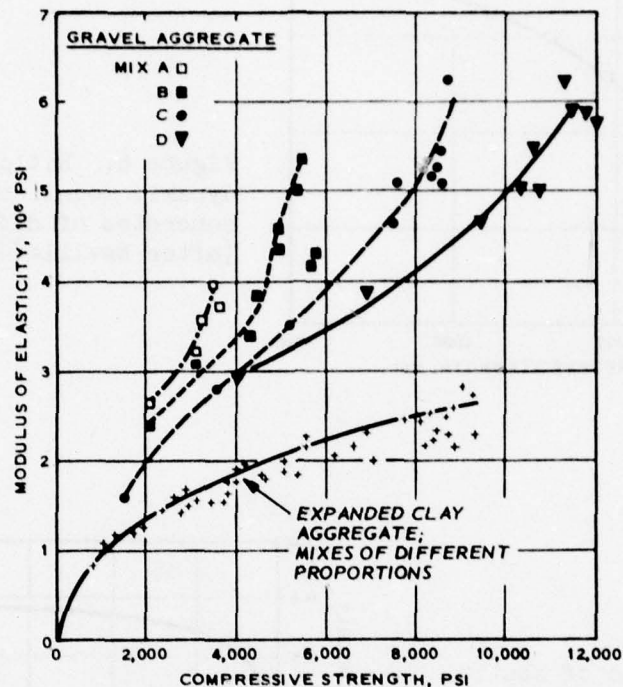
Figure 5. Static modulus of elasticity of concretes
made with gravel aggregate and expanded clay aggre-
gate and tested at different ages up to one year
(after Neville[22])

dynamic E refers to almost purely elastic effects and is unaffected
by creep. Therefore, the dynamic E is approximately equal to the E_i
determined in the static test and is, therefore, appreciably higher
than E_s . Figure 6 shows that the ratio of static to dynamic E in-
creases with an increase in the strength of the concrete. For a given
mix, the ratio increases also with age (Figure 7). The static modulus
was determined at 1000 psi in Figures 6 and 7. Figure 8 shows that
the dynamic E determined by transverse vibration of cylinders in-
creases almost linearly with an increase in their compressive
strength.

Orchard[23] (ν). Orchard stated that ν of concrete varies with
the richness of the mix (proportion of cement to total aggregate, by

25
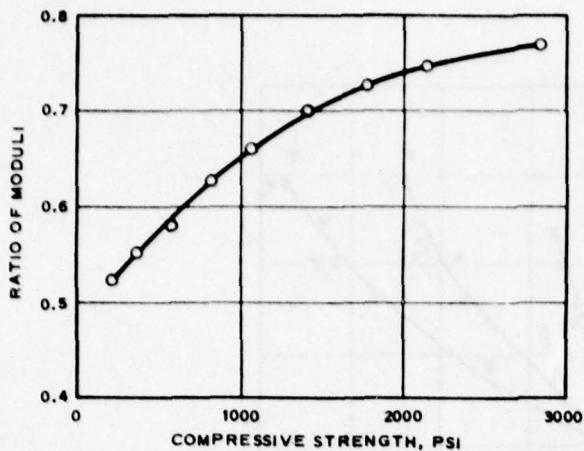
Figure 6. Ratio of static and dynamic moduli of elasticity of concretes of different strengths (after Neville[22])

Figure 7. Ratio of static and dynamic moduli of elasticity of concrete at different ages (after Neville[22])

Figure 8. Relation between the dynamic modulus of elasticity, determined by transverse vibration of cylinders, and their compressive strength (after Neville[22])

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weight). It is approximately 0.20 for a 1 to 4-1/2 mix, 0.10 for a
1 to 6 mix, and 0.08 for a 1 to 9 mix.

WES[8] (E). Table 3 presents results of quasi-static and dynamic
(single concentrated load) tests by WES to determine E values for low-
and high-strength concrete at three different levels of confining stress,
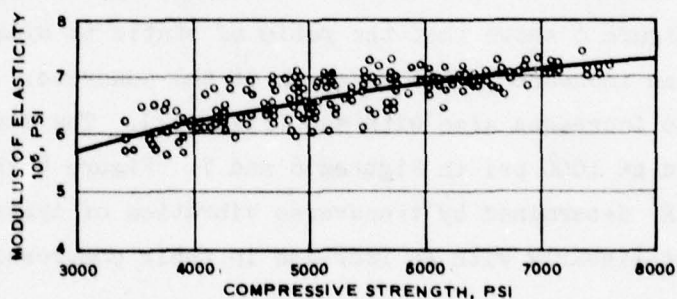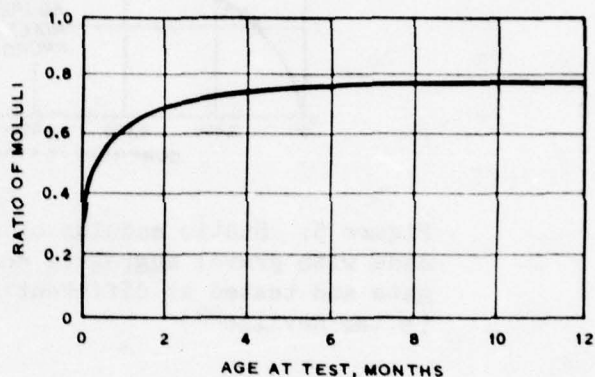σ_3 . The quasi-static values, E_{cs} , were determined using a standard
triaxial compression device, and the dynamic values, E_{cd} , were deter-
mined using the MIT device. These test results show that the E values
under the test conditions varied by a maximum factor of 1.4 when the
confining pressure changed from 0 to 500 psi, by a maximum factor of
1.7 between low- and high-strength concrete, and by a maximum factor of
1.2 between quasi-static and dynamic test methods. Dynamic testing con-
sistently yielded higher elastic modulus values than did static testing.

AC WEARING SURFACES

Glynn and Kirwan[13] (E). These researchers suggested that the
modulus of AC is influenced by mix proportions as well as temperature
but made no study of this point.

Morgan and Scala[24] (E). From observations of tests performed
under rolling wheels in which they examined the strain gage output, Mor-
gan and Scala decided that E is influenced by stress level, loading
rate, and temperature.

Other Researchers (E). Several other researchers also have
suggested that temperature, stress level, loading rate, and mix propor-
tions have significant influence on E in AC.

Klomp and Niesman[25] (E). Figure 9 shows a relationship developed
for modulus, temperature, and frequency of loading by Klomp and Niesman.
Modulus decreases very rapidly with an increase in temperature, less
rapidly with a decrease in rate of loading (vehicle speed).

Izatt et al.[26] (E). Izatt et al. developed a relationship
between elastic modulus and temperature, which reveals that for the
range in temperatures that exists in an AC wearing surface the modulus
can vary by a factor of ten (Figure 10).

Table 3

Summary of Triaxial Test Results (After Lundeen[8])

Confining Pressure (σ_3) psi	Low-Strength Concrete			High-Strength Concrete		
	Dynamic Modulus of Elasticity, E_{cd},* psi	Static Modulus of Elasticity, E_{cs},** psi	E_{cd}/E_{cs}	Dynamic Modulus of Elasticity, E_{cd}, psi	Static Modulus of Elasticity, E_{cs}, psi	E_{cd}/E_{cs}
0	1,950,000	1,650,000	1.18	3,000,000	2,810,000	1.07
250	2,120,000	2,080,000	1.02	3,590,000	3,030,000	1.18
500	2,560,000	2,370,000	1.08	3,430,000	3,330,000	1.03

* Determined using MIT device.
** Determined using standard triaxial device.

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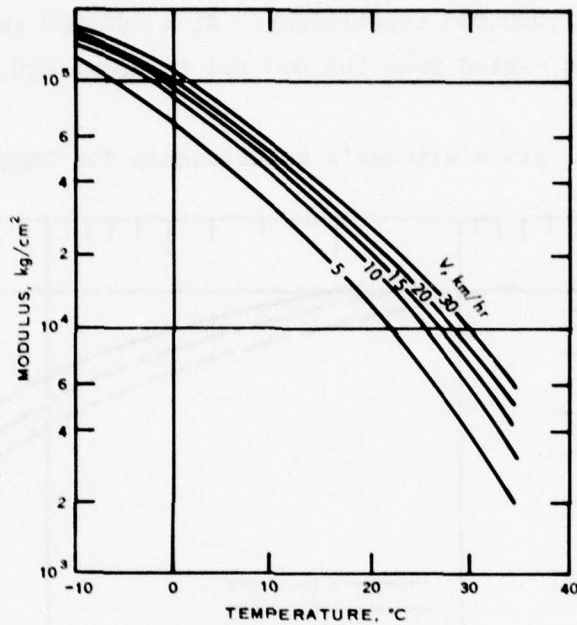
Figure 9. Asphalt concrete modulus re-
lated to temperature and rate of loading
(vehicle speed) (after Klomp and
Niesman[25])
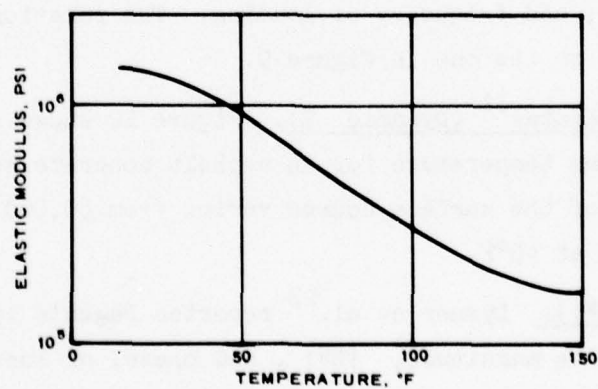
Figure 10. Variation of elastic modulus of
AC with temperature (after Izatt et al.[26])

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Witczak[27] (E). For a hot environment, Witczak selected limiting design modulus values of from 200,000 psi at 100 strain repetitions to 100,000 psi at 1,000,000 repetitions. At 1,000,000 repetitions, the limiting modulus varied from 100,000 psi to about 230,000 psi for cold climates.

Figure 11 gives Witczak's relationship for temperature, asphalt
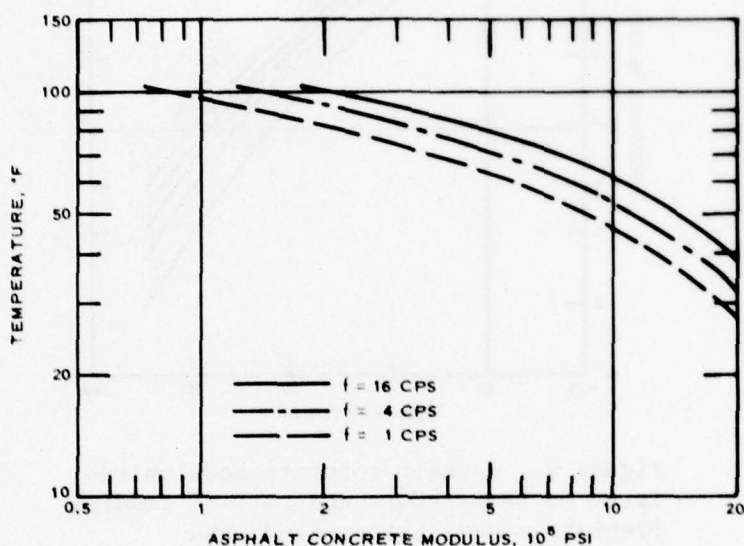
Figure 11.   Effect of temperature and frequency upon typical asphalt concrete dynamic modulus (after Witczak[27])

concrete modulus, and frequency of loading. The relationship is qualitatively similar to the one in Figure 9.

Cook and Krukar[11] (Dynamic E). Figure 12 shows a plot of dynamic modulus versus temperature for an asphalt concrete surface course. The dynamic E of the surface course varies from 60,000 psi at 100°F to 1,300,000 psi at 40°F.

Pagen ($|E^*|$). Lysmer et al.[28] reported Pagen's 1965 work relating the absolute magnitude, $|E^*|$, and phase, or loss, angle, ϕ_L, of the complex modulus to the frequency of loading, for a typical AC at 77°F. Figure 13 gives this relationship. The absolute value, $|E^*|$, increases from 0 at 0 frequency to approximately 700,000 psi at a frequency of 100 rad/sec (15.9 Hz).

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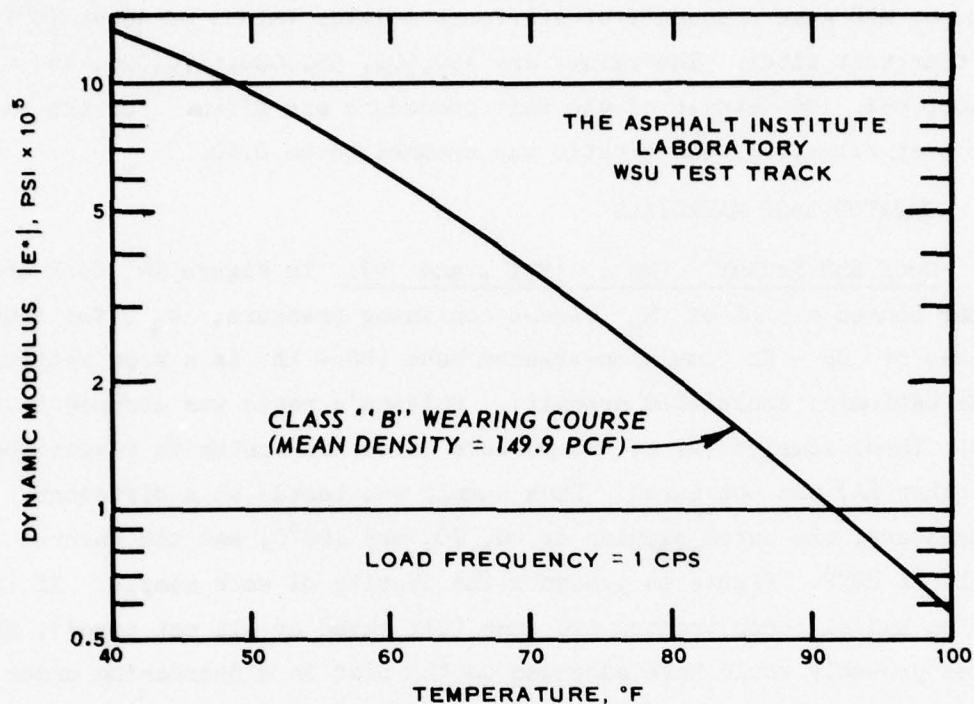
Figure 12. Dynamic modulus versus temperature re-
lationships for asphalt treated base and asphalt
concrete surface course (after Cook and Krukar[11])
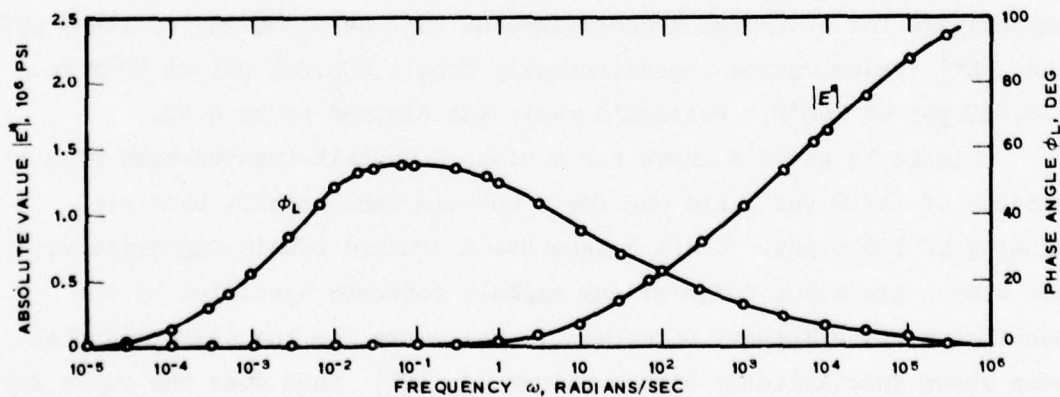
Figure 13. Magnitude and phase angle of complex elastic modulus,
E , as a function of angular frequency at 77°F (after Pagen[28])

MRD[21] (Stiffness Modulus, ν). In the evaluation of San Francisco Airport, MRD gave a summary of stiffness modulus values of AC at 70°F for four test sites. The values are 450,000, 550,000, 570,000, and 600,000 psi. No details of the test procedure are given. For the same four test sites, Poisson's ratio was assumed to be 0.40.

TREATED BASE MATERIALS

Cook and Krukar[11] (M_R, $|E^*|$, and ν). In Figure 14, Cook and Krukar showed a plot of M_R versus confining pressure, σ_3, for four samples of SS - Kh emulsion-treated base (SS - kh is a slow setting grade cationic, emulsified asphalt). Poisson's ratio was assumed to be 0.40. Three samples (B, C, and D) were cured six months in plastic bags, the other (A) was not cured. Each sample was tested at a different temperature; the cured samples at 40, 70, and 100°F, and the uncured sample at 80°F. Figure 14 presents the density of each sample. If the samples had all been treated the same (all cured or all not cured), the curves probably would have ascended on the plot in a decreasing order of temperature, i.e., D, A, C, and D (see Figure 11 for temperature effect on E). This did not occur, however, probably because curing raised the M_R values of curves B, C, and D. The effects of density are not readily apparent.

Figure 15 shows a curve of dynamic modulus, $|E^*|$, versus temperature for a special asphalt-treated base at a density of 144.9 pcf. The $|E^*|$ value varies logarithmically from 1,600,000 psi at 40°F to 110,000 psi at 100°F. Poisson's ratio was assumed to be 0.40.

Figure 16 gives a curve for a class F asphalt-treated base with a density of 147.2 pcf, and one for a hot-mix sand asphalt base with density of 116.6 pcf. Class F base has a crushed basalt aggregate with the widest gradation range of any asphalt concrete specified by the Washington State Highway Department. The curve for the class F asphalt base shows substantially higher values of $|E^*|$ than does the curve for the hot-mix sand asphalt base because the density of the former material is considerably higher. The effect of different aggregate gradations is
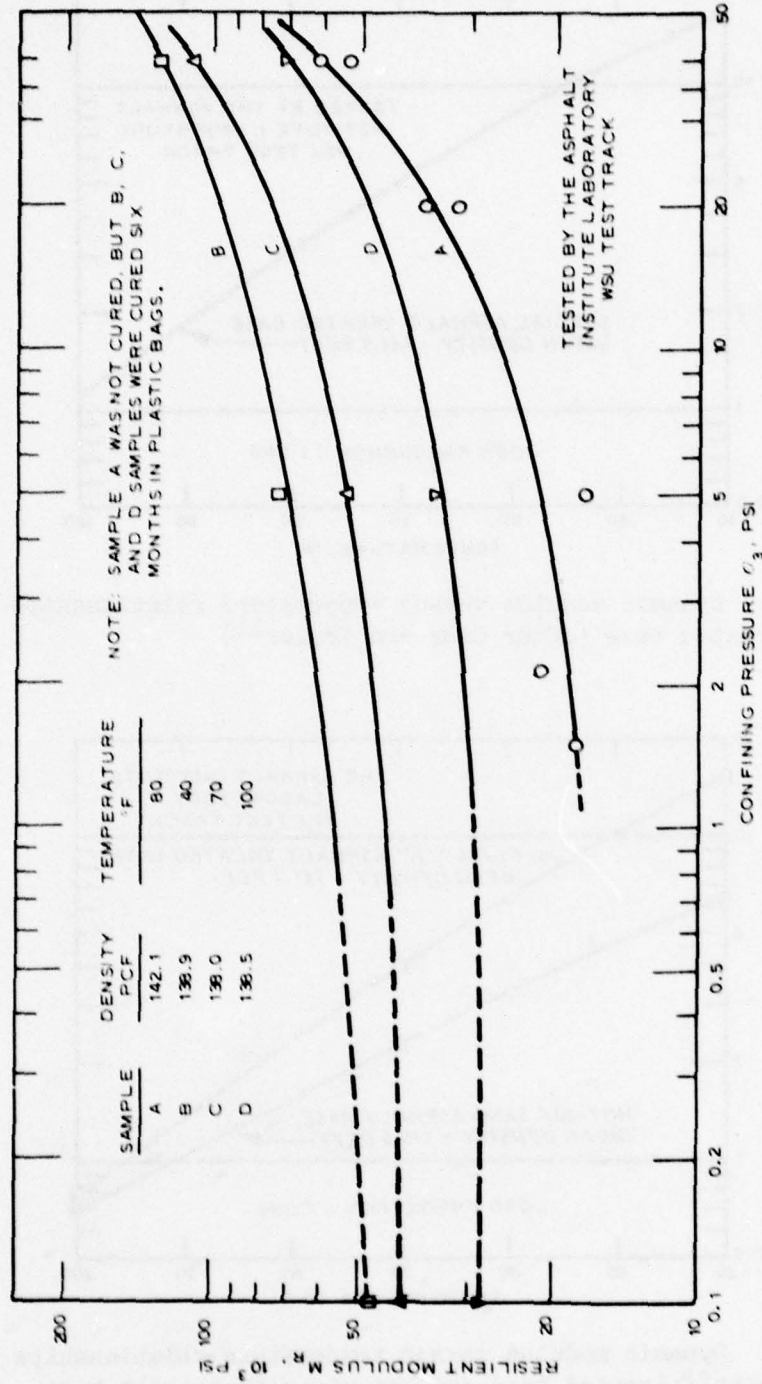
Figure 14. Resilient modulus versus confining pressure relationship for SS - Kh emulsion-treated base (after Cook and Krukarll)
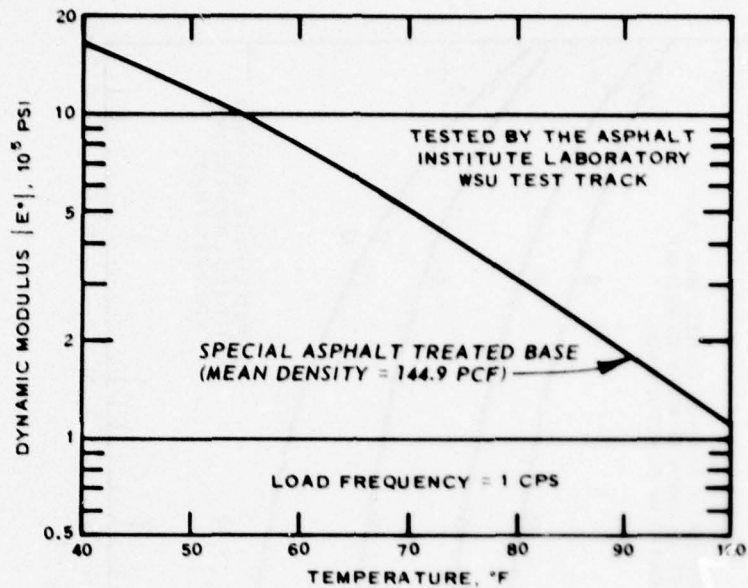
Figure 15. Dynamic modulus versus temperature relationships for asphalt-treated base (after Cook and Krukar[11])
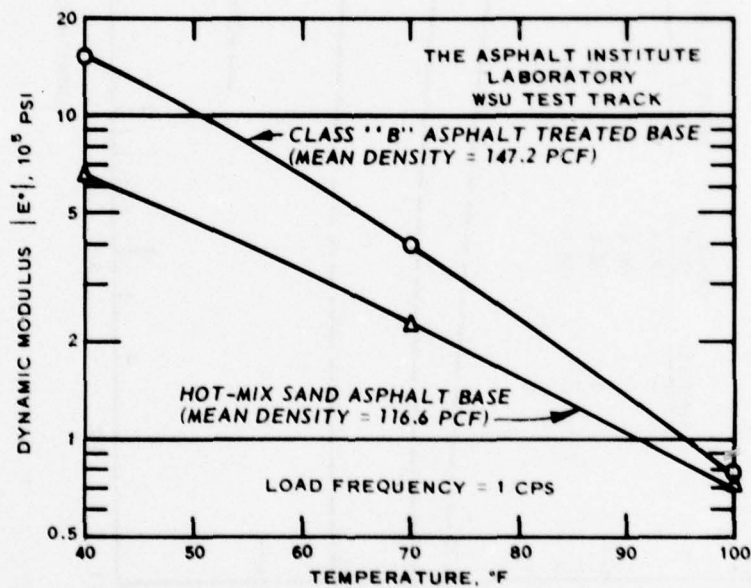
Figure 16. Dynamic modulus versus temperature relationships for class F asphalt-treated base and hot-mix sand asphalt base (after Cook and Krukar[11])

also probably reflected in the different $|E^*|$ values. Poisson's ratio was assumed to be 0.40.

Barker et al.[29] (E , ν). Because the tensile strength of cement- and lime-stabilized material is lower than the compressive strength, Barker et al. determined the properties of these materials in indirect tensile testing. Cement- and lime-stabilized clay test specimens were 4.0 in. in diameter by 2.5 in. in height. Clayey gravelly sand material was molded into a 6-in.-diam by 4.5-in.-high specimen. The specimens under study were impact compacted to field conditions of density and moisture. Accurate measurement of horizontal deformations was not possible; therefore, Poisson's ratios were not obtained in the series of tests described in Reference 29. Barker found marked increases in E with increases in cement content. Table 4 lists the modulus values

Table 4

Tensile Strength and Modulus of Elasticity of Cement-Stabilized Materials (After Barker et al.[29])

Material	Indirect Tensile Strength, psi	Modulus of Elasticity E , psi
Lean clay		
3.0 percent cement	14.7	8,400
10.0 percent cement	42.5	46,900
3.5 percent lime	7.3	13,500
Clayey gravelly sand		
3.0 percent cement	18.4	31,100
10.0 percent cement	141.0	223,000

determined in the testing. Poisson's ratio was assumed to be 0.25 for all materials shown in Table 4.

MRD[21] (E , ν). MRD found the cement-treated base at San Francisco International Airport to have an elastic modulus of 1,100,000 psi and a Poisson's ratio of 0.30. These values were determined using ASTM Designation C469-65.[6]

UNTREATED BASE MATERIALS

Dorman and Klomp[14] (Dynamic E). Untreated bases are subject to
the same kinds of variations in elastic constants as are subgrade mate-
rials, discussed later. One kind of variation that should be mentioned
here, however, is the apparent dependency of the dynamic elastic modulus
of an untreated base material on the modulus of the underlying subgrade.
Figure 17 gives the relationship that Dorman and Klomp found from wave
velocity measurements. The ratios of modulus of elasticity of the
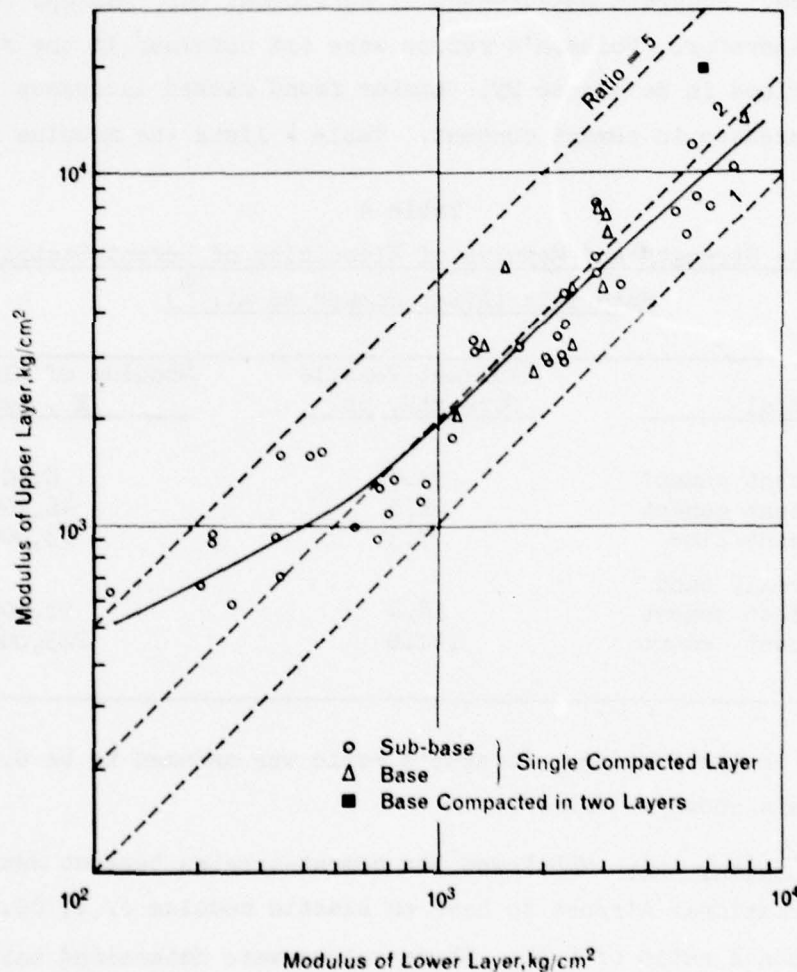
Figure 17. Dynamic moduli of unbound granular
layers (after Dorman and Klomp[14])

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untreated base materials to modulus of elasticity of the subgrades for the many cases tested all fall in the range from 1 to 5. The average ratio is approximately 2.

Cook and Krukar[11] (M_R). Cook and Krukar performed M_R tests on samples of an untreated base material (crushed basaltic rock) at three different water contents (W/C), 7.4 percent, 3.0 percent (dried in air), and saturated under vacuum. All samples were compacted at 7.4 percent W/C.

Figure 18 shows the relationship of M_R to confining pressure, σ_3, for the three samples. The curve for the sample with the lowest W/C (3.0 percent) plots well above the curves for the other two samples, which are nearly coincident. M_R increases with increasing confining pressure, and there appears to be a tendency for all curves to coalesce at very high confining pressures.

Figure 19 presents the same samples on a background of M_R versus bulk stress, θ. Bulk stress, or the first stress invariant, is the sum of the principal stresses ($\theta = \sigma_1 + \sigma_2 + \sigma_3$). The driest samples continues to show the highest values of M_R, and M_R increases with increasing θ.

Glynn and Kirwan[13] (M_R). Figures 20-22 show the relationship of modulus of resilient axial deformation (same as M_R) to cell pressure σ_3, i.e., confining pressure for a gravel with a dry density of 127.8 pcf and a moisture content of 9.4 percent, a gravel with a dry density of 131.4 pcf and a moisture content of 7.15 percent, and a crushed stone with a dry density of 128.1 pcf and a moisture content of 7.85 percent, respectively. The deviator stress ($\sigma_1 - \sigma_3$; 10 or 20 psi in Figure 20, and 10, 20, or 30 psi in Figures 21 and 22) is indicated for each point plotted. While not overwhelming, the evidence indicates that M_R increases with increasing density and decreasing moisture content and is not significantly affected by deviator stress.

Allen[30] (M_R). Allen has summarized the results of repeated load triaxial tests to determine M_R for several untreated base materials
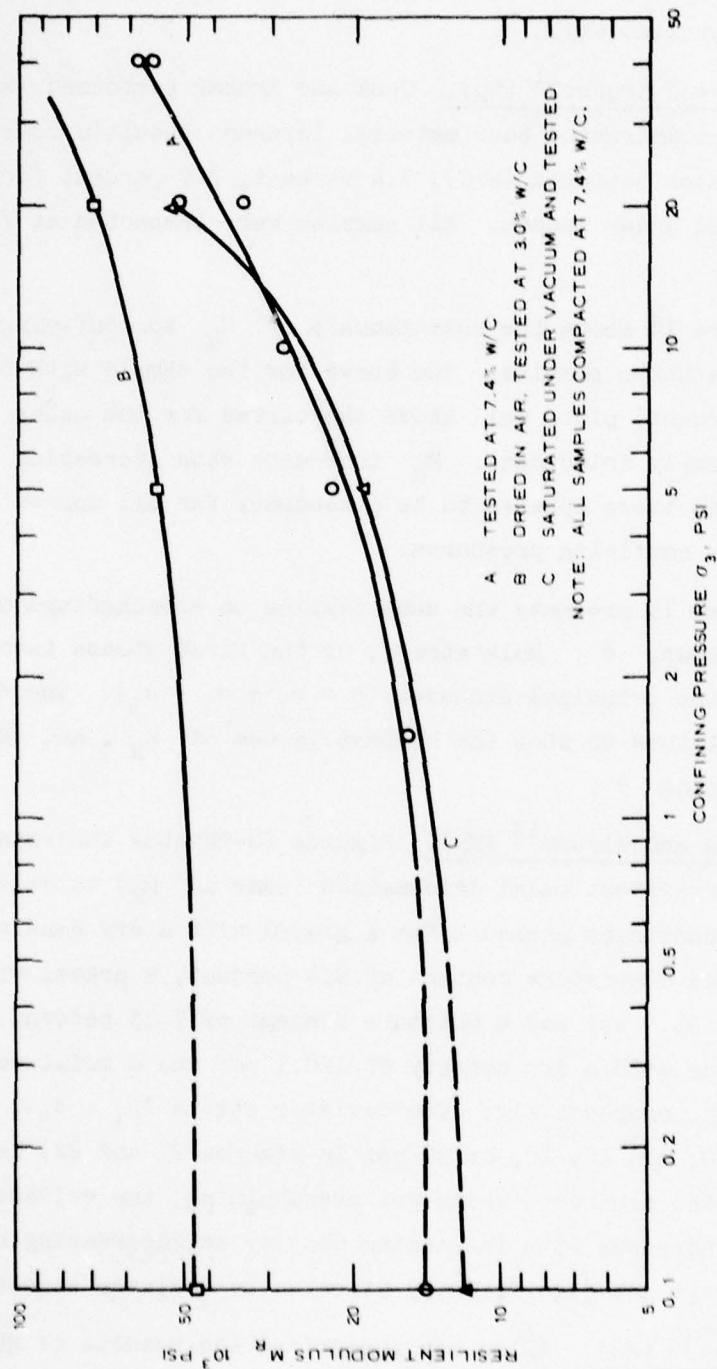
Figure 18. Resilient modulus versus confining pressure relationship for untreated base material (after Cook and Krukar[11])

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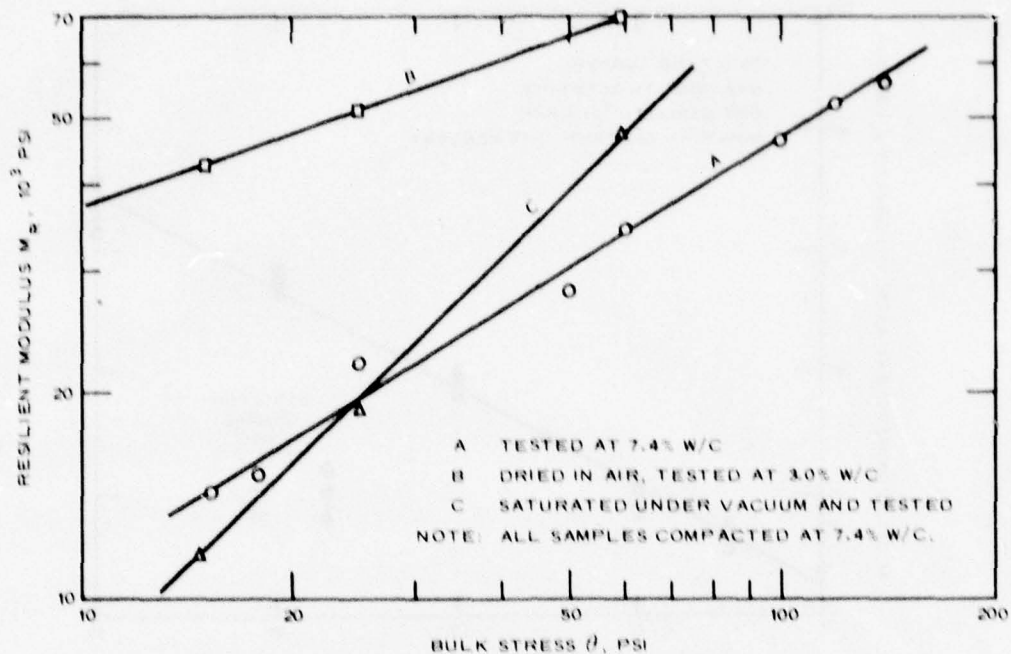
Figure 19. Resilient modulus versus bulk stress relationship for untreated base material (after Cook and Krukar[11])
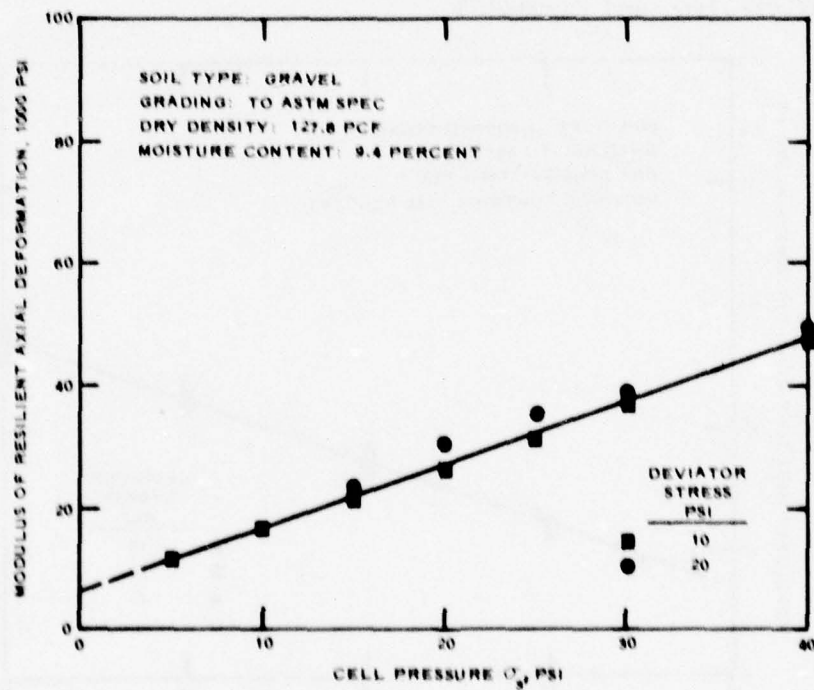
Figure 20. Modulus versus cell pressure (σ_3), test series 3 (after Glynn and Kirwan[13])
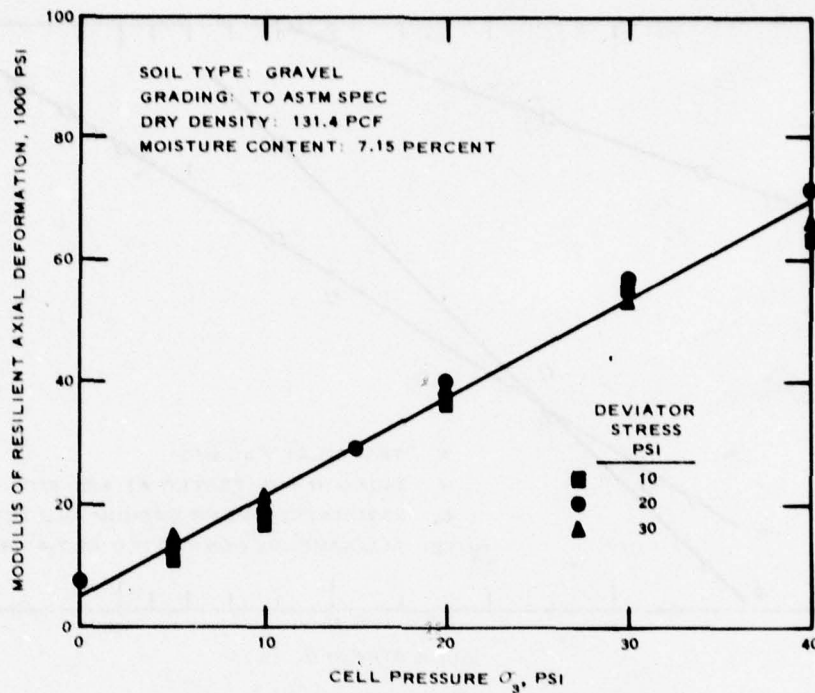
Figure 21. Modulus versus cell pressure (σ_3), test series 4 (after Glynn and Kirwan[13])
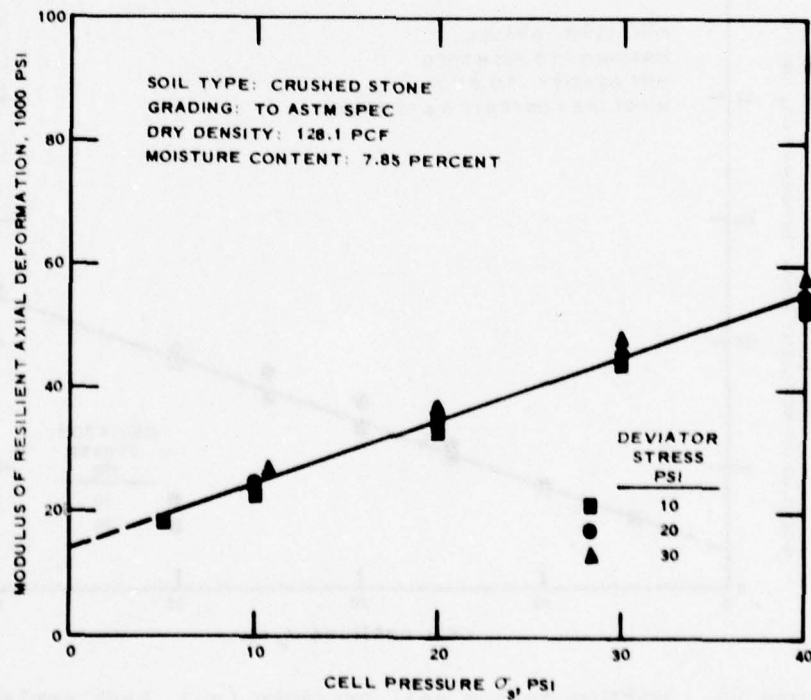
Figure 22. Modulus versus cell pressure (σ_3), test series 5 (after Glynn and Kirwan[13])

in Table 5. This table presents the researcher(s), the material tested, the range of M_R values found, and comments on the test conditions. The variation in values of M_R for a given material and between materials is seen to be quite wide in some cases. From the table, it appears that stress level had a greater effect on M_R than did the other test factors (frequency and duration of load, moisture content, gradation, void ratio, dry density, and rate of deformation); however, such a conclusion on the basis of the data shown can only be considered tentative.

Table 5

Summary of Repeated-Load Triaxial Tests to Evaluate the Resilient

Properties of Granular Materials (After Allen[30])

	Researcher	Material	Resilient Modulus, psi	Comments
1.	Seed and Chan	Silty sand	21,300 to 27,300	Varied frequency and duration of load
2.	Haynes and Yoder	Gravel and crushed stone	28,000 to 63,000	Varied moisture content and gradation
3.	Biarez	Rounded aggregate	16,700 to 54,500	Varied stress levels and void ratio
4.	Trollope, Lee and Morris	Poorly graded dry sand	35,000 to 95,000	Varied stress levels, dry density, rate of deformation
5.	Dunlap	Well graded aggregate	30,000 to 160,000	Varied stress levels
6.	Mitry	Dry gravel	$7,000\sigma_3^{0.55}$ to $1,900\theta^{0.61}$	Varied stress levels
7.	Schiffley	Crushed gravel	$13,000\sigma_3^{0.50}$ to $9,000\sigma_3^{0.50}$	Varied moisture content
8.	Kasianchuk	Aggregate base aggregate subbase	$3,830\theta^{0.53}$ to $2,900\theta^{0.47}$	Varied stress levels
9.	Hicks and Finn	Aggregate base	$5,400\theta^{0.50}$ to $21,000\theta^{0.50}$	Varied moisture content

NOTES: σ_3 is confining pressure; θ is bulk stress = $\sigma_1 + \sigma_2 + \sigma_3$.

Allen[30] (ν). Allen has summarized Hicks' study of some of the factors that influence the resilient Poisson's ratio of untreated base materials from tests similar to those conducted at WES. In Figure 23, a plot of resilient ν against the principal stress ratio, σ_1/σ_3, shows that the ν for the dry specimen is approximately twice that of the wet one and that the various confining pressures had some effect on
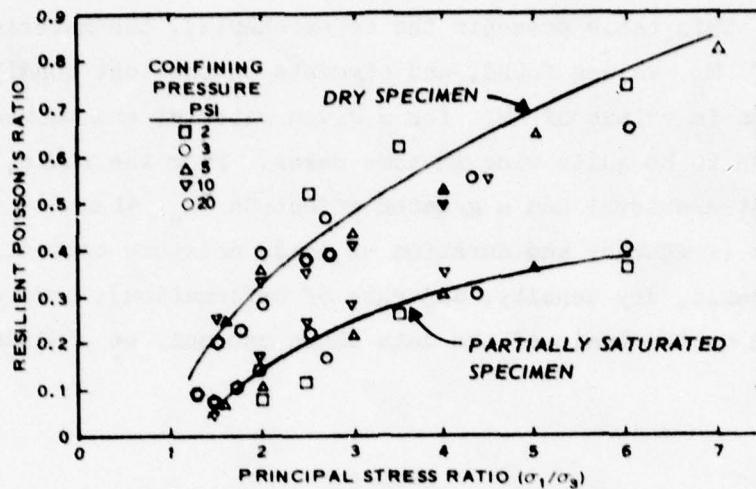
Figure 23. Secant Poisson's ratio as a function of principal stress ratio with partially crushed aggregate, low density, coarse grading (after Hicks; summarized by Allen[30])

the results. In Figure 24a, Allen (Hicks) shows two plots, one for a coarsely graded material and the other for a medium-graded material. Each contains curves for samples at 0 and 100 percent saturation and at an intermediate saturation. A relative density is indicated for each curve. Figure 24b also shows ν versus σ_1/σ_3 plots for a coarsely graded and a finely graded material. In these plots, the samples are dry, and relative densities are indicated.

On the basis of examination of the four plots, it can be concluded tentatively that water content is the most significant factor (the drier the sample, the higher the ν) and that increases in relative density usually correspond to decreases in resilient ν.

Hardin[15] (G). In tests on 19 gravel samples, Hardin extended his earlier work with soils to arrive at the following expression for gravels, which summarized the factors that cause variations in the shear moduli of coarse materials.

$$G_{max} = Y \left[2 + D_5 (\overline{\sigma}_o^{-0.2/3}) \right] \frac{(2.973 - e)^2}{(1 + e)} \overline{\sigma}_o^{1/2}$$

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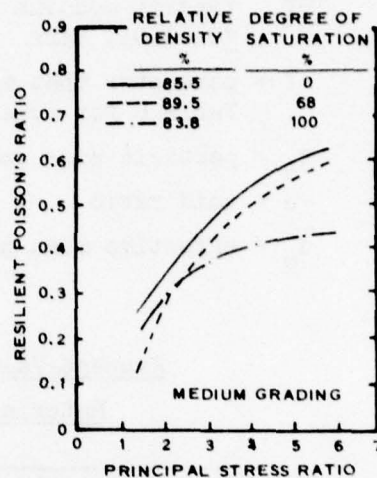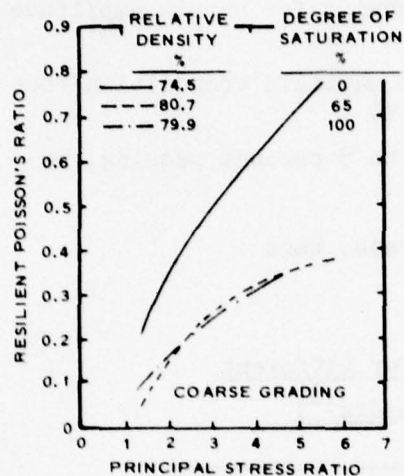
a. EFFECT OF DEGREE OF SATURATION ON THE RELATIONSHIP
BETWEEN RESILIENT POISSON'S RATIO AND PRINCIPAL STRESS
RATIO (σ_1/σ_3) WITH PARTIALLY CRUSHED AGGREGATE
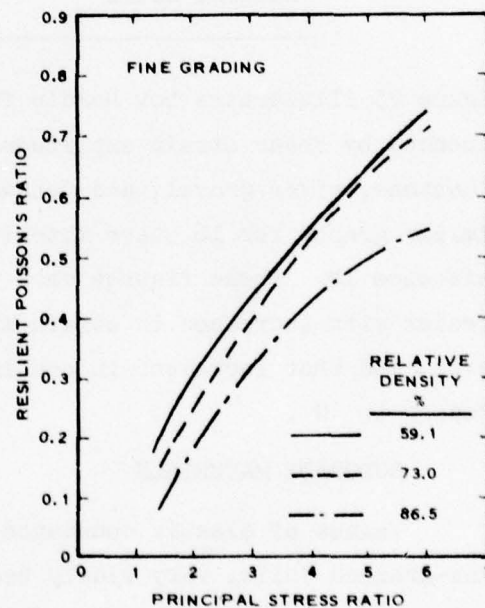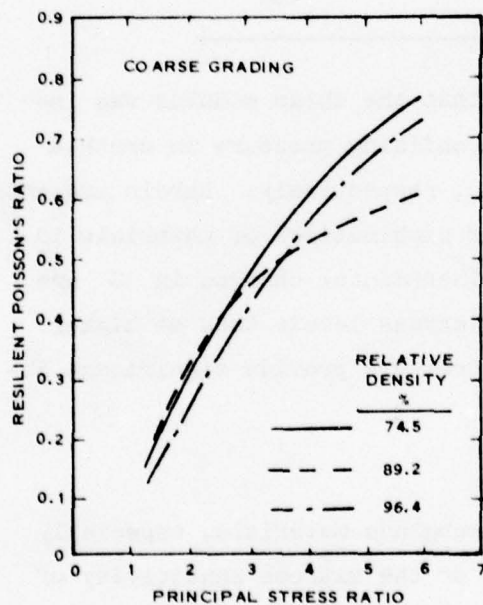
b. EFFECT OF DENSITY ON THE RELATIONSHIP BETWEEN
RESILIENT POISSON'S RATIO AND PRINCIPAL STRESS
RATIO (σ_1/σ_3) WITH PARTIALLY CRUSHED AGGREGATE
AND DRY SAMPLES

Figure 24. Resilient Poisson's ratio versus principal stress
ratio (after Hicks; summarized by Allen[30])

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where

G_{max} = maximum shear modulus, which is equal to the initial tangent modulus or secant modulus for strain amplitude ≤ 0.00001, bars

Y = parameter that may depend on particle composition (see Table 6 for some values of Y)

D_5 = particle size corresponding to 5 percent passing, mm

e = void ratio

$\bar{\sigma}_o$ = effective mean principal stress, bars

Table 6

Average Values of Y for Different Materials (After Hardin[18])

Material	Value of Y
Crushed limestone	146
River gravel	143
Standard Ottawa sand	110
Natural soils	132

Figure 25 illustrates how Hardin found that the shear modulus was influenced by shear strain amplitude and confining pressure in crushed limestone, river gravel, and Ottawa sand, respectively. Hardin presents similar graphs for 16 other materials or combinations of materials in Reference 18. These figures show that incremental changes in G are greater with increases in strain at low stress levels than at higher levels and that increases in confining pressure provide significant increases in G.

SUBGRADE MATERIALS

Values of elastic constants for subgrade materials, especially fine-grained soils, vary widely because of the extreme sensitivity of such values to soil properties (type, water content, etc.) and to in situ and/or test conditions (confining pressure, strain amplitude, etc.). It is important to note that elastic constants determined in the laboratory often are not accurate for field conditions. Terzaghi and Peck[31]

Figure 25. Shear modulus versus shear strain amplitude in crushed limestone, river gravel, and Ottawa sand (after Hardin[18])

and Bowles[32] agree that the stress-strain characteristics of granular
materials are essentially the same in the field and the laboratory;
however, for soils with cohesion, the elastic moduli are many times
greater in the field than indicated by the laboratory tests. It also
is important to note that stress-strain curves of soils are not straight
lines; they generally are concaved downward and, therefore, have no
definite elastic limit.

Much of the discussion on values and variations of elastic con-
stants in subgrade materials that follows is fundamental or qualitative.
Quantitative data are presented, where available.

<u>Terzaghi and Peck.</u>[31] Terzaghi and Peck used the illustration in
Figure 26 to demonstrate the stress-strain relationships of a typical

Figure 26. Stress versus
strain for an unconfined
clay specimen (after
Terzaghi and Peck[31])

saturated clay soil. In this illus-
tration, the abscissa and ordinate
represent the vertical stress on an
unconfined clay specimen and the
corresponding vertical strain,
respectively. The curve Oc shows
the manner in which the strain in-
creases while the stress is in-
creased at a constant rate. If
the load is held constant at some
value, the sample continues to get
shorter, as indicated by the **horizon-
tal** line ab. The rate of shortening
decreases with time and finally becomes equal to zero, provided the
shearing stress on the potential surface of failure is smaller than the
stress required to produce creep. The resumption of loading at the
original rate leads to a curve that joins the main branch Oc without
any break.

If the load is removed at the same rate at which it was previously
applied, the elastic recovery is smaller than the preceding com-
pression. If the load is again applied, the recompression curve joins

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the main branch without any breaks, and the decompression and recompression curves enclose a hysteresis loop. As soon as the stress becomes equal to the unconfined compressive strength, q_u, of the material, the specimen fails by shearing or bulging.

Since for most soils the stress-strain curve Oc in Figure 26 is curved throughout its entire length, the relation between stress and strain for soils, unlike that of elastic materials, cannot be expressed by a single numerical value E. In order to compare the stress-strain properties of different soils or those of the same soil under different conditions, one of the following four quantities may be used: the initial tangent modulus E_i, the tangent modulus E_t, the secant modulus E_s, or the hysteresis modulus E_h. These quantities are equal, respectively, to the slopes (stress per unit of strain) of the dash-dotted lines in Figure 26. The secant modulus E_s represents the average slope of the stress-strain curve for the range in stress between zero and some arbitrary value P_a.

If an undisturbed clay specimen is first tested and then re-molded at unaltered W/C and tested again, it may be observed that the values of q_u and E are very much smaller for the remolded than for the undisturbed material, but the general character of the stress-strain diagram remains unaltered. The magnitude of the decrease of strength and stiffness depends on the degree of sensitivity of the clay. If the remolded specimens are allowed to age without change in water content, their strength and stiffness increase at a rate that decreases with time, but it is doubtful whether they would ever attain values corresponding to those for the undisturbed specimens.

The stress-strain characteristics of partly confined, normally loaded clays are similar to those of fine-grained and saturated loose sands. However, at a given confining pressure, the values of confined compressive strength, q_c, and E for clay are very much smaller than the corresponding values for sand. If a unidirectional stress is added to an all-around pressure in clay, the relation between this stress and the corresponding strain is similar to that shown in Figure 27 for loose sand.

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Figure 27 shows a plot of initial tangent modulus versus confining pressure, p_c, for dense and loose sands. The E_i of confined, normally loaded clays increases like that of a loose sand, in simple proportion to p_c; that is

$$E_i = Cp_c$$
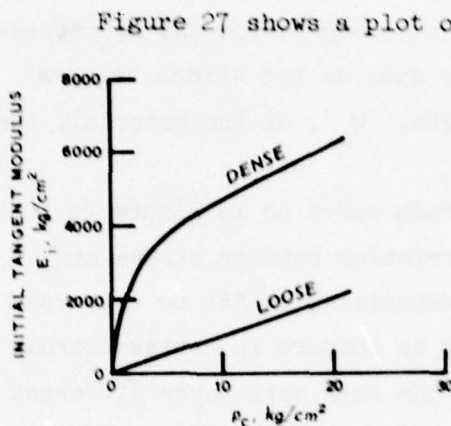
Figure 27. Initial tangent modulus versus confining pressures for sand (after Terzaghi and Peck[31])

The value of C depends on the type of clay and on the conditions of drainage. If the clay is first completely consolidated under p_c and the unidirectional stress is added without permitting the water content of the specimen to change, the value of C ranges from about 10 for highly colloidal clays to 100 for silty or slightly sandy clays.

Lambe and Whitman[33] (E and ν). Lambe and Whitman determined that it is difficult to find the E_i accurately because the slope of the stress-strain curve changes rapidly even at small strains. They also found that the E_i modulus as determined from the first loading of a triaxial test usually is much less than the modulus computed from wave velocity.

Lambe and Whitman stated that the effect of the actual state of stress on modulus is not clear, but the best available information shows that modulus depends on the average of the initial principal stresses; that is,

$$E \sim \sqrt{\sigma_v \frac{1 + 2K_o}{3}}$$

where

σ_v = vertical stress

K_o = the coefficient of lateral stress at rest

The equation is true only when $1/2 < K_o < 2$ and when the factor of safety against failure is two or greater.

The elastic modulus decreases and Poisson's ratio increases as strain level increases. Lambe and Whitman presented a relationship between σ_v, σ_h, and ϵ_v, which is

$$\sigma_v - \sigma_h = \sqrt{\frac{\epsilon_v}{a + b\epsilon_v}}$$

where a and b are constants and subscripts h and v mean "horizontal" and "vertical," respectively. The general form of this empirical expression indicates that the modulus depends heavily on stress level.

Lambe and Whitman made the following general statements:

a. The modulus of a soil decreases with:

(1) An increase in deviator stress.

(2) A soil disturbance.

b. The modulus of a soil increases with:

(1) An increase in consolidation stress.

(2) An increase in overconsolidation ratio.

(3) An increase in aging.

(4) An increase in strain rate.

Lambe and Whitman also stated that during the early range of strains for which the concepts of theory of elasticity are useful, the Poisson's ratio is varying with strain and for sands becomes constant only for large strains that imply failure; then it has a value greater than 0.5. Poisson's ratio is less than 0.5 only during the early stages of strain where the specimen decreases in volume. Because of this behavior, it is difficult to make an exact evaluation of ν. For early stages of a first loading of a sand, when particle rearrangements are important, ν typically has values of about 0.1 to 0.2. During cyclic loading, ν becomes more of a constant, with values from 0.3 to 0.4. However, satisfactory estimates of ν of low saturated and fully

saturated soils are 0.35 and 0.5, respectively, for most computations.*

Barkan[10] (E). Barkan concluded that the E modulus of sand containing no silt or clay does not change drastically with grain size. Table 7 shows how E varies with sands of different grain sizes.

Table 7

Grain Size of Sand and E Moduli
(After Barkan[10])

Grain Size of Sand, mm	E kg/cm^2
1.25-1.50	450
1.00-1.25	520
0.60-0.80	620
0.35-0.60	480
0.30-0.35	480
0.20-0.30	620

For clay, Barkan found that the E modulus depended on water content, grain size, and porosity. Figure 28 shows how E values obtained
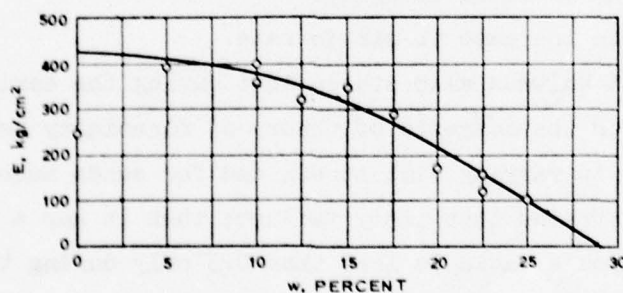
Figure 28. E versus water content for a clay (after Barkan[10])

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* Although values ≥0.5 for Poisson's ratio have been observed in the laboratory, these upper limits are not valid for use in elastic theory since the bulk modulus of elasticity, defined as $E/3(1 - 2\nu)$, calculates to infinity if $\nu = 0.5$.

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from compression tests on clay cubes varied with water content. An empirical formula that expresses the E modulus of clay in terms of water content is

$$E = E_o \left(1 - \frac{W^2}{W_o^2} \right)$$

where

E_o = value of modulus with zero moisture content

W = moisture content of clay at time of interest

W_o = moisture content of the clay for which the modulus is very small

Figure 29 presents Barkan's relationship between E of a clay and its void ratio, e. In a change of void ratio from 0.32 to 0.93, the value of E changes fourteenfold, from 1700 to 120 kg/cm^2.

Table 8 gives the values of E modulus determined by Barkan for nine different soils. It is apparent that soil name alone is no clue to an E value.

Barkan summarized his study of E values for soils with the following conclusions:

Figure 29. E versus void ratio for a clay (after Barkan[10])

> On the basis of these data, it is possible to conclude that Young's modulus for clayey soils depends to a large degree on physiomechanical properties. While the values of E for sands change comparatively little, those for clays and clays with silt and sand may change greatly depending on the moisture content, the void ratio, and, to a lesser degree, the grain size. Since the data now available are not sufficient for generalizations concerning the effects of these characteristics on Young's modulus, it is not possible to recommend any values of E for cohesive soils for use in design computations. It is necessary to determine the design values of E in each case by direct testing of undisturbed soil taken from the construction site.

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Table 8

Elastic Moduli for Nine Soils (After Barkan[10])

Soil Description	E , kg/cm^2
Plastic silty clay with sand and organic silt	310
Brown saturated silty clay with sand	440
Dense silty clay with some sand	2,950
Medium moist sand	540
Gray sand with gravel	540
Fine saturated sand	850
Medium sand	830
Loess	1,000-1,300
Loessial soil	1,200

Nielsen[34] (M_R). As indicated by this reference, for some soils the resilient modulus can be significantly greater than the modulus determined from a single static test and is an aeolotropic property in homogeneous deposits of fine-grained soils. Factors that influence the resilient modulus are method of compaction, initial compaction moisture, and stress history.

Monismith et al.[35] (M_R). These researchers listed the following factors as influencing the resilient moduli of soils:

 a. Fine-grained soils--stress intensity, number of stress applications, age at initial loading (thixotropy), compaction method, compaction density, water content, and subsequent changes in these.

 b. Untreated granular materials--confining pressure, stress level, duration of stress application, rate of application, rate of deformation, type and gradation of aggregate, void ratio, and degree of saturation.

The influence of stress intensity is the most important factor considered.

Robnett and Thompson[36] (E_r). The findings of Robnett and Thompson reinforce the opinion of Monismith as to the importance of stress intensity in resilient modulus testing. They state that it is apparent

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that any adequate characterization of subgrade soil support must recognize the stress dependent nature of the resilient modulus, E_r . They also recognized the important influence of compaction moisture content and density on the resilient properties of soil.

Figure 30 presents the load-time relationship of the resilient modulus testing equipment used by Robnett and Thompson.

Figure 31 shows resilient modulus in relationship to degree of saturation for eight different soils. The plot indicates that the resilient moduli decreases with an increasing degree of saturation.

Figures 32 and 33 show the influence of compaction moisture content on resilient modulus. Figure 32 is a graph of E_r versus applied deviator stress, with $\sigma_3 = 0$, for three soil types. For each soil type, relationships are shown for water contents of optimum and optimum plus 2 percent. At an applied

Figure 30. Typical specimen load-time relation developed by the resilient testing equipment (after Robnett and Thompson[36])

deviator stress of 5 psi (Figure 32), decreasing the moisture content by 2 percent increased E_r by 27 to 63 percent for the three soils. The effect of moisture content is fairly constant with applied deviator stress. Figure 33, which is a graph of E_r versus compaction moisture content relative to optimum, shows the effects of moisture content on seven soil types in addition to those shown in Figure 32. The E_r's of all 10 soil types decrease, some more drastically than others, with increasing moisture content.

Figure 34 shows the influence of density on the curves of resilient moduli versus applied deviator stress (axial stress since

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Figure 31. Resilient modulus versus degree of saturation for eight soils (after Robnett and Thompson36)
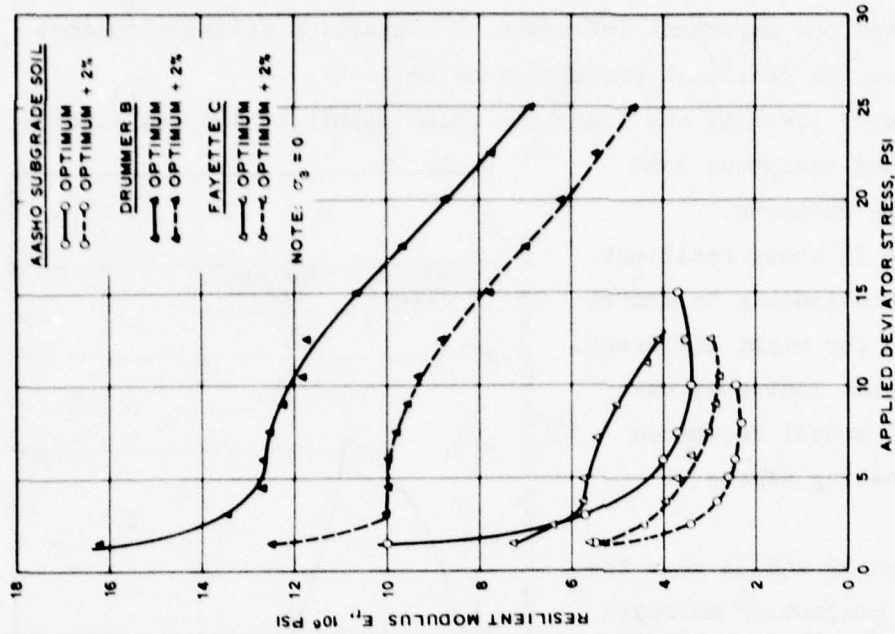
Figure 32. Relation between resilient moduli and applied deviator stress for three soils prepared at optimum and 2 percent above optimum and compacted to 95 percent of standard density (after Robnett and Thompson36)

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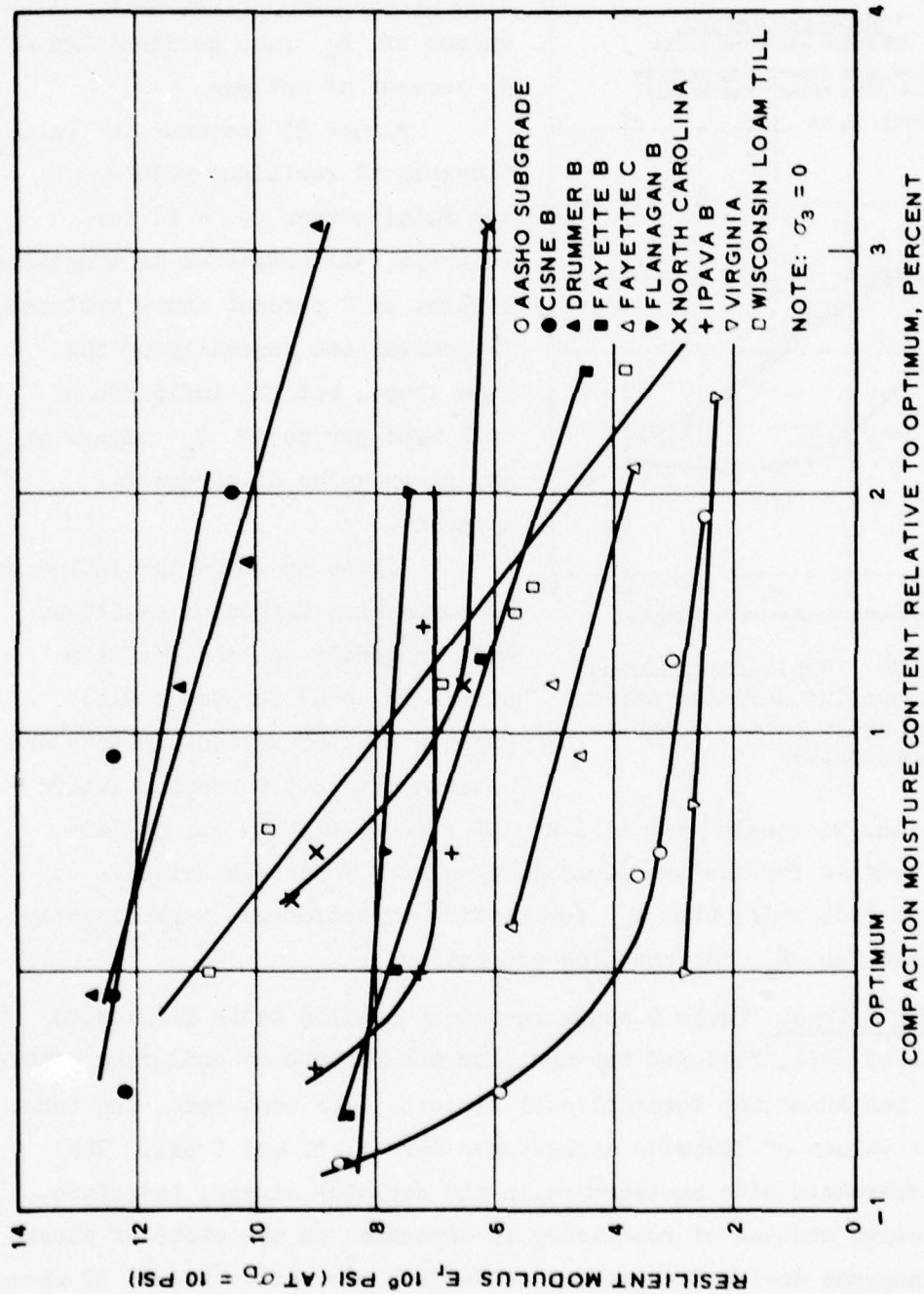
Figure 33. Relation between resilient moduli (at $\sigma_D = 10$ psi) and compaction moisture content for phase 1 soil samples compacted to 95 percent AASHO T-99 density (after Robnett and Thompson[36])

FLANAGAN B
OPTIMUM MOISTURE + 1.7%

▸--◂ 95% AASHO T-99 DENSITY
▸—◂ 100% AASHO T-99 DENSITY

WISCONSIN LOAM TILL
OPTIMUM MOISTURE + 2.5%

◻--◻ 95% AASHO T-99 DENSITY
◻—◻ 100% AASHO T-99 DENSITY

NOTE: $\sigma_3 = 0$
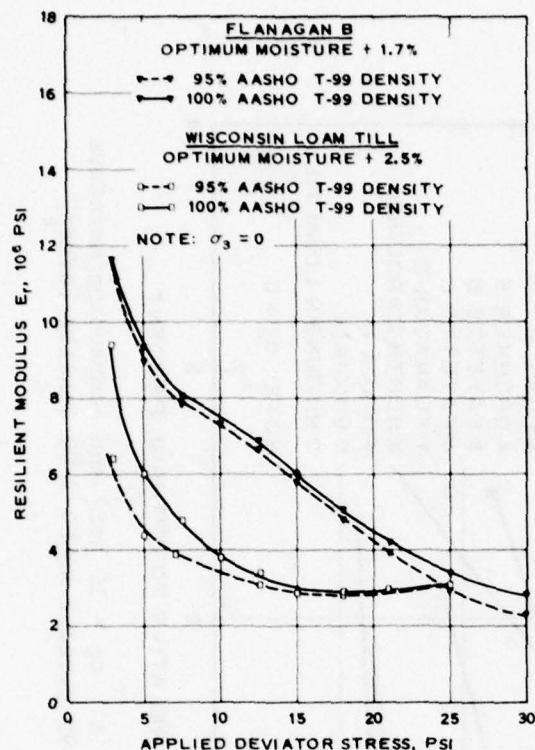
Figure 34. Resilient modulus versus applied deviator stress for two soils (after Robnett and Thompson [36])

σ_3 was zero) for two soils. It is apparent that densities of 100 percent of optimum correspond to higher values of E_r than do densities of 95 percent of optimum.

Figure 35 presents the relationship of resilient modulus E_r and axial stress ($\sigma_3 = 0$) for 10 soils, all compacted at a moisture content of 2 percent above optimum. The curves are generally of the same shape, but the influence of soil type per se on E_r values at any given value of stress is apparent.

Figure 36 shows the influence of compaction method on resilient modulus versus applied deviator stress ($\sigma_3 = 0$) for two soils: Fayette B at 95 percent density and optimum plus 2.5 percent moisture content, and Wisconsin loam till at 100 percent density and optimum plus an average for the two field samples of 2.7 percent moisture content. For each soil, the E_r for static compaction was significantly higher than the E_r for kneading compaction.

MRD[21] (M_R). Table 9 shows resilient modulus tests results on two types of soil, fill and bay mud, for the purpose of analyzing conditions at San Francisco International Airport. For each test, the three different values of deviator stress used were 1, 3, and 5 psi. The modulus decreased with an increase in the deviator stress; therefore, the effective modulus of resiliency is dependent on the state of stress of the subgrade during the passage of various aircraft. Figure 37 shows the dependency of M_R on repeated deviator stress for four samples from Table 9.

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Figure 36. Typical influence of compaction method on resilience (after Robnett and Thompson[36])

Figure 35. Relation between resilient moduli and applied stress for phase 1 soil samples compacted at optimum +2 percent moisture to 95 percent AASHO T-99 density (after Robnett and Thompson[36])

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Table 9

Summary of Resilient Modulus Test Results (After Materials Research and Development[21])

Sample No.	Resilient Modulus Deviator Stress, psi		
	1	3	5
Fill			
1L-2-2	4,050	2,930	2,780
1R-1-1	7,860	7,080	6,380
10L-1-1	4,780	3,920	3,790
10R-1-1	6,400	5,070	4,580
10R-6-1	3,210	2,110	1,900
F-1-1	3,300	2,610	2,760
Bay Mud			
1L-1-3	1,340	840	710
1L-2-3	810	670	650
1L-3-3	970	690	625
1L-6-2	1,280	870	785
1R-1-3	1,675	1,400	1,150
10L-1-2	2,065	1,440	1,170
10R-1-2	2,680	1,640	1,230
S-1-2	2,040	1,330	960
S-2-3	1,170	700	635
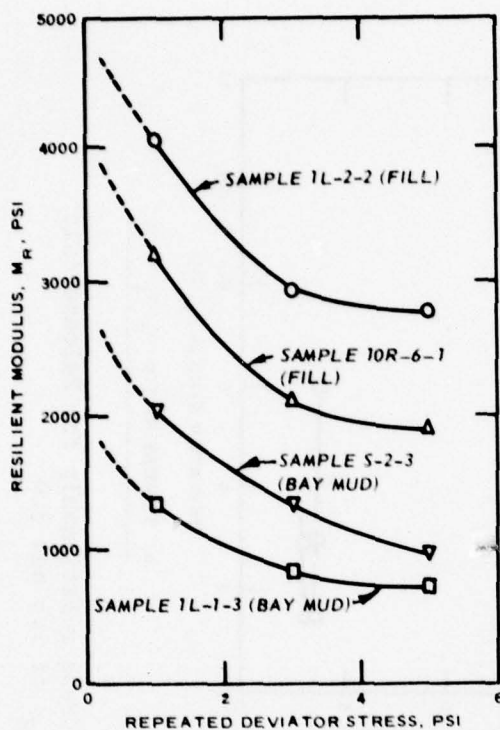
Figure 37. Typical examples of the stress sensitivity of the resilient modulus of soils (after Materials Research and Development[21])

Cook and Krukar[11] (M_R). Figure 38 presents two plots showing the results of repeated load triaxial compression tests on 6-in.-diam by 12-in.-high specimens, which were used in Cook and Krukar's analysis of subgrade soil (Palouse silt) under test tracks at WSU. These plots illustrate relationships of resilient modulus to deviator stress and are both for the same soil, at the same four moisture contents and densities on each. The difference between them is that the stress ratios, σ_d/σ_3, were 1.5 and 3.0 for Figures 38a and 38b, respectively. The shapes of the curves (A, B, C, and D) are nearly the same from one to the other; however, in-creasing the stress ratio from 1.5 to 3.0 decreased M_R for all four soil conditions. The changes in M_R with deviator stress are not con-sistent. M_R decreases with increases in deviator stress for curves A and C and increases with increases in deviator stress for curves B and D.

Foster and Heukelom[37] (E). Table 10 presents modulus values determined for six soils by Foster and Heukelom using wave velocity tests. The E values range from a low of 80 kg/cm^2 to a high of 9000 kg/cm^2; peats and clays have considerably lower values than do the more granular soils.

Yoder[20] (ν). Yoder indicated that Poisson's ratio for a soil is a "tenuous value which has not been determined experimentally with any degree of exactitude."

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Figure 38. Resilient modulus deviator stress relationship for Palouse silt for stress ratios (σ_d/σ_3) of 1.5 and 3.0

Table 10

Wave Velocities and Subgrade Modulus Values Observed on Various

Soils (After Foster and Heukelom[37])

Type of Soil	Wave Velocity m/sec	Subgrade Modulus, E	
		kg/cm^2	psi
Peat	50-110	80-330	1,130-4,670
Clay	90-150	440-1,200	6,230-17,000
Sand	120-180	800-1,800	11,300-25,500
Sandy clay	150-200	1,200-2,200	17,000-31,100
Clay gravel	190-320	2,100-6,000	29,700-84,900
Moraine*	250-400	3,500-9,000	49,500-127,000

* Glacial deposit.

Morgan and Scala[24] (ν). According to Morgan and Scala, the
theoretical upper limit of 0.5 for ν is appropriate to assume for a
saturated clay loaded under undrained conditions. Sands should be
assigned a value from 0.3 to 0.4. For first loadings, values from 0.11
to 0.91 were determined for dense sands initially loaded to high stress
in a conventional triaxial test. While looking for evidence that ν
was influenced by stress level, they found that little work had been
done on the subject. Their report indicated that Brown and Pell found
no systematic variation of ν with stress on a clay subgrade and that
values varied randomly between 0.2 and 0.6.*

Barkan[10] (ν). Working with Katsenelenbogen, Barkan studied the
effects of the moisture content of clay and an admixture of clay on
ν in compression and in a consolidometer. Their study revealed that
ν does not depend on moisture content but that increasing sand content
has the effect of decreasing ν . They also found the average value for
pure clay to be 0.50 and for clay with 30 percent sand to be 0.42. They
further generalized that ν is smaller for sands than for clays and

* Ibid. p. 50.

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suggested that ν for clays lies close to 0.5 and for sands from 0.30 to 0.35.

Barkan calculated values of ν of 0.3 for sands and from 0.41 to 0.43 for clays. He reported that G. I. Polrovsky computed ν from 0.38 to 0.40 for artificially prepared clay and that Ramspeck determined values of ν on the basis of wave velocity measurements and found values of 0.5 for moist clay, 0.44 for loess, and from 0.31 to 0.47 for sandy soils. Barkan also stated that N. A. Tsytovich recommends values from 0.15 to 0.25 for sandy soils, 0.30 to 0.35 for clay with some sand and silt, and 0.35 to 0.40 for clays.

Bowles[32] (ν). Bowles suggested the values of ν in Table 11.

Table 11

Typical Range of Values for Poisson's Ratio,

ν (After Bowles[32])

Type of Soil	ν
Clay, saturated	0.4-0.5
Clay, unsaturated	0.1-0.3
Sandy clay	0.2-0.3
Silt	0.3-0.35
Sand (dense)	0.2-0.4
Coarse (void ratio = 0.4-0.7)	0.15
Fine-grained (void ratio = 0.4-0.7)	0.25
Rock	0.1-0.4*

* Depends somewhat on type of rock.

These vary from 0.1 for rock and unsaturated clay to 0.5 for saturated clay.

Lysmer et al.[28] (G). The shear modulus of a soil can be computed from the results of Rayleigh wave velocity measurements using the following equation:

$$\gamma = \sqrt[n]{\frac{G}{\rho}}$$

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where

γ = Rayleigh wave velocity

n = factor depending upon Poisson's ratio ν

G = shear modulus

ρ = mass density

ρ and ν must be determined by conventional means.

Lysmer et al. reported that Hardin and Drnevich studied the factors affecting the shear moduli of soils and concluded that the most significant were strain amplitude, effective mean principal stress, void ratio, number of cycles of loading, and degree of saturation of cohesive soils. Of less importance were octahedral shear stress, overconsolidation ratio, effective stress strength parameters, and time effects. Lysmer et al. also made the following comments on the factors that influence the shear moduli of soils:

> The interrelationship of the parameters is very complex and presents a difficult problem if definitive and comprehensive expressions are sought to describe the behavior of soils subjected to dynamic loading. Laboratory programs have been conducted using forced and free longitudinal, torsional, shear and triaxial compression vibratory testing. All investigations have shown that the modulus values and damping factors for soils are strongly influenced by the strain amplitude. While many of the other factors, especially confining pressure and void ratio, are of great importance in laboratory studies, in situ vibratory testing in the field has the advantage that the need to account for the influence of a majority of these parameters is eliminated. The effects of sample disturbance are also eliminated. This fact is of major importance when saturated clays are involved.

Hardin[38] (G). Hardin revealed that the shear modulus of soils may vary between the extreme limits of values commonly less than 1,000 psi to values greater than 50,000 psi. The relationship between shear modulus and strain level is extremely variable but can be simplified by normalization as outlined in Reference 38. The normalized values are, however, still variable because a given strain does not have the same effect on all soils or on the same soil under different states

of stress. Also, the normalized relationship is affected by percent
saturation, number of cycles, and rate of loading. Table 12 gives
values of maximum shear moduli for 19 different soils. The largest
value is 14 times the smallest value listed in the table.

RELATIONSHIPS OF ELASTIC CONSTANTS
TO OTHER PARAMETERS IN SOILS

Since laboratory and field procedures for directly determining
elastic constants are relatively complicated and expensive, it is
desirable to express the constants in terms of more conveniently mea-
sured material properties or in terms of results of more readily per-
formed tests. Many researchers have attempted to do this, and the work
of several of them (not already covered in the section of the report
called "Values and Variations of Elastic Constants") is outlined (but
not necessarily completely described) in the following paragraphs. No
significant relationships of this type were found for wearing surfaces
or treated base materials; accordingly, the following discussion per-
tains solely to untreated base materials and subgrade materials, i.e.
soils. The elastic constant(s) discussed in a paragraph is (are) shown
in parentheses at the end of the paragraph heading. Quantitative data
and symbols are given in the same terms as used by the referenced
authors.

EQUATIONS RELATING MODULUS
AND STATE OF STRESS

Hicks[39] and Dunlap[40] (M_R). Two expressions from Hicks and one
from Dunlap that relate resilient modulus to the state of stress exist-
ing in a material are:

<u>a</u>. $M_R = K_1 \sigma_3^{K_2}$ (Hicks).

<u>b</u>. $M_R = K_1' \theta^{K_2'}$ (Hicks).

<u>c</u>. $M = K_2'' + 2K_3'' \sigma_r$ (Dunlap).

Table 13 lists the values of K_1, K_2, K_1', K_2', K_1'', and K_2''.
The three factors, σ_3, θ, and σ_r, represent confining stress,

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Table 12

Values of Maximum Shear Modulus (After Hardin[38])

Soil Identification	Percent Saturation	G max* psi
WES sand	Dry	18,830
St. John's sand	100	12,580
Air Force silty sand	33	8,050
Air Force silty clay	91	5,200
Vicksburg loess	73	12,130
Vanceburg	91	14,290
Allen	44	11,000
Kentucky 55	100	6,020
Longhorn	70	18,360
West Virginia shale	69	11,740
Virginia clay	94	11,290
Dover	86	16,810
Prestonsburg sand	47	11,220
Kirtland No. 10-36	40	12,840
Louisiana clay	100	1,350
San Francisco clay	100	4,800
Ellsworth	66	10,030
Cheeks	98	9,790
Nevada clay	99	4,940

* G max = maximum shear modulus = the initial tangent modulus or secant modulus for strain amplitude $\geq$ 0.00001.

Table 13

Material Constant Values Proposed for Various Granular Materials (After Hicks[39] and Dunlap[40])

Description of Material	Constants	

Expression: $M_R = K_1 \sigma_3^{K_2}$

	K_1	K_2
Dry, partially crushed gravel	10,094	0.580
Dry, crushed gravel	13,126	0.550
Partially saturated, partially crushed gravel	7,650	0.591
Partially saturated, crushed gravel	8,813	0.569
Saturated, partially crushed gravel	9,894	0.528
San Diego base	12,225	0.540
Gonzales Bypass base	15,000	0.480
Gonzales Bypass subbase	10,000	0.400
Morro Bay base	11,800	0.390
Morro Bay subbase	6,310	0.430

Expression: $M_R = K_1' \theta^{K_2'}$

	K_1'	K_2'
San Diego base	3,933	0.61
Dry, crushed gravel	2,156	0.71
Partially saturated, crushed gravel	2,033	0.67
Morro Bay subbase	2,900	0.47
Morro Bay base	3,030	0.53

Expression: $M = K_2'' + 2K_3'' \sigma_r$

	K_2''	K_3''
Crushed limestone	4,856	390
Crushed limestone after 36,000 repetitions	37,710	1082

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bulk stress that equals $\sigma_1 + \sigma_2 + \sigma_3$, and radial stress in a triaxial test, respectively.

Morgan and Scala[24] (E_s , M_R , and E_t). Morgan and Scala reported the following equations attributable to Brown and Pell, Seed et al., and Holden:

    <u>a</u>. $E_s = 2040J_1^{0.57}$ (Brown and Pell) where E_s = calculated value of in situ secant modulus and J_1 = first stress invariant = $\sigma_1 + \sigma_2 + \sigma_3$.

    <u>b</u>. $M_R = 1900J_1^{0.61}$ (Seed et al.) where M_R = secant modulus of resiliency at a constant confining pressure determined from a triaxial test and $J_1 = \sigma_1 + \sigma_2 + \sigma_3$.

    <u>c</u>. $E_t = a + b\sigma_{oct} - c\tau_{oct}$ (Holden) where E_t = tangent modulus of resiliency, a , b , and c = constants, $\sigma_{oct} = 1/3 J_1$ $= 1/3(\sigma_v + 2\sigma_r)$ from a triaxial test, and τ_{oct} $=\sqrt{2/3(\sigma_v - \sigma_r)}$. Subcripts v and r refer to vertical and radial, respectively.

Nielsen[34] (M). The USN Civil Engineering Laboratory (USNCEL) acknowledged Dunlap's equation relating resilient modulus to the state of stress in a material as useful to explain the response of granular soils to repeated loadings. USNCEL also pointed out that Dunlap's modulus is the slope of the initial portion of the stress-strain curve in a triaxial test and is defined as follows:

$$M = k_2 + 2k_3\sigma_r$$

where

      M = modulus of deformation measured in the direction of the applied stress, σ_z

    k_2 = modulus of deformation when $\sigma_r = 0$

    k_3 = constant of proportionality expressing the influence of σ_r on M

    σ_r = radial (lateral) stress in triaxial test

Figure 39 illustrates the equation. For a single, slowly applied normal stress that continually increased until the sample fails, molding moisture and unit weight have significant influence on the deformation constants, k_2 and k_3 . Under repetitive stresses, these constants are
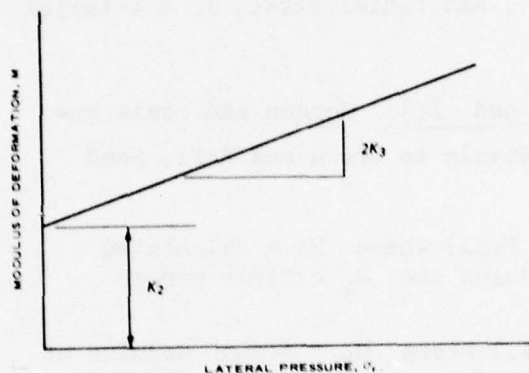
Figure 39. Relationship between modulus of deformation and lateral pressure proposed for granular material (after Dunlap[40])

influenced by the number of load repetitions applied to the specimen.

Glynn and Kirwan[13] (E_r). Glynn and Kirwan offer the general expression, suggested by earlier work of Barkan:

$$E_r = E_o (1 + \alpha\sigma)$$

where

E_r = resilient modulus in a confined state

E_o = initial modulus in an unconfined state

α = constant for any one soil

σ = stress component

Wang et al.[41] For resilient modulus of a highly plastic soil, Wang et al. used

$$M_R = K_1 + (K_2 - \sigma_d)K_3 \quad \text{for} \quad \sigma_d < K_2$$

$$M_R = K_1 + (\sigma_d - K_2)K_4 \quad \text{for} \quad \sigma_d > K_2$$

where

K_1, K_2, K_3, and K_4 = material constants

σ_d = deviator stress

RESILIENT MODULUS OF CLAY IN TERMS
OF PLASTICITY INDEX (PI), DENSITY,
AND MOISTURE CONTENT

Glynn and Kirwan[13] have developed a relationship between

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resilient modulus, PI, moisture content, and density (Figure 40). The
term r_w is the ratio of the actual moisture content to the optimum
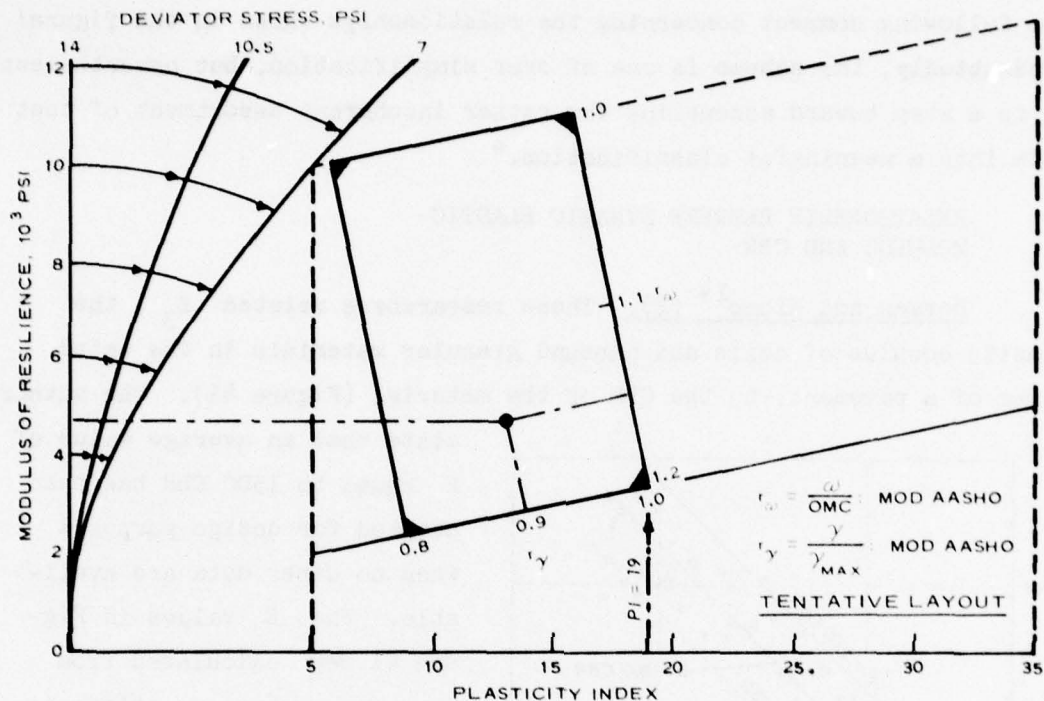
Figure 40. Dynamic stress response of clays
(after Glynn and Kirwan[13])

moisture content, and r_γ is the ratio of dry density to the maximum
dry density of the modified AASHO test. Glynn and Kirwan did not offer
an example showing how to use Figure 40. Apparently, the rectangle on
which scales for r_γ and r_w are presented must be moved along the axis
on which r_γ is shown until the value of 1.0 for r_γ lines up with the
measured PI. The lines designated by long and short dashes in Figure 40
are for a clay with a PI of 19, r_w of 1.15, and r_γ of 0.9. For
these parameters, the modulus of resilience is approximately 4900 psi for
a deviator stress of 7 psi; 4800 psi for a deviator stress of 10.5 psi;
and 4700 psi for a deviator stress of 14 psi. Figure 40 is limited to
deviator stresses between 7 and 14 psi, densities from 80 to 100 percent
of modified AASHO, moisture contents from 100 to 120 percent of optimum,
and PI's from 5 to 25. However, it is comprehensive in that a method is

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suggested whereby resilient moduli can be established from fairly simple soils testing. It also considers moisture content, density, and stress, which are three major influences on resilient moduli. The authors offer the following comment concerning the relationships shown in the figure: "Undoubtedly, the scheme is one of over simplification, but nevertheless, it is a step toward assembling the rather incoherent assortment of test data into a meaningful classification."

RELATIONSHIP BETWEEN DYNAMIC ELASTIC MODULUS AND CBR

Dorman and Klomp[14] (E). These researchers related E_3, the elastic modulus of soils and unbound granular materials in the third layer of a pavement, to the CBR of the material (Figure 41). The authors state that an average value of E equal to 1500 CBR has been adopted for design purposes when no other data are available. The E values in Figure 41 were calculated from wave propagation or stiffness measurements.
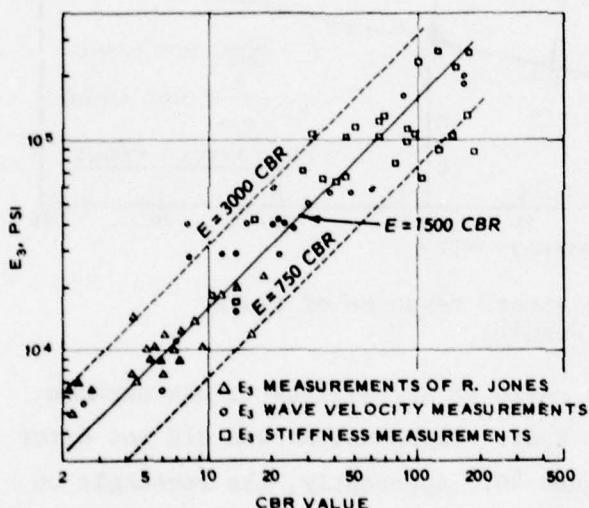
Figure 41. Relation between dynamic modulus and CBR (after Dorman and Klomp[14])

Robnett and Thompson[36] (E_r). These authors developed a graph, Figure 42, of soaked CBR versus resilient modulus, E_r, determined at a stress level equivalent to the calculated vertical compressive stress under a pavement. They concluded from this study that no relationship existed between E_r and CBR and observed that soils of similar CBR can exhibit substantially different resiliencies.

RELATIONSHIP BETWEEN MODULUS OF ELASTICITY AND PLATE BEARING VALUE

The Asphalt Institute[12] developed an approximate relationship

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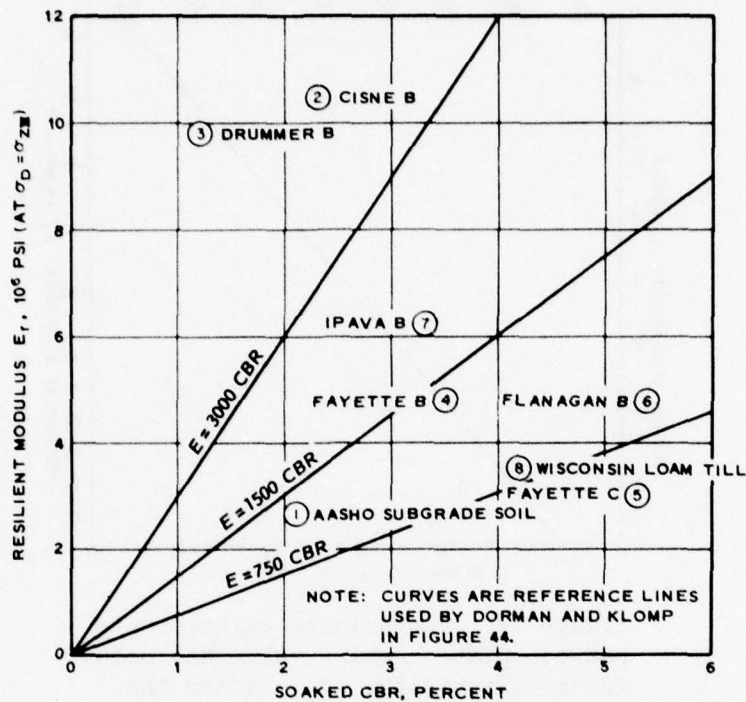
Figure 42. Resilient modulus versus soaked CBR
(after Robnett and Thompson[36])

between subgrade modulus of elasticity and bearing value (Figure 43).
The plate bearing test uses a 30-in.-diam bearing plate, 0.5-in. deflec-
tion, and 10 repetitions of load. The equation of the line in Figure 43
is E_s(psi) = 480 (bearing value) - 5000 .

EQUATION FOR SHEAR MODULUS
OF SOILS

Hardin's equation for the shear modulus of gravels has been pre-
sented earlier in the report. His summary equation for the shear modu-
lus of soils[18] (excluding gravels and gravelly soils) is

$$\frac{G}{G_{max}} = \frac{1}{1 + \gamma_h}$$

with
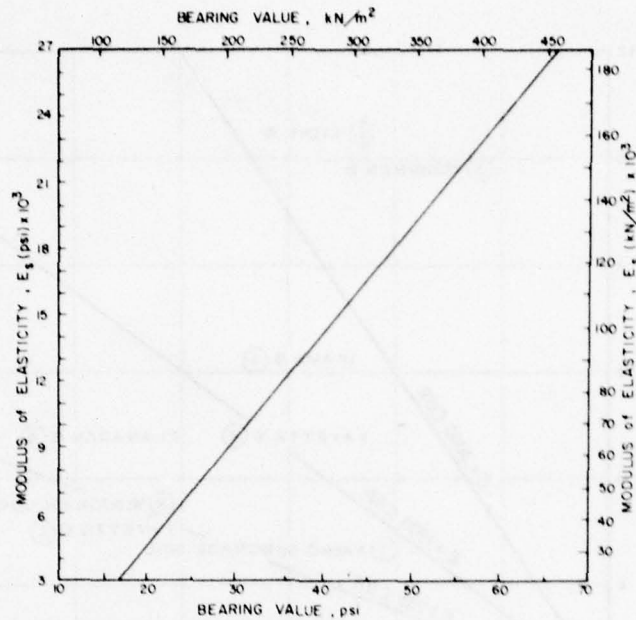
BEARING VALUE, kN/m²

Figure 43. Approximate relationship
between plate bearing value and modu-
lus of elasticity, E_s (after the
Asphalt Institute[12])

$$\gamma_h = \frac{\gamma}{\gamma_r} \left[1 + ae_{xp} - \left(\frac{\gamma}{\gamma_r}\right)^{0.4} \right]$$

where γ_h = hyperbolic strain

   G = secant shear modulus for a given strain amplitude and
       cycle

   G_{max} = maximum shear modulus, initial tangent modulus, or secant
       modulus for strain amplitude ≤ 0.00001

   γ = shear strain

   γ_r = reference strain = τ_{max}/G_{max} , defined by the intersection
       of the initial tangent line and strength asymptote in Fig-
       ure 44 and empirically related to G_{max}

   e_{xp} = base of natural logarithms

   a = defined by one of the following equations, depending on
       the type of soil

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Figure 44. Schematic single
shear-stress strain relation
(after Hardin[18])
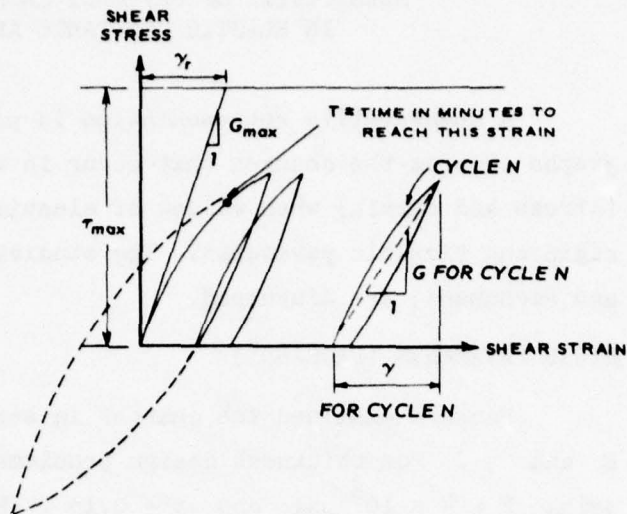
$$
a = \begin{cases}
\left[\left(\dfrac{3.85}{N}\right) - 0.85\right] T^{0.025} & \text{for clean dry sand} \\[2em]
1.6(1 + 0.02S)\,\dfrac{T^{0.2}}{N^{0.6}} & \text{for nonplastic soils with fines and low-plasticity soils} \\[2em]
0.2(1 + 0.2S)\,\dfrac{T^{0.75}}{N^{0.15}} & \text{for high-plasticity soils with liquid limit > 50}
\end{cases}
$$

where

N = number of cycles of loading

T = time in minutes to reach a normalized strain equal to one

S = percent saturation

For pavement evaluation, Hardin suggested that the type of soil (including PI and particle size) and S would have to be estimated from available knowledge of the subgrade or from a core sample; G_{max} would be measured by nondestructive vibratory testing; N would come from traffic records; γ would be determined by the finite-element analysis; and T would depend on the speed of the aircraft. (The complete procedure for determining G also requires the measurement or estimation of the void ratio, e , and involves several intermediate steps not shown in this report.)

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SENSITIVITY OF PAVEMENT RESPONSES TO CHANGES
IN ELASTIC CONSTANTS AND THICKNESS

A quantitative representation is provided in the following para-
graphs showing the changes that occur in values of pavement responses
(stress and strain) when values of elastic constants are changed in
rigid and flexible pavements. The studies of two researchers, Packard
and Pichumani, are discussed.

RIGID PAVEMENTS (PACKARD[4])

Packard examined the changes in stress that occur with changes in
E and ν. For thickness design problems in PCC, Packard recommended
using $E = 4 \times 10^6$ psi and $\nu = 0.15$. He gave the following approxi-
mate effects of variations in E and ν from the recommended values.

CHANGE IN E ($\nu = 0.15$)

A reduction in E from 4,000,000 to 3,000,000 psi decreases the
stress 5 percent. An increase in E from 4,000,000 to 5,000,000 psi
increases stress 4 percent.

CHANGE IN ν ($E = 4 \times 10^6$ psi)

An increase in ν from 0.15 to 0.20 and from 0.15 to 0.25 in-
creases stress 4 and 8 percent, respectively.

RIGID PAVEMENTS (PICHUMANI[5])

Pichumani made a parametric study of both rigid and flexible pave-
ment systems, using the Airfield Pavement (AFPAV) Code to show the
effect of input material properties on theoretical predictions of pave-
ment response. The AFPAV Code is an analytical model based on the fi-
nite element structural analysis technique and was developed for the Air
Force Weapons Laboratory (AFWL) by J. E. Crawford. The following dis-
cussion is an excerption from Pichumani's report.

BASE CONDITIONS OF STUDY

Load. The load used was the static load of a C-5A aircraft, i.e.,
30 kips on each wheel of a 12-wheel assembly.

Parameters Varied. Table 14 presents the rigid pavement param-
eter variations studied.

VARIATION IN E OF SURFACE COURSE

Unchanged Parameters (Table 15). In the surface course, thick-
ness, h , was 10 in. and ν was 0.20; in the subgrade, E was
6000 psi and ν was 0.45. The surface course rested directly on the
subgrade.

Results (Table 15). A decrease in surface course E from
6,600,000 to 3,000,000 psi resulted in an increase in surface deflection
from 0.098 to 0.116 in. (18 percent).

VARIATION IN h OF SURFACE COURSE

Unchanged Parameters (Table 15). In the surface course, E was
6,600,000 psi and ν was 0.20; in the subgrade, E was 6,000 psi and
ν was 0.45. The surface course rested directly on the subgrade.

Results (Table 15). When surface course h was increased from
10 to 14 in., the corresponding change in surface deflection was from
0.098 to 0.081 in. (a decrease of 17 percent).

VARIATION IN E OF SUBGRADE

Unchanged Parameters (Table 16). The surface course rested
directly on the subgrade. The surface course E was 6,600,000 psi with
ν of 0.20 and h of 10 in.; the subgrade ν was 0.45.

Results (Table 16). A change in subgrade E from 6000 to
3000 psi increased surface deflection from 0.098 to 0.166 in., a 69 per-
cent increase; a further decrease of E to 1000 psi caused the deflec-
tion to increase to 0.387 in., a 295 percent increase over the deflec-
tion of 0.098 when E was 6000 psi.

EFFECT OF BASE COURSE

In this permutation, a base course was inserted between the sur-
face course and the subgrade.

Unchanged Parameters (Table 17). The constant parameters were

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Table 14

Rigid Pavement Parameter Variations Studied (After Pichumani[5])

| Surface Course Parameters | | | | Subgrade Parameters | | |
Thickness h, in.	Modulus of Elasticity, E psi	Poisson's Ratio ν	Layer	Thickness h, in.	Modulus of Elasticity E, psi	Poisson's Ratio ν
10.0	6,600,000	0.20	1	600	6,000	0.45
10.0	5,000,000	0.20	1	600	6,000	0.45
10.0	3,000,000	0.20	1	600	6,000	0.45
10.0	6,600,000	0.20	1	600	6,000	0.45
12.0	6,600,000	0.20	1	600	6,000	0.45
14.0	6,600,000	0.20	1	600	6,000	0.45
10.0	6,600,000	0.20	1	600	6,000	0.45
10.0	6,600,000	0.20	1	600	3,000	0.45
10.0	6,600,000	0.20	1	600	1,000	0.45
10.0	6,600,000	0.20	1	12	24,000	0.35
			2	588	6,000	0.45
10.0	6,600,000	0.20	1	18	36,000	0.35
			2	582	6,000	0.45
10.0	6,600,000	0.20	1	24	60,000	0.35
			2	576	6,000	0.45

Table 15

Effect of Parameter Variations in Surface Course on Rigid

Pavement Deflection (After Pichumani[5])

Parameters		Maximum Elastic Surface Deflection,	Remarks
E , psi	h , in.	in.	
6,600,000	10	0.098	
5,000,000	10	0.105	E modulus only varied
3,000,000	10	0.116	
6,600,000	10	0.098	
6,600,000	12	0.088	Thickness, h , only varied
6,600,000	14	0.081	

Note: The surface course rested directly on the subgrade; the E
modulus and Poisson's ratio of the subgrade were 6000 psi and
0.45, respectively.

Table 16

Effect of Parameter Variations in Subgrade on

Rigid Pavement Deflection (After Pichumani[5])

Parameters		Maximum Elastic Surface Deflection in.
E psi	ν	
6,000	0.45	0.098
3,000	0.45	0.166
1,000	0.45	0.387

Note: The surface course with E modulus
= 6,600,000 psi and Poisson's ratio, ν ,
= 0.20 rested directly on the subgrade.

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Table 17

Effect of Base Course on Rigid Pavement Response
(After Pichumani[5])

	Parameters		Maximum Elastic Surface Deflection in.
h , in.	E , psi	ν	
12	24,000	0.35	0.093
18	36,000	0.35	0.090
24	60,000	0.35	0.084

Note: The surface course parameters were E modulus = 6,600,000 psi, Poisson's ratio, ν , = 0.20, and thickness, h , = 10 in.; the subgrade parameters were E = 6,000 psi and ν = 0.45.

E = 6,600,000 psi, ν = 0.20 , and h = 10 in. for the surface course; E = 6,000 psi and ν = 0.45 for the subgrade; and ν = 0.35 for the base course.

Results (Table 17). Variations were made in E and h of the base course. When the subgrade E was 24,000 psi and its h was 12 in., the surface deflection was 0.093 in. This value is only slightly lower than in the comparable "no base course" case (0.098 in.). When the base course E was 60,000 psi and its h was 24 in., the resulting pavement response (0.084 in.) was very close to the 0.081-in. deflection in the no base course case where the E of the surface course was the same (6,600,000 psi), but its h was 4 in. thicker, i.e. 14 in. thick (compare bottom data line of Table 17 with bottom data line of Table 15).

FLEXIBLE PAVEMENTS (PICHUMANI[5])

BASE CONDITIONS OF STUDY

Load. The load was the same as for the rigid pavement study, i.e. the static load of a C-5A.

Parameters Varied. Table 18 lists the values of E , ν , and h in the surface course, base course, subbase, and subgrade assumed as

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Table 18

Assumed Layer Properties of Flexible Pavement

(After Pichumani[5])

Layer	E Modulus psi	Poisson's Ratio, ν	Thickness h , in.
Surface course	150,000	0.25	3
Base course	50,000	0.30	6
Subbase	25,000	0.35	24
Subgrade	5,000	0.45	∞

reference values in this study. Table 19 presents the variations of these values studied. Only the value of one parameter in one layer was varied for each permutation. All other values remained the same as given in Table 18.

EFFECT OF E IN EACH LAYER

Tables 20 and 21 show that the surface elastic deflections were not changed significantly by variations in the E modulus of the surface course, base course, and subbase within the range of values presented in Table 19. However, Table 22 indicates that variations in Young's modulus of the subgrade affected the surface deflection very significantly. For example, a 67 percent reduction in the E modulus of the surface course (i.e. from 1,500,000 to 500,000 psi) resulted in an increase of only 8 percent in surface deflection (from 0.157 to 0.170 in.), while a 50 percent reduction in the subgrade E modulus (from 5,000 to 2,500 psi) increased the surface deflection by more than 60 percent (from 0.183 to 0.299 in.).

Pichumani observed that even though the E modulus values of the base course and subbase may not be very significant for surface elastic deflections, they are very important in determining the state of stress in the upper layers of the pavement system. Therefore, it is necessary to determine the E moduli of the various layers as accurately as possible since structural failure of the pavement system can be caused

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Table 19
Flexible Pavement Parameter Variations Studied
(After Pichumani[5])

Layer	Parameter Varied Symbol	Unit	Values of Parameter Varied
Surface course	E	psi	1,500,000; 500,000; 150,000
Surface course	ν	--	0.20; 0.25; 0.30
Surface course	h	in.	3.0; 4.5; 6.0
Base course	E	psi	75,000; 50,000; 37,500
Base course	ν	--	0.25; 0.30; 0.35
Base course	h	in.	6.0; 9.0; 12.0
Subbase	E	psi	37,500; 25,000; 12,500
Subbase	ν	--	0.30; 0.35; 0.40
Subbase	h	in.	15.0; 24.0; 33.0
Subgrade	E	psi	5,000; 2,500; 1,250
Subgrade	ν	--	0.40; 0.45; 0.495
Existing ground profile	E	psi	5,000; 50,000; 500,000

Note: E = modulus of elasticity ; ν = Poisson's ratio ; h = layer thickness.

Table 20

Effect of Parameter Variations in Surface Course
on Flexible Pavement Deflection
(After Pichumani[5])

Parameters			Maximum Elastic Surface Deflection in.	Remarks
E , psi	ν	h , in.		
1,500,000	0.25	3.0	0.157	E modulus only varied
500,000	0.25	3.0	0.170	
150,000	0.25	3.0	0.183	
150,000	0.20	3.0	0.183	Poisson's ratio, ν , only varied
150,000	0.25	3.0	0.183	
150,000	0.30	3.0	0.182	
150,000	0.25	3.0	0.183	Thickness, h , only varied
150,000	0.25	4.5	0.174	
150,000	0.25	6.0	0.168	

Note: The properties of the other layers are given in Table 18.

Table 21

Effect of Parameter Variations in Base Course
and Subbase on Flexible Pavement Deflection
(After Pichumani[5])

Parameters			Maximum Elastic Surface Deflection	
E , psi	ν	h , in.	in.	Remarks

Base Course

E , psi	ν	h , in.	in.	Remarks
37,500	0.30	6	0.187	E modulus only varied
50,000	0.30	6	0.183	
75,000	0.30	6	0.178	
50,000	0.25	6	0.183	Poisson's ratio, ν , only varied
50,000	0.30	6	0.183	
50,000	0.35	6	0.182	
50,000	0.30	6	0.183	Thickness, h , only varied
50,000	0.30	9	0.174	
50,000	0.30	12	0.167	

Subbase

E , psi	ν	h , in.	in.	Remarks
12,500	0.35	24	0.207	E modulus only varied
25,000	0.35	24	0.183	
37,500	0.35	24	0.171	
25,000	0.30	24	0.182	Poisson's ratio, ν , only varied
25,000	0.35	24	0.183	
25,000	0.40	24	0.183	
25,000	0.35	15	0.197	Thickness, h , only varied
25,000	0.35	24	0.183	
25,000	0.35	33	0.168	

Note: The properties of the other layers are given in Table 18.

Table 22

Effect of Parameter Variations in Subgrade on Flexible
Pavement Deflection (After Pichumani[5])

| Parameters | | Maximum Elastic Surface Deflection | |
E , psi	ν	in.	Remarks
5,000	0.450	0.183	E modulus only varied
2,500	0.450	0.299	
1,250	0.450	0.493	
5,000	0.400	0.215	Poisson's ratio, ν , only varied
5,000	0.450	0.183	
5,000	0.495	0.141	

Note: The properties of the other layers are given in Table 18.

by critical load stresses exceeding the inherent strength of the pavement materials.

EFFECT OF ν IN EACH LAYER

Tables 20 and 21 show that surface deflections were not significantly affected by ν of the surface course, base course, or subbase for the range of thicknesses studied; however, Table 22 shows that ν of the subgrade influenced the deflections significantly. A ν of 0.40 for the subgrade gave a maximum surface deflection of 0.215 in., while a ν of 0.495 gave a maximum deflection of only 0.141 in.

EFFECT OF h IN UPPER LAYERS

Increasing the thicknesses of the upper pavement layers (i.e., the surface course (Table 20), base course (Table 21), and subbase (Table 21)) decreased the surface deflections. For example in Table 21, when the subbase thickness (originally 24 in.) was altered to 15 and 33 in., an increase of 8 percent (0.183 to 0.197 in.) and a decrease of 8 percent (0.183 to 0.168 in.) in surface deflection, respectively, resulted.

EFFECT OF E OF EXISTING GROUND

In the finite element idealization of the pavement system, the total depth of the pavement structure was assumed to be 50 ft, and the subgrade was considered to have a constant E modulus. The total built-up depth of an existing WES flexible pavement test section was only 12 ft. All other parameters of the WES section were exactly the same as assumed by Pichumani in this exercise. The E modulus of the ground below the built-up subgrade was not known. However, based on the reported CBR value of 2 to 4, the E modulus of the ground was assumed to be the same as that of the subgrade. To illustrate the importance of knowing the E modulus of the existing ground below a depth of 12 ft, a parametric study assuming three different moduli values (5,000, 50,000, and 500,000 psi) was conducted. The maximum surface deflection of 0.183 in. with an E modulus of 5,000 psi was reduced by one third to 0.121 in. when the existing ground was assumed to have an E modulus of 50,000 psi. An E modulus of 500,000 psi below 12 ft further reduced the maximum surface deflection to 0.108 in., a reduction of about 40 percent. Therefore, it appears that it is necessary to determine the properties of the pavement system to an appreciable depth.

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RELATIONSHIPS BETWEEN VIBRATORY TEST
RESULTS AND ELASTIC CONSTANTS

In the following paragraphs, brief discussions are given of the principal mathematical models of pavement response to vibratory tests, vibrator wave patterns, comments on relating vibrator results and elastic constants, extrapolation of constants from low to high stress levels, and results of preliminary studies to establish a relationship between the WES 16-kip vibrator test results and elastic modulus.

MATHEMATICAL MODELS

In the study of how the elastic constants of E , ν , or G are to be extracted from vibratory test results, it is obviously desirable to know how the soil or pavement is affected by the vibratory load. The two types of models that describe the response are the lumped mass model and the linear elastic half-space model. The primary difference between these two models is that the former one assumes that there is a finite mass of soil, which acts as a spring for the foundation and vibrates together with it, while the latter model assumes no finite mass of soil. Several particular models that fit into one of these two types are described briefly in the following paragraphs.

LINEAR LUMPED MASS MODEL
(TSCHEBOTARIOFF[42])

For the lumped mass model, Tschebotarioff presented the equation

$$f_n = \frac{1}{2\pi} \sqrt{\frac{K'Ag}{W_s + W_v}}$$

where

f_n = natural frequency of system

K' = dynamic modulus of soil reaction, or volume spring coefficient

A = contact area between base of foundation and soil

g = acceleration of gravity

W_s = weight of vibrating soil

W_v = weight of vibratory loading device

Figure 45 shows the conditions of loading, and Figure 46 illustrates how the soil vibrates together with the vibrator. Tschebotarioff offers that there cannot be a clearly defined limit for the soil mass W_s as indicated by Figure 45. Figure 46 shows that the motions of the soil are more complex than the motion of the vibrator. He suggests that the term W_s should be considered as referring to an equivalent weight of soil with no clearly defined physical boundaries.

The pertinent point to be made by the equation for f_n is that the elastic constants of the soil are assumed present in the terms W_s and K' ; however, the process by which they can be extracted from this simple model, which is for a single-mass system and assumes no damping, is not clear. Then, of course, the equation assumes a homogeneous soil mass. Working with a layered pavement system obviously renders the system much more complicated.

LINEAR LUMPED MASS MODEL (YANG[43])

Yang uses the linear spring-mass-dashpot model to describe the frequency sweep measurements done with a vibrator. He computes an effective E value for the entire pavement and subgrade system by calculating the area under the measured frequency response curve. By neglecting damping and inertial terms, he arrives at the following expression for the E modulus:

$$E = \frac{1}{2a}\left(\frac{1}{\frac{1}{2F}\int_1^\infty \frac{z(\mu)}{\mu}\,d\mu}\right)$$

where

a = radius of vibrator baseplate

F = force amplitude

$z(\mu)$ = peak-to-peak response

μ = ratio of frequency to natural frequency

Yang applies the single lumped-mass spring model, which exhibits only one resonance peak, to the entire multipeaked frequency response spectrum.

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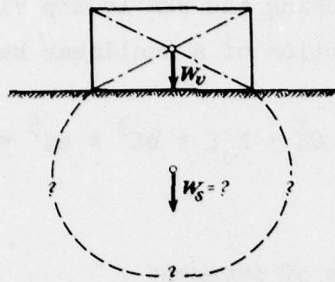
Figure 45. Sketch illustrating the equation
$$f_n = 1/2\pi\sqrt{K'Ag/W_s + W_v}$$
(after Tschebotarioff[42])
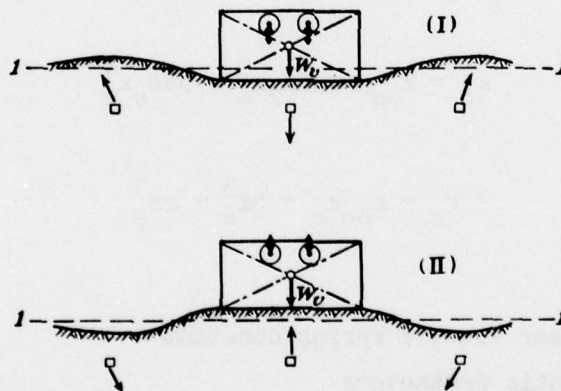
Figure 46. Probable nature of the soil-surface deformation around a vibrator inducing vertical oscillations (after Tschebotarioff[42])

NONLINEAR LUMPED MASS MODEL WEISS[2]

In his study of the dynamic stiffness of pavements, Weiss developed a nonlinear model that describes the response of pavement systems to vibratory loading. The nonlinear dynamic load-deflection curves that are measured using the WES 16-kip vibrator are described by solving the equation of motion of a nonlinear harmonic oscillator

$$m\ddot{\xi} + C\ddot{\xi} + k_o\xi + b\xi^3 + e\xi^5 = F_D$$

where

m = effective mass of pavement

ξ = dynamic deflection of the pavement surface

C = damping constant

k_o = spring constant that depends on static load of vibrator

b = third-order nonlinear plastic parameter

e = fifth-order nonlinear elastic parameter

F_D = dynamic load

The spring constant k_o is related to the static load by the equations

$$k_o = k_{oo} + 3b\varepsilon_2 x_e^2 + 5e\varepsilon_e x_e^4$$

$$F_s = k_{oo}x_e + bx_e^3 + ex_e^5$$

where

k_{oo} = linear elastic spring constant

$\varepsilon_2, \varepsilon_4$ = elastic parameters

x_e = static elastic deflection of pavement surface beneath the vibrator baseplate

F_s = static load of vibrator

The elastic constants appear in the expressions for the elastic parameters k_{oo} , b , and e . For example, for a subgrade the expression for the parameter k_{oo} is

$$k_{oo} = \frac{2\pi a^2 Q_1 G_1 \Psi}{\ell_o}$$

where

    k_{oo} = linear elastic parameter of a nonlinear pavement

    a = radius of the vibrator baseplate

    Q = function of Poisson's ratio

    G = shear modulus of homogeneous elastic half-space

    Ψ = expansion parameter

    ℓ_o = finite depth of influence of the static strain field

More general expressions for k_{oo} , b , and e for the case of a
layered pavement appear in Reference 2. Weiss's procedure was developed
to describe the measured nonlinear dynamic load-deflection curves. How-
ever, Weiss[3] has also developed a linear model of the frequency response
spectrum while a nonlinear model of the frequency response spectrum is
under study. The linear model is found to be inadequate to describe the
frequency response spectrum.

    The nonlinear dynamical theory of Weiss can also be applied to
the laboratory resilient modulus measurement.[3] This theory gives an
analytical expression for the resilient modulus in terms of the dynamic
deviator stress, static confining pressure, frequency, and a set of
basic soil parameters including the Young's modulus. The nonlinear
dynamical theory can be used to extract the static elastic Young's
modulus value from the measured dynamic resilient modulus.

FINITE ELEMENT ELASTIC HALF-SPACE MODEL (WAAS[44])

    Waas has developed a finite element computer program that cal-
culates the response of a foundation resting on a linear layered elastic
half-space to a dynamic load applied to the top of the foundation. A
rigid lower layer is required by the finite element approach. By this
method, torsional and vertical vibrations of circular footings on, or
embedded in, homogeneous and inhomogeneous soil or pavement layers over
rock are studied. The approach is a numerical method for layered

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elastic media, which extends the earlier result of Lysmer[45] for the homogeneous elastic half-space.

The method assumes that the equation of motion to be solved for each finite element is

$$m\ddot{\mu} + C\dot{\mu} + k\mu = \underline{P}(t)$$

where

   m = mass of finite element

   μ = displacement of a nodal joint

   C = viscoelastic damping constant

   k = spring constant

   $\underline{P}$ = driving force

   t = time

The displacements, damping constants, masses, and spring constants of the finite element nodes are collected into a displacement matrix (μ), a mass matrix (m), and a stiffness matrix (k). The stiffness matrix is complex because it includes the viscoelastic effects, i.e., it includes both k and C. The matrix form of the equations of motion are solved subject to the boundary conditions imposed by the particular program. The result is a theoretical frequency response spectrum that depends on the values of the elastic moduli of the layered system and on the depth of the lower rigid layer.

LINEAR ELASTIC HALF-SPACE MODEL (LYSMER ET AL.[28])

A linear elastic half-space model was advanced by Lysmer et al. in their development of a nondestructive pavement evaluation method based on the wave propagation method. Figure 47 shows the idealized system assumed for the half-space

$P = P_{max}\, e^{i\omega t}$

Sinusoidally varying force due to rotating masses

Ballasting Mass

Pavement reaction

$R = R_{max}\, e^{i(\omega t - \alpha)}$

Figure 47. Schematic of force system for Royal Dutch Shell Machine (after Lysmer et al.[28])

model. Pertinent equations are as follows:

$$P = P_{max}e^{i\omega t}$$

$$R = R_{max}e^{i(\omega t - \alpha)}$$

$$z = z_{max}e^{i(\omega t - \beta)}$$

where

   P = force due to rotating masses

   e = base of natural logarithms

   i = -1

   ω = angular frequency of rotating masses

   t = time

   R = pavement reaction

   α = phase lag of R behind P

   z = deflection of pavement under center of loading

   β = phase lag of z behind P

The elastic constants in the half-space model are contained in the R_{max}
and α terms in a very complex relationship developed by Lysmer.

VIBRATOR WAVE PATTERNS

   The motion of the soil or pavement surface in direct contact with
a vibratory baseplate is in phase with the plate and vibrates with it;
however, the soil motion is out of phase and complicated at positions
away from the plate. Additional comments on vibrator wave patterns are
given below.

   LYSMER ET AL.

Lysmer et al.[28] stated that two wave types can exist in elastic
media, dilational (P-waves) and rotational (S-waves or shear waves).
Under special conditions, the two basic wave types combine to form other
waves with special names, such as Rayleigh waves, Love waves, and
Stonely waves.

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BARKAN AND WES

Barkan[10] disclosed that when a vibrator is placed on the soil and vibrates as a solid body with one degree of freedom, then theoretically the resonance curve of the forced vertical vibrations has a maximum. However, measured resonance curves have several peaks. This phenomenon of multiple peaks is exhibited in Figure 48, which shows graphs of deflection versus frequency for nine test sites, all on rigid pavements. Data for these graphs were collected using the WES 16-kip vibrator at a constant dynamic load of 10 kips. Within the given frequency range of 5 to 100 Hz, five or six peaks are generally apparent. Barkan says that some investigators explain these multiple peaks by coincidences of the natural frequencies of the soil layers and the frequencies of propagating waves.

WES

Figure 49 shows a comparison of pressures on a pavement surface created by a wheel of a 727 aircraft and the WES 16-kip vibrator. The pressure over a point created by the aircraft builds up from zero as the tire begins to approach the point, reaches a maximum of 172 psi as the center of the tire is over the point, and decreases back to zero as the tire leaves the point. The pressure created by the vibrator is a combination of static and dynamic pressures. When the load mass is lowered to the pavement, a static pressure of 63 psi is created. If the dynamic load is 15 kips, the resulting total maximum pressure is 122 psi and the minimum pressure is 4 psi. The maximum surface pressure that can be created by the vibrator is 71 percent of the pressure of one wheel of the 727 aircraft (122 psi ÷ 172 psi). To duplicate the time that a point is loaded by the 727 moving at 20 mph, the vibrator must be operated at a frequency of approximately 17 Hz; however, the vibrator cannot be operated for one cycle of loading, which would simulate the passage of an aircraft, due to mechanical and control limitations. Also, only the pavement response to the dynamic load can be measured with the existing system that uses velocity transducers.

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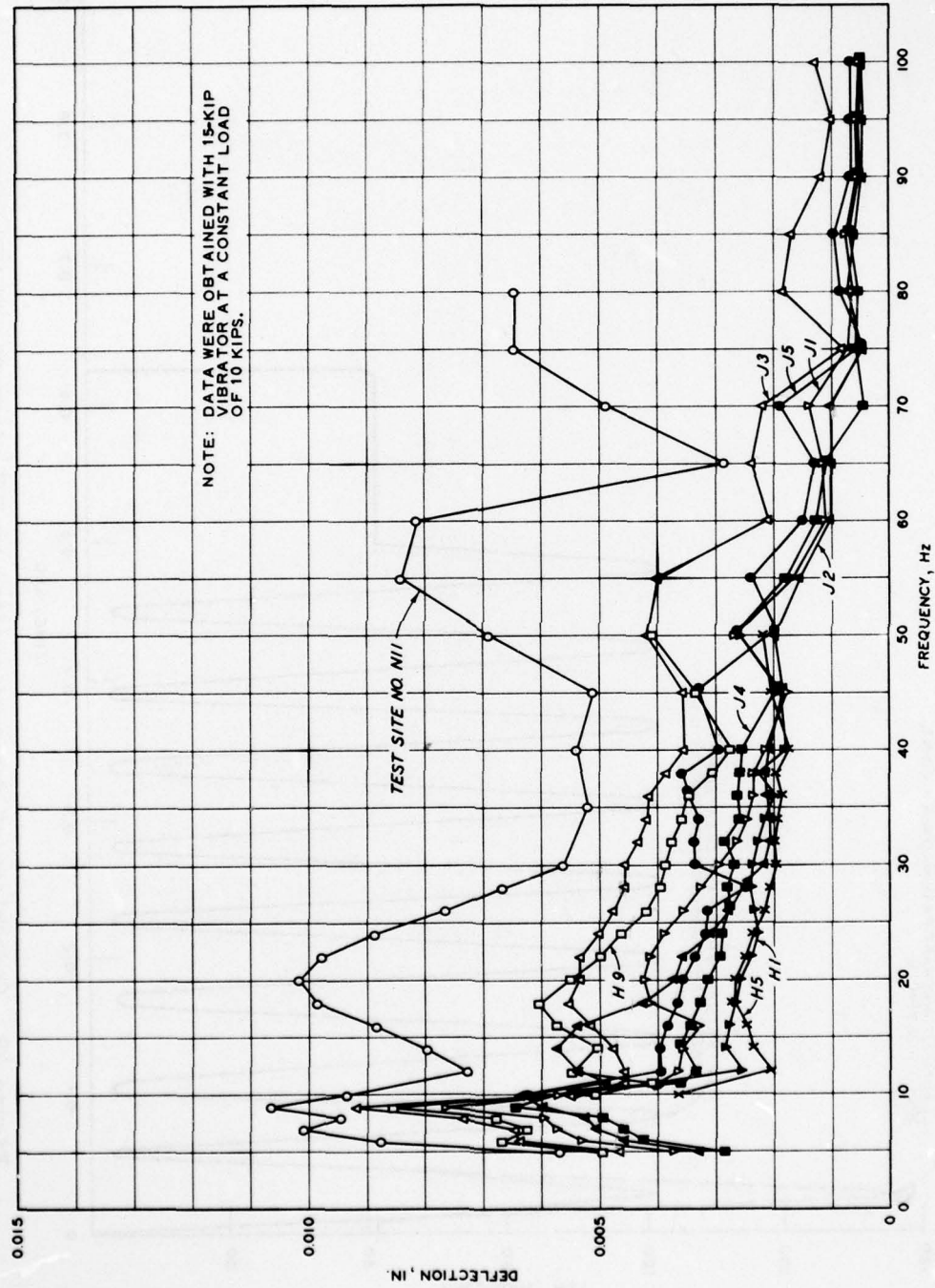
Figure 48. Deflection versus frequency for rigid pavements

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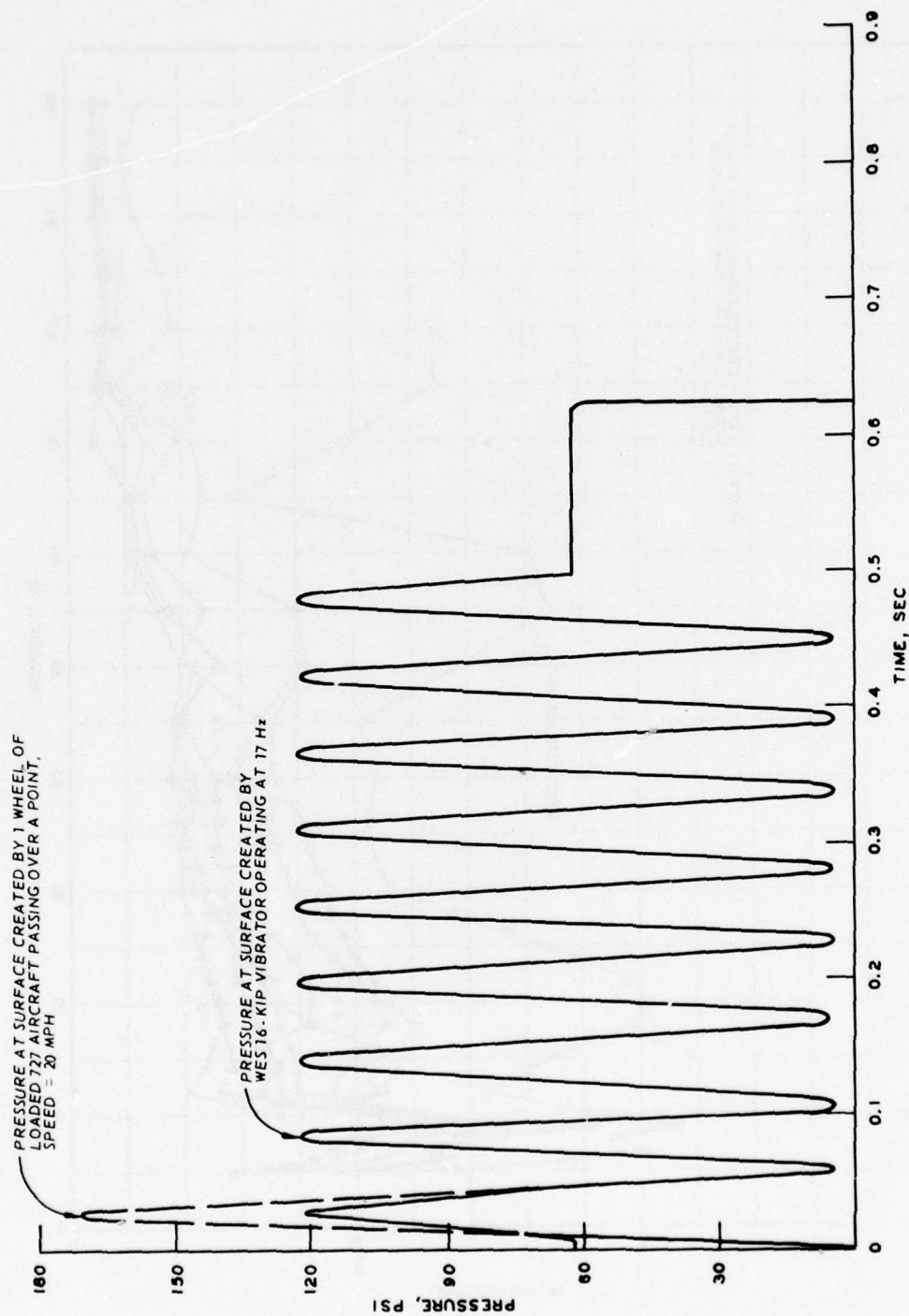
Figure 49. Comparison of pressure created by 727 aircraft and WES vibrator

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The surface pressures created by one wheel of most other air-
craft are smaller than for the 727; for example, the surface pressures
created by one wheel of the DC-9 and the 737 in psi are 165 and 151,
respectively. The tests to determine the resilient moduli of pavement
and subgrade materials can be designed to reproduce the axial pressures
created by any aircraft or the vibrator. However, there are differences
between the state of stresses in the field under the vibrator or air-
craft and the resilient modulus test. First, in resilient modulus test-
ing, the specimen is confined in the longitudinal direction, and stress
is constant at any point in the test specimen; however, under the air-
craft and the vibrator, the stress attenuates with depth. Secondly,
the confining, or chamber pressure, in the resilience tests can be care-
fully controlled to any level; however, the confining pressures under
the aircraft and vibrator can only be assumed. Little is known about
the configuration or the mass of the soil influenced by vibratory
loading.

COMMENTS ON RELATING VIBRATOR RESULTS
AND ELASTIC CONSTANTS

TRANSPORTATION RESEARCH BOARD
CONFERENCE[46]

At the Transportation Research Board Conference held in September
1973, discussion group C reviewed the 10 overlay design procedures based
on deflection, curvature, or stiffness known to be in use in the United
States. The group concluded that priority one in research needs should
be given to establishing the "fundamental deflection basin relationship
to performance and material properties." Further, the group observed
that most of the evaluation procedures had several features in common,
but none of the procedures was universally applicable due to the varia-
tions in materials, environment, and measuring techniques. They found
that users of the procedures believed that they had insufficient feed-
back data to validate and imporve the procedures at that time.

A. J. SCALA[47]

In his discussion of Cogill's presentation on utilization of

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surface deflection measurements in the evaluation of highways, A. J. Scala states: "The interpretation of the deflection bowl of a pavement under loading in terms of elastic moduli or strength coefficients of the various layers would be of inestimable value in both the design and evluation of pavements." He further lists four pertinent references (Texas Transportation Institute, Utah State Highway Department, N. I. Viswani, and C. P. Valkering) of which he recommended the Texas system as the most useful model at that time.

The Texas Transportation Institute system[48] is based on the Burmister equation

$$\frac{4\pi E_1}{3P} wr = \int_{x=0}^{\infty} V \cdot J_0(x) \, dx$$

where

E_1 = Young's modulus of upper layer

P = vertical load at point 0

w = vertical displacement

r = cylindrical coordinate

$V = 1 + 4Nme^{-2m} - N^2 e^{-4m}/1 - 2N(1 + 2m^2)e^{-2m} + N^2 e^{-4m}$,

where e = base of natural logarithms

m = parameter

$N = (1 - E_2/E_1)/(1 + E_2/E_1) = (E_1 - E_2)/(E_1 + E_2)$

where E_2 = Young's modulus of lower layer

$J_0(x)$ = Bessel function of the first kind and zero order with argument x

A computer program, ELASTIC MODULUS, computes the elastic moduli of each layer of a two-layer system. The required inputs are the thickness of the upper layer and deflections measured by a Dynaflect on the pavement surface. Poisson's ratio is assumed as 0.5 for both layers.

In ELASTIC MODULUS, the equation used is

$$\frac{w_1 r_1}{w_2 r_2} = \frac{F\left(\dfrac{E_2}{E_1}, \dfrac{r_1}{h}\right)}{F\left(\dfrac{E_2}{E_1}, \dfrac{r_2}{h}\right)}$$

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where

w_1 = deflection measured at a distance r_1 from the load

F = function

E_2 = Young's modulus of the lower layer

E_1 = Young's modulus of the upper layer

h = thickness of upper layer

w_2 = deflection measured at a distance r_2 from the load

The only unknown is E_2/E_1. By a convergent process of trial and error, a value of E_2/E_1 usually can be found that satisfies the equation and E_1 is calculated from

$$\frac{4\pi E_1}{3P} w_1 r_1 \approx F\left(\frac{E_2}{E_1}, \ \frac{r_1}{h}\right)$$

where

P = vertical force acting at a point in the horizontal surface of a two-layer elastic half-space

$E_2 = E_1(E_2/E_1)$

The authors warn that for calculations involving deflections under a heavy vehicle the moduli computed by ELASTIC MODULUS should be reduced by approximately one half.

EXTRAPOLATION OF G FROM LOW TO
HIGH STRESS LEVELS

LYSMER ET AL.

Lysmer et al.[28] recognized that because strain amplitudes induced by vibratory testing are smaller than those induced by aircraft, the results from vibratory in situ tests can be expected to yield higher values of moduli and indicate less damping than would be appropriate for use in the functional evaluation of structures. Lysmer discusses a proposed method in which G and damping properties obtained from in situ vibratory tests may be used to obtain similar properties for the description of behavior at strain amplitudes greater than those induced by the test. The method is contained in Figure 50, which is a graph of the ratio of shear modulus at shear strain γ to shear modulus at

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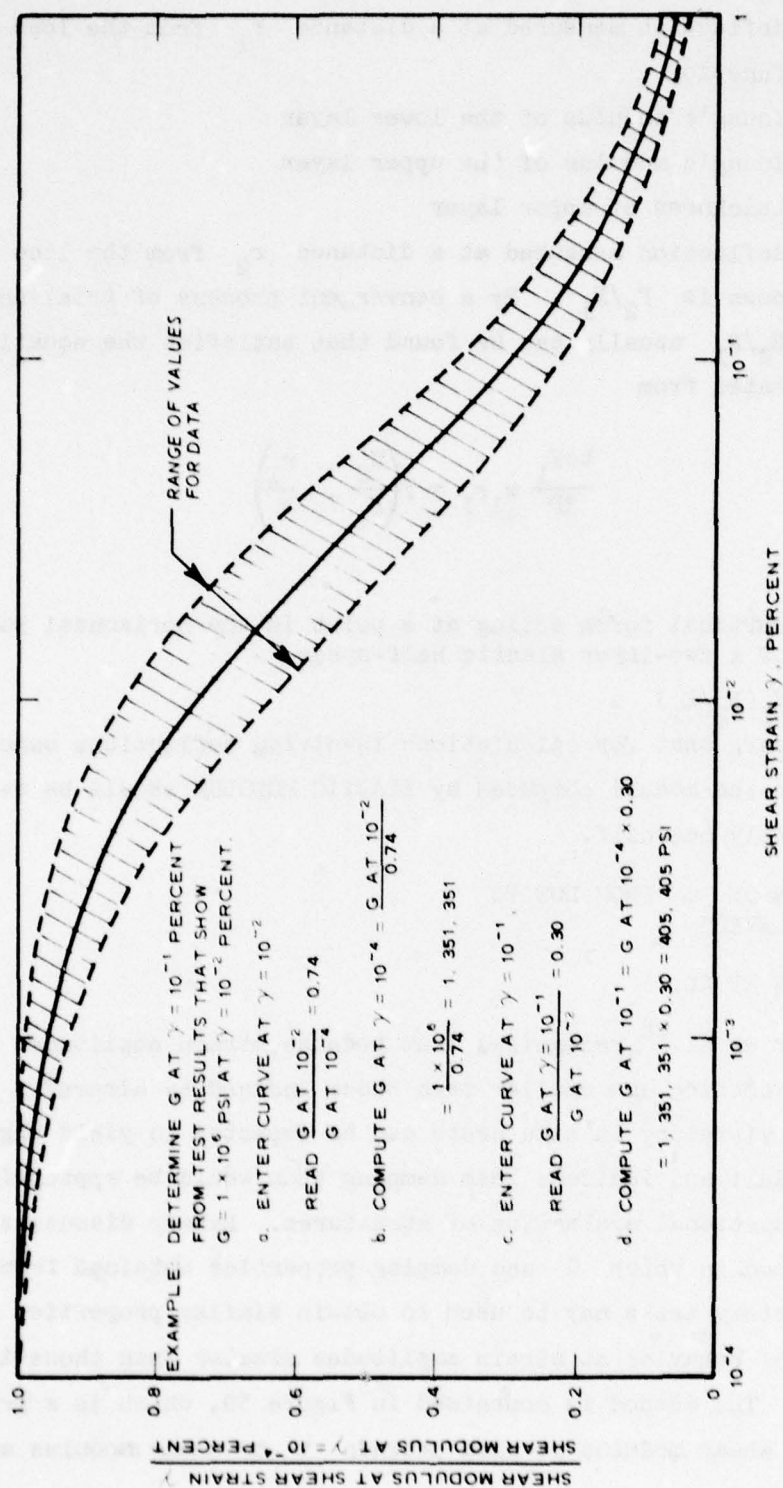
Figure 50. Variation of shear modulus with shear strain for sands (after Lysmer et al.[28])

$\gamma = 10^{-4}$ in percent versus shear strain γ in percent. The modulus versus shear-strain relationship can be approximated for any sand by determining the modulus at any strain and reducing this value for other strains by using the average line in Figure 50.

Lysmer found that information about the dynamic response of soils other than sands is limited and suggested that moduli obtained from a test on gravelly soils may be modified to approximate values at other strain amplitudes in accordance with the method proposed for sands.

HARDIN

Hardin's procedure[18] whereby shear moduli at any strain level may be calculated from results at a given strain level is illustrated in Figure 25.

PRELIMINARY RELATIONSHIP ESTABLISHED BETWEEN WES 16-KIP VIBRATOR AND ELASTIC MODULUS

DYNAMIC STIFFNESS MODULUS

The DSM is measured with the WES 16-kip vibrator and proposed for use by WES in a procedure for evaluating airport pavements. It is defined as the ratio of load to deflection, when the load is a specific, steady-state vibratory load. A complete description of procedures for determining DSM and applying it in the evaluation of pavements is given in Reference 1. Figure 51 shows a sample computation for DSM.

EQUATION FOR DSM

As part of this literature review, a special study was made to establish a preliminary relationship between the WES 16-kip vibrator results and elastic modulus. The following equation gave the best results:

$$DSM = 1.909526(t_1 \times E_1) + 0.508105(t_2 \times E_2)$$

$$+ 27.809179(E_S) - 871,471$$
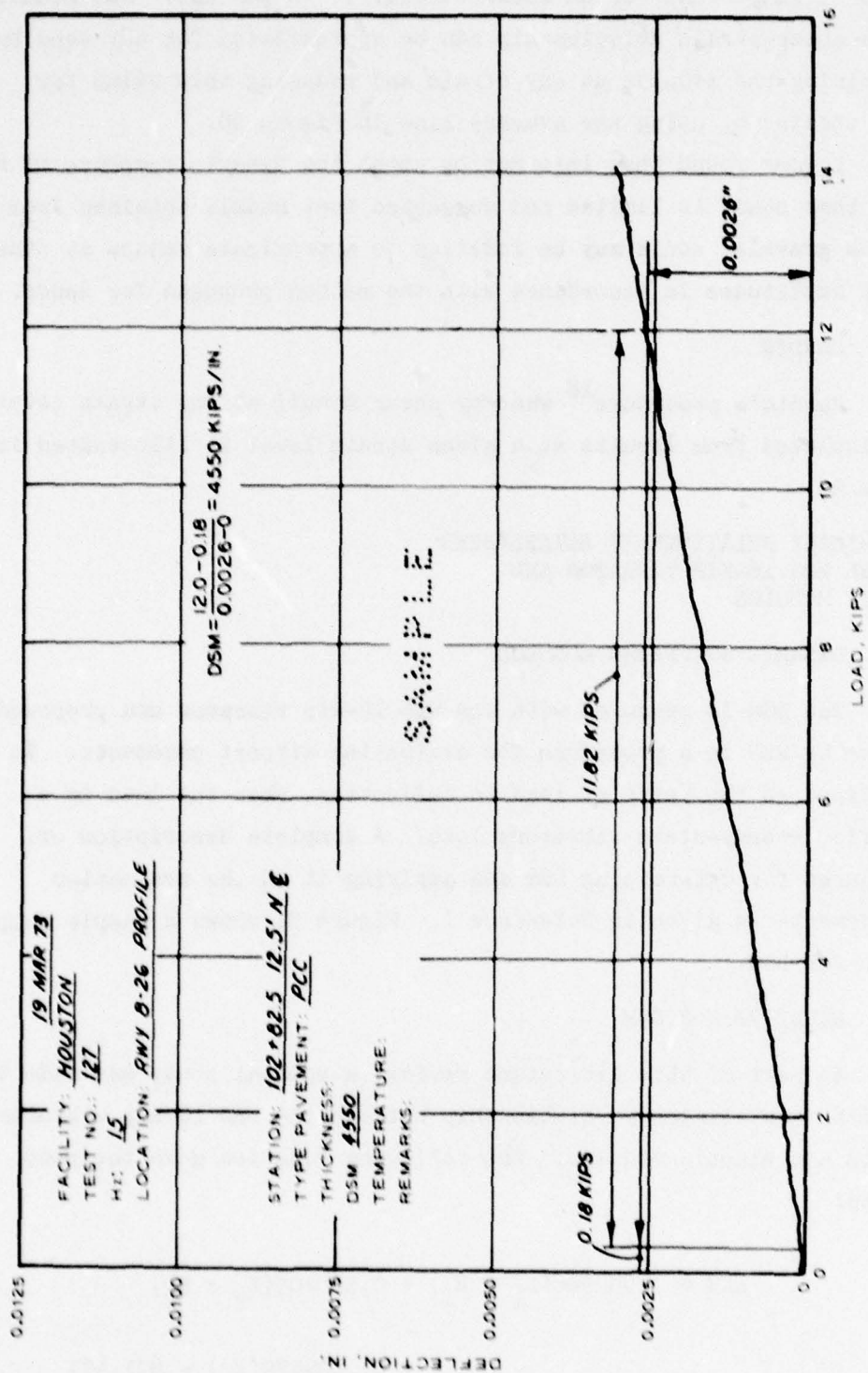
Figure 51. Sample DSM calculation

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FACILITY: *HOUSTON*

TEST NO.: *12*

Ht:

LOCATION *RWY 8-26 PROFILE*

STATION: *102+82.5 12.5'N ₵*

TYPE PAVEMENT: *PCC*

THICKNESS:

DSM *4550*

TEMPERATURE:

REMARKS:

$$DSM = \frac{12.0 - 0.18}{0.0026 - 0} = 4550 \text{ KIPS/IN.}$$

SAMPLE

0.0026"

11.82 KIPS

0.18 KIPS

LOAD, KIPS

DEFLECTION, IN.

where

DSM = dynamic stiffness modulus, lb/in.

t_1 = thickness of bound pavement material in layer one, in.

E_1 = average modulus of bound pavement material in layer one, psi

t_2 = thickness of unbound pavement material in layer two, in.

E_2 = average modulus of unbound pavement material in layer two, psi

E_S = modulus of subgrade, psi

The regression coefficient, 27.809179, has to have the dimension of inches, and the constant, 871,471, must be pounds per inch, but the other constants are dimensionless.

Table 23 shows the pavement layer properties and the DSM's used to establish the regression equation. DSM's were adjusted to what their values would have been at a mean pavement temperature of 70°F. Modulus values were computed for the bound pavement and for the unbound pavement materials, respectively, from Figures 52 and 53, which are unpublished works by Dr. Walter Barker of WES. The modulus of the subgrade was taken as $E_S = 1500 \times CBR$.

The regression equation is not satisfactory for estimating the E values of pavements dissimilar to those used in its development, especially E values of subgrades per se. An example will show how the equation can be abused. For the case of no pavement or base, the equation would reduce to

$$DSM = 27.809179 E_S - 871,471$$

or

$$E_S = \frac{871,471 + DSM}{27.809179}$$

The value of DSM cannot be negative; therefore, the smallest value of E_S that could result (when DSM = 0) is 31,337 psi. Since actual values of E_S are often considerably less than 31,000 psi, this example

Table 23

Pavement Layer Properties and Dynamic Stiffness
Moduli (DSM)

Test Site No.	Bituminous Material Thickness t_1 , in.	Bituminous Material Modulus E_1 , psi	Granular Material Thickness t_2 , in.	Granular Material Modulus E_2 , psi	Subgrade Modulus E_S , psi	DSM lb/in.
N18	3.25	58,000	6.0	33,000	27,000	630,000
N20	6.0	56,000	6.0	33,000	27,000	1,170,000
N22	6.0	56,000	7.0	33,500	27,000	740,000
N23	6.0	56,000	7.0	33,500	27,000	700,000
W1	9.0	82,000	5.0	34,000	30,000	1,080,000
W2	12.0	91,000	6.0	33,000	30,000	2,060,000
B1	17.5	79,250	6.0	27,000	21,000	2,680,000
B2	12.0	70,000	9.0	31,500	21,000	920,000
B3	10.5	67,000	10.5	35,500	25,500	1,170,000
P1	10.0	65,500	8.0	38,000	10,500	1,020,000
P2	8.5	62,500	7.5	26,000	9,000	560,000
P5	6.0	56,000	6.0	46,000	22,500	790,000
P13	15.0	98,000	5.0	34,000	30,000	3,000,000
P14	14.0	95,000	5.0	34,000	30,000	2,630,000
NV1	13.0	72,000	19.0	63,750	18,000	2,460,000
NV2	13.0	72,000	25.0	79,750	18,000	1,880,000
NV3	8.0	56,000	12.0	53,000	18,000	570,000
NV4	18.0	79,500	12.0	66,000	18,000	3,160,000
S4	9.0	82,000	15.0	21,500	6,000	350,000
S7	3.0	56,000	21.0	52,000	6,000	570,000
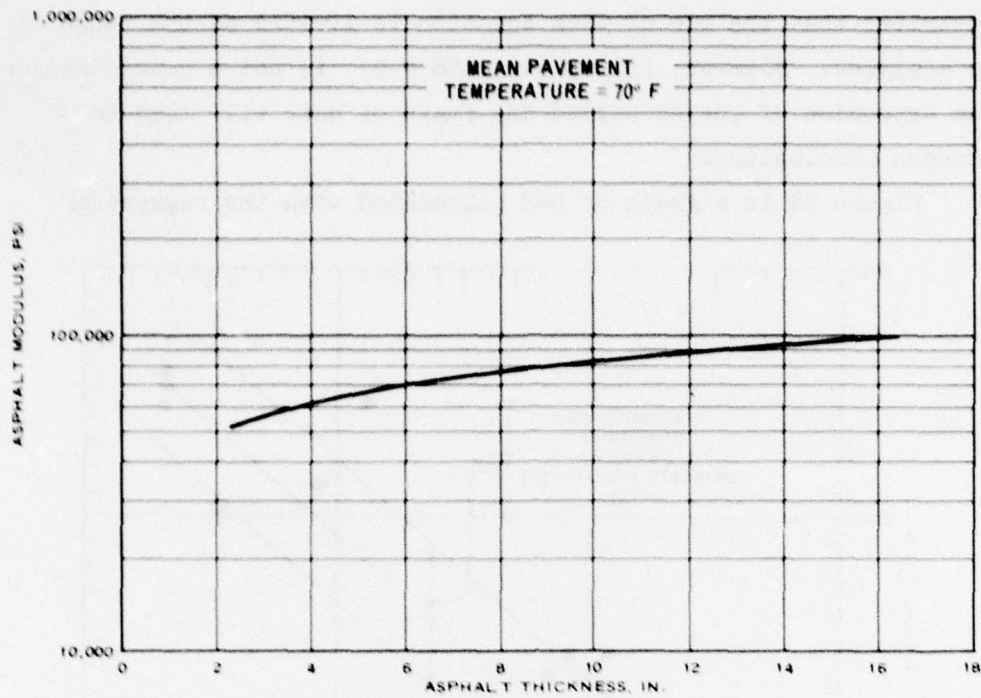
Figure 52. Asphalt modulus versus asphalt thickness
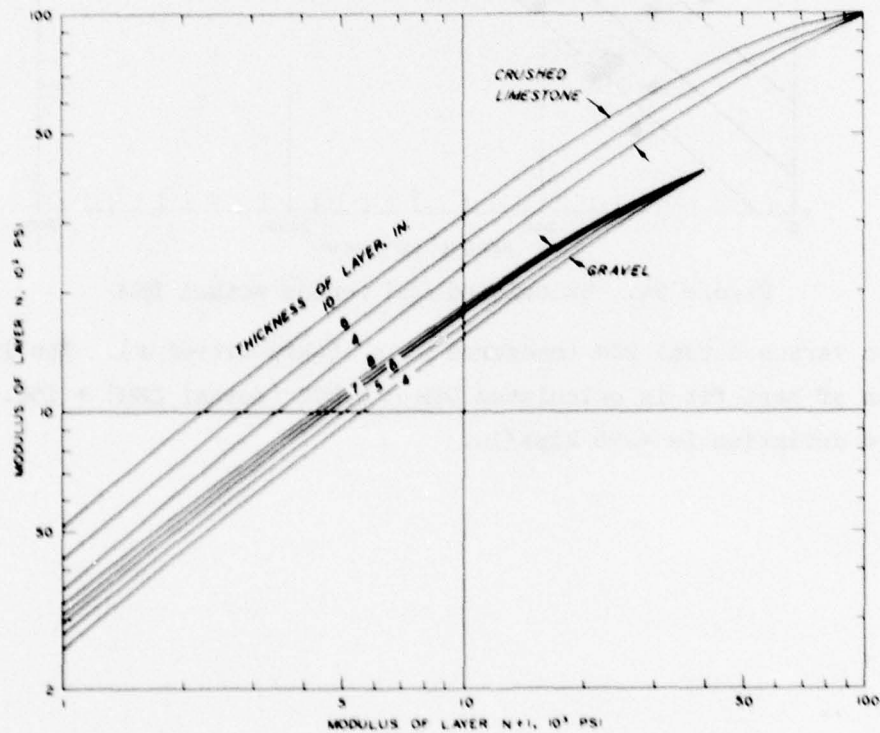
Figure 53. Modulus of layer N versus modulus
of layer N+1

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illustrates that the use of this equation is limited to the range of data analyzed. However, in reality, DSM = 0 , is not a consideration since extension of curves beyond the range of data will lead to erroneous conclusions.

Figure 54 is a graph of DSM calculated with the regression
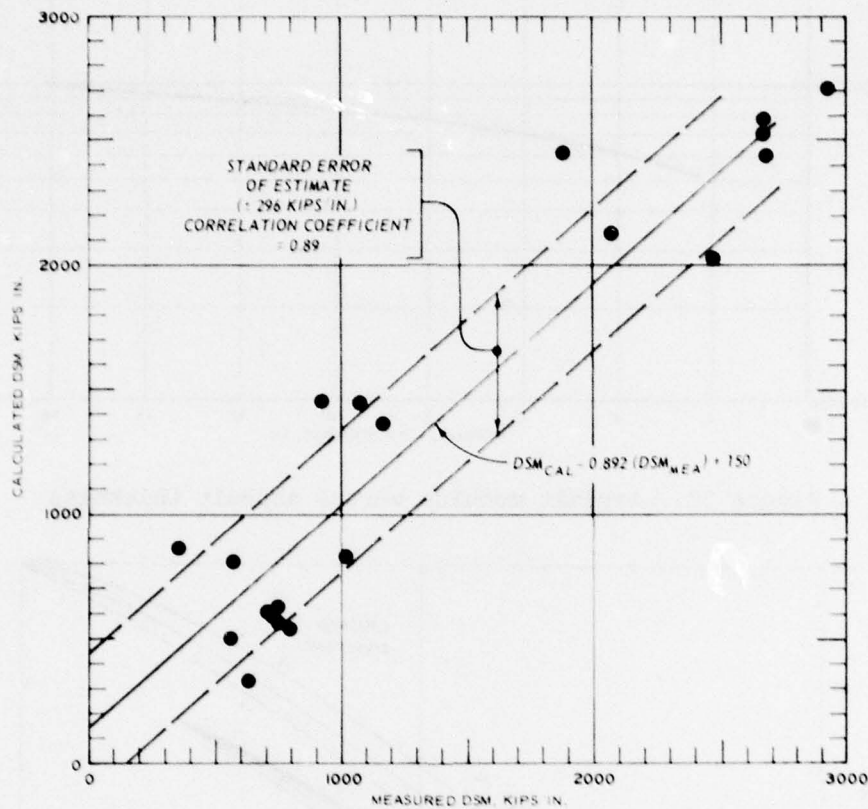
Figure 54. Calculated DSM versus actual DSM

equation versus actual DSM (measured with 16-kip vibrator). The linear equation of best fit is calculated DSM = 0.892 (actual DSM) + 150. The standard deviation is ±296 kips/in.

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SUMMARY DISCUSSION

To this point, this study has been concerned almost entirely with the direct presentation of pertinent information from the literature and has essentially refrained from evaluation or discussion of that information. Thus in the following paragraphs, an effort is made to summarize, evaluate, or otherwise discuss the information that has been presented.

REVIEW OF ELASTIC CONSTANTS

ELASTIC CONSTANT TEST METHODS
AND SPECIFIC VALUES

The test methods reviewed in this study can be grouped into five general categories:

a. Compression testing.

b. Flexure testing.

c. Tension testing.

d. Wave velocity testing.

e. Natural frequency testing.

A summary reference, or index, to the test methods and specific values of elastic constants reported herein is given in Table 24. This table shows the elastic constant symbol (as used by the researcher); the material; the researcher or reference; the figure and table; the type of test and category of test (listed above); and the test characteristics (whether quasi-static or dynamic, and whether state of stress and loading rate frequency were considered).

Table 25 presents the minimum and maximum values cited herein for elastic constants. It is evident that the specific values for each constant can have an extremely wide range. It is doubtful that the elastic constants determined by one test procedure are compatible with those determined by another test procedure. Certainly, test procedures that do not account for factors that cause variations in the elastic constants will produce values for the same materials that are different from those obtained from test procedures that do account for the factors.

Table 24

Summary of References to Test Methods for and Specifi

Elastic Constant Symbol		Material	Researcher or Reference	Figure or Table
				Elastic Modulus
		Wearing Surface		
E	PCC		ASTM C469-65, MRD	
E	PCC		CRD-C21	
E	PCC		ASTM C597-71	
E	PCC		Murillo, CRD-C21-58	Table 2
Dynamic E	PCC		Barkan	Table 1
Dynamic E	PCC		CRD-C18-59	
E	PCC		Popov	
E , Dynamic E	PCC		Neville	Figs. 5-8
E , Dynamic E	PCC		WES	Table 3
$|E^*|$	AC		WSU, AI	
M_R , flexural stiffness	AC		Glynn and Kirwan	
E	AC		Klomp and Niesman	Fig. 9
E	AC		Izatt et al.	Fig. 10
E	AC		Witczak	Fig. 11
Dynamic E	AC		Cook and Krukar (WSU)	Fig. 12
$|E^*|$	AC		Pagen	Fig. 13
Stiffness modulus	AC		MRD	
		Treated Base		
M_R , $|E^*|$	Emulsion		Cook and Krukar	Fig. 12, 14, 15, 16
E	Cement, lime		Barker et al.	Table 4
E	Cement		MRD, ASTM 469-65	
		Untreated Base		
M_R				
Dynamic E	Various		Dorman and Klomp	Fig. 17
M_R	Crushed basaltic rock		Cook and Krukar	Fig. 18, 19
M_R	Gravel, untreated stone		Glynn and Kirwan	Fig. 20, 21, 22
M_R	Various		Allen	Table 5
		Subgrade		
M_R			WES	
Dynamic E			Dorman and Klomp	
E_i	Clay, sand		Terzaghi and Peck	Fig. 28
				(Continued)

* The five categories are (a) compression testing, (b) flexure testing, (c) tension testing, (d) wave veloc

Table 24

Figure or Table	Type of Test	Category*	Characteristics of Test Quasi-Static	Dynamic	State of Stress	Loading Rate Frequency
Elastic Modulus						
	Compression	a	x			
	Flexure	b	x			
	Pulse Velocity	d		x		
Table 2	Rupture, three-point beam	a	x			
Table 1	Sound generator	d		x		x
	Natural frequency	e		x		
Figs. 5-8	Natural frequency	e		x		
Table 3	Triaxial, compression, impact	a	x	x	x	x
	Compression, sinusoidal stress	a		x		x
	Flexure of thin, circular slabs	b		x		x
Fig. 9						
Fig. 10						
Fig. 11				x		
Fig. 12				x		
Fig. 13				x		x
	Stiffness	b				
Fig. 12, 14, 15, 16						
Table 4	Tension	c				
	Compression	a	x			
	Repetitive triaxial compression	a		x	x	
Fig. 17	Wave velocity	d		x		
Fig. 18, 19	Repetitive triaxial compression	a		x	x	x
Fig. 20, 21, 22	Repetitive triaxial compression	a		x	x	x
Table 5	Repetitive triaxial compression	a		x	x	
	Repetitive triaxial compression	a		x	x	x
	Wave velocity	d		x		
Fig. 28	Triaxial compression	a	x		x	

(Continued)

ng, (d) wave velocity testing, and (e) natural frequency testing.

(Sheet 1 of 3)

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Table 24 (Continued

Elastic Constant Symbol	Material	Researcher or Reference	Figure or Table
			Elastic Modulus (Con

Subgrade (Continued)

E		Lambe and Whitman	
E	Sand, clay	Barkan	
E_r		Robnett and Thompson	
H_R	Fill, mud	MRD	
M_R		Cook and Krukar	
E_m		Foster and Heukelom	
E	All materials singly or in combination	WES	

Poisson's Ratio

Wearing Surface

μ	PCC	ASTM C469-65	
μ	PCC	CRD-C21	
μ	PCC	CRD-C18-59	
μ	PCC	Yoder and PCA	
μ	PCC	MRD	
μ	PCC	Neville	
μ	AC	Glynn and Kirwan (TTI)	
μ	AC	Orchard	

Treated Base

μ	Emulsion	Cook and Krukar	
μ	Cement, lime	Barker et al.	
μ	Cement	MRD, ASTM 469-65	

Untreated Base

μ	Granular	Allen	Fig. 23, 24

Subgrade

μ	Clay	Terzaghi and Peck	
μ	Sand	Lambe and Whitman	
μ	Saturated clay, sand	Morgan and Scala	
μ	Clay, clay admixtures	Barkan w/Katsenelenbogan	
μ	Sand, clay	Barkan	
μ	Artificial clay	Barkan (Pokrovsky)	
μ	Clay, loess, sandy soils	Barkan (Ramspeck)	
μ	Sandy; silty, sandy clay; clay	Barkan (Tsytovich)	
μ	Rock to saturated clay	Bowles	Table 11

(Continued)

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Figure or Table	Type of Test	Category	Quasi-Static	Dynamic	State of Stress	Loading Rate Frequency
			Characteristics of Test			

stic Modulus (Continued)

Figure or Table	Type of Test	Category	Quasi-Static	Dynamic	State of Stress	Loading Rate Frequency
	Triaxial compression	a	x		x	
	Compression on clay cubes	a	x			
	Repetitive triaxial	a		x	x	x
	Repetitive triaxial	a		x	x	x
	Repetitive triaxial	a		x	x	x
	Wave velocity	d		x		
	Vibratory (DSM)	a		x		x

Poisson's Ratio

	Compression	a	x			
	Flexure	b	x			
	Natural frequency	e		x		
	Compression	a	x	x		
	Compression	a	x			
Fig. 23, 24	Repetitive triaxial compression	a		x	x	x
	Repetitive triaxial compression	a		x	x	x
	Triaxial compression	a	x		x	
	Wave velocity			x		

Table 11
(Continued)

(Sheet 2 of 3)

Table 24 (Concluded)

Elastic Constant Symbol	Material	Researcher or Reference	Figure or Table
			Shear Modulus
	Wearing Surface		
G	PCC	CRD-C18-59	
	Untreated Base		
G		Hardin	
G	Crushed limestone, gravel, sand	Hardin	Fig. 25, 26, 27
	Subgrade		
G		Hardin	
G		Hardin	Table 12

Figure or Table	Type of Test	Catetory	Characteristics of Test		State of Stress	Loading Rate Frequency
			Quasi-Static	Dynamic		
ar Modulus						
	Natural frequency	e		x		
	Resonant column device	e		x		
ig. 25, 26, 27	Triaxial compression	a	x			
able 12	Resonant column device	e		x		

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Table 25

Summary of Measured Extreme Values of Elastic Constants

Material	E , psi		M_R , psi		E^* , psi		ν , psi		G , psi		Stiffness Modulus, psi	
	Minimum	Maximum	Minimum	Maximum	Minimum	Maximum	Minimum	Maximum	Minimum	Maximum	Minimum	Maximum
PCC Wearing Surface	800,000	6,300,000	---	---	---	---	---	0.21 (static)	---	---	---	---
PCC Wearing Surface	363,000†	1,290,000†	---	---	---	---	---	0.24 (dynamic)	---	---	---	---
AC Wearing Surface	28,000	2,100,000	---	---	60,000	2,000,000	---	0.40††	---	---	450,000	600,000
Treated Base	8,400	1,100,000	18,000	140,000	70,000	1,600,000	0.25††	0.40††	---	---	---	---
Untreated Base	---	---	1,900	160,000	---	---	0.05	0.85	3,625	30,450	---	---
Subgrade	0	127,000	1,900	37,000	---	---	0.20 (clay)	0.60	1,000	50,000	0	0
Subgrade	---	---	---	---	---	---	0.1 (sand)	0.91	---	---	---	---

† At rupture.
†† Assumed.

The point is that values for the elastic constants cannot be randomly extracted from reports or books without qualification as can, for example, the unit weight of water at 70°F. Another point is that test procedures should not be "mixed." For example, in the computer analysis of a two-layered pavement structure, it is not good practice to choose an elastic modulus for one layer based on a wave velocity test and the modulus of the other layer based on a quasi-static triaxial compression test.

FACTORS THAT AFFECT VALUES OF
ELASTIC CONSTANTS

The inelastic behavior of pavement materials, especially untreated base materials and subgrades, i.e., soils, is universally recognized as the basic cause of the wide variations that occur in values of elastic constants and the difficulties that arise in the measurement of the constants, their use in characterizing pavement materials, and in the ultimate use of elastic theory as a basis for the development of airport pavement design and evaluation methodology. Yet, no better approach to a solution of these problems has evidenced itself than the precalculated assumption of elastic behavior, i.e., the assumption that Hooke's law will be used, knowing that some inaccuracy is a likely consequence. As one researcher, Barkan,[10] puts it, "...the assumption of a relationship more complicated than Hooke's law will lead to the necessity of employing a nonlinear theory of elasticity operating with nonlinear differential equations. The solution of these equations, even in the simplest problems, leads to considerable difficulties..." He further states that the application of the nonlinear relations becomes practically impossible, and it is necessary to restrict the analysis by the assumption that the material, especially the soil, strictly follows Hooke's law. However, the problem of applying nonlinear relations can be resolved by use of finite element programs. A linear relationship can be assumed within each element, and a nonlinear relationship can be applied from element to element. Nevertheless, finite element solutions only approximate nonlinear elastic solutions, but the approximations may

be of acceptable accuracy in some cases. It must be kept in mind that the numerical values of elastic soil constants should be selected with due consideration of the influence of the simplifying assumptions.

Many specific causes of variations in values of elastic constants were cited in this literature review. These may be classified into six general categories, as follows:

    a. Frequency and duration of loading.

    b. State of stresses.

    c. Age of material.

    d. Compositions and material properties.

    e. Temperature.

    f. Strain.

Table 26 contains summary information regarding the factors that affect values of elastic constants. This table shows the elastic constant symbol (as used by the researcher); the material; the researcher or reference; the figure and table; the factor(s) affecting the elastic constant values; and the category (listed above) of the factor. A discussion, by categories, follows.

Frequency and Duration of Loading. The studies of Morgan and Scala,[24] Klomp and Niesman,[25] Witzcak,[27] and Pagen[28] showed that the elastic modulus of AC wearing surfaces is highly dependent on the frequency of loading. For example (Figure 11), at a temperature of 70°F, the AC moduli at frequencies of 1, 4, and 16 cps are 370,000, 570,000, and 770,000 psi, respectively.

Allen[30] reported that Seed and Chan (Table 5) indicated an effect of frequency and duration of loading on the M_R of silty sand (Table 5) but did not elucidate further. Monismith et al.[35] also mentioned that rate of application is an influencing factor on the M_R of untreated granular materials.

Graphs of deflection versus frequency of loading obtained from vibrator test devices on all materials, including homogenous soils, exhibit pronounced peaks and troughs (Figure 48) for rigid pavements. For test site No. N11, changing the frequency from 20 to 40 Hz reduced

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Table 26

Summary of References to Factors That Affect Values of Elastic Constants

Elastic Constant Symbol	Material	Researcher or Reference	Figure or Table	Factor Affecting Elastic Constant	Category*
Elastic Modulus					
Wearing Surface					
E	PCC	Popov	Table 1	Water/cement ratio	d
Dynamic E	PCC	Bartan	Fig. 5, 7	Age, composition	c, d
E, Dynamic E	PCC	Neville	Table 3	Age, E of aggregate	c, d
E, Dynamic E	PCC	WES (Lundeen)		Confining pressure, static vs dynamic	b, a
E	AC	Glynn & Kirvan		Mix proportions, temperature	d, e
E	AC	Morgan & Scala	Fig. 9	Stress level loading rate, temperature	b, a, e
E	AC	Klomp & Niesman	Fig. 10	Temperature, loading rate	e, a
E	AC	Izatt et al.	Fig. 11	Temperature	e
E	AC	Witczak	Fig. 12	Temperature, loading rate	e, a
E	AC	Cook & Kruxar		Temperature	e
E	AC	Pagen	Fig. 13	Loading rate	a
Treated Base					
M_R	Emulsion	Cook & Kruxar	Fig. 14	Temperature, curing, confining pressure	e, c, b
|E|	Asphalt, hot-mix sand	Cook & Kruxar	Fig. 15, 16	Temperature, density	e, d
E	Cement, lime	Barter et al.	Table 4	Cement, lime content	d
Untreated Base					
Dynamic E	Various	Dorman & Klomp	Fig. 17	E of underlying layer	b, d
M_R	Crushed basaltic rock	Cook & Kruxar	Fig. 18	Confining pressure, water content	b, d
M_R	Gravel	Glynn & Kirvan	Fig. 19	Bulk stress, water content	b, d
M_R	Crushed stone		Fig. 20, 21	Cell pressure, density, water content	b, d, d
M_R	Various	Allen	Fig. 22	Cell pressure	b
Subgrade					
E, E_4, E_5, E_N	Clay, sand		Table 5	Stress level, frequency and duration of load, moisture content, gradation, void ratio, dry density, rate of deformation	b, a, d, d, d, d, d, f
		Terzaghi & Peck	Fig. 28	Stress-strain	b, f
				Remolding, age	d, c
E		Lambe & Whitman	Fig. 29	Confining pressure, clay type, drainage	b, d, d
				Initial principal stresses	b

(Continued)

* The six categories are (a) frequency and duration of loading, (b) state of stresses, (c) age of material, (d) compositions and material properties, (e) temperature, and (f) strain.

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Table 26 (Continued)

Elastic Constant Symbol	Material	Researcher or Reference	Figure or Table	Factor Affecting Elastic Constant	Category
	Subgrade (Continued)				
E	Lambe & Whitman	Fig. 29		Strain level	f
				Deviator stress, soil disturbance,	b, d,
				Consolidation stress, overconsolidation ratio	b, b,
				Aging, strain rate	c, f
E	Sand	Barkan	Table 7	Grain size	d
	Clay		Fig. 30	Water content	d
	Clay		Fig. 31	Void ratio	d
	Various		Table 8	Soil type	d
	Sands, fine-grained soils	Nielsen (USNCEL)		Four factors above	d
M_R	Fine-grained soils	Monismith et al.		Compaction, moisture, stress history	d, d, b
M_R	Fine-grained soils			Stress intensity, number of applications, age,	b, a, c,
M_R	Granular			Compaction method, density, water content	d, d, d
				Confining pressure, stress level, duration,	b, b, a,
				Rate of stress application, rate of deforma-	a, f, d
				tion, aggregate type, void ratio, saturation	d, d, d
E_r	Various	Robnett & Thompson	Fig. 33	Degree of saturation	d
			Fig. 34	Deviator stress, moisture content	b, d
			Fig. 35	Moisture content	d
			Fig. 36	Deviator stress, density	b, d
			Fig. 37	Deviator stress	b, d
			Fig. 38	Deviator stress, compaction	b, d
M_R	Fill, sand	MED	Table 9, Fig. 39	Deviator stress	b, d
M_R	Silt	Cook & Kruxar	Fig. 40, 41	Deviator stress, stress ratio	b, b
M_R		Hicks	Table 13	State of stress	b
M		Dunlap	Table 13	State of stress	b
E_a		Morgan & Scala (Brown & Pell)		State of stress	b
M_R		Morgan & Scala (Seed et al.)		State of stress	b
E_n		Morgan & Scala (Holden)		State of stress	b
M		Nielsen (USNCEL)		Lateral stress	b
E_r		Glynn & Kirwan	Fig. 42	State of stress	b
M_R		Wang et al.		State of stress	b
M_R		Glynn & Kirwan	Fig. 43	PI, density, moisture content	d, d, d
				Poisson's Ratio	
	Wearing Surface				
v	PCC	Neville		Static vs dynamic	a
v	PCC	Orchard		Richness of mix	d

(Continued)

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Table 26 (Concluded)

Elastic Constant Symbol	Material	Researcher or Reference	Figure or Table	Factor Affecting Elastic Constant	Category
			Poisson's Ratio (Continued)		
	Untreated Base				
ν	Granular	Allen	Fig. 23	Stress ratio, water content	b, d
			Fig. 24	Stress ratio, grading, density, moisture	b, d, d, d
	Subgrade				
ν	Sand	Lambe & Whitman		Stress level	f
ν	Clay, sand	Morgan & Scala		Soil type	d
ν	Clay, clay with sand	Bartan		Soil type	d
ν	Rock to saturated clay	Bowles		Soil type	d
			Shear Modulus		
	Untreated Base				
G	Gravel	Hardin		Particle composition, size, stress, void ratio	d, d, b, d
				Strain, confining pressure	f, b
	Subgrade				
G		Lysmer et al. (Hardin & Drnevich)		Mass density	d
G		Lysmer et al. (Hardin & Drnevich)		Strain, principal stress, void ratio,	f, b, a,
				Number of cycles of loading, degree of saturation	a, d
G		Lysmer et al. (Monismith)		Strain amplitude, confining pressure, void ratio	f, b, d
G		Hardin	Table 12	Strain, saturation, number of cycles, loading rate	f, d, a, a
	Soils, excluding gravels	Hardin		Strain, soil type	f, d

the deflection by nearly one half (from 0.0102 to 0.0054 in.). Obviously, tests on the same material at different frequencies will not produce the same results.

State of Stresses. Work at WES[8] and MRD,[21] as well as the research of Hardin,[15] Cook and Krukar,[11] Glynn and Kirwan,[13] and others, showed that modulus values of materials depend on the state of the stresses or the confining pressures. Since the shape or mass of soil influenced by vibratory testing is indefinite, it is difficult to determine the pressure on the in-place soils during vibratory testing.

A dynamic theory for computations of stresses in soils beneath vibratory loading was not found; therefore, for the present, a static theory must be used as an approximation.

The variable nature of the elastic constants of soil and the variability introduced by the different test methods emphasize the desirability of in situ determinations of these constants. It has been recognized for years that the behavior of soils in the laboratory, especially clays, can be radically different from behavior in the field even when tests are performed on undisturbed specimens under carefully controlled conditions. With the knowledge being gained by contemporary researchers, it should soon be possible to design vibratory tests to exactly simulate in situ stress conditions created by aircraft and thus yield more precise results.

Of the equations relating resilient modulus and state of stress in base course materials, Barker et al.,[29] in his work with the structural properties of stabilized layers, found Hicks' expression

$$M_R = K_1 \sigma_3^{K_2}$$

to be most useful. The values of the constants K_1 and K_2 are listed for 10 categories of materials in Table 13. However, five of these categories are for gravels in five different states of crushed condition or saturation, and the other five categories of materials are classified by regional nomenclature. Therefore, the application of this equation

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is limited until the list of constants is expanded.

The equations relating moduli and states of stress in soils are useful in describing the changes in moduli with change in conditions; however, application of the equations is limited because of the effect on the modulus of soils (especially clays) of the many variables, such as moisture content, porosity, and particle composition.

Age of Material. The influence of age on the elastic modulus of PCC wearing surfaces was recognized by Barker. According to his data, the aging of six concrete samples from 7 to 28 days raised the E values of five of them from about 7 to 26 percent (Table 1). The E value of one of the samples actually decreased 6 percent. No explanation of this apparent anomaly was given. The aging of concrete samples probably resulted in some dehydration, mainly in the early stages of aging, and the dehydration tended to increase the cementation forces and thus the E value. Beneficial effects of aging will continue at a reduced rate after 28 days.

Apparently, age is a significant factor in the resilient moduli of emulsion-treated base (Figure 14). Uncured test specimen A, which was tested at 80°F, exhibited significantly lower moduli values than did test specimen B, which was cured for 6 months and tested at 100°F.

Monismith et al.[35] stated that the age of a fine-grained soil at initial loading is a factor that influences the value of its modulus of resiliency. No attempt was made to quantify that influence. Several researchers in addition to Monismith acknowledge age effects but offer no detailed discussion. Apparently, the researchers are discussing an influence that is independent of all the factors listed in Table 26 that affects elastic constants. The only other factors that come to mind are settlement and molecular bond.

Compositions and Material Properties. The relationship, developed by Glynn and Kirwan (Figure 40), between resilient modulus, stress, PI, moisture content, and density of clays, appears promising because it considers four factors that have an important influence on the modulus value, and also because it expresses the modulus in terms of

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basic soil properties that are simple to determine.

Dorman and Klomp's relationship[14] of CBR to subgrade E (Figure 41) is useful because it represents in situ conditions when the CBR is measured in place. However, Figure 41 shows that there is significant scatter in the data. For a CBR of 20, the E values range in the approximate limit from 15,000 to 60,000 psi. Robnett and Thompson[36] concluded that there is no relationship between E_r and CBR because of the scatter observed in their graph (Figure 42). Scatter in Figure 42 is comparable to that in Figure 41. The scatter evident in the CBR-E relationship does not nullify the possibility of using CBR as an estimator of E. However, the variation in E for a given CBR should be considered when this relationship is used. When modulus testing is feasible, CBR tests are not recommended as substitutes.

The relationship proposed by the Asphalt Institute[12] in Figure 43 ties the subgrade modulus of elasticity to plate bearing value. However, Crawford et al.[49] found that to approximately compute the elastic layer deformation using a Westergaard idealization, subbase k values measured in the field must be reduced by 90 percent. Crawford's findings indicate that the subgrade moduli may also require adjustment before serving as input for the layered-elastic programs.

Hardin's equations for shear moduli of soils and gravels appear to be more complete in their treatment of influencing variables than do the equations found in other references. These equations could be useful in verification and correlation work; however, it is not clear at this time if they can be adapted for the study at hand. The equations also require the determination of some index properties of soil.

Temperature. The effect of temperature on the modulus of elasticity of AC has long been recognized. An idea of how profound this effect can be may be gained by inspecting Figure 9. It can be seen that a change of 40°C (from -10 to 30°C) caused a decrease in elastic modulus of about 150,000 kg/cm^2 (from 160,000 to 10,000 kg/cm^2). Other graphic evidence of temperature effects on the elastic modulus of AC wearing surfaces can be seen in Figures 10, 11, 12, 14, 15, and 16.

No references were examined that showed the effect of temperature on PCC wearing surfaces, base materials, or subgrade materials. It is known, however, that temperature does affect the elastic constants of these materials but to a lesser degree than it does those of AC wearing surfaces.

Strain. One has only to inspect Figure 26, a plot of stress versus strain for an unconfined clay specimen (after Terzaghi and Peck[31]), to recognize the extreme importance of strain on the value of an elastic constant in clay. Allen reported that Trollope, Lee, and Morris (Table 5) considered rate of deformation to be an influencing factor on the M_R of poorly graded dry sand. Lambe and Whitman[33] stated that E decreases and ν increases as strain level increases and that E increases with an increase in strain rate. Monismith et al. listed rate of deformation as an influence on M_R of untreated granular materials. Hardin showed a logarithmically decreasing G with an increase in strain amplitude. Lysmer et al.[28] listed strain amplitude as one of the most significant factors affecting G.

SENSITIVITY OF PAVEMENT RESPONSES TO
CHANGES IN ELASTIC CONSTANTS AND
THICKNESS

Pichumani's studies[5] provide data for a summary discussion of the effect of changes in elastic constants on surface deflection. Table 27 shows the factors by which E and ν may vary in flexible and rigid pavement layers to produce less than 10 percent change in the calculated surface deflections. The factors were derived by interpolation of results obtained by Pichumani with the AFPAV program. Although values are only approximate, this table shows the relative importance of accurate E and ν values for the pavement layers and subgrade.

RIGID PAVEMENTS

For rigid pavements, the E of the surface course is more critical than for flexible pavements, and accurate values for E's of the subgrade are just as important for rigid as for flexible pavements.

Table 27

Approximate Factor by Which Elastic Constants May Vary
from True Value and Produce Less Than 10 Percent
Error in Computed Surface Deflection

Pavement Type	Course	Factor E	Factor ν
Rigid	Surface	1.43	Not studied
	Subgrade	1.03	Not studied
Flexible	Surface	2.42	10.0
	Base	3.11	10.0
	Subbase	2.16	10.0
	Subgrade	1.05	1.07

There is no data on sensitivity of calculations to variations in ν for rigid pavements.

FLEXIBLE PAVEMENTS

In flexible pavements, calculations of surface deflection are not highly sensitive to either E values or Poisson's ratios of the surface, base, or subbase course, but both the E value and the Poisson's ratio of the subgrade must not vary more than 5 and 7 percent, respectively, from their true values, if less than 10 percent error in surface deflection is to be maintained.

Pichumani cautions that although the E values for flexible pavement base and subbase courses may not be highly significant for surface deflections, they are important in determining the state of stress in the upper layers of the pavement system.

RELATIONSHIPS BETWEEN VIBRATORY TEST RESULTS AND MATERIAL PARAMETERS

MATHEMATICAL MODELS

The method of Tschebotarioff[42] applies the linear spring-mass-dashpot model to one resonant peak of the frequency response spectrum. Weiss[3] has used a similar linear lumped-mass spring model to measured resonant peaks and has found that the linear model is inadequate to

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describe the resonant peak and to determine the subgrade Young's modulus; a nonlinear dynamic theory of the frequency-response spectrum is required. Yang[43] uses the linear lumped-mass spring model to obtain the E modulus of a pavement from the entire measured frequency response spectrum including all resonant peaks.

Waas[44] and Lysmer[45] have developed a linear elastic half-space model for calculating the response of a foundation resting on a layered elastic half-space. Waas describes the frequency response spectrum in terms of a numerical procedure based on the finite element method. The dynamic load-deflection curves predicted by this model are always linear because of the assumption of the linear elastic half-space.

A nonlinear harmonic oscillator model has been developed by Weiss[2,3] to describe the measured nonlinear dynamic load-deflection curves and to extract the value of the subgrade Young's modulus from these curves. This model also describes the laboratory resilient modulus test and can obtain the value of the Young's modulus from the measured resilient modulus. The value of the Young's modulus obtained from the field and laboratory tests should agree approximately with the Shell equation, E = 1500 CBR.

The wave propagation method of vibratory nondestructive testing of pavements has been studied recently by Lysmer et al.[28] Wave solutions of the basic dynamical elastic equations of motion are obtained subject to the boundary conditions at the pavement surface and at the interfaces of the pavement layers. Rayleigh wave dispersion curves giving phase velocity versus wavelength are obtained using a mechanical vibrator, and these curves exhibit discontinuous branches, which must be described theoretically by solving the secular determinant equation arising from the boundary conditions.

The wave propagation method has these disadvantages: (a) it requires a complicated theoretical description, which may not fully describe the physical situation; and (b) it predicts elastic moduli under very small stress and strain conditions, so that these elastic moduli cannot be used directly for pavement calculations that require elastic moduli under actual loading conditions.

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The linear regression equation for DSM also indicates that the elastic moduli of the various pavement layers and subgrade are related to the surface deflection in a complicated fashion. In addition, in any regression equation containing several terms for each pavement layer, there is no assurance that the contribution made by each pavement layer is exactly indicated by the equation.

VIBRATOR WAVE PATTERNS

The utilization of deflection basin measurements in attempts to extract elastic constants from vibratory test results is not considered practical for several reasons. The first reason is that the motion of soil away from the vibrator baseplate is complicated and not understood. The fact that the ground at some distance away from the baseplate may be out of phase with the baseplate by various degrees makes the data difficult to interpret. The second reason is that there are two or more types of waves that radiate away from the baseplate during vibration. Although vibratory devices have peak detectors that produce data which plot in neat "bowl-shaped" graphs, there is no certainty that the pavement always bends to conform to the graph of the pavement surface. The third reason is that deflection rapidly decreases in magnitude as distance from the baseplate increases. In many cases, the ability of the velocity transducers to produce accurate deflection measurements will be exceeded. A fourth reason for not using deflection basins is the complication of different wavelengths created by vibrations on different pavement sections and at different frequencies. In many cases, it will not be known if in the horizontal range of the transducers, a quarter, half, full, or some unknown fraction of a wavelength is being reflected by the measurements.

The Texas Transportation Institute procedure for extracting elastic constants from vibratory test results is useful but has three shortcomings. First, it uses deflection basin measurements that are difficult to analyze. Second, it relies on the static Burmister theory to describe dynamic loading; and third, the moduli values produced

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require multiplication by a judgment factor to be realistic for use in calculations involving heavy loads.

EXTRAPOLATION OF CONSTANTS FROM LOW TO HIGH STRESS LEVELS

The significance of stress level on modulus value is recognized, and Hardin's method[18] of extrapolating shear moduli at one strain level to any strain level appears to be the best available procedure. This procedure will allow the calculation of appropriate modulus values for stresses created by aircraft of any size.

PRELIMINARY RELATIONSHIPS ESTABLISHED BETWEEN WES 16-KIP VIBRATOR AND ELASTIC CONSTANTS

The empirical relationship between the DSM calculated for the WES 16-kip vibrator and the elastic modulus of the pavement layers apparently is valid since calculated values of DSM correlate well with measured values; however, the material properties of the layers are not known with enough precision to establish a reliable working relationship. Also, the empirical equation is not universally applicable as indicated by the fact that the minimum subgrade modulus value which can be computed is higher than values that have often been measured.

The surface pressures created by the 16-kip vibrator can be made equal to those created by aircraft by changing the baseplate size. However, the characteristics of the existing vibrator that cannot be altered to simulate aircraft loadings are: (a) one cycle of loading is not possible; (b) the static weight is fixed; and (c) it is possible to determine only the deflection created by the peak dynamic load.

During the course of this literature review, the reviewer constantly had in mind the objective of the overall program, namely, to develop a more rational and analytical method for evaluating airport pavements based on elastic theory and using elastic constants as determined from vibratory test results. As a consequence, it was inevitable that the reviewer be prompted to certain relevant impressions and ideas. These thoughts, some from the direct results of the data examined and others largely intuitive or even obvious, are offered here in the form of "Recommendations," in the hope that they will be of value in planning and/or implementing the ongoing study and future studies to develop improved methodology. As a final recommendation, a suggested breakdown of the ongoing study into phases or work areas is offered.

REVIEW OF ELASTIC CONSTANTS

MEASURING ELASTIC CONSTANTS

State of stresses for field testing should be duplicated in the laboratory. The effect of the inelastic behavior of materials on elastic modulus can be minimized by always using the same modulus (E_i , E_t , E_s , or E_h) in laboratory work.

VARIATIONS IN ELASTIC CONSTANTS

Frequency and Duration of Loading. The nonlinear model developed by Weiss[2,3] allows the extraction of elastic modulus values from field vibratory test results. These modulus values are independent of frequency as well as static and dynamic loads of the vibrator. Axial stresses and confining stresses are equal to in situ conditions with no load. On a stress-strain curve for any material, the elastic modulus determined by Weiss would correspond to the initial tangent modulus, E_i . Therefore, repetitive load laboratory testing done in conjunction with the Weiss work should be performed at frequencies as low as possible to minimize any possible disparities between quasi-static and dynamic test methods.

State of Stresses. Ahlvin and Foster[50] developed influence diagrams for stresses imposed in semi-infinite elastic media by a uniform circular surface load, which can be used to approximate stresses induced by vibratory loading. Figures 55 and 56 are the diagrams for vertical normal stress, σ_z , and horizontal normal stress, σ_r . Because these diagrams are for a semi-infinite media, and the present concern is with stresses beneath multilayered pavements, another approximation is necessary. This involves converting the pavement layers to equivalent thicknesses of subgrade as suggested in TM 5-312.[51] For example, assume stresses are desired for a point in a subgrade with a unit weight of γ_s at a distance h_s below the top of the subgrade. The subgrade is covered with a material with a unit weight of γ_1 , which has a thickness, t . The depth, z , to be used with the influence diagrams is computed as

$$z = h_s + \frac{\gamma_1}{\gamma_s} (t)$$

Age of Material. When age could be an important factor in modulus or vibrator test results, the effect will have to be ignored. Insufficient information exists on age effects to allow even approximate quantification of its effects on any material.

Compositions and Material Properties. Wide ranges in behavior due to compositions and properties of materials, especially soils, make the assignment of values of E , ν , and G from tabulated values nothing but guesswork. Hardin's method[15] for determining shear modulus of soils and gravels appears to consider all pertinent factors, and similar procedures will have to be adopted for all pavement materials; however, time will not allow the development of these procedures for the methodology presently being developed for FAA by WES. Therefore, it is recommended that methods for estimating elastic constants for the evaluation methodology be developed based on existing equations that account for in situ material properties and stress levels.

Hicks' list[39] of constants K_1 and K_2 should be expanded. His
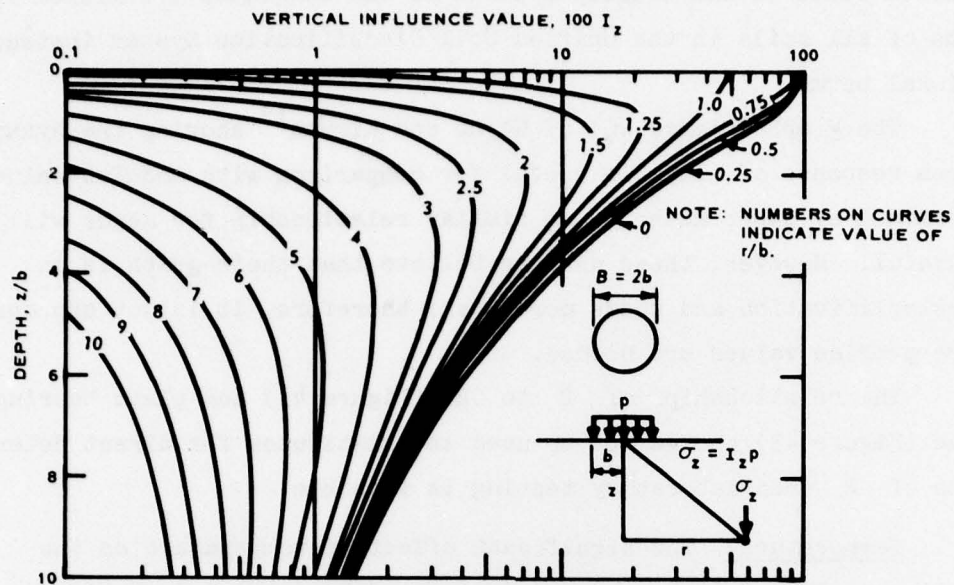
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VERTICAL INFLUENCE VALUE, 100 I_z

Figure 55. Influence diagram for vertical normal stress σ_z at various points within an elastic half-space under a uniformly loaded circular area (after Ahlvin and Foster[50])

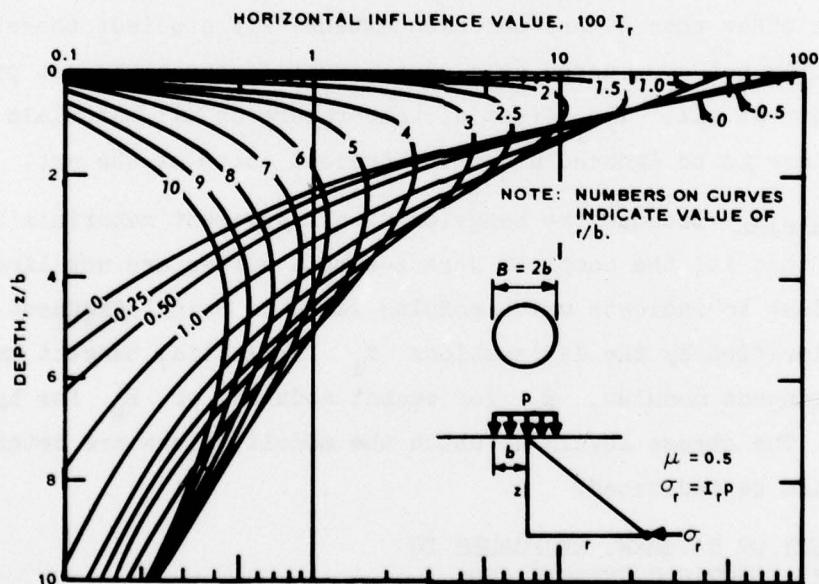HORIZONTAL INFLUENCE VALUE, 100 I_r

Figure 56. Influence diagram for horizontal normal stress σ_r at various points within an elastic half-space under a uniformly loaded circular area (after Ahlvin and Foster[50])

equation could be universally applied if the constants are listed in terms of all soils in the Unified Soil Classification System instead of regional terms.

The graph (Figure 40) of Glynn and Kirwan[13] showing the dynamic stress response of clays is useful for comparison with modulus values determined by other methods. A similar relationship for sands will also be useful. However, these authors believe that their graph is an oversimplification and needs more work; therefore, it is not appropriate where precise values are needed.

The relationships of E to CBR (Figure 41) and plate bearing value (Figure 43) should not be used as substitutes for direct determinations of E when laboratory testing is feasible.

Temperature. The significant effect of temperature on the elastic modulus of AC has been demonstrated by several investigators. It is obvious that temperature should be accounted for in field and laboratory tests on AC. Modulus tests performed in the laboratory should duplicate field temperature conditions. The effect of temperature on materials other than AC has not been extensively studied; therefore, it is evidently not considered as a significant factor or is not practical to consider at all. The effect of temperature on all materials except AC will have to be ignored under the present state of the art.

Strain. Because the behavior of all pavement materials is inelastic, that is, the complete stress-strain curves are not linear, it is important to indicate which modulus value is being discussed. This can be clarified by the designations E_i for initial tangent modulus, E_t for tangent modulus, E_s for secant modulus, or E_h for hysteresis modulus. The stress levels at which the moduli values are determined should also be indicated.

SENSITIVITY OF PAVEMENT RESPONSES TO
CHANGES IN ELASTIC CONSTANTS AND
THICKNESS

Pichumani's study[5] of the sensitivity of computed pavement deflections to changes in elastic constants and thickness showed that

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the E values of the layers of flexible pavement could be in error by a factor of 2 or 3 and not significantly affect the computed deflection. Also, the E value of the PCC could be in error by a factor of about 1.5 and not significantly affect the deflections computed for rigid pavements. However, he varied only one parameter at the time, but in practice, the effect of choosing erroneous values for all pavement layers, including a subbase for rigid pavement, should be considered. Consequently, it is imperative, if accurate results are to be obtained from the layered elastic method, that values of E and ν be estimated with more precision than he indicates is necessary. A comprehensive sensitivity study of any proposed pavement evaluation methodology should be made.

Pichumani also showed the importance of accurate values for ν of subgrade materials. However, work by several researchers indicates that ν is difficult to determine even under carefully controlled laboratory conditions. Therefore, it is recommended that a procedure for estimating ν be adopted.

RELATIONSHIPS BETWEEN VIBRATORY TEST RESULTS AND MATERIAL PARAMETERS

MATHEMATICAL MODELS

Weiss's nonlinear mathematical model,[2,3] which describes the response of a pavement structure to vibratory loading of the WES 16-kip vibrator, appears to be the most practical and accurate procedure to adopt for the determination of elastic constants. The Weiss model uses DSM graphs or frequency response curves; therefore, other forms of non-destructive test results, such as deflection basins or wave velocity measurements, need not be considered at this time.

VIBRATOR WAVE PATTERNS

The effect of the input wave shape on the response of a material is not known. The response of the pavement system and subgrade to sinusoidal loading of the vibrator and of test laboratory specimens to haversine loading will be assumed to approximate the response to loading by the vibrator.

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EXTRAPOLATION OF CONSTANTS FROM
LOW TO HIGH STRESS LEVELS

Hardin's method[18] for extrapolation of shear moduli to any stress
level should be included in the proposed evaluation methodology.

PRELIMINARY RELATIONSHIP BETWEEN WES
VIBRATOR AND ELASTIC CONSTANTS

The empirical equation that relates DSM, E , and t adds to the
evidence that vibratory test results and the elastic moduli of materials
are related. A better relationship could be established if pavement and
subgrade constants were known with more precision. However, it is
recommended that efforts be directed at developing and complementing the
Weiss theory[2,3] rather than improving an existing empirical relationship.

BREAKDOWN OF ONGOING STUDY FOR DE-
VELOPMENT OF EVALUATION PROCEDURE
BASED ON LAYERED ELASTIC THEORY
AND VIBRATORY TEST RESULTS

The ongoing study can be divided into four areas of work:

a. Field and laboratory work that will consist of performing
    nondestructive testing and laboratory testing on several
    different types of soil in order to provide data with which
    to verify the theoretical procedures proposed by Weiss.

b. Choosing the best methods to estimate the elastic moduli and
    Poisson's ratios of pavement layers, as substitutes for
    difficult testing methods.

c. Further investigation of the procedures for:

    (1) Extrapolating modulus values to any strain level.

    (2) Relating pavement response characteristics to the
        vibrator to pavement response characteristics to the
        aircraft.

    (3) Calculating stresses in soils due to vibratory loading.

    (4) Determining relationships between quasi-static and
        dynamic conditions.

d. Finally, the pertinent information contained in this report;
    the results of the three areas of work listed as a, b, and
    c above; the Weiss theory; and the reverse of a design
    procedure based on layered elastic theory will be formed into
    a pavement evaluation procedure.

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APPENDIX A: ABBREVIATIONS AND SYMBOLS

Recurring abbreviations and sumbols are listed below. Symbols used only once or repeatedly in close proximity, and defined in that proximity, are not shown in the list. Symbols of this type appear in some tables and figures. To be certain of the significance of symbols in tables and figures, the reader should refer to the pertinent text.

AC	Asphaltic concrete
AI	Asphalt Institute
DSM	Dynamic stiffness modulus
E	Young's modulus; elastic modulus
E_h	Hysteresis modulus
E_i	Initial tangent modulus
E_r	Resilient modulus
E_s	Secant modulus
E_t	Tangent modulus
E^*	Complex modulus
$|E^*|$	Magnitude of complex modulus
FAA	Federal Aviation Administration
G	Shear modulus
h	Thickness of a pavement layer
MIT	Massachusetts Institute of Technology
M_R	Resilient modulus
MRD	Materials Research and Development
PCC	Portland cement concrete
USNCEL	USN Civil Engineering Laboratory
WES	U. S. Army Engineer Waterways Experiment Station
γ	Shear strain
ϵ	Strain
$\epsilon_x, \epsilon_y, \epsilon_z$	Strains along the axes, x, y, and z
λ	Wavelength
ν	Poisson's ratio
σ	Normal stress

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$\sigma_x, \sigma_y, \sigma_z$ — Normal stresses acting along the axes, x, y, and z

$\sigma_1, \sigma_2, \sigma_3$ — Normal stresses acting along the axes, 1, 2, and 3 of an orthogonal coordinate system

τ — Shear stress

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