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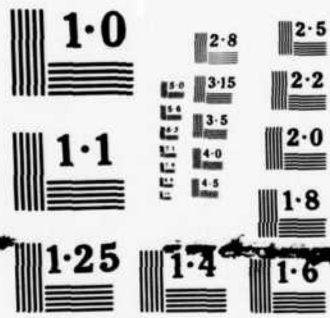

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Temperature Correlations in Turbulent Boundary Layers

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7 June 1978

Interim Report

APPROVED FOR PUBLIC RELEASE;
DISTRIBUTION UNLIMITED

Prepared for
AIR FORCE WEAPONS LABORATORY
Kirtland Air Force Base, N. Mex. 87117

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This interim report was submitted by The Aerospace Corporation, El Segundo, CA 90245, under Contract No. F04701-77-C-0078 with the Space and Missile Systems Organization, Deputy for Advanced Space Programs, P.O. Box 92960, Worldway Postal Center, Los Angeles, CA 90009. It was reviewed and approved for The Aerospace Corporation by A. H. Silver, Director, Electronics Research Laboratory. Lieutenant Dara Batki, SAMSO/YCPT, was the project officer for Advanced Space Programs.

This report has been reviewed by the Information Office (OI) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nations.

This technical report has been reviewed and is approved for publication. Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER SAMSO TR-78-90	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) TEMPERATURE CORRELATIONS IN TURBULENT BOUNDARY LAYERS		5. TYPE OF REPORT & PERIOD COVERED Interim rept.
		6. PERFORMING ORG. REPORT NUMBER TR-0078(3606)-1
7. AUTHOR(s) William C. Meecham (Consultant)		8. CONTRACT OR GRANT NUMBER(s) F04701-77-C-0078
9. PERFORMING ORGANIZATION NAME AND ADDRESS The Aerospace Corporation El Segundo, Calif. 90245		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Weapons Laboratory Kirtland Air Force Base, N. Mex. 87117		12. REPORT DATE 7 June 1978
		13. NUMBER OF PAGES 35 39p.
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) Space and Missile Systems Organization Air Force Systems Command Los Angeles, Calif. 90009		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Turbulent Boundary Layers
Temperature Correlations in Layers
Temperature Correlations
Temperature Fluctuation Effects

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Temperature fluctuation effects are examined in heated turbulent boundary layers. A modified Lagrangian integral procedure is used. The temperature fluctuations are matched at the laminar sublayer by using standard heat transfer theory to obtain their value. It is found that the temperature correlations are similar to the velocity correlations. They are, of course, at maximum near the laminar sublayer, decrease as one passes through that layer, and vanish beyond the boundary layer. The temperature → next pag

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SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

19. KEY WORDS (Continued)

20. ABSTRACT (Continued)

correlation is an integral of the velocity correlations, which are assumed in this treatment to be given. It appears from this theory that they can have a small negative correlation at distances of a few local velocity scale lengths. There is a very weak Mach number dependence in the result.

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PREFACE

The author acknowledges the significant support of Dr. M. T. Tavis in this work.

CONTENTS

I. INTRODUCTION

Recently, a considerable interest has developed in the temperature fluctuations occurring within heated boundary layers. The specific important applications include the effects of such temperature fluctuations, notably through attendant density fluctuations, upon laser beam propagation through boundary layers. Primarily, such problems arise for beams being sent through boundary layers occurring about jet planes, flying at considerable altitude, and at speeds near (the ambient) Mach 1. The density changes that affect laser propagation can be caused, of course, either by the temperature fluctuations, considered here, or by pressure fluctuations; the latter effects have been treated elsewhere[1] by techniques similar to those used in this paper. We assume in this work that the temperature effects dominate, and, we neglect the pressure fluctuation effects.

Usually work on heated boundary layers has been directed toward finding rms temperature fluctuations (among other quantities) at the most. Here for the laser propagation problem, and for other problems as well, the temperature correlation function is needed. For simple rms values of the temperature fluctuation it would be possible to use some variation of mixing-length methods. But of course for the correlation function much more information is needed. Our approach here (involving beginning with the usual equations of motion for the fluid) uses an approximate Lagrangian integration procedure in order to obtain the required results.

The basic procedure to be followed consists of beginning with the energy equation for fluid flow. In that equation we shall generally neglect the effects of thermal conduction within the heated turbulent boundary layer, with the exception of the region very near the wall bounding the layer; in this (laminar) sublayer and regions contiguous to it, there is heat generated by the viscous action, as a result of the very high velocity-shear occurring very near the wall. This heat generation, given quantitatively by the dissipation function, is included in the problem as part of the heat flux occurring within the boundary layer. In addition, there is a rapid deceleration of the fluid near the wall through viscous action; since this process is not adiabatic, there is some irreversible heat generation, also to be included in the heat flux. And, finally, in problems of this type the wall temperature is in general different than the ambient temperature. In the application cited above, it is considerably higher than the external atmospheric temperatures. In our treatment, this heated wall is also taken into account. This effect can, again, be included as part of the heat flux within the boundary layer.

The boundary layer near a flat plate or wall is initiated at the leading edge and grows slowly as one progresses downstream (in the x-direction). In our treatment, we assume (as is usual) that the layer growth rate is slow. The process does not change appreciably, in regions of interest, in the downstream direction. The derivatives of important quantities within the flow field are much greater in the normal direction (the z-direction) than in the x-direction. For such a flow, the velocity fluctuations are approximately statistically homogeneous in planes parallel to the wall. Further, even for flows approaching Mach 1, it is well known that the velocity fluctuations, which are particularly important in the treatment here, are at relatively small Mach number: the rms velocity is considerably below the mean flow velocity, typically no more than 10% of that velocity. Thus, for the fluctuations, one can treat the problem as incompressible. We carry that assumption here but, of course, allow temperature variations to produce attendant density variations. In effect, we are adopting the familiar assumption that the dynamics of the process are but slightly influenced by compressible effects. It will be seen in the development that the temperature fluctuations under our assumptions can be determined from the velocity fluctuations by an integration process. We do not attempt here to solve for velocity fluctuations; we suppose that they are given.

In this treatment, we assume that the total convected heat flux is a given quantity and that quantity will be determined from familiar fluid mechanics theory.

The needed velocity correlations are taken from statistically homegeneous and isotropic flows (notably wind tunnel experiments), but the correlations are weighted with the rms velocity fluctuations at the measuring points for the velocity field. The scale for the correlation functions is proportional to the distance from the wall so long as one lies within the boundary layer and is δ, the displacement thickness of the boundary layer, outside that layer.

We proceed to calculate the temperature space-correlation (and its variance). We must be given the turbulence intensity level in this treatment. A separate calculation (using standard methods from heat transfer theory) is made to obtain the convective heat flux, just above the layer δ_s.

II. THEORY

A sketch of the boundary layer problem is shown in Fig. 1, where the average velocity $\bar{u}$ depends only on z, following our parallel flow assumption for the turbulent boundary layer. For this problem, the free stream velocity is $\bar{u}$ and δ is the displacement thickness of the boundary layer defined by

$$\delta = \bar{u}_\infty^{-1} \int_0^\infty [\bar{u}_\infty - \bar{u}(z)]\, dz \tag{1}$$

The average temperature, also a function of z alone, is $\overline{T}(z)$. The temperature of the wall is T_w. The laminar sublayer is of thickness δ_s. In Fig. 1, the heating effect of the dissipation and the imperfect temperature recovery, confined to regions near the sublayer, is shown to cause an increase in the average temperature. The ambient value of the temperature, T_∞, is shown for z large. The temperature T_w is assumed here to exceed the ambient value though, of course, if it were less than that value the treatment would be the same. The energy equation for compressible fluid flow is given[2] by

$$\rho C_p \frac{DT^*}{Dt} = \frac{Dp^*}{Dt} + \nabla \cdot (k \nabla T^*) + \mu\Phi \tag{2}$$

where the dissipation function Φ is given by

$$\Phi = 2\left[\left(u_x^*\right)^2 + \left(v_y^*\right)^2 + \left(w_z^*\right)^2\right] + \left(v_x^* + u_y^*\right)^2 + \left(w_y^* + v_2^*\right)^2$$
$$+ \left(u_z^* + w_x^*\right)^2 - \frac{2}{3}(\nabla \cdot \underline{u}^*)^2 \tag{3}$$

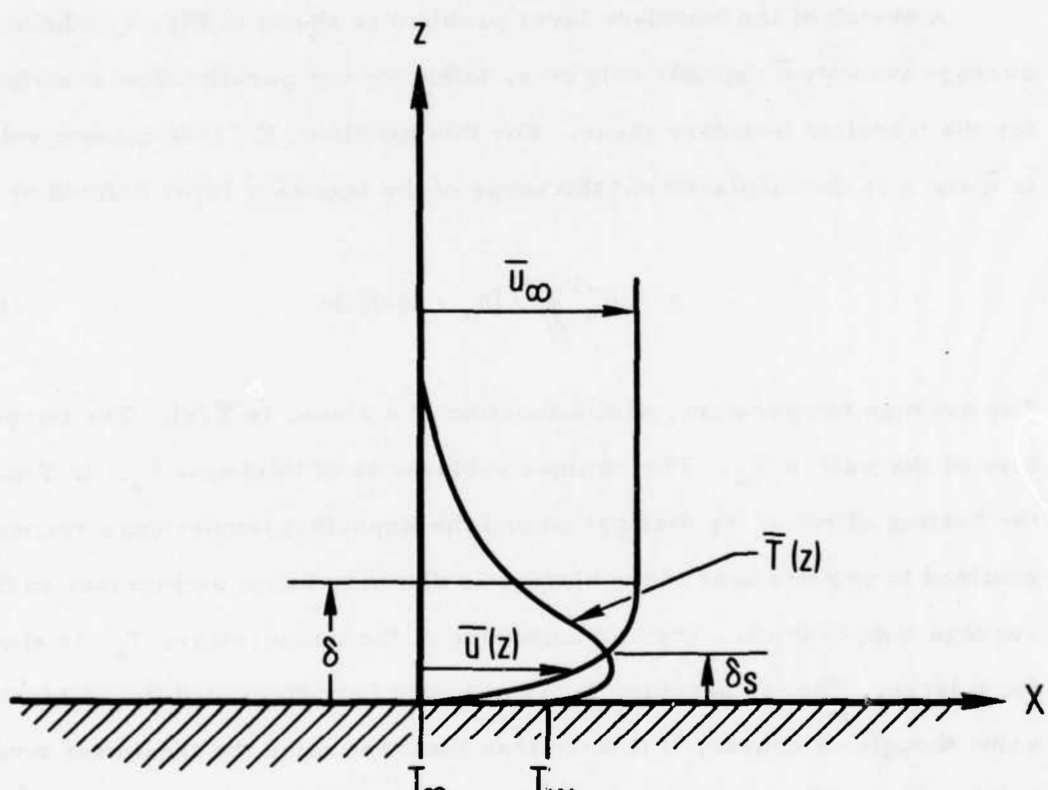

Figure 1. Thermal Boundary Layer. The relative size of δ_s and δ is exaggerated.

with

$$\underline{u}^* = (u^*, v^*, w^*)$$

and where subscripts x, y, and z indicate partial derivatives here and later. We also use the corresponding component notation for other fields later. For incompressible flow Eq. (2) reduces to (for approximately constant thermal conductivity k)

$$\rho C_p \frac{DT^*}{Dt} = k\nabla^2 T^* + \mu\Phi \tag{4}$$

where Φ is given by Eq. (3) without the last term. In these equations, ρ, assumed here to be approximately constant, is the density, C_p is the specific heat at constant pressure, per unit mass, and μ is the viscosity of the fluid. Further, D/Dt is the material derivative given by

$$DT^*/Dt = \partial T^*/\partial t + \underline{u}^* . \nabla T^* \tag{5}$$

or, for incompressible flow, if convenient, we can write

$$DT^*/Dt = \partial T^*/\partial t + \nabla \cdot (\underline{u}^* T^*)$$

The change in density is assumed small in our work. In Eq. (4), strictly speaking, for incompressible flow there is but one specific heat. However, we assume here that there is a response of the medium to the temperature changes set in near the wall. The appropriate specific heat is that at constant pressure, for such a slight variation from incompressible flow.

The dissipation function is, as suggested earlier, small throughout most of the boundary layer and may be neglected there. In the region δ_s near the wall, because of the very high shear-velocity, there may be an appreciable generation of heat. Accordingly, we treat the region just outside the viscous sublayer and take into consideration the heat generated within that layer when calculating the total heat transfer through the boundary layer. Furthermore, we neglect (for either air or water) the very slight heat conduction term, which is small compared with the convective effects, outside the sublayer δ_s. In dimensionless form, we know that the heat conduction is of order the viscous dissipation. For large Reynolds number flows being considered here, these two effects are in general small except for special regimes: specifically, the heat conduction is of order the convective effect, near the laminar sublayer δ_s. The result of these considerations is that our governing equation becomes

$$DT^*/Dt = 0 \qquad (6)$$

Thus, there is no change in temperature (no heat exchange) following the flow. Under the stated assumptions, Eq. (6) may be expected to apply everywhere outside δ_s — through the boundary layer and out to regions beyond the layer.

III. DERIVATION OF AVERAGES AND CORRELATION FUNCTIONS

We begin by writing the field quantities in terms of their averages and fluctuations about those averages.

$$T^* = \bar{T} + T; \; \underline{u}^* = \bar{\underline{u}} + \underline{u} \qquad (7)$$

with, by definition,

$$\langle T \rangle = \langle \underline{u} \rangle = 0$$

By incompressibility we see that

$$\nabla \cdot \underline{u}^* = \nabla \cdot \bar{\underline{u}} = \nabla \cdot \underline{u} = 0 \qquad (8)$$

The overbar indicates the (time) average; we assume that the process is statistically stationary. Substituting Eq. (7) into Eq. (6) and averaging the resulting equation, we find that

$$\partial \bar{T}/\partial t + \nabla \cdot (\bar{\underline{u}}\bar{T} + \overline{\underline{u}T}) = 0 \qquad (9)$$

Note that the heat flux q convected in the normal direction, away from the wall, is given by

$$q/C_p\rho = \overline{wT} \qquad (10)$$

One can see this relation by considering the following: If a parcel of fluid with slightly higher temperature is transported upward, that is with w positive, there will be an increase in heat in the upper region, and conversely. Thus, the net heat flux flowing upward is proportional to $\overline{wT}$.

We again substitute Eq. (7) into Eq. (6) and subtract Eq. (9) to find the fluctuation relation, using Eq. (8).

$$\dot{T} + (\underline{\bar{u}} + \underline{u}) . \nabla T = - \nabla . (\underline{u}\bar{T} - \overline{\underline{u}T}) \tag{11}$$

Equation (11) states that the material derivative of the temperature fluctuation (the time rate of change of that fluctuation following the flow) is equal to the two terms on the right side. Of those terms the first is considerably larger than the second, involving as it does the average temperature rather than the fluctuation; we neglect the second term here. Before attempting to integrate this equation, some discussion is necessary. Begin with a parcel of fluid at constant temperature, that is, with no fluctuation; such constant temperature boundaries occur at the wall and in the free stream. This parcel of fluid then diffuses by what amounts to a random walk into the boundary layer, arriving at the point of interest. This diffusion path is nearly horizontal but of course not exactly so. The vertical distance through which it diffuses in time T_D is of order Δz,

$$(\Delta z)^2 = \left\langle \left(\int_0^{T_D} w(t)dt \right)^2 \right\rangle \cong (w')^2 \, T_D \, t_o \tag{12}$$

where t_o is the characteristic time for the turbulence process and the prime as usual indicates the rms value of the vertical velocity fluctuation.

The characteristic time for the turbulent process is known to be approximately

$$t_o = \delta / w'$$

The distance Δz, which is required here for the diffusion process, is quite complicated. Near the wall the distance is essentially z, the distance from the wall, and the fluid which arrives consists mainly of hot elements of fluid. On the other hand, near the edge of the boundary layer, that is, z near δ, the diffusion distance is something like δ-z and the fluid that arrives comes mainly from the cold, free stream. Finally, no fluid with temperature fluctuation arrives at points outside the boundary layer, so that there the diffusion time may be taken as zero. We use for the distance over which the diffusion occurs, the convenient, approximate expression

$$\begin{aligned} \Delta z^{-2} &= z^{-2} + (\delta - z)^{-2}, \quad z < \delta \\ &= 0 \qquad\qquad\qquad\;\;, \quad z > \delta . \end{aligned} \tag{13}$$

Combining the simple expressions, we obtain for the time over which the diffusion occurs, the result

$$T_D = \left|[z^{-2} + (\delta - z)^{-2}]\, \delta w'\right|^{-1}, \quad z < \delta \tag{14}$$

and $T_D = 0$ for $z > \delta$. Using the incompressibility condition on $\underline{u}$ and the parallel flow assumption, we can write for Eq. (11), neglecting the term $- \nabla \cdot \overline{uT}$,

$$DT/Dt = -w\overline{T}_z \tag{15}$$

where the subscript indicates differentiation; now, we formally integrate that equation to find

$$T(\underline{r}) = - \overline{T}_z \int_0^{T_D} w[x - \overline{u}(z)\tilde{t},\ y,\ z;\ t - \tilde{t}]d\tilde{t} \tag{16}$$

We suppose that the parcel of fluid began in a region where there was no temperature fluctuation. In integrating along the path of the fluid as shown in Eq. (16) we have dropped the velocity fluctuation in the argument of 2 since it in general is much smaller than the mean flow used there. Dropping the velocity fluctuation has the effect of integrating the function w along a line parallel to the x-axis rather than following the actual path taken by the fluid particle. This, of course, introduces some error in the result (see Appendix A). The various accumulated errors in the treatment will be handled by introducing a multiplicative constant, which will be adjusted in order to give the proper heat flux near the laminar sublayer.

We proceed to construct the temperature correlation function. To do this, we write Eq. (16) at the point $\underline{r}'$ rather than $\underline{r}$ as here, then do the same at the point $\underline{r}''$. Multiplying the resulting equations together and averaging, we obtain

$$\begin{aligned} \tau(\underline{r}', \underline{r}'') &= \langle T(\underline{r}')\, T(\underline{r}'') \rangle \\ &= \overline{T}_{z'} \overline{T}_{z''} \int_0^{T_D} dt' \int_{t'}^{T_D - t'} dt \\ &\quad \times R_{33}[x - \overline{u}(z')(t + t') + \overline{u}(z')t',\ y;\ z',\ z'',\ t] \end{aligned} \tag{17}$$

In Eq. (17) we used the parallel flow assumption, that the velocity correlation is statistically homogeneous in planes parallel to the wall. As a consequence, the correlation is a function of the difference arguments in the x- and y- directions. That correlation is defined by the relation

$$R_{ij}(\underline{r}', \underline{r}'') = \langle u_i(\underline{r}')\, u_j(\underline{r}'')\rangle \tag{18}$$

where we employ the notation

$$\underline{r} = \underline{r}'' - \underline{r}' \tag{19}$$

We also changed one variable of time integration from t'' to $t = t'' - t'$. For Eq. (17) we know that the scale of the correlation function in space is of order $M \sim \delta$, and in time is of order t_o, given by Eq. (12). It is seen, then, that the integral is controlled by the integration of the time dependence in the x-argument. The time dependence in the time argument itself has a much larger scale. Using that fact we can integrate Eq. (17) and obtain the result

$$\tau(\underline{r}', \underline{r}'') = \bar{T}_{z'}\, \bar{T}_{z''}\, \frac{T_D}{\bar{u}(z'')} K^2 \int_{-\infty}^{\infty} R_{33}\,(\xi, y; z', z'')\, d\xi \tag{20}$$

where R_{33} may be taken to be the simultaneous (z-directed) velocity correlation. In Eq. (20), there is no x-dependence for the temperature correlation until the separation between the measuring points in the x-direction exceeds (the erroneous x-scale) $\bar{u}T_D \sim \bar{u}M/w'$. Thus, the scale in the x-direction would be considerably greater than the scale in the other directions. This, as is pointed out in Appendix A, is a result of the approximation made in Eq. (16). Further as pointed out there, a multiplicative constant is introduced by the approximation, inserted in Eq. (20) and called K^2.

In order to proceed with the calculation, we need the correlation R_{33}; we make the plausible assumption that it is approximately given by

$$R_{33}(\underline{r}', \underline{r}'') = w'(z')\, w'(z'')\, R^*_{33}(\underline{r}) \qquad (21)$$

The R^*_{ij} is the normalized, second-order velocity correlation for incompressible, homogeneous and isotropic turbulence. These correlations have been much measured experimentally. It is seen that the proposed velocity correlation for this problem is a weighted, statistically homogeneous correlation, the weighting being taken at the measuring points for the correlation function. There is a question as to whether an incompressible flow field may be represented using such weighting functions. It is shown in Appendix B that this is possible. By our parallel flow assumption the rms velocities, being average quantities, depend only on z. From the theory of homogeneous and isotropic turbulence[3] we know that the normalized correlation can be written

$$R^*_{ij} = -\frac{1}{2r}\frac{\partial f\left(\frac{r}{M}\right)}{\partial r} r_i r_j + \left(f + 1/2\, r \frac{\partial f}{\partial r}\right)\delta_{ij} \qquad (22)$$

so

$$R^*_{33} = f\left(\frac{r}{M}\right) + \frac{r}{2M} f'\left(\frac{r}{M}\right)\left(1 - \frac{z^2}{r^2}\right) \tag{23}$$

with

$$f'\left(\frac{r}{M}\right) = \frac{\partial f\left(\frac{r}{M}\right)}{\partial (r/M)}$$

where M is the integral scale of the turbulence, and f is the longitudinal correlation. Referring again to Ref. 3, the transverse correlation is related to the longitudinal

$$g = f + \frac{1}{2}\left(\frac{r}{M}\right) f' \tag{24}$$

Consider M, the scale of the turbulence. Near the wall, the scale is given by the distance from the wall. For the two-point correlations considered here the scale must be given by the lesser distance. As we proceed through the boundary layer, finally the scale approaches the thickness of the boundary layer and outside that layer we can expect that any disturbances which remain will retain the scale of the boundary layer thickness. We accordingly choose for the function M, the following form

$$M^{-2}(z', z'') = (z')^{-2} + (z'')^{-2} + \delta^{-2} \tag{25}$$

Substituting from Eqs. (22) and (23) in Eq. (20) and defining the normalized coordinates, we obtain

$$\rho = r/M, \ \xi = x/M, \ \eta = y/M, \ \zeta = z/M \tag{26}$$

where x, y, and z are the difference arguments defined above, we find Eq. (27)

$$\tau(\underline{r}', \underline{r}'') = \overline{T}_{z'} \overline{T}_{z''} \frac{T_D}{\overline{u}(z'')} K^2 w'(z') w'(z'') M(z', z'')$$

$$\times 2 \int_{\sqrt{\eta^2 + \zeta^2}}^{\infty} \frac{\rho d\rho}{\sqrt{\rho^2 - \eta^2 - \zeta^2}} \left[f(\rho) + \frac{\rho}{2} f'(\rho) \left(1 - \frac{\zeta^2}{\rho^2} \right) \right] \tag{27}$$

In order to determine the constant K we adjust the temperature fluctuation at the laminar sublayer to equal the value obtained using heat transfer theory found in Appendix C, Eq. (C13). To do so, let $\underline{r}'$ equal $\underline{r}''$ equal δ_s. We see from Eq. (13) that for such small values of z

$$\Delta z^2 \simeq \delta_s^2 \tag{28}$$

Then $\overline{T}_{z}, \delta_s$ may be approximated by $T_w - T_\infty$. We obtain for the mean square temperature fluctuation, the form

$$T'^2(\delta_s) = (T_w - T_\infty)^2 K^2 \frac{w'(\delta_s)}{\overline{u}_\infty} \alpha \tag{29}$$

or, using the nominal value $[w'(\delta_s)/\bar{u}_\infty] = 0.05$ (Ref. 7)

$$T'(\delta_s) = (T_w - T_\infty)\,K(0.05)^{1/2}\,\alpha^{1/2} \tag{30}$$

Here, α is the integral

$$\alpha = \int_0^\infty f(\xi)d\xi = 0.5 \tag{31}$$

the numerical value for this integral is obtained below from a large Reynolds number calculation for the longitudinal correlation function f. As suggested, we compare this result with that obtained from heat transfer theory given in Eq. (C13). The result for K is

$$K = 0.16\,(M_\infty\,x)^{-1/5} \tag{32}$$

The rms temperature fluctuation throughout the layer is,

$$T'(z') = 0.12\,T_{z'}(z')\,\Delta z\left(\frac{w'(z')}{\bar{u}(z')}\right)^{1/2}\left(M_\infty\,\frac{x}{x_o}\right)^{-1/5} \tag{33}$$

from Eq. (27), where x_o is 1m and x is measured from the beginning of the boundary layer.

For simplicity, we assume (without great error) that the mean temperature, the turbulent velocity fluctuation, and the mean velocity do not vary greatly in going from z′ to z″, then the normalized temperature correlation function, found by dividing Eq. (27) by the square of Eq. (33), is

$$\frac{\tau(\underline{r}', \underline{r}'')}{[T'(\underline{r}')]^2} = 2\int_{\sqrt{\eta^2+\zeta^2}}^{\infty} \frac{\rho d\rho}{\sqrt{\rho^2 - \eta^2 - \zeta^2}}\left[f(\rho) + \frac{\rho}{2}\, f'(\rho)\left(1 - \frac{\zeta^2}{\rho^2}\right)\right]$$

$$= I_1(\eta^2 + \zeta^2) + \zeta^2\, I_2(\eta^2 + \zeta^2) \tag{34}$$

where

$$I_1 = 2\int_{\sqrt{\eta^2+\zeta^2}}^{\infty} \frac{\rho d\rho}{\sqrt{\rho^2 - \eta^2 - \zeta^2}}\left[f + \frac{\rho}{2} f'\right]$$

$$I_2 = 2\int_{\sqrt{\eta^2+\zeta^2}}^{\infty} \frac{\rho d\rho}{\sqrt{\rho^2 - \eta^2 - \zeta^2}}\left[-\frac{\rho}{2} f'\right]$$

IV. DISCUSSION OF RESULTS AND CONCLUSIONS

The standard deviation of the temperature (the rms fluctuation) given in Eq. (33) has a constant chosen to match the expected standard deviation of the temperature near the laminar sublayer, as found using heat transfer theory. This is discussed in Appendix C. We see from Eq. (33), again near the sublayer, that the temperature standard deviation is

$$T'(\delta_s) = 0.027(T_w - T_\infty)\left(M_\infty \frac{x}{x_o}\right)^{-1/5} \tag{35}$$

with a nominal value used for the turbulence intensity and $T_{Z'}(Z')\Delta Z$ replaced by $T_w - T_\infty$. This result is approximately a few percent of the temperature contrast. The Mach number dependence is very weak, probably quite difficult to find experimentally. The dependence on distance downstream from the leading edge likewise is very weak.

The normalized temperature correlation is given in Eq. (34). It seems plausible that the x-dependence of the correlation, not found from the present analysis, will be essentially similar to the dependence in the y-direction. We propose replacing $\eta^2 + \zeta^2$ in Eq. (34) by σ^2 defined by

$$\sigma^2 = (x^2 + y^2 + z^2)/M^2 \tag{36}$$

where, recall, M is the local scale of the turbulence, approximately the distance from the wall when close to the wall, and δ is the displacement thickness of the boundary layer as one moves out into that layer. Then the normalized correlation may be written following Eq. (34)

$$\frac{T(r', r'')}{[T'(z')]^2} = I_1(\sigma^2) + \zeta^2 I_2(\sigma^2) \tag{37}$$

To calculate these integrals we need to know the normalized longitudinal velocity correlation f. This function, for the large Reynolds number flows of interest here, could be taken from experiments; however, we must differentiate the function once in constructing the required integrals. That makes the process somewhat more difficult. Therefore, we have chosen to take the function f from recent large Reynolds number calculations. The longitudinal correlation function of Hogge and Meecham has proper continuity and differentiation is not a problem. The results of the computation of the two integrals I_1 and I_2 are presented in Fig. 2. Specifically, for the two measuring points at the same height z' from the plane, the correlation function is just I_1. It is interesting to note that there is a small negative lobe in normalized value of about -0.2 at a distance in the x-y plane of 2 to 3 local correlation lengths. This arises from the known negative lobe of the transverse velocity correlation, on which this integral is dependent. If we look at the correlation in the vertical direction, that is $x = y = 0$, we need $I_1(\zeta^2) + \zeta^2 I_2(\zeta^2)$. Again, it appears that a negative correlation will develop, according to our results, though it is not large.

The density fluctuation, which is needed for optical propagation through the heated boundary layer, may be obtained from the ideal gas law. Recall

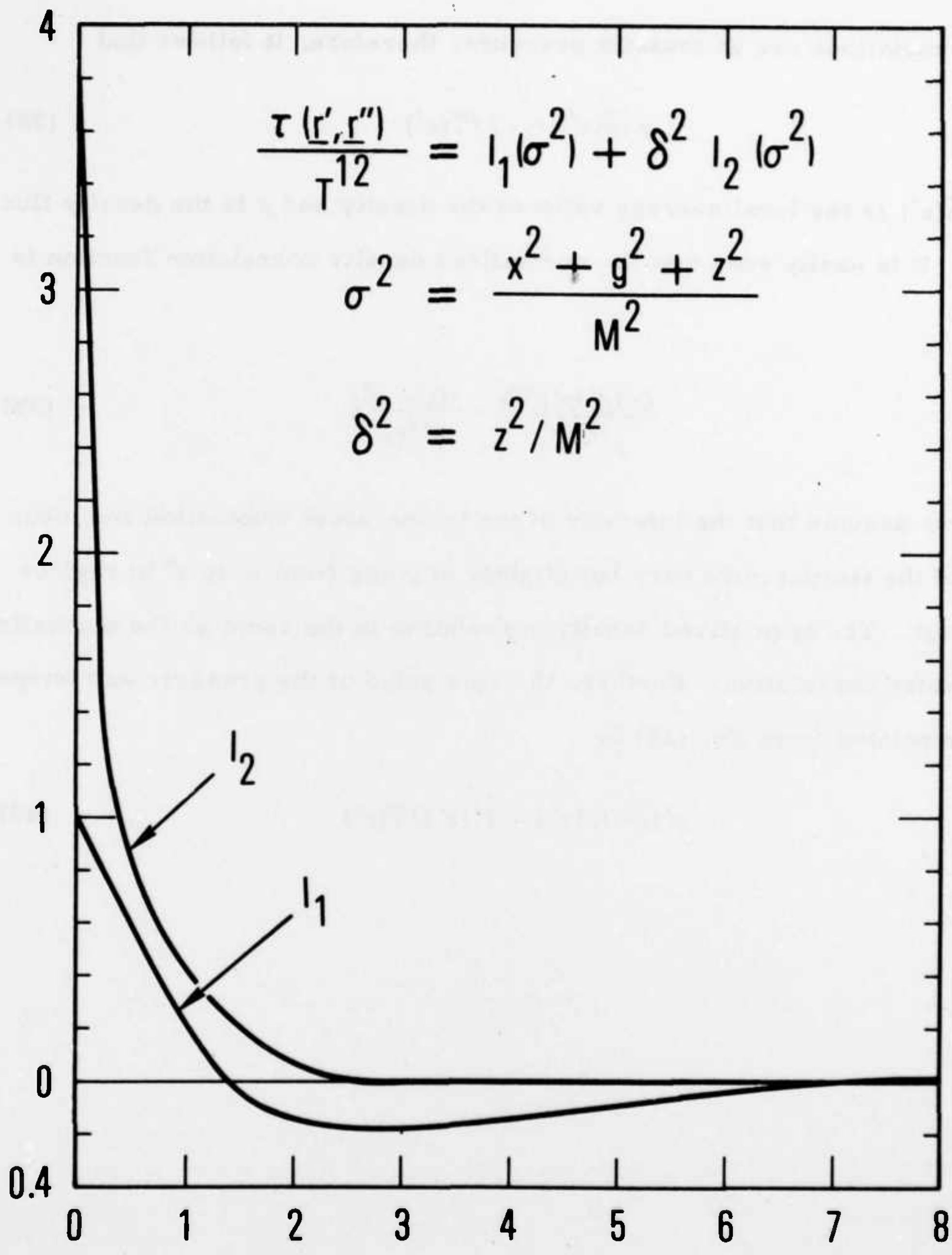

Figure 2. Plot of Integrals I_1 and I_2

that the variations are at constant pressure, therefore, it follows that

$$\rho/\bar{\rho}(z') = -T/\bar{T}(z') \cdot \tag{38}$$

Here, $\bar{\rho}(z')$ is the local average value of the density and ρ is the density fluctuation. It is easily seen that the normalized density correlation function is given by

$$\frac{\langle \rho(\underline{r}')\rho(\underline{r}'')\rangle}{\rho'^2(z')} = \frac{\tau(\underline{r}', \underline{r}'')}{T'^2(z')} \tag{39}$$

Again, we assume that the intensity of the temperature fluctuation and mean values of the temperature vary but slightly in going from z' to z'' in regions of interest. The normalized *density* correlation is the same as the normalized temperature correlation. Further, the rms value of the pressure and temperature are related from Eq. (38) by

$$\rho'(z')/\bar{\rho}(z') = T'(z')/\bar{T}(z') \tag{40}$$

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APPENDIX A. THE EVALUATION OF THE LAGRANGIAN INTEGRAL, Eq. (16)

It is seen in the body of the text that we attempt to evaluate a Lagrangian integral, that is, an integral of the velocity fluctuation following the flow. The exact form of that integral can be written

$$T(\underline{r}) = -\overline{T}_z(z)\int_o^{T_D} w\left[x - \int_o^{T_D}(\overline{u} + u)\, d\tilde{t},\ y - \int_o^{T_D} v d\tilde{t},\right.$$

$$\left. z - \int_o^{T_D} w d\tilde{t};\ t - \tilde{t}\right] d\tilde{t},$$

$$\overline{u} = \overline{u}\left[z - \int_o^{T_D} w d\tilde{t}\right] \qquad \text{(A1)}$$

where

$$u = u\left[x - \int_o^{T_D}(\overline{u} + u)\, d\tilde{t},\ y - \int_o^{T_D} v d\tilde{t},\ z - \int_o^{T_D} w d\tilde{t};t - \tilde{t}\right] \qquad \text{(A2)}$$

and similarly for v and w. Such a Lagrangian integral is clearly extremely complicated. There is a continued implicit dependence upon the Eulerian space variables x, y, z, and the time t. In Eq. (16), we broke off that extended implicit dependence by neglecting, in the arguments, the velocity fluctuation $\underline{u}$.

We attempt here to find the qualitative effect of that approximation. The most important such effect can be seen in Eq. (A2). By neglecting w in the argument of the function $\overline{u}$, we neglect changes in the vertical position which may cause large changes in the value of the average flow velocity experienced. Referring to Eq. (A1) it is seen that these in turn cause relatively large changes in the x argument of the function w. This introduces a random fluctuation in the x-direction which has been neglected. A consequence of that neglect is that the correlation of the temperature fluctuation in the x-direction assumes, incorrectly, large scales in that direction as was seen in the body. Further, as a result of our inability to deal exactly with the complicated integration represented by Eq. (A1) to (A2), there will be errors in the amplitude of the predicted fluctuation, mostly on the large side, since we have suppressed some of the fluctuating character of the integral in (A1). As a consequence, it was seen in the text that it was necessary to introduce an empirical coefficient in order to match the known temperature fluctuations near the laminar sublayer.

APPENDIX B. DISCUSSION OF INCOMPRESSIBILITY OF THE VELOCITY CORRELATION R_{ij}

There is a question concerning whether the form Eq. (22) is a proper one for an inhomogeneous, incompressible velocity field. We know that, for homogeneous turbulence, Eq. (23) will yield such an incompressible field, but the weighting functions appearing in Eq. (22) complicate the discussion. Consider the relation

$$R_{ij}(\underline{r}', \underline{r}'') = -1/2\left(\frac{\partial}{\partial r_k'} \frac{\partial}{\partial r_k''} \delta_{ij} - \frac{\partial}{\partial r_j'} \frac{\partial}{\partial r_i''}\right) w'(z')w'(z'')M^2(z', z'') \times \int_0^{\rho} \rho' f(\rho')\, d\rho' \tag{B1}$$

where we use the summation convention: repeated indices should be summed from one to three. For incompressible flow the divergence of the correlation function must vanish;[3] thus

$$\frac{\partial}{\partial r_i'} R_{ij}(r', \underline{r}'') = \frac{\partial}{\partial r_j''} R_{ij}(\underline{r}', \underline{r}'') = 0 \tag{B2}$$

It is readily verified that Eq. (B1) has this property. Furthermore, it is easily seen that, for statistically homogeneous turbulence, where w' and M are constant, Eq. (B1) reduces to the homogeneous form given in Eq. (23). Finally for the z-component correlation we find the form proposed in Eq. (23). Thus, the correlation Eq. (B1) is incompressible, reduces to the homogeneous

correlation, and does yield the simple form proposed in Eq. (23). It is seen that correlations other than R_{33} have coefficients involving the turbulence-intensity weighting function, w', in a more complicated way.

APPENDIX C. CALCULATION OF THE CONVECTED HEAT FLUX q AND $T'(\delta_s)$

The basic equation for the calculation of the convected heat flux is

$$q(\delta_s) = h(T_{aw} - T_\infty) \quad , \quad \text{for } T_{aw} > T_w$$
$$= h(T_w - T_\infty) \quad , \quad \text{for } T_{aw} < T_w \tag{C1}$$

essentially Newton's law of cooling. Here T_{aw} is the temperature which would be attained by a thermally insulated, adiabatic, wall, as a result of the heat generation within the laminar sublayer. The constant of proportionality h is obtained from the empirical relation

$$h = Pr^{*-2/3} \rho^* u_\infty C_p^* \, 0.0288(Re_x^*)^{-1/5} \tag{C2}$$

A discussion of this type of heat transfer calculation may be found in Ref. 4. The usual calculation is for the heat transfer at the wall (what is needed in previous applications). We use here the same coefficient h, since the convective process needed is in the vicinity of δ_s. This transfer coefficient applies to turbulent boundary layers: $Re_x > 5 \times 10^5$, the case here. It will be seen that the ratio $h/C_p\rho$ (which we need) is but weakly dependent on the parameters of the calculation.

The starred quantities in Eq. (C2) are all referred to a reference temperature given by

$$T^* = T_\infty + 0.5(T_w - T_\infty) + 0.22(T_{aw} - T_\infty) \tag{C3}$$

The Prandtl number Pr is, for air, 0.71, and the Reynolds number is defined

$$Re_x^* = \frac{\bar{u}_\infty x}{\nu^*} \tag{C4}$$

The stagnation temperature T_o is defined by

$$\frac{T_o}{T_\infty} = 1 + \frac{\gamma - 1}{2} M_\infty^2 \tag{C5}$$

where γ is the ratio of specific heats. A recovery factor giving the effect upon the heat transfer of the process of bringing the high speed fluid to rest (through viscous action) at the wall is

$$r = \frac{T_{aw} - T_\infty}{T_o - T_\infty} = Pr^{1/3} \tag{C6}$$

The procedure for the calculation of q is: first, find the recovery factor r and the stagnation temperature T_o; then determine T_{aw}, the reference temperature T*, the Newton's law constant h, and finally use Eq. (C1) to find q.

It should be noted that, for flows such as this, the pressure remains essentially constant throughout the boundary layer, for this calculation.

For our purpose, we can simplify; divide by $C_p\rho(T_{aw})$ and separate out the $M_\infty \equiv u_\infty/a_\infty$ dependence,

$$\frac{h}{C_p\rho(T_{aw})} = Pr^{-2/3} M_\infty^{4/5}\ 0.0288(a_\infty/\nu^*)^{-1/5} a_\infty x^{-1/5} \tag{C7}$$

where x is measured from the point of initiation of the boundary layer and we use the fact that Pr is nearly constant.

Consider an example: a jet plane flies at an altitude of 28 Kft, at $M_\infty = 0.9$ with $T_w = 72°F$. At this altitude $T_\infty = -40°F$, $p_\infty = 0.30$ atmospheres; we want to know h at x = 20 ft (we use, as usual, English units). From the relations given above we find the intermediate values $a_\infty = 1006$ ft/sec, $T_o = 488°R$, $r = 0.892$, $T_{aw} = 481°F$, $T* = 489°R$, $Re^*_x = 4.65 \times 10^7$, $\nu* = 4.33 \times 10^{-4}$ ft^2/sec. We have, for Eq. (C7), converting to MKS

$$\begin{aligned} h/C_p\rho &= 1.52 \times 10^{-3} M_\infty^{4/5} a_\infty x^{-1/5} \\ &= 1.40 \times 10^{-3} a_\infty x^{-1/5}, \text{ in MKS} \end{aligned} \tag{C8}$$

We see $T_w > T_{aw}$, so from Eq. (C1)

$$\begin{aligned} q/C_p\rho &= 1.52 \times 10^{-3} M_\infty^{4/5} a_\infty x^{-1/5} (T_w - T_\infty)°C - m/s \\ &= 18.6°C - m/s \end{aligned} \tag{C9}$$

Consider a second example, a nominal wind tunnel experiment, take $T_\infty = 0°F$, $M_\infty = 0.8$, p_∞ = atmospheric pressure, $T_w = 70°F$. We find $T_o = 519°R$, $T_{aw} = 513°R$, $a_\infty = 1053$ ft/sec, $T^* = 507°R$, $\nu^* = 1.50 \times 10^{-4}$ ft^2/sec; choose x = 3.7 ft., then, $Re_x^* = 2.08 \times 10^7$. We have

$$h/C_p\rho = 1.22 \times 10^{-3} M_\infty^{4/5} a_\infty x^{-1/5}, \text{ MKS} \tag{C10}$$

We see $T_w > T_{aw}$; thus,

$$q/C_p\rho = 12.4°C - m/s \tag{C11}$$

We note from Eqs. (C8) and (C10) that the coefficient of $M_\infty^{4/5} a_\infty x^{-1/5}$ changes little in these quite different applications; we might for many applications of the above type take it as constant. Thus, for such application one can use

$$q/C_p\rho = 1.3 \times 10^{-3} M_\infty^{4/5} a_\infty x^{-1/5} (T_w - T_\infty)°C - m/s \tag{C12}$$

in MKS (and °C).

Note also that

$$\frac{q}{C_p\rho} = \langle wT \rangle \sim w' T' \tag{C13}$$

if $\langle wT(\underline{r} = 0)\rangle \sim w' T'$. There is the possibility that the average is much smaller than that product; in statistically homogeneous and isotropic, incompressible turbulence $\langle wT(\underline{r})\rangle = 0$, the whole correlation vanishes though, of course, w' and T' do not. All things considered, however, Eq. (C13) seems plausible. So using $w'/u_\infty \approx 0.05$, we obtain

$$\begin{aligned} T'(\delta_s) &= \frac{1.3 \times 10^{-3} M_\infty^{4/5} a_\infty (x/x_o)^{-1/5}}{0.05\,\bar{u}_\infty} (T_w - T_\infty) \\ &= 0.026\, M_\infty (x/x_o)^{-1/5} (T_w - T_\infty) \quad , \quad x_o = 1\text{ m} \end{aligned} \tag{C14}$$

The maximum temperature fluctuation is a few percent of the temperature contrast, as one might have assumed. These results are clearly not valid for small Mach numbers or small x.

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