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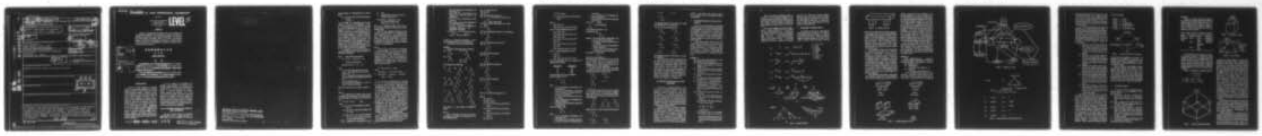

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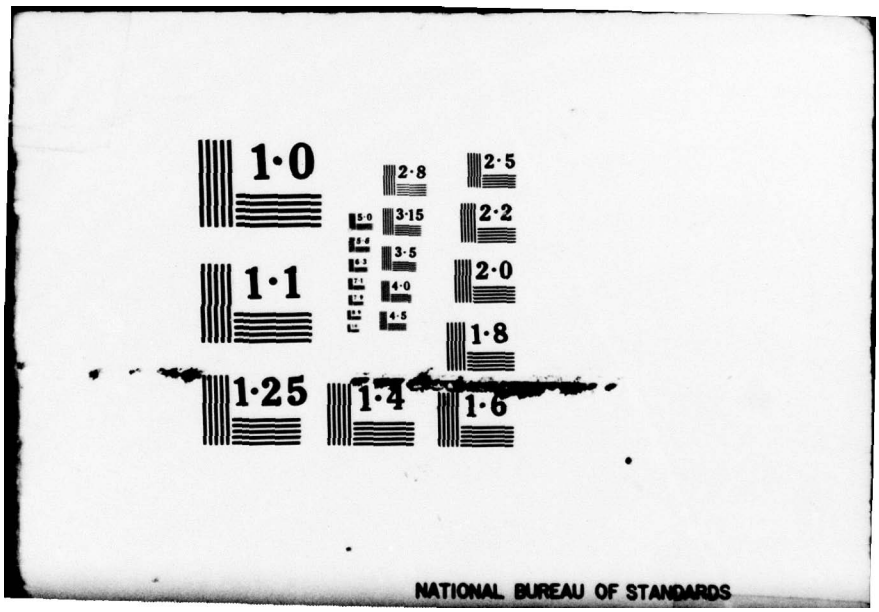
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INFERENCE OF HIGH DIMENSIONAL GRAMMARS*

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LEVEL II

ABSTRACT

Inference of high-dimensional grammars such as tree grammars and web grammars is discussed. The k-tail inference procedure for finite-state grammars is extended to the case of regular tree grammars. The behavior of the k-tail procedure with variable values of k is studied. The derivation diagram of context-free web languages is introduced. A "semantic teacher" is used for the inference of web grammars. Application examples in picture and scene analysis are presented.

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推演高階語法之方法

傅京孫

美國普渡大學電機工程系

摘　要

本文討論高階語法之推演，所謂高階語法指數狀語法 (Tree Grammar) 和衛普語法 (Web Grammar)。吾人將有限狀態語法 (Finite-State Grammar) 之 k 尾 (k-tail) 推演過程延伸至普通之數狀語法，並研究 k 尾程序對各種不同 k 所表現之特性。

本文亦介紹一種與前後文無關之衛普語法 (Contest-Tree Web Language) 之推演圖解。爲了推演衛普語法，吾人利用所謂之語意指導 (Semantic teacher)，本人最後介紹此方法在圖形識別和景相分析上之應用。

INTRODUCTION

The use of formal linguistics in modeling natural and programming languages and describing physical patterns and data structures has recently received increasing attention. Grammar or syntax rules are employed to describe the syntax of languages or the structural relations of patterns or data. In order to model a language or to describe a class of patterns or data structures under study more realistically, it is hoped that the grammar used can be directly inferred from a set of sample sentences or a set of sample patterns (or data). Grammatical inference is the problem of learning a grammar based on a set of sample sentences. Potential applications of grammatical inference include areas of pattern recognition, information retrieval, programming language design, translation and compiling, graphics languages, man-machine communication, and artificial intelligence.

In (1–3), inference of nonstochastic and stochastic string grammars was surveyed and a heuristic inference procedure for tree grammars was proposed in (4). In this paper, the k-tail presented. An inference procedure for transition network grammars was proposed in (4). In this paper, the k-tail inference procedure for finite-state grammar (5) is extended to the case of regular tree grammars. The behavior of the k-tail tree grammar inference method for varying values of k is studied. A web grammar interpretation of Winston's structure learning is discussed and an inference procedure for context-free web grammars is suggested.

K-TAIL INFERENCE METHOD FOR REGULAR TREE GRAMMARS

The k-tail inference method for finite-state string grammars requires an integer parameter k as input along with the presentation of (positive)

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training samples (5). Sublanguages S_w are created where

$S_w = [\, x/wx$ is a string in the (positive) training set and $\neg|x| \leq k \,]$

$|x|$ is the length of x. Equivalent S_w sets are then combined to form the i^{th} sublanguage. A rule, $A_i \rightarrow tA_j$ is produced if there is a string w such that S_w is the i^{th} sublanguage and S_{wt} is the j^{th} sublanguage. The rule $A_i \rightarrow t$ is produced if there is a string w such that S_w is the i^{th} sublanguage and wt is in the training sample. For strings, the exactness of the grammar produced for any given training set can be adjusted by varying k from 0 up to the length of the longest string in the training set. The inferred languages vary correspondingly from something close to the universal language to the presentation itself. Thus, any method restricted to k = 1 will infer grammars which generate languages which are very "loose" in their fit of the sample set.

It is possible to extend the k-tail method for finite-state string grammars to regular tree grammars. The method is as follows:

Step 1. Form the following collection:

$$C_t = [\, (\tau_1.\tau_2, \ldots \tau_m) \mid t\tau_1\tau_2 \ldots \tau_m \text{ is a tree in the training set and } |\tau_\ell| \leq k \text{ for } \ell = 1,2, \ldots m \,]$$

where

t is a tree with a single special frontier node.

$\tau_1, \tau_2, \ldots \tau_m$ are any trees that can occur in positions 1,2, . . . m.

$t\tau_1\tau_2 \ldots \tau_m$ is the tree formed by concatenating τ_ℓ at the ℓ^{th} position of the special frontier node of t,

$|\tau_\ell|$ is the depth of $\tau_\ell + 1$

m is the number of descendants of t and is not fixed to any particular integer.

Note that t, the empty tree, is possibly a member of C_t.

Step 2.

The collection C_t of tuples of trees can be partitioned into subcollections of m-tuples where m is a fixed integer for all elements of each subcollection.

$$C_t = C_{t0} \cup C_{t1} \cup \ldots \cup C_{tm}$$

where

$C_{t0} = [\, \epsilon \,]$ if t is in the training set, otherwise $C_{t0} = \phi$

$C_{t1} = [(\tau_1) \mid t\tau_1$ is a tree in the training set and $|\tau_1| \leq k]$

$C_{t2} = [(\tau_1, \tau_2) \mid t\tau_1\tau_2$ is a tree in the training set and $|\tau_1| \leq k$ and $:|\tau_2| \leq k \,]$ (Note here that the subscript indicates the position a tree occupies and that τ_2 in C_{t2}

- is not
- necessarily always the same tree nor is it the same tree as τ_1 in C_{t1}.)

$$C_{tm} = [(\tau_1, \tau_2, \ldots \tau_m) \mid t\tau_1\tau_2 \ldots \tau_m \text{ is a tree in the training set and } |\tau_\ell| \leq k \text{ for } \ell = 1,2, \ldots m]$$

Thus, C_t is a collection of tuples of trees and C_{ti} is a collection of i-tuples of trees where i is a fixed, specified integer.

Each of these collections defines all of the i-tuples of k-tail trees that are in the training set with root attached to the tree, t, at its special frontier node. The collections are separated in this way because an i-tuple and a j-tuple where $i \neq j$ cannot be generated by the same rule. Thus, we will now demonstrate the procedure that should be applied to each of the subcollections.

Step 3.

The next step is one which is not necessary in the case of strings. It is necessary here because a node can have several descendants and it may be that only certain ordered combinations of descondants are allowed. Thus, each subcollection of i-tuples of trees, C_{ti}, must be further divided into subcollections of i-tuples, each of which can be expressed as the cartesian product of i sets of trees. Thus, C_{ti} may be written:

$$C_{t1} = C_{ti1} \cup C_{ti2} \cup \ldots \cup C_{tin}$$

where

$$C_{tij} = [(\tau_1, \tau_2, \ldots \tau_i) \mid \tau_1 \in S_{j1}, \tau_2 \in S_{j2}, \ldots]$$

or

$$C_{tij} = [(\tau_1, \tau_2, \ldots \tau_i) \mid (\tau_1, \tau_2, \ldots \tau_m) \in S_{j1} \times S_{j2} \times \ldots \times S_{ji} \,]$$

That is, each C_{tij} is characterized by i sets, $S_{j\ell}$, ($\ell = 1,2, \ldots i$), of trees from which the ℓ^{th} member of an i-tuple must be selected. These $S_{j\ell}$ sets are sublanguages of trees and may be regarded as a set of trees generated by a particular nonterminal of the tree grammar. The difficult part of this step is to find those sets $S_{j\ell}$ which efficiently characterize the C_{tij}. First of all, the resulting grouping is not unique. One possible grouping would be that in which each C_{tij} has one element. This would not be a good choice because each C_{tij} will result in a grammar rule. Thus, this choice would result in a large number of rules. Since there are a finite number of elements in C_{ti}, there are a finite number of groupings and each of these can be tried. It is not necessary that the C_{tij} be disjointed. A particular grouping would be optimum if it introduced a minimum number of new $S_{j\ell}$ sublanguages.

Now the rules for the grammar can be constructed. Equivalent $S_{j\ell}$ sublanguages are combined and a nonterminal is assigned corresponding to each distinct sublanguage. Now a rule $A_{j\ell} \rightarrow x\, A_{n1} A_{n2} \ldots A_{nm}$ is produced if there is a tree t such that:

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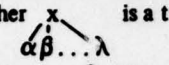
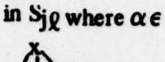
1. $A_{j\ell}$ is the nonterminal corresponding to the sublanguage $S_{j\ell}$.
2. There exists a C_{tij} that contains the sublanguage $S_{j\ell}$ in the ℓ^{th} position of its specification.
3. tx is a tree with x concatenated at the ℓ^{th} position of t.
4. There exists a C_{txmn} which is specified by the sublanguages $S_{n1}, S_{n2}, \ldots S_{nm}$.
5. $A_{n1}, A_{n2}, \ldots A_{nm}$ are the nonterminals corresponding to the $S_{n1}, S_{n2}, \ldots S_{nm}$ sublanguages, respectively.
6. Either  is a tree in $S_{j\ell}$ where $\alpha \in S_{n1}$, $\beta \in S_{n2}, \ldots \lambda \in S_{nm}$ or  $> k$.

A rule $A_{j\ell} \rightarrow x$ is produced if conditions 1, 2 and 3 above are satisfied and tx is in the training set.

To illustrate consider the following example:

Example 1:

Consider the following regular tree grammar:

(1) $S \rightarrow \$$ 

(2) $B \rightarrow b$ 

(3) $B \rightarrow b$ 

(4) $A \rightarrow a$

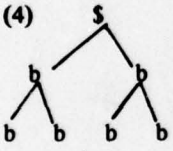
(5) $B \rightarrow b$

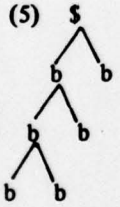
The training set is the following:

(1) 

(2) 

(3) 

(4) 

(5) 

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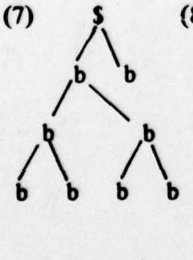
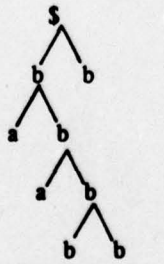
(7) 

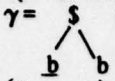
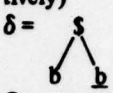
(8) 

(9) 

Now assume k = 1 and construct the grammar as follows:

Step 1:

(Note: Greek letters are used here to specify the distinct trees which were all represented by t in the explanation above.)

Let $\alpha = \epsilon$ (the empty tree)
Then $C_\alpha = [\phi]$
Let $\beta = \$$
Then $C_\beta = [(b,b)]$ (from sample 1)
Let $\gamma =$  (the underline denotes where the $(\tau_1 \ldots \tau_i)$ are concatenated)
Then $C_\gamma = [\epsilon, (b,b,)]$ (from samples 1 & 2, respectively)
Let $\delta =$ 
Then $C_\delta = [\epsilon, (b,b)]$ (from sample 1 & 3, respectively)
Let $\rho =$ 
Then $C_\rho = [\epsilon, (b,b)]$ (from saples 2 & 5)
Let $\eta =$ 
Then $C_\eta = [\epsilon]$ (from sample 8)
Let $\theta =$ 
Then $C_\theta = [(a,b)]$ (from sample 8)
Let $\lambda =$ 
Then $C_\lambda = [\epsilon]$ (from sample 8)
Let $\mu =$ 
Then $C_\mu = [\epsilon, (b,b)]$ (from samples 8 & 9)

Step 2:

$C_\alpha = C_{\alpha 0} = \phi$

$C_\beta = C_{\beta 2} = [(b,b)]$

$C_\gamma = C_{\gamma 0} \cup C_{\gamma 2}$ where $C_{\gamma 0} = [\epsilon]$ and $C_{\gamma 2} = [(b,b)]$

$C_\delta = C_{\delta 0} \cup \epsilon_{\delta 2}$ where $C_{\delta 0} = [\epsilon]$ and $C_{\delta 2} = [(b,b)]$

$C_\rho = C_{\rho 0} \cup C_{\rho 2}$ where $C_{\rho 0} = [\alpha]$ and $C_{\rho 2} = [(b,b)]$

$C_\eta = C_{\eta 0} = [\epsilon]$

$C_\epsilon = C_{\theta 2} = [(a,b)]$

$C_\mu = C_{\mu 0} \cup C_{\mu 2}$ where $C_{\mu 0} = [\epsilon]$ and $C_{\mu 2} = [(b,b)]$

Step 3:

$C_{\alpha 0} = C_{\alpha 01} = \phi$

$C_{\beta 2} = C_{\beta 21} = [(\tau_1,\tau_2) \mid \tau_1 \in B, \tau_2 \in B]$

where B is the sublanguage of trees = [b]

$C_{\gamma 0} = C_{\gamma 01} = [\epsilon]$

$C_{\gamma 2} = C_{\beta 21} = [(\tau_1,\tau_2) \mid \tau_1 \in B, \tau_2 \in B]$

$C_{\delta 0} = C_{\delta 01} = [\epsilon]$

$C_{\delta 2} = C_{\delta 21} = [(\tau_1,\tau_2) \mid \tau_1 \in B, \tau_2 \in B]$

$C_{\delta 0} = [\epsilon]$

$C_{\delta 2} = C_{\delta 21} = [(\tau_1,\tau_2) \mid \tau_1 \in B, \tau_2 \in B]$

$C_{\eta 0} = [\epsilon]$

$C_{02} = C_{O21} = [(\tau_1,\tau_2) \mid \tau_1 \in A, \tau_2 \in B]$

where A is the sublanguage of trees = [a]

$C_{\lambda 0} = [\epsilon]$

$C_{\mu 0} = [\epsilon]$

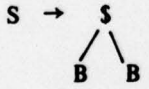
$C_{\mu 2} = C_{\mu 21} = [(\tau_1,\tau_2) \mid \tau_1 \in B, \tau_2 \in B]$

Now the nonterminals and their equivalent sublanguages are enumerated

Nonterminal	Sublanguagee
S	ϕ
A	[a]
B	[b]
E	[ϵ]

Now the grammar rules can be constructed:

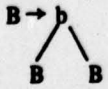
From the relation $\beta = \alpha\$$:



Note:

1. S is the nonterminal corresponding to the sublanguage, ϕ.
2. $C_{\alpha 01}$ has the sublanguage ϕ concatenated at its 1st position. (i.e., there are no trees of depth 0 in the training set.)
3. $\beta = \alpha\$$ is a tree with $ concatenated in the 1st position.
4. $C_{\beta 21}$ is specified by the sublanguages [b] and [b], respectively.
5. B and B are the nonterminals corresponding to [b] and [b].

From the relation $\gamma = \beta b$:



Note:

1. B is the nonterminal corresponding to the sublanguage, [b].
2. $C_{\beta 21}$ has [b] in the 1st position (at depth 2).
3. $\gamma = \beta b$ is a tree with b concatenated in the 1st position.
4. $C_{\gamma 21}$ is specified by the sublanguages [b] and [b], respectively.
5. B and B are the nonterminals corresponding to [b] and [b].

Also B → b because

is in the training set.

Now consider the relation $\theta = \gamma ab$

This yields the rule:

Because 1. B is the nonterminal corresponding to [b].

2. $C_{\gamma 21}$ has [b] in its 1st position.
3. $\theta = \gamma ab$ is a tree with b concatenated in its 2nd position.
4. $C_{\theta 21}$ is specified by the sublanguages [a] and [b], respectively.
5. A and B are the nonterminals corresponding to [a] and [b], respectively.

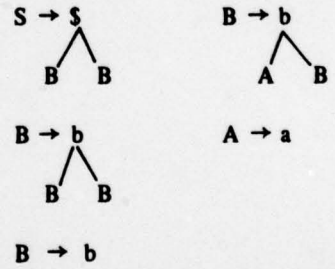
The relation $\lambda = \theta ab$ yields

A → a

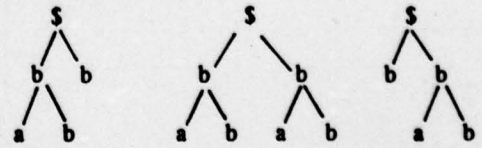
because λ is in the training set.

Now notice that the nonterminal E does not appear on the left-hand side of any rule and can be ignored. This is because it corresponds to the sublanguages, [ϵ], which means the tree has terminated without further descendants.

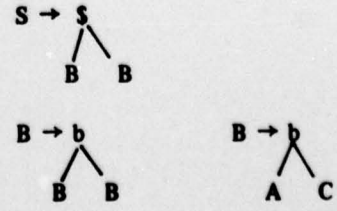
Further, tests with subtrees from the training set will show that all the rules have now been found. The entire production set is shown below:

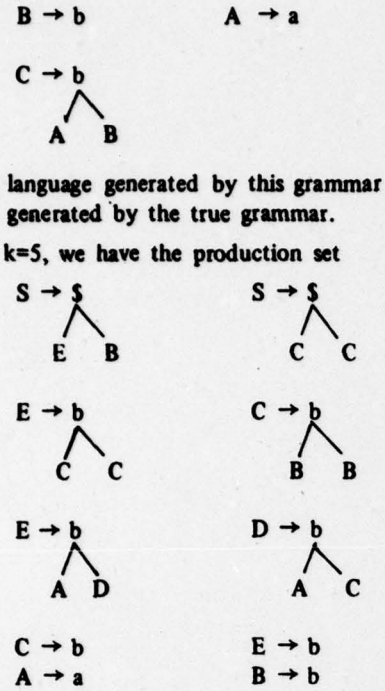


Note that this grammar generates all of the samples in the training set and in fact generates a language larger than the real one. For example, this grammar would generate the following trees which are not in the real language:



Similarly, for k=2, we have the inferred production set





The language generated by this grammar is exactly that generated by the true grammar.

For k=5, we have the production set

The language generated by this grammar is exactly the training set.

The tree grammar inference methods of Bhargava and Fu (6) and Gonzalez and Thomason (7) are similar in that they both assume recursiveness whenever there is the slightest evidence of it. It is in this sense that they are similar to the k tail method with k=1. In the k-tail method, when k=1, the "loosest" nontrivial grammar is produced. In many cases, this will be the same grammar as produced by both methods. The k-tail method will produce more satisfactory grammars when $k > 1$ and when the training set is of adequate size.

AN INFERENCE PROCEDURE FOR WEB GRAMMARS

In his work on language identification in the limit, Gold (8) noted the importance of correctly ordering the information sequence. Most other grammatical inference researchers have also noted this importance. An interesting demonstration of the need to carefully select the training sequence is the work by Winston (9). The purpose of the work was to develop a system which could learn structural descriptions of scenes by analyzing specially selected examples. This work is now formalized and related to the grammar inference problem.

The basic idea will be to correlate the derivation diagram of a web grammar with the semantic net used by Winston. They by following the steps used by Winston on the semantic net and finding equivalent steps for the derivation diagram, the method can be translated into web grammar terminology. The result will be a grammatical inference procedure for web grammars which can be applied more generally than in the specific block world considered. A brief review of the derivation diagram of web grammars (10) will be required to support this discussion.

1. The Derivation Diagram of Context-Free Web Grammars

Study of the context-free class of web languages reveals that many of the formal language properties of string language also hold for the corresponding web languages. One example is the existence for context-free web grammars of a structure similar to a derivation tree for context-free string grammars (10). The definition of this structure, called a derivation diagram is now given and an example is given in Figure 1.

A new, unique relation called the direct descendant relation is introduced. For a pair of nodes (n_1,n_2) connected by this relation, n_2 is called the direct descendant of n_1. n_1 is called the direct ancestor of n_2. A node n_k is called a descendant of n_1 if there is a sequence $n_1 \ldots n_k$ such that n_{i+1} is a direct descendant of n_i. n_1 is called an ancestor of n_k.

Definition 1:

D, a web, is a derivation diagram for a context-free web grammar $G=(V_N,V_T,P,S)$ if:

(1) There is one node called the root with no ancestors whose label is S, the start symbol of G.

(2) All other nodes have exactly one direct ancestor and every node is a descendant of the root.

(3) Every node has a label which is a symbol in V_N.

(4) If a node n has at least one descendant and has label A, then A must be in V_N.

(5) If nodes $n_1,n_2,\ldots,n_k$ are the direct descendants of node n with labels $A_1,A_2,\ldots,A_k$ respectively, $A \rightarrow \beta$ must be a production of P of G where $N_\beta=n_1,n_2,\ldots,n_k$ and the A_i is the label of the node n_i in β, $i=1,\ldots,k$.

(6) n_i and n_j are connected by relation r if and only if

a) one is the direct descendant of the other and r is the direct descendent relation or

b) n_i and n_j are both direct descendants of A, $A \rightarrow \beta$ is a rule in P and $\overset{A_i}{n_i} \xrightarrow{r} \overset{A_j}{n_j}$ is a subweb of β or

c) n_j and some node n_k are connected by relation r and n_i is the direct descendant of n_k through the rule $A \rightarrow \beta$, a rule in P and the r between n_i and n_j results from the embedding mapping ϕ of A.

There are two kinds of subdiagrams which are of interest. The first, called the *skeleton* of the derivation diagram, is obtained by keeping all nodes and all direct descendant relations and erasing all other relations. The result shown in Figure 1(c) nicely illustrated the basic structure of the *derivation.*

The second subdiagram of interest is called a *section.* If m_i is a frontier node of the skeleton (i.e., has no descendants), let $n_0, \ldots, m_i$ be a path to m_i from the root node, n_0 along only descendant edges. Let $m_1, m_2, \ldots, m_k$ be all of the frontier nodes. Then a set C of nodes of the derivation diagram is a crosscut set if $C \cap [n_0, \ldots, m_i]$ is a singleton for all $1 \leq i \leq k$. A crosscut set, C, together with all of the edges of the derivation diagram between nodes of C is called a *section.* Naturally, only those edges are kept which are connected to two nodes which are both kept. A section, illustrated in Figure 1(d), nicely illustrates the basic structure of *sentential forms.*

2. Interpretation of Winston's System

An example of the type of scene Winston's system analyzes is shown in Figure 2. The sequence of examples Winston found necessary to train the system is shown in Figure 3. Notice that Winston's method uses negative samples in the form of "near misses" as shown in scene 2 and scene 3. The des-

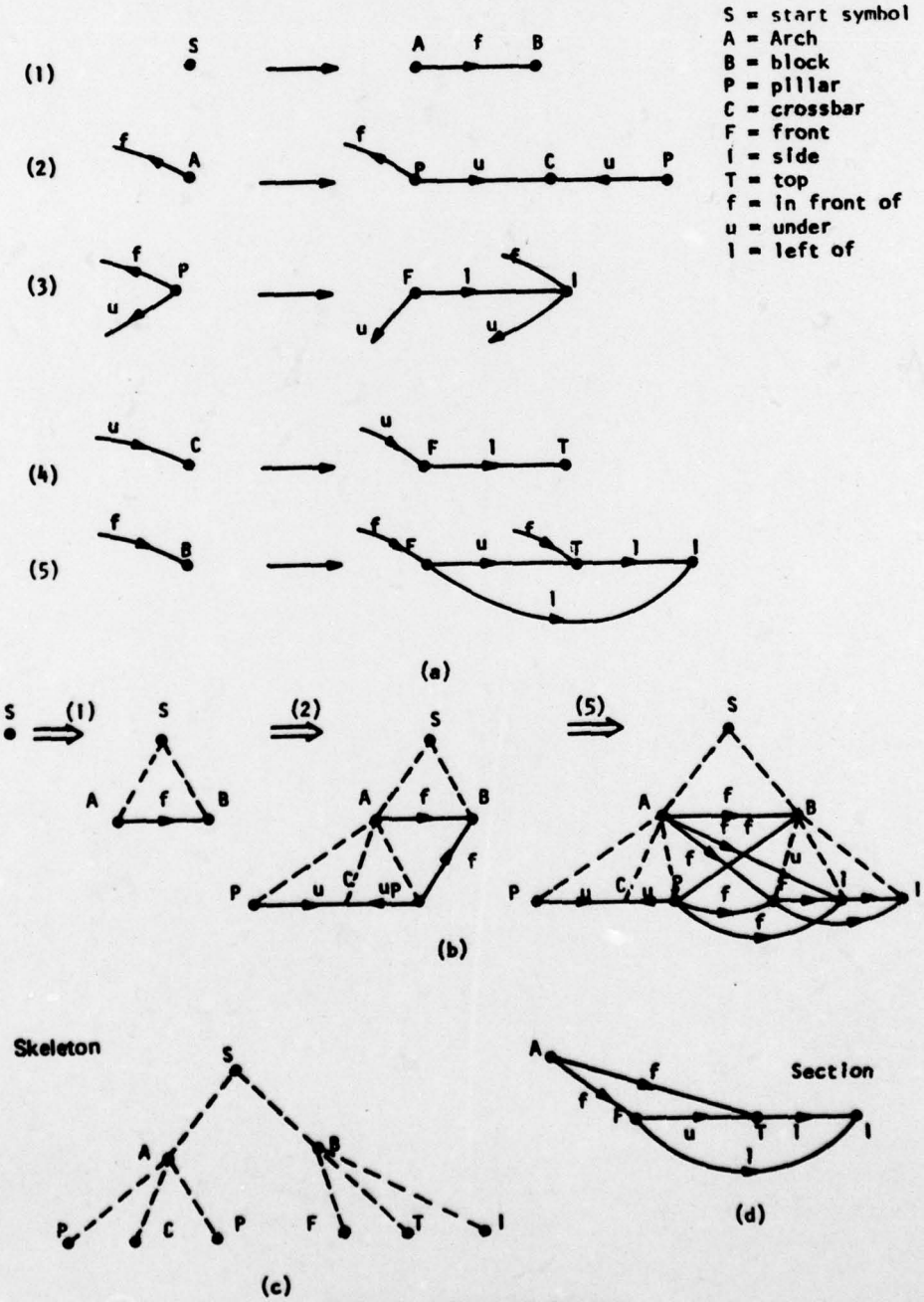


Figure 1. A Derivation Diagram

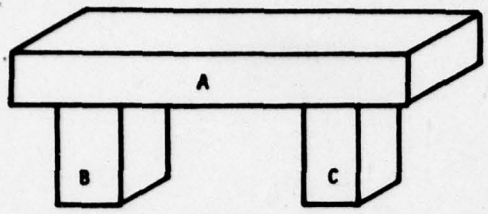


Figure 2. An Example of an Arch

cription that is finally learned is shown in Figure 4. It is assumed that all of the concepts illustrated (except ARCH) have already been learned. Each sample in the training sequence is constructed so that it has only one difference from the already learned description. Scene 2 illustrates that the supports of the arch must not abut. Scene 3 illustrates that A must be supported by B and C. Scene 4 illustrates that a more general object than a BRICK may be used as a top.

The description in Figure 4 can be interpreted as a hierarchical graph model and as a derivation diagram of a web grammar. As such, it can be converted to a web grammar. Some of the rules of this grammar are shown in Figure 5. These rules are created from Figure 5 by generating a rule when a relationship such as "a–kind-of" or "one-part-is" is encountered in the diagram. Thus, the grammar will have a derivation diagram similar to Figure 4. In this case, the system is learning one rule. That is, it is trying to find the predicate which describes the right side of rule (1). If this predicate can be learned, it can then be used to analyze higher order patterns containing it.

Mony important nonterminals in a web grammar will not occur in recursive rules. These nonterminals will be important because they represent important semantic concepts which give "meaning" to the structural descriptions. To learn an individual rule in a web grammar, the system must be able to learn the most general description possible for each object most general description possible for each object on the right-hand side. Assuming the form of the rule is known (this is generally learned from the first sample), then learning the exact rule becomes a matter of finding how much each object may be generalized. In this case, the original description of ARCH might contain the objects A, B, and C; that is, an exact description of this particular scene. This description would be of little general use because no slightly different arch could be identified. Even the appropriate parse of this scene is not known because grammars describing it might be ambiguous.

In a general formalism an object like A is described by properties like orientation and shape. These properties allow successive generalization to occur according to what values of a particular property are important. The structure which describes and systematizes the generalization process is called the property lattice.

Definition 2:

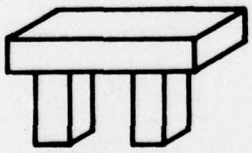
A set of elements $C=[c_1,c_2, \ldots]$ is said to be *partially ordered* (hierarchical) if there exists a relation ($\leqslant$) defined on the elements of C which is:

(1) Reflexive: $c \leqslant c$.

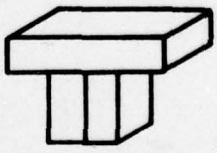
(2) Antisymmetric:
$c_1 \leqslant c_2$ *and* $c_2 \leqslant c_1$ *implies* $c_2 = c_2$;

(3) Transitive:
$c_1 \leqslant c_2$ and $c_2 \leqslant c_3$ implies $c_1 \leqslant c_3$.

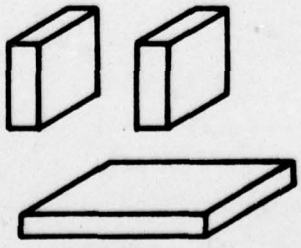
If C is a partially ordered set and X is any subset of C, then $a \epsilon \theta$ is a *lower bound* of X if $a \leqslant x$ for all $x \epsilon X$ and a is an upper bound of X if $x \leqslant a$ for all $x \epsilon X$. A lower bound b of X is called the



SCENE 1
AN ARCH



SCENE 2
NOT AN ARCH



SCENE 3
NOT AN ARCH

SCENE 4
AN ARCH

Figure 3. A Training Sequence for an Arch

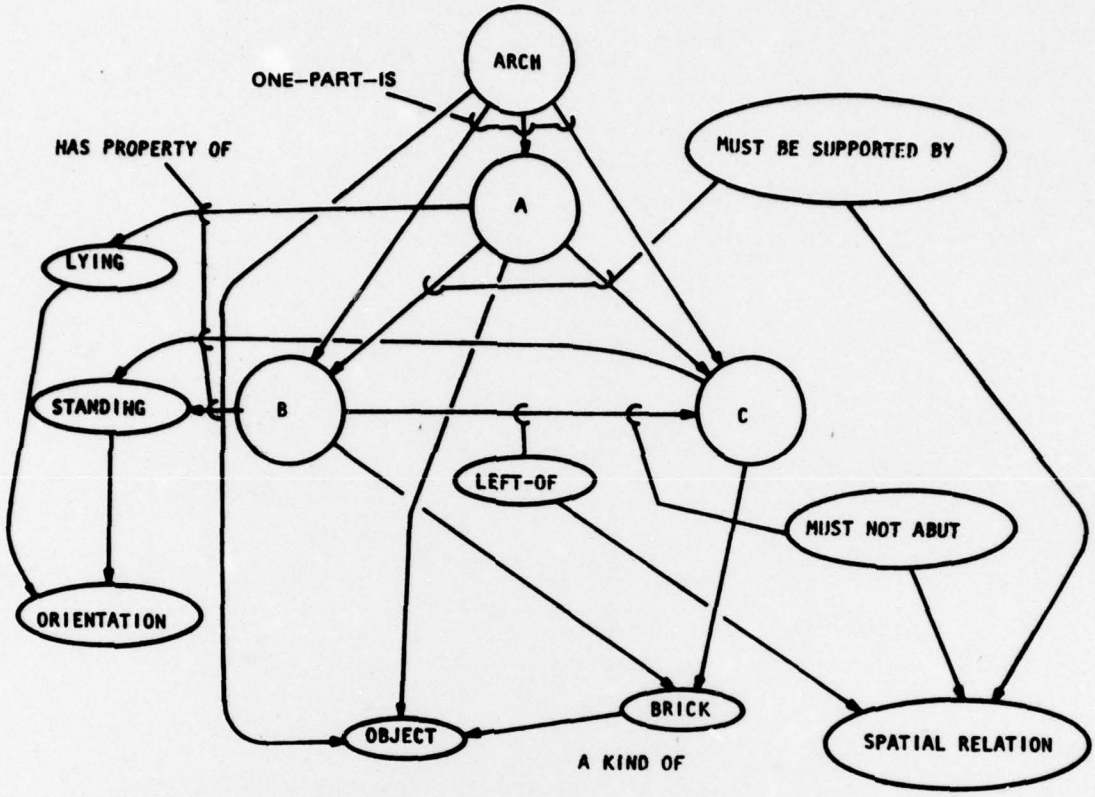
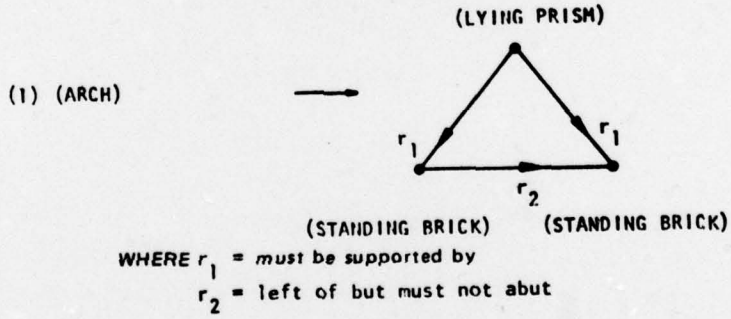


Figure 4. Derivation of an Arch



(2) (OBJECT) ⟶ A

(3) (OBJECT) ⟶ (BRICK)

(4) (OBJECT) ⟶ (ARCH)

(5) (BRICK) ⟶ B

(6) (BRICK) ⟶ C

Figure 5. Some Web Grammar Rules Describing an Arch

greatest lower bound (g.l.b.) of X if for every a that is a lower bound of X, a $\leq$ b. Similarly, an upper bound d of X is called the *least upper bound* if for every e that is an upper bound of X, d $\leq$ e. A partially ordered set of C in which any two elements have a least upper bound and a greatest lower bound is called a *lattice.*

In the case of concept learning here, the elements of C are called *concepts* and consist of subsets of samples containing certain property values. The partial order relation considered is set inclusion. The purpose of the learning procedure will be to find that concept which contains all of the samples showing allowed property values and none of the samples having disallowed property values. The procedure to be used in learning a concept will be as follows:

(1) Whenever a set of positive samples are given, then all lower bounds of the set in the lattice are allowed as the possible concept. The least upper bound of the set and all its lower bounds are also allowed.

(2) Whenever a set of negative samples are given, then all upper bounds of the set in the lattice are disallowed as the possible concept. The greatest lower bound of the set and all its upper bounds are also diallowed.

(3) Whenever a new positive sample is given, then the new allowed part of the lattice is the set of all lower bounds of the least upper bound of the new example and the previously learned least upper bound.

(4) Whenever a new negative sample is given, then the new disallowed part of the lattice is the set of all upper bounds of the greatest lower bound of the new example and the previously learned greatest lower bound.

(5) When all of the points in the lattice are either allowed or disallowed, the correct concept is the least upper bound of the allowed part of the lattice and is said to have been learned.

The purpose of this study will be to see how the lattice can help in selecting a good training set and to see how grammars can help in setting up the lattice. In many practical cases, properties are neither all independent nor all dependent. In these cases, the property lattice is more nouniform. Fortunately, the property lattice can be constructed from the grammar if the grammar is in the right form as is shown in Figure 6. Note in this case that a (*STANDING TRIANGULAR PRISM*) is not allowed by the grammar so the higher order concepts (STANDING) and (TRIANGULAR PRISM) are also not present. How, the number and selection of samples necessary to learn a concept in this lattice can be investigated. To generalize to the concept (PRISM), 2 positive samples (STANDING BRICK) and (LYING TRIA PRISM) must be given. To generalize only to (BRICK) or (LYING), all three samples (2 positive and 1 negative) must be given. To generalize to (STANDING BRICK) only, two samples must be given.

Givon the Grammon:

(PRISM) → (BRICK)
(PRISM) → (LYING)
(BRICK) → (LYING BRICK)
(BRICK) → (STANDING BRICK)
(LYING) → (LYING BRICK)
(LYING) → (LYING TRIA PRISM)

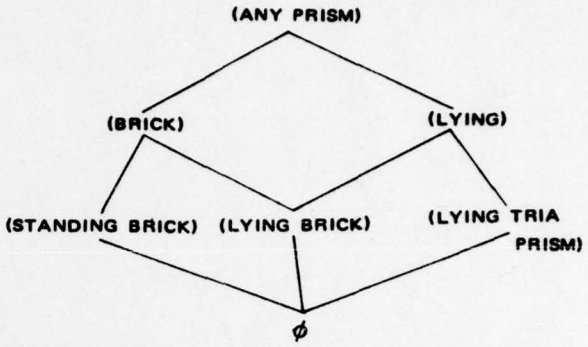


Figure 6. A Lattice Constructed From A Grammar

Thus, by using the grammaticaly formalism for lower order concepts, such as (PRISM), a more efficient lattice structure can be set up. If this lattice is big enough, there is less necessity for a "near miss" to be so near because samples which are more different will still have a least upper bound and greatest lower bound in the lattice. This lattice structure can help in the selection of proper training samples for higher order concepts such as (ARCH).

3. An Inference Procedure

In terms of formal grammatical inference, Winston's procedure, as just formalized, can be stated as follows:

(1) Assume that a given set of properties and predicate forms are known to be appropriate from a priori information about the application.

(2) Given a sample, get all possible parses of it with these forms and arrange the parse nonterminals in a property lattice.

(3) Then, by giving a sequence of appropriate positive and negative samples, and using least upper and greatest lower bound operations in the lattice, converge to the correct parse common to all positive samples and including no negative samples.

(4) Construct the grammar rule reflecting this parse.

An example of applying this procedure to a Winston-like problem is now given.

Example 2:

Assume we are given a problem in which the only objects are rectangular prisms and the only properties detectable are size, shape, and color. Furthermore, assume that green cubes do not exist. A lattice illustrating these properties is shown in Figure 7. The objects, properties, and relations are summarized below in Table 1.

Table 1 Objects, Properties, and Relations for Example 2

Object	Properties	Values	Relations
Rectangular Prisms	Size	Larger	Supported by
		Smaller	Larger-smaller
	Shape	Cube	Same color
		Rectangular Prism	
	Color	Red	
		Green	

We now wish to learn the concept of a pyramid. For illustrative purposes it is assumed that a legal pyramid can have cubes or rectangular prisms but supporting objects can only be red in color. That is only the top object can be green. To being, a positive sample of a pyramid (shown in Figure 8) is presented and the pattern is parsed. The parse or derivation diagram or semantic net resulting is shown in Figure 9.

Now, by presenting an appropriate sequcene

Object Lattice

P = (P1, P2)

P1 = COLOR, 0 = RED, 1 = GREEN

P2 = SHAPE, 0 = CUBE, 1 = RECTANGLE

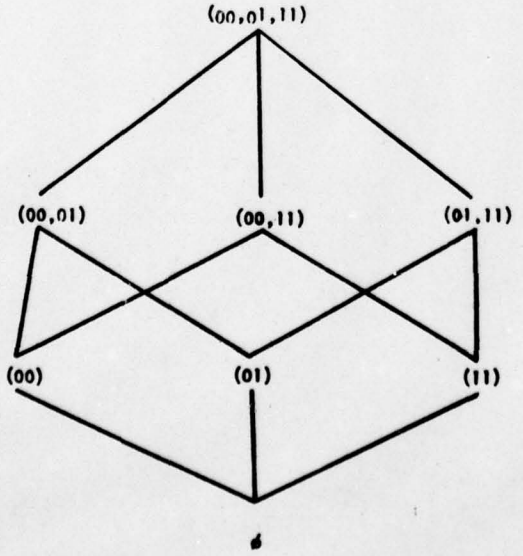


Figure 7. Lattice for Properties of Example 2

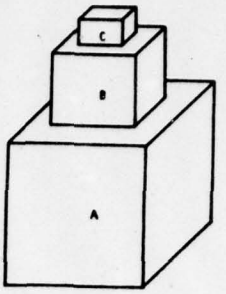


Figure 8. An Example of a Pyramid

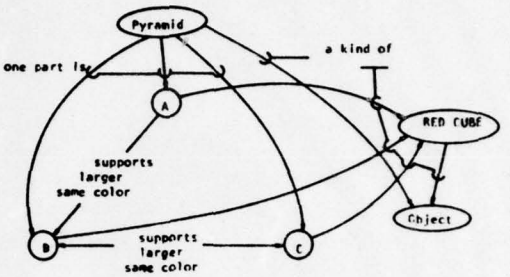
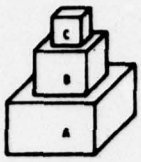


Figure 9. Parse of Figure 8

of positive and negative samples, the teacher must illustrate the most general object or relation which is allowed in each position. This example will concentrate just on the objects and for the moment ignore the fact that the relations must be learned also. The supporting objects in the pyramid can be any shape but must be red. This is illustrated by the (00,01) entry in the lattice. This can be illustrated by three samples: 00 and 01 as positive samples and 11 as a negative sample. The top object can be green. Since a cube cannot be green, this is illustrated by the (00, 01, 11) entry in the lattice. This state in the lattice can be learned by presenting 00, 01, and 11 as positive samples. Thus, for each individual object, three samples must be given. But, since these can occur in various combinations with the other objects, a total of 27 combinations must be presented to completely learn the definition of the pyramid. The samples are shown in Figure 10. Note that if the objects can be considered independent only seven samples need to be given. These sample. are shown with asterisks in Figure 10.

The derivation diagram which is finally learned is shown in Figure 11. The grammar rule learned is extracted from this diagram by putting the ancestor of the "One-part-is" relation on the left-hand side and the descendants on the right-hand side. This rule is shown in Figure 12. The embedding of this rule is somewhat arbitrary.

Several conclusions can be drawn from this example. First, if there are several properties involved and these properties take on several values and it is necessary to learn a pattern containing several objects, then many samples must be used in training unless some heuristic assumption is made. Second, if one part of the pattern can be assumed indepedent of



Sample Code for C 00
Code for B = 00
Code for A 00

Samples which must be presented:

Positive Samples:

00* 00* 00* 00 01* 01 01 01
00 00 01 01 00 00 01 01
00 01 00 01 00 01 00 01

11* 11 11 11
00 00 01 01
00 01 00 01

Negative Samples:

00* 00 00* 00 00 01 01 01 01 01
00 01 11 11 11 00 01 11 11 11
11 11 00 01 11 11 11 00 01 11

11 11 11 11 11
00 01 11 11 11
11 11 00 01 11

Figure 10 Training Samples

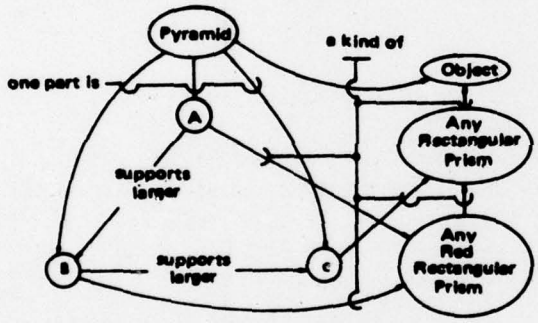


Figure 11. Final Derivation Diagram

Figure 12. The Resulting Grammar Rule

other parts, the number of samples needed to learn it can be greatly reduced. Third, this method as shown does not specify the embedding.

CONCLUSIONS AND REMARKS

This paper presents some preliminary results in the inference of tree and web grammars. It is hoped that the preliminary results will stimulate new and better inference methods for high-dimensional grammars, particularly concerning the quality of inference (or the "goodness of fit") and the applicability to real-world problems. A proposal for inferring web grammar from pictorial patterns can be found in [10].

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