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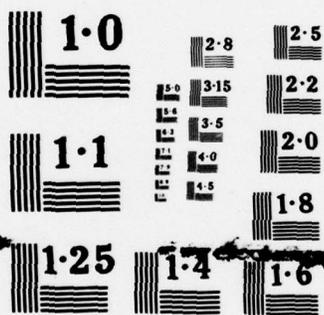
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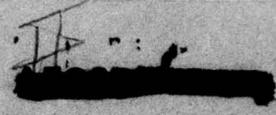


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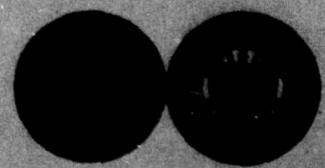
**SUMMARY OF RESEARCH ON LINEAR AND NONLINEAR
EFFECTS ON VIBRATION OF ELASTIC STRUCTURES**

Lawrence W. Rehfield



May 1978

GEORGIA INSTITUTE OF TECHNOLOGY
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| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This report contains a brief summary of research on linear and nonlinear effects in the vibration of elastic structures, supported by Grant AFOSR-73-2479 and performed during the period 1 Jan 73 to 31 Dec 77. The research includes studies of nonlinear vibration of structures, boundary restraint effects in vibration and hygrothermal effects on dynamic behavior of resin matrix composites. A listing of reports and publications that have resulted from this activity is included. | | | |

SUMMARY OF RESEARCH ON LINEAR AND NONLINEAR
EFFECTS ON VIBRATION OF ELASTIC STRUCTURES

Final Scientific Report
Grant AFOSR-73-2479

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INTRODUCTION

This report contains a brief summary of research on structural vibration sponsored by the United States Air Force Office of Scientific Research under Grant AFOSR-73-2479. The work has been performed at the School of Aerospace Engineering, Georgia Institute of Technology during the period January 1, 1973 to December 31, 1977. Professor Lawrence W. Rehfield has been the Principal Investigator. This research program is unusual in that a transition from purely theoretical tasks to mostly experimental ones has taken place in the course of the five-year period. This transition occurred in a natural fashion as the need for experimental data in a number of areas became apparent.

The research began with a small scale theoretical investigation of the influence of nonlinear and boundary restraint effects on the vibration of structural elements. The scope of the nonlinear studies broadened to include elements of composite materials. Also, it became apparent that little experimental data on nonlinear structural vibration appeared in the open literature; an experimental task was initiated in late 1974 to provide nonlinear vibration data on beams, therefore. The boundary restraint studies were focused upon the interaction between boundary restraint and element curvature; considerable understanding of this complex issue was gained, but concise, general conclusions proved to be elusive.

Our efforts were redirected in 1976 in response to a pressing problem confronting the aerospace community. The environmental effects of moisture/elevated temperature were identified as a major problem for resin matrix composite structures in late 1975. A shift in emphasis from theory and analysis to laboratory development for basic experiments related to the

influence of environmental conditions on dynamic behavior began. The subsequent vibration data produced is the first of its kind.

The scope of the research activity obviously has varied over wide limits during the five-year period. Consequently, a summary cannot be written which unifies the work around a theme more specific than structural vibration. The work described in the following should be viewed, therefore, in the evolutionary context in which it was performed.

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RESEARCH ACCOMPLISHMENTS

Introductory Remarks

As indicated in the introduction, the research scope covers a range of activities. For convenience, the work will be placed into three categories. Each category will be treated separately, although there is overlapping among the categories. The first category is nonlinear vibration of structures; this activity is complete in the sense that the entire spectrum from basic concepts to experimental confirmation of theoretical predictions has been explored. Another category is boundary restraint effects; this work, although important, failed to produce a unified, consistent way of conceptualizing general situations. The third category is hygrothermal effects on dynamic behavior of resin matrix composites; only preliminary results are available at this time. This work will continue under a subsequent program.

Nonlinear Vibration of Structures

As efforts continue to produce more highly optimized structures, some effects which have been historically of secondary importance in structural design will tend to assume a primary role. Often these effects are associated with nonlinearity of the structural response. Consequently, much of the research effort has been devoted to exploring the nature and importance of nonlinearity in structural vibrations.

Nonlinear effects in structural dynamics may manifest themselves in subtle ways such as by simple "drift" of natural frequencies with increasing amplitude or by altering the time history of transient response. They also can produce quite dramatic effects that cannot be explained or predicted on the basis of linear theory; among such effects are jump phenomena, dynamic buckling (oil canning) in the presence of prestress or geometric curvature, subharmonic or superharmonic resonances at frequencies far

different from the exciting frequency, and bounded limit cycle response. Both the subtle and dramatic effects have been observed in actual structures and have been treated in an ad hoc manner by analysts and engineers. The objective has been to unify and synthesize the treatment of nonlinear effects. In large measure, it has been achieved.

A general theory for large amplitude, free, undamped vibration of linearly elastic structures appears in Reference 1. This pioneering work establishes the framework for all subsequent theoretical effort in nonlinear vibration. The theory is based upon Hamilton's principle and a perturbation procedure and is in much the same spirit as the theory of initial postbuckling behavior due to Koiter.^{2,3} It provides information regarding the first order effects of finite displacements upon the frequency and dynamic stresses arising in free, undamped vibration of structures. Solutions to the governing equations are sought as a power series in the amplitude of the linear vibration mode and higher order effects are systematically generated by successive perturbation equations. The first applications were to homogeneous uniform beams and rectangular plates.¹

The general theory has been applied also to the free vibration of shallow arches as an initial study of the effect of curvature on nonlinear vibration behavior.⁴ It has been learned that as the rise of the arch increases, the free vibration behavior in the fundamental mode first exhibits a softening trend due to curvature. This trend is reversed as the rise parameter exceeds a critical value, and a neutral limit is ultimately approached for large values of rise.

The theory subsequently was extended to include forced vibrations of linearly elastic structures in References 5 and 6. It has been assumed that the structure is excited by a disturbing force which is a harmonic function of time and is spatially distributed so as to excite only a

single mode. General results are presented^{5,6} along with applications to the vibration of beams and rectangular plates. Other solution methods have been compared with the general theory for the beam and the results are found to be in complete agreement.

A further extension of the theory was accomplished to include the effects of nonlinear elastic constitutive relations in both free and forced vibration.⁷ As a first approximation, it was assumed that tensile and compressive behavior of the materials are similar and that the stress-strain relations can be adequately represented as polynomial functions. The theory was first applied to the undamped free vibration of a simply supported beam with immovable ends. Numerical results have been presented for several materials, for beams of varying slenderness, and for fundamental and higher vibration modes to illustrate the nonlinear material effects (NLME's). The results obtained indicate that NLME's are extremely important and dominantly influence the behavior for higher modes and short beams. They also suggest that the second order theory, in which the frequency is altered by NLME's but the stress distribution is the same as for Hookean elastic materials, is fully adequate for characterizing the elastic vibration of structures made of conventional homogeneous materials.

In Reference 8, an approximate method for analyzing nonlinear vibration of elastic structures is presented. It is based upon the general theory^{1,6,7} and permits the engineer to obtain a rapid assessment of nonlinear effects in the fundamental mode of vibration. It represents a logical extension of Rayleigh's method to nonlinear problems. Applications to a number of uniform beam problems⁸ have been made to illustrate the use of the method. The results obtained also serve to further emphasize the importance of NLME's and boundary restraint effects in structural vibrations.

In early 1976, many aspects of the moisture/elevated temperature or hygrothermal problem of resin matrix composites were not adequately

defined. The combined difficulties posed by moisture/temperature simulation and the variety of composite lay-ups and material systems of interest resulted in an incomplete, fragmented data base. Test methods had been developed on an ad hoc basis, so comparison and correlation of data was often not rationally possible. Consequently, the scope and magnitude of the effects still lacked sufficient definition. One effect that was documented, however, is the degradation of compressive stiffness. A pioneering paper⁹ on the effects of nonlinear material behavior on the vibration of resin matrix composites was presented at the 17th AIAA/ASME Structures, Structural Dynamics and Materials Conference. In this paper, an estimate of the effect of compressive stiffness degradation on the vibration of [0] laminated beams is provided. It is concluded that this effect will not produce important consequences for fiber-dominated flexural vibrations. It is further concluded that only matrix-dominated flexure, such as that experienced during lateral vibration of angle ply panels, can potentially exhibit important nonlinear material effects.

It was recognized early in this nonlinear vibration program that experimental work is scarce and greatly needed. There are difficulties, however, in performing experiments in nonlinear parametric ranges. For example, conventional electromagnetic shakers, which are designed for resonance testing, are both stroke and force limited; consequently, it is not a straight-forward matter to excite beam specimens to amplitudes that are sufficiently large to produce nonlinear effects over the required range of frequencies. A number of exploratory experimental setups were tried and found inadequate. The first successful experimental approach is described in Reference 10. The unique feature about this approach is the use of longitudinal rather than lateral excitation to produce large amplitude motion in beams. The phenomenon produced, parametric excitation,

is intrinsically nonlinear in character. Consequently, it is not possible to determine linear theory, small amplitude response with this experimental set up. Since linear as well as nonlinear characteristics of the vibration are needed for a complete description, an alternative is needed.

A second independent experimental effort was under taken in early 1977 under the direction of Dr. Giora Maymon. This approach is based upon the general theory ^{1,6,7} developed previously. A useful indicator of the magnitude and nature (hardening or softening) of nonlinear stiffness-related effects is the frequency-amplitude relation for undamped, free vibration in a given vibration mode. For most structures, this relation is of the form

$$\frac{\omega_r^2}{\omega_{or}^2} = 1 + B_r (\xi_r)^2 \quad (1)$$

ω_r is the response frequency, ω_{or} is the natural frequency according to linear vibration theory, and ξ_r is the normalized amplitude of the r-th vibration mode. B_r is the coefficient of nonlinearity for the r-th mode; its sign and magnitude characterize the nonlinear effects.

Attention is directed to vibration of elastic structures that respond by simple jump behavior. A typical nonlinear response curve is shown in Figure 1. While the response curve for a linear system is almost symmetric about the undamped natural frequency, this curve is distorted and "overhangs" (to the right (hardening behavior) in the case depicted in Figure 1). Also, multivalued amplitudes exist for the same value of frequency. Jumps occur if the frequency is varied; a "jump down" occurs as the frequency is increased. A "jump up" takes place as the frequency is decreased. The "backbone curve" indicated in Figure 1 corresponds to undamped, free vibration. It is mathematically described by Equation (1). If the exciting force magnitude is varied, a family of response curves similar to

the one shown are produced. Each such curve will have a response point corresponding to where the jump down occurs. If the damping of the system is small and independent of amplitude, these points corresponding to "jump down" will, for practical experimental purposes, also correspond to the maximum amplitude at a given level of excitation and to the intercept of the response curve with the backbone curve. Consequently, the maximum amplitudes achieved in a test are interpreted as being on the backbone curve and corresponding to the "jump down" frequency. This is the essential basis for the experimental method.

In order to evaluate this method of experimentally determining nonlinear effects, a simple aluminum clamped-clamped beam with immovable ends was chosen as the structural system. The beam is excited laterally with two small exciters at points close to the clamped ends; these excitation points were selected so that only a small force is needed to excite rather large amplitudes. The exciters are attached to the beam specimen through small universal joints to avoid any side loads during flexural rotations. Miniature piezoelectric load cells sensed excitation levels. Accelerations were measured by a small accelerometer and frequency by a digital counter. The experimental setup is shown schematically in Figure 2.

A typical experimentally determined response curve is shown in Figure 3. The 'open' symbols correspond to a scan with increasing frequency, while the corresponding 'solid' symbols refer to a scan with decreasing frequency. The jumps are clearly and easily discernible. The primary results of interest are shown in Figure 4. If Equation (1) describes the structural system, a plot of frequency squared vs. amplitude squared will be linear. Furthermore, the intercept with the frequency squared axis provides the linear theory natural frequency and the slope gives the coefficient of nonlinearity. Figure 4 is self explanatory. The natural frequency has been determined also in a separate small amplitude transient test; the two experiments are in

agreement.

The coefficient of nonlinearity determined from the data presented in Figure 4 is five percent larger than the theoretical value given in Reference 11. Consequently, the data confirms the theory and the physical approximations upon which the experimental method is based. These results have been presented as Reference 12.

In addition to the above experimental approach, three methods for determining the damping coefficient for a nonlinear vibrating structural system have been found.¹³ All three are based upon the fact that the frequencies and amplitudes of the motion that correspond to jumps are well-defined, easily determined points. The methods, therefore, have no counterparts for linear systems.

It is seen that the nonlinear vibration work reached a state of completeness and maturity for simple structural systems with Reference 12. Issues such as modal coupling or interaction remain as topics for future research.

Boundary Restraint Effects

The engineer in seeking to avoid or solve a vibration problem most often will alter structural stiffness to achieve the desired results.

Stiffness of an element is changed by

- (1) selecting a new material
- (2) selecting another gauge of the same material
- (3) altering and/or introducing a favorable prestress field in the element
- (4) modifying element boundary restraint
- (5) altering and/or introducing favorable geometric curvature
- (6) redesigning the element

Several of these actions may be taken in a given situation depending

upon the stage in the design process that the problem is discovered, governing economic factors and the particular performance requirements of the system. Since unwanted resonance and fatigue are usually the concern, increased stiffness is normally prescribed.

Stringent weight requirements for flight vehicle airframe structure dictate that optimal solutions be found to vibration problems whenever possible. For this reason, items (4) and (5) and their interaction were investigated using linear theory. The addition of a small amount of fastener weight, for example, to increase boundary restraint may effectively result in a large increase in natural frequencies for structural elements with geometric curvature. Evidence supporting this contention for curved panels is provided by results published in Reference 14.

The analysis of Reference 14 demonstrates beyond doubt the importance of boundary restraint and a great sensitivity of the response to seemingly minor changes in the details of the restraint. In order to harness the potential benefits for achieving desirable results in actual structures, additional basic knowledge and an understanding of the governing mechanisms must be acquired. A step in this direction was taken in 1974 by the thorough study of the vibration of ring segments reported in Reference 15.

Circular ring segments, in addition to being of practical interest in connection with floating ring stiffened shell design, are the simplest structural elements upon which to study the influence of boundary restraint and curvature. Results are presented in Reference 15 for vibration modes which are symmetric and anti-symmetric about the segment semi-span, for limiting cases of bending and extensional restraint and also for finite extensional elastic end springs. Definite patterns in the vibration behavior have been found. It was learned that the phenomena of frequency reversal among anti-symmetric and symmetric vibration modes as curvature is increased (the symmetric mode's corresponding frequency increases

with curvature to the point when it becomes greater than an anti-symmetric mode's frequency, producing an exchange or reversal of position of the modes with regard to frequency) very nearly corresponds to a maximum value of extensional strain energy in the symmetric mode. The correspondence is not an exact one, but so nearly so that it tends to identify the ratio of extensional to total strain energy as a measure of merit in evaluating frequency reversal phenomena and for evaluating the beneficial, stiffening effect of geometric curvature.

The above hypothesis was tested by returning to a further study of curved panels to see if the ring segment observations have wider applicability. Unfortunately, they do not. Although the extensional strain energy ratio is a significant parameter, all efforts to draw general conclusions beyond those reported in References 14 and 15 were unsuccessful. The investigation, therefore, was terminated at this point.

Hygrothermal Effects on the Dynamic Behavior of Resin Matrix Composites

Epoxy resins used as matrix materials in advanced structural composites experience degradation of mechanical properties due to moisture absorption and elevated temperature environments. It is this fact that caused a redirection of the research effort in 1976. Considerable recent and current work has been directed toward documenting the extent of this degradation for simple layup configurations and states of stress. Primary emphasis has been placed upon static performance. An exception is the work pursued under this grant.

As indicated previously, a preliminary estimate of the impact of hygrothermal effects on vibration of beams is given in Reference 9. This has been followed by the pioneering experimental study by Maymon, Briley and Rehfield¹⁶ of the influence of moisture/temperature effects on dynamic structural behavior of epoxy matrix composite beams in flexure.

This study showed the contrast between two limiting reference conditions -- dry at room temperature (77°F) and moisture saturation at 200°F. Changes in fundamental natural frequency, an overall, practical measure of stiffness, and damping were determined experimentally. The objective was to establish a data base to inspire confidence in the use of graphite/epoxy composites in hostile, dynamic environments.

In addition to providing the first data on hygrothermal effects on damping, there is another aspect of this work that warrants special mention. An effort was made to approximate the environmental conditions during the test. The vibration testing technique was dictated by the need for simplicity, repeatability and preservation of the environmental conditions. In order to preserve the environmental state to the maximum extent, the excitation technique selected for vibration testing was a transient one. The advantage obtained is a short duration test. The beams are mounted in cantilever fashion in a fixture with prescribed tip deflection. This setup is shown in Figure 5. The tip is suddenly released producing transient response of the specimen that is dominated by the fundamental mode of vibration. Response is sensed by an accelerometer mounted near the specimen tip. The data are processed by means of a minicomputer-based Fourier analyzer which incorporates an analog to digital converter. The digitized accelerometer output is printed out, and the fundamental frequency and corresponding damping coefficient determined by simple analyses. Frequency is determined simply by counting the number of peak values contained in a suitably chosen time interval. Values for the damping coefficient are obtained by the logarithmic decay method.

Three distinct composite layups have been considered. The layups are $[0]_2, [+45]_2$, and $[0_2, +45_2, 90_1, -45_1]_s$. They were each 12-ply thick and symmetric and will be referred to as type A, B, and C, respectively.

The material is graphite/epoxy T300/5208. Types A, and B are limiting types of fundamental interest, while C is representative of practical vehicle skin panels. The A-specimens respond in a fiber controlled mode of vibration, while the B-specimens respond in a matrix controlled mode. The results indicate that stiffness is reversibly lowered in matrix controlled modes of deformation and that damping characteristics are altered substantially in both fiber and matrix controlled modes due to environmental conditioning. The C-specimens exhibit stiffness characteristics that appear to be fiber controlled (unchanged); the damping, however, is similar in nature to matrix controlled behavior. The hot, wet environment increases damping for A-type specimens, while it decreases damping markedly for B- and C-type ones. The latter is consistent with simple logic based upon decreasing effective internal viscosity of the epoxy matrix material with increasing moisture and temperature. The former could be related to increased relative motion between fibers and matrix at the interface.

The initial phase of the study of hygrothermal effects on dynamic behavior appears in Reference 16. This work will be continued and broadened in scope under a new contract.

CONCLUDING REMARKS

The research work summarized in the foregoing pages represents a long journey in the minds of the participants. Basic theoretical issues as well as practical experimentation have been encountered at different points along the way. As mentioned in the introduction, this is an unusual research program because of this variation in scope. Mr. William J. Walker of AFOSR provided the opportunity for response to the problem posed by hygrothermal effects in resin matrix composites. Such a challenging opportunity is rare; it is gratefully acknowledged.

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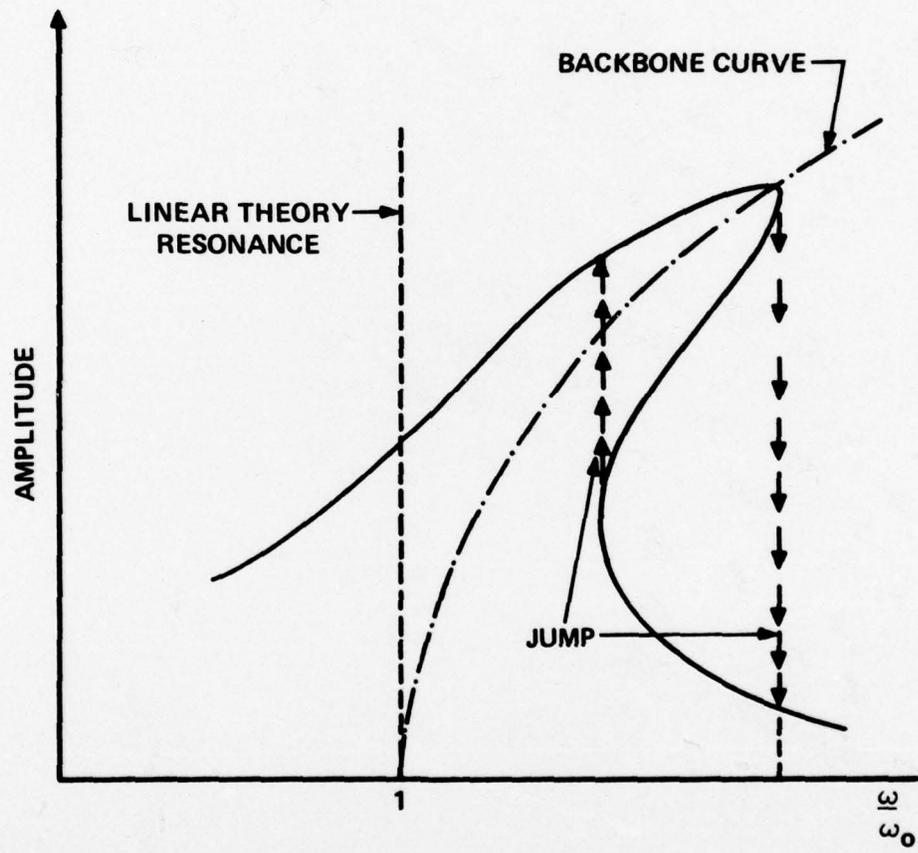


Figure 1. Typical Nonlinear Forced Response Curve for a Hardening Elastic System

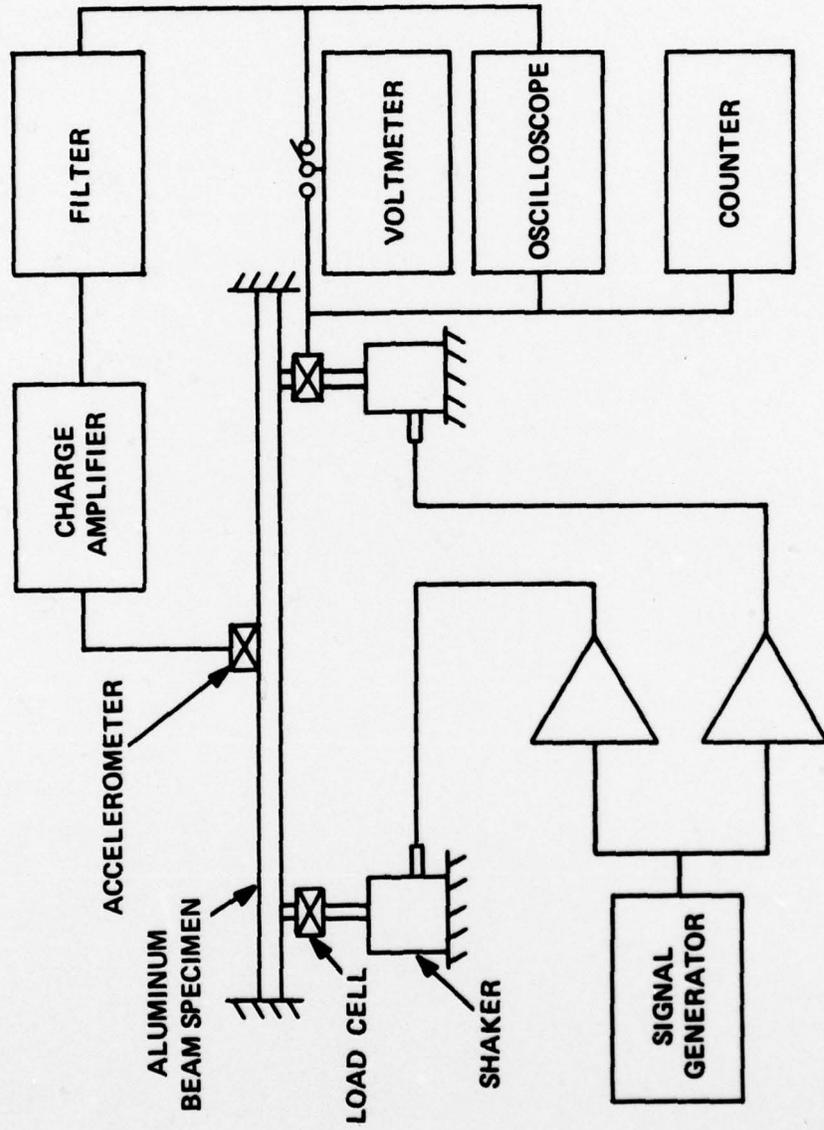


Figure 2. Schematic of Experimental Setup for Nonlinear Vibration Study

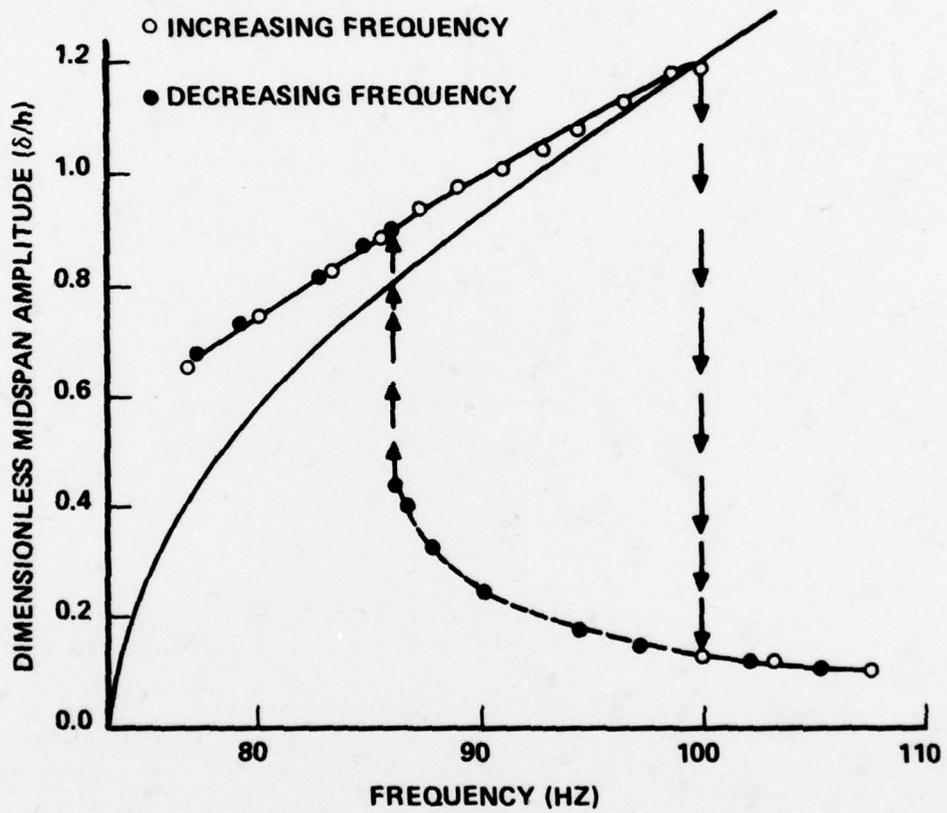


Figure 3. Typical Experimentally Determined Nonlinear Forced Response Data

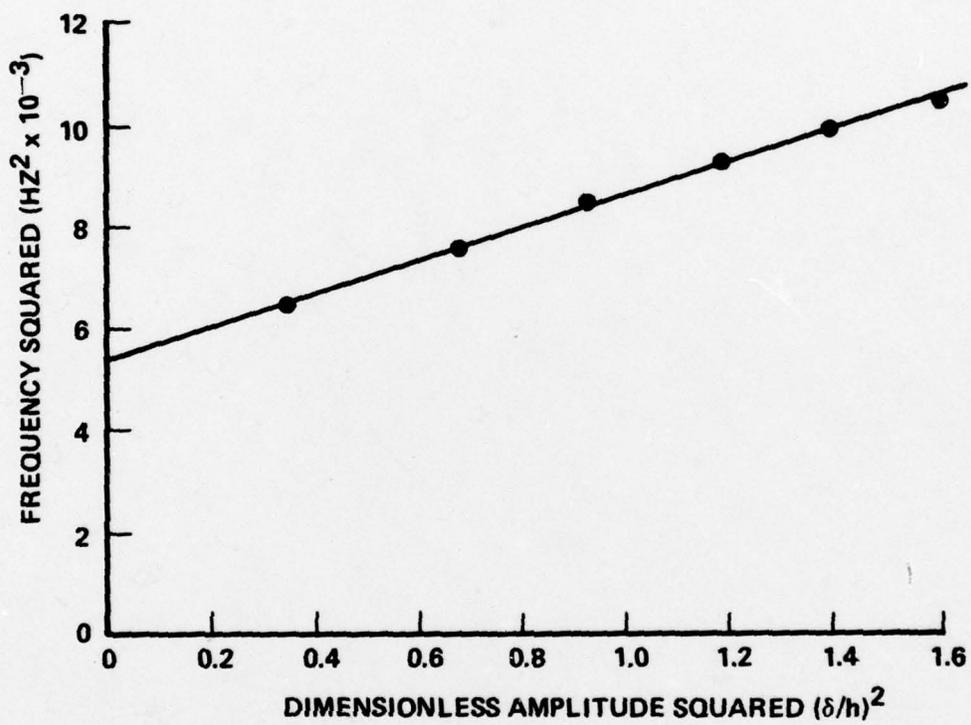


Figure 4. Nonlinear Stiffness Plot
(Frequency Squared vs. Amplitude Squared)

Figure 5. Vibration Test Fixture With a Mounted Beam Specimen

