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UNSTEADY BURNING OF SOLID PROPELLANTS

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A theoretical analysis is presented of unsteady solid propellant combustion, particularly combustion stability and extinction by rapid depressurization. It is assumed that the solid decomposes by a pyrolosis law and the gaseous products react exothermically following an Arrhenius law. For large values of this non-dimensional activation energy the gas-phase combustion, turns out to be quasisteady. The characteristic response time of the solid to gas-phase perturbations turns out to be large compared to the charcteristic residence time in the heat-up zone of the solid, their ratio being of the order of the non-dimensional activation energy

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in the gas-phase. A linear stability criterion has been obtained that gives stable burning for steady burning rates above and below two limiting values; the width of the unstable region increases with the activation energy of the pyrolisis law and becomes zero for a finite value of that activation energy. A nonlinear stability analysis has been performed that shows the existence of a limit cycle for unsteady burning conditions; in a part of the cycle the burning is slow and the solid responses as a whole to variations in gas phase burning; during other part the characteristic time is short and the heat content of the solid is constant. The dynamic extinction process has also been investigated.

ABSTRACT

A theoretical analysis is presented of unsteady solid propellant combustion, particularly combustion stability and extinction by rapid depressurization. It is assumed that the solid decomposes by a pyrolosis law and the gaseous products react exothermically following an Arrhenius law. For large values of this non-dimensional activation energy the gas-phase combustion, turns out to be quasisteady. The characteristic response time of the solid to gas-phase perturbations turns out to be large compared to the characteristic residence time in the heat-up zone of the solid, their ratio being of the order of the non-dimensional activation energy in the gas-phase. A linear stability criterion has been obtained that gives stable burning for steady burning rates above and below two limiting values;the width of the unstable region increases with the activation energy of the pyrolisis law, and becomes zero for a finite value of that activation energy. A nonlinear stability analysis has been performed that shows the existence of a limit cycle for unsteady burning conditions; in a part of the cycle the burning is slow and the solid responses as a whole to variations in gas phase burning; during other part the characteristic time is short and the heat content of the solid is constant. The dynamic extinction process has also been investigated.

N O M E N C L A T U R E

A Amplitude of limit cycle, eq.(64).

B Preexponential factor, and constant of integration (eq.52).

c Specific heat.

C Constant of integration (eq.52).

D Diffusion coeficient.

E Activation energy of the pyrolisis.

E_g Activation energy of the gas-phase reaction.

E' Dimensionless activation energy of the gas-phase reaction.

l Dimensionless heat flux at the surface, see Eq. (14).

L Heat of vaporization.

L_e Lewis number.

m Burning rate.

n Exponent of fuel mass fraction in chemical reaction rate.

p Pressure.

q Total heat content of the solid (eq. 71).

$\bar{q}$ Total heat content of the solid (eq.67).

Q Heat released per unit mass of fuel.

R Universal gas constant.

T Temperature.

t Time.

t_c Characteristic time, see Eq.(43).

u Velocity

x Space coordinate.

Y Fuel mass fraction.

y Dimensionless space coordinate.

α Thermal difussivity, and dimensionless parameter (eq.62).

γ Dimensionless steady state surface temperature, see Eq.(38).

δ Dimensionless width of the unstable region. Eq.(57).

ε Dimensionless inverse activation energy.

θ Nondimensional temperature.

λ Thermal conductivity.

μ Dimensionless burning rate.

μ' Dimensionless burning rate (eq. 58).

ξ Nondimensional space coordinate.

π Nondimensional pressure history.

ρ Density.

τ_1 Dimensionless time, see Eq.(19).

τ Dimensionless time, see Eq.(30).

$\bar{\tau}$ Dimensionless time, see Eq.(61).

ω See Eq.(38) and (46).

SUBSCRIPTS

f Flame.

g Gas.

s Surface.

o Steady state conditions.

∞ Infinity in the solid.

1. Introduction

An understanding of the unsteady burning of solid propellants provides insight into such important problems as the pressurization, depressurization and stability of a solid-rocket motors.

Typical models used for the analysis are those presented in references (1) to (12).

Research in the stability of combustion in a rocket motor, has been active over the past twenty years, since irregular pulses in chamber pressure were observed to develop[6] instead of the expected smooth pressure-time history. These irregular pulses are generally accompanied by more regular, small-amplitude, pressure oscillations, with frecuencies of the order of the natural vibrational frecuencies of sound waves in the chamber. Combustion instability leads to inefficient operation of rocket motors and even to mechanical failure of the propellant.

A considerable theoretical effort has been devoted towards the understanding of this phenomenon. However, there are so many different effects that may influence the stability of a burning solid as to prevent a consensus on the theoretical description of the phenomenon.

An excellent review of the analyses of the small amplitude pressure oscillations has been developed by Culick[7]. These analyses calculate the admittance function of the burning surface which gives the burning rate response to a pressure disturbance, indicating therefore whether pressure oscillations are amplified

or attenuated. Most of the analyses assume the gas phase to be quasisteady, in the sense that the gas phase adjusts very quickly to changes in conditions when compared to the response of the solid, and differ mainly in the assumptions used to calculate the heat transfer from the gas to the solid phase. However, as Culick points out, the majority of the results lead to the same two-parameter form of the admittance function, with different definitions of the two parameters. A stability boundary in the space which coordinates the two parameters entering the admittance function, is defined as the curve where the admittance function becomes infinite, so that a small pressure change causes a large fluctuation in burning rate. Points above this stability boundary produce unstable solutions and points below this boundary provide stable steady solutions.

Interest in the response of a burning solid to an externally applied pressure variation stems from the possibility of extinction by a rapid pressure decay[3]. This dynamic extinction process is useful in order to design solid propellant rockets with stop-restart capabilities, of interest in connection with space applications.

Nearly all the theoretical models developed to explain dynamic extinction[9-14] invoke some kind of quasisteady approximation for the gas-phase. However, as pointed out in reference (12) some of these analyses[9-11] have interpreted incorrectly this assumption by implying that the heat feebback from the flame to the solid is a steady state function of the instantaneous pressure only.

The present paper is an attempt towards analyzing unsteady

processes in solid propellant burning by means of asymptotic techniques based on the assumption that the nondimensional activation energy of the gas-phase reaction is large. Attention is paid both to the problem of the stability of the steady state and to the response of the burning propellant to an externally imposed pressure variation.

We consider a one-dimensional model in which a condensed material gasifies by a rate-controlled surface process and then reacts in the gas phase. This gas phase reaction is described by an Arrhenius law, and we consider the limit in which the nondimensional activation energy is large. The quasisteady assumption is used to describe the gas-phase, so that we may use the results obtained by Williams[3] and Buckmaster et al[15], when analyzing the quasisteady burning of a solid in the limit of high activation energy. These analyses yield the burning rate and the heat feedback to the solid as functions of the pressure and the flame temperature, and these relations are then used to analyze the unsteady response of the condensed phase. It is found that the characteristic response time of the solid is large, of the order of the nondimensional activation energy, so that the temperature profiles are quasisteady in first approximation and the transient term is a perturbation of the quasisteady solution which may be calculated from the quasisteady profiles. In this way a differential equation is derived which describes the evolution with time of the burning rate as a function of the nondimensional surface temperature, the nondimensional pyrolisis activation energy and the pressure-time history. The analysis of this equation yields the stability condition of the steady state solution. It is found that burning conditions are stable outside a range of values of steady burning rates. The amplitude of this unstable range

increases with the activation energy of the pyrolisis, and becomes zero for a finite value of that activation energy. It turns out that instability is related to the fact that the total heat content of the solid is a decreasing function of the burning rate under steady conditions.

Then, the existence of non-linear oscillations for conditions corresponding to instability under steady conditions is investigated. However, the equation obtained for the burning rate shows that, as the unstability boundary is crossed, the time derivative becomes infinity revealing that during part of the oscillation period there is a characteristic time much shorter than the one corresponding to unsteady response of the solid. The previously obtained equation is conveniently modified, and then it is found that the nonlinear oscillations due to instability are amplified until they reach a limit cycle that is characterized by two intervals of time; one of then is long and during it the temperature profiles in the solid are quasisteady, in the other the characteristic time is much shorter and the solid has no time to change its heat content. The amplitude of this limit cycle increases with the activation energy of the pyrolisis. For values of this activation energy close to the one that corresponds to the disappearance of the unstability region, the amplitude of the limit cycle becomes zero, and the period of this cycle decreases, disappearing the difference between the two time intervals, now the variation of the burning rate with time is smooth and in the limit tends to be sinusoidal.

The differential equation describing the quasisteady evolution of the burning rate with time is then used to obtain the response of the solid to a pressure decrease and a possible criterion for dynamic extinction is obtained.

The formulation of the problem presented here and some of its preliminary ideas were presented in a previous Interim Scientific Report[16], and for a better understanding have been included again in this final report.

2. Formulation

We consider a one-dimensional model with the solid occupying the half space $x < 0$ and the gaseous phase the region $x > 0$. For convenience, the origin is fixed at the surface of the regressing solid. Figure 1 is a schematic representation of the process and shows the effects accounted for in this study.

Surface gasification is assumed to occur by an irreversible pyrolisis process which is described by an Arrhenius law. A one step over-all exothermic reaction takes place in the (premixed) gas. The present model has been used because it has been successfull in describing the steady-state deflagration of several propellants.

The equation of conservation of momentum reduces to the statement that the pressure is approximately uniform throughout the region treated but varies with time[1], and we will also use the well justified assumptions that the work associated with viscous and external forces is negligible and we will use Fick's law to calculate the diffusion velocities.

With these assumptions, the conservation equations of mass, concentration and energy in the condensed phase and in the gas phase, become respectively.

$$\frac{\partial \rho_g}{\partial t} + \frac{\partial (\rho_g u)}{\partial x} = 0 \tag{1}$$

$$\frac{\partial Y}{\partial t} + u \frac{\partial Y}{\partial x} - \frac{1}{\rho_g} \frac{\partial}{\partial x} \left(\rho_g D_g \frac{\partial Y}{\partial x} \right) = - B p^{n_Y n} \exp\left(- \frac{E_g}{RT}\right) \tag{2}$$

$$c_g \frac{\partial T}{\partial t} + c_g \mu \frac{\partial T}{\partial x} - \frac{1}{\rho_g} \frac{\partial}{\partial x} \left(\lambda_g \frac{\partial T}{\partial x} \right) - \frac{1}{\rho_g} \frac{\partial p}{\partial t} = Q B \, p^{n_Y n} \exp\left(\frac{E_g}{RT}\right) \tag{3}$$

$$\rho c \frac{\partial T}{\partial t} + mc \frac{\partial T}{\partial x} = \frac{\partial}{\partial x} \left(\lambda \frac{\partial T}{\partial x}\right) \qquad (4)$$

where all symbols are defined in the nomenclature.

These equations, together with the equation of state should determine ρ_g, u, Y, T, T_g.

The boundary conditions are

$$T(t, 0^+) = T(t, 0^-) = T_s \qquad (5)$$

$$-\lambda \frac{\partial T}{\partial x}\Big|_{s^-} + \lambda_g \frac{\partial T}{\partial x}\Big|_{s^+} = m\left[(c_g - c)T_s + L\right] \qquad (6)$$

$$\rho_g D_g \frac{\partial Y}{\partial x}\Big|_{s^+} = m(Y_{s^+} - 1) \qquad (7)$$

$$T = T_\infty \quad \text{at} \quad x \to -\infty \,, \quad Y = 0 \,, \quad T = T_f \quad \text{at} \quad x \to \infty \qquad (8)$$

The pyrolisis process is assumed to follow an Arrhenius law

$$m = B' \exp(-E/RT_s) \qquad (9)$$

where m is the mass flux relative to the burning surface, and E the activation energy of the pyrolisis process.

3. Gas-phase analysis

In most studies of unsteady solid propellant burning, the gas-phase is assumed to be quasisteady in the sense that the response time in the gas is short compared to the response time of the condensed phase. The ratio of these characteristic times is of the order of the ratio of the thermal responsivities of solid and gas, which is usually small[2]. The quasisteady assumption will not be valid when analyzing the response of a burning solid to

very high frecuency pressure oscillations[1].

When the gas-phase is considered to be quasisteady, the analysis is greatly simplified since the conservation equations are uncoupled. All the time derivate terms may be neglected so that equation (1) reduces to $\rho_g u = m$. In addition the dp/dt term in equation (3) may be neglected. Under these conditions the gas-phase equations may be solved in the limit of high activation energy of the gas-phase reaction. This quasisteady solution has been derived by Williams[3] in the case of Lewis number unity and by Buckmaster et al[15]., for arbitrary Lewis number, Le.

Rather than repeat those analyses we will only state the results needed in the following sections. The reader is referred to reference 3 and 15 for details. The analysis in reference (15) considers Le constant but arbitrary. In the limit of high activation energy, it is found that the reaction term is only significant in a thin flame where T is close to T_f. In terms of the nondimensional variables

$$\theta = c_g T/Q_g \qquad \xi = \int_o^x (m/\alpha)dx \tag{10}$$

the solution outside of the flame sheet becomes simply

$$\left.\begin{aligned} \theta &= \theta_s - 1 + 1 \exp(\xi) \\ Y &= 1 - \exp\{L_e(\xi - \xi_f)\} \end{aligned}\right\} \quad \text{for } \xi < \xi_f \tag{11}$$

$$\left.\begin{aligned} \theta &= \theta_f \\ Y &= 0 \end{aligned}\right. \quad \text{for} \quad \xi > \xi_f \tag{12}$$

where ξ_f, the location of the flame sheet, is given to leading order by

$$\xi_f = \ln (1/1) \tag{13}$$

The parameter

$$l = \frac{d\theta}{d\xi}\Big|_s \qquad (14)$$

is the dimensionless heat conducted out of the gas at the interface. The effect of the solid and the pyrolisis are concealed in l. An overall energy balance in the gas phase, provides a relationship between θ_s, θ_f and l.

$$l = 1 + \theta_s - \theta_f \qquad (15)$$

Since the flame sheet must lie in the gas, equation (13) provides limitations in the possible values of l

$$0 < l = 1 + \theta_s - \theta_f < 1 \qquad (16)$$

When $l \to 0$ the flame sheet moves to infinity and no heat reaches the condensed phase from the flame. When $l \to 1$ the flame sheet approaches the surface and all the heat generated at the flame goes to the solid.

Solutions for $l \to 0$ and $l \to 1$ are presented in reference (15)

The reaction zone is located in the vicinity of ξ_f, where $(\xi - \xi_f)$ is of order θ_f^4/E'^2. The parameter E' is the nondimensional activation energy $E' = E_g c_g/RQ$. In the reaction zone the temperature differs from the flame temperature by a small quantity of order $T_f^2 R/E_g$. To leading order, only the reactive and diffusive terms are important in this zone. Solution of the energy equation with the appropriate matching conditions to the frozen solution outside of the flame sheet, provides the burning rate eigen-value.

In this way an expression for the burning rate is obtained

involving the pressure and the flame temperature, namely

$$m = \left|2\Gamma(n+1)\alpha B\theta_f^{2n+2}/E'^{n+1}\right|^{1/2} \exp(-\frac{E'}{2\theta_f})p^{n/2} \qquad (17)$$

For large activation energy the effect of the exponential term is dominant so that the square root term may be taken as constant when analyzing small changes in the flame temperature. Equation (17) coincides with the Denison-Baum formula[5].

4. Condensed Phase Analysis

The characteristic time in the condensed phase is short compared with the characteristic time in the gas phase[2]. Therefore, the condensed phase should not be considered quasisteady. However, a brief description of the quasisteady solution will be presented before considering the unsteady analysis.

In terms of the nondimensional variables for the solid

$$\theta = \frac{T-T_\infty}{T_{so}-T_\infty} \qquad y = \frac{m_o c}{\lambda} x \qquad (18)$$

$$\tau_1 = \frac{m_o^2 c}{\lambda\rho} t \qquad \mu = \frac{m}{m_o} \qquad (19)$$

Eq. (4) may be written

$$\frac{\partial\theta}{\partial\tau_1} + \mu\frac{\partial\theta}{\partial y} = \frac{\partial^2\theta}{\partial y^2} \qquad (20)$$

The subscript o refers to the initial, steady condition. The boundary condition (6) may be written,

by using Eqs. (14) and (15) as

$$\left.\frac{\partial\theta}{\partial y}\right|_s = \frac{\mu}{c(T_{so}-T_\infty)}\left[cT_\infty - c_gT_f + Q - L + c\theta_s(T_{so}-T_\infty)\right] \qquad (21)$$

To solve the quasisteady problem, we may neglect the time derivative term in Eq. (20). The solution to Eq. (20) is simply

$$\theta = \theta_s \exp(\mu y) \qquad (22)$$

so that from Eq. (21).

$$c_gT_{fo} = cT_\infty + Q - L \qquad (23)$$

This equation can also be obtained by an overall energy balance and shows that the flame temperature is constant in a quasisteady process. For a given T_∞, Eq. (23) gives the value of T_{fo} which can be used in the Denison-Baum formula, Eq. (17), to calculate m. The pyrolisis law, Eq. (9), yields T_s, so that Eqs. (11), (12) and (22) describe the complete temperature profile.

Figure (2) shows the burning rate m under steady conditions; as a function of pressure for a fixed value of T_∞ and therefore of T_{fo}. This curve is calculated by using Eqs. (17) and (23).

The surface temperature T_s is a parameter along the curves of figure (2), since it is related to the burning rate m_o through the pyrolisis law. However, T_s is limited by the inequalities (16) so that only a portion of the curve applies. At P_∞ the flame sheet has moved off to infinity. At P_s the flame sheet has reached the surface.

We will now analyze the evolution with time of an initially steady temperature profile during an unsteady process. After some manipulations, Eqs. (20) and (21) provide the following relationships

$$\left.\frac{\partial\theta}{\partial y}\right|_s - \mu\theta_s = \int_{-\infty}^{o} \frac{\partial\dot{\theta}}{\partial\tau_1}\, dy = \mu\,\frac{c_g}{c}\,\frac{T_{fc}-T_f}{T_{so}-T_\infty} \qquad (24)$$

where use has been made of Eq. (23). The left hand side of the preceding equation represents the difference between the heat flux existing during an unsteady process and the one existing if the process was quasisteady. We will consider unsteady processes that result in changes of order unity in the burning rate with respect to the one existing under quasisteady conditions. Equation (17) shows that in the limit of high activation energy of the gas phase reaction, small changes in the flame temperature of order RT_f^2/E_g, produce variations of order unity in the burning rate. Therefore, the right hand side of Eq.(24) is small, so that the heat conducted to the solid during an unsteady process differs by a small quantity from the heat conducted if the process was quasisteady.

The second equality of Eq.(24) indicates that the variation with time of the heat content in the heat up zone of the solid is small of order RT_f^2/E_g. When this heat content decreases, the solid appears from the gas phase as a heat source, and therefore the flame temperature increases.

It is necessary to wait times of order E_g/RT_f^2 to produce changes of order unity in the heat content of the heat up zone in the solid. The characteristic response time of the solid is therefore long compared with the characteristic residence

time in the heat up zone.

The Denison-Baum formula, Eq. (17), yields

$$\mu = \left(\frac{T_f}{T_{fo}}\right)^{n+1} \left(\frac{P}{P_o}\right)^{n/2} \exp\left[\frac{E_g}{2RT_{fo}} - \frac{E_g}{2RT_f}\right] \tag{26}$$

which in the limit of high values of $E_g/2RT_{fo}^2$ becomes

$$\frac{\mu}{\pi} = \exp\left\{\frac{E_g(T_f - T_{fo})}{2RT_{fo}^2}\right\} \tag{27}$$

where

$$\pi = \left(\frac{P}{P_o}\right)^{n/2} \tag{28}$$

Let's define a small parameter ε, as

$$\varepsilon = \frac{c_g}{c} \frac{2RT_{fo}^2}{E_g(T_{so} - T_\infty)} \tag{29}$$

and introduce as nondimensional time variable

$$\tau = \varepsilon\tau_1 \tag{30}$$

Equation (27) may be used to express Eq. (20) and the boundary condition (24) in terms of the new time variable τ, as

$$\varepsilon \frac{\partial\theta}{\partial\tau} + \mu \frac{\partial\theta}{\partial y} = \frac{\partial^2\theta}{\partial y^2} \tag{31}$$

$$\left.\frac{\partial\theta}{\partial y}\right|_s = \mu\left[\theta_s - \varepsilon \ln \frac{\mu}{\pi}\right] \tag{32}$$

The value of the surface temperature may be written in terms of the burning rate μ, through the pyrolisis law, Eq.(9), resulting:

$$\theta_s = \frac{\frac{\omega}{\gamma-1}\ln\mu+1}{1-\omega\ln\mu} \tag{33}$$

where

$$\gamma = \frac{T_{so}}{T_\infty} \quad , \quad \omega = \frac{RT_{so}}{E} \tag{34}$$

Introducing the expansions

$$\theta = \theta_0 + \varepsilon\theta_1 + \ldots \tag{35}$$

$$\mu = \mu_0 + \varepsilon\mu_1 + \ldots \tag{35'}$$

in Eqs. (31) and (32) the following equations defining θ_0 are obtained

$$\mu_0 \frac{\partial\theta_0}{\partial y} = \frac{\partial^2\theta_0}{\partial y^2} \quad , \quad \left.\frac{\partial\theta_0}{\partial y}\right|_s = \mu_0\theta_{0s} \tag{36}$$

Whose solution is the quasisteady solution

$$\theta_0 = \theta_{s0}\exp(\mu_0 y) \tag{37}$$

Integrating eq.(31) with boundary condition (32) the following integral condition is obtained:

$$\int_{-\infty}^{0} \frac{\partial\theta}{\partial\tau}\,dy = -\mu\ln\frac{\mu}{\pi} \tag{38}$$

On using the first approximation for θ, the integral in the left hand side may be calculated, thus obtaining

$$\frac{d}{d\tau}\left(\frac{\theta_{s0}}{\mu_0}\right) = -\mu_0 \ln \frac{\mu_0}{\pi} \tag{39}$$

where θ_{s0}/μ_0 represents the total heat content of the solid under steady state conditions. Now the pyrolisis law is used to relate the surface temperature to the burning rate, obtaining:

$$\frac{d\mu_0}{d\tau} = -\frac{(\gamma-1)(1-\omega\ln\mu_0)\mu_0^3}{\gamma\omega-\gamma+1+\omega(\gamma-2)\ln\mu_0+\omega^2\ln^2\mu_0} \ln\left(\frac{\mu_0}{\pi}\right) \tag{40}$$

which describes the evolution with time of the burning rate during an unsteady process, in terms of two parameters γ and ω. For a quasisteady evolution $d\mu_0/d\tau$ vanishes and therefore $\mu_0 = \pi$.

The value of the burning rate, μ_0, obtained from Eq.(40), may be used in the pyrolisis law to deduce the surface temperature history, and through Eq.(37) the complete temperature profile.

5. Stability Analysis.

In this section an analysis is presented of the stability of the steady deflagration of a solid which undergoes an Arrhenius type gas phase reaction with large activation energy.

The pressure is considered to be constant, so that $\pi=1$. The burning rate of the steady solution is m_o, so that initially

$\mu_0=1$. Let's assume that at $\tau=0$ a perturbation changes the value of the burning rate so that $\mu_0=1+\mu'$. Eq.(40) shows that

$$\frac{d\mu'}{d\tau} = - \frac{\mu'}{\frac{\gamma\omega}{\gamma-1} - 1} \tag{41}$$

Therefore the stability of the steady state solution depends on the value of the parameter $\frac{\gamma\omega}{\gamma-1}$. If this parameter is greater than one, the solution is stable, being unstable when it is smaller than one. In the stable case the perturbations decrease exponentially. The parameter

$$\frac{\gamma-1}{\gamma\omega} = \frac{E(T_{so}-T)}{RT_{so}^2} = A \tag{42}$$

is identical with the parameter A used by Denison and Baum[5] when analyzing the unstable burning of solid propellants. These authors develop a linearized analysis of the response of a burning solid to a pressure disturbance using a model which parallels the one used in the present analysis. The difference however, is that in our analysis we retain the nonlinear effects that were linearized in reference (5). In figure (1) of their paper, Denison-Baum show the stability boundary which separates regions of stable from regions of unstable burning. Two parameters A and α, define the stability of a given solution. The parameter α is essentially identical to our parameter ε, and in that figure (1) we observe that for $\alpha \to 0$ the solutions are stable for $A < 1$ and unstable for $A > 1$. Therefore our stability criterion coincides with the one of Denison and Baum.

The instability of the one-dimensional deflagration may be

clearly interpreted, when the activation energy of the pyrolisis is large (ω small). Let's assume that at a certain time a perturbation causes the burning rate to increase (decrease) with respect to its steady value. Since the activation energy of the pyrolisis is large, the surface temperature will remain nearly constant. However, the width of the heat up zone in the solid will decrease (increase), so that the total heat content of the solid will decrease (increase). The variation of the thermal energy of the solid produces an increase (decrease) in the flame temperature, as may be seen from Eq. (24), which will further increase (decrease) the burning rate. This selfaccelerating behaviour results in instability of the one-dimensional deflagration. When the activation energy of the pyrolisis is not large, there are two effects which control the response of the burning solid to a perturbation in the burning rate. When the burning rate increases the width of the heat up zone decreases and the surface temperature increases. The decrease in width tends to decrease the heat content of the solid thus producing instability, and the increase in surface temperature tends to increase the heat content of the solid thus producing stability. The stability of the solution depends on which of these two effects dominates.

If the reference pressure P_o is such that π is different from one, the perturbation of the steady solution will be

$$\mu_0 = \pi + \mu'$$

and Eq.(40) will show that

$$\frac{d\mu'}{d\tau} = - F(\pi)\mu' \qquad (43)$$

where $F(\pi)$ is the first factor on the left hand side of Eq.(40) with $\pi = \mu_0$. An alternative form of writing (43) is, from Eq.(39)

$$\left.\frac{d(\theta_{s0}/\mu_0)}{d\mu_0}\right|_{\mu_0=\pi} \frac{d\mu'}{d\tau} = -\mu' \qquad (43')$$

When the stability of other points besides the one that corresponds to $\pi=1$ is studied, it is necessary to analyze the sign of the first factor on the right hand side of Eq(43'); when it is positive the steady state is stable. As it may be deduced from Eqs.(38) and (43'), the steady state is stable when the total heat content of the solid (under steady conditions) increases as the burning rate is increased; this criterion is valid for any pyrolisis law.

For the pyrolisis law given by Eq.(33) the stability is related to the sign of $F(\pi)$ in Eq.(43) or the first factor of the left hand side of Eq.(40). In this factor the numerator vanishes for the limiting values of μ_0

$$\mu_0 = \exp(1/\omega) \quad \text{and} \quad \mu_0 = 0 \qquad (44)$$

which correspond to θ_s tending to infinity (eq.(33)), and the burning rate tending to zero respectively. It will be considered that μ_0 is outside these limits. The stability criterion is then given by the sign of the denominator that vanishes for:

$$\ln\mu_0 = -\frac{1}{2\omega}\left[\gamma-2 \mp \sqrt{\gamma(\gamma-1/\omega)}\right] \qquad (45)$$

For values of $\mu_0=\pi$ above the larger root or below the lower root of Eq.(45) the steady state solution is stable.

There is a certain arbitrariness in the choice of the value of P_o, (Eq.(28)), that establishes the steady state solution for $\pi=1$. For the following analysis it is more convenient to choose $T_{so}=2T_\infty$. Then from Eq.(38).

$$\gamma = 2 \quad , \quad \omega = \frac{2RT_\infty}{E} \tag{46}$$

Equation (45) becomes

$$\ln\mu_0 = \pm \frac{1}{\omega}\sqrt{1-2\omega} \tag{47}$$

With this choice of the reference value of T_{so}, the steady state solution that corresponds to $\pi=1$ is unstable and is located in the middle of the unstable region (if we consider $\ln\mu$ as the appropiate variable).

For values of the activation energy of the pyrolisis smaller than a finite value:

$$E < 4RT_\infty \quad , \quad \omega > 1/2$$

there are no unstable states. The thickness of the unstable region increases as the activation energy of the pyrolisis increases.

The pyrolisis law, Eqs.(33) and (9), in the new variables becomes:

$$\theta_s = \frac{1+\omega \ln\mu}{1-\omega \ln\mu} \qquad (48)$$

6. Non linear stability analysis

The purpose of this section is to show that unstable steady states produce non-linear oscillations of finite amplitude around the steady state. Equation (40), however, is not sufficient to describe these oscillations. A small perturbation around the steady state will start to grow exponentially as shows Eq.(43) ($F<0$). As this perturbation becomes larger, the linear analysis fails and equation (40) should be used. This Eq. (40) gives a monotonic increase of μ_0, and when it gets close to the value given by Eq.(47) its derivative becomes infinite and the present analysis fails.

Let us retain in Eqs.(35 and 35') the terms of order ε and it will be shown that these terms will introduce a second order derivative of μ in equation (40), which generates oscillations of μ with time. Additionaly, to retain the terms of order ε will make the first derivatives of μ finite in its whole range of variation, and it will be shown that Eq.(40) will be valid only for some intervals of those oscillations.

Retaining terms of order ε in Eq.(35) and using Eq.(37).

$$\theta = \theta_s \exp(\mu y) + \varepsilon\theta_1 \qquad (49)$$

where θ_1 should satisfy the conditions

$$\theta_1 = 0 \quad \text{for} \quad y = 0 \quad \text{and} \quad y \to -\infty \qquad (50)$$

Introducing Eq.(49) into Eq.(31) and neglecting terms of order ε^2, the following equation is obtained

$$\varepsilon \exp(\mu y)\left(\frac{d\theta_s}{d\tau} + y\theta_s \frac{d\mu}{d\tau}\right) + \varepsilon\mu \frac{\partial\theta_1}{\partial y} = \varepsilon \frac{\partial^2\theta_1}{\partial y^2} \qquad (51)$$

The solution of Eqs. (50) and (51) is:

$$\theta_1 = (By + Cy^2) \exp \mu y \qquad (52)$$

where

$$B = \frac{1}{\mu} \frac{d\theta_s}{d\tau} - \frac{\theta_s}{\mu^2} \frac{d\mu}{d\tau} \qquad (53)$$

$$C = \frac{1}{2} \frac{\theta_s}{\mu} \frac{d\mu}{d\tau} \qquad (54)$$

This solution should also satisfy the boundary condition (32). As it was done previously, this condition is substituted by an integral condition obtained by integrating Eq.(31) between $-\infty$ and 0, and using Eq.(32).

$$\not{\varepsilon} \int_{-\infty}^{0} \frac{\partial\theta}{\partial\tau}\, dy = -\not{\varepsilon}\mu \ln \frac{\mu}{\pi} + O(\varepsilon^2) \qquad (55)$$

Where the last term is included because in the gas-phase analysis terms of order ε^2 were neglected. By introducing Eqs.(49), (52), (53) and (54) into (55) the following equation for μ is obtained :

$$-\varepsilon \frac{d}{d\tau}\left[\frac{1}{\mu}\frac{d}{d\mu}\left(\frac{\theta_s}{\mu^2}\right)\frac{d\mu}{d\tau}\right] + \frac{d}{d\tau}\left[\frac{\theta_s}{\mu}\right] = -\mu \ln\frac{\mu}{\pi} + O(\varepsilon) + O\left(\varepsilon^2 \frac{d^3\mu}{d\tau^3}\right) \qquad (56)$$

Where θ_s is a function of μ given by the pyrolisis law (eqs.(37) or (48)), although in principle Eq.(56) is valid for any pyrolisis law. If terms of order ε are neglected, this Eq. becomes the same as Eq.(39) or Eq.(40). In principle the term of order ε on the right hand side which has not been calculated and is due to higher order contributions of the gas phase reaction, is as important as the new calculated term (the first one), unless, as it is in the present case, the time scale is very small in regions where the first term should be retained. The last term on the right hand side represents the influence of higher order terms that would appear if this procedure would be continued.

6.1. Case for which the instability region is narow

Before considering the more general case, let us study the case for which ω is close to 0.5.

$$\omega = \frac{1}{2}(1-\delta) \qquad (57)$$

$$\delta << 1$$

Then, from equation (47) the instability range will be

$$\ln\mu = \pm 2\sqrt{\delta}$$

Since we are going to study values of μ located near or in the

instability range, we make the change of variables

$$\mu = 1 + 2\mu'\sqrt{\delta} \tag{58}$$

The point of steady burning corresponds to $\mu_o = \pi$ and we assume this point to be in or near the instability region

$$\pi = 1 + 2\mu'_o\sqrt{\delta} \tag{59}$$

Then, equation (56) in the new variables becomes, after using equation (48)

$$\varepsilon\delta^{1/2}\frac{d^2\mu'}{d\tau^2} + 2\delta^{3/2}(\mu'^2-1)\frac{d\mu'}{d\tau} + 2\delta^{1/2}(\mu'-\mu'_o) + \tag{60}$$

$$+ O(\varepsilon\delta\frac{d^2\mu'}{d\tau^2}) + O(\delta^2\frac{d\mu'}{d\tau}) + O(\varepsilon) + O(\varepsilon\delta^{1/2}\frac{d^3\mu'}{d\tau^3}) = 0$$

In this case the significant limit corresponds to a choice of the new time scale that will be shorter than the previous one

$$\bar{\tau} = \tau\sqrt{2/\varepsilon} \tag{61}$$

and δ should be small, of order $\sqrt{\varepsilon}$. Introducing the parameter α:

$$\delta = \alpha\sqrt{\varepsilon/2} \tag{62}$$

Then, equation (60) becomes:

$$\frac{d^2\mu'}{d\bar{\tau}^2} + \alpha(\mu'^2-1)\frac{d\mu'}{d\bar{\tau}} + (\mu' - \mu'_o) = 0 \qquad (63)$$

and all remaining terms become negligible. Equation (63) is a Lienard type equation, that reduces to Van der Pol's equation when $\mu'_o=0$. For $|\mu'_o|>1$ the steady state solution is stable since we are outside the instability region. For $|\mu'_o|<1$ the steady state is unstable for a small perturbation; however, as the perturbation grows it tends towards a limit cycle of finite amplitude. This means that if we maintain a constant pressure that would correspond to a steady state burning in the unstable region, the burning rate would exhibit an oscillating behavior of finite amplitude.

If μ'_o is close to the instability boundary, (μ'_o close to one) the amplitude of the limit cycle will be small and the solution of (63) for the limit cycle will be of small amplitude, and can be aproximated by:

$$\mu' = \mu'_o + A \operatorname{sen} \bar{\tau} \qquad (64)$$

Then to impose the condition that the secular terms of equation (63) will vanish, let us multiply eq.(63) by $(\frac{d\mu'}{d\bar{\tau}})$ and integrate through a cycle:

$$\alpha \int_o^{2\pi} (\mu'^2-1)(\frac{d\mu'}{d\bar{\tau}})^2 \, d\bar{\tau} = 0$$

obtaining

$$A = 2\sqrt{1-\mu_o'^2} \tag{65}$$

Results (64) and (65) for the limit cycle are valid only for α small or $|\mu_o'|$ close to one.

When α is large, the solution approaches that which corresponds to an instability region of finite amplitude, since from equation (62) δ will become larger (see also eq.(57)). Now one may explore the limit α large in eq.(63), (still maintaining the limit δ small).

Rewritting eq.(63) in the form (defining a new variable $\bar{q}$)(*):

$$\alpha \frac{d\bar{q}}{d\bar{\tau}} = \mu_o' - \mu' \tag{66}$$

$$\frac{1}{\alpha} \frac{d\mu}{d\bar{\tau}} = \bar{q} - \left(\frac{\mu'^3}{3} - \mu'\right) \tag{67}$$

and analyzing the phase plane corresponding to equations (66) and (67) for $|\mu_o'|<1$ and α large (see fig.3a)

$$\alpha^2 \frac{d\bar{q}}{d\mu} = \frac{\mu_o' - \mu'}{\bar{q} - \left(\frac{\mu'^3}{3} - \mu'\right)} \tag{68}$$

(*) $\bar{q}$ is the total heat content of the solid. The meaning of this new variable and the physical interpretation of the results are reserved for the next section, where they will become more clear.

it can be seen that if the steady burning point is in the unstable branch BD, then it will go out from that point and approach the branch AB or CD and from there it would finally approach the limit cycle ABCD. This limit cycle has an amplitude (corresponding to points C and A) of:

$$\mu' = \pm 2$$

While we are in branches AB or CD in the stable region, the solution is given by:

$$\alpha \frac{d}{d\bar{\tau}} \left(\frac{\mu'^3}{3} - \mu'\right) = \mu'_o - \mu' \qquad (69)$$

which is esentially equation (40) corresponding to the case in which the second derivative of μ with respect to time is negligible. Then, the characteristic time is long; $\bar{\tau}$ is of order α. In branches BC and DA on the other hand "$\bar{q}$" remains approximately constant, equal to $\pm 2/3$, then, the characteristic time is short of order $\bar{\tau} \sim 1/\alpha$, for the variation of μ, given now by equation (67).

In figure 3b the phase plane is presented for the case in which μ'_o is in the stable region.

This method for α large fails when $|\mu'_o|$ is sufficiently close to 1; then the limit cycle would be given by equations (64) and (65). Figure 4 presents the results of the numerical integration of equation (63); giving the maximum and minimum value of μ' in the limit cycle for different values of μ'_o and α

6.2 Case for which the instability region is finite.

To extend this analysis to the case when the instability range has a thickness of order unity, the method presented on section 6.1 for α large will be used. Assuming equation (56) to be valid and rewritting it in a form similar to that of equations (66) and (67), the following relations are obtained

$$\frac{dq}{d\tau} = -\mu \ln \frac{\mu}{\pi} \tag{70}$$

$$q = -\varepsilon \frac{1}{\mu} \frac{d(\theta_s/\mu^2)}{d\mu} \frac{d\mu}{d\tau} + \frac{\theta_s}{\mu} \tag{71}$$

From equations (55) and (70) it can be seen that "q" represents the total heat content of the solid. As long as the time τ is of order unity, the first term of the right hand side is small and, $q=\theta_s/\mu$, as it happened for the quasisteady solution (equation (39)). However, if during the limit cycle there is a period in which $\tau \sim \varepsilon$ (that is $\tau_1 \sim 1$, the characteristic time for heat conduction in the solid, see equations (19) and (30)), then the total heat content of the solid, q, would be given by the complete equation (71). Nevertheless, equation (55) expresses that the characteristic time to transfer across the solid surface the heat needed to change the total heat content of the solid is of order τ. In order words if equation (70) is rewritten with the characteristic time τ_1 it becomes:

$$\frac{dq}{d\tau_1} = -\varepsilon\mu \ln \frac{\mu}{\pi} \tag{70'}$$

that means that q remains almost constant during periods of order τ_1; during this period there is no time for transfer of the heat needed to change the total heat content of the solid.

Figure (5) shows schematically θ_s/μ as a function of μ, and by analogy with figure 3a one may assume that the limit cycle is given by ABCD. In branch AB the process is quasi-steady and corresponds to making ε equal to zero in equation (71), or using Eqs.(39) or (40). In branch BC q is constant, and the characteristic time is τ_1, μ changes very rapidly and there is no time for the heat content of the solid to change, this process continues until we are well in the stable region in the point where μ has the value that makes under steady conditions $q_c=q_B$. Figure 5b shows schematically the way in which μ will change as a function of time.

The previous qualitative results are independent of the pyrolisis law. We are now going to calculate the maximum and minimum values of μ (μ_A, μ_C) for the pyrolisis law expressed by equation (48).

For point A of figure 5:

$$\frac{(1-\sqrt{1-2\omega})(1+\omega\ln\mu)}{(1+\sqrt{1-2\omega})(1-\omega\ln\mu)} \exp\left(\frac{\sqrt{1-2\omega}}{\omega} - \ln\mu\right) = 1 \qquad (72)$$

For point C it can be shown that

$$\ln\mu_A = -\ln\mu_C$$

The solution of eq.(72) depends only on parameter ω and is presented in figure 6.

For ω close to 0.5

$$\omega \ln \mu_A = -2\sqrt{1-2\omega}$$

that corresponds to the solution of section 6.1 for α large. The other limiting value of ω corresponds to a large value of the pyrolisis activation energy, then ω tends to zero, and $\ln \mu_A \to -1/\omega$:

$$\omega \ln \mu_A = -1 + \frac{4\omega}{(1-\sqrt{1-2\omega})^2} \exp\left(-\frac{2}{1-\sqrt{1-2\omega}}\right) \qquad (73)$$

This means that for ω small

$$\ln \mu \to \pm 1/\omega$$

and from equation (48):

$$\theta_s \to 0, \infty$$

Obviously, then, the pyrolisis law is probably not correct, however, this is an indication that the solid is igniting and extinguishing alternatively in its oscillations.

In Eq.(72) is probably not well justified to neglect terms such as $0(\varepsilon^2 \frac{d^3\mu}{d\tau^3})$ of Eq.(56), that will originate terms of order $\varepsilon^2 \frac{d^2\mu}{d\tau^2}$ in the right hand side of eq(71). This is

justified in the analysis of section 6.1 when the unstability region is thin. Then, the fast transition regions corresponding to branches BC and DA of figure 5 will not be appropiately described by Eq.(72) with q constant. A different approach will be needed that will probably consist in calculating more accurately the total heat content of the solid when μ and θ_s change rapidly (subject to the pyrolisis law, eq.(48)) and impose the condition that this heat content does not change in branches BC and DA.

7. Response of a burning solid to pressure variations

In a previous report, ref (16), corresponding to this grant, an analysis was presented of the response of a solid to an imposed pressure variation; the evolution with time of the burning rate was calculated by using Eq.(40) with the non-dimensional pressure, π, being a known function of time defined by Eq.(28). Here we shall only reproduce the main conclusions obtained in that report, ref. (16).

It was shown in section 4 that the characteristic response time of the solid is

$$t_c = \frac{\lambda\rho}{m_o^2 C_f} \frac{E_g(T_{so}-T_\infty)}{2RT_{fo}^2} \qquad (74)$$

Therefore, if the characteristic time of pressure variation is small compared with it, the pressure variation is a step function. If the time of pressure variation is long compared to the characteristic time shown in Eq.(74), the response of the solid may be considered as a sequence of stationary states.

In the present analysis only depressurization problems will be considered, so that π will decrease from some initial value π_I at time 0, to its final value π_F. In addition it is assumed that the initial solution is stable:

$$\text{either } \pi_I > \pi_1 \quad \text{where} \quad \pi_1 = \exp\left(\frac{1}{\omega}\sqrt{1-2\omega}\right) \tag{75}$$

$$\text{or} \quad \pi_I < \pi_2 \quad \text{where} \quad \pi_2 = \exp\left(-\frac{1}{\omega}\sqrt{1-2\omega}\right) \tag{76}$$

where π_1 and π_2 are the values of the pressure that limit the unstable region, Eq.(47).

Two possible types of evolutions are possible a) and b):

a) The unstable region is not crossed during depressurization. This is the case when:

a1) The unstable region does not exist

$$4RT_\infty > E$$

a2) The initial and final values of pressure are both above the unstable region.

$$\pi_I > \pi_F > \pi_1$$

a3) Both values of pressure are below the unstable region

$$\pi_2 > \pi_I > \pi_F$$

In this case a) the evolution of the burning rate, μ, given by Eq.(40) is smooth and it tends monotonically to its final steady value $\pi_F = \mu_F$.

b) The unstable region is crossed during depressurization. This is the case when

b1) $\pi_I > \pi_1 > \pi_2 > \pi_F$ the final steady state is stable.

b2) $\pi_I > \pi_1 > \pi_F > \pi_2$ the final steady state is unstable.

In this case b) Eq.(40) has a singularity at $\mu = \pi_1$ and at a time such that $\pi(\tau) = \pi_1$. The character of the singularity depends on the value of the shape of the pressure time function curve when the instability region is entered at $\pi=\pi_1$. When

$$\left|\left(\frac{d\pi}{d\tau}\right)_{\pi=\pi_1}\right| < \frac{1}{2\omega}\,\frac{(1-\sqrt{1-2\omega})^2\,\exp\left(\frac{3}{\omega}\sqrt{1-2\omega}\right)}{\sqrt{1-2\omega}} \qquad (77)$$

this singularity has a node-like character and Eq.(40) is able to give a transition across the singular point. On the other hand when condition (77) is not satisfied, the singularity has a spiral character and the solution ceases to be valid when μ approaches π_1, since the variation of burning rate with time is so rapid, that is no longer valid to assume that the derivative term is only a perturbation of the quasisteady solution. A different type of analysis, (may be similar to that presented in section 6 of this report), will be required to study the behaviour of the solution near $\mu=\pi_1$. However, since μ is decreasing very rapidly to zero, the condition oposite to (77) may be identified as a criterion for dynamic extinction caused

by a rapid depressurization process.

When condition b1 is satisfied there is another singular point at $\mu=\pi_2$ that is always a saddle through which the solution will exit from the unstable region.

Any perturbation occurring while the value of μ is in the unstable region will grow with time leading to the non-linear oscillations studied in section 6. It is possible that this study on depresurization is not appropiately described by Eq.(40), and should be remade taking into account higher order unsteady effects similar to those retained in section 6 when studying non-linear oscillations.

8. Conclusions

An asymptotic analysis has been developed of the unsteady burning of a solid propellant in the limit of high activation energy of the gas phase reaction. The analysis shows that the ratio of the characteristic response time of the solid, and the characteristic residence time in the heat-up zone is of the order of the nondimensional activation energy of the gas-phase reaction. This slow response of the solid allows to suppose that the temperature distribution is in a first approximation quasisteady, and by integrating the condensed phase energy equation, an equation is derived whose solution yields the evolution with time of the burning rate.

It is found that for values of the activation energy of the pyrolisis sufficiently large:

$$E > 4RT_\infty$$

there are, for other parameters fixed, a range of non-dimensional pressures, eq.(28), such that

$$-\frac{1}{\omega}\sqrt{1-2\omega} < \ln\pi < \frac{1}{\omega}\sqrt{1-2\omega}\ , \quad \omega = \frac{2RT_\infty}{E}$$

,in it,steady burning will be unstable. This criterion corresponds to that of Denison y Baum[5] in the limit of high activation energy of the gas phase reaction.

The evolution with time of the burning rate for a fixed pressure corresponding to instability has been investigated; the non-linear equation obtained from the quasisteady temperature distribution is not appropiate to study this evolution because it gives derivatives of the burning rate with respect to time that are infinite when the unstable region is crossed. This suggests that during part of the evolution of the burning rate the appropiate time scale should be shorter. Higher order terms are then retained when calculating the temperature distribution in the solid and a new equation for the burning rate evolution is obtained; this new equation contains second order derivatives of the burning rate which g nerate oscillations of the burning rate with time. This new equation is first solved in the limit that the instability interval is small; and it is found that when the width of this interval is of the order of fourth root of the inverse of the non-dimensional activation energy of the gas-phase, there is a significative limit in which all the retained terms are of the same order and the expansion procedure is justified. Then, instability leads to oscillations that correspond to a limit cycle that is described by an equation of the type of Van der Pool equation slightly modified. When the width of the instability region is much smaller than the

above-mentioned limit, the oscillations have a sinusoidal character, and when it is much larger, two different time intervals appear during the oscillations period, one interval is large and the other short, corresponding this last one to a sudden jump in the burning rate. Based on this last result, the limit cycle is investigated for the case that width of the instability region is finite and the corresponding oscillations are of the form shown schematically in fig. 5b. During part of the oscillation period the characteristic time is of the order of the characteristic response time of the solid and much larger than the characteristic time for heat transfer, the temperature profile in the solid is quasisteady, and the burning rate changes slowly until it reaches the instability region; then, the characteristic time becomes much shorter, of the order of the characteristic time for heat transfer, and the temperature distribution in the solid is no longer quasisteady. However, during this short time, which is much shorter than the characteristic response time of the solid, there is no time for changing the total heat content of the solid. In the intervals BC and DA of fig. 5a the burning rate changes very rapidly, maintining the total heat content of the solid, and goes across the unsteady region until it is found in the oposite stable region a burning rate that would correspond to a quasisteady temperature distribution in the solid of the same total heat content.

The expansion procedure indicated in this work is not appropiate to describe the structure of the fast transition regions BC and DA of fig. 5a (for a finite width of the instability region), because in this regions the problem in the solid is esentially unsteady, and any method that would

consider the time derivatives of the temperature as a perturbation in the energy equation of the solid, would not be valid. The appropiate method would consist in solving this energy equation subject to the condition that the total heat content of the solid is constant, when the burning rate is changing.

The analysis presented in this work makes use of a pyrolisis law of the form given by eq.(9)(or eq.(48), in non-dimensional variables). However, the method only uses this law for specific applications and is in principle valid for any other law that would relate the burning rate with the surface temperature. Instability, as shown in fig. 5a would correspond to an interval of burning rates in which the total heat content of the solid, under steady conditions, would decrease as the burning rate increases.

There are other possible mechanisms, besides the one examined here, that would prevent the growth of the oscillations such as three dimensional effects.

The analysis has also been applied to study the response of a burning solid subjected to a depressurization process, and assuming that the steady solution at the initial pressure is stable. For this analysis it was used Eq.(40) that corresponds to quasisteady temperature distribution in the solid.

It is found that for depressurization processes such that the initial, intermediate, and final pressure, would all correspond to steady stable burning, the variation of burning rate with time is smooth and monotonous, even for step-like changes in pressure, although in this last case the burning rate does not adjust instantaneously to the new pressure, and approaches

it in a long time. The absence of dynamic extinction appears to coincide with the suggestion of T'ien[13], who considers that heat losses is the mechanism responsible for dynamic extinction. Nevertheless, our analysis seems to indicate that there is a condition that may be identified with the onset of dynamic extinction if during the process of depressurization the unstable region is crossed. A mathematical singularity occurs when, during the depressurization process, the instability region is entered; and the character of this singularity depends on the slope of imposed pressure-time curve at the singularity. When this slope is larger than a certain critical value defined in Eq. (77), the singularity has a spiral-like character that originates derivatives of the burning rate with respect to time that are infinite. This, and the fact that any small perturbation will be amplified in the instability region, indicate the need of retaining in some way (which may be similar to the one used for studying non-linear oscillations) unsteady effects in the temperature distribution within the solid when, during the depressurization process, the instability region is crossed.

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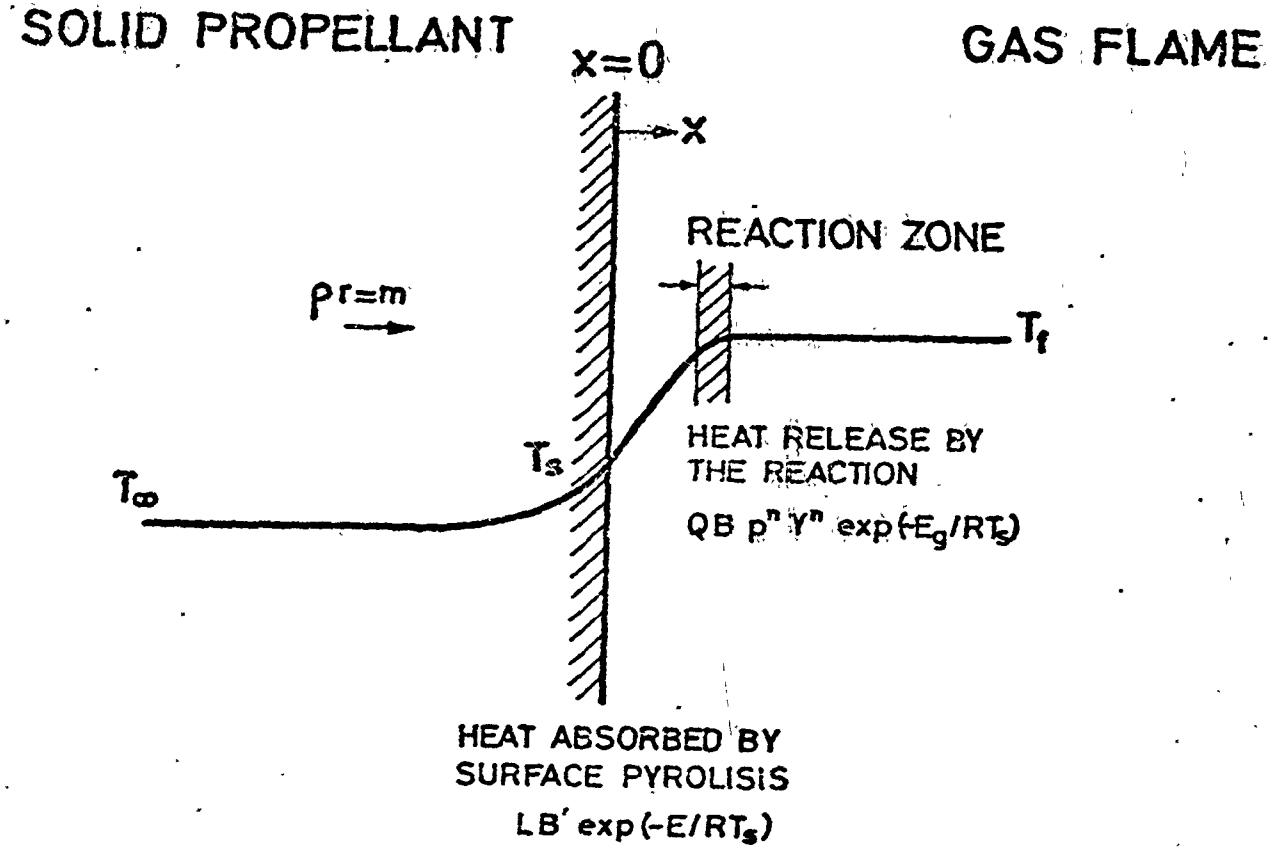
LIST OF FIGURES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T_f
T_s
T_∞
HEAT RELEASE BY
THE REACTION
$QB\, p^n Y^n \exp(-E_g/RT_s)$
HEAT ABSORBED BY
SURFACE PYROLISIS
$LB' \exp(-E/RT_s)$

Figure 1

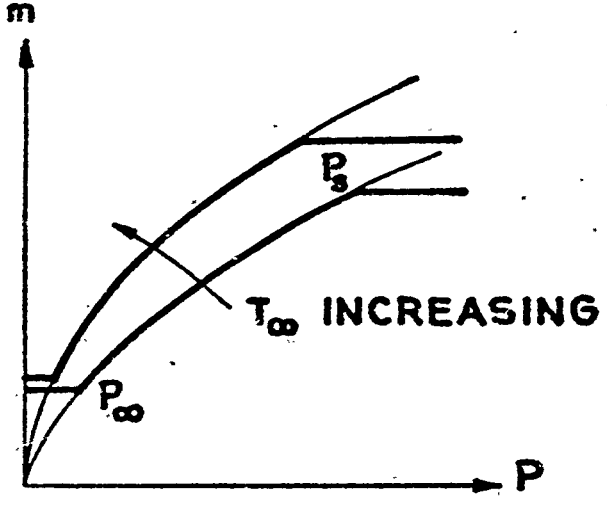

Figure 2

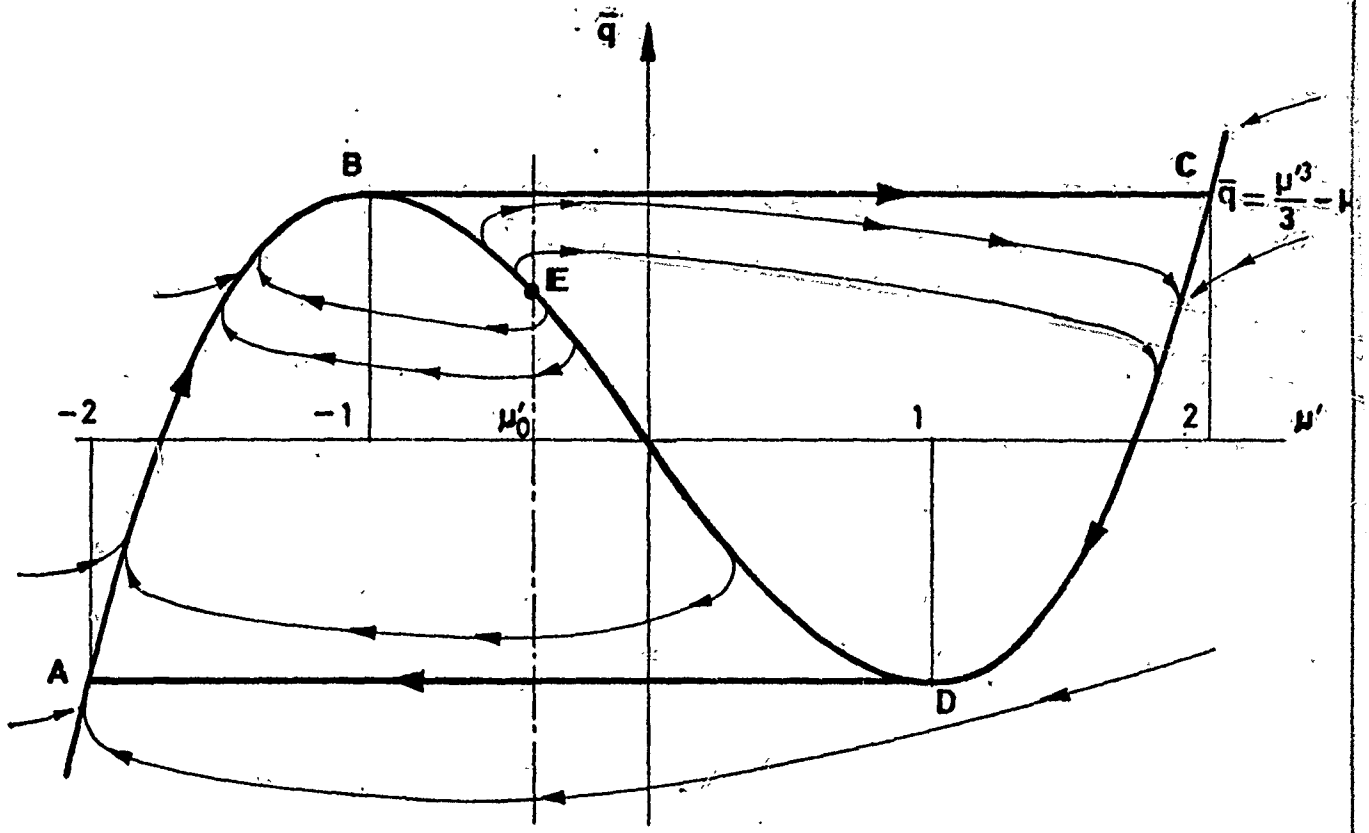

Figure 3a

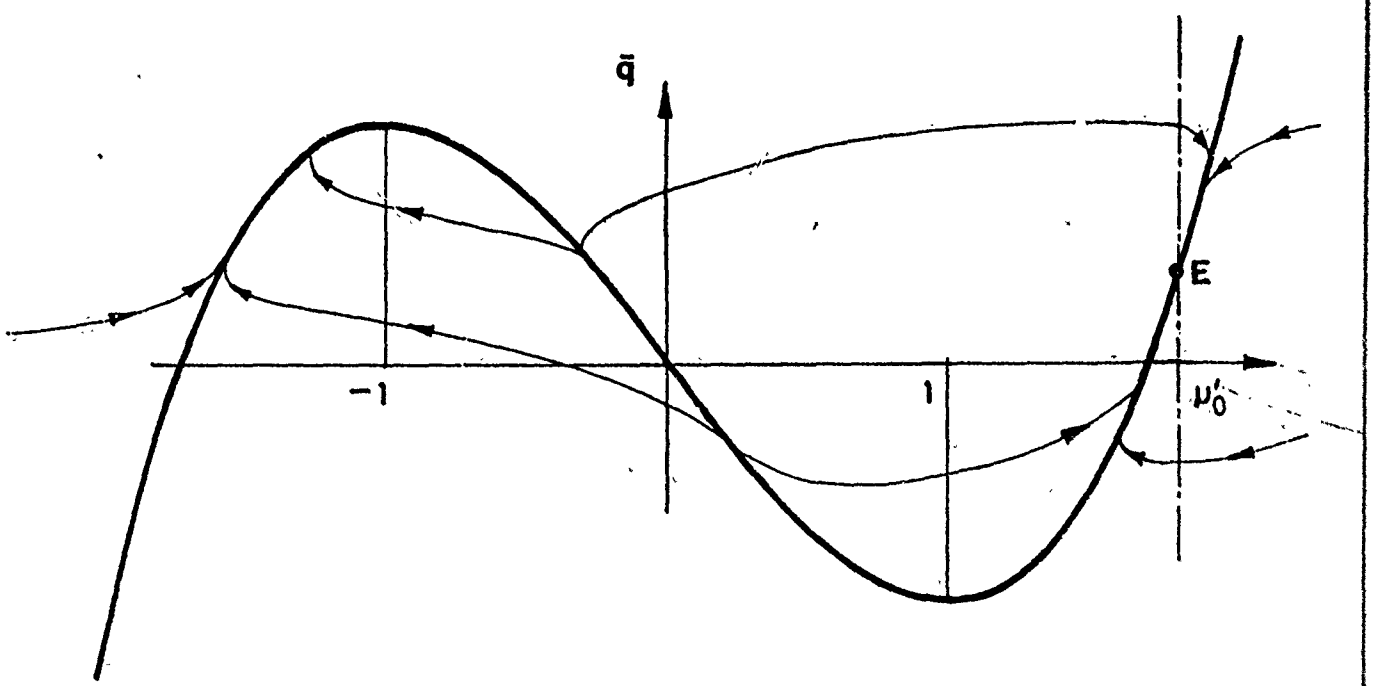

Figure 3b

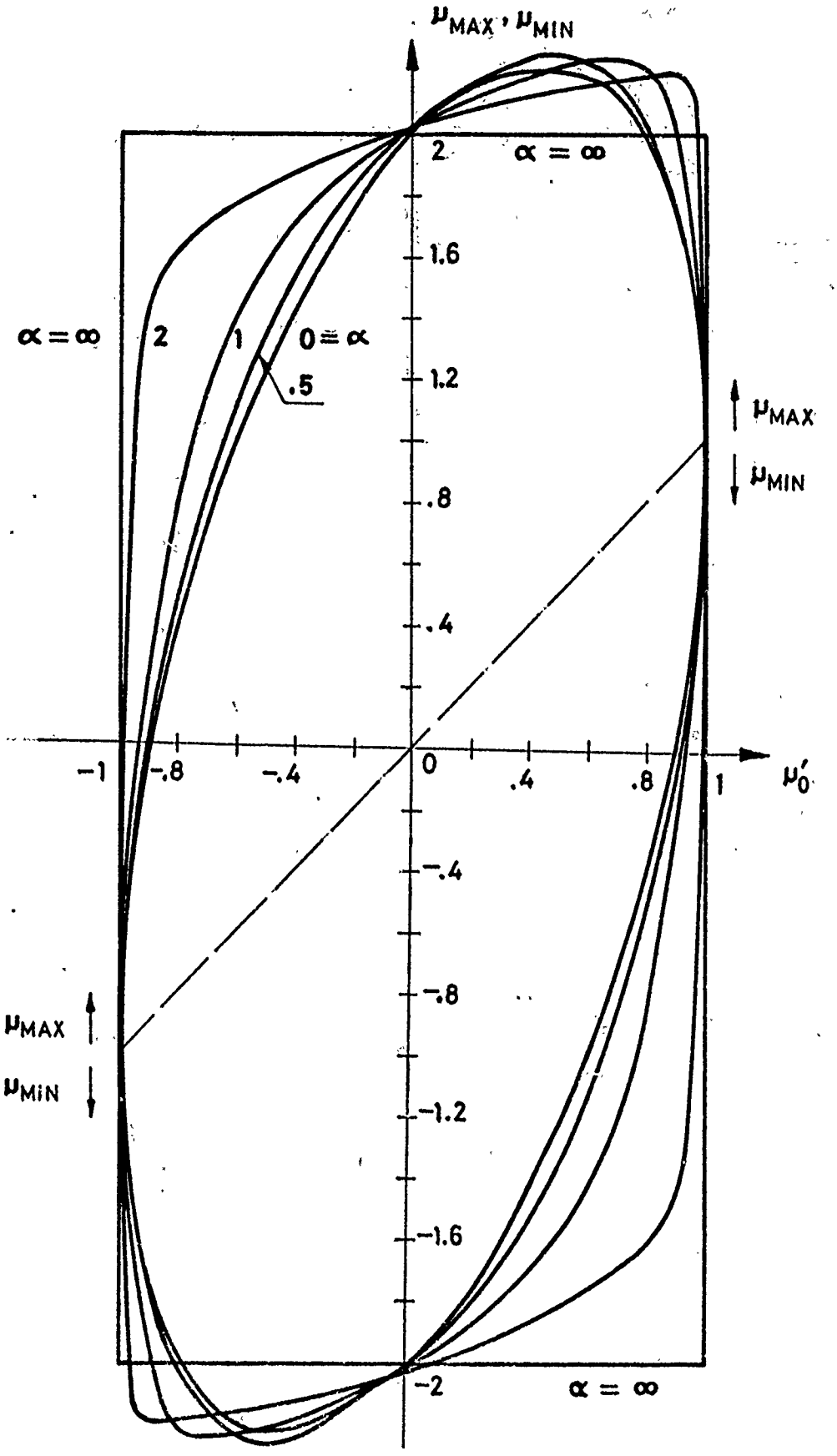
μ_{MAX} , μ_{MIN}
2
$\alpha = \infty$
1.6
$\alpha = \infty$
2
1
$0 \equiv \alpha$
.5
1.2
μ_{MAX}
μ_{MIN}
.8
.4
−1
−.8
−.4
0
.4
.8
1
μ_0'
−.4
−.8
μ_{MAX}
μ_{MIN}
−1.2
−1.6
−2
$\alpha = \infty$

Figure 4

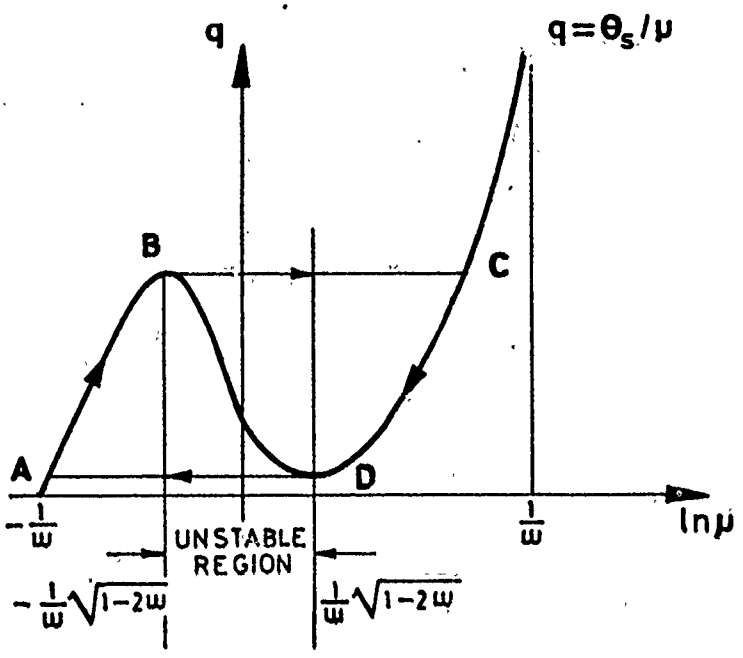

Figure 5a

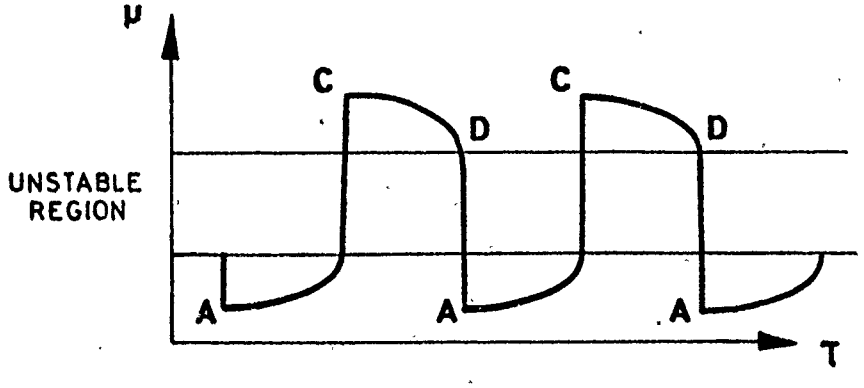

Figure 5b

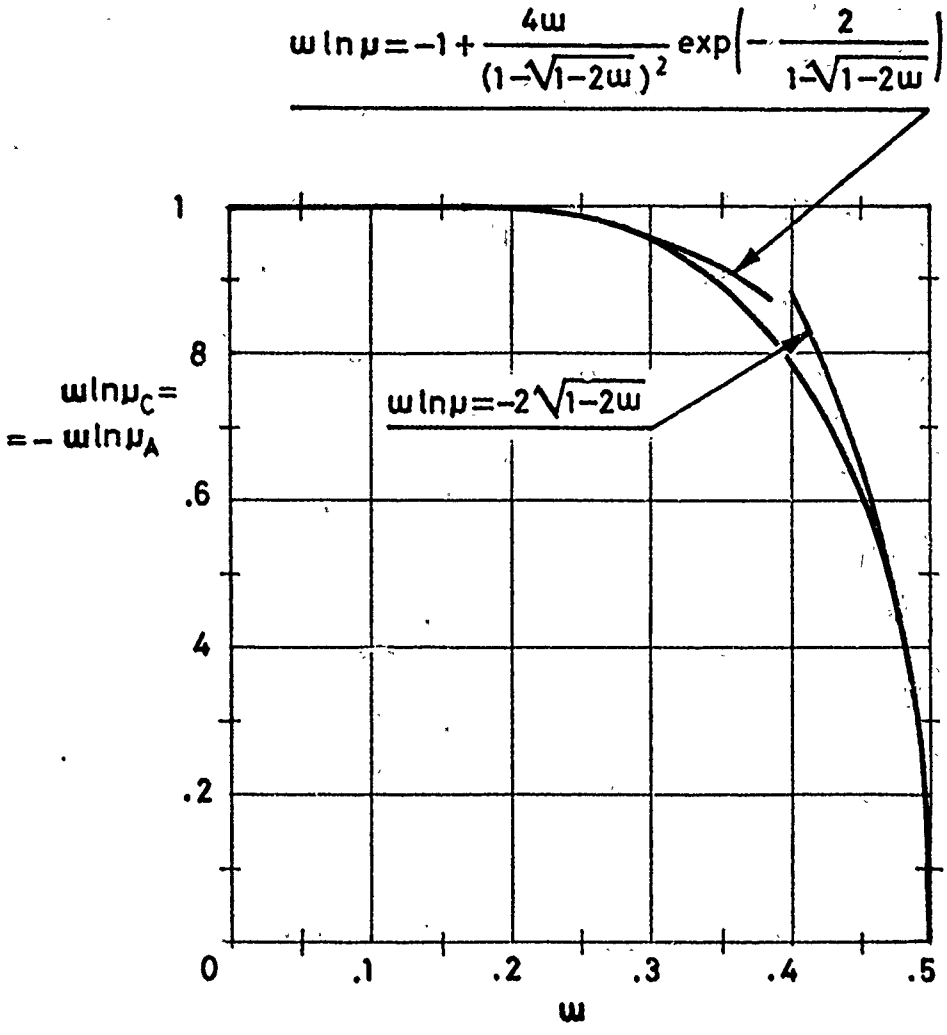
$w \ln \mu = -1 + \frac{4w}{(1-\sqrt{1-2w})^2} \exp\left(-\frac{2}{1-\sqrt{1-2w}}\right)$
$w \ln \mu = -2\sqrt{1-2w}$
$w \ln \mu_C = -w \ln \mu_A$
1
.8
.6
.4
.2
0
.1
.2
.3
.4
.5
w

Figure 6