1.0

1.1

1.25 1.4 1.6

2.8 2.5

3.2 2.2

3.6

4.0 2.0

1.8

MICROCOPY RESOLUTION TEST CHART

NATIONAL BUREAU OF STANDARDS 1963-A

FOR FURTHER TRAN

AFOSR-TR- 78 - 1115

AD A055941

AD No.
DDC FILE COPY

JUN 30 1978

NIELSEN ENGINEERING
AND RESEARCH, INC.

78 06 27 057

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFOSR-TR- 78-1115	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) MODIFICATIONS AND IMPROVEMENTS IN A STRUCTURAL OPTIMIZATION SCHEME BASED ON AN OPTIMALITY CRITERION		5. TYPE OF REPORT & PERIOD COVERED FINAL rept. 1 Mar 77 - 28 Feb 78
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) S. C. MCINTOSH, JR		8. CONTRACT OR GRANT NUMBER(s) F49620-77-C-0055
9. PERFORMING ORGANIZATION NAME AND ADDRESS NIELSEN ENGINEERING & RESEARCH, INC 510 CLYDE AVE MOUNTAIN VIEW, CA 94043		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 2307B1 61102F
11. CONTROLLING OFFICE NAME AND ADDRESS AIR FORCE OFFICE OF SCIENTIFIC RESEARCH/NA BLDG 410 BOLLING AIR FORCE BASE, D C 20332		12. REPORT DATE June 1978
		13. NUMBER OF PAGES 78
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) 80 p.		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

OPTIMIZATION
STRUCTURAL DESIGN
AEROELASTICITY

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Three optimization algorithms based on optimality criteria are compared with an algorithm based on mathematical programming. Two of the optimality-criterion algorithms are adapted from previous work and the third is new. The example problem used is a simple 12-design-variable rectangular wing with three separate equality behavioral constraints: a fixed deflection at the tip under a transverse load, a fixed fundamental free-vibration frequency, and a fixed subsonic flutter speed. Minimum-gage constraints were also imposed. All of the algorithms gave virtually identical optimal designs. The optimality-criterion algorithms were → next page
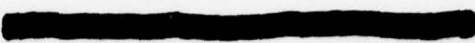
DD FORM 1473
1 JAN 73

389 783

generally more efficient than the mathematical-programming algorithm, although comparisons with more complicated problems are needed to establish clearly the degree of superiority. The problems currently being considered are discussed, and the research plan for the next year is briefly described.

MODIFICATIONS AND IMPROVEMENTS IN A
STRUCTURAL OPTIMIZATION SCHEME BASED
ON AN OPTIMALITY CRITERION

1 March 1977 to 28 February 1978
Contract F49620-77-C-0055
1 June 1978

Prepared by: _S. C. McIntosh, Jr._
S. C. McIntosh, Jr.
Principal Investigator

Approved by: _S. B. Spangler_
S. B. Spangler
Vice President

78 06 27 057

TABLE OF CONTENTS

NOMENCLATURE

A	set of active design-variable numbers
b	reference length (ie, wing semichord)
$[B]$	system matrix
c	constraint value
C_i	redesign multiplier
e_1, e_2	exponents in "energy-density" redesign algorithm
$(e_v)_i$	"energy-density" for ith design variable
e_{av}	average "energy-density"
$\{F\}$	nodal forces
$\{F^{(r)}\}$	dummy-load nodal forces
g	damping parameter
$[GA], [GK], [GM]$	generalized aerodynamic, stiffness, and mass matrices, respectively
$[GK_i], [GM_i]$	derivatives of $[GK]$ and $[GM]$ with respect to ith design variable
j	imaginary unit, $(-1)^{1/2}$
J	merit function
k	reduced frequency, $\omega b / U_\infty$
K	mass-reduction factor in Segenreich algorithm
$[K]$	discrete stiffness matrix
$[K_o]$	invariant portion of $[K]$
$[K_i]$	derivative of $[K]$ with respect to ith design variable
$\bar{m}$	total mass
m_o	invariant portion of total mass
m	varying portion of total mass
m_i	derivative of m with respect to ith design variable
M_∞	Mach number

NOMENCLATURE (CONCLUDED)

$[M]$	discrete mass matrix
$[M_o]$	invariant portion of $[M]$
$[M_i]$	derivative of $[M]$ with respect to ith design variable
N	number of design variables
NA	number of active design variables
$\{p\}$	multipliers
$\{\bar{p}\}$	multipliers; adjoint eigenvector
$\{q\}$	coordinates, generalized or nodal
$\{\bar{q}\}$	generalized coordinates
$\{q^{(r)}\}$	eigenvector for rth mode of free vibration
t_i	design variable
u_r	displacement to be constrained
$\{u\}$	displacements resulting from loads $\{F\}$
$\{u^{(r)}\}$	displacements resulting from loads $\{F^{(r)}\}$
U_∞	speed
α	parameter in Kiusalaas recursion relation
$\alpha^{(0)}, \alpha_x$	parameters in Rizzi algorithm
ε	convergence parameter
λ	multiplier
ν	iteration number
ω	frequency
ω_r	frequency to be constrained
Ω	eigenvalue
$\bar{\Omega}$	complex eigenvalue, $(1 + jg)/\omega^2$
$(\ )_{des}$	desired value

MODIFICATIONS AND IMPROVEMENTS IN A
STRUCTURAL OPTIMIZATION SCHEME BASED
ON AN OPTIMALITY CRITERION

1. INTRODUCTION

While recent years have seen continuing increases in the variety
and scope of applications of structural optimization technology within
the aerospace industry, there are still some significant problems to
be overcome before this technology can be routinely applied wherever
it is needed in all stages of aerospace vehicle design. Among these
problems are the need for efficient treatment of large numbers of
design variables (in the thousands, for example) with many different
constraints. Indeed, it may well be that the number of different
design requirements with which industry is faced is growing more
rapidly than the number of constraints that is being incorporated in
the most recent structural optimization schemes.

In an effort to aid in the resolution of these problems, Nielsen
Engineering & Research, Inc. (NEAR) has undertaken a research program
under the sponsorship (Contract F49620-77-C-0055) of the Air Force
Office of Scientific Research (AFOSR). Of specific interest is the
extension of an optimality-criterion algorithm (refs. 1-3) to large
problems involving multiple constraints, where the constraints include
both strength and stiffness requirements. Before proceeding with this
algorithm, however, it was considered advisable to conduct some effi-
ciency comparisons in order to identify other algorithms with equal
or perhaps superior qualities. Other tasks that were to be undertaken
during the first year involved a study of the accuracy of computing
flutter-speed or flutter-eigenvalue gradients with fixed modes defining
generalized coordinates and possibly the incorporation of multiple
equality behavioral constraints. The sections that follow describe
the first year's accomplishments in more detail, the conclusions to be
drawn from this work, and the work planned for the next year. One
archive publication, covering a portion of the year's accomplishments,
will appear in 1978. A preprint of this paper is appended.

2. ALGORITHMS BASED ON OPTIMALITY CRITERIA

2.1 Optimality Criteria Versus Mathematical Programming

Optimization algorithms are more or less arbitrarily classed according to the philosophy underlying their derivation. Mathematical-programming (MP) algorithms employ a relatively sophisticated set of calculations at each design point in order to produce a design change that will simultaneously improve the merit function (i.e., reduce the total weight) without violating any of the constraints. Optimality-criterion (OC) algorithms are derived, generally heuristically, from conditions that must be satisfied at the optimum design. MP algorithms can generally be proven to converge, while OC algorithms cannot. On the other hand, OC algorithms are less involved than MP algorithms from a computational standpoint. However, the fundamental reason for interest in OC algorithms is to be found in their potential for application to very large problems, based originally on the success of the stress-ratio or fully-stressed-design algorithm in treating stress constraints on members numbering in the thousands.

2.2 Optimality Criterion and Recursion Relation for an Equality Behavioral Constraint

OC algorithms come in many forms. To fix ideas, let us consider a single behavioral constraint on a structure to be optimized. The structure is characterized by N design variables t_i, and its mass $\bar{m}$ to be minimized is assumed to be a linear function of these variables:

$$\bar{m}(t_i) = m_o + m(t_i) = m_o + \sum_{i=1}^{N} m_i t_i \tag{1}$$

The assumption of linearity is not necessary, but it covers a broad class of finite-element models where the design variables are plate thicknesses, bar areas, or the like. In addition, there is some mass m_o that is not available for optimization, which may represent fasteners, lightly loaded structure, fuel, etc. The behavioral constraint is written simply as

$$c(t_i) = 0 \tag{2}$$

and there may also be minimum-gage constraints of the form

$$t_i \geq (t_i)_{min}, \quad i = 1, 2, \ldots, N \tag{3}$$

One simple form of an optimality criterion can be obtained from variations of the merit function

$$J = m + \lambda c \tag{4}$$

In particular, the requirement of vanishing variations of J with respect to the active design variables leads to

$$\frac{1}{m_i} \frac{\partial c}{\partial t_i} = -\frac{1}{\lambda}, \quad \forall \, i \, \epsilon \, A \tag{5}$$

Here A denotes the set of design variables that are active--i.e., not at their minimum values--at the optimum. A number of recursion relations have been introduced for iteratively resizing the structure in order to arrive at a design that satisfies equation (5). (A very revealing discussion of the relationships among many of these relations may be found in ref. 4.) One that has been widely used is due to Kiusalaas (ref. 5):

$$t_i^{\nu+1} = C_i^\nu \, t_i^\nu \tag{6}$$

where

$$C_i^\nu = \alpha^\nu - (1 - \alpha^\nu) \lambda^\nu \left(\frac{1}{m_i} \frac{\partial c}{\partial t_i} \right) \tag{7}$$

Here ν is the iteration number, and α^ν is a parameter that ranges in value from -1 to $+1$. It may or may not vary with each iteration. Note that as the optimum is approached, $\lambda^\nu \left(\frac{1}{m_i} \frac{\partial c}{\partial t_i} \right)^\nu \to -1$, and $C_i^\nu \to 1$ no matter what value α^ν takes.

2.3 Two Resizing Algorithms

There are in turn a number of ways of devising resizing algorithms based on equations 6 and 7. One procedure, from references 1-3, determines α^ν so that a given reduction in mass is achieved, while the

multiplier λ^ν is determined from the requirement that the constraint value c^ν be always driven to zero. To first order, the change in the constraint value is given by

$$c^{\nu+1} - c^\nu = \sum_{i=1}^{N} \left(\frac{\partial c}{\partial t_i}\right)^\nu \left(t_i^{\nu+1} - t_i^\nu\right) \tag{8}$$

With $\Delta m^\nu = \sum_{i=1}^{N} m_i (t_i^{\nu+1} - t_i^\nu)$, equations (5)-(8) can be manipulated to yield, for $c^{\nu+1} = 0$,

$$\alpha^\nu = 1 + \frac{\Delta m^\nu + c^\nu \beta_1^\nu / \beta_2^\nu}{m^\nu - (\beta_1^\nu)^2 / \beta_2^\nu} \tag{9}$$

$$\lambda^\nu = -\frac{\beta_1^\nu + c^\nu/(\alpha^\nu - 1)}{\beta_2^\nu} \tag{10}$$

where $\beta_1^\nu = \sum_{i=1}^{N} \left(\frac{\partial c}{\partial t_i} t_i\right)^\nu$, and $\beta_2^\nu = \sum_{i=1}^{N} \frac{1}{m_i} \left[\left(\frac{\partial c}{\partial t_i}\right)^\nu\right]^2 t_i^\nu$. The mass change Δm^ν is specified as

$$\Delta m^\nu = -Km^\nu \tag{11}$$

where K is chosen from a table of user-selected values arranged in descending order. The largest value is chosen first, and α^ν, λ^ν, and the c_i^ν are calculated for the active set A. If any active design variable is changed by more than 25%, the next lower value of K is chosen, and the process is repeated until no design-variable change exceeds the 25% limit. Next, the new values of the active design variables are compared to their minimum gages. If $t_i^{\nu+1} < (t_i)_{min}$, the next lower value for K is selected, and the redesign process is repeated. If $t_i^{\nu+1} < (t_i)_{min}$ for the smallest value of K, then for that design variable $t_i^{\nu+1} = (t_i)_{min}$, and it is relegated to the passive set. A final pass through the redesign step is now required,

since $\Delta t_i = (t_i)_{min} - t_i^\nu$, and equations (9) and (10) must be altered accordingly to reflect revised expressions for $c^{\nu+1} - c^\nu$ and Δm^ν (see reference 1 for details). If the active-passive identities are unchanged, then a new iteration is begun. Convergence is checked in two ways. First, if K is at its minimum value and an active design variable is calculated to change by more than 25%, the design is declared final, subject to a check on the active-passive identities. This test is essentially a "diminishing returns" criterion, since a mass reduction of a given amount requires larger and larger changes in the design variables as the optimum is approached. The second test involves the proximity of the redesign factors c_i^ν to unity:

$$|c_i^\nu - 1| < \epsilon \tag{12}$$

Here ϵ is a user-supplied convergence parameter. If this test is satisfied, the design is declared final, again subject to a check on the active-passive identities. This check is to ensure that there are no passive design variables that should be reintroduced into the active set. It is based on the Kuhn-Tucker optimality conditions and is described in detail in references 1 and 2. A flow diagram for this algorithm is given in Appendix A.

A variant on the above algorithm involves simply scheduling α, rather than computing it for a scheduled set of mass reductions (refs. 6 and 7). Also, the minimum-gage constraints are handled differently. At each iteration following the one where a design variable reaches its minimum value, the factor c_i^ν for this variable is still evaluated. If $c_i^\nu > 1$, the variable is reintroduced immediately into the active set. (Here also special steps must be taken in the calculation of λ^ν to account for design-variable changes associated with entry to or exit from the passive set. Details are in references 6 and 7.) In this algorithm, then, active-passive identities are continually checked, but there is very little computation associated with determining α^ν. This parameter is scheduled by the following formula, which replaces equation (9):

$$\alpha^\nu = \alpha^{\nu-1}\alpha_x = \alpha^{(0)}(\alpha_x)^{\nu-1} \tag{13}$$

Typically, $\alpha^{(0)}$ will be chosen to be near unity - say, 0.9. Then at each iteration the current value of α is obtained by multiplying the previous value by a factor α_x, which is slightly less than unity - say, 0.95. The reduction of α at each step is equivalent to increasing step sizes as the optimum is approached, in order to accelerate convergence. Convergence is evaluated by checking the uniformity of the weighted derivatives, according to equation (5):

$$\left| 1 + \lambda^\nu \left(\frac{1}{m_i} \frac{\partial c}{\partial t_i} \right)^\nu \right| < \varepsilon, \quad \forall\, i \,\varepsilon\, A \tag{14}$$

The convergence parameter ε is user-supplied. A flow diagram for this algorithm is also given in Appendix A.

2.4 An Alternate Approach to an Optimality Criterion

The form of the optimality criterion derived in subsection 2.2 is very general, and this generality is easily extended to multiple constraints (ref. 5). However, it will prove instructive to consider at least one other approach, which involves using the equations governing the structure for the particular constraint being considered. In an effort to retain some generality, let the equations be written as

$$[B(\Omega, t_i)]\{q\} = \{0\} \tag{15}$$

Here $\{q\}$ is a column of unknowns which may be discrete displacements or modal coordinates, and the coefficients of the equations are given in $[B]$, which is a function of the design variables t_i and possibly an eigenvalue Ω as well. In the case of an aeroelastic constraint, such as a fixed flutter speed, then equation (15) will be complex; for a constraint on a natural frequency, the equations will be real. The merit function J can now be written as

$$J = m + \text{Re}(\lfloor p \rfloor [B]\{q\}) \tag{16}$$

Here the scalar multiplier λ is replaced by a set of multipliers $\{p\}$, and the real part of the triple matrix product must be taken if

equations (15) are complex (ref. 8). At the optimum, the merit function J is stationary with respect to _independent_ variations of the t_i, {p}, and {q}. If the eigenvalue Ω is not fixed by the constraint, then variations of it must be considered also. Applying these conditions yields

$$[B]\{q\} = \lfloor 0 \rfloor \tag{17}$$

$$\lfloor p \rfloor [B] = \lfloor 0 \rfloor, \text{ or } [B]^T\{p\} = \{0\} \tag{18}$$

$$Re(\lfloor p \rfloor \left(\frac{\partial B}{\partial \Omega}\right)\{q\}) = 0 \tag{19}$$

$$m_i + Re(\lfloor p \rfloor \left(\frac{\partial B}{\partial t_i}\right)\{q\}) = 0 \tag{20}$$

Equation (20) is the optimality criterion. The term involving the triple matrix product resembles an energy density, so this could be referred to as an "energy-density" form of an optimality criterion. (Specific expressions for various types of constraints will be presented in Section 3 below.) Writing $(e_v)_i = \frac{1}{m_i} Re(\lfloor p \rfloor \left(\frac{\partial B}{\partial t_i}\right)\{q\})$ permits equation (20) to be recast as

$$(e_v)_i = -1, \quad \forall\ i\ \varepsilon\ A \tag{21}$$

As noted here, in the presence of side (e.g., minimum-gage) constraints only the indexes for the active design variables are to be considered.

2.5 "Energy-density" Recursion Relation and Redesign Algorithm

The number of recursion relations that can be devised to satisfy equation (21) is limited only by the designer's imagination. One choice is a variation of the recursion relations developed in references 9 and 10:

$$t_i^{\nu+1} = C_i^\nu\ t_i^\nu$$

where

$$C_i^\nu = \left| \frac{(e_v)_i^\nu}{(e_{av})^\nu} \right|^{e_1} (1 + c^\nu)^{e_2} \tag{22}$$

and

$$(e_{av})^{\nu} = \frac{1}{NA} \sum_{i \varepsilon A} (e_v)_i^{\nu} \tag{23}$$

This relation differs from those in references 9 and 10 in several important respects. In reference 9, which is concerned with altering a strength design to meet a flutter-speed constraint, only the strain-energy density is used to define the $(e_v)_i$, rather than the complete, or exact, expression represented by equation (20). Also, the denominator in the "energy-density" ratio is obtained by considering only those elements whose $(e_v)_i$ exceed the average (e_{av}). This has the effect of permitting only increases in design variables, so that a particular design variable that is too large for strength requirements in the redesigned structure cannot be reduced. This limitation is removed in reference 10, but "energy-densities" are defined as $(e_v)_i = \frac{1}{m_i} \left| \lfloor p \rfloor \frac{\partial B}{\partial t_i} \{q\} \right|$, which is also different from that required to satisfy equation (20). Hence neither of these two procedures is capable of converging to the "exact" optimum, whereas a redesign algorithm based on equations (6), (22), and (23) will do so.

In equation (23), the average $(e_{av})^{\nu}$ is determined by averaging the energy densities only for those design variables in the active set. (NA is the number of active design variables.) In equation (22), e_1 is typically less than unity (for example, 0.5) and e_2 is greater than unity (say, 2.0). Since the individual $(e_v)_i$ may differ in sign from e_{av}, the absolute value of the "energy-density" ratio is required to avoid numerical problems. This ratio also implies that a design variable should be decreased only if its "energy density" is less than the average. In the factor involving the current constraint value c^{ν}, $c^{\nu} > 0$ represents an infeasible value, requiring an increase in the design variables.

A redesign algorithm based on equations (6), (22), and (23) is quite simple. Minimum-gage constraints are treated by simply relegating a design variable to the passive set whenever $t_i^{\nu+1} \leq (t_i)_{min}$. At each step, all C_i^{ν} are calculated, so whenever a particular C_i^{ν} is greater than unity for a passive design variable, that variable is reintroduced into the active set and the redesign procedure is invoked

again. This approach to the identification of the active and passive sets is identical to that followed in the algorithm of references 6 and 7. Convergence is evaluated by equation (12). A flow diagram for this "energy-density" algorithm is given in Appendix A.

3. CONSTRAINT EVALUATIONS AND DERIVATIVE OR "ENERGY-DENSITY" CALCULATIONS

The algorithms discussed in Section 2 above were coded as separate routines that require constraint evaluations and calculations of the derivatives or the "energy-densities". In this section, specific forms for the constraints considered--displacements, natural frequencies, and flutter speeds--are presented. Also, the equivalence of the two forms of optimality criteria--equation (21) or equation (5)--is discussed.

3.1 Displacement Constraint

The displacements of a structure loaded by a set of forces {F} are found by solving

$$[K]\{u\} = \{F\} \tag{24}$$

where [K] is the discrete stiffness matrix, {F} is a column of nodal applied forces, and {u} is a column of nodal displacements. One of these displacements, u_r, is to be constrained, so that $c^\nu = \dfrac{u_r}{(u_r)_{des}} - 1$. In the derivative calculation, the dummy-load method is used. This involves calculating the displacements $\{u^{(r)}\}$ due to a dummy load set $\{F^{(r)}\}$, where $F_j^{(r)} = \delta_{jr}$:

$$[K]\{u^{(r)}\} = \{F^{(r)}\} \tag{25}$$

It is now assumed that the stiffness matrix [K] is a linear function of the design variables and can be written as

$$[K] = [K_o] + \sum_{i=1}^{N} t_i [K_i] \tag{26}$$

The justification for this assumption follows the same line of reasoning used in justifying equation (1). The constraint derivative can then be shown to be (see, for example, ref. 5)

$$\frac{\partial c}{\partial t_i} = \frac{1}{(u_r)_{des}} \frac{\partial u_r}{\partial t_i} = - \frac{1}{(u_r)_{des}} \lfloor u \rfloor [K_i] \{u^{(r)}\} \tag{27}$$

The first form of the optimality criterion, equation (5), becomes

$$\frac{1}{m_i (u_r)_{des}} \left[\lfloor u \rfloor [K_i] \{u^{(r)}\} \right] = \frac{1}{\lambda}, \quad \forall\ i\ \varepsilon\ A \tag{28}$$

While the form of the constraint equation in this case does not correspond to that used in developing the second form of the optimality criterion, equation (21), it is nevertheless possible to make use of the recursion relations, equations (6), (22), and (23), by replacing $(e_v)_i$ with $\frac{\partial c}{\partial t_i}$. This is reminiscent of the recursion relation finally chosen for FASTOP (ref. 11), in the sense that it can be viewed as a heuristically derived recursion relation based on satisfying the optimality criterion given by equation (5).

Constraint evaluation in this case is straightforward. The derivative matrices $[K_i]$ are invariant, as a result of the linearity assumption, so equation (26) is used to update the stiffness matrix at each step.

The displacement u_r is found by solving equations (24) for $\{u\}$. The dummy-load displacements $\{u^{(r)}\}$ are found by solving equations (25), and $\frac{\partial c}{\partial t_i}$ is calculated from equation (27). This information is then passed to any of the optimization subroutines.

3.2 Frequency Constraint

The equation of motion for free vibration can be written in discrete form as:

$$([K] - \omega^2 [M]) \{u\} = \{0\} \tag{29}$$

The mass matrix is also assumed to be a linear function of the design variables:

$$[M] = [M_o] + \sum_{i=1}^{N} t_i [M_i] \tag{30}$$

The order of [M] and [K] is typically large, and it is often more
efficient to rewrite equation (29) in terms of modal coordinates,
where a set of natural modes of the initial design is used to define
generalized coordinates for all subsequent designs. (Although this
could lead to inaccuracies if the optimal design is substantially
different from the initial one, periodic updating of the modes or
simply using a few more modes of the initial design should be
sufficient to avoid severe problems. This is an open question that
still needs to be answered.) With $[\phi]$ defined as the modal matrix,
modal coordinates are defined as $\{u\} = [\phi]\{q\}$, and equation (29)
becomes

$$([GK] - \omega^2 [GM])\{q\} = \{0\}, \tag{31}$$

with

$$\left.\begin{array}{l} [GK] = [\phi]^T[K][\phi] \\ [GM] = [\phi]^T[M][\phi] \end{array}\right\} \tag{32}$$

The linearity assumption embodied in equations (30) and (26) also
carries over to the generalized mass and stiffness matrices, so that
derivative generalized matrices $[GM_i]$ and $[GK_i]$ may be defined by
obvious analogy. With fixed modes defining generalized coordinates,
these are also invariant and of much smaller order than their discrete
counterparts.

If ω_r is the frequency to be fixed, and $(\omega)_{des}$ is the desired
value of this frequency, then $c = \dfrac{(\omega)_{des}}{\omega_r} - 1$, and $\dfrac{\partial c}{\partial t_i} = - \dfrac{(\omega)_{des}}{\omega_r^2} \dfrac{\partial \omega_r}{\partial t_i}$.
To calculate $\dfrac{\partial \omega_r}{\partial t_i}$, equation (31) is written for the rth mode, differ-
entiated with respect to t_i, and then premultiplied by $\lfloor q^{(r)} \rfloor$, the rth
eigenvector. Solving then for $\dfrac{\partial \omega_r}{\partial t_i}$ gives

$$\frac{\partial \omega_r}{\partial t_i} = \frac{\lfloor q^{(r)} \rfloor [GK_i]\{q^{(r)}\} - \omega_r^2 \lfloor q^{(r)} \rfloor [GM_i]\{q^{(r)}\}}{2\omega_r \lfloor q^{(r)} \rfloor [GM]\{q^{(r)}\}} \tag{33}$$

and the optimality criterion of equation (5) reads

$$\lfloor q^{(r)} \rfloor [GK_i]\{q^{(r)}\} - \omega_r^2 \lfloor q^{(r)} \rfloor [GM_i]\{q^{(r)}\} = C, \quad \forall \, i \, \epsilon \, A \qquad (34)$$

where, with $\omega_r = (\omega)_{des}$,

$$C = \frac{2m_i \omega_r^2}{\lambda} \lfloor q^{(r)} \rfloor [GM]\{q^{(r)}\} \qquad (35)$$

The left-hand side of equation (34) can be termed a "specific Lagrangian density", since it is the difference between the peak strain energy and the peak kinetic energy in the constrained mode per unit value of each active design variable. Note also that equation (34) is homogeneous with respect to the eigenvector $\{q^{(r)}\}$, so it is invariant with respect to the normalization of $\{q^{(r)}\}$.

For the "energy-density" form, the matrix [B] is identified with $[GK] - \omega_r^2 [GM]$, and $\{p\}$ and $\{q\}$ are both $\{q^{(r)}\}$, since [B] is symmetric (see equations (17) and (18)). The eigenvalue ω_r is fixed, so there is no "free" eigenvalue Ω, and equation (19) does not apply. Upon writing out $(e_v)_i$, equation (21) becomes

$$\lfloor q^{(r)} \rfloor [GK_i]\{q^{(r)}\} - \omega_r^2 \lfloor q^{(r)} \rfloor [GM_i]\{q^{(r)}\} = -m_i, \quad \forall \, i \, \epsilon \, A \qquad (36)$$

In this equation, $\{q^{(r)}\}$ can be normalized to give an arbitrary value on the right-hand side, so equations (34) and (36) are in fact equivalent.

For any of the optimization routines, constraint evaluation is straightforward. Equation (31) is solved for ω_r and $\{q^{(r)}\}$, and then the $\frac{\partial c}{\partial t_i}$ or $(e_v)_i$ are calculated. The mass and stiffness matrices are updated with the current values of the design variables by equations identical in form to equations (30) and (26).

3.3 Flutter-Speed Constraint

The governing equations for flutter, written in modal coordinates, have the form

$$([GM] + [GA] - \bar{\Omega}[GK])\{\bar{q}\} = \{0\} \qquad (37)$$

Here [GM] and [GK] are the generalized mass and stiffness matrices
defined previously, and [GA] is the generalized aerodynamic matrix.
The modal coordinates $\{\bar{q}\}$ are again defined by a fixed set of modes,
so [GA] is a complex function of reduced frequency k and Mach number
M_∞. (The altitude is assumed given.) The eigenvalue $\bar{\Omega}$ is written
in terms of the frequency ω and damping parameter g as $(1 + jg)/\omega^2$,
and flutter is determined by that combination of k and M_∞ which gives
g = 0 and the lowest flutter speed U_∞, which is obtained from k and ω.
A final step involves varying the Mach number until there is com-
patibility among the altitude, the Mach number, and the flutter speed.

The flutter constraint will be enforced by requiring that g = 0
for a given combination of altitude, Mach number, and speed. The
derivative $\frac{\partial g}{\partial t_i}$ is calculated in the manner of references 1 and 2:

$$\frac{\partial g}{\partial t_i} = \left[(R_2^i - \omega^2 R_1^i - g I_2^i)(2g R_3 + 2 I_3 + \frac{b\omega^3}{U_\infty} I_4) \right.$$
$$\left. - (2 R_3 - 2g I_3 + \frac{b\omega^3}{U_\infty} R_4)(I_2^i - \omega^2 I_1^i + g R_2^i) \right]/D \tag{38}$$

where

$$D = (2g R_3 + 2 I_3 + \frac{b\omega^3}{U_\infty} I_4) I_3 + (2 R_3 - 2g I_3 + \frac{b\omega^3}{U_\infty} R_4) R_3 \tag{39}$$

$$R_1^i = \text{Re}(\lfloor \bar{p} \rfloor [GM_i]\{\bar{q}\}) \tag{40}$$

$$R_2^i = \text{Re}(\lfloor \bar{p} \rfloor [GK_i]\{\bar{q}\}) \tag{41}$$

$$R_3 = \text{Re}(\lfloor \bar{p} \rfloor [GK]\{\bar{q}\}) \tag{42}$$

$$R_4 = \text{Re}(\lfloor \bar{p} \rfloor \left[\frac{\partial GA}{\partial k}\right]\{\bar{q}\}) \tag{43}$$

and I_1^i, I_2^i, I_3, and I_4 are the corresponding imaginary parts. It is
also understood that $\{\bar{q}\}$ is the eigenvector for the critical flutter
mode, and $\{\bar{p}\}$ is the eigenvector for the adjoint problem

$$([GM] + [GA]^T - \bar{\Omega}[GK])\{\bar{p}\} = \{0\} \tag{44}$$

For R_4 and I_4, derivatives of the elements of the aerodynamic matrix [GA] are needed with respect to reduced frequency k. These are obtained approximately by calculating [GA] for a band of reduced frequencies in the range of interest and then fitting these data with polynomials in k. The fitting procedure used here is the one discussed at length in reference 12. The derivatives are then obtained by differentiating the polynomials. For a flutter constraint, then, the optimality criterion is obtained from equation (5) by identifying c with g:

$$\frac{1}{m_i} \frac{\partial g}{\partial t_i} = -\frac{1}{\lambda}, \qquad \forall \ i \ \epsilon \ A \tag{45}$$

with $\frac{\partial g}{\partial t_i}$ given by equation (38).

For the "energy-density" form, [B] is identified with [GM] + [GA] $-\bar{\Omega}$[GK], {p} with {$\bar{p}$}, and {q} with {$\bar{q}$}, so the optimality criterion in this form is

$$(e_v)_i = \frac{1}{m_i} \ \text{Re}(\lfloor \bar{p} \rfloor ([GM_i] - \bar{\Omega}[GK_i])\{\bar{q}\}) = -1, \qquad \forall \ i \ \epsilon \ A \tag{46}$$

This expression is much simpler than equation (45), so it appears that an algorithm based on computing $(e_v)_i$ rather than $\frac{\partial g}{\partial t_i}$ would be more efficient, since less computation is involved, and derivatives of the aerodynamic matrix need not be calculated. However, this is only part of the story, since the convergence characteristics of the algorithm also influence the efficiency.

It is worth noting that the "energy-density" formulation provides an additional relation when a "free" eigenvalue is involved. In this case, the "free" eigenvalue is the flutter frequency, which is unconstrained. Thus equation (19), with Ω identified with ω, is applicable. By making use of the actual expression for [B] and the definitions of equations (40)-(43) and their imaginary counterparts, equation (19) can be manipulated to give another condition to be satisfied at the optimum:

$$2R_3 - 2gI_3 + \frac{b\omega^3}{U_\infty} R_4 = 0 \tag{47}$$

If this relation is inserted into equations (38) and (39), equation
(45) is greatly simplified and reads

$$\frac{1}{m_i} (\omega^2 R_1^i - R_2^i + g I_2^i) = \frac{I_3}{\lambda} , \qquad \forall \ i \ \epsilon \ A \qquad (48)$$

Equation (46), when rewritten with the definitions of equations (40)
and (41) and their imaginary counterparts, becomes

$$\frac{1}{m_i} (\omega^2 R_1^i - R_2^i + g I_2^i) = -\omega^2, \qquad \forall \ i \ \epsilon \ A \qquad (49)$$

Since the normalization of $\{\bar{p}\}$ and $\{\bar{q}\}$ is arbitrary, it is always
possible to renormalize these eigenvectors so that the right-hand
side of equation (49) is identical to the right-hand side of equation
(48). Hence the two optimality criteria are formally equivalent.
However, it must be emphasized that the derivative $\frac{\partial g}{\partial t_i}$ is in general
given by equation (38); the simplified expression is valid only at
the optimum.

For any of the redesign algorithms, constraint evaluation is
accomplished by solving equation (37). Equation (44) is solved to
obtain $\{\bar{p}\}$. The generalized mass and stiffness matrices are updated
as described in subsection 3.2. Since the flutter frequency will
vary as the redesign progresses, the reduced frequency k must be
updated in order to ensure compatibility between its value and the
frequency computed from $\bar{\Omega}$. This is most easily done iteratively.
The value of k from the previous design is used initially to determine
[GA], and a new frequency is computed from the eigenvalue $\bar{\Omega}$. If this
frequency differs sufficiently from that of the previous design, the
newly calculated frequency is used to recalculate k, and the analysis
is repeated. (This procedure implies, of course, that [GA] has been
determined for an appropriate range of k values, as discussed above.)
The current constraint value, g^ν, and either $\left[\frac{\partial g}{\partial t_i}\right]$ or $(e_\nu)_i^\nu$ are then
calculated.

4. NUMERICAL EXAMPLES

For comparison purposes, three separate redesign algorithms were coded--two based on equations (6) and (7), and an "energy-density" one based on equations (6), (22), and (23). The first two differ in the selection of α and in the treatment of side constraints; one (Segenreich) is based on the algorithm in references 1 and 2, while the other (Rizzi) is based on the algorithm in references 6 and 7. These are described in subsection 2.3. The third algorithm ("energy-density") is described in subsection 2.5. A fourth algorithm (ref. 13), based on the method of feasible directions, was also included in order to provide some comparisons with an MP method. Program CONMIN, described in reference 13, can use analytically computed gradients or can calculate gradients by finite differencing. Since analytical gradients were already being calculated, they were used for CONMIN as well.

4.1 Rectangular Wing

The first example problem involves a simple rectangular wing structure whose dimensions are given in figure 1. This wing was first used in reference 14 and has since been treated by other researchers. A very simple finite-element model was created, involving two cover sheets, two spar webs, one rib, and four spar caps in each of three bays in the structural box. The spar caps were represented by axial elements, and the other members were represented by in-plane elements. There are 12 design variables, whose numbers and initial values are given in Table 1. In all cases, minimum-gage constraints of one quarter of these values were imposed. No weight or stiffness is assigned to any portion of the wing except the structural box, and the initial weight is 88.45 kg. This is the initial configuration for all of the constraints considered. All of the computing was performed on an IBM 370/168 computer.

For the displacement constraint, transverse loads of 445 N were applied at the six nodes on one side of the wing, and the transverse displacement at the tip nodes was calculated to be 1.465 cm. This

displacement was constrained to be 1.524 cm. Iteration histories for
the various algorithms used are given in figure 2, and relative CPU
times are given in Table 2, along with the final values of the active
design variables, the weight, and the constraint parameter
$c = u_r/(u_r)_{des} - 1$. All of the final designs are essentially iden-
tical, with the only active variables being the web thicknesses in
bays 1 and 2. The Rizzi and "energy-density" algorithms took less
than one third the CPU time taken by CONMIN, with the Segenreich
algorithm having an intermediate time. The iteration histories
reveal that all of the OC algorithms nearly reach the optimum weight
within ten iterations. However, it was necessary to have these
algorithms continue in order to ensure that all of the passive
variables were identified and the constraint satisfied. For all
except the "energy-density" algorithm, the intermediate designs
followed the constraint boundary rather closely, since such behavior
is enforced at each step. The "energy-density" algorithm is somewhat
looser in this respect and appears to proceed in two stages - a first
stage where most of the weight is removed, and a second where the
constraint is satisfied. Parameters chosen for the various algorithms
are as follows: For the Segenreich algorithm, the sequence of weight-
reduction factors K_i was (0.2, 0.1, 0.05, 0.025, 0.01, 0.005); for
the Rizzi algorithm, $\alpha^{(0)} = 0.90$, $\alpha_x = 0.95$; for the "energy-density"
algorithm, $e_1 = 0.5$, $e_2 = 2.0$; for all of these algorithms, the con-
vergence parameter ε was 0.001; and for CONMIN, either the default
values or the values recommended in reference 13 were used for the
various parameters required.

For a frequency constraint, the fundamental frequency of free
vibration was constrained to be 68.0 rad/sec, or 10.82 Hz. This is
slightly higher than the calculated frequency of 67.16 rad/sec, or
10.69 Hz, for the initial design. Since both the inertial and stiff-
ness properties of the wing are linearly proportional to the design
variables and there is no nonactive mass or stiffness, constraining
the frequency to be identical to that of the initial design would
result in a trivial problem, since the mass and stiffness matrices

can be scaled by an arbitrary factor without affecting the frequency.
The optimum would therefore be given by the design with all design
variables at their minimum values. With a slightly altered frequency
constraint, the optimum design should have at least one active design
variable, and this in fact is the case, as can be seen in Table 3.
The constraint parameter c is here $(\omega)_{des}/\omega_r - 1$. The iteration
histories are given in figure 3. For the Segenreich algorithm, the
table of weight-reduction factors was (0.1, 0.05, 0.025, 0.01, 0.005,
0.002); for the Rizzi algorithm, $\alpha^{(0)} = 0.95$, $\alpha_x = 1.0$; the parameters
for the "energy-density" algorithms and for CONMIN were identical to
those used for the displacement constraint. The CPU-time comparison
shows that this time only the "energy-density" algorithm is faster
than CONMIN, although the differences are not great. The "energy-
density" algorithm actually results in an increase in weight for a
few iterations before it converges, and the Rizzi algorithm does not
display the same rapid approach to the vicinity of the optimum weight
as it did for the displacement constraint. The latter phenomenon is
undoubtedly a result of keeping α constant, which was necessary in
order to achieve convergence.

For the flutter constraint, the six transverse modes of the
initial design were used to define generalized coordinates, and sub-
sonic generalized aerodynamic forces were calculated with the doublet
lattice method (the program described in reference 12 was used). A
flutter Mach number of 0.717 was calculated at an altitude of 1,372 m,
corresponding to a speed of 240 m/sec. This flutter point was imposed
as the constraint, with c = g. CPU-time comparisons among the four
algorithms and the final design information are given in Table 4, and
the iteration histories are given in figure 4. The parameters used
in the various algorithms are the same as those used for the displace-
ment constraint, with two exceptions - $\epsilon = 0.002$ in the Rizzi algorithm,
and $e_1 = 0.1$ in the "energy-density" algorithm. The exponent e_1 had to
be reduced from the originally selected value of 0.5 in order to prevent
divergence. The iteration history reflects this reduction, in that the
approach to the optimum weight is more gradual than it was in the

previous examples. As a check on the accuracy of using fixed modes, the final design was reanalyzed for flutter with new natural modes. This new analysis gave a flutter speed 6% lower than that calculated with the original modes. Since the maximum number of transverse modes available, six, was used in both cases, it was not possible to see whether retaining more modes initially would narrow this difference.

4.2 Rectangular Wing with Increased Degrees of Freedom and Internal Fuel

In an effort to obtain a slightly more complex problem with more design variables active at the optimum, the rectangular wing was remodeled with an increased number of elements, as indicated in figure 5. Each bay was divided into two with a set of nodes at the midspan of the bay. New ribs were not added, however, so only the number of cover sheets, spar webs, and spar caps was doubled. This resulted in a total of 21 design variables, whose numbering and initial values are given in Table 5. In addition, nonstructural mass to represent internal fuel was distributed uniformly within the structural box. The initial weight was increased in this manner by 110.58 kg to a total of 199.13 kg. Minimum-gage constraints of 25% of the initial values were again imposed on the design variables.

The initial design was analyzed for flutter with generalized coordinates defined by 12 transverse vibration modes, and a flutter speed of 231 m/sec was calculated at an altitude of 1,372 m, corresponding to a Mach number of 0.689. As before, doublet-lattice generalized aerodynamic forces were calculated with the program described in reference 12. This flutter speed was imposed as the behavioral constraint, and the wing was optimized with the Rizzi algorithm, with $\alpha^{(0)} = 0.90$, $\alpha_x = 0.95$, and $\varepsilon = 0.005$. The iteration history is shown in figure 6, and the final design information is given in Table 6. In addition to the webs that were active before at the optimum, there are now some cover sheets that are also involved, but most of the design variables are still at their minimum values. The final design was also reanalyzed for flutter with new modes, and the recalculated flutter speed is 9% lower than that calculated with the original modes. In this case, also, all of the modes available were used for both calculations.

Attempts to obtain optimum solutions with the other algorithms were not successful, and at this point it was decided to use another example that would be more representative of a real design and that might also have more active design variables. This is described below.

4.3 Delta Wing

The next example structure to be considered is a biconvex-airfoil delta wing, shown in planform in figure 7. This wing is example 2 of reference 15. It can have up to 102 design variables, and lower totals can be created through linking, so problems of varying complexity in terms of the optimization task can be treated while the fully modeled structure is used for analyses. The structural planform is built up from triangular in-plane elements separated by very stiff axial members at each mode that model the core. Mass to represent internal fuel is distributed in the core.

The structure has been modeled and is still in the process of being analyzed, so there are no optimization results that can be given here.

5. CONCLUDING REMARKS

5.1 Conclusions From Results to Date

From the results with the simple model of the rectangular wing, it can be seen that the OC algorithms generally performed better than the MP algorithm in terms of relative CPU time, with the Rizzi algorithm doing the best. However, the need for the user to experiment with these parameters in any new application will have a great influence on the utility of the algorithm being used, which could well overshadow any advantages the algorithm might have in computational efficiency. For the final comparisons, therefore, it would be desirable to have recommended or default values for the parameters in various applications and not change them unless it is absolutely necessary.

In retrospect, it has also become apparent that the convergence criteria for the various algorithms may not have been as consistent as they should have. Consider, for example, the relationship between the convergence criterion for the Rizzi algorithm (eq. (14)) and that for the "energy-density" and Segenreich algorithms (eq. (12)). The Rizzi and Segenreich criteria can be related through the recursion relation, equation (7). This can be manipulated to yield

$$1 - C_i^\nu = \left\{ 1 + \lambda^\nu \left(\frac{1}{m_i} \frac{\partial c}{\partial t_i} \right)^\nu \right\} (1 - \alpha^\nu) \tag{50}$$

If ϵ_1 is the convergence parameter for the Segenreich algorithm and ϵ_2 that for the Rizzi algorithm, then

$$\epsilon_1 = |1 - \alpha^\nu| \epsilon_2 \tag{51}$$

With $\alpha^{(0)} = 0.90$ and $\alpha_x = 0.95$, after 25 iterations $\alpha = 0.263$, so $\epsilon_1 = 0.737\epsilon_2$. On the other hand, if $\alpha_x = 1.0$, then $\epsilon_1 = 0.1\epsilon_2$. This means that, for equivalent convergence criteria, in these two cases, ϵ_1 should be 74% or 10% of ϵ_2. With the rectangular-wing example, inconsistencies in these criteria affect CPU time more than the final answer, since the optimum appears to be located in a very shallow depression in design space. It is hoped that the delta-wing structure, or others that may be analyzed, will have more sharply defined optima.

5.2 Future Work

During the next year, comparisons of the various algorithms will be continued with the delta-wing example described above. This example will also be used to test the accuracy of computing gradients with fixed modes. The most promising algorithm will then be extended to treat multiple behavioral constraints, and alternative strategies for handling these constraints will be evaluated.

REFERENCES

1. Segenreich, S. A.: Weight Optimization Under Flutter Constraint.
    Ph.D. Thesis, Stanford University, Stanford, CA, Jan. 1975.

2. Segenreich, S. A. and McIntosh, S. C., Jr.: Weight Minimization
    of Structures for Fixed Flutter Speed via an Optimality Criterion.
    AIAA Paper No. 75-779, presented at AIAA/ASME/SAE 16th Structures,
    Structural Dynamics, and Materials Conference, Denver, CO, May
    27-29, 1975.

3. Segenreich, S. A. and McIntosh, S. C., Jr.: Weight Optimization
    Under Multiple Equality Constraints Using an Optimality Criterion.
    Paper presented at AIAA/ASME/SAE 17th Structures, Structural
    Dynamics, and Materials Conference, King of Prussia, PA, May
    5-7, 1976.

4. Khot, N. S., Berke, L., and Venkayya, U. B.: Comparison of
    Optimality Criteria Algorithms for Minimum Weight Design of
    Structures. AIAA Paper No. 78-469, presented at AIAA/ASME
    19th Structures, Structural Dynamics, and Materials Conference,
    Bethesda, MD, Apr. 3-5, 1978.

5. Kiusalaas, J.: Minimum Weight Design of Structures via Optimality
    Criteria. NASA TN D-7115, Dec. 1972.

6. Rizzi, P.: The Optimization of Structures with Complex Constraints
    via a General Optimality Criteria Method. Ph.D. Thesis, Stanford
    University, Stanford, CA, July 1976.

7. Rizzi, P.: Optimization of Multi-Constrained Structures Based
    on Optimality Criteria. Paper presented at AIAA/ASME/SAE 17th
    Structures, Structural Dynamics, and Materials Conference, King
    of Prussia, PA, May 5-7, 1976.

8. Turner, M. J.: Optimization of Structures to Satisfy Flutter
    Requirements. AIAA Jour., Vol. 7, No. 5, May 1969, pp. 945-951.

9. Siegel, S.: A Flutter Optimization Program for Aircraft Structural
    Design. AIAA Paper No. 72-795, presented at AIAA 4th Aircraft
    Design, Flight Test, and Operations Meeting, Los Angeles, CA,
    Aug. 7-9, 1972.

10. Haftka, R. J., Starnes, J. H., Jr., Barton, F. W., and Dixon, S. C.:
    Comparison of Two Types of Structural Optimization Procedures for
    Flutter Requirements. AIAA Jour., Vol. 13, No. 10, Oct. 1975,
    pp. 1333-1339.

11. Wilkinson, K., Lerner, E., and Taylor, R. F.: Practical Design of Minimum Weight Aircraft Structures for Strength and Flutter Requirements. Jour. Aircraft, Vol. 13, No. 8, Aug. 1976, pp. 614-624.

12. Gwin, L. B. and McIntosh, S. C., Jr.: A Method of Minimum-Weight Synthesis for Flutter Requirements, Part I - Analytical Investigation. AFFDL TR 72-22, Part I, June 1972.

13. Vanderplaats, G. N.: CONMIN - A Fortran Program for Constrained Function Minimization. User's Manual. NASA TM X-62,282, 1973.

14. Rudisill, C. S. and Bhatia, K. G.: Optimization of Complex Structures to Satisfy Flutter Requirements. AIAA Jour., Vol. 9, No. 8, Aug. 1971, pp. 1487-1491.

15. Haftka, R. J. and Starnes, J. H., Jr.: WIDOWAC (Wing Design Optimization with Aeroelastic Constraints): Program Manual. NASA TM X-3071, Oct. 1974.

TABLE 1. DESIGN-VARIABLE NUMBERING AND INITIAL VALUES FOR THE RECTANGULAR WING.

Bay	Spar-Cap Areas		Web Thicknesses		Cover Thicknesses		Rib Thicknesses	
	DV	Initial Value, cm^2	DV	Initial Value, cm	DV	Initial Value, cm	DV	Initial Value, cm
1	t_1	12.90	t_4	0.2032	t_7	0.1016	t_{10}	0.1016
2	t_2	12.90	t_5	0.2032	t_8	0.1016	t_{11}	0.1016
3	t_3	12.90	t_6	0.2032	t_9	0.1016	t_{12}	0.1016

TABLE 2. RELATIVE CPU TIMES AND FINAL DESIGN INFORMATION FOR THE OPTIMAL RECTANGULAR WING WITH A DISPLACEMENT CONSTRAINT.

Algorithm	CPU	Weight, kg	Active Design Variables		Constraint	Iterations
			t_4, cm	t_5, cm		
Segenreich (Refs. 1 and 2)	0.379	25.85	0.3590	0.1744	-0.00044	27
Rizzi (Refs. 6 and 7)	0.303	25.85	0.3596	0.1744	-0.00071	22
"Energy-density"	0.315	25.86	0.3602	0.1746	-0.0018	24
CONMIN (Ref. 13)	1.00	25.83	0.3618	0.1699	-0.0084	9

TABLE 3. RELATIVE CPU TIMES AND FINAL DESIGN INFORMATION FOR THE OPTIMAL RECTANGULAR WING WITH A FREQUENCY CONSTRAINT.

Algorithm	CPU	Weight, kg	Active Design Variable- t_4, cm	Constraint	Iterations
Segenreich (Refs. 1 and 2)	1.10	22.12	0.05319	0.0000	23
Rizzi (Refs. 6 and 7)	1.04	22.12	0.05319	0.0000	28
"Energy-density"	0.744	22.12	0.05324	-0.0002	16
CONMIN (Ref. 13)	1.00	22.13	0.05420	-0.0051	7

TABLE 4. RELATIVE CPU TIMES AND FINAL DESIGN INFORMATION FOR THE OPTIMAL
RECTANGULAR WING WITH A FREQUENCY CONSTRAINT.

Algorithm	CPU	Weight, kg	Active Design Variable- t_4, cm	Constraint	Iterations
Segenreich (Refs. 1 and 2)	0.952	22.85	0.1376	-0.000091	49
Rizzi (Refs. 6 and 7)	0.360	22.86	0.1383	-0.00079	11
"Energy-density"	0.705	22.85	0.1376	-0.00027	39
CONMIN (Ref. 13)	1.00	22.88	0.1409	-0.0038	12

TABLE 5. DESIGN-VARIABLE NUMBERING AND INITIAL VALUES FOR THE RECTANGULAR WING WITH INCREASED NUMBER OF ELEMENTS.

Bay	Spar-Cap Areas		Web Thicknesses		Cover Thicknesses		Rib Thicknesses	
	DV	Initial Value, cm^2	DV	Initial Value, cm	DV	Initial Value, cm	DV	Initial Value, cm
1	t_1	12.90	t_7	0.2032	t_{13}	0.1016		
2	t_2	12.90	t_8	0.2032	t_{14}	0.1016	t_{19}	0.1016
3	t_3	12.90	t_9	0.2032	t_{15}	0.1016		
4	t_4	12.90	t_{10}	0.2032	t_{16}	0.1016	t_{20}	0.1016
5	t_5	12.90	t_{11}	0.2032	t_{17}	0.1016		
6	t_6	12.90	t_{12}	0.2032	t_{18}	0.1016	t_{21}	0.1016

TABLE 6.   FINAL DESIGN INFORMATION FOR THE RECTANGULAR WING WITH
           INCREASED NUMBER OF ELEMENTS AND A FLUTTER CONSTRAINT.

Weight, kg: 143.2

Active design variables, cm: $t_7 = 0.5469$, $t_8 = 0.3195$, $t_9 = 0.07341$

$t_{15} = 0.1569$, $t_{16} = 0.1301$, $t_{17} = 0.04879$

Constraint: -0.715×10^{-4}

Iterations: 24
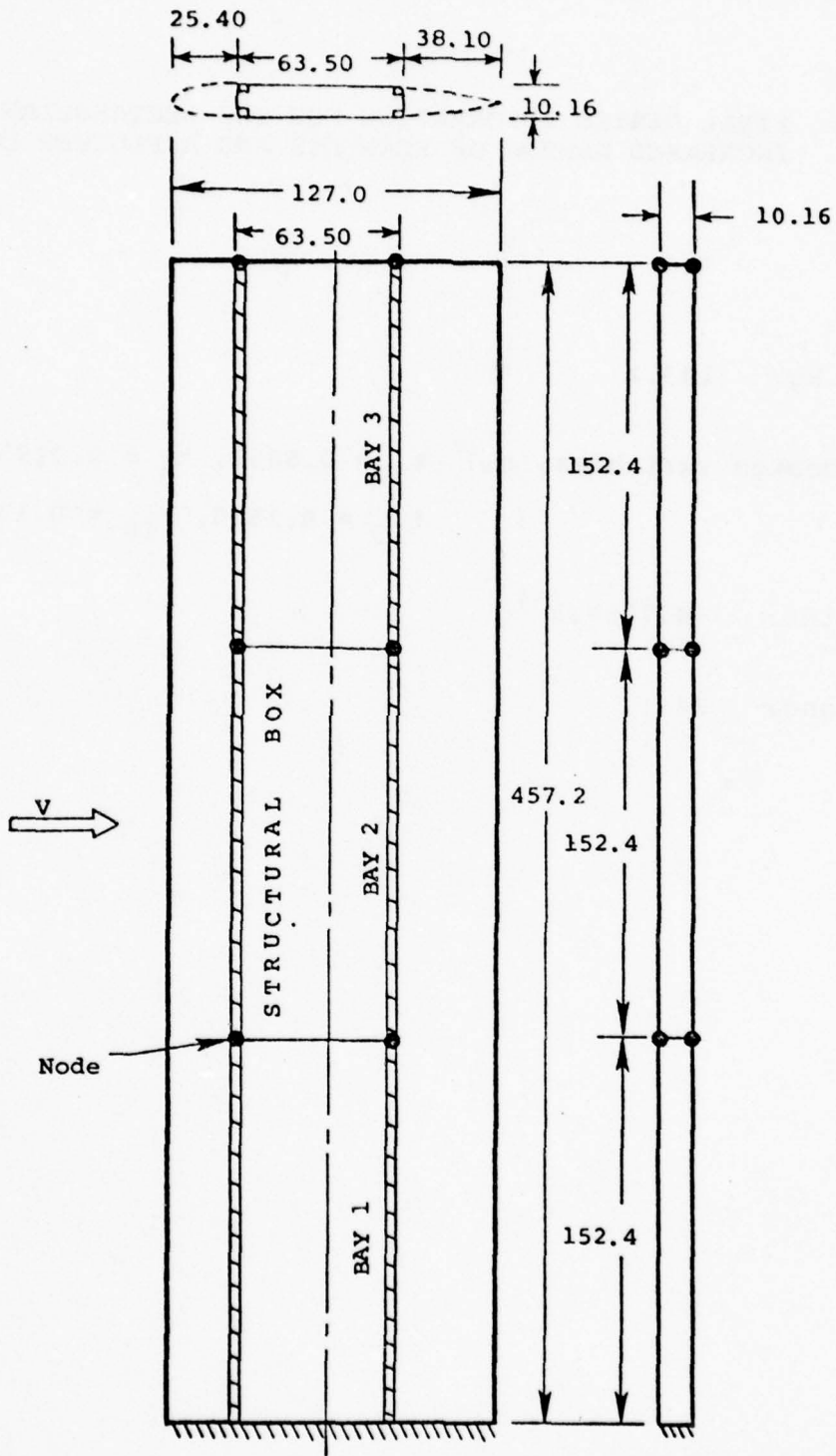
Figure 1.- Layout of rectangular wing. All dimensions are in cm.
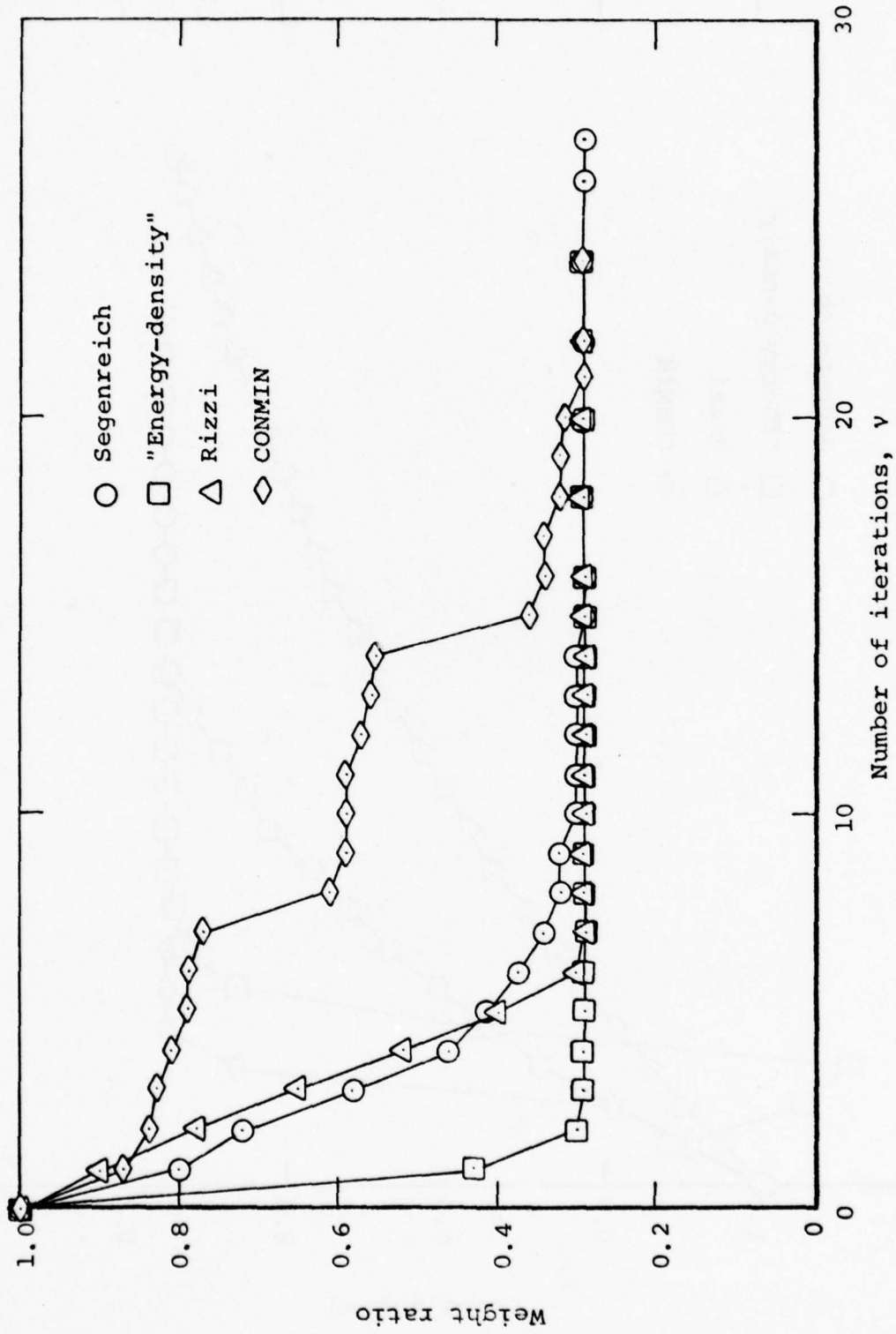
Figure 2.- Iteration histories for the rectangular wing with a displacement constraint.
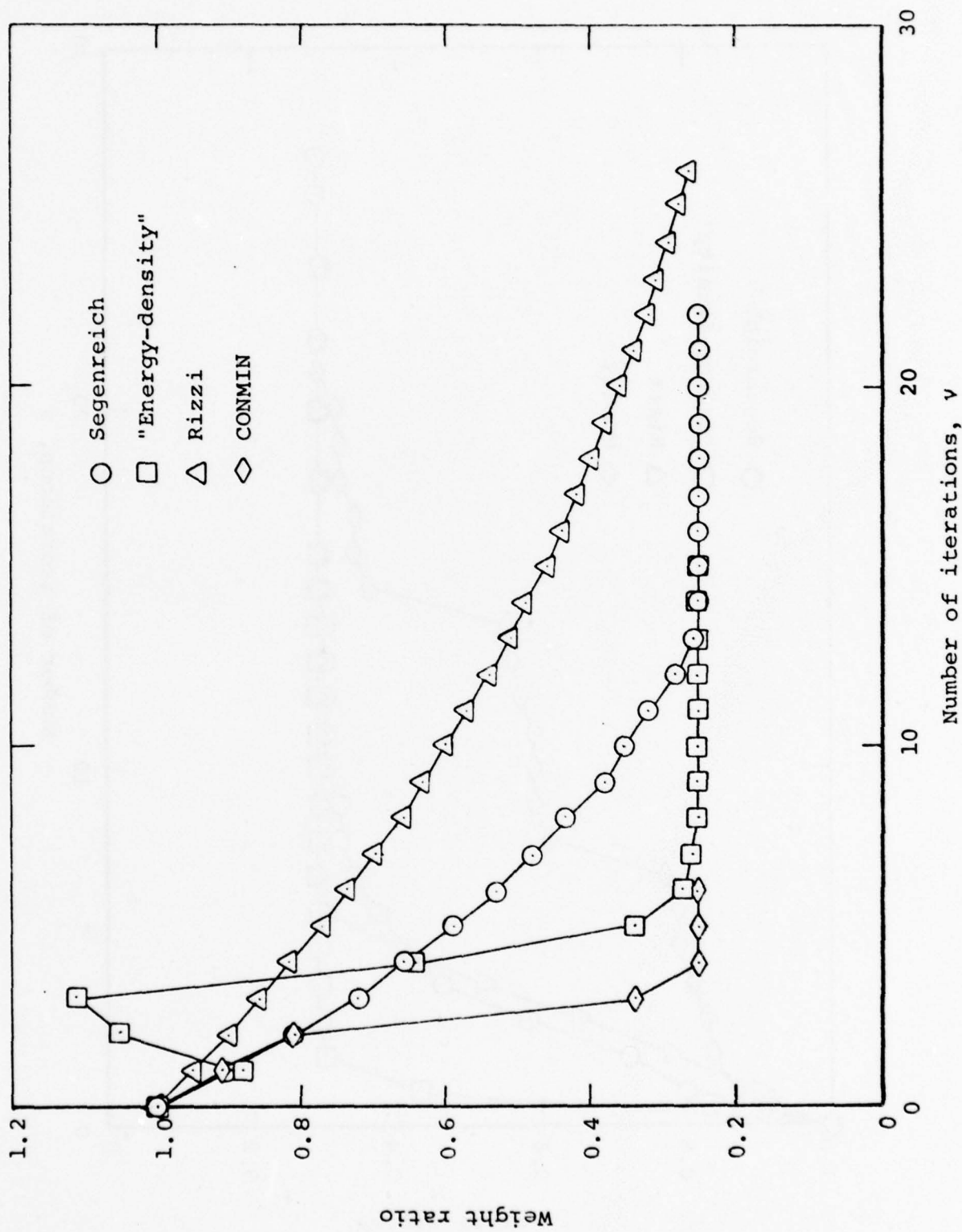
Figure 3.- Iteration histories for the rectangular wing with a frequency constraint.
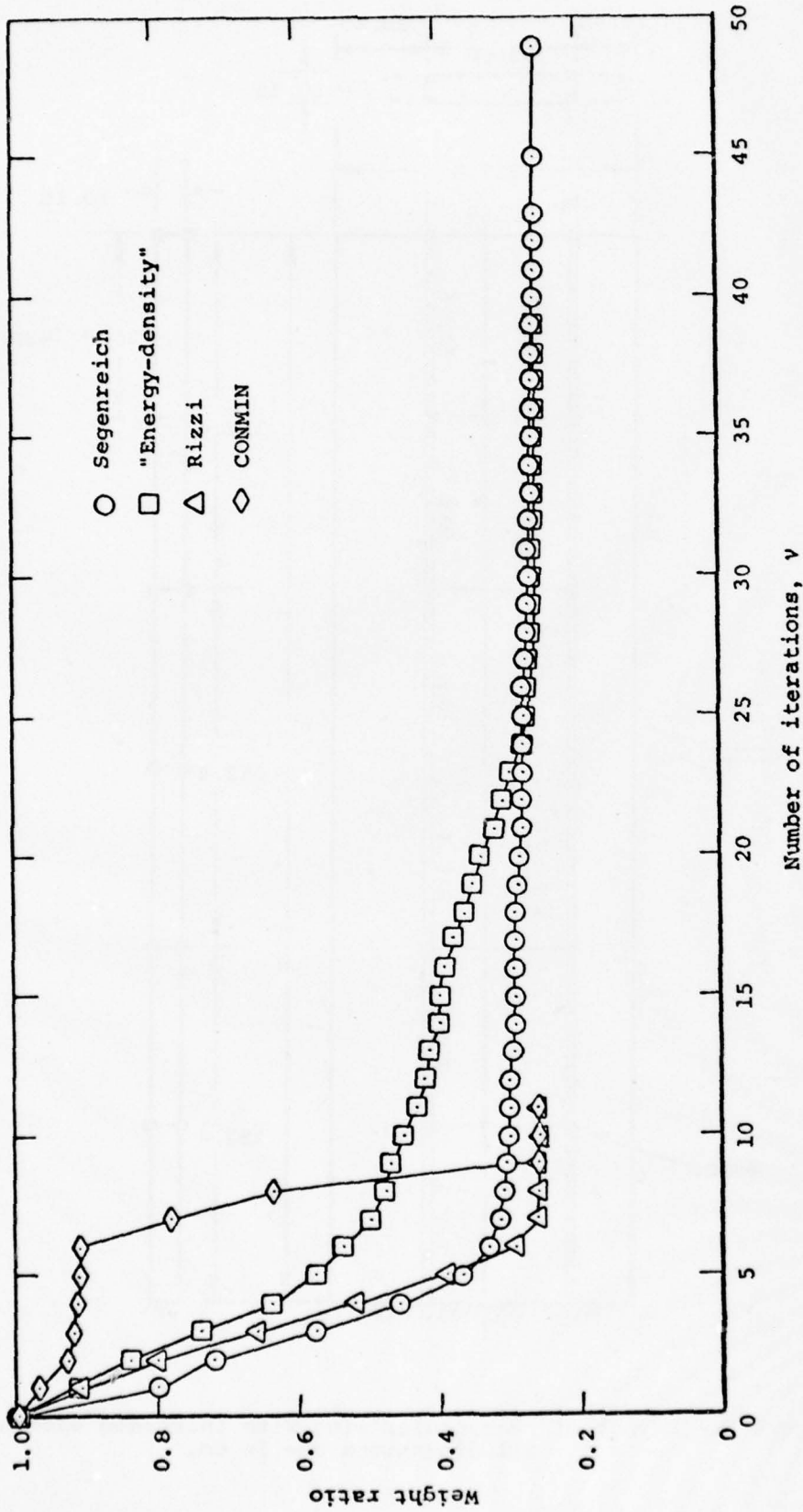
Figure 4.- Iteration histories for the rectangular wing with a flutter constraint.
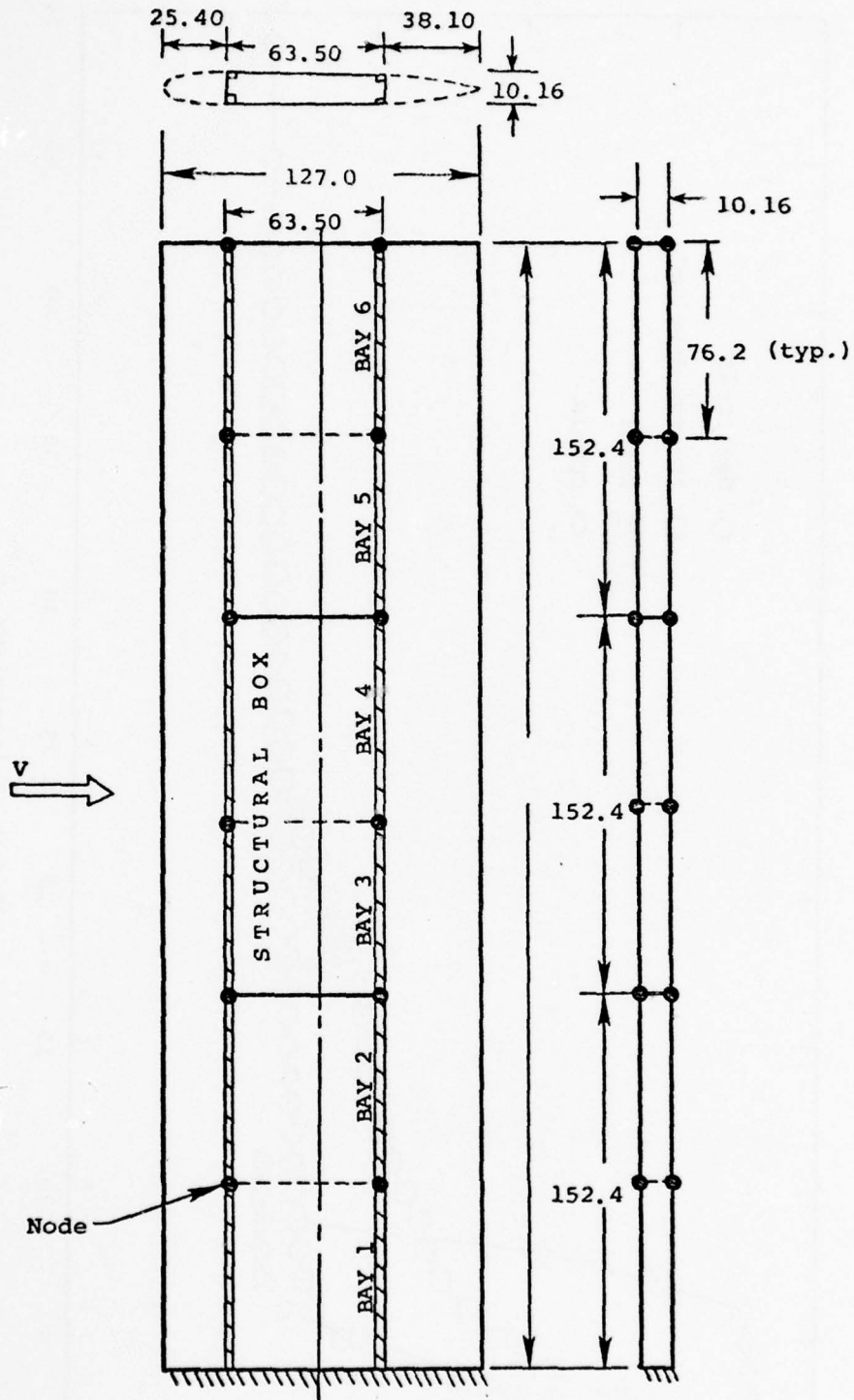
Figure 5.- Layout of rectangular wing with increased elements.
All dimensions are in cm.
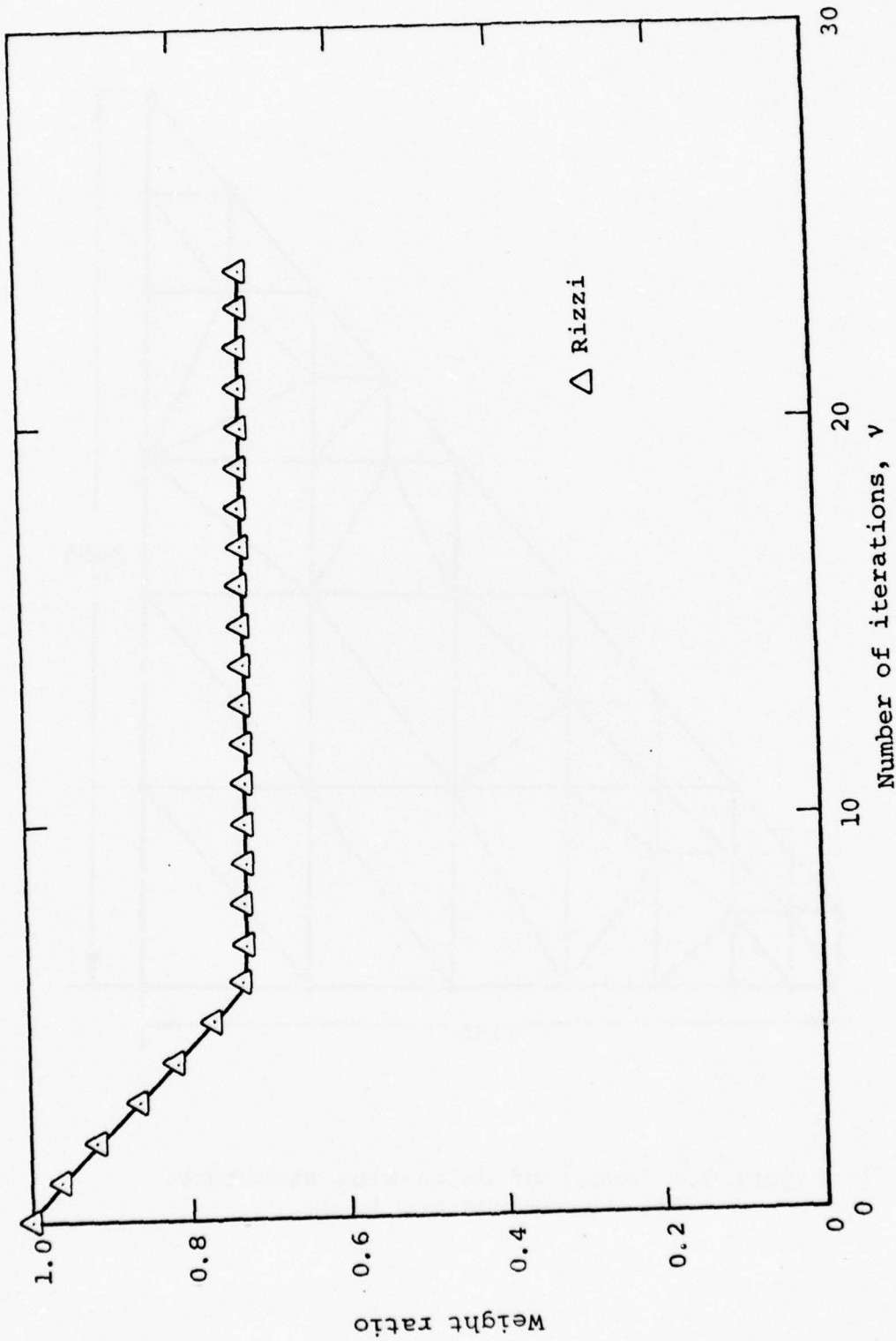
Figure 6.- Iteration history for the increased-element rectangular wing with a flutter constraint.
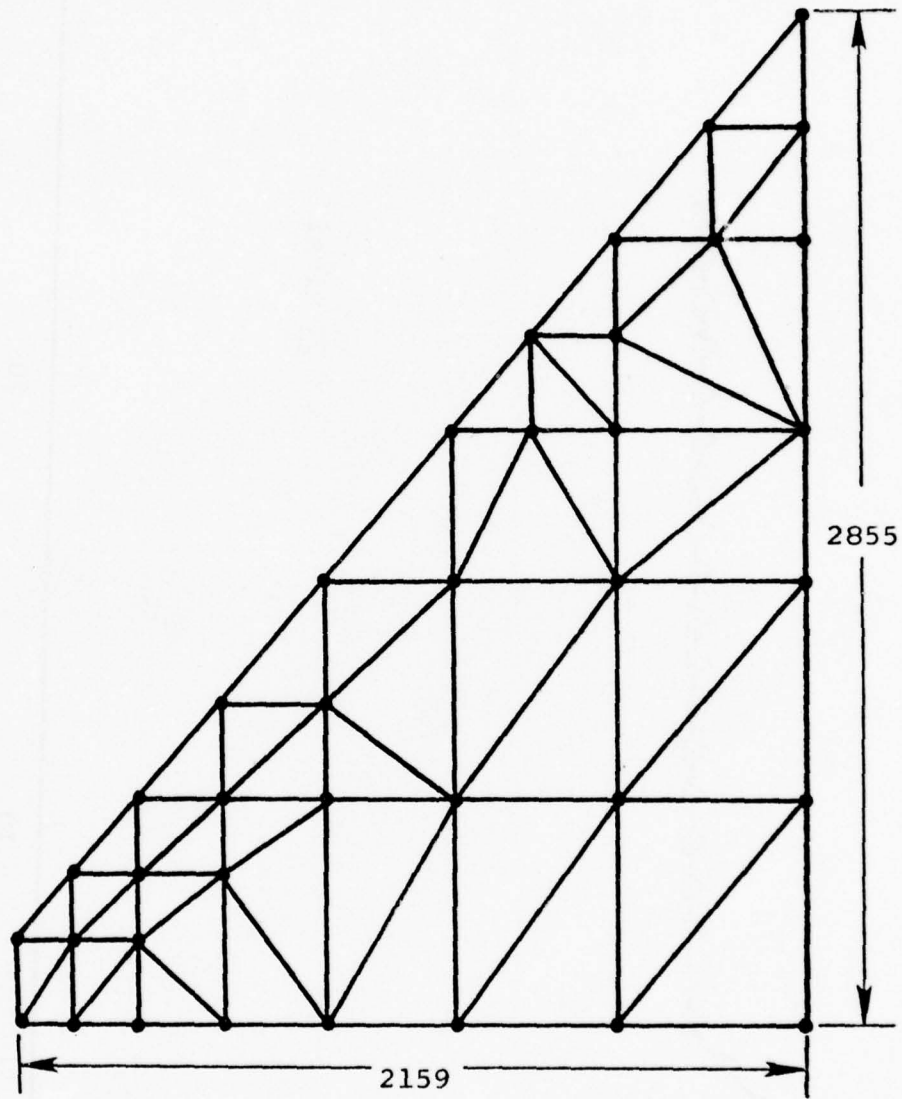
Figure 7.- Layout of delta-wing structure.
All dimensions are in cm.

APPENDIX A

Start

Input data;
initialize variables

$\nu = 1$

Evaluate constraint c^ν,
total mass m^ν, and
derivatives $\left(\frac{\partial c}{\partial t_i}\right)^\nu$
(Separate subroutine)

$K = (K)_{max}$

①

Calculate α^ν, λ^ν

Calculate C_i^ν, $t_i^{\nu+1}$
Δt_i^ν, $\forall i \in A$

Choose next
value of K

$t_i^{\nu+1} = (t_i)_{min}$
Revise A

$|\Delta t_i^\nu|/t_i^\nu > 0.25?$ Yes

Choose next
value of K

$K = (K)_{min}?$ No

Yes

No

$K = (K)_{min}?$ No

Yes

$t_i^{\nu+1} \geq (t_i)_{min}?$

Yes

$|C_i^\nu - 1.0| < \epsilon?$
$\forall i \in A$ Yes

Reintroduce any
passive variables?   No

Yes

①

No

$t_i^\nu = t_i^{\nu+1}$
$\forall i \in A$

Print final
results

$\nu = \nu + 1$

Stop

No $\nu > (\nu)_{max}?$ Print message   Stop

Flow chart for algorithm of references 1-3 (Segenreich).

Start

Input data, initialize variables

$v = 1$

$\alpha = \alpha^{(o)}$
Assume all variables active

$\alpha^v = \alpha_x \alpha^{v-1}$
Use current active-passive identities

Evaluate constraint c^v, total mass m^v, and derivatives $\left(\dfrac{\partial c}{\partial t_i}\right)^v$

(Separate subroutine)

Stop

Calculate λ^v and C_i^v

Revise active-passive identities

Print message

No

$t_i^{v+1} < (t_i)_{min}$ for active variable or $C_i^v > 1.0$ for passive variable?

Yes

Yes

$v > (v)_{max}$?

NO

$t_i^v = t_i^{v+1}$
$\forall i \in A$

$v = v + 1$

No

$\left| 1 + \lambda^v \left(\dfrac{1}{m_i} \dfrac{\partial c}{\partial t_i}\right)^v \right| < \varepsilon$?
$\forall i \in A$

Yes

Print final results

Stop

Flow chart for algorithm of references 6 and 7 (Rizzi).

Start

Input data,
initialize variables

$\nu = 1$

Evaluate constraint c^ν,
total mass m^ν, and
"densities" $(e_\nu)_i^\gamma$
(Separate subroutine)

Compute $(e_{av})^\nu$ and
c_i^ν

$c_i^\nu > 1$
for
passive
variable?

Yes

Add to
active set A

No

Compute $t_i^{\nu+1}$,
$\forall i \in A$

$t_i^{\nu+1} < (t_i)_{min}$?

Yes

$t_i^{\nu+1} = (t_i)_{min}$
Add to
passive set

No

$|c_i^\nu - 1.0| < \epsilon$?
$\forall i \in A$

No

Yes

Print final results

Stop

$t_i^\nu = t_i^{\nu+1}$
$\forall i \in A$

$\nu = \nu + 1$

$\nu > (\nu)_{max}$?

No

Yes

Print message

Stop

Flow chart for "energy-density" algorithm.

APPENDIX B

ON THE OPTIMIZATION OF DISCRETE STRUCTURES

WITH AEROELASTIC CONSTRAINTS[*]

by

Samuel C. McIntosh, Jr., Research Engineer,
Nielsen Engineering & Research, Inc., Mountain View, Calif.

and

Holt Ashley, Professor, Aeronautics and Astronautics,
Stanford University, Calif., U.S.A.

Dedicated to Professor John H. Argyris

on the Occasion of his 65th Birthday

Abstract

It is observed that modern optimal design of structures represents a confluence of two streams of theoretical development: Matric finite-element approximation on the digital computer — a technology of which Professor Argyris is one of the founders; and practical application of the variational calculus. The present paper addresses optimization problems wherein complicated constraints involving dynamic aeroelastic behavior are prominent. Search procedures based on optimality criteria are believed to offer special advantages relative to such problems.

With the principal constraint formulated in terms of the "V-g method" of flutter analysis, three search schemes are applied to the minimum-weight redesign of a particular wing. The first scheme is based on the method of feasible directions and is representative of mathematical-programming methods. The other two are derived from necessary conditions for a local optimum and can be classed as optimality-criteria schemes. Although the results are by no means definitive, they do suggest that a heuristic redesign algorithm based on an optimality criterion may be the best candidate for incorporation in a more general design procedure capable of treating multiple constraints with large numbers of design variables.

The paper's final section undertakes to show how optimality criteria might be constructed when the aeroelastic constraint is written in the time domain. Three special forms of the aerodynamic generalized forces are considered: quasi-steady, quasi-steady with dissipation omitted, and fully unsteady. The resulting criteria for the first and last cases are based on an unproven hypothesis, but it is suggested that their simplicity merits a trial application.

1

1. INTRODUCTION

The design of realistic, efficient structures by procedures based
on the mathematics of optimization is today a routine tool in the manu-
facture of large aircraft and other devices where a combination of light
weight with high reliability is necessary for success. This situation
comes about in consequence of the merging of two streams of analytical
development. The first of these involves the use of finite-element
approximations to built-up structural arrangements, together with matrix
theory and the digital computer. Although hundreds of names might be
mentioned, no one has contributed more to the foundations or to the
current useful state of finite-element methods than Professor Argyris.
In addition to numerous research papers, his several books (e.g.,
Argyris [1]) and his valuable series of articles with Professor Kelsey
in Aircraft Engineering are classics of the field. The computer program
ASKA, developed with his colleagues at Institut für statik und Dynamik
der Flugkonstruktionen in Stuttgart, is in daily use throughout the world
on problems which include the class discussed in this paper.

The second stream derives from variational calculus and the concept
of extrema. Based on the construction of necessary or sufficient conditions
that must be met by the optimum (usually, minimum-weight) design, its
practical aspect consists in the formulation of ever-more-efficient search
methods, which are intended to bring a trial or starting configuration to
within acceptable convergence of the goal by the smallest number of
steps or the least cycles of computer operation. An excellent summary,
with innumerable examples, of the mathematics of optimization is contained
in Bryson and Ho [2]. The AIAA Structures Design Lecture by Schmit [3]
reviews the rich history of aeronautical applications.

2

The present research addresses a small corner of dynamic structural optimization: one where constraints relating to flutter of a wing or other dynamic aeroelastic performance must be imposed along with conditions of a more conventional nature, such as those relating to stress under load, deflection, minimum dimensions of structural elements, etc. This general topic was reviewed recently by Stroud [4] — a survey which perhaps relieves the present authors of responsibility for extensive literature citations. Special recognition should be given, however, to the paper by Turner [5], wherein optimization with rigorous flutter requirements was first formulated in a discrete, finite-element framework.

The focus here is on a single constraint involving aeroelastic stability. Section II begins with a very familiar statement of the flutter problem for a linear system with a finite number of degrees of freedom (cf. Bisplinghoff et al. [6], Sect. 9-5). The structure's motion is assumed in advance to be a simple harmonic time function, and through the artificial introduction of energy dissipation one seeks the actual speed of neutral stability for flight under given atmospheric conditions.

Because flutter calculation is so time consuming in cases of aeronautical interest, there is here a special reason for identifying search methods that require this step as infrequently as possible. It is the authors' opinion that schemes which fall under the heading of optimality criteria offer the best prospect. Accordingly, a relatively simple wing structure, subjected to a single constraint on its flutter performance, is analyzed in several ways and the computer costs are compared. The chosen methods range from well-known and generally-available routines, based on mathematical programming, to a pure criterion approach that is believed to incorporate some new features. While the latter is

3

similar in form to those employed by Siegel [7] and Haftka et al. [8], it alone is capable of converging to a true local optimum. As will be seen, this permits comparisons of efficiency among the different schemes that involve both the same initial design and essentially the same final design.

Section III contains no numerical results but is an exploration of how aeroelastic optimization might be carried out in circumstances when it is undesirable to prescribe simple harmonic motion. Such computations appear more feasible today because of investigations like those of Vepa [9] and Edwards [10], wherein means are described for adapting existing aerodynamic theory to the unsteady flows produced by general small motions of wings or bodies.

The aim of Sect. III is to produce optimality criteria under various approximations to the aerodynamic terms in the equations of motion. Although no such scheme may be regarded as proven until after its successful application to meaningful designs, nevertheless these proposals are deemed worthy of trial. In the process of their development, the concept of the adjoint system plays a significant role. A curious discovery is mentioned, which relates to this adjoint in circumstances where "aerodynamic memory" must be accounted for.

II. COMPARISONS OF DIFFERENT OPTIMIZATION METHODS ON A WING STRUCTURE WITH FIXED FLUTTER SPEED

In his lecture, Schmit [3] has discussed the different philosophies underlying optimization schemes derived from mathematical programming and from optimality criteria. Basically, the mathematical-programming algorithms make use of information from the current design and calculate a design

4

change that will alter the objective in the required direction without violating any of the constraints. Optimality-criterion algorithms are derived — often purely heuristically — from the necessary conditions for optimality, or an approximation thereto. Either approach may or may not require that derivatives of the constraints be calculated. For a complicated behavioral constraint, such as a constraint on flutter speed, the computation of derivatives may introduce unnecessary penalties in terms of computation time. The purpose of this section is to introduce an algorithm that does not require derivatives and to present some comparisons with other algorithms on a relatively simple system governed by a single equality constraint on the flutter speed. Additionally, the new algorithm will be seen to have the capability of converging to the "exact" optimum, so that the comparisons are more meaningful. Although virtually every new or revised scheme that has appeared has been compared with other schemes, there are very few instances (e.g., Haftka et al. [8]) where the comparisons have been made on the same computer system.

2.1. Statement of the Flutter-Speed Constraint

Consider now a lifting surface whose deformation is approximated by superposition of a finite number of free-vibration modes, which may or may not be normal modes. (Typically, they will be the normal modes of an initial design, which for the sake of simplicity will be retained as primitive modes during the optimization.) These modes define generalized coordinates $\xi_i(t)$, $i = 1, \ldots, n$. With the assumption of simple harmonic motion in time t, the governing equations for flutter become

$$\left([M] + [\bar{A}] - \bar{\Omega} [K] \right) \{\bar{\xi}\} = \{0\} \tag{2.1}$$

5

Here $[M]$ and $[K]$ are symmetric $n \times n$ generalized mass and stiffness matrices, respectively, $[\overline{A}]$ is a matrix of oscillatory generalized aerodynamic forces, and $\xi_i(t) = \overline{\xi}_i e^{i\omega t}$. The matrix $[\overline{A}]$ is a complex function of Mach number and reduced frequency. In the "V-g" method, (2.1) is solved for fixed Mach number and a number of reduced frequencies to give a set of eigenvalues $\overline{\Omega}_i$, and eigenvectors $\{\overline{\xi}\}_i$, $i = 1, \ldots, n$. With $\overline{\Omega}_i = (1 + jg_i)/\omega_i^2$, $(j = \sqrt{-1})$, a corresponding set of frequencies ω_i and artificial damping parameters g_i can be calculated. From each frequency and the reduced frequency, an airspeed V can be determined, and the roots of (2.1) can then be plotted as curves of V vs. g for varying values of reduced frequency. The lowest speed at which a root makes the transition from negative g, denoting stability, to positive g, denoting instability, is the critical flutter speed. A final step involves repeating this process for other values of Mach number until the values of the critical flutter speed, the Mach number, and the sound speed at the chosen altitude are compatible.

For the purposes of optimization, there are several ways in which the constraint may be imposed. One possibility is to work directly with the speed itself; this has been done successfully in a number of instances ([11], [12]). Another possibility that offers certain advantages is to fix the speed (and therefore the Mach number) and to constrain the value of g (cf. Segenreich and McIntosh [13]). This practice will be followed here, and the constraint for flutter is then stated simply as

$$g \le 0 \qquad\qquad\qquad (2.2)$$

6

for the critical root (or mode), with speed and Mach number fixed. In the examples that follow, this constraint will be the only behavioral constraint, so (2.2) can really be regarded as an equality constraint.

2.2. A Heuristic Redesign Algorithm Based on an Optimality Criterion

The finite-element representations of many structures involve elements whose stiffness and inertial properties depend linearly on the design variables, which are commonly plate thicknesses, spar-cap cross-sectional areas, etc. This linear dependence will therefore be assumed here, and the weight to be minimized will be written in the form

$$J_o = \sum_{i=1}^{N} a_i t_i \tag{2.3}$$

There may, in fact, be some mass that is invariant with respect to the optimization, but it is not necessary to include it in (2.3). In accordance with Turner [5] and Haftka et al. [8], the equations of motion (2.1), written for the critical mode, are premultiplied by a row matrix of Lagrange multipliers or adjoint variables, and the real part of this quantity is adjoined to J_o to give

$$J(\{\overline{q}\}, \{\overline{\xi}\}, \overline{\Omega}, t_i) = J_o + \text{Re}(\lfloor \overline{q} \rfloor [B] \{\overline{\xi}\}) \tag{2.4}$$

where

$$[B] = [M] + [\overline{A}] - \overline{\Omega}[K] \tag{2.5}$$

Necessary conditions for optimality are given by the vanishing of variations of J with respect to $\overline{\Omega}$, t_i, and the elements of $\{\overline{q}\}$ and $\{\overline{\xi}\}$. These yield, respectively,

7

$$\text{Re} \left(\lfloor \overline{q} \rfloor \left[\frac{\partial B}{\partial \overline{\Omega}} \right] \{\overline{\xi}\} \right) = 0 \tag{2.6}$$

$$a_i + \text{Re} \left(\lfloor \overline{q} \rfloor \left[\frac{\partial B}{\partial t_i} \right] \{\overline{\xi}\} \right) = 0, \quad i = 1, \ldots, N \tag{2.7}$$

$$[B] \{\overline{\xi}\} = \{0\} \tag{2.8}$$

$$\left(\lfloor \overline{q} \rfloor [B] \right)^T = [B]^T \{\overline{q}\} = \{0\} \tag{2.9}$$

Since $\overline{\Omega} = 1/\omega^2$ on the constraint boundary where $g = 0$, (2.6) is equiv-alent to the vanishing of the variation of J with respect to the flutter frequency ω, and it can be viewed as a relation giving the flutter frequency at the optimum design. The original constraint equations are reproduced by (2.8), while (2.9) defines the adjoint equations. The optimality criterion is given by (2.7). Under the aforementioned assump-tion of linear dependence of the inertial and stiffness properties, $[M]$ and $[K]$ can be written as

$$[M] = \sum_{i=1}^{N} t_i [M_i] \tag{2.10}$$

$$[K] = \sum_{i=1}^{N} t_i [K_i] \tag{2.11}$$

and (2.7) becomes

$$(e_v)_i = \frac{1}{a_i} \text{Re} \langle \lfloor \overline{q} \rfloor \left([M_i] - \overline{\Omega} [K_i] \right) \{\overline{\xi}\} \rangle = -1 \tag{2.12}$$

The left-hand sides of (2.12) resemble energy densities, and the optimality criterion is therefore seen to require that all of these "energy densities" have the same value. If minimum-gage constraints on the design variables are also specified, then some design variables may become passive — i.e., equal to their minimum allowable values—during redesign. If this occurs, then (2.12) must hold only for the active design variables.

One can create any number of iterative redesign schemes based on (2.12). One particularly attractive candidate is the following:

$$t_i^{\nu+1} = c_i^\nu \, t_i^\nu \tag{2.13}$$

$$c_i^\nu = \left| \frac{(e_v)_i^\nu}{e_{av}^\nu} \right|^{e_1} (1+g^\nu)^{e_2} \tag{2.14}$$

Here ν is the iteration number, and e_{av} is the average of the $(e_v)_i$ for all the active variables. The absolute value of the "energy-density" ratio is required, since the exponent e_1 is typically less than unity and the $(e_v)_i$ may be either positive or negative. Note that as the "energy-density" ratios approach unity, the constraint factor $(1+g)^{e_2}$ serves to ensure that the constraint $g = 0$ will be satisfied. The form of (2.14) is derived from two assumptions:

    (a) If $|(e_v)_i| > |e_{av}|$, the corresponding design variable should be increased, and

    (b) If the current design is not feasible $(g^\nu > 0)$, all design variables should be increased.

Convergence of this iterative formula cannot be proven, and there is no guarantee that the formula will be capable of equalizing not only the magnitudes but also the signs of the $(e_v)_i$, which is a necessary condition for optimality according to (2.12). The formula (2.14) is similar to that used by Haftka et al. [8] , except that the absolute value rather than the real part was used in defining the $(e_v)_i$ in (2.12). In effect, this means that the algorithm used in [8] attempts to satisfy an approximate rather than an exact optimality criterion, and the final designs obtained with this algorithm did not correspond to the final designs obtained with other methods.

9

The analysis and redesign algorithm proposed herein proceeds in
the following steps:

(1) The current value of the constraint is found by solving (2.8)
    and obtaining g, ω, and $\{\overline{\xi}\}$ for the critical mode. Since
    the aerodynamic matrix $[\overline{A}]$ will vary with frequency, it is
    required that the frequency used in defining $[\overline{A}]$ coincide with
    the frequency calculated from $\overline{\Omega}$. This is achieved iteratively.
    The frequency from the previous design is used initially to
    define $[\overline{A}]$ and is then compared with the frequency calculated
    from $\overline{\Omega}$. If these two frequencies are not in agreement to within
    a specified limit, the frequency computed from $\overline{\Omega}$ is used to
    determine a new generalized aerodynamic matrix $[\overline{A}]$ and the
    process is repeated.

(2) The adjoint equations (2.9) are solved, and $\{\overline{q}\}$ for the
    critical mode is obtained.

(3) The densities $(e_v)_i^\nu$ are calculated as in (2.12) for the
    current active set of design variables, and e_{av}^ν and the
    C_i^ν are calculated. The C_i^ν are calculated for all i.

(4) The new active design variables $t_i^{\nu+1}$ are calculated. If
    any of these is less than its specified minimum value, it is
    set to that minimum value and is relegated to the passive set.

(5) If for a passive variable $C_i^\nu > 1.0$, this variable is reintro-
    duced to the active set and steps (3) – (5) are repeated until
    the active-passive identities are stable. Once this stability
    is achieved, the new set of design variables $t_i^{\nu+1}$ is taken
    as the next design.

(6) Convergence is checked by testing the C_i^ν for the active
set. If $|1 - C_i^\nu| < \epsilon$ for all of the C_i^ν tested, the new
design is declared final. If not, the algorithm is repeated.
The convergence parameter ϵ is set by the user.

Before a numerical example is given, it will be instructive to consider
briefly an alternative optimality criterion and to explore its relationship
to the criterion derived above. This criterion is derived from variations
of the function

$$J(\lambda, t_i) = J_o + \lambda g \tag{2.15}$$

Vanishing of the variations of J with respect to the design variables
yields the optimality criterion

$$\frac{1}{a_i} \frac{\partial g}{\partial t_i} = -\frac{1}{\lambda} \tag{2.16}$$

Once again, this equality holds for those design variables that are active
at the optimum. Following Segenreich and McIntosh [13], one can write for
the derivatives

$$\frac{\partial g}{\partial t_i} = \left[(R_2^i - \omega^2 R_1^i - g I_2^i)(2g R_3 + 2I_3 + \frac{b\omega^3}{V} I_4) \right.$$
$$\left. - (2R_3 - 2g I_3 + \frac{b\omega^3}{V} R_4)(I_2^i - \omega^2 I_1^i + g R_2^i) \right] /D \tag{2.17}$$

where

$$D = (2g R_3 + 2I_3 + \frac{b\omega^3}{V} I_4) I_3 + (2R_3 - 2g I_3 + \frac{b\omega^3}{V} R_4) R_3, \tag{2.18}$$

$$R_1^i = \text{Re} (\lfloor \overline{q} \rfloor [M_i] \{\overline{\xi}\}), \tag{2.19}$$

$$R_2^i = \text{Re} (\lfloor \overline{q} \rfloor [K_i] \{\overline{\xi}\}), \tag{2.20}$$

11

$$R_3 = \text{Re} (\lfloor \overline{q} \rfloor [K] \{ \overline{\xi} \}), \tag{2.21}$$

$$R_4 = \text{Re} (\lfloor \overline{q} \rfloor \left[\frac{\partial \overline{A}}{\partial k} \right] \{ \overline{\xi} \}), \tag{2.22}$$

I_1^i, I_2^i, I_3, and I_4 are the corresponding imaginary parts, b is a reference length, and $k = \omega b/V$ is the reduced frequency. At first glance, it would seem that (2.16) bears little resemblance to its counterpart (2.12). However, by making use of (2.6), which is not obtained from the formulation immediately above, it can be shown that <u>at the optimum</u>

$$2R_3 - 2g\, I_3 + \frac{b\omega^3}{V} R_4 = 0 \tag{2.23}$$

Making use of this relation in (2.18) and (2.17) produces a vastly simplified expression for $\frac{\partial g}{\partial t_i}$, which when inserted into (2.16) gives

$$\frac{1}{a_i} (\omega^2 R_1^i - R_2^i + g\, I_2^i) = \frac{I_3}{\lambda} \tag{2.24}$$

Since $\overline{\Omega} = (1 + jg)/\omega^2$, (2.12) can be rewritten with the definitions (2.19)-(2.22) to give

$$\frac{1}{a_i} (\omega^2 R_1^i - R_2^i + g\, I_2^i) = -\omega^2 \tag{2.25}$$

Since the quantities R_1^i, etc. involve triple matrix products with $\{q\}$ and $\{\overline{\xi}\}$, and since the normalization of $\{q\}$ and $\{\overline{\xi}\}$ is arbitrary, (2.24) and (2.25) are in fact equivalent expressions. It must be emphasized, however, that the simplified expression for $\frac{\partial g}{\partial t_i}$ is valid only at the optimum, so that an optimization scheme that requires derivative calculations must use the full expression (2.17).

12

2.3. A Numerical Example

In order to illustrate some of the ideas discussed above, consider the rectangular wing whose dimensions are given in Figure 1. This wing was created by Rudisill and Bhatia [11] and has since been considered by Segenreich and McIntosh [13], among others. The structural box has three bays. In each bay there are two cover sheets, two spar webs, and one rib, all modeled by in-plane rectangular elements, and four spar caps, modeled by axial elements. The design variables are numbered as shown in Table 1. The initial design values were: $t_1 - t_3 = 12.90 \text{ cm}^2$, $t_4 - t_6 = 0.2032$ cm., $t_7 - t_{12} = 0.1016$ cm. The weight of the initial design was 88.45 kg. Minimum-gage constraints were imposed at one quarter of the initial values. For the flutter calculations, the first six transverse free-vibration modes of the initial design were used to define generalized coordinates. The doublet-lattice method [14] was used to calculate generalized aerodynamic forces, and a flutter Mach number of 0.717 was calculated at an altitude of 1,372 meters. This flutter point was imposed as the primary behavioral constraint. Optimal designs were calculated with three methods — the "energy-density" method described in 3.2, the method used by Segenreich and McIntosh [13], and a method developed by Vanderplaats [15]. The method in [13] is derived from an algorithm due to Kiusalaas [16], which is based on an optimality criterion of the form (2.16) and requires calculation of the derivatives $\frac{\partial g}{\partial t_i}$ at each iteration. The method in [15] is based on a mathematical-programming procedure known as the method of feasible directions [17].

The final designs from all three methods were essentially identical. The only active design variable was t_4, the thickness of the front and rear spar webs in bay 1. The final designs, constraint values, and relative

13

computer CPU times are given in Table 2. All computations were performed on an IBM 370/168 computer at Stanford University. Both Siegel [7] and Haftka et al. [8] used $e_1 = 0.5$, $e_2 = 2.0$ in their computations. With these values, the algorithm of Sect. 2.2 diverged, and it was necessary to reduce e_1 to 0.15 in (2.14) before convergence could be demonstrated.

Before the flutter constraint was considered, the same structure was optimized with a displacement and a natural-frequency constraint. For these problems, it was possible to leave $\dot{e}_1$ at 0.5. All three methods produced almost identical final designs and CPU-time comparisons similar to those in Table 1.

The final design was reanalyzed for flutter with new (normal) modes in order to ensure that the root that was constrained was still the critical root. This was found to be the case; the critical branches of the initial and final designs are compared in Fig. 2. Using the normal modes of the initial design as primitive modes during redesign is seen to cause an error of 5.5% in the estimation of the flutter speed in comparison with the flutter speed calculated with normal modes of the final design. In more complex practical applications of the method, especially when the initial design is far from optimal, one should occasionally re-calculate the normal modes. They are then to be used as the basis for subsequent iterations, and slight mismatches of the final flutter speed can thus be avoided.

14

III. OPTIMALITY CRITERIA BASED ON AEROELASTIC EQUATIONS IN THE TIME AND LAPLACE-TRANSFORM DOMAINS

Since optimality criteria are closely related to variational principles from which the associated equations of motion can be derived, the first stage of this investigation looks at these principles and at the adjoint systems which are an inherent feature when nonconservative effects are present. The starting point, as in Sect. II, is a reference design whose behavior can be adequately represented by the generalized coordinates $\xi_i(t)$, i = 1, ... n, of some subset of its natural modes of free vibration. The generalized masses and stiffnesses are described by symmetric nxn matrices [M] and [K], respectively. At the beginning of optimization, these matrices may be diagonal. But, as the search proceeds, the generalized coordinates and associated eigenfunctions are left unchanged. Therefore $[M(m_k)]$ and $[K(m_k)]$ become full matrices, functions of the N added or subtracted masses m_k that constitute the vector of design variables. The dependence on m_k may, in one or both instances, be linear (cf. Turner [5]), but this is not necessary to the development.

For the analysis of homogeneous aeroelastic phenomena like flutter stability, the equations of motion governing the column matrix $\{\xi(t)\}$ contain aerodynamic generalized forces, linearly dependent on this matrix. Two cases are considered in what follows.

3.1. Quasi-Steady Aerodynamics

In this approximation (cf. Sect. 5-6 of Bisplinghoff et al. [6] and Pines [18], among many other sources of information) there is negligible "memory," so that the airloads are algebraically related to [†] $\{\xi\}$ and $\{\dot{\xi}\}$.

[†] The "apparent mass" effect might also be included through a $\{\ddot{\xi}\}$ term, but this refinement would contribute nothing of significance.

One useful form of the equations then reads

$$[M]\{\ddot{\xi}\} + [K]\{\xi\} - \Big([D]\{\dot{\xi}\} + [E]\{\xi\}\Big) = 0 \qquad (3.1)$$

Here $[D]$ and $[E]$ are nonsymmetric matrices of real constants for a given flight condition. It proves desirable to extract from these terms a quantity with dimensions of force, containing the aeroelastic eigenvalue Q_∞. This eigenvalue may be the actual flight dynamic pressure or perhaps Mach number, etc. [cf. the parameter λ_0 used by Weisshaar ([19], Chapter 6) in panel flutter optimization]. Equations (3.1) then become

$$[M]\{\ddot{\xi}\} + [K]\{\xi\} - Q_\infty S \Big(\overline{c}/V \;\; [d]\{\dot{\xi}\} + [e]\{\xi\}\Big) = 0 , \qquad (3.2)$$

where $[d]$ and $[e]$ would normally be dimensionless. S is a reference area such as that of the wing planform projection; reference wing chordlength $\overline{c}$ and flight speed V are employed to cancel the dimensions of the time derivative.

Let $q_j(t)$, $j = 1, \cdots , n$, be generalized coordinates for the adjoint system. The equations governing the column matrix of the q_j are

$$[M]\{\ddot{q}\} + [K]\{q\} - Q_\infty S \Big(-\overline{c}/V [d]^T \{\dot{q}\} + [e]^T \{q\}\Big) = 0 \qquad (3.3)$$

Here superscript T denotes the transpose of a square matrix. One way of deriving (3.3) is to require stationarity, with respect to variations in the elements of $\{\xi\}$ and $\{q\}$, of a generalized Hamilton's principle which might be written

$$H = \int_{t_1}^{t_2} \langle - \lfloor \dot{q} \rfloor [M]\{\dot{\xi}\} + \lfloor q \rfloor [K]\{\xi\} + Q_\infty S \Big(\overline{c}/V \Big)\lfloor \dot{q} \rfloor [d]\{\xi\}$$
$$- Q_\infty S \lfloor q \rfloor [e]\{\xi\} \rangle dt \qquad (3.4)$$

16

The open brackets mean a row matrix. In the usual way, "initial" conditions are prescribed at the time limits t_1 and t_2 to prevent nonzero terms from arising during integration by parts. It is remarked that H — like the optimality criteria based on it, to be proposed below — is not unique. Thus the third term in the integrand could have its sign changed and the time differentiation transferred from $\lfloor q \rfloor$ to $\{\xi\}$.

3.2. Linear Aerodynamics with "Memory."

When analyzing stability, experience shows that one must usually account for the unsteady influences of the past history of a wing or body's motion. They are due to the presence of a vortex wake shed as the result of prior changes in the circulation "bound" to a wing, to the finite speed of sound propagation in the gas, or to both of these effects. One must then replace the aerodynamic terms in (3.1) by a convolution

$$[A] * \{\xi\} \equiv - \int_0^t [A(t-\tau)] \{\xi(\tau)\} \, d\tau \tag{3.5}$$

In this form, the motion is assumed to have begun at $t = 0$. The elements $A_{ij}(t)$ of $[A]$ are indicial time functions, giving the generalized force exerted on one degree of freedom i due to impulsive motion in another j. In most cases these elements are known, without approximation, only in terms of their Laplace transforms $\overline{A}_{ij}(s)$.

Replacing $[A]$ by a dimensionless equivalent $[a]$ as before, one is led to the fully unsteady generalization of (3.2):

$$[M]\{\ddot{\xi}\} + K\{\xi\} - Q_\infty S \int_0^t [a(t-\tau)] \{\xi(\tau)\} \, d\tau = 0 \tag{3.6}$$

Clearly, (3.6) can be derived from stationarity, with respect to the elements of $\lfloor q \rfloor$, of

17

$$H = \int_{t_1}^{t_2} \langle - \lfloor \dot{q} \rfloor [M]\{\dot{\xi}\} + \lfloor q \rfloor [K]\{\xi\}$$

$$- Q_\infty S \lfloor q(t) \rfloor \int_0^t [a(t-\tau)]\{\xi(\tau)\}\, d\tau \rangle \, dt \tag{3.7}$$

It is of interest to examine the adjoint equation, the one governing $\{q(t)\}$. To this end and before a meaningful variation of H with respect to ξ_i can be carried out, the order of integrations in (3.7) must be inverted and the variables t and τ interchanged. Since the result should not depend on the choice of t_1 and t_2, it seems convenient to replace these quantities with the starting time $t_1 = 0$ and with $t_2 = \infty$. After these choices have been made, simple manipulations lead to

$$[M]\{\ddot{q}\} + [K]\{q\} - Q_\infty S \int_t^\infty [a(\tau-t)]\{q(\tau)\}\, d\tau = 0 \tag{3.8}$$

Albeit there are other ways of writing the adjoint equation, (3.8) has a certain intrigue. If one attempts to give physical meaning to the aerodynamic term, it would seem to imply that the state at instant t is affected by events subsequent in time. Such speculation is apparently not fruitful. As in other nonself-adjoint systems, there is often a great deal of artificiality in the adjoint problem, and this feature is here exacerbated by the presence of a convolution in (3.6). The convolution theorem of Laplace transformation, incidentally, does not apply to the integral in (3.8).

In an effort to gain insight, an elementary problem with a single degree of freedom $\xi(t)$ was constructed from (3.7) and (3.8). The single indicial function was taken as a simple lag,

$$A(t) = a_0 e^{-\sigma t} \tag{3.9}$$

Even in this case, the solution of (3.6) and initial conditions

18

$$M\ddot{\xi} + M\omega^2 \xi - a_o \int_0^t e^{-\sigma(t-\tau)} \xi(\tau) \, d\tau \qquad (3.10a)$$

$$\xi(0) = \xi_o \; ; \quad \dot{\xi}(0) = 0, \qquad (3.10b)$$

is not particularly convenient to write out. The special choice $a_o = K\sigma = M\omega^2\sigma$ causes a pole-zero cancellation, however, yielding the elementary result

$$\frac{\xi(t)}{\xi_o} = \frac{\omega}{\omega_n} e^{-\sigma t/2} \sin(\omega_n t + \psi), \qquad (3.11a)$$

$$\text{with} \quad \omega_n = \omega \sqrt{1 - (\sigma/2\omega)^2} \qquad (3.11b)$$

$$\psi = \tan^{-1}(2\omega_n/\sigma) \qquad (3.11c)$$

The corresponding adjoint $q(t)$, also with

$$q(0) = q_o; \quad \dot{q}(0) = 0 \qquad (3.12)$$

cannot be calculated by Laplace transformation. Merely assuming it in the form of (3.11a), however, yields

$$q(t)/q_o = \xi(t)/\xi_o \qquad (3.13)$$

This is physically reasonable and not unexpected, because the system and its adjoint should have the same eigenvalues.

3.3. The Search for Optimality Criteria.

As an aid to deriving criteria that might enable efficient search routines while aeroelastic constraints are enforced, the ideas in Sect. 5b of Plaut's review [20] furnished an excellent lead. This presentation is, accordingly, broken into three parts, the first dealing just with a discrete version of the same kind of nonconservative system treated by that author.

19

3.3a. Minimum-Weight Structure with Prescribed Flutter Stability Boundary of the Frequency-Merging Type.

In the quasi-steady Eq. (3.2), specialize further by assuming

$$[d] = 0 \tag{3.14}$$

This elimination of dissipative mechanisms ensures that the eigenvalues of

$$[M]\{\ddot{\xi}\} + [K]\{\xi\} - Q_\infty S [e]\{\xi\} = 0 \tag{3.15}$$

remain purely imaginary as parameter Q_∞ is increased from zero. Because of the asymmetry of $[e]$, however, there is some $Q_\infty = Q_{\infty_C}$ at which a pair becomes equal and produces "instability by merging of frequencies" (cf. Pines [18]).

In a form slightly different from that in Sect. II, a typical optimization problem may be stated as follows:

$$\text{Minimize } J_o = \sum_{k=1}^{N} m_k , \tag{3.16}$$

with $\{\xi\}$ a solution of (3.15) such that

$$Q_{\infty_C} \geq Q_o \tag{3.17}$$

and (for example)

$$m_k \geq m_{k_o} , \quad k = 1, 2, \ldots N . \tag{3.18}$$

Observe that the m_{k_o} can be inferred from specified minimum dimensions of structural members; if J_o represents the reduction of total weight from an initial design, they will be negative numbers. Equation (3.15), its adjoint, and the generalized Hamiltonian integrand (cf. 3.4) are rewritten with

20

the substitutions $s^2 = -\Omega$,

$$\xi_i(t) = \bar{\xi}_i e^{st} \quad \text{and} \quad q_j(t) = \bar{q}_j e^{st} : \tag{3.19}$$

$$\left(-\Omega [M] + [K] - Q_\infty S[e]\right)\{\bar{\xi}\} = 0 \tag{3.20}$$

$$\left(-\Omega [M] + [K] - Q_\infty S[e]^T\right)\{\bar{q}\} = 0 \tag{3.21}$$

$$\bar{h} = \lfloor\bar{q}\rfloor [K]\{\bar{\xi}\} + \Omega \lfloor\bar{q}\rfloor [M]\{\bar{\xi}\} - Q_\infty S \lfloor\bar{q}\rfloor [e]\{\bar{\xi}\} \tag{3.22}$$

[Note that, although H is nonlinear in q and ξ, it is expected that the eigenvectors will be real up to $Q_{\infty c}$. Therefore, there is no question of taking real parts in (3.22).]

Plaut [20] points out that the characteristic equation of (3.20) or (3.21) has the form

$$F(\Omega_i, Q_\infty) = 0 \tag{3.23}$$

He reasons that, with two frequencies merged at $\Omega = \Omega_c$ and $Q = Q_{\infty c}$, it should be true that

$$\left.\frac{d Q_\infty}{d\Omega}\right|_{\substack{\Omega = \Omega_c \\ Q_\infty = Q_{\infty c}}} = 0 \tag{3.24}$$

He suggests premultiplying (3.20) by $\lfloor\bar{q}\rfloor$ and solving for Q_∞:

$$Q_\infty = \frac{-\Omega\lfloor\bar{q}\rfloor [M]\{\bar{\xi}\} + \lfloor\bar{q}\rfloor [K]\{\bar{\xi}\}}{S \lfloor\bar{q}\rfloor [e]\{\bar{\xi}\}} \tag{3.25}$$

Applying (3.24) to (3.25) then produces a relation

21

$$\lfloor \overline{q} \rfloor [M] \{\overline{\xi}\} = 0 ,$$
(3.26)

which must hold at the critical condition.

The manner of constructing an optimality criterion is to adjoin $\overline{h}$ to the performance index J_o with a Lagrange multiplier λ ,

$$J \left(\{\overline{\xi}\} , \{\overline{q}\}, \Omega , m_k \right) = \sum_{k=1}^{N} m_k$$
$$+ \lambda \left(\lfloor \overline{q} \rfloor [K(m_k)] \{\overline{\xi}\} + \Omega \lfloor \overline{q} \rfloor [M(m_k)] \{\overline{\xi}\} - Q_\infty S \lfloor \overline{q} \rfloor [e] \{\overline{\xi}\} \right)$$
(3.27)

The variations (or partial derivatives) of J with respect to the elements of $\{\overline{\xi}\}$ and $\{\overline{q}\}$ produce (3.21) and (3.20), respectively. The vanishing of the variation with respect to Ω is assured by (3.26). Finally, variations on the design variables m_k yield

$$- 1 = \lambda \left(\lfloor \overline{q} \rfloor \left[\frac{\partial K}{\partial m_k} \right] \{\overline{\xi}\} + \Omega \lfloor \overline{q} \rfloor \left[\frac{\partial M}{\partial m_k} \right] \{\overline{\xi}\} \right)$$
(3.28)

Note that the equality, for all $k = 1, \ldots, N$, of the quantities in parentheses in (3.28) furnishes the desired criterion. Just as in Sect. II the "energy ratios" are employed to adjust the individual masses during progress toward an optimum, so these quantities should be able to do the job here without the need for differentiations with respect to eigenvalues. That such criteria do lead to meaningful designs is implied by the simple examples given in Plaut [20].

3.3b. More General Quasi-Steady Airloads

The physical and adjoint equations of motion in this case are the full (3.2) and (3.3). To be specific, let it be required that, at a given Q_∞, the least stable of the aeroelastic eigenvalues has a given degree of stability — a condition that might be expressed

22

$$\text{Re }\{s_i\} \leq - s_o, \quad \text{for all} \quad i = 1, 2 \ldots .2n \tag{3.29}$$

Here s_o is a positive real constant. Under assumption (3.19), the real or complex-conjugate pairs of s_i are eigenvalues of

$$\left\langle s^2 [M] + [K] - Q_\infty S \left(\frac{s\overline{c}}{V} [d] + [e]\right)\right\rangle \{\overline{\xi}\} = 0 \tag{3.30}$$

and also of the corresponding equations from (3.3). Subject to these constraints and to a prescribed Q_∞, the optimization problem then consists of (3.16), (3.18) and (3.29).

By analogy with (3.27), it is now hypothesized that an appropriate formulation requires the vanishing of variations with respect to ξ_i, q_j, m_k and s of

$$J\left(\{\overline{\xi}\}, \{\overline{q}\}, s, m_k\right) = \sum_{k=1}^{N} m_k + \text{Re} \left\langle \lambda \left(\lfloor \overline{q} \rfloor [K]\{\overline{\xi}\} - s^2 \lfloor \overline{q} \rfloor [M]\{\overline{\xi}\}\right.\right.$$

$$\left.\left. - Q_\infty S \left(\frac{\overline{cs}}{V}\right) \lfloor \overline{q} \rfloor [d]\{\overline{\xi}\} + Q_\infty S \lfloor \overline{q} \rfloor [e]\{\overline{\xi}\}\right) \right\rangle \tag{3.31}$$

Here, of course, λ, the eigenvectors and s_i are all complex numbers. The line of reasoning whereby the real part is taken in (3.31) follows directly that given first by Turner [5].

As before, two additional relations (beside the physical and adjoint equations) are obtained from variations of (3.31).

$$- 1 = \text{Re} \left\langle \lambda \left(\lfloor \overline{q} \rfloor \left[\frac{\partial K}{\partial m_k}\right]\{\overline{\xi}\} - s^2 \lfloor \overline{q} \rfloor \left[\frac{\partial M}{\partial m_k}\right]\{\overline{\xi}\}\right) \right\rangle \tag{3.32}$$

for all $k = 1, \ldots, N$. Also

$$\text{Re} \left\langle \lambda \left(2s \lfloor \overline{q} \rfloor [M]\{\overline{\xi}\} + Q_\infty S \frac{\overline{c}}{V} \lfloor \overline{q} \rfloor [d]\{\overline{\xi}\}\right) \right\rangle = 0 \tag{3.33}$$

23

The solution procedure that is proposed to be tried starts from calculating, at a given design step, all eigenvalues and vectors of (3.30) and its adjoint. In general (3.32) and (3.33) will not be satisfied. For a particular eigenvalue s (the least stable), they amount to $N + 1$ real equations involving $N + 2$ variables: the N m_k's, $Re\{\lambda\}$ and $Im \{\lambda\}$. Using the known m_k for the given design, one might select a single one of (3.32) plus (3.33) and calculate λ . With λ available, the remaining members of (3.32) can be used to adjust the various m_k up or down, aiming for a design in which the quantities within the parentheses of (3.32) are equal for all k.

One difficulty remains: that of how to drive the real part of the least stable of the s_i toward s_o at each step. This might be done by numerically differentiating the stability determinant of (3.30) with respect to the "most active" of the design variables m_k. This m_k could be altered by just the amount needed to bring $Re \{s_i\}$ to s_o. Then the aforementioned optimality criterion from (3.32) provides a way of correcting all other m_k. (Note that "passive" m_k, which already coincide with their minima prescribed by (3.18), are omitted.)

Obviously, the changes in the other m_k (besides the "most active") are going to throw off the system stability. Nevertheless, these changes become progressively smaller as the optimization progresses, so choosing the particular index k whose mass seems to be varying most rapidly may furnish a sufficiently powerful control on $Re \{s_i\}$.

24

3.3c. Fully Unsteady Airloads

Surprisingly, this case appears to be a fairly straightforward extension of the treatment in Sect. 3.3b. The key step seems to be to assume that the motion has gone on for a long time prior to $t = 0$. In this case, (3.6) and (3.8) must be modified to

$$[M]\{\ddot{\xi}\} + [K]\{\xi\} - Q_\infty S \int_{-\infty}^{t} [a(t-\tau)] \{\xi(\tau)\} \, d\tau = 0 \qquad (3.34)$$

$$[M] \{\ddot{q}\} + [K]\{q\} - Q_\infty S \int_{-\infty}^{\infty} [a(\tau - t)]^T \{q(\tau)\} \, d\tau = 0 \qquad (3.35)$$

Leaving aside certain questions of existence, one again chooses the homogeneous solutions (3.19). This substitution into (3.34), with the variable change $\tau_1 = t - \tau$, then leads to the algebraic system

$$\left([K] + s^2 [M] - Q_\infty S \, [\overline{a}\,(s)] \right) \{\overline{\xi}\} = 0 , \qquad (3.36)$$

where the $\overline{a}_{ij}(s)$ are the Laplace transforms of the aerodynamic indicial functions — exactly the quantities which the Edwards [10] investigation shows how to calculate.

Inserting (3.19) into (3.34) gives rise to the following manipulations of the aerodynamic integral:

$$\int_{-\infty}^{\infty} [a(\tau - t)]^T \{q(\tau)\} \, d\tau = \left(\int_{t}^{\infty} e^{s(\tau - t)} [a(\tau - t)]^T \, d\tau \right) \{\overline{q}\} \, e^{st}$$

$$= \left(\int_{0}^{\infty} e^{s\tau_2} [a(\tau_2)]^T \, d\tau_2 \right) \{\overline{q}\} \, e^{st} = [\overline{a}\,(-s)]^T \{\overline{q}\} \, e^{st} \qquad (3.37)$$

In (3.37) $\tau_2 = \tau - t$, and the elements of the final square matrix are simply the Laplace transforms of a_{ij}, with the sign of s reversed. Substituting (3.37) into (3.35), one obtains

25

$$\left([K] + s^2 [M] - Q_\infty S [\overline{a}(-s)]^T \right) \{\overline{q}\} = 0 \tag{3.38}$$

The steps in the proposed-optimality-criterion approach parallel rather closely those starting from (3.31). The expressions containing matrices [d], [e] and their tranposes must, of course, be adjusted by the insertion of $[\overline{a}(s)]$ and $[\overline{a}(-s)]^T$, as appropriate. The quantity whose variations yield the needed data is

$$J = \sum_{k=1}^N m_k + \text{Re} \left\langle \lambda \left(\lfloor \overline{q} \rfloor [K] \{\overline{\xi}\} - s^2 \lfloor \overline{q} \rfloor [M] \{\overline{\xi}\} - Q_\infty S \lfloor \overline{q} \rfloor [\overline{a}(s)] \{\overline{\xi}\} \right) \right\rangle \tag{3.39}$$

The principal complication over the quasi-steady case seems to be that (3.33) is replaced by a more elaborate formula containing derivatives of the elements of $[\overline{a}(s)]$. Although these derivatives might have to be obtained numerically in the "exact" formulation, one suspects that their computation will be greatly simplified by adopting the Padé approximants of Vepa [9].

26

IV.   CONCLUDING REMARKS

Although the example considered in Sect. V is a very simple one, the
results are encouraging enough to support additional investigations with
the new algorithm described in Sect. 2.2. It will, of course, be necessary
to apply all of the methods employed in Sect. 2.3 to more realistic problems
before any quantitative assessments can be given concerning relative
efficiency. In particular, it should be noted that the weight reduction
achieved — some 74% of the initial weight — is not truly representative
of what would occur in actual practice. Ultimately the methods need to be
tested for multiple behavioral constraints. The mathematical-programming
procedure CONMIN [15] is already capable of treating such problems, and
the work of Segenreich et al. [21] and Rizzi [22] will provide interesting
possibilities for introducing a multiple-constraint capability into the
algorithm of Sect. 2.2.

Having thus demonstrated, by means of a familiar example, the attrac-
tiveness of optimality criteria in connection with an analytically-complicated
design constraint like flutter, the paper proceeds into a more speculative
area. It cannot be too strongly emphasized that expressions like (3.31)
and (3.39) are hypothesized. The similarity of criteria derived from them
to proven counterparts like (3.27), however, lends credence to the proposal
that these schemes be accorded a fair trial. One may cite the interesting
and successful work of Siegel [7] as evidence that complete mathematical
rigor is not always necessary in a procedure for design optimization.

REFERENCES

1. J.H. Argyris. _Recent Advances in Matrix Methods of Structural Analysis_. Pergamon Press, London (1964).

2. A.E. Bryson, Jr. and Y.-C. Ho. _Applied Optimal Control_, Revised Printing, a Halsted Press Book. John Wiley & Sons, New York (1975).

3. L.A. Schmit, Jr. Structural Synthesis from Abstract Concept to Practical Tool. Annual Structures Design Lecture, presented at AIAA/ASME 18th Structures, Structural Dynamics, and Materials Conf., San Diego (1977).

4. W.J. Stroud. Automated Structural Design with Aeroelastic Constraints: A Review and Assessment of the State of the Art. _Structural Optimization Symposium_, AMD-Vol. 7. ASME, New York (1974), pp. 77-118.

5. M.J. Turner. Optimization of Structures to Satisfy Flutter Requirements. _AIAA J._ 7, 945-951 (1969).

6. R.L. Bisplinghoff, H. Ashley, and R.L. Halfman. _Aeroelasticity_. Addison-Wesley Publishing Co., Reading, Mass. (1955).

7. S. Siegel. A Flutter Optimization Program for Aircraft Structural Design. AIAA Paper No. 72-795 (1972).

8. R.T. Haftka, J.H. Starnes, Jr., F.W. Barton, and S.C. Dixon. Comparison of Two Types of Structural Optimization Procedures for Flutter Requirements. _AIAA J._ 13, 1333-1339 (1975).

9. R. Vepa. Finite State Modeling of Aeroelastic Systems. NASA CR-2779 (1977).

10. J.W. Edwards. Unsteady Aerodynamic Modeling and Active Aeroelastic Control. SUDAAR 504, Dept. of Aero. and Astro., Stanford Univ., Stanford Calif. (1977). (A summary paper by Edwards, Ashley, and Breakwell will appear shortly in AIAA J.)

28

11. C.S. Rudisill and K.G. Bhatia. Optimization of Complex Structures to Satisfy Flutter Requirements. *AIAA J.* 9, 1487-1491 (1971).

12. L.B. Gwin and R.F. Taylor. A General Method for Flutter Optimization. *AIAA J.* 11, 1613-1617 (1973).

13. S.A. Segenreich and S.C. McIntosh, Weight Minimization of Structures for Fixed Flutter Speed via an Optimality Criterion. AIAA Paper No. 75-779 (1975).

14. E. Albano and W.P. Rodden. A Doublet Lattice Method for Calculating Lift Distributions on Oscillating Surfaces in Subsonic Flows. *AIAA J.* 7, 279-285 (1969).

15. G.N. Vanderplaats. CONMIN - A Fortran Program for Constrained Function Minimization. User's Manual. NASA TM X-62,282 (1973).

16. J. Kiusalaas. Minimum Weight Design of Structures via Optimality Criteria. NASA TN D-7115 (1972).

17. G. Zoutendijk. *Methods of Feasible Directions*. Elsevier Publishing Co., Amsterdam (1960).

18. S. Pines. An Elementary Explanation of the Flutter Mechanism. *Proceedings of National Specialists Meeting on Dynamics and Aeroelasticity*. IAS, New York (1958), pp. 52-58.

19. T.A. Weisshaar. An Application of Control Theory Methods to the Optimization of Structures Having Dynamic or Aeroelastic Constraints. SUDAAR 412, Dept. of Aero. and Astro., Stanford Univ., Stanford, Calif. (1970).

20. R.H. Plaut. On the Stability and Optimal Design of Elastic Structures. *Stability: Special Lectures*. Solid Mechanics Div., Univ. of Waterloo, Waterloo, Canada (1972), pp. 547-577.

21. S.A. Segenreich, N.A. Zouain, and J. Herskovits. An Optimality
    Criteria Method Based on Slack Variables Concept for Large Scale
    Structural Optimization. Paper presented at Symposium on Application
    of Computer Methods in Engineering, Univ. of Southern California,
    Los Angeles, Calif. (1977).

22. P. Rizzi. Optimization of Multi-Constrained Structures Based on
    Optimality Criteria. *Proceedings AIAA/ASME/SAE 17th Structures,
    Structural Dynamics, and Materials Conf.* AIAA, New York (1976),
    pp. 448-462.

Bay No.	Spar-Cap Areas	Web Thickness	Cover Thickness	Rib Thickness
1	t_1	t_4	t_7	t_{10}
2	t_2	t_5	t_8	t_{11}
3	t_3	t_6	t_9	t_{12}

Table 1. Design-variable numbering for rectangular wing.

Method	g	t_4, cm.	Weight, kg.	No. of Itera-tions	Rel. CPU Time
Vanderplaats [15]	-3.8×10^{-3}	0.1409	22.8S	12	1.4
Segenreich and McIntosh [13]	-9.1×10^{-5}	0.1376	22.85	49	1.4
"Energy-Density" Ratio	-2.4×10^{-4}	0.1376	22.85	38	1.0

Table 2. Comparison of Results for the Rectangular Wing

FIGURE CAPTIONS

Figure 1. Layout of rectangular wing. All dimensions are in cm.

Figure 2. Behavior of critical modes in V-g plane for initial and final
designs. M = 0.717, altitude 1,372 m, 6 modes.

STRUCTURAL BOX

BAY 1

BAY 2

BAY 3

V

25.40

38.10

63.50

127.0

63.50

10.16

10.16

152.4

457.2

152.4

152.4