Comparison of Soil Moduli Obtained from Non-Destructive Dynamic Testing and Constitutive Equations

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This report describes an effort to determine the degree of agreement between moduli of airfield granular base materials as measured by a non-destructive (vibratory) pavement evaluation procedure and as predicted from constitutive equations that relate moduli to states of stress induced in the granular material by aircraft loading. The results indicated that for the tensile state of stress that is induced by heavy aircraft loads, Young's modulus of the granular materials considered in the study was approximately inversely proportional to the square root of the state of stress. This could be postulated from the knowledge that

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) for compressive state of stress, Young's modulus is approximately directly proportion to the square root of the state of stress. It was concluded that the moduli of the pavement granular base materials studied which were measured in the field in the Air Force non-destructive pavement evaluation procedure and then reduced to account for low strain levels are in agreement with the moduli determined from empirical constitutive equations.

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COMPARISON OF SOIL MODULI OBTAINED FROM NON-DESTRUCTIVE DYNAMIC TESTING AND CONSTITUTIVE EQUATIONS

William H. Highter

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LIST OF SYMBOLS

E	-	Young's Modulus
ν	-	Poisson's ratio
G	-	Shear Modulus
K	-	Parameter defined in Equation 2
n	-	Parameter defined in Equation 2
G_{max}	-	Maximum shear modulus
γ	-	Shear strain
γ_r	-	Reference shear strain
τ_{max}	-	Maximum shear stress
D	-	Parameter representing type of soil
S	-	Degree of saturation
N	-	Number of load cycles
T	-	Time to reach reference strain, γ_r
γ_h	-	Hyperbolic shear strain
e	-	Void ratio
E_r	-	Resilient Young's Modulus
K_1	-	Parameter defined in Equation 4
n_1	-	Parameter defined in Equation 4
θ	-	First stress invariant
$\sigma_1, \sigma_2, \sigma_3$	-	Principal stresses
K_2	-	Parameter defined in Equation 5
n_2	-	Parameter defined in Equation 5
K_3	-	Parameter defined in Equation 6
n_3	-	Parameter defined in Equation 6
PI	-	Plasticity Index

LIST OF SYMBOLS (CONCLUDED)

σ_{Z1}	-	Vertical normal stress at bottom of top layer of three layer system
σ_{Z2}	-	Vertical normal stress at bottom of middle layer of three layer system
σ_{R1}	-	Horizontal normal stress at bottom of top layer of three layer system
σ_{R2}	-	Horizontal normal stress at bottom of middle layer of three layer system
σ_{R3}	-	Horizontal normal stress at top of deepest layer of a three layer system

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ABSTRACT

The objective of this study was to determine if the value of Young's modulus of soil obtained from field vibratory test data and Young's modulus obtained from considering the state of stress induced by aircraft loads are compatible. Agreement between the moduli obtained by the two independent procedures would provide independent validation of the elastic parameters used in the Air Force pavement evaluation procedure.

The Air Force is presently using a pavement evaluation procedure which utilizes parameters obtained from non-destructive pavement testing performed in the field. In the field tests, the pavement system is subjected to vibratory loading at low stress increments and hence low strain amplitudes. A shear modulus is obtained from the test data, but since the shear modulus of soil is dependent on strain amplitude, the modulus obtained must be reduced to account for the relatively high strain induced by aircraft loads. By making this adjustment and assuming the soil is elastic and Poisson's ratio is known, Young's modulus can be obtained for use in the non-destructive pavement evaluation procedure.

The method used in the present study was to calculate the stresses induced in a pavement system by aircraft static wheel loads. Elastic parameters determined by the Air Force Civil and Environmental Engineering Development Office at two Air Force Bases were used in the analysis. With the state of stress known, a least squares technique was used to determine the parameters of well known constitutive equations which relate state of stress to Young's

modulus for granular soils.

The results indicated that for the tensile state of stress that is induced by heavy aircraft loads, Young's modulus of the granular materials considered in the study was approximately inversely proportional to the square root of the state of stress. This could be postulated from the knowledge that for compressive states of stress, Young's modulus is approximately directly proportional to the square root of the state of stress. It was concluded that the moduli of the pavement granular base materials studied which were measured in the field in the Air Force non-destructive pavement evaluation procedure and then reduced to account for low strain levels are in agreement with the moduli determined from empirical constitutive equations.

SECTION 1

INTRODUCTION

In the past structural evaluation of in-service pavements involved field tests that disturbed and even destroyed some portions of the pavement system under study. Frequently the pavement wearing course was cored and test pits were excavated to determine in situ soil properties including unit weight, water content, and CBR. The destructive nature of the tests and the interruption to traffic operations were primary motivators for developing non-destructive pavement evaluation techniques.

The Air Force Civil and Environmental Engineering Development Office (CEEDO) is currently using a non-destructive pavement evaluation procedure which is incumbent upon determining elastic properties of the pavement system. A vibratory load is applied at the pavement surface and the shear modulus of the underlying foundation soil is determined. Because the shear modulus of soil is dependent on the magnitude of strain amplitude (or shear stress) and the level of strain amplitude induced by the vibratory load is much less than that created by aircraft loads, the value of the shear modulus obtained is much greater than that corresponding to the in-service loads.

Hardin (Reference 1) has developed a method for reducing the shear modulus of soils with increasing strain amplitude. Compared to values measured in the laboratory, the procedure gives reasonably accurate results for a wide variety of soil types and conditions.

Having a value of the shear modulus and assuming the pavement system is elastic and an appropriate value of Poisson's ratio, an estimate of Young's modulus can be obtained from

$$E = 2(1+\nu)G \tag{1}$$

where, for the soil, E is Young's modulus

ν is Poisson's ratio

and G is the shear modulus.

It has been known for some time that Young's modulus of soil is not a constant but is a function of the state of stress, in particular, the confining stress. Several investigators have proposed relationships between induced stress and Young's modulus of the form

$$E = K\,\theta^{n} \tag{2}$$

where

K and n are parameters which depend on soil type and condition

and θ is the first stress invariant.

By determining stresses induced in the foundation soil by in-service loads, and applying Equation (2), Young's modulus can be determined.

The primary objective of this research was to compate the parameters of Equation (2) as determined from a least squares fit technique knowing: i) E from field testing; and, ii) θ by calculating stress induced in several pavement systems by aircraft static wheel loads; to values previously found in laboratory research. The parameters obtained would then be checked against those found by other investigators from laboratory testing. Successful completion of the research would provide an independent validation of elastic parameters used in the Air Force pavement evaluation procedure.

SECTION II

BACKGROUND

Efforts to characterize the behavior of soil mathematically by unique load-deflection, axial stress-axial strain, or shear stress-shear strain relationships have met with little success in the past because such relationships depend on the stress (or strain) level to which the soil is subjected, the number and frequency of cycles of stress (or strain) and whether or not the soil is undrained, partially drained or fully drained during shear. Also the soil type and degree of saturation affect the stress-strain behavior. It is necessary, then, in determining values of parameters to characterize soil to be certain that the parameter values are consistent with the levels of the influencing factors that are extant in the in-service environment.

If it is assumed that the soil response is elastic at least for small numbers of load cycles, there are two distinct approaches to determine elastic properties: measuring the in-service response to a known loading and calculating the elastic parameters that provide the relationship between the known input and measured response; on the basis of laboratory tests define a relationship between measured elastic parameters and the independent variables that affect the elastic parameters. Presently the Air Force non-destructive pavement evaluation procedure is based on the first approach which is discussed in the following subsection. The second approach is used in the present research and is discussed in the subsection entitled Constitutive Equations.

HARDIN METHOD OF REDUCING SHEAR MODULUS FOR INCREASING STRAIN AMPLITUDE

Curves depicting load-deflection, axial stress-axial strain or shear stress-shear strain relationships for soils are distinctly non-linear even for single cycles of load. Inspection of Figure 1 shows (1) that a modulus defined by a tangent to any load cycle decreases with increasing level of stress (or strain) and, (2) that a representative modulus is obtained only after the soil is subjected to many cycles of load. Since the vibratory load applied during field testing as part of the Air Force non-destructive pavement evaluation procedure induces stresses (and strains) in the soil much smaller than those induced by an aircraft during ground operations, the modulus must be adjusted (reduced) appropriately to be commensurate with the larger loading. Furthermore, since a representative modulus is not obtained until after several cycles of load, a second adjustment must be made. Hardin (Reference 1) has suggested a method for making those adjustments to account for the effect of strain amplitude on the shear modulus of soils.

Hardin's approach can most easily be understood by referring to Figure 2. The maximum value of the shear modulus, G_{max}, is obtained from a non-destructive field vibratory test. The variability in shear modulus and strain level is reduced by the normalization, G/G_{max}. The reduction in variability due to normalization is significant for a given soil under similar states of stress, but great variability still exists because different soils do not mobilize the same shear stress for a given shear strain and a single soil will not mobilize the same shear

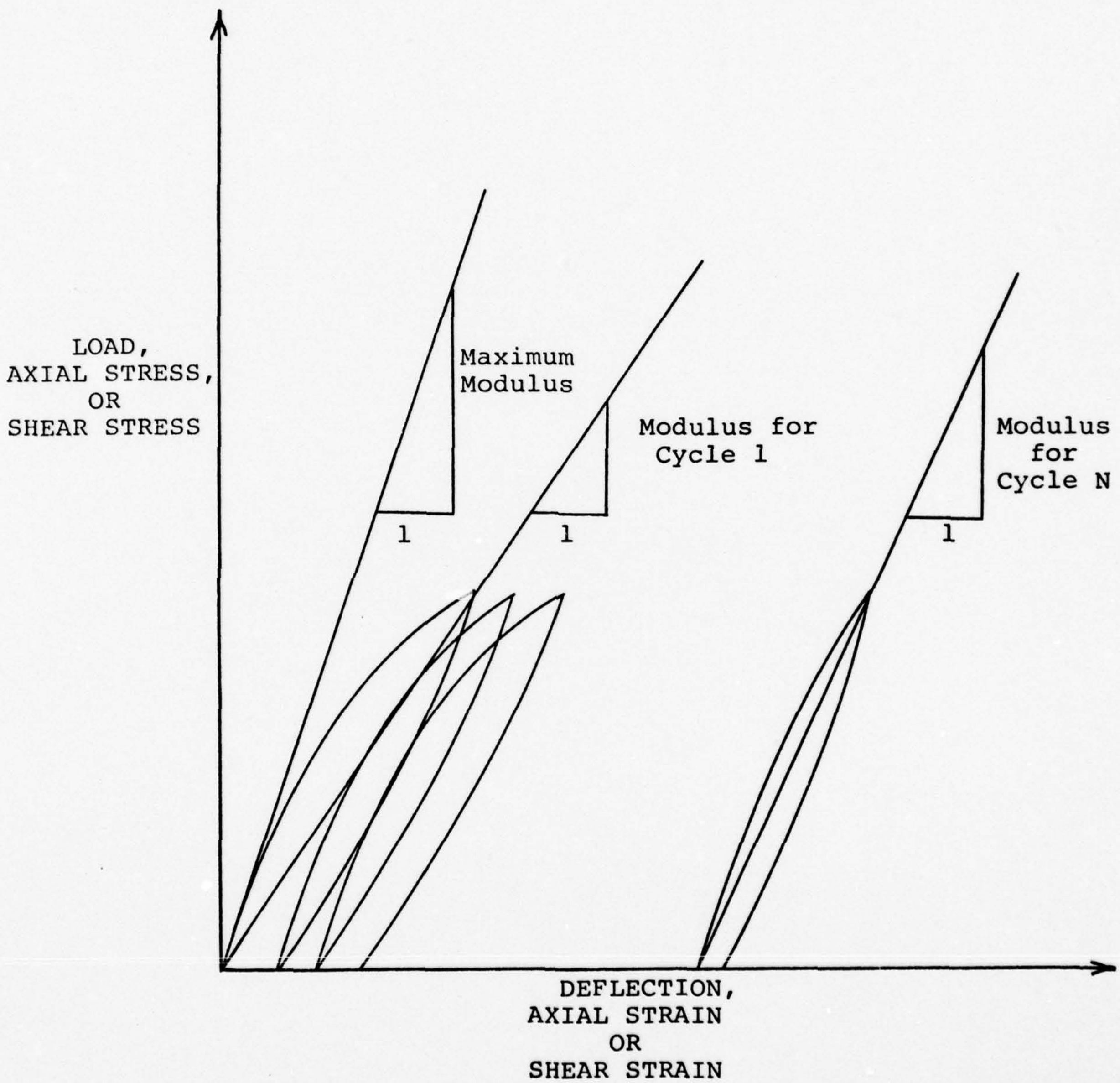

FIGURE 1 SCHEMATIC OF LOAD-DEFLECTION, AXIAL STRESS-AXIAL STRAIN, OR SHEAR STRESS-SHEAR STRAIN RELATIONSHIP FOR SOIL.

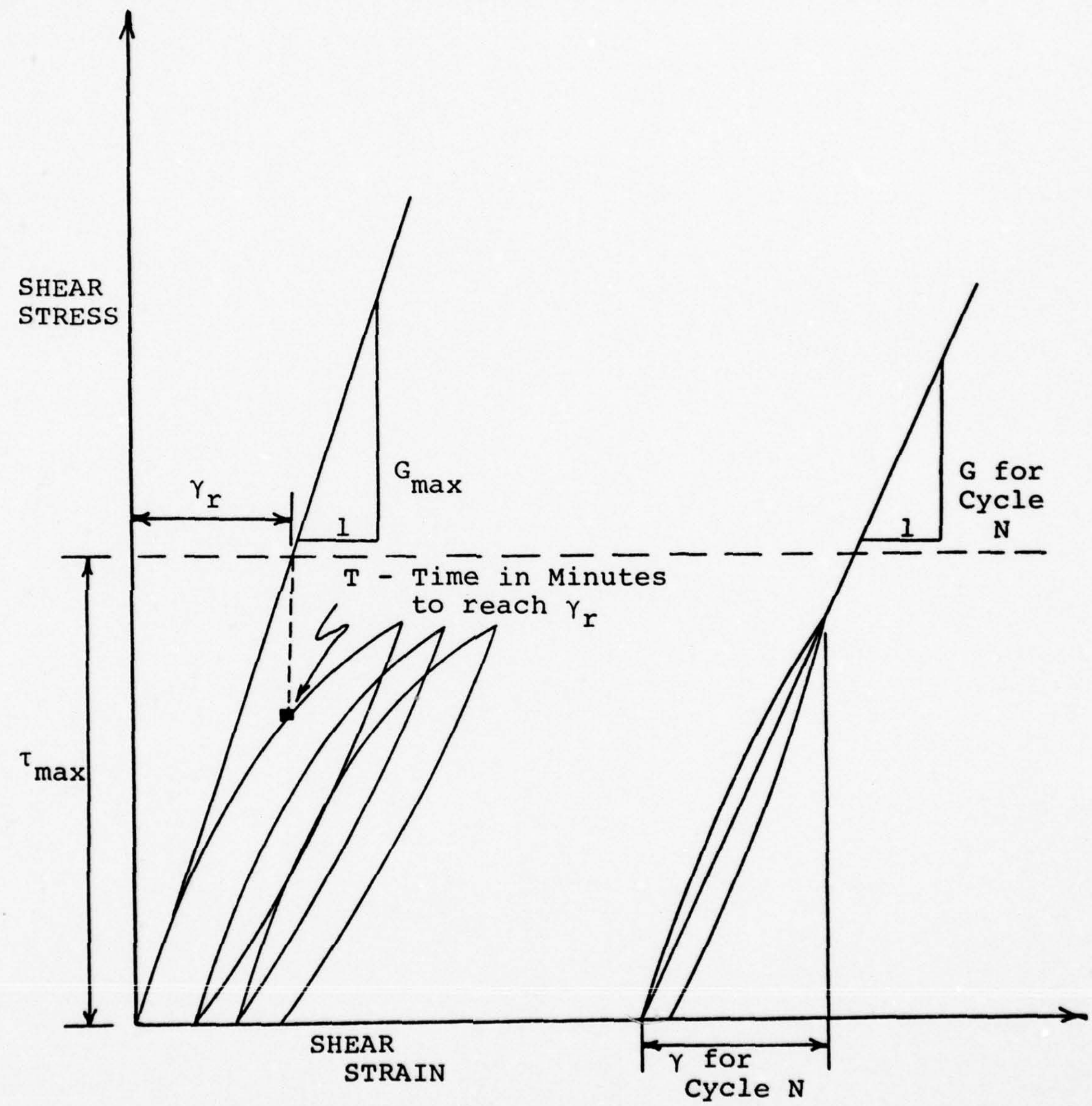

FIGURE 2 SCHEMATIC OF SHEAR STRESS-SHEAR STRAIN RELATION (From Reference 1).

stress for a given shear strain if the state of stress on the soil element changes. To define a more nearly unique relationship between shear modulus and shear strain, Hardin suggested normalization of the shear strain γ, by dividing by the reference strain, γ_r where $\gamma_r = \tau_{max}/G_{max}$ (see Figure 2). The variability in the relationship between the normalized strain, γ/γ_r and the normalized modulus, G/G_{max}, is much less than the variability in the relationship between γ and G.

Essentially, the normalization removes the dependency on state of stress from the shear strain-shear modulus relationship for a given soil (D) at a given degree of saturation (S), sheared a given number of cycles (N), and when the reference strain, γ_r, is reached in a given time (T). According to Hardin, the variability due to D, S, N and T can be reduced and a single relationship for all soils and conditions can be obtained by defining a hyperbolic strain. The hyperbolic strain is a function of soil type, D, degree of saturation, S, number of shear stress-shear strain cycles, N, time to reach reference strain, T, and the normalized shear strain γ/γ_r. Hardin proposed the relationship between the normalized shear modulus and hyperbolic strain as:

$$\frac{G}{G_{max}} = \frac{1}{1+\gamma_h} \tag{3}$$

where γ_h is the hyperbolic strain and G and G_{max} are as shown in Figure 2.

Reference 1 gives several charts for solving the equations for G knowing G_{max}, D, S, N, T, γ and e (void ratio). Experimental evidence suggests that the procedure suggested by Hardin gives very good results except for highly plastic soils having a liquid limit greater than 50%. The reason for this is probably due to the fact that the relationship used by Hardin between the effective angle of shearing resistance and plasticity index for clays in finding the reference strain is approximate and apparently was not intended to be used for extremely plastic clays (References 2 and 3). However, Hardin's method is a valuable contribution especially because τ_{max} (Figure 2) need not be measured in the field — a very difficult undertaking — and data obtained from a non-destructive field vibration test can be utilized.

CONSTITUTIVE EQUATIONS

As can be seen from Figures 1 and 2, after many cycles of load, the modulus (G in Figure 2) attains a nearly constant value. If axial stress-axial strain data from a static cycle triaxial test on granular soil are plotted, the shape of the curve for each cycle will be very similar to the curves in Figure 2 and the nearly constant Young's modulus obtained after several load cycles is termed the resilient Young's modulus, E_r.

It has been found that the following factors have an effect on the resilient parameters of highway base materials and also can be expected to affect the resilient parameters of airfield base materials: duration of applied load; number of load repetitions; state of stress; dry unit weight; degree of saturation; percent of particles smaller than the No. 200 sieve (0.074 mm); and, whether the confining pressure is held constant or is allowed to vary.

Barksdale (Reference 4) used non-linear and linear finite element analysis to estimate the time duration that points within the pavement system were stressed as a function of vehicle speed. For speeds of 1 to 45 miles per hour, the analysis indicated that pulse time was inversely proportional to vehicle speed. Vehicle speed also affects the magnitude of the stress pulse. Field measurements have shown that as the speed increases, stresses and deflections decrease due to inertial forces and viscous effects (Reference 5). Hicks and Monismith (Reference 6) found stress durations in the range of 0.10 to 0.25 seconds had no observable influence on the resilient modulus.

Several investigators (e.g. Allen and Thompson (Reference 7); Hicks and Monismith (Reference 6) found that after 100-200 cycles of load in a cyclic triaxial test an increase in the number of load repetitions up to at least 25,000 cycles had a negligible effect on the resilient parameters for partially saturated soils. This was also true for saturated soils if the principal stress ratio did not exceed 6 or 7 or if the samples were subjected to 1000-2000 cycles of stress in the drained state before closing the drainage valve.

The applied state of stress has been found by several investigators to be the greatest single influence on the resilient modulus. Moreover, the confining pressure has a greater influence than the time varying axial load (e.g. Barksdale and Hicks (Reference 8). Several researchers have found that the resilient modulus is a function of the first stress invariant:

$$E_r = K_1 \theta^{n_1} \tag{4}$$

where

E_r is the resilient modulus

K_1, n_1 are constants determined by analyzing laboratory data for several stress levels

and

θ is the first stress invariant ($\theta = \sigma_1+\sigma_2+\sigma_3$). For a triaxial test with constant cell pressure, $\theta = \sigma_1+2\sigma_3 = \sigma_{axial} + 2\sigma_{confining}$; $\sigma_1, \sigma_2, \sigma_3$ are principal stresses).

The importance of the confining pressure, σ_3, is indicated by the following relationship suggested or used by Allen and Thompson (Reference 7), Barksdale (Reference 4), and Hicks and Monismith (Reference 6):

$$E_r = K_2 \sigma_3^{n_2} \tag{5}$$

where

K_2 and n_2 are constants determined by analysis of laboratory data for several confining pressures

and

E_r and σ_3 are as previously defined.

A third constitutive equation similar to Equation (4) has also been suggested (Reference 2):

$$E_r = K_3 \left(\frac{\theta}{3}\right)^{n_3} \tag{6}$$

where K_3 and n_3 are constants and E_r and θ are as previously defined.

Although most researchers have determined the resilient modulus of base materials using a constant confining pressure, Allen and Thompson (Reference 7) varied the cell pressure as well as the axial stress during cyclic triaxial testing and reported that the resilient modulus computed from constant cell pressure tests exceeded the resilient modulus values determined from variable cell pressure tests for most stress levels. The magnitude of the difference was a function of stress and was not constant. However, Brown and Hyde (Reference 9) found that similar values of the resilient modulus were obtained from cyclic and constant *confining (cell) pressure* tests when the constant stress was equal to the mean of the cyclic value.

Investigators have encountered problems in determining values of the resilient Poisson's ratio from repetitive triaxial tests (e.g. Monismith, Ogawa, and Freene; Reference 10) partly because of specimen creep. Because of specimen creep and since the response of a pavement is relatively insensitive to variations in the residual Poisson's ratio over its typical range, it has been suggested that estimated values of resilient Poisson's ratio be used as an approximation in engineering analysis (Transportation Research Board, Reference 11).

In summary, the same factors which influence the shear modulus, G, also affect the resilient modulus, E_r. This is to be expected from Equation (1). The thrust of this research is to determine the parameters from Equations (4), (5) and (6) and to compare these values to those reported in the literature. Details of the analysis performed and results are given in the following section.

compare these values to those reported in the literature. Details of the analysis performed and results are given in the following section.

SECTION III

ANALYSIS AND RESULTS

ANALYSIS

To determine if the moduli of the granular base material as obtained by the non-destructive field technique in conjunction with the Hardin Method (Reference 1) could be approximated by a constitutive relationship of the form

$$E_r = K_1 \theta^{n_1} \tag{4}$$

$$E_r = K_2 \sigma_3^{n_2} \tag{5}$$

or

$$E_r = K_3 \left(\frac{\theta}{3}\right)^{n_3} \tag{6}$$

the following approach was used:

a. Obtain weraring course, base and subgrade thicknesses from several field sections where non-destructive pavement evaluations had been performed;

b. Obtain the moduli of the various pavement system layers for the same field sections as determined by the non-destructive pavement evaluation technique;

c. Using elastic layer theory, determine the stresses induced by an aircraft in the base material by a given aircraft;

d. Knowing E_r (step b) and θ (step c) find K_1 and n_1 in Equation (4) by least-squares fit; and,

e. Determine if the values of K_1 and n_1 obtained in step (d) are similar to those obtained from laboratory testing by other researchers.

Pavement system geometric parameters (step a) and corresponding moduli (step b) for nine pavement sections were obtained from the Air Force Civil and Environmental Engineering Development Office (CEEDO), Tyndall Air Force Base, Florida. These data are tabulated in the Appendix and were obtained from field testing carried out by CEEDO at Carswell Air Force Base, Texas and Dyess Air Force Base, Texas.

Data from only four (test sections 2, 4, 8 and 9) of the nine test sections were used in this study because (a) five of the nine test sections had granular bases, and (b) one of these five (test section 6) was a Portland cement concrete pavement which has been overlayed by asphaltic concrete. Because the technique used by the Air Force to determine moduli of layered systems cannot be used when the moduli increases with depth, the moduli of the Portland cement concrete layer which was below the asphaltic concrete layer had to be assumed. The assumed value as reported by CEEDO was 3,000,000 psi. Rather than using assumed values in the analysis, data from test section 6 were not used and thus only measured data were utilized in the analysis.

The Air Force has many different types of aircraft in its inventory representing a wide range of wheel loads, tire-pavement contact areas, and tire air pressures. All of these factors influence the magnitude of stresses induced in pavements. Because of these factors, the stresses created in the base material at the four test sections by two very different Air Force aircraft were determined. The aircraft chosen were the F-4E and the C-135A. The F-4E has a maximum main gear wheel load of 27 kips, a tire pressure of 265 psi, and a contact area of 102 square inches. The C-135A has a maximum main gear wheel load of 33 kips, a tire pressure of 143 psi, and a contact area of 230 square inches (Reference 12). In the stress analysis, it was assumed that contact areas were circular.

For each of the two aircraft the induced vertical and horizontal stresses were determined for each of the four test sections. In calculating the stresses, it was assumed that the pavement sections consisted of three layers: a wearing course; a base; and a subgrade. The wearing course and base were of finite thicknesses as given in the Appendix and the subgrade was of infinite thickness. Each of the three layers was assumed to be a homogenous, isotropic, elastic solid and thus each layer could be characterized by two parameters: Young's modulus and Poisson's ratio. For each layer Poisson's ratio was assumed to be 0.5 and the Young's modulus used was that measured by the non-destructive pavement evaluation technique. It was assumed that there was full friction i.e. no slip at the interface boundaries between layers. Reference 13 is an annotated bibliography containing references of various contributions to boundary value problems leading to solutions for stresses and displacements in layered systems. The stresses determined in the present study were obtained from the tabulated solutions of Jones (Reference 14) and the graphical solutions of Peattie (Reference 15) for 3 layer systems. Both Jones' solution and Peattie's solution are also given in Reference 16.

Table 1 is a tabulation of the normal stresses found at various points beneath the center of the load in the four pavement test sections investigated for the two aircraft. It was necessary to interpolate the solutions of Peattie and Jones but no extrapolations were carried out. Interpolation is permissible but extrapolation is not allowed (References 14 and 15). Examination of Table 1 shows that, as expected, the vertical normal stresses are compressive and decrease with depth and the horizontal normal stresses are tensile and also decrease with depth. The normal vertical stresses induced by the two aircraft at each test section were generally within 20% of one another. At each test section the differences in the magnitude between both the vertical stresses and the horizontal stresses induced by the two aircraft decreased with depth.

Table 2 lists the first stress invariant ($\theta = \sigma_{Z2} + 2\sigma_{R2}$) in the granular base material at the interface between the granular base and the underlying fine grain subgrade soil. For both aircraft at all four test sections this parameter is negative indicating a tensile stress. A least squares fit was performed on Equation (4) using the absolute value of θ for each aircraft at each test section and the Young's modulus measured by the nondestructive pavement evaluation technique. The parameters determined from the least squares were K_1 = 142,696 psi and $n_1 = -0.46$ and the predictive equation is

$$E = 142{,}696\ \theta^{-0.46} \tag{7}$$

TABLE 1

Stresses Beneath the Center of the Loaded Area for Two Aircraft and Four Test Sections

Test Section	Aircraft	σ_{Z1} (psi)	σ_{R1} (psi)	σ_{Z2} (psi)	σ_{R2} (psi)	σ_{R3} (psi)
2	F-4E	10.1	-46.4	5.6	- 9.3	-0.3
	C-135A	9.7	-54.1	6.1	-12.3	-0.9
4	F-4E	10.1	-46.4	5.6	- 9.3	-0.3
	C-135A	9.7	-54.1	6.1	-12.3	-0.9
8	F-4E	19.1	-94.6	4.8	- 9.0	-0.5
	C-135A	25.9	-80.9	5.3	-10.0	-0.7
9	F-4E	20.1	-93.3	10.1	-18.0	-2.4
	C-135A	21.6	-90.7	8.9	-21.0	-4.1

Positive stresses designate compression.

Negative stresses designate tension.

Explanation of Symbols used in Table 1

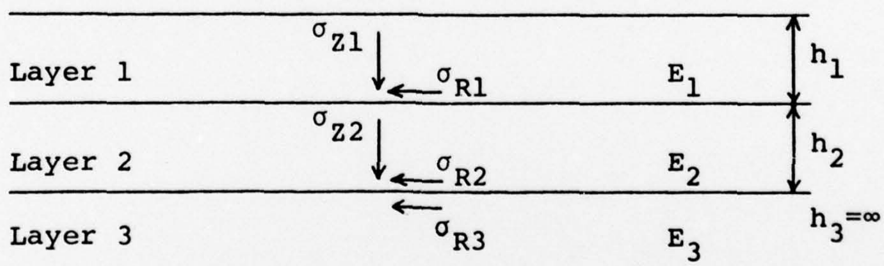

TABLE 2

Results of Least Squares Fit on Constitutive Equation $E = 142{,}696\ \theta^{-0.46}$

Test Section	Aircraft	$\theta^* = \sigma_{Z2} + 2\sigma_{R2}$ (psi)	E_2 Measured (psi)	E_2 Predicted (psi)
2	F-4E	-13.0	45,719	43,868
2	C-135A	-18.5	45,719	37,274
4	F-4E	-13.0	45,719	43,868
4	C-135A	-18.5	45,719	37,274
8	F-4E	-13.2	35,346	43,546
8	C-135A	-14.7	35,346	41,442
9	F-4E	-25.9	28,314	31,943
9	C-135A	-33.1	28,314	28,553

*The absolute value of ($\theta = \sigma_{Z2} + 2\sigma_{R2}$) is used in the constitutive equation.

The values of E predicted from this equation are also tabulated in Table 2. The average error between measured values of the granular base modulus E_2 and the values predicted by Equation (7) is 12.4%.

Table 3 gives the horizontal stress in the granular base material at the interface between the base and subgrade soils. The stress induced by both aircraft is tensile at all four test sections. A least squares fit was performed on Equation (5) using the absolute value of the horizontal stress, σ_{R2}, caused by the aircraft and the Young's modulus measured by the non-destructive pavement evaluation technique. The analysis yielded K_2 = 114,636 psi and n_2 = -0.46 and the predictive equation is:

$$E = 114{,}636\ \sigma_{R2}^{-0.46} \tag{8}$$

The predicted values of E for both aircraft at the four test sections are tabulated in Table 3. The average error between the measured values of the granular base moduli and those predicted by Equation (8) is 12.5%.

A variation in Equation (4) suggested in the literature (Reference 2) is to use the average confining stress i.e., one-third of the first stress invariant, θ. The results of a least squares fit on the equation

$$E = K_3 \left(\frac{\theta}{3}\right)^{n_3} \tag{6}$$

yields K_3 = 91,200 psi and n_3 = -0.49 or

TABLE 3

Results of Least Squares Fit on Constitutive Equation $E = 114{,}636\ \sigma_{R2}^{-0.46}$

Test Section	Aircraft	σ_{R2}* (psi)	E_2 Measured (psi)	E_2 Predicted (psi)
2	F-4E	- 9.3	45,719	41,098
	C-135A	-12.3	45,719	36,138
4	F-4E	- 9.3	45,719	41,098
	C-135A	-12.3	45,719	36,138
8	F-4E	- 9.0	35,346	41,722
	C-135A	-10.0	35,346	39,749
9	F-4E	-18.0	28,314	30,322
	C-135A	-21.0	28,314	28,255

*The absolute value of σ_{R2} is used in the constitutive equation.

$$E = 91{,}200 \left(\frac{\sigma_{Z2}+2\sigma_{R2}}{3}\right)^{-0.49} \quad (9)$$

A tabulation of the results is given in Table 4. The average percent error between measured and predicted moduli is 12.2%.

DISCUSSION OF RESULTS

It has been suggested (References 2 and 4) that the coefficients n_1, n_2 and n_3 in Equations (4), (5) and (6), respectively are usually 0.5 when θ and σ_3 are positive i.e., the stresses are compressive. This indicates that the modulus of granular soil increases directly with the square root of the confining pressure or first stress invariant when these stresses are compressive. The magnitude of K_1, K_2 and K_3 in Equations (4), (5) and (6), respectively are functions of properties of the granular soil other than the state of stress to which the soil is subjected in its service environment.

Although a literature search did not indicate the magnitude (or sign) of n_1, n_2 and n_3 in Equations (4), (5) and (6) respectively, if σ_{R2} were tensile or if the first stress invariant were tensile, it is not unreasonable to postulate that the modulus would vary inversely with the square root of σ_{R2} or the first stress invariant. Thus, as these stresses become more highly tensile, the modulus would decrease; conversely, it is known that as this stress becomes more highly compressive, the modulus would increase.

As seen from Equations (7), (8) and (9), respectively, the values of n_1, n_2 and n_3 calculated from the least squares analysis

TABLE 4

Results of Least Squares Fit on Constitutive

Equation $E = 91{,}200 \left(\frac{\sigma_{Z2} + 2\sigma_{R2}}{3}\right)^{-0.49}$

Test Section	Aircraft	$\frac{\sigma_{Z2}+2\sigma_{R2}}{3}$* (psi)	E_2 Measured (psi)	E_2 Predicted (psi)
2	F-4E	- 4.33	45,719	44,475
	C-135A	- 6.17	45,719	37,390
4	F-4E	- 4.33	45,719	44,475
	C-135A	- 6.17	45,719	37,390
8	F-4E	- 4.40	35,346	44,127
	C-135A	- 4.90	35,346	41,860
9	F-4E	- 8.63	28,314	31,721
	C-135A	-11.03	28,314	28,127

*The absolute value of $\left(\frac{\sigma_{Z2}+2\sigma_{R2}}{3}\right)$ is used in the constitutive equation.

are -0.46, -0.46 and 0.49. These values are very close to -0.5 which would be postulated. On this basis it appears that the moduli measured in the field in the Air Force non-destructive pavement evaluation procedure and then reduced by the Hardin procedure described earlier (Reference 1) are in agreement with the empirical constitutive equations. It should be noted that the average percent error between measured moduli and those predicted by Equations (7), (8) and (9) are well within the error that can be expected when using the Hardin procedure (Reference 1).

Equations (7), (8) and (9) are strictly applicable only for the F-4E and C-135A aircraft operating on test sections designated 2, 4, 8 and 9 (see Appendix). These equations were used only to determine the degree of agreement between soil moduli measured in the non-destructive pavement evaluation procedure and empirical constitutive equations.

SECTION IV

SUMMARY AND CONCLUSIONS

Parameters were determined for three empirical constitutive equations which relate state of stress induced by surface load in a granular soil mass to Young's modulus. Layer thicknesses and elastic parameters measured in the field by non-destructive testing carried out by the Air Force Civil and Environmental Engineering Development Office at four test sites were used to calculate stresses in a granular base induced by F-4E and C-135A aircraft. The parameters in the constitutive equations were determined by a least squares fit technique. It was found that for a tensile state of stress, the modulus varied inversely with approximately the square root of the horizontal normal stress and also with approximately the square root of the first stress invariant. This relationship can be postulated from the knowledge that for compressive states of stress, the modulus varies directly with the square root of the horizontal normal (confining) stress and also with the square root of the first stress invariant. On this basis it was concluded that the moduli of pavement granular base materials measured in the field in the Air Force non-destructive pavement evaluation procedure and then reduced according to the procedure suggested by Hardin are in agreement with the moduli determined from empirical constitutive equations.

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APPENDIX

PAVEMENT SYSTEM DATA FROM FIELD NON-DESTRUCTIVE VIBRATORY LOADING

The 9 sets of data contained in the appendix were obtained from the Air Force Civil and Environmental Engineering Development Office, Tyndall Air Force Base, Florida. The data sets represent pavement sections having Portland cement concrete surfaces and Portland cement concrete overlain by asphaltic concrete. The data are from airfields at Dyess Air Force Base, Texas and Carswell Air Force Base, Texas. The data were obtained for pavement evaluation purposes by the Air Force. Values of Poisson's ratio ν, for all layers were assumed.

Data Set 1 Data Point 347-03 Carswell Air Force Base, Texas

Layer	Material	Thickness[3] (inches)	E (psi)	ν	S	e	PI
1	AC[1]	5	153,577	0.25	-	-	-
2	PCC[2]	10	3,000,000	0.15	-	-	-
3	Fine grain soil	129	2096	0.43	80%	0.45	18%

Data Set 2 Data Point 347-04 Carswell Air Force Base, Texas

Layer	Material	Thickness (inches)	E (psi)	ν	S	e	PI
1	PCC	24	321,732	0.15	-	-	-
2	Granular soil	7	45,719	0.25	80%	0.22	NP
3	Fine grain soil	113	10,345	0.43	80%	0.45	15%

[1] Asphaltic Concrete

[2] Portland Cement Concrete

[3] For CEEDO computer program purposes, the thickness of the deepest soil layer is assigned so that the total thickness of each pavement section is 144 inches.

Data Set 3 Data Point 317-03 Carswell Air Force Base, Texas

Layer	Material	Thickness (inches)	E (psi)	ν	S	e	PI
1	AC	4	191,976	0.25	-	-	-
2	PCC	18	3,000,000	0.15	-	-	-
3	Fine grain soil	122	5388	0.43	80%	0.40	20%

Data Set 4 Data Point 347-04 Carswell Air Force Base, Texas

Layer	Material	Thickness (inches)	E (psi)	ν	S	e	PI
1	PCC	24	321,732	0.15	-	-	-
2	Granular soil	7	45,719	0.25	80%	0.22	NP
3	Fine grain soil	133	10,345	0.43	80%	0.45	15%

Data Set 5 Data Point 317-02 Carswell Air Force Base, Texas

Layer	Material	Thickness (inches)	E (psi)	ν	S	e	PI
1	AC	4	92,832	0.25	-	-	-
2	PCC	16	3,000,000	0.15	-	-	-
3	Fine grain soil	124	3448	0.43	80%	0.45	15%

Data Set 6 Data Point 347-01 Carswell Air Force Base, Texas

Layer	Material	Thickness (inches)	E (psi)	ν	S	e	PI
1	AC	5	79,504	0.25	-	-	-
2	PCC	10	3,000,000	0.15	-	-	-
3	Granular soil	6	5428	0.25	80%	0.22	NP
4	Fine grain soil	123	1242	0.43	80%	0.45	10%

Data Set 7 Data Point 318-01 Carswell Air Force Base, Texas

Layer	Material	Thickness (inches)	E (psi)	ν	S	e	PI
1	PCC	18	280,207	0.15	-	-	-
2	Fine grain soil	126	7511	0.43	80%	0.40	15%

Data Set 8 Data Point Runway 34 Dyess Air Force Base, Texas

Layer	Material	Thickness (inches)	E (psi)	ν	S	e	PI
1	PCC	16	285,659	0.15	-	-	-
2	Granular soil	19	35,346	0.25	80%	0.22	NP
3	Fine grain soil	109	7981	0.43	80%	0.45	15%

Data Set 9 Data Point Apron C Dyess Air Force Base, Texas

Layer	Material	Thickness (Inches)	E (psi)	ν	S	e	PI
1	PCC	16	201,834	0.15	-	-	-
2	Granular Soil - some fines	6	28314	0.25	80%	0.22	7%
3	Fine Grain Soil	122	8430	0.43	80%	0.45	15%