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T. C. HUANG, K. S. SHEN
and VINOD K. NAGPAL
Department of Engineering Mechanics
University of Wisconsin—Madison

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Department of Engineering Mechanics
College of Engineering
University of Wisconsin—Madison
Madison, Wisconsin

UNIVERSITY OF WISCONSIN MADISON

PROBABILISTIC FACTORS IN RANDOM FATIGUE

T. C. Huang, K. S. Shen and Vinod K. Nagpal
Department of Engineering Mechanics
University of Wisconsin-Madison
Madison, Wisconsin 53706

Abstract

To predict random fatigue life, the popular cumulative damage criterion, which is based on constant amplitude sinusoidal fatigue tests, is known to be inaccurate. Therefore, actual random fatigue experiments are proposed to establish new and reliable damage criterion. In this report, all conceivable probabilistic factors which affect random fatigue life are explored and mathematically analyzed. Based on these factors the random fatigue experiments will be planned and conducted.

INTRODUCTION

There are two major approaches for investigating random fatigue: the cumulative damage approach and the phenomelogical approach. The main concept of cumulative damage criterion, proposed by Miner [1] is based on the results of constant amplitude sinusiodal fatigue tests. His hypothesis is characterized by the following equation

$$\sum_i (n_i/N_i) < c$$

in which n_i represents the actual number of cycles at a given stress level, N_i the fatigue life at that same stress level, and c is a constant. Miner proposed that c should have a value of 1.0. It is shown [2] that the limited experimental results which have been used to evaluate this hypothesis show a range for c from 0.3 to about 3.0. Since sinusoidal fatigue results are readily available this criterion became very popular even though it was repeatedly proved [3-6] inaccurate to predict failure.

The phenomelogical approach involves actual random fatigue experiments. Some of them use statistical experiment design for both deterministic and random factors and most of them use statistical and variance analyses of results [7-11]. The present report explores all conceivable random factors which affect random fatigue life. These factors are then mathematically analyzed. The mathematical expressions of these factors facilitate the design of random excitation signals and data processing of random responses. The final purpose is to establish new and reliable damage criteria for random fatigue.

ELASTIC AND PLASTIC RANDOM FATIGUE

The stress-strain curves of tensile tests and the SN-curves of fatigue tests indicate three stress levels that may be significant to random fatigue.

They are fatigue run-out stress, the yield point stress, and ultimate stress, represented by σ_f, σ_y, and σ_u respectively. The corresponding strain levels are denoted by ε_f, ε_y, and ε_u. The yield point divides the elastic and plastic regions. The ultimate stress, not rupture stress, is used for both brittle and ductile materials. In the case of brittle materials the ultimate and rupture stresses are the same. Although ductile materials have distinct ultimate and rupture stresses, they behave like brittle materials under dynamic loadings. Unless all the peaks of strain signals of random response are under ε_f, three types of failure problems can occur. The first two types are random fatigue problems.

If all the peaks of random strain signals are below ε_y, and there are peaks between ε_f and ε_y, the corresponding fatigue is elastic random fatigue. If all the peaks are below ε_u, and there are peaks between ε_y and ε_u, the corresponding fatigue is plastic random fatigue. Once the strain is beyond ε_u, this is a single dangerous state of first excursion failure.

PROBABILISTIC FACTORS

Based on the discussion of elastic and plastic random fatigue, probabilistic factors conceived to affect elastic and plastic fatigue are listed as follows.

(1) mean strain

(2) variance of strain

(3) zero strain upcrossings

(4) ε_f and ε_y level upcrossings

(5) duration of excursion between zero crossings

(6) duration of excursion beyond ε_f and ε_y levels

(7) peak strain probability density functions - band width

(8) average peak strain amplitude beyond ε_f and ε_y levels.

MATHEMATICAL ANALYSIS OF PROBABILISTIC FACTORS

For each random signal the probabilistic factors will be mathematically analyzed prior to the random fatigue experiments. The statistical expressions of these factors other than mean and variance are derived according to Rice [12]. The derived equations are general and applicable to random variables and random processes under the conditions prescribed in the derivation; and they will be applied to our random strain signals.

1. Mean Strain

The mean or expected value of a continuous variable X of a random process $X(t)$, expressed as $E[X]$, is defined as

$$E[X] = \int_{-\infty}^{\infty} x p_X(x)\, dx \qquad (1)$$

where $p_X(x)$ is the probability density function. It is assumed that the integral does exist. In other words $|x| p_X(x)$ may be integrated in the interval $(-\infty,\infty)$. $E[X]$ is also denoted by μ_X. The mean value represents the steady part of a random signal which consists of steady and fluctuating components.

2. Variance of Strain

The expected value of the square of the difference between random variable X and mean μ_X is defined as variance which is denoted by σ_X^2. Therefore

$$\sigma_X^2 = E[(X-\mu_X)^2]$$

The above equation, after expansion, can be written as

$$\sigma_X^2 = E[X^2] - \mu_X^2 \qquad (2)$$

In the derivation of equation (2) the following expression

$$\int_{-\infty}^{\infty} p_X(x)\, dx = 1$$

is used. The variance is a measure of dispersion and is always greater than zero. The dispersion can be measured by taking the positive square root of σ_X^2. σ_X is referred to as the standard deviation, or root mean square value (r.m.s.).

3. Zero Strain Upcrossings

Strain reversal is considered as an important factor which affects fatigue. Whereas in sinusoidal fatigue it is represented by the number of cycles, in random fatigue it is represented by the number of zero strain crossings.

To investigate the expected number of crossings of a Gaussian random process $x(t)$ with realizations $x_1(t)$ at an arbitrary level x_0 in the time interval $\Delta t = t_2 - t_1$, it is necessary to construct a counting functional n. The unit step function $u(t)$ is used for this purpose.

Let a realization of a new random process $Y(t)$ be defined as

$$Y_1(t) = u\{x_1(t) - x_0\}$$

Differentiating with respect to time gives

$$\dot{Y}_1(t) = \dot{x}_1(t)\, \delta\{x_1(t) - x_0\}$$

which vanishes everywhere except when $x_1(t) = x_0$, at which point there exists a spike of unit area directed upward or downward depending on whether $\dot{x}_1(t)$ is positive or negative. The counting functional, i.e., the number of crossings per unit time at t, for this typical realization of the random process $x(t)$, given by Middleton [13] as

$$n(x_0, t) = |\dot{x}_1(t)|\, \delta\{x_1(t) - x_0\}$$

includes both upward and downward crossings. The total number of crossings in the interval (t_1,t_2) is therefore

$$N(x_0,t_1,t_2) = \int_{t_1}^{t_2} |\dot{x}_1(t)|\ \delta\{x_1(t)-x_0\}\ dt$$

The expected number of crossings in this typical realization $x_1(t)$ of the Gaussian random process $x(t)$ is given by

$$E[N(x_0,t_1,t_2)] = \int_{t_1}^{t_2} E[|\dot{x}(t)|\ \delta\{x(t)-x_0\}]dt$$

$$= \int_{t_1}^{t_2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |\dot{x}|\ \delta(x-x_0)p_{x\dot{x}}(x,\dot{x})\ dx\ d\dot{x}\ dt$$

$$= (t_2-t_1)\int_{-\infty}^{\infty} |\dot{x}|p_{x\dot{x}}(x_0,\dot{x})\ d\dot{x}$$

where $p_{x\dot{x}}(x,\dot{x})$ is the joint probability density function. Since each upcrossing is followed by a downcrossing the expected number of upcrossings per unit time is given by

$$E[N_+(x_0)] = \frac{1}{2}\frac{E[N(x_0,t_1,t_2)]}{t_2-t_1}$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} |\dot{x}|p_{x\dot{x}}(x_0,\dot{x})\ d\dot{x}$$

From the definition of probability density function, we have

$$p_{x\dot{x}}(x,\dot{x}) = \frac{1}{2\pi\sqrt{m_0m_2(1-\rho^2)}}\ \exp\left[\frac{-1}{2(1-\rho^2)}\left(\frac{x^2}{m_0} - \frac{2\rho x\dot{x}}{\sqrt{m_0m_2}} + \frac{\dot{x}^2}{m_2}\right)\right]$$

in which

$$\rho = \frac{E[x(t)\dot{x}(t)]}{\sqrt{m_0m_2}}$$

$$m_0 = E[x^2(t)]$$

$$m_2 = E[\dot{x}^2(t)]$$

In the frequency domain, m_0 and m_2 are defined as the moments of spectral density function expressed as

$$m_n = \int_0^\infty \omega^n \Phi_{xx}(\omega)\, d\omega \qquad n = 0,1,2,....$$

where ω is the angular frequency and $\Phi_{xx}(\omega)$ is the one sided spectral density function.

For ergordic process the expectations are equal to the corresponding temporal averages. Thus

$$E[x(t)x(t+\tau)] = R_{xx}(\tau) = \langle x(t)x(t+\tau)\rangle$$

where $R_{xx}(\tau)$ is the autocorrelation function and $\langle\ \rangle$ indicates the temporal average. We also have

$$R_{xx}(\tau) = R_{xx}(-\tau)$$

Let the dot be used to indicate differentiation with respect to τ. Then

$$\dot{R}_{xx}(\tau) = R_{x\dot{x}}(\tau) = R_{\dot{x}x}(-\tau)$$

For $\tau = 0$

$$R_{x\dot{x}}(0) = \langle x(t)\dot{x}(t)\rangle$$

$$R_{\dot{x}x}(0) = \langle -\dot{x}(t)x(t)\rangle$$

Therefore

$$\dot{R}_{xx}(0) = R_{x\dot{x}}(0) = R_{\dot{x}x}(0) = 0$$

Since

$$E[x(t)\dot{x}(t)] = R_{x\dot{x}}(0) = 0$$

then

$$\rho = 0$$

Therefore

$$p_{x\dot{x}}(x,\dot{x}) = \frac{1}{2\pi\sqrt{m_0 m_2}} \exp\left[-\frac{1}{2}\left(\frac{x^2}{m_0} + \frac{\dot{x}^2}{m_2}\right)\right]$$

and the equation for $E[N_+(x_0)]$ becomes

$$E[N_+(x_0)] = \frac{1}{2\pi}\sqrt{\frac{m_2}{m_0}}\; e^{-(x_0^2/2m_0)}$$

When $x_0 = 0$ the expected number of zero upcrossings per unit time is obtained as

$$E[N_+(0)] = \frac{1}{2\pi}\sqrt{\frac{m_2}{m_0}} \tag{3}$$

4. ε_f and ε_y Level Upcrossings

The expected number of upcrossings at the level x_0, as derived above, is

$$E[N_+(x_0)] = \frac{1}{2\pi}\sqrt{\frac{m_2}{m_0}}\; e^{-(x_0^2/2m_0)}$$

Therefore the equations for the expected number of upcrossings of a random strain signal at the levels ε_f and ε_y per unit time are obtained, respectively, as follows.

$$E[N_+(\varepsilon_f)] = \frac{1}{2\pi}\sqrt{\frac{m_2}{m_0}}\; e^{-(\varepsilon_f^2/2m_0)} \tag{4a}$$

$$E[N_+(\varepsilon_y)] = \frac{1}{2\pi}\sqrt{\frac{m_2}{m_0}}\; e^{-(\varepsilon_y^2/2m_0)} \tag{4b}$$

5. Duration of Excursion Between Zero Crossings

The duration of the excursion is the time interval during which the oscillation exceeds a specific level. A particular case is the duration of excursion between zero crossings.

Let an arbitrary level x_0 be exceeded by the realization $x_1(t)$ of the stationary random process $x(t)$ at an upcrossing when the oscillation

has positive slope $\dot{x}(t_1) \geq 0$ while at a downcrossing the oscillation has negative slope $\dot{x}(t_2) < 0$. Define a realization of a new random process as

$$Y_1(t_1,t_2) = u\{x_1(t_1)-x_0\}u\{\dot{x}_1(t_1)-0\}u\{x_1(t_2)-x_0\}u\{-\dot{x}_1(t_2)-0\}$$

in which u is a step function. Differentiating $Y_1(t_1,t_2)$ with respect to t_1 and then with respect to t_2 gives four terms. One of these terms is

$$\dot{x}_1(t_1)\delta\{x_1(t_1)-x_0\}u\{\dot{x}_1(t_1)-0\}\dot{x}_1(t_2)\delta\{x_1(t_2)-x_0\}u\{-\dot{x}_1(t_2)-0\}$$

which represents a spike of unit area when $x_1(t_1) = x_1(t_2) = x_0$, $\dot{x}_1(t_1) \geq 0$ and $\dot{x}_1(t_2) < 0$. It can be used as the counting functional representing the number of crossings per unit time whenever $x_1(t_1) = x_1(t_2) = x_0$ and provided that $\dot{x}_1(t_1) \geq 0$ and $\dot{x}_1(t_2) < 0$. Let this counting functional be denoted as $n(x_0,x_0,\dot{x}_1,\dot{x}_2,t)$. Then

$$n(x_0,x_0,\dot{x}_1,\dot{x}_2,t) = \dot{x}_1(t_1)\dot{x}_1(t_2)\delta\{x_1(t_1)-x_0\}u\{\dot{x}_1(t_1)-0\}$$
$$\delta\{x_1(t_2)-x_0\}u\{-\dot{x}_1(t_2)-0\}$$

For a zero mean valued Gaussian random process, let $t_1 = t$ and $t_2 = t + \tau$, the total number of crossings for duration of excursion τ at level x_0 during time interval $t_2 - t_1$ is

$$N(x_0,x_0,\dot{x},\dot{x}_\tau,t_1,t_2) = -\int_{t_1}^{t_2} n(x_0,x_0,\dot{x},\dot{x}_\tau,t)\,dt$$

where $\dot{x} = \dot{x}(t)$, $\dot{x}_\tau = \dot{x}(t+\tau)$. The expected number of the duration of excursion at level x_0 for the time interval $t_2 - t_1$ or τ is

$$E[N(x_0,x_0,\dot{x},\dot{x}_\tau,t_1,t_2)]$$
$$= -\int_{t_1}^{t_2} E[\dot{x}_1(t_1)\dot{x}_1(t_2)\delta\{x_1(t_1)-x_0\}u\{\dot{x}_1(t_1)-0\}\delta\{x_1(t_2)-x_0\}u\{-\dot{x}_1(t_2)-0\}]\,dt$$

$$= -(t_2-t_1) \int_{-\infty}^{0} \int_{0}^{\infty} \dot{x}\, \dot{x}_\tau\, p_x(x_0,x_0,\dot{x},\dot{x}_\tau)\, d\dot{x}\, d\dot{x}_\tau$$

The expected number of excursion interval τ per unit time is

$$E[N(\tau)] = - \int_{-\infty}^{0} \int_{0}^{\infty} \dot{x}\, \dot{x}_\tau p_x(x_0,x_0,\dot{x},\dot{x}_\tau)\, d\dot{x}\, d\dot{x}_\tau$$

In the above equation the four variable density function is given by

$$p_x(x_0,x_0,\dot{x},\dot{x}_\tau) = \frac{1}{4\pi^2|\Delta_4|^{1/2}}\ e^{-ax_0^2/|\Delta_4|} \exp[-\frac{1}{|\Delta_4|}\ [bx_0(\dot{x}-\dot{x}_\tau) + d\dot{x}\dot{x}_\tau$$

$$+ \frac{e}{2}\ (\dot{x}^2+\dot{x}_\tau^2)]$$

where Δ_4 is a four variable variance-covariance matrix, expressed as

$$\Delta_4 = \begin{bmatrix} m_0 & m_0(\tau) & 0 & m_1(\tau) \\ m_0(\tau) & m_0 & -m_1(\tau) & 0 \\ 0 & -m_1(\tau) & m_2 & m_2(\tau) \\ m_1(\tau) & 0 & m_2(\tau) & m_2 \end{bmatrix}$$

and

$$a = \{m_2-m_2(\tau)\}[\{m_0-m_0(\tau)\}\{m_2+m_2(\tau)\} - m_1^2(\tau)]$$

$$b = m_1(\tau)[\{m_0-m_0(\tau)\}\{m_2+m_2(\tau)\} - m_1^2(\tau)]$$

$$d = -m_2(\tau)\{m_0^2-m_0^2(\tau)\} + m_0(\tau)m_1^2(\tau)$$

$$e = m_2\{m_0^2-m_0^2(\tau)\} - m_0m_1^2(\tau)$$

$$m_0(\tau) = E[x(t)x(t+\tau)] = R_{xx}(\tau)$$

$$m_1(\tau) = E[x(t)\dot{x}(t+\tau)] = R_{x\dot{x}}(\tau)$$

$$m_2(\tau) = E[\dot{x}(t)\dot{x}(t+\tau)] = R_{\dot{x}\dot{x}}(\tau)$$

The determinant of Δ_4 is expanded and simplified as

$$|\Delta_4| = [\{m_0+m_0(\tau)\}\{m_2-m_2(\tau)\} - m_1^2(\tau)][\{m_0-m_0(\tau)\}\{m_2+m_2(\tau)\} - m_1^2(\tau)]$$

$$= \frac{e^2-d^2}{m_0^2-m_0^2(\tau)}$$

The probability density function for the excursion interval τ between zero crossings for which $x_0 = 0$ is defined as

$$p_0(\tau) = \frac{E[N(\tau)]_{x_0=0}}{E[N_+(0)]}$$

where

$$E[N_+(0)] = \frac{1}{2\pi}\sqrt{\frac{m_2}{m_0}}$$

$$E[N(\tau)]_{x_o=0} = \frac{1}{4\pi^2|\Delta_4|^{\frac{1}{2}}} \int_0^{-\infty}\int_0^{\infty} \dot{x}\dot{x}_\tau \exp\left[-\frac{1}{|\Delta_4|}\{d\dot{x}\dot{x}_\tau + \frac{e}{2}(\dot{x}^2+\dot{x}_\tau^2)\}\right] d\dot{x}\, d\dot{x}_\tau$$

Rice has evaluated the above integral for $E[N(\tau)]_{x_0=0}$ and gives

$$p_o(\tau) = \frac{1}{2\pi}\sqrt{\frac{m_0}{m_2}}\,(e^2-d^2)^{\frac{1}{2}}\,[m_0^2-m_0^2(\tau)]^{-\frac{3}{2}}\,[1 + h\cot^{-1}(-h)]$$

where

$$h = d(e^2-d^2)^{-\frac{1}{2}}$$

Therefore

$$E[\tau]_{x_0=0} = \int_0^{\infty} \tau p_0(\tau)d\tau = \int_0^{\tau_{max}} \tau p_0(\tau)d\tau \qquad (5)$$

6. Duration of Excursion Beyond ε_f and ε_y Levels

The probability density function for excursion interval τ between an upcrossing and a downcrossing at level x_0 is defined as

$$p(\tau) = \frac{E[N(\tau)]}{E[N_+(x_0)]}$$

where

$$E[N_+(x_0)] = \frac{1}{2\pi}\sqrt{\frac{m_2}{m_0}}\; e^{-(x_0^2/2m_0)}$$

$$E[N(\tau)] = -\int_{-\infty}^{0}\int_{0}^{\infty} \dot{x}\dot{x}_\tau p_x(x_0,x_0,\dot{x},\dot{x}_\tau)\, d\dot{x}\, d\dot{x}_\tau$$

as obtained previously. Instead of a tedious numerical integration of the above equation for $E[N(\tau)]$, Tikhonov [14] derives an approximate expression for the probability density function for the duration of excursion at an arbitrary level x_0 as

$$p(\tau) = p_0(\tau)\exp(-a^2/4)$$

where

$$a = x_0/\sigma_X$$

Then

$$E[N(\tau)] = \int_0^\infty \tau p(\tau)d\tau = \int_0^{\tau_{max}} \tau p(\tau)d\tau \qquad (6)$$

7. Peak Strain Probability Density Function - Band Width

A maximum of a typical realization $x_1(t)$ of the Gaussian random process $x(t)$ occurs provided that the realization $\dot{x}_1(t)$ of the random process $\dot{x}(t)$ is zero and the realization $\ddot{x}_1(t)$ of the random process $\ddot{x}(t)$ is negative, i.e. $\dot{x}_1(t) = 0$, $\ddot{x}_1(t) \leq 0$.

Define the counting functional as

$$n(x_0,0,t) = -\ddot{x}_1(t)\,\delta\{\dot{x}_1(t) - 0\}u\{x_1(t) - x_0\}$$

which represents a spike of unit area whenever $x_1(t) \geq x_0$, $\dot{x}_1(t) = 0$, and $\ddot{x}_1(t) \leq 0$. The total number of peaks in the time interval $\Delta t = t_2 - t_1$ is

$$N(x_0,0,t_1,t_2) = -\int_{t_1}^{t_2} \ddot{x}_1(t)\ \delta\{\dot{x}_1(t) - 0\}u\{x_1(t) - x_0\}\ dt$$

Then the expected number of peaks in this typical realization $x_1(t)$ of the Gaussian process $x(t)$ is given by

$$E[N(x_0,0,t_1,t_2)] = -\int_{t_1}^{t_2} E[\ddot{x}_1(t)\ \delta\{\dot{x}_1(t) - 0\}u\{x_1(t) - x_0\}]\ dt$$

The expected number of peaks greater than x_0 per unit time for a zero mean valued Gaussian random process $x(t)$ is

$$E[N(x_0,0)] = -\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} d\dot{x} \int_{-\infty}^{\infty} \ddot{x}\delta\{\dot{x}-0\}u\{x-x_0\}p_{x\dot{x}\ddot{x}}(x,\dot{x},\ddot{x})\ d\ddot{x}$$

$$= -\int_{x_0}^{\infty} dx \int_{-\infty}^{0} \ddot{x}p_{x\dot{x}\ddot{x}}(x,0,\ddot{x})\ d\ddot{x}$$

In the above equation, the probability density function is derived as

$$p_{x\dot{x}\ddot{x}}(x,0,\ddot{x}) = \frac{1}{(2\pi)^{3/2}\sqrt{|\Delta_3|}}\ \exp\left[-\frac{1}{2|\Delta_3|}\ (m_2m_4x^2 + 2m_2^2x\ddot{x} + m_0m_2\ddot{x}^2)\right]$$

in which Δ_3 is a three variable variance-covariance matrix, expressed as

$$\Delta_3 = \begin{bmatrix} m_0 & 0 & -m_2 \\ 0 & m_2 & 0 \\ -m_2 & 0 & m_4 \end{bmatrix}$$

and

$$|\Delta_3| = m_2(m_0m_4 - m_2^2)$$

$$m_0 = E[x^2(t)]$$

$$m_2 = E[\dot{x}^2(t)]$$

$$m_4 = E[\ddot{x}^2(t)]$$

In the derivation of probability density function the following relations are used:

$$E[x(t)\dot{x}(t)] = E[\dot{x}(t)\ddot{x}(t)] = 0$$

$$E[x(t)\ddot{x}(t)] = -E[\dot{x}^2(t)] = -m_2$$

Previously, m_0, m_2, m_4 have been defined in the frequency domain as the moments of spectral density function.

Let $x_0 = -\infty$, than the expected total number of maximum per unit time of all the peaks regardless of their amplitude is obtained by

$$E[N(-\infty,0)] = -\int_{-\infty}^{\infty} dx \int_{-\infty}^{0} \ddot{x} p_{x\dot{x}\ddot{x}}(x,0,\ddot{x})\, d\ddot{x}$$

Substituting the expression of $p_{x\dot{x}\ddot{x}}(x,0,\ddot{x})$ in terms of moments into the above equation and integrating we obtain

$$E[N(-\infty,0)] = \frac{1}{2\pi}\sqrt{\frac{m_4}{m_2}}$$

The expected number of maximum per unit time in the range $(y,y+\delta y)$ is

$$E[N(y,0)] = -\int_{y}^{y+\delta y} dx \int_{-\infty}^{0} \ddot{x} p_{x\dot{x}\ddot{x}}(x,0,\ddot{x})\, d\ddot{x}$$

$$= -\delta y \int_{-\infty}^{0} \ddot{x} p_{x\dot{x}\ddot{x}}(y,0,\ddot{x})\, d\ddot{x}$$

From definition, the probability that a peak lies in the range $(y,y+\delta y)$ is

$$p[y < \text{peaks} \leq y + \delta y] = p_Y(y)\delta y = \frac{E[N(y,0)]}{E[N(-\infty,0)]}$$

from which

$$p_Y(y) = -2\pi \sqrt{\frac{m_2}{m_4}} \int_{-\infty}^{0} \ddot{x} p_{x\dot{x}\ddot{x}}(y,0,\ddot{x})\, d\ddot{x}$$

Substituting the probability density function $p_{x\dot{x}\ddot{x}}$ and integrating gives the following probability density function

$$p_Y(y) = \frac{\varepsilon}{\sqrt{2\pi m_0}} e^{-y_2/2m_0\varepsilon^2} + \frac{(1-\varepsilon^2)^{\frac{1}{2}}}{m_0}\, y\, e^{-y^2/2m_0} [0.5 + \operatorname{erf}\{\frac{y}{\varepsilon}(\frac{1-\varepsilon^2}{m_0})^{\frac{1}{2}}\}]$$

where

$$\varepsilon^2 = 1 - \frac{m_2^2}{m_0 m_4}$$

$$\operatorname{erf}(y) = \frac{1}{\sqrt{2\pi}} \int_0^y e^{-z^2/2}\, dz$$

From integration table

$$\operatorname{erf}\begin{Bmatrix} 0 \\ \infty \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0.5 \end{Bmatrix}$$

The quantity ε which can be used as a factor to classify the probability density functions and evaluated by

$$\varepsilon = \sqrt{1 - \frac{m_2^2}{m_0 m_4}} \qquad (7)$$

is referred to as the band width.

8. Average Peak Amplitude Beyond ε_f and ε_y Levels

Let N be the total number of signal peaks in a given time interval and n_0 be the number of peaks with magnitude greater than a given level x_0 in the same time interval. Then the number n is defined as

$$n = \frac{N}{n_0}$$

The probability that the peaks exceed an amplitude x_0 is given by

$$P[X > x_0] = \frac{1}{n} = \int_{x_0}^{\infty} p_X(x)\, dx$$

As the peak distribution of a random process is described by the Rayleigh probability density function, so that

$$\frac{1}{n} = \int_{x_0}^{\infty} \frac{x}{m_0}\, e^{-x^2/2m_0}\, dx$$

from which n is obtained after integration as

$$n = e^{x_0^2/2m_0}$$

The average value of the $\frac{1}{n}$ th highest peaks, denoted as $\bar{x}_{\frac{1}{n}}$ is expressed as

$$\bar{x}_{\frac{1}{n}} = n \int_{x_0}^{\infty} x p_X(x) dx$$

$$= n \int_{x_0}^{\infty} \frac{x^2}{m_0}\, e^{-x^2/2m_0}\, dx$$

After integration

$$\bar{x}_{\frac{1}{n}} = n\sqrt{2m_0}\, \{\frac{1}{n}\, (\ell n\ n)^{\frac{1}{2}} + \sqrt{\pi}\, [0.5-\text{erf}(2\ell n\ n)^{\frac{1}{2}}]\}$$

where

$$\text{erf}\ x = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-y^2/2}\, dy$$

Finally

$$\bar{x}_{\frac{1}{n}} = e^{x_0^2/2m_0} \sqrt{2m_0}\, [e^{-x_0^2/2m_0} (x_0^2/2m_0)^{\frac{1}{2}} + \sqrt{\pi}\, \{0.5-\text{erf}(x_0^2/m_0)^{\frac{1}{2}}\}] \qquad (8)$$

for average peak amplitude beyond ε_f and ε_y levels provided x_0 is

equal to ε_f and ε_y, respectively.

CONCLUSION

After defining elastic and plastic random fatigue in this report, all probabilistic factors which are conceived to affect random fatigue are analytically studied. In addition to the mathematical definitions of mean and variance all other factors are mathematically analyzed and expressed in terms of moments of spectral density functions. Based on some or all of these probabilistic factors random fatigue experiments will be planned and conducted.

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To predict random fatigue life, the popular cumulative damage criterion, which is based on constant amplitude sinusoidal fatigue tests, is known to be inaccurate. Therefore, actual random fatigue experiments are proposed to establish new and reliable damage criterion. In this report, all conceivable probabilistic factors which affect random fatigue life are explored and mathematically analyzed. Based on these factors the random fatigue experiments will be planned and conducted.

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