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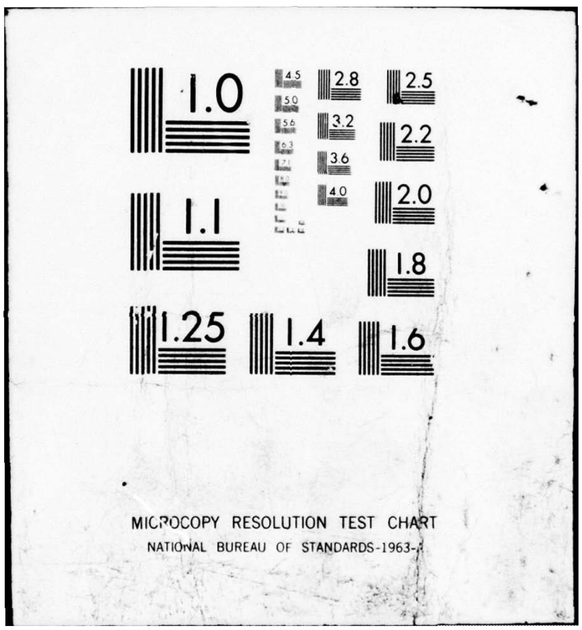
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DYNAMO ACTION IN A THIN SLAB

by

D. D. Barbosa

NASA Research Grant NGL 05-020-272
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SU-IPR Report No. 742

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ABSTRACT

We consider the dynamo action produced by convection of a partially-ionized, electrically conducting gas in a magnetic field. The model consists of two thin, Cartesian unipolar inductors connected in series by the magnetic field. For the case of a uniform magnetic field we compute the total current system generated by an arbitrary gas flow; for the case of a non-uniform field, we compute only the field-aligned coupling current. Application is made to the solar atmosphere.

I. INTRODUCTION

Magnetic fields play a significant role in a wide variety of planetary, interplanetary, and astrophysical contexts. One important possibility is that plasma motion across field lines can serve as a dynamo to convert mechanical energy into electric and magnetic energy. Dynamo effects are conspicuous in the Earth's magnetosphere and ionosphere, Jupiter's magnetosphere, and on the solar surface. Accurate modelling of these processes will advance our understanding of solar system plasma physics a great deal.

Solar active regions are known to be permeated by strong ($\sim$ kG) magnetic fields in the photosphere. The large energy output of flares is generally attributed to the rapid conversion of magnetic energy of non-potential ($\nabla \times \vec{B} \neq 0$) fields in the corona into particle energy. In the low $\beta = nkT/B^2$ coronal plasma above a well-developed active region, field-aligned currents can flow which close in the photosphere and lower chromosphere. The electromotive force for these coronal currents can be understood as arising from the convective motions of the partially ionized gas in the photosphere at a pressure $p \sim 0.1$ atm. If we accept a value of $\eta_0 \sim 10$ ohm cm for the intrinsic resistivity of the photosphere, then the gas has electrical conducting properties equivalent to a semi-conductor with a conductivity somewhat better than sea water. Photospheric "neutral winds" [Sen and White, 1972] can blow across the magnetic field and impart stresses on it causing it to be twisted or sheared. The flare is then a rapid release of this free energy.

The simplest approach to the dynamo problem is to assume infinite conductivity in the photosphere for which the field lines are frozen into the gas. For time scales less than the resistive time constant, this is probably an accurate picture. Finite resistivity is generally incorporated by solving a convection/diffusion equation for the magnetic field.

However, there are advantages to an alternate perspective of the problem given by a source-theoretic analysis. Here, one solves directly for the currents using, say, Ohm's law and self-consistently adjusts the magnetic field. This approach has the advantage of enabling one to assess and incorporate anomalous plasma and kinetic effects (e.g., current-driven plasma instabilities) and also to handle tricky boundary conditions and constraints which can arise when the total system consists of quite different behaving plasmas. Alfven and Falthammer (1963) have described three classes of plasmas with radically different properties and when combinations of these occur in a system, source theory is well suited for the analysis on a local as well as global basis.

Sen and White (1972) adopted the picture of the photosphere as an MHD dynamo and considered the currents driven by radial gas flow from the center of a symmetric sunspot. They only considered those currents which flow in the photosphere and their analysis could not describe any twisting or shearing of the field. Heyvaerts (1974), however, provided the key idea that the magnetic field could act as a shunt and allow a large current flow out of the photosphere and into the corona. This aspect characterizes the photosphere as a full MHD generator [Brogan, 1968] capable of also storing magnetic energy through a coronal current system.

This picture is attractive for flares because if photospheric faculae and chromospheric plages are interpreted as resulting from Joule dissipation of dynamo currents [Sen and White, 1972], then in view of Svestka's (1976) description "A flare appears either as a brightening of a part of existing plage (the most common case) or as the formation of new bright areas in places where the plage did not exist before", the plage is a signature of dynamo action (either as a generator or a load) and its coronal magnetic field can be highly non-potential and susceptible to a flare instability.

The purpose of this paper is to synthesize the work of Sen and White (1972) and Heyvaerts (1974) in a model which allows more general gas flows. The very low electrical resistance of the corona compared to the photosphere suggests a very simple geometrical model of the system consisting of 2 + 1 dimensions can be used. We consider the dynamo action in a relatively thin (100-1000 km) horizontal layer on the top of the photosphere and allow currents to pass with little resistance into the corona and dissipate in a photospheric load at the conjugate magnetic footpoints. The model can be viewed as simply two resistive unipolar inductors connected in series.

The most complete analysis of this problem would require that we self-consistently compute the generated magnetic field. But for simplicity we shall assume 1) steady-state conditions and 2) the magnetic Reynolds number $R_m < 1$. The second condition puts us in the regime of conventional MHD (electric) generators [Brogan, 1968]. However, the analysis for $R_m \gg 1$ is inherently non-linear and our simplified linear treatment should contain useful information even for $R_m \sim 1$. It may well be that an MHD or tearing instability prevents a very large deviation from the potential magnetic field configuration.

In Section II we describe the model of two coupled parallel-plane unipolar inductors. In Section III we compute the Hall, Pedersen, and field-aligned currents for an arbitrary two-dimensional neutral wind velocity flow for the case of uniform $\vec{B}$. In Section IV we consider an inhomogeneous $\vec{B}$ and compute only the field-aligned current. In Section V we present concluding remarks.

II. MODEL

Heyvaerts' (1974) shunt model of the photosphere is shown schematically in Fig. 1(a) for a simple bipolar magnetic field configuration. The dynamo slabs are linked by the magnetic field and because of the low electrical resistivity of the fully ionized, low β plasma in the corona the field lines are near iso-potential contours of the electric field. We have modified his model slightly and will consider the system sketched in Fig. 1(b). This configuration simplifies the analysis somewhat; it also lends more generality to the problem both in execution and application to other space and astrophysical circumstances. We are only interested in the current systems possible and the generated magnetic fields may differ for 1(a) and 1(b) because of geometrical considerations. We have also neglected effects due to horizontal components of the magnetic field in the slabs for the sake of simplicity.

We assume a neutral wind flows in each slab of effective width $\ell_{a,b}$ for which the two dimensional (anisotropic) electrical conductivity tensor $\underset{\sim}{\sigma}$ perpendicular to the magnetic field is known. The thickness of the slab is treated as small so that the potential drop along the magnetic field is small everywhere. The magnetic field lines are equipotentials of the electric field and conduct current between each slab with no resistance; in the interface region, the magnetic field lines are perfect insulators in the perpendicular direction.

III. HOMOGENEOUS MAGNETIC FIELD

A. CURRENTS

If the magnetic field is uniform in the dynamo region, the analysis yields differential equations with constant coefficients and a complete solution for the currents may be obtained. For two dimensional cartesian coordinates we write the conductivity tensor in a slab as

$$\sigma = \sigma_1 \iota - \sigma_2 \epsilon = \sigma_1 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \sigma_2 \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \qquad (1)$$

which defines the matrices ι and ϵ. The scalar quantities σ_1 and σ_2 are the Pedersen and Hall conductivities respectively. If we can neglect the effects of electron-ion collisions, the conductivities may be written as

$$\sigma_1 = n_0 e^2 \left[\frac{\nu_{en}}{m(\nu_{en}^2 + \Omega_{ce}^2)} + \frac{\nu_{in}}{M(\nu_{in}^2 + \Omega_{ci}^2)} \right] \simeq \frac{n_0 e^2}{M \nu_{in}} (1 + \alpha) \qquad (2)$$

$$\sigma_2 = n_0 e^2 \left[\frac{\Omega_{ce}}{m(\nu_{en}^2 + \Omega_{ce}^2)} - \frac{\Omega_{ci}}{M(\nu_{in}^2 + \Omega_{ci}^2)} \right] \simeq \frac{n_0 e^2}{m \Omega_{ce}} \ . \qquad (3)$$

Here we have assumed a simple gas where the charge carriers are electrons of mass m and singly charged ions of mass M with densities $n_e \simeq n_i = n_0$. The approximate forms of (2) and (3) result for the dynamo inequalities [Sen and White, 1972]

$$\nu_{en}/\Omega_{ce} \ll 1 \ , \quad \nu_{in}/\Omega_{ci} \gg 1 \qquad (4)$$

which can occur in the photosphere in the presence of kG magnetic fields. When $\alpha \equiv \frac{\nu_{en}\nu_{in}}{\Omega_{ce}\Omega_{ci}} < 1$ the ion slip regime of conventional MHD generators occurs [Swift-Hook, 1965]. We also may use

$$\sigma_0 = n_0 e^2 \left[\frac{1}{m\nu_{en}} + \frac{1}{M\nu_{in}} \right] \tag{5}$$

and

$$\sigma_3 = \sigma_1 \left[1 + (\sigma_2/\sigma_1)^2 \right] \tag{6}$$

for the intrinsic (field-aligned) and Cowling conductivities in the photosphere. A thorough discussion of electrical conductivity may be found in the reviews of Chapman (1956) and Braginskii (1965). An excellent review of MHD generator physics can be found in Swift-Hook (1965).

Referring to Fig. 1(b), if the field-aligned current density falls off rapidly inside the slab and in the vicinity of $z = 0$, we may approximate $J_z(x,y,z) \simeq J_z(x,y)H(z)$ where $H(z)$ is the Heaviside step function, then integration of the charge conservation equation yields

$$\int dz \left(\frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z} \right) = 0 \tag{7}$$

so that

$$J_z(x,y) = - \left(\frac{\partial K_x}{\partial x} + \frac{\partial K_y}{\partial y} \right) \equiv - \nabla \cdot K , \tag{8}$$

where

$$K = \begin{pmatrix} K_x \\ K_y \end{pmatrix}$$

and

$$K_{x,y} = \int dz \; J_{x,y}(x,y,z) \tag{9}$$

is the surface current density in the slab across the magnetic field. All spatial dependence of the variables now occurs in the (x,y) plane and we shall make use of the (scalar) two dimensional curl operation defined by

$$\nabla \times V \equiv \nabla \cdot \epsilon V = \frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y} .$$

For the case of a uniform magnetic field, slab a maps identically onto b and we have

$$J_z(x,y) = -\nabla \cdot K_a(x,y) = \nabla \cdot K_b(x,y) . \tag{10}$$

If for each slab we define the height-integrated conductivity as

$$\Sigma = \int dz\, \sigma \sim \sigma \ell , \tag{11}$$

where ℓ measures the effective thickness of each slab, then Ohm's law may be written in matrix form as

$$K_{a,b} = \Sigma_{a,b} \left[E + \frac{B}{c} \epsilon V_{a,b} \right] . \tag{12}$$

$E(x,y)$ is the electric field which in steady state satisfies

$$\left. \begin{aligned} \nabla \times E &= 0 \\ \nabla \cdot E &= 4\pi\rho \end{aligned} \right\} \tag{13}$$

and the second term in the brackets is the electromotive field resulting from a neutral wind velocity field $V_{a,b}(x,y)$ in each slab.

We now add the expressions in (12) for a and b and using (10) and (13) we obtain

$$\nabla \cdot E = \frac{-B/c}{s_{1a}+s_{1b}} \nabla \cdot \left[(s_{2a}V_a + s_{2b}V_b) + \epsilon\, (s_{1a}V_a + s_{1b}V_b) \right] , \tag{14}$$

where we have defined analogous to (1)

$$\Sigma = s_1 \iota - s_2 \epsilon \tag{15}$$

and have used the relation $\epsilon\epsilon = -\iota$.

It is convenient, at this point, to introduce the potential functions, ψ and χ , such that

$$V = V_\psi + V_\chi = -\nabla\psi + \epsilon\nabla\chi . \qquad (16)$$

V_ψ and V_χ are the irrotational and solenoidal components of the neutral velocity field satisfying

$$\left.\begin{aligned} \nabla^2\psi &= -\nabla\cdot V \\ \nabla^2\chi &= -\nabla\times V \end{aligned}\right\} . \qquad (17)$$

Defining the electric potential to be $E = -\nabla\phi$, we obtain from (14)

$$\nabla^2\phi = \frac{-B/c}{s_{1a}+s_{1b}}\left[s_{2a}\nabla^2\psi_a + s_{1a}\nabla^2\chi_a + s_{2b}\nabla^2\psi_b + s_{1b}\nabla^2\chi_b\right] . \qquad (18)$$

The solution to (18) is given as

$$\phi = \frac{-B/c}{s_{1a}+s_{1b}}\left[s_{2a}\psi_a + s_{1a}\chi_a + s_{2b}\psi_b + s_{1b}\chi_b\right] + \Lambda . \qquad (19)$$

Λ is the homogeneous solution satisfying $\nabla^2\Lambda = 0$ and represents the effects of any externally imposed boundary conditions in the (x,y) plane. However, for the geometry of Fig. 1(b) it is assumed that all sources of the electric field have been accounted for by specification of $V_{a,b}$. We may, therefore, take $\Lambda = 0$ and the electric field is given as

$$E = \frac{B/c}{s_{1a}+s_{1b}}\left[s_{2a}\nabla\psi_a + s_{1a}\nabla\chi_a + s_{2b}\nabla\psi_b + s_{1b}\nabla\chi_b\right] . \qquad (20)$$

Inserting this expression into (12) we have finally

$$K_a = \frac{B/c}{s_{1a}+s_{1b}} \left\{ \iota \left[s_{1a}(s_{2b}\nabla\psi_b + s_{1b}\nabla\chi_b) - s_{1b}(s_{2a}\nabla\psi_a + s_{1a}\nabla\chi_a) \right] \right.$$

$$\left. -\epsilon \left[s_{2a}(s_{2b}\nabla\psi_b + s_{1b}\nabla\chi_b) + (s_{1a}^2 + s_{2a}^2 + s_{1a}s_{1b})\nabla\psi_a - s_{1b}s_{2a}\nabla\chi_a \right] \right\} \tag{21}$$

$$K_b = \frac{B/c}{s_{1a}+s_{1b}} \left\{ -\iota \left[s_{1a}(s_{2b}\nabla\psi_b + s_{1b}\nabla\chi_b) - s_{1b}(s_{2a}\nabla\psi_a + s_{1a}\nabla\chi_a) \right] \right.$$

$$\left. -\epsilon \left[s_{2b}(s_{2a}\nabla\psi_a + s_{1a}\nabla\chi_a) + (s_{1b}^2 + s_{2b}^2 + s_{1a}s_{1b})\nabla\psi_b - s_{1a}s_{2b}\nabla\chi_b \right] \right\} \tag{22}$$

$$J_z = \frac{B/c}{s_{1a}+s_{1b}} \left\{ s_{1b}(s_{2a}\nabla^2\psi_a + s_{1a}\nabla^2\chi_a) - s_{1a}(s_{2b}\nabla^2\psi_b + s_{1b}\nabla^2\chi_b) \right\} \tag{23}$$

We remark that all ambipolar effects are automatically taken account of in the solution for the polarization electric field (20). The representations in (21) and (22) show explicitly the irrotational and solenoidal components of K in each slab, the former giving rise to the field-aligned current (23) linking both slabs either of which can be a "generator/motor" or just simply a "load".

If we rewrite (12) for each slab (suppressing subscripts) as

$$\vec{E} + \frac{1}{c}\vec{V} \times \vec{B} = \underset{\approx}{\Sigma}^{-1} \cdot \vec{K} \tag{24}$$

and make use of the fact that

$$\vec{K} \cdot \underset{\approx}{\Sigma}^{-1} \cdot \vec{K} = \frac{1}{s_3} K^2 \quad , \tag{25}$$

then scalar multiplication of (24) with $\vec{K}$ yields

$$\frac{1}{c}\vec{V} \cdot (\vec{K} \times \vec{B}) + \frac{1}{s_3} K^2 = \vec{K} \cdot \vec{E} \ . \tag{26}$$

If we multiply (26) by an infinitesimal element of area dA , then the first term is the work done by the height-integrated volume force $\frac{1}{c}\vec{K} \times \vec{B}$ at velocity $\vec{V}$ and, if negative, represents the local power generated by the slab as a seat of electromotive force. The second term represents ohmic losses and the right-hand side gives the electrical power into dA . A negative value of $\vec{K} \cdot \vec{E}$ represents electrical power out of dA and when this occurs a negative value of $\vec{V} \cdot (\vec{K} \times \vec{B})$ is required; the slab is then locally a power generator [Swift-Hook, 1965].

We note that in the most general case the quantity $(\vec{K}_a + \vec{K}_b)\cdot\vec{E} \neq 0$ at a point (x,y). This seems to violate conservation of electrical energy. However, it is easy to show that if we integrate over the entire area of the system, $\int dA\, (\vec{K}_a + \vec{K}_b)\cdot\vec{E} = 0$. This enforces conservation of total electrical energy and indicates that different points in a slab communicate power through the Poynting flux $\vec{S} = \frac{c}{4\pi}\vec{E} \times \vec{B}$.

B. DECOMPOSITION OF THE VELOCITY FIELD

The solutions (20)-(23) have been readily obtained because of the possibility of the decomposition in (16). The Fourier expansion of

$$\nabla^2 G = -4\pi\delta(x-x')\delta(y-y') \tag{27}$$

leads to the Green's function in two dimensions $(k = |k_\perp|)$

$$\tilde{G}(k) = \frac{4\pi}{k^2} \ . \tag{28}$$

Thus, for a given neutral velocity field in each slab, the solution of (17) is

$$\begin{Bmatrix} \psi \\ \chi \end{Bmatrix} = \int \frac{d\vec{k}_\perp}{(2\pi)^2} \frac{1}{k^2} e^{i\vec{k}_\perp\cdot\vec{x}_\perp} \int d\vec{x}_\perp{}' \, e^{-i\vec{k}_\perp\cdot\vec{x}_\perp'} \begin{Bmatrix} \nabla'\cdot V \\ \nabla' x V \end{Bmatrix} , \tag{29}$$

and the formal solution of the problem is complete.

C. SIMPLE GAS FLOWS

Consideration of some elementary gas flows in special circumstances will illuminate much of the physics contained in (20)-(23). We take $V_b = 0$ and let $V_a = V$ for the following cases of interest:

(i) OPEN TERMINALS ($\Sigma_b = 0$)

Current flow is restricted to slab a and $J_z = K_b = 0$. The electric field (20) reduces to

$$\vec{E} = -\frac{s_{2a}}{s_{1a}} \frac{B}{c} \vec{V}_\psi - \frac{1}{c} \vec{V}_\chi \times \vec{B} \tag{30}$$

and the surface current (21) is

$$\vec{K}_a = \frac{1}{c} s_{3a} \vec{V}_\psi \times \vec{B} . \tag{31}$$

The magnetic force density on the gas is given as

$$\frac{1}{c} \vec{K} \times \vec{B} = -\frac{B^2}{c^2} s_{3a} \vec{V}_\psi \tag{32}$$

while the ohmic losses and electrical power density reduce to

$$\frac{1}{s_{3a}} K^2 = \frac{B^2}{c^2} s_{3a} V_\psi^2 \tag{33}$$

$$\vec{K} \cdot \vec{E} = -\frac{B^2}{c^2} s_{3a} \vec{V}_\psi \cdot \vec{V}_\chi . \tag{34}$$

The quantities in (33) and (34) will, in general, differ when dA is a seat of EMF. We could "redefine" the electric field $\vec{E} \to \vec{E}'$ to absorb the $\vec{V} \times \vec{B}$ term in (12); then, of course, $\frac{1}{s_3} K^2 = \vec{K} \cdot \vec{E}'$. However, we prefer to distinguish between the electromotive field and the electric field satisfying (13) which, in principle, can unambiguously be determined by hooking up a potentiometer across the slab's magnetic field terminals.

We note that only the irrotational component of $\vec{V}$ experiences a braking force in (32). The solenoidal part of $\vec{V}$ flows freely due to the absence of any electrical load in slab b. We also note that the surface current in (31), which is manifestly solenoidal, is driven by the irrotational component of $\vec{V}$ by means of the "enhanced" or Cowling conductivity s_3. Clearly, a polarization electric field (30) has built up which enhances the current flow above that expected from the Pedersen conductivity alone. If, also $\Sigma_b = \Sigma_a$, the polarization field will drive, in addition to other currents, a strong component $= \frac{1}{2c} \frac{s_2^2}{s_1} \vec{V}_\psi \times \vec{B}$ in slab b. By analogy to the effect observed in the earth's equatorial ionosphere [Cunnold, 1978] we may consider the current in (31) driven by $\vec{V}_\psi$ through the Cowling conductivity as a two-dimensional electrojet. Although the effects are similar, in fact the earth's electrojet (s) is much more complicated owing to the complexities of the three-dimensional field/ionosphere configuration.

(ii) SHORTED TERMINALS ($s_{1b} \to \infty$, $\frac{s_{2b}}{s_{1b}} \to 0$)

Of course $E = 0$ and the surface currents are

$$\left.\begin{aligned} \vec{K}_a &= \frac{B}{c} s_{2a} \vec{V} + \frac{1}{c} s_{1a} \vec{V} \times \vec{B} \\ \vec{K}_b &= -\frac{B}{c} s_{2a} \vec{V}_\psi - \frac{1}{c} s_{1a} \vec{V}_\chi \times \vec{B} \end{aligned}\right\} . \tag{35}$$

It is interesting here to compare the body forces

$$\left.\begin{aligned} \vec{F}_a &= \frac{B}{c^2} s_{2a} \vec{V}_\psi \times \vec{B} - \frac{B^2}{c^2} s_{1a} \vec{V}_\psi - \frac{B^2}{c^2} s_{1a} \vec{V}_\chi + \frac{B}{c^2} s_{2a} \vec{V}_\chi \times \vec{B} \\ \vec{F}_b &= -\frac{B}{c^2} s_{2a} \vec{V}_\psi \times \vec{B} + \frac{B^2}{c^2} s_{1a} \vec{V}_\chi \end{aligned}\right\} . \tag{36}$$

For an irrotational flow, the magnetic field opposes the gas flow and slab b exerts a force perpendicular to the gas flow; a solenoidal flow is braked by slab b and the magnetic field exerts a force perpendicular to the gas flow.

(iii) SOLENOIDAL FLOW ($\nabla \cdot V = 0$)

The currents are given as

$$\left.\begin{aligned} \vec{K}_a &= \frac{1}{c}\,\frac{s_{1b}}{s_{1a}+s_{1b}}\left[Bs_{2a}\vec{V}_\chi + s_{1a}\vec{V}_\chi \times \vec{B}\right] \\ \vec{K}_b &= -\frac{1}{c}\,\frac{s_{1a}}{s_{1a}+s_{1b}}\left[Bs_{2b}\vec{V}_\chi + s_{1b}\vec{V}_\chi \times \vec{B}\right] \end{aligned}\right\} \tag{37}$$

and

$$J_z = \frac{B}{c}\,\frac{s_{1a}s_{1b}}{s_{1a}+s_{1b}}\,\nabla^2 \chi\ . \tag{38}$$

The electric field is

$$\vec{E} = -\frac{1}{c}\,\frac{s_{1a}}{s_{1a}+s_{1b}}\,\vec{V}_\chi \times \vec{B}\ , \tag{39}$$

and becomes $\vec{E} \to -\frac{1}{c}\vec{V}_\chi \times \vec{B}$ only when either $s_{1b} \to 0$ or $s_{1a} \to \infty$. Heyvaerts' (1974) model falls into this class of photospheric flows which are good candidates for shearing, say, a bipolar magnetic arcade.

(iv) IRROTATIONAL FLOW ($\nabla \times V = 0$)

Sen and White's (1972) model falls into this class and, in light of recent solar observations of sub-arcsecond, kilogauss magnetic flux elements, we reexamine their model. The currents are given as

$$\left.\begin{aligned}\vec{K}_a &= \frac{1}{c}\,\frac{1}{s_{1a}+s_{1b}}\left[Bs_{2a}s_{1b}\vec{V}_\psi + (s_{1a}^2+s_{2a}^2+s_{1a}s_{1b})\vec{V}_\psi \times \vec{B}\right] \\ \vec{K}_b &= \frac{1}{c}\,\frac{s_{2a}}{s_{1a}+s_{1b}}\left[-Bs_{1b}\vec{V}_\psi + s_{2b}\vec{V}_\psi \times \vec{B}\right]\end{aligned}\right\} \qquad (40)$$

and

$$J_z = \frac{B}{c}\,\frac{s_{2a}\,s_{1b}}{s_{1a}+s_{1b}}\,\nabla^2\psi \; . \qquad (41)$$

The corresponding electric field is

$$\vec{E} = -\frac{B}{c}\,\frac{s_{2a}}{s_{1a}+s_{1b}}\,\vec{V}_\psi \; . \qquad (42)$$

Irrotational, radial gas flow may be described by the expression

$$\vec{V} = V(\rho)\left[H(\rho-\rho_1) - H(\rho-\rho_2)\right]\hat{\rho} \; ,$$

where $\rho_1 < \rho_2$ defines the inner and outer boundaries of a flow annulus. Sen and White (1972) considered an outflow, but we shall instead consider an inflow, $V(\rho) < 0$.

If we take $\Sigma_b = 0$, the above expressions reduce to the results of Sen and White (1972) for the ion slip regime they considered. In this situation the electrons are strongly magnetized and the ions unmagnetized. Physically, the gas carries ions inwards until a polarization field builds up to cancel any radial electric current, $K_\rho = 0$. A small azimuthal ion current flows in the $\vec{V} \times \vec{B}$ direction. However, in response to the polarization electric field, electrons $\vec{E} \times \vec{B}$ drift and enhance the azimuthal current considerably. This intense azimuthal current system (primarily of electrons) was termed a Hall current by Sen and White in reference to the direction of the electric field; however, in the language of MHD generators, a current flow in the $\vec{V} \times \vec{B}$ direction is conventionally a Faraday or Pedersen current [Swift-Hook, 1965; Brogan, 1968] in reference to the direction of the electromotive field. It is clear that either convention is acceptable so long as one makes explicit his reference to either a driving electric or electromotive field.

Since inward gas flow enhances a "seed" magnetic field, the question is can a self-consistent magnetic dynamo be set up. If the electrojet current is confined to a thin sheet, it is straightforward to show that in steady-state, the velocity required to maintain the magnetic field must satisfy the order of magnitude estimate (esu)

$$\frac{V}{c} \sim \frac{c}{2\pi\sigma_3 \ell} \quad . \tag{43}$$

This result is in agreement with Cowling's (1957) estimate (aside from a factor of two); however, we emphasize that the conductivity to be used is the Cowling conductivity (6) and the scale length is the thickness of the current sheet not the perpendicular scale length, r . For a current sheet where $\ell \ll r$ the magnetic field does not depend sensitively on the vertical distribution of current inside the sheet so long as the surface current density is held constant, $K \propto \sigma_3 \ell V$. Thus, the relevant combination must be as given in (43). If we take values of $\sigma_3 \sim 10^{11}$ esu and $\ell \sim 100$ km, Equation (43) yields $V = 1.5$ m/sec. This value is 10^9 times larger than Cowling's (1957) estimate since his conductivity was a coronal value and he used a very large perpendicular scale length.

The very short lifetime ($\lesssim$ hrs) of sub-arcsecond magnetic flux elements (fluxules) [Harvey, 1977; Stenflo, 1976; Parker, 1978, and his references] strongly suggests that they are a solar surface phenomenon and are generated in the photosphere (or not far from it). Parker's (1978) analysis demonstrated that a superadiabatic effect occurring beneath the fluxule could lower the temperature of the core and thereby reduce the core gas pressure. An effect of this type can produce an MHD magnetic generator where the core is a low pressure, MHD sump. Observations clearly show gas downdrafts into the core and we may expect also a smaller horizontal velocity component across the field into the core.

Parker's analysis also raises an important point concerning the electric conductivity. The fact that ion-neutral collisions are important in the quiet photosphere has already been appreciated by Kopecky and Kuklin (1969) and by Sen and White (1972). In this regime the conductivity is very sensitive to gas temperature relating directly to the degree of ionization. A cool core may even lower the conductivity by as much as two orders of magnitude from the value quoted earlier and place the effective conductivity in the range $\sigma_3 \sim 10^9 - 10^{11}$ esu [Kopecky and Kopecky, 1971]. The perpendicular velocity of 150 m/s required by (43) for this case is then not far from observed downdraft velocities $\sim$ km/s.

The question of whether a steady state can be reached is also relevant. We shall show in a subsequent paper that for a current sheet where the thickness and perpendicular scale length satisfy $\ell \ll r$, the magnetic resistive decay time is approximately

$$\tau_{DIFF} \simeq \frac{2\pi\sigma_3 \ell r}{c^2} \quad . \tag{44}$$

For a radial gas inflow the gas convection time is clearly

$$\tau_{CONV} \sim r/v \quad . \tag{45}$$

Thus, the magnetic Reynolds number for a current sheet in the plane of the photosphere is

$$R_m = \tau_{DIFF}/\tau_{CONV} \sim \frac{2\pi\sigma_3 \ell v}{c^2} \ . \qquad (46)$$

We note that the self-consistency relation (43) is equivalent to $R_m \sim 1$ for a current sheet. Even for values $\ell \sim r \sim 100$ km, Equation (44) yields decay times consistent with observation [Stenflo, 1976]. The actual lifetime of the fluxules is probably determined by MHD considerations: e.g., how long can the low pressure core be maintained? An MHD instability could also develop to destroy the dynamo on a time scale related to (44).

IV. INHOMOGENEOUS MAGNETIC FIELD

We shall restrict our attention to the simplest case of a vertical magnetic field $B(\vec{x}_\perp)$ which is the same in both slabs and also treat $\Sigma_a = \Sigma_b = \Sigma(\vec{x}_\perp)$. If we take the divergence of (12) for each slab and add, we obtain using (10)

$$\nabla \cdot \left[\Sigma E\right] = -\frac{1}{2} \nabla \cdot \left[\Sigma \frac{B}{c} \epsilon(V_a + V_b)\right] . \tag{47}$$

The field-aligned current density is thus

$$J_z(x,y) = \frac{1}{2} \nabla \cdot \left[\Sigma \frac{B}{c} \epsilon(V_b - \dot{V}_a)\right] . \tag{48}$$

If we use the approximate forms of (2) and (3) for the dynamo inequalities (4) and treat n_0 as a constant, Equation (48) can be cast in the form

$$J_z(x,y) = \frac{1}{2} n_0 e\ell \left\{ \nabla\cdot(V_b - V_a) + \nabla x \left[\frac{\Omega_{ci}}{\nu_i}(1+\alpha)(V_b - V_a)\right]\right\} , \tag{49}$$

this result and (48) reducing to (23) for $\nabla\Sigma = \nabla B = 0$. For the symmetric case we have treated, the polarization field need not be computed to obtain J_z ; the surface currents, of course, require a solution for E . Equation (49) gives the interesting result that even for a constant velocity field, a field-aligned current can be generated through $\vec{V} \times \nabla \Omega_{ci}$, where $\Omega_{ci} = eB/Mc$, when $\alpha \ll 1$.

V. CONCLUDING REMARKS

We have considered the currents generated in two coupled unipolar inductors. For simplicity we have concentrated on the case of a uniform magnetic field for which all the currents may be computed in 2 + 1 Cartesian geometry with relative ease. More accurate modelling is needed to encompass the more realistic case of non-uniform $\vec{B}$. We have indicated in Section IV how gradients in magnetic field strength can also generate field-aligned currents in the presence of neutral wind flows.

For application to the solar atmosphere we have considered the broad aspects of two basic categories of problems: 1) in the presence of a strong, large-scale magnetic field (with a subphotospheric primary current source) can photospheric winds distort the field and store magnetic energy in the corona, to serve as free energy for a flare? and 2) can the photosphere generate magnetic fields of itself? We have presented some evidence that the answer to both questions is yes.

In regard to question (1), Heyvaerts (1974) has given quantitative arguments that the photosphere can be a coronal current generator using reasonable values for parameters. He has also issued the caveat that for the case of a thin dynamo, the thickness of the slab is important in dimensional analyses. We have attempted to generalize his work for an arbitrary gas flow. In the case of a bipolar magnetic arcade, if the dynamo action is far removed from the polarity inversion line, our solution should be valid; in the vicinity of the inversion line, edge effects can occur that can be accounted for by a modification of Equation (29) for a semi-infinite plane.

In regard to question (2), we believe that electrodynamically there is no inconsistency in interpreting small-scale magnetic flux elements as photospheric dynamos. The magnetohydrodynamics, of course, must still be done to get an overall, self-consistent, gas flow solution. The central problem in tying everything together must be an accurate determination of the electrical conductivity tensor $\sigma_i (i=0-3)$ for each solar surface phenomenon under consideration.

For application to other space environments, the coupled inductor model can provide order of magnitude estimates for many effects if not just valuable insight. The important point is that the model attempts to account for the complete current circuit and in doing so the source-theoretic approach can handle a problem where local dynamo action can produce a global effect that shows up a large distance away. Field-theoretic approaches that concentrate locally on the physics in a small volume must properly take account of boundary conditions far away. For instance, the influence of Σ_b on a dynamo in slab a is manifest throughout (20)-(23). With appropriate scale factors that effect the transformation from a $\leftrightarrow$ b [viz., Equation (10)], slab b may represent a planet's ionosphere. Slab a can be used to describe the motion of, say, the ionosphere of Io orbiting Jupiter. A generalized Ohm's law may be constructed to replace (12) for slab a and the model may be able to provide a crude description of the complex magnetosphere/ionosphere interaction. We are pursuing this line of reasoning in the hope that temporal, electromagnetic induction effects may be incorporated into the model.

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DYNAMO ACTION IN A THIN SLAB

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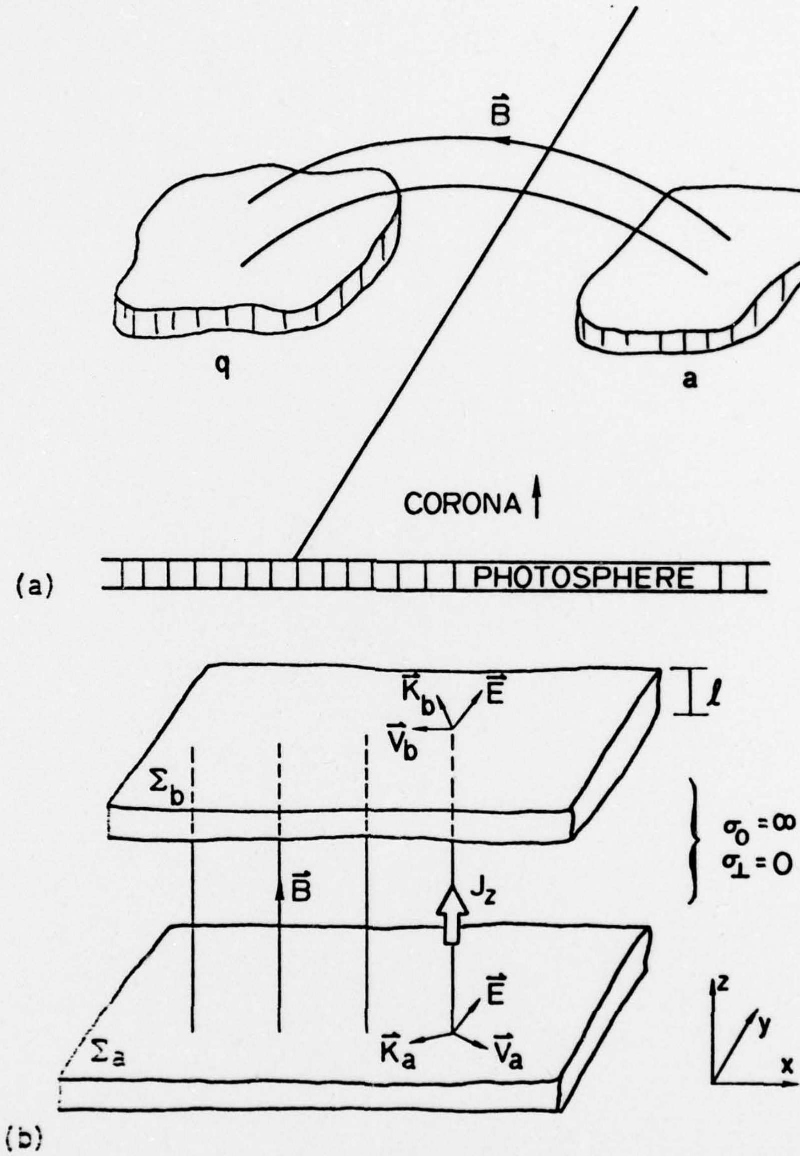
FIGURE CAPTION

Figure 1. Dynamo action in two thin, coupled slabs.

(a) Heyvaert's (1974) model for a bipolar magnetic configuration. Photospheric convection generates a field-aligned coronal current system which can twist or shear the field and provide magnetic free energy for a flare.

(b) Two resistive, Cartesian unipolar inductors connected in series.



$\vec{B}$
q
a
CORONA
(a)
PHOTOSPHERE
$\vec{K}_b$
$\vec{E}$
ℓ
$\vec{V}_b$
Σ_b
$\sigma_0 = \infty$
$\sigma_\perp = 0$
$\vec{B}$
J_z
$\vec{E}$
z
y
x
Σ_a
$\vec{K}_a$
$\vec{V}_a$
(b)

Figure 1

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We consider the dynamo action produced by convection of a partially-ionized, electrically conducting gas in a magnetic field. The model consists of two thin, Cartesian unipolar inductors connected in series by the magnetic field. For the case of a uniform magnetic field ~~we compute~~ the total current system generated by an arbitrary gas flow; for the case of a non-uniform field, ~~we compute~~ only the field-aligned coupling current. Application is made to the solar atmosphere.

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