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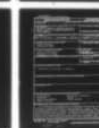   MAY 78   J D HOVANESIAN, Y Y HUNG, A J DURELLI   N00014-76-C-0487
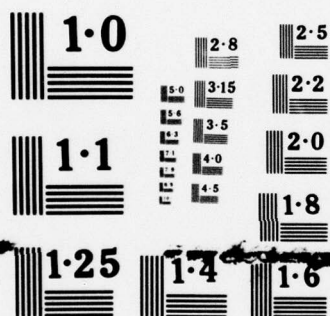
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BY

J. B. NOVANINSTAN, Y. J. HUNG AND A. J. DURELLI

NEW OPTICAL METHOD TO DETERMINE VIBRATION-INDUCED STRAINS

WITH VARIABLE SENSITIVITY AFTER RECORDING

by

J. D. Hovanesian, Y. Y. Hung and A. J. Durelli

Sponsored by

School of Engineering
Oakland University
Rochester, Michigan 48063

May 1978

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Previous Technical Reports to the Office of Naval Research

1. A. J. Durelli, "Development of Experimental Stress Analysis Methods to
    Determine Stresses and Strains in Solid Propellant Grains"--June 1962.
    Developments in the manufacturing of grain-propellant models are
    reported. Two methods are given: a) cementing routed layers and
    b) casting.

2. A. J. Durelli and V. J. Parks, "New Method to Determine Restrained
    Shrinkage Stresses in Propellant Grain Models"--October 1962.
    The birefringence exhibited in the curing process of a partially
    restrained polyurethane rubber is used to determine the stress associated
    with restrained shrinkage in models of solid propellant grains partially
    bonded to the case.

3. A. J. Durelli, "Recent Advances in the Application of Photoelasticity in
    the Missile Industry"--October 1962.
    Two- and three-dimensional photoelastic analysis of grains loaded by
    pressure and by temperature are presented. Some applications to the
    optimization of fillet contours and to the redesign of case joints are
    also included.

4. A. J. Durelli and V. J. Parks, "Experimental Solution of Some Mixed
    Boundary Value Problems"--April 1964.
    Means of applying known displacements and known stresses to the boundaries
    of models used in experimental stress analysis are given. The applica-
    tion of some of these methods to the analysis of stresses in the field
    of solid propellant grains is illustrated. The presence of the "pinching
    effect" is discussed.

5. A. J. Durelli, "Brief Review of the State of the Art and Expected Advance
    in Experimental Stress and Strain Analysis of Solid Propellant Grains"--
    April 1964.
    A brief review is made of the state of the experimental stress and strain
    analysis of solid propellant grains. A discussion of the prospects for
    the next fifteen years is added.

6. A. J. Durelli, "Experimental Strain and Stress Analysis of Solid Propellant
    Rocket Motors"--March 1965.
    A review is made of the experimental methods used to strain-analyze solid
    propellant rocket motor shells and grains when subjected to different
    loading conditions. Methods directed at the determination of strains in
    actual rockets are included.

7. L. Ferrer, V. J. Parks and A. J. Durelli, "An Experimental Method to Analyze
    Gravitational Stresses in Two-Dimensional Problems"--October 1965.
    Photoelasticity and moiré methods are used to solve two-dimensional
    problems in which gravity-stresses are present.

8.   A. J. Durelli, V. J. Parks and C. J. del Rio, "Stresses in a Square Slab Bonded on One Face to a Rigid Plate and Shrunk"--November 1965.
A square epoxy slab was bonded to a rigid plate on one of its faces in the process of curing. In the same process the photoelastic effects associated with a state of restrained shrinkage were "frozen-in." Three-dimensional photoelasticity was used in the analysis.

9.   A. J. Durelli, V. J. Parks and C. J. del Rio, "Experimental Determination of Stresses and Displacements in Thick-Wall Cylinders of Complicated Shape"--April 1966.
Photoelasticity and moiré are used to analyze a thre-dimensional rocket shape with a star shaped core subjected to internal pressure.

10.   V. J. Parks, A. J. Durelli and L. Ferrer, "Gravitational Stresses Determined Using Immersion Techniques"--July 1966.
The methods presented in Technical Report No. 7 above are extended to three-dimensions. Immersion is used to increase response.

11.   A. J. Durelli and V. J. Parks, "Experimental Stress Analysis of Loaded Boundaries in Two-Dimensional Second Boundary Value Problems"-- February 1967.
The pinching effect that occurs in two-dimensional bonding problems, noted in Reports 2 and 4 above, is analyzed in some detail.

12.   A. J. Durelli, V. J. Parks, H. C. Feng and F. Chiang, "Strains and Stresses in Matrices with Inserts"--May 1967.
Stresses and strains along the interfaces, and near the fiber ends, for different fiber end configurations, are studied in detail.

13.   A. J. Durelli, V. J. Parks and S. Uribe, "Optimization of a Slot End Configuration in a Finite Plate Subjected to Uniformly Distributed Load"--June 1967.
Two-dimensional photoelasticity was used to study various elliptical ends to a slot, and determine which would give the lowest stress concentration for a load normal to the slot length.

14.   A. J. Durelli, V. J. Parks and Han-Chow Lee, "Stresses in a Split Cylinder Bonded to a Case and Subjected to Restrained Shrinkage"-- January 1968.
A three-dimensional photoelastic study that describes a method and shows results for the stresses on the free boundaries and at the bonded interface of a solid propellant rocket.

15.   A. J. Durelli, "Experimental Stress Analysis Activities in Selected European Laboratories"--August 1968.
This report has been written following a trip conducted by the author through several European countries. A list is given of many of the laboratories doing important experimental stress analysis work and of the people interested in this kind of work. An attempt has been made to abstract the main characteristics of the methods used in some of the countries visited.

16. V. J. Parks, A. J. Durelli and L. Ferrer, "Constant Acceleration Stresses in a Composite Body"--October 1968.
Use of the immersion analogy to determine gravitational stresses in two-dimensional bodies made of materials with different properties.

17. A. J. Durelli, J. A. Clark and A. Kochev, "Experimental Analysis of High Frequency Stress Waves in a Ring"--October 1968.
A method for the complete experimental determination of dynamic stress distributions in a ring is demonstrated. Photoelastic data is supplemented by measurements with a capacitance gage used as a dynamic lateral extensometer.

18. J. A. Clark and A. J. Durelli, "A Modified Method of Holographic Interferometry for Static and Dynamic Photoelasticity"--April 1968.
A simplified absolute retardation approach to photoelastic analysis is described. Dynamic isopachics are presented.

19. J. A. Clark and A. J. Durelli, "Photoelastic Analysis of Flexural Waves in a Bar"--May 1969.
A complete direct, full-field optical determination of dynamic stress distribution is illustrated. The method is applied to the study of flexural waves propagating in a urethane rubber bar. Results are compared with approximate theories of flexural waves.

20. J. A. Clark and A. J. Durelli, "Optical Analysis of Vibrations in Continuous Media"--June 1969.
Optical methods of vibration analysis are described which are independent of assumptions associated with theories of wave propagation. Methods are illustrated with studies of transverse waves in prestressed bars, snap loading of bars and motion of a fluid surrounding a vibrating bar.

21. V. J. Parks, A. J. Durelli, K. Chandrashekhara and T. L. Chen, "Stress Distribution Around a Circular Bar, with Flat and Spherical Ends, Embedded in a Matrix in a Triaxial Stress Field"--July 1969.
A three-dimensional photoelastic method to determine stresses in composite materials is applied to this basic shape. The analyses of models with different loads are combined to obtain stresses for the triaxial cases.

22. A. J. Durelli, V. J. Parks and L. Ferrer, "Stresses in Solid and Hollow Spheres Subjected to Gravity or to Normal Surface Tractions"--October 1969.
The method described in Report No. 10 above is applied to two specific problems. An approach is suggested to extend the solutions to a class of surface traction problems.

23. J. A. Clark and A. J. Durelli, "Separation of Additive and Subtractive Moiré Patterns"--December 1969.
A spatial filtering technique for adding and subtracting images of several gratings is described and employed to determine the whole field of Cartesian shears and rigid rotations.

24. R. J. Sanford and A. J. Durelli, "Interpretation of Fringes in Stress-Holo-Interferometry"--July 1970.
    Errors associated with interpreting stress-holo-interferometry patterns as the superposition of isopachics (with half order fringe shifts) and isochromatics are analyzed theoretically and illustrated with computer generated holographic interference patterns.

25. J. A. Clark, A. J. Durelli and P. A. Laura, "On the Effect of Initial Stress on the Propagation of Flexural Waves in Elastic Rectangular Bars"--December 1970.
    Experimental analysis of the propagation of flexural waves in prismatic, elastic bars with and without prestressing. The effects of prestressing by axial tension, axial compression and pure bending are illustrated.

26. A. J. Durelli and J. A. Clark, "Experimental Analysis of Stresses in a Buoy-Cable System Using a Birefringent Fluid"--February 1971.
    An extension of the method of photoviscous analysis is presented which permits quantitative studies of strains associated with steady state vibrations of immersed structures. The method is applied in an investigation of one form of behavior of buoy-cable systems loaded by the action of surface waves.

27. A. J. Durelli and T. L. Chen, "Displacements and Finite-Strain Fields in a Sphere Subjected to Large Deformations"--February 1972.
    Displacements and strains (ranging from 0.001 to 0.50) are determined in a polyurethane sphere subjected to several levels of diametral compression. A 500 lines-per-inch grating was embedded in a meridian plane of the sphere and moiré effect produced with a non-deformed master. The maximum applied vertical displacement reduced the diameter of the sphere by 27 per cent.

28. A. J. Durelli and S. Machida, "Stresses and Strain in a Disk with Variable Modulus of Elasticity"--March 1972.
    A transparent material with variable modulus of elasticity has been manufactured that exhibits good photoelastic properties and can also be strain analyzed by moiré. The results obtained suggests that the stress distribution in the homogeneous disk. It also indicates that the strain fields in both cases are very different, but that it is possible, approximately, to obtain the stress field from the strain field using the value of E at every point, and Hooke's law.

29. A. J. Durelli and J. Buitrago, "State of Stress and Strain in a Rectangular Belt Pulled Over a Cylindrical Pulley"--June 1972.
    Two- and three-dimensional photoelasticity as well as electrical strain gages, dial gages and micrometers are used to determine the stress distribution in a belt-pulley system. Contact and tangential stress for various contact angles and friction coefficients are given.

30. T. L. Chen and A. J. Durelli, "Stress Field in a Sphere Subjected to Large Deformations"--June 1972.
Strain fields obtained in a sphere subjected to large diametral compressions from a previous paper were converted into stress fields using two approaches. First, the concept of strain-energy function for an isotropic elastic body was used. Then the stress field was determined with the Hookean type natural stress-natural strain relation. The results so obtained were also compared.

31. A. J. Durelli, V. J. Parks and H. M. Hasseem, "Helices Under Load"--July 1973.
Previous solutions for the case of close coiled helical springs and for helices made of thin bars are extended. The complete solution is presented in graphs for the use of designers. The theoretical development is correlated with experiments.

32. T. L. Chen and A. J. Durelli, "Displacements and Finite Strain Fields in a Hollow Sphere Subjected to Large Elastic Deformations"--September 1973.
The same methods described in No. 27, were applied to a hollow sphere with an inner diameter one half the outer diameter. The hollow sphere was loaded up to a strain of 30 per cent on the meridian plane and a reduction of the diameter by 20 per cent.

33. A. J. Durelli, H. H. Hasseem and V. J. Parks, "New Experimental Method in Three-Dimensional Elastostatics"--December 1973.
A new material is reported which is unique among three-dimensional stress-freezing materials, in that, in its heated (or rubbery) state it has a Poisson's ratio which is appreciably lower than 0.5. For a loaded model, made of this material, the unique property allows the direct determination of stresses from strain measurements taken at interior points in the model.

34. J. Wolak and V. J. Parks, "Evaluation of Large Strains in Industrial Applications"--April 1974.
It was shown that Mohr's circle permits the transformation of strain from one axis of reference to another, irrespective of the magnitude of the strain, and leads to the evaluation of the principal strain components from the measurement of direct strain in three directions.

35. A. J. Durelli, "Experimental Stress Analysis Activities in Selected European Laboratories"--April 1975.
Continuation of Report No. 15 after a visit to Belgium, Holland, Germany, France, Turkey, England and Scotland.

36. A. J. Durelli, V. J. Parks and J. O. Bühler-Vidal,"Linear and Non-linear Elastic and Plastic Strains in a Plate with a Big Hole Loaded Axially in its Plane"--July 1975.
Strain analysis of the ligament of a plate with a big hole indicates that both geometric and material non-linearity may take place. The strain concentration factor was found to vary from 1 to 2 depending on the level of deformation.

37. A. J. Durelli, V. Pavlin, J. O. Bühler-Vidal and G. Ome, "Elastostatics of a Cubic Box Subjected to Concentrated Loads"--August 1975.
Analysis of experimental strain, stress and deflection of a cubic box subjected to concentrated loads applied at the center of two opposite faces. The ratio between the inside span and the wall thickness was varied between approximately 5 and 121.

38. A. J. Durelli, V. J. Parks and J. O. Bühler-Vidal, "Elastostatics of Cubic Boxes Subjected to Pressure"--March 1976.
Experimental analysis of strain, stress and deflections in a cubic box subjected to either internal or external pressure. Inside span-to-wall thickness ratio varied from 5 to 14.

39. Y. Y. Hung, J. D. Hovanesian and A. J. Durelli, "New Optical Method to Determine Vibration-Induced Strains with Variable Sensitivity After Recording"--November 1976.
A steady state vibrating object is illuminated with coherent light and its image slightly misfocused. The resulting specklegram is "time-integrated" as when Fourier filtered gives derivatives of the vibrational amplitude.

40. Y. Y. Hung, C. Y. Liang, J. D. Hovanesian and A. J. Durelli, "Cyclic Stress Studies by Time-Averaged Photoelasticity"--November 1976.
"Time-averaged isochromatics" are formed when the photographic film is exposed for more than one period. Fringes represent amplitudes of the oscillating stress according to the zeroth order Bessel function.

41. Y. Y. Hung, C. Y. Liang, J. D. Hovanesian and A. J. Durelli, "Time-Averaged Shadow Moiré Method for Studying Vibrations"--November 1976.
Time-averaged shadow moiré permits the determination of the amplitude distribution of the deflection of a steady vibrating plate.

42. J. Buitrago and A. J. Durelli, "On the Interpretation of Shadow-Moiré Fringes"--April 1977.
Possible rotations and translations of the grating are considered in a general expression to interpret shadow-moiré fringes and on the sensitivity of the method. Application to an inverted perforated tube.

43. J. der Hovanesian, "18th Polish Solid Mechanics Conference." Published in European Scientific Notes of the Office of Naval Research, in London, England, Dec. 31, 1976.
Comments on the planning and organization of, and scientific content of paper presented at the 18th Polish Solid Mechanics Conference held in Wisla-Jawornik from September 7-14, 1976.

44. A. J. Durelli, "The Difficult Choice,"--May 1977.
The advantages and limitations of methods available for the analyses of displacements, strain, and stresses are considered. Comments are made on several theoretical approaches, in particular approximate methods, and attention is concentrated on experimental methods: photo-elasticity, moiré, brittle and photoelastic coatings, gages, grids, holography and speckle to solve two- and three-dimensional problems in elasticity, plasticity, dynamics and anisotropy.

45. C. Y. Liang, Y. Y. Hung, A. J. Durelli and J. D. Hovanesian,
"Direct Determination of Flexural Strains in Plates Using Projected
Gratings,"--June 1977.
The method requires the rotation of one photograph of the deformed
grating over a copy of itself. The moiré produced yields strains by
optical double differentiation of deflections. Applied to projected
gratings the idea permits the study of plates subjected to much larger
deflections than the ones that can be studied with holograms.

46. A. J. Durelli, K. Brown and P. Yee, "Optimization of Geometric
Discontinuities in Stress Fields"--March 1978.
The concept of "coefficient of efficiency" is introduced to evaluate
the degree of optimization. An ideal design of the inside boundary of
a tube subjected to diametral compression is developed which decreases
its maximum stress by 25%, at the time it also decreases its weight by
10%. The efficiency coefficient is increased from 0.59 to 0.95.
Tests with a brittle material show an increase in strength of 20%. An
ideal design of the boundary of the hole in a plate subjected to axial
load reduces the maximum stresses by 26% and increases the coefficient
of efficiency from 0.54 to 0.90.

Abstract

A steady-state vibrating object is illuminated with coherent light
and its image is slightly misfocused in the film plane of a camera. The
resulting processed film is called a "time-integrated specklegram." When
the specklegram is Fourier filtered, it exhibits fringes depicting deriva-
tives of the vibrational amplitude. The direction of the spatial derivative,
as well as the fringe sensitivity may be easily and continuously varied
during the Fourier filtering process. Besides the above mentioned advantages,
this new method is also much less demanding than holographic interferometry
with respect to vibration isolation, optical set-up time, illuminating source
coherence, required film resolution, etc.

NEW OPTICAL METHOD TO DETERMINE VIBRATION-INDUCED STRAINS
WITH VARIABLE SENSITIVITY AFTER RECORDING

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Abstract: A steady-state vibrating object is illuminated with coherent light and its image is slightly misfocused in the film plane of a camera. The resulting processed film is called a "time-integrated specklegram." When the specklegram is Fourier filtered, it exhibits fringes depicting derivatives of the vibrational amplitude. The direction of the spatial derivative, as well as the fringe sensitivity may be easily and continuously varied during the Fourier filtering process. Besides the above mentioned advantages, this new method is also much less demanding than holographic interferometry with respect to vibration isolation, optical set-up time, illuminating source coherence, required film resolution, etc.

Introduction: Optical methods used to study vibrations experimentally are noncontacting and generally full-field. They may be grouped into three main categories: moiré, holographic interferometry, and laser speckle.

A brief review of the moiré methods to visualize nodes and antinodes in a vibrating plate can be found in reference [1]. A projected grating method [2,3] was also developed for studying vibrational amplitudes. The grating is observed when projected onto a vibrating object. Photographing the perturbated grating with an exposure time of several vibrational periods yields time-integrated moiré fringes depicting modal amplitude of the steady state vibration. These fringes can also be seen by eye in real-time if the vibrational frequency is approximately 20 H_z or more. All the moiré methods, however, are limited to studies of vibrations of relatively large amplitudes.

Powell and Stetson [4] and other investigators [5,6,7] have demonstrated that time-averaged holography is well suited for analysis of steady-state vibrations with very small amplitudes. In the reconstruction of the hologram which recorded a steady-state vibrating object, a time-averaged fringe pattern is observed which is described by a zero-order Bessell function containing the

vibrational amplitude in its argument. Nodal areas are also clearly indicated by the regions of the brightest reconstruction. An extension of this technique, the modulated holography [8,9], can be used to reduce the sensitivity, and to determine the relative phase. Shortcomings of these techniques, however, are those associated with holographic interferometry, such as stringent requirement of mechanical and thermal stabilities, the fringe localization problem, etc.

The speckle pattern produced by the random scattering of a diffused object when illuminated by coherent light acts as an information carrier about the object surface. By simply watching the speckle pattern on a surface while it is undergoing steady state oscillation, one can observe nodal patterns in real time [10,11,12]. This provides a quick and simple way of scanning for the appropriate frequencies to be studied in greater detail. Butters and Leendertz [13] demonstrated that a time-averaged speckled fringe pattern depicting vibrational amplitudes similar to that of time-averaged holography could be obtained. The order of the sensitivity of their technique is about the same as that of holography, but the technique is less demanding on film resolution and speckle fringes localize on the object surface. Another virtue of speckle interferometry over holographic interferometry is that it permits in-plane displacements to be measured independently [14,15,16]. This advantage is particularly useful for inplane vibration analysis [17,18].

The speckle-shearing interferometer introduced by Hung and Taylor [19] measures derivatives of vibrational amplitudes directly; thus it greatly facilitates the analysis of stresses resulting from vibrations. An improved version was reported by Hung, Rowlands and Daniel [20]. The object under study was illuminated by a beam of coherent light and was imaged by a camera having a mask with four apertures in front of the lens. A photographic plate was exposed in the misfocused image plane while the object was undergoing steady-state vibration. The purpose of misfocusing was to produce a shearing effect. The processed plate was Fourier filtered to yield four families of fringes, each depicting the derivatives of the vibrational amplitudes with respect to the direction of the line joining the centers of each aperture pair. The sensitivity of the method was mainly controlled by the amount of shearing. However, both the derivative directions as well as the sensitivity were fixed after recording.

Since strains due to vibrational bending are proportional to the second derivatives of the amplitudes, the proposed technique requires only one differentiation to obtain strains in contrast to those amplitude measuring techniques which need to differentiate the data twice.

The examples presented in Fig. 4 were prepared at low levels of sensitivity in order to show the principle of this technique. For actual quantitative studies, the systems should be adjusted to yield higher fringe densities in order to facilitate accurate determination of the second derivatives.

Description of the Method: Figure 1 illustrates the schematic diagram for recording. The object is illuminated by a collimated beam of coherent light inclined at an angle θ to the viewing direction. It is imaged by a camera with the aperture fully opened. A photographic plate is placed at a small distance S from the image plane. While the object is undergoing steady state oscillations the photographic plate is exposed with an exposure
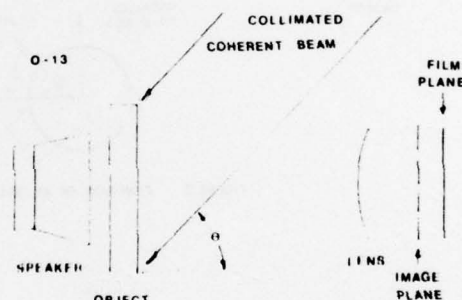

FIGURE 1:   SCHEMATIC   DIAGRAM   FOR
RECORDING

time equal to many vibrational periods. The processed photographic plate is referred to as a "time integrated specklegram." It will be shown in the following that a fringe pattern contouring the derivatives of the vibrational amplitudes with respect to any <u>desired</u> direction can be extracted from the specklegram with <u>variable</u> sensitivity by a Fourier filtering technique.

Analysis of Recording Process: The full aperture of the lens can be considered as an infinitely dense collection of small aperture pairs. Let us focus on the function of one aperture pair. Let the line joining the centers of the aperture pair be parallel to the r-direction as shown in Fig. 2. and d be their separation. The two images of the structure, one focused by each

aperature, are coincident in the image plane. Due to the film plane being misfocused, these two images are sheared laterally in the r-direction. The amount of shearing δr' in the film plane is

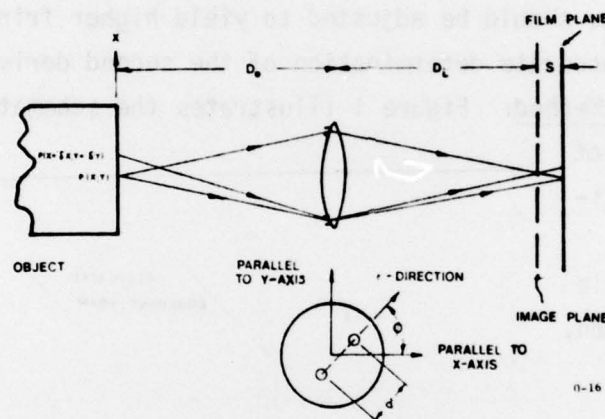$$\delta r' = \frac{S}{D_i} d \tag{1}$$

FIGURE 2   FUNCTION OF AN APERTURE-PAIR

where D_i is the distance of the image plane from the lens, and S is the distance from the focal plane. The equivalent shearing δr in the object plane is

$$\delta r \simeq \frac{\delta r'}{m} = \frac{S}{mD_i} d \tag{2}$$

where m is the magnification. Obviously this arrangement brings the ray scattered from a point P(x,y) to meet the ray scattered from a neighboring point P(x+δx, y+δy) in the film plane where δx = δr cosφ, and δy = δr sinφ. If the two apertures are small, they can be considered as a two-point source, and the resulting interference is a grating whose lines are perpendicular to the r-direction. The frequency f' of the grating is given by [15,16]:

$$f' = \frac{d}{\lambda(D_i + S)} \tag{3}$$

where λ = wavelength of the laser light.
Hence the equivalent frequency f in the object is:

$$f = \frac{dm}{\lambda(D_i+S)} = \frac{m^2 D_i}{\lambda S(D_i S)} \delta r \qquad (4)$$

The ruling is distorted by the randomness of the speckle pattern. It can be shown that the intensity distribution $I(x,y)$ of the speckle grating is represented by [19,20]:

$$I(x,y) = a^2[1 + \cos(2\pi fr + \chi)]* \qquad (5)$$

where χ is the random speckle phase; a is the light amplitude;
$x = r\cos\phi$ and $y = r\sin\phi$

It has been assumed in the above analysis that the object is in its stationary position. Suppose that the object is undergoing steady state sinusoidal vibration with a displacement only in the z-direction represented by:

$$W(x,y,t) = A(x,y)\cos(\omega t+\alpha) \qquad (6)$$

where

$W(x,y,t)$ is the displacement in the z-direction; $A(x,y)$ is the amplitude function; ω the circular frequency and α the arbitrary phase.

The relative displacement $\delta W(t)$ between the point $P(x,y)$ and the neighboring point $P(x+\delta x, y+\delta y)$ is thus:

$$\delta W(x,y,t) = W(x+\delta x, y+\delta y, t) - W(x,y,t) \qquad (7)$$

This relative displacement will cause a relative optical path change and hence a phase change Δ between the light scattered from the two points. Assuming that the object size is small compared to D_0, Δ can be shown to be approximately given by:

$$\Delta = \frac{2\pi}{\lambda}(1+\cos\theta)\delta W(x,y,t) \qquad (8)$$

which results in a local optical shift of the speckle-grating [4]. Thus equation (5) in which $I(x,y)$ is now time dependent becomes:

$$I(x,y,t) = a^2[1 + \cos(2\pi fr + \chi + \Delta)] \qquad (9)$$

The exposure $E(x,y)$ on the film for a number of vibrational period is therefore an integration of the intensity over the exposure time T given by:

$$E(x,y) - k\int_0^T I(x,y,t)dt \qquad (10)$$

---

*For simplicity, the magnification is assumed to be unity and so the image plane would take the same coordinates as the object plane except it is inverted. This assumption is carried on hereafter.

where k is the photographic film constant

The above integration yields:

$$E(x,y) = kTa^2\{1 + \cos(2\pi fr + \chi) \cdot J_0[\beta]\} \qquad (11)$$

where

$$\beta = \frac{2\pi}{\lambda}(1 + \cos\theta)[A(x+\delta x, y+\delta y) - A(x,y)] \qquad (12)$$

Equation (11) represents a carrier of approximately constant frequency modulated by $J_0[\beta]$, a Bessel function of zero-order. Nulling of the carrier which will be identified as moiré fringes occurs when β equals to the null roots of the zero-order Bessel function. Equation (12) may be rewritten as:

$$\beta = \frac{2\pi}{\lambda}(1 + \cos\theta)\,\delta r[\frac{A(x+\delta x, y+\delta y) - A(x,y)}{\delta r}] = \frac{2\pi}{\lambda}(1 + \cos\theta)\,\delta r[\frac{\delta A(x,y)}{\delta r}] \qquad (13)$$

If δr, the lateral shear is small, $(\frac{\delta A}{\delta r})$ approaches $(\frac{\partial A}{\partial r})$. Hence the fringe pattern depicted by Eq. (11) is a contour map of the derivatives of the vibrational amplitudes with respect to the r-direction.

It is seen in Eq. (13) that δr is a sensitivity factor, and it depends upon d, the separation of the aperture centers. Also δr is related to the frequency of ruling by Eq. (4). Thus an aperture-pair with greater separation produces a greater shearing and hence greater carrier frequency. The direction of the shearing is parallel to the line joining the aperture-pair centers, i.e., it is perpendicular to the lines of the ruling. With the orientation of the aperture-pair varied, both the shearing direction and the ruling orientation are changed accordingly.

We have just shown that an aperture-pair is indeed an interferometer measuring the derivatives of the vibrational amplitudes with respect to a direction parallel to the line joining the aperture-pair centers and with sensitivity depending upon the aperture separation. This information is represented by a family of fringes formed by moiré of two slightly mismatched speckle-rulings interpreted by Eq. (11). Since the whole lens aperture consists of many aperture pairs with separation varying from 0 to 2R, the diameter of the lens, and with the orientation varying from $\phi = 0$ to $\phi = 360°$, the rulings recorded on the specklegram have all orientations and frequency varying from 0 to $R/\lambda(D_i+S)$. Each ruling forms a fringe family of its own. Therefore a specklegram contains a superposition of a multiplicity of moiré fringe families. Each fringe family depicts vibrational amplitudes with respect to a direction perpendicular to the ruling-line and with sensitivity

depending on the ruling frequency. Thus the derivatives of the vibrational amplitudes with respect to all directions are recorded in the specklegram with a variable range of sensitivity after recording, the upper limit depending on the highest frequency which the recording lens is capable of passing.

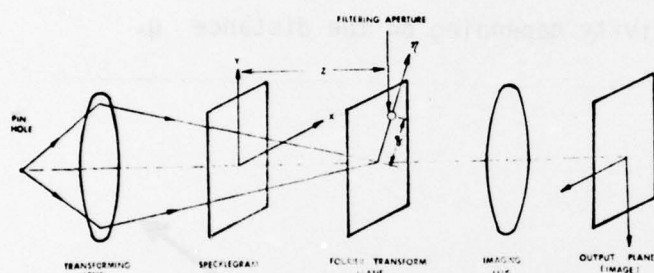Readout Process by Fourier Filtering: The device shown in Fig. 3 is employed.

FIGURE 3 FOURIER FILTERING SCHEME FOR EXTRACTING DISPLACEMENT-DERIVATIVE FRINGES

A beam from a laser is expanded by a microscopic objective and is spatially filtered by the pinhole. The beam diverged from the pin hole is focused by a lens to a point lying in a plane known as the Fourier filtering plane. The time-integrated specklegram is placed in the input plane as shown and is imaged by a camera behind the Fourier filtering plane. It will be shown that by inserting a small aperture off-center in the η-direction, a fringe pattern depicting the vibrational amplitude derivative with respect to the η-direction is displayed in the image plane of the camera (output plane) with a sensitivity proportional to the distance of the aperture from the optical center.

It can be shown that what appears in the Fourier filtering plane of Fig. 3 is the Fourier transform of the amplitude transmittance of the specklegram multiplied by a quadratic phase term.[21] This represents a transformation from a spatial domain to a frequency domain. In other words, gratings of different frequency are separated in the Fourier filtering plane. If a small aperture is located at a distance q from the optical axis in the η-direction, the grating orthogonal to the η-direction has a frequency

$$f = \frac{q}{\lambda' Z} \tag{14}$$

where λ' = wavelength of light used. As analyzed above this grating corresponds to a shearing δr in the η-direction given by:

$$\delta r = \frac{q}{\lambda' Z} \frac{\lambda S(D_i + S)}{D_i} \qquad (15)$$

However, areas corresponding to the nulling of the carrier as described in
Eq. (11) make no contribution and will appear dark in the output plane.
Therefore, a fringe pattern is observed on the image that depicts the amplitude-derivative with respect to the η-direction accompanied by the sensitivity
factor δr. By varying the η-direction, any desired derivative direction can
be chosen with an adjustable sensitivity depending on the distance q.
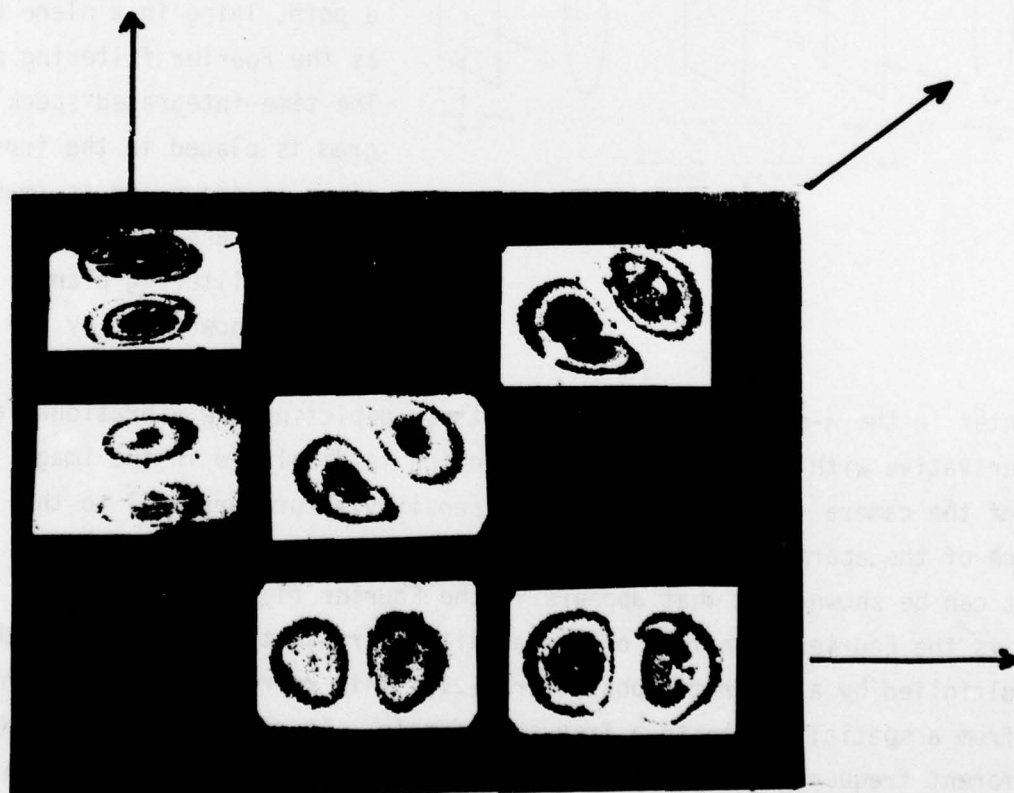
O-14

FIGURE 4:
FRINGE PATTERNS DEPICTING DERIVATIVES OF VIBRATIONAL
AMPLITUDES OF THE RECTANGULAR PLATE, PHOTOGRAPHED FROM
THE VARIOUS POSITIONS IN THE FILTERING PLANE

Experiments: A rectangular plate, 3.75 cm x 5.0 cm, clamped along its boundaries, was chosen for demonstration. The plate was driven by an audio speaker to vibrate its fundamental frequency. The set-up shown in Fig. 1 was used for recording whereby a Spectra Physics 125 laser was used for the illuminating source. The processed time-integrated specklegram was Fourier filtered with the scheme of Fig. 3. The specklegrams were recorded on Agfa 10E75 film, typically using exposure times of 1-5 seconds. Photographic processing included 5 minutes in Kodak D-19, 1 minute stop, 5 minutes Fix and 10 minutes wash. Figure 4 shows the fringe patterns extracted from the specklegram depicting the derivatives of the vibrational amplitudes with respect to the various directions as shown, and with different sensitivities.

Summary and Conclusion: A new optical technique has been described which permits derivatives of vibrational amplitudes to be measured directly. One outstanding feature of the technique is that it provides variability after recording, both in derivative direction as well as sensitivity. The method can be viewed as a holographic interferometric method in which every point on the object acts as a reference for all its neighboring points with the advantage that several limitations associated with holography have been alleviated. These include: (1) the experimental set up is simpler; (2) the stability of the equipment does not have to be as great, (3) the film resolution does not have to be as high and (4) there is greater range of sensitivity. A shortcoming of the technique is that fringe visibility is generally poor.

It should be noted that the mechanical shift of speckle-gratings due to lateral displacement (displacement orthogonal to line of sight) is assumed negligible. Should the lateral displacement be appreciable, the mechanical shift must be considered. The combined effect of the optical and mechanical shift of speckle-rulings will be analyzed in a future paper.

and N. Perrone and N. Basdekas of ONR for their support. The manuscript reproduction has been prepared by M. Wood.

The authors also wish to acknowledge that a similar technique is currently being developed independently by Professor F. P. Chiang of the State University of New York at Stony Brook.

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1. REPORT NUMBER 47	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) NEW OPTICAL METHOD TO DETERMINE VIBRATION-INDUCED STRAINS WITH VARIABLE SENSITIVITY AFTER RECORDING		5. TYPE OF REPORT & PERIOD COVERED
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) J. D. Hovanesian, Y. Y. Hung and A. J. Durelli		8. CONTRACT OR GRANT NUMBER(s) No. N00014-76-C-0487
9. PERFORMING ORGANIZATION NAME AND ADDRESS Oakland University Rochester, MI 48063		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS Office of Naval Research Department of the Navy Washington, D.C. 20025		12. REPORT DATE May 1978
		13. NUMBER OF PAGES 15 24p.
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Distribution of this report is unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Optical Methods
Vibrations
Speckle Interferometry
Fourier filtering

Derivatives of Deflections
Strains
Speckle Shearing

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A steady-state vibrating object is illuminated with coherent light and its image is slightly misfocused in the film plane of a camera. The resulting processed film is called a "time-integrated specklegram." When the specklegram is Fourier filtered, it exhibits fringes depicting derivatives of the vibrational amplitude. The direction of the spatial derivative, as well as the fringe sensitivity may be easily and continuously varied during the Fourier filtering process. Besides the above mentioned advantages, —

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this new method is also much less demanding than holographic interferometry with respect to vibration isolation, optical set-up time, illuminating source coherence, required film resolution, etc.