



. HER TRAN TRA .. Unclassified SECURITY CLASSIFICATION OF THIS PAGE (When Date Entered) READ INSTRUCTIONS BEFORE COMPLETING FORM REPORT DOCUMENTATION PAGE 2. JOVT ACCUSSION NOT 3. RECIPIENT'S CATALOG NUMBER 1. REPORT NUMBER ARD 11013.4-M TYPE OF REPORT & PERIOD COVERED TITLE (and Subtitio) Final Report. Topics in Functional Analysis 1 Jan 73 - 31 Dec 77. RER PERFORMING ORG. REPORT NO 7. AUTHOR(.) S. CONTRACT OR GRANT NUMBER(.) <u>'C</u> C. Ionescu/Tulcea 0 DAAG29-73 0-0005 DAHC 04-In PROGRAM ELEMENT, PROJECT. TAR . PERFORMING ORGANIZATION NAME AND ADDRESS Northwestern University Evanston, Illinois 60201 11. CONTROLLING OFFICE NAME AND ADDRESS 12. REPORT DATE U. S. Army Research Office 11 May 178 **P.** 0. Box 12211 NUMBER OF PAGES 27709 Research Triangle Park, NC 14. MONITORING AGENCY NAME & ADDRESS(I different from Controlling Office) 15. SECURITY CLASS. (of this report) unclassified 15. DECLASSIFICATION/DOWNGRADING 16. DISTRIBUTION STATEMENT (of this Report) JUN 20 1978 Approved for public release; distribution unlimited. 17. DISTRIBUTION STATEMENT (of the obstract entered in Black 20, if different from Report) 18. SUPPLEMENTARY NOTES The findings in this report are not to be construed as an official Department of the Army position, unless so designated by other authorized documents. 19. KEY WORDS (Continue on reverse side if necessary and identify by block number) FILF 10. ABSTRACT (Continue on reverse side if necessary and identify by block number) Topics investigated include lifting theory, stochastic processes and potential theory, ergodic theory, information theory, and measures on topological spaces and on infinite dimensional manifolds. A bibliography and brief summary are presented. 78 DD 1 JAN 73 1473 EDITION OF I NOV 65 IS OBSOLETE Unclassified

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FINAL REPORT

- (1) Contract DAHCO4-68-0005/DAAG29-73-C-0005 (Unclassified)
- (2) Period covered by the contract: April 15, 1973 to April 15, 1978.
- (3) Author of the report: PROFESSOR C. IONESCU TULCEA

This report is divided into the following sections:

(1) PERSONNEL

(2) LIST OF RESEARCH PAPERS AND BOOKS

(3) COMMENTS CONCERNING THE RESEARCH PERFORMED UNDER THIS CONTRACT

1. Personnel

Professor C. Ionescu Tulcea functioned as Principal Investigator. Professor Alexandra Bellow was the Research Associate.

The support given by the contract was essential for the development of the research of the afore named investigators (see for instance the list of publications).

Among other personnel supported briefly during certain periods by this contract, we mention the following:

Professor <u>R. Theodorescu</u>, from Laval University, worked on various problems concerning concentration functions, information <u>theory</u> and stochastic processes

(mainly on stochastic processes of infinite order used in learning theory).

2

Professor <u>N. Dinculeanu</u>, from the University of Florida, Gainesville, Florida, worked with the Principal Investigator on manuscript of the volume of Functional Analysis prepared by the Principal Investigator.

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BIBLIOGRAPHY

List of research papers prepared under the contract

- <u>On pointwise convergence, compactness and equicontinuity in the lifting</u> <u>topology</u>, (I), Zeitschrift fur Wahrscheinlichkeitstheorie, 26 (1973).
- 2. <u>On pointwise convergence, compactness and equicontinuity (II)</u>, Advances in Mathematics, 12 (1974).
 - On measurability, pointwise convergence and compactness, Invited Address given at the A.M.S. Meeting, April 1973 (Evanston), Bulletin of the A.M.S., 80 (1974).
 - Pointwise convergence in terms of convergence in expectation, (in collaboration with D. Austin and G. Edgar), Zeitschrift fur Wahrscheinlichkeitstheorie (1974).
 - <u>Concentration and Information</u>, Technical report (by Professor R. Theodorescu), April 1974.
- 6. Measures and Premeasures, Technical report, October 1975.
- An L^p-Inequality with application to Ergodic Theory, Houston Journal of Mathematics 1975.
- 8. Ergodic Theorems (I), Technical report, May 1976.
- 9. <u>A problem in L^P-space</u>, Proceedings of the International Conference on "Measure Theory" held at Oberwolfach (June 1975) Springer-Verlag Lecture Notes (1976).
- Stability properties of the class of asymptotic martingales, Bulletin A.M.S., 1976 Vol. 82, March No. 2.
- On vector-valued asymptotic martingales, Proceedings of the National Academy of Sciences, June 1976.
- 12. Functional analysis, Parts I & II, Mimeographed notes.
- <u>Several stability properties of the class of asymptotic martingales</u>, to appear in Zeit. Wahrscheinlichkeitstheorie.

BIBLIOGRAPHY continued

- 14. <u>A class of vector-valued asymptotic martingales for which strong almost sure</u> convergence obtains (in preparation).
- 15. Ergodic Theorems (II), In preparation.
- <u>Characterization of nuclear spaces in terms of vector valued measures</u>, in preparation.
- 17. Game Theory notes (21), In preparation.

Although not directly related to the work done under the contract we mention here the book:

18. Casino Gambling (by V. Graham and C. Ionescu Tulcea), Von Nostrand Reinhold 1976.

THE RESEARCH PROGRAM

The Research program followed the initial program closely. Of course new ideas, connected with the main program, were also considered.

1. Papers (1), (2), (3) and (4) contain the results concerning lifting theory, which were obtained by the personnel supported by this contract.

We observe here that a huge amount of work has been done in this field during the last years. All of it is based on the Springer-Verlag monograph TOPICS IN THE THEORY OF LIFTING (1969), prepared by the personnel on this contract (while supported by a previous Army contract).

2. Professor Radu Theodorescu and the Principal investigator have done a certain amount of work on stochastic process during the early period covered by this contract. Part of this work appeared in (5).

3. In papers (8) and (15) an interesting ergodic theorem is proved and discussed. This theorem does not require neither the linearity of the considered operators nor the linearity of the basic space. The theorem contains as a particular case all the mean ergodic theorems for linear operators, found in the literature.

It was quite unexpected to see that the contraction theorem (in metric compact spaces) is also a particular case of our theorem.

4. Various types of Kernels (encountered in Probability theory and Potential theory) were studied by the research associate.

5. The Principal investigator has (almost) completed a monograph on Functional Analysis (see (12)). The manuscript has more than 400 pages. SpringerVerlag has expressed (repeatedly) interest in its publication. We hope to interest two former students of the Principal investigator (now Professors at Munster University and University of Florida, Gainsville) in its publication. They will add a few notes, the bibliography and take care of the publication arrangements.

For us the publication of this volume (which took a number of years to prepare) does not present, at this time, any incentive.

6. In the recent years the Principal investigator worked in Game theory and its applications (this field has applications in economics, military, gambling, etc.).

REMARKS

During the present Academic year the Principal investigator is on Leave from Northwestern University. He has visited U. of Maryland and University of Florida, Gainsville. He received many other invitations, for instance from U. of Munster, W. Germany. Unfortunately most of the invitations had to be declined due to cost reasons.

