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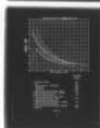
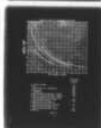
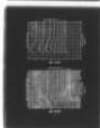
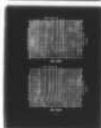
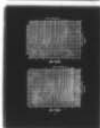
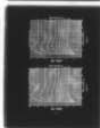
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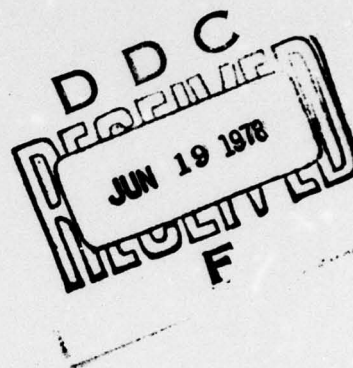
*Cover: Test truck on river ice. (Photograph by  
Stephen L. DenHartog.)*

# CRREL Report 78-3

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## *Bearing capacity of river ice for vehicles*

Donald E. Nevel



April 1978

CORPS OF ENGINEERS, U.S. ARMY  
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) The mathematical theory for the bearing capacity of river ice for vehicles is presented. The floating ice sheet is assumed to have simple supports at the shore line. Solutions are presented for loads uniformly distributed over circular and rectangular areas. Numerical evaluations are made for a number of vehicles and the results presented in graphical form. ABSTRACT 037 200		

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## PREFACE

This report was prepared by Dr. Donald E. Nevel, Research Physical Scientist, Applied Research Branch, Experimental Engineering Division, U.S. Army Cold Regions Research and Engineering Laboratory. The report was published under DA Task 1T062112A13001, *Cold Regions Research - Applied Research and Engineering*.

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### CONVERSION FACTORS: U.S. CUSTOMARY TO METRIC (SI) UNITS OF MEASUREMENT

These conversion factors include all the significant digits given in the conversion tables in the ASTM *Metric Practice Guide* (E 380), which has been approved for use by the Department of Defense. Converted values should be rounded to have the same precision as the original (see E 380).

<i>Multiply</i>	<i>By</i>	<i>To obtain</i>
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foot	0.3048*	meter
pound	0.4535924	kilogram
ton	907.1847	kilogram

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## BEARING CAPACITY OF RIVER ICE FOR VEHICLES

Donald E. Nevel

### INTRODUCTION

In cold environments vehicles may cross a river over the ice. If the river ice is not thick enough to support the vehicles, an ice bridge may be built by flooding the surface of the ice with a few inches of water and waiting for the water to freeze. By repeating this procedure, the thickness of the ice can be increased faster than by the natural growth of ice. This procedure should not be repeated indefinitely since there is a maximum ice thickness which can be obtained. This maximum thickness depends upon the local climatic environment. If snow is on the ice it should not be removed. However, if the snow depth is greater than 5 in., the snow should be compacted before being flooded.

In the past, two types of ice bridges have been built. For one type, the ice was flooded within a confined strip from shore to shore. For the other, the ice was flooded freely from shore to shore. G. Frankenstein of CRREL has shown that free flooding is easier to do and produces a stronger bridge than confined flooding. This was demonstrated during a school for crossing river ice given at Fort Wainwright, Alaska, in November 1965 by G. Frankenstein, R. Garner (also of CRREL), and the author. Hence, it is expected that future ice bridges will be built by free flooding.

In a previous report by the author (Nevel 1965b), equations describing the stresses in an ice bridge built by confined flooding were presented. Although these equations will probably not be used for practical applications, eq 15c and 15d of this 1965 report are incorrect. These equations, respectively, should be

$$D \left[ \frac{\partial^2 w}{\partial y^2} + \sigma \frac{\partial^2 w}{\partial x^2} \right]_{y=d} = \bar{D} \left[ \frac{\partial^2 \bar{w}}{\partial \bar{y}^2} + \sigma \frac{\partial^2 \bar{w}}{\partial \bar{x}^2} \right]_{\bar{y}=0}$$
$$D \left[ \frac{\partial^3 w}{\partial y^3} + (2 - \sigma) \frac{\partial^3 w}{\partial x^2 \partial y} \right]_{y=d} = \bar{D} \left[ \frac{\partial^3 \bar{w}}{\partial \bar{y}^3} + (2 - \sigma) \frac{\partial^3 \bar{w}}{\partial \bar{x}^2 \partial \bar{y}} \right]_{\bar{y}=0}$$

Additional equations which result from these two equations are also incorrect.

For the bridge built by free flooding, the ice thickness varies but its variations are not precisely known. Free flooding, however, produces a very wide bridge which may be approximated by an infinite width bridge. Therefore, for practical applications the bearing capacity of a river that has a uniform ice thickness will be determined. The result will then represent the bearing capacity of the frozen river without an ice bridge or with an ice bridge built by free flooding. Because of the formation of shoreline cracks, a simple support is the best representation of the boundary conditions along each shore of a river. Hence, we want to analyze an infinite strip on an elastic foundation simply supported on opposite sides.

## THEORY

### Solution by superposition

A.D. Kerr (1959) used the method of images to solve a number of floating ice sheet problems which have simply supported boundary conditions. One of his solutions considers a simply supported strip on an elastic foundation with a concentrated load. His method is easily extended for distributed loads.

For a single distributed load on an infinite ice sheet, the deflection for a point not under the load is given by M. Wyman (1950) as

$$w(R) = \frac{P}{\pi k \ell^2} \frac{\ell}{b} [\text{ber}'(b/\ell) \text{ker } R - \text{bei}'(b/\ell) \text{kei } R] \quad (1)$$

where  $P$  = total load

$k$  = density of water

$b$  = radius of loading

$\ell^4 = Eh^3/12k(1 - \sigma^2)$

$R = r/\ell$

$r$  = distance from the center of loading to the point under discussion

$h$  = ice thickness

$E$  = modulus of elasticity

$\sigma$  = Poisson's ratio.

Consider an infinite ice sheet with two distributed loads, one acting downward and one acting upward. The perpendicular bisector of the line joining these two loads is a line of zero deflection. The bending moments along this line are also zero. Hence this perpendicular bisector acts as a simple support in the ice sheet. This is demonstrated in Figure 1 where one load acts downward (positive) at  $x_0, y_0$  and another load acts upward (negative) at  $-x_0, y_0$ . The perpendicular bisector is the  $y$  axis which acts as a simple support.

In order to make the line  $x = L$  a simple support in Figure 1, an infinite set of these positive and negative loads must be placed on the ice sheet. The first two such sets are shown in Figure 1.

Hence by superposition, the deflection at point  $Q$  is given by

$$w(x, y) = \sum_{n=-\infty}^{\infty} [w(R_n) - w(\bar{R}_n)] \quad (2)$$

where

$$R_n^2 = (x - x_0 - 2L_n)^2 + (y - y_0)^2$$

$$\bar{R}_n^2 = (x + x_0 - 2L_n)^2 + (y - y_0)^2.$$

The angles associated with these distances are

$$\tan \theta = \frac{y - y_0}{x - x_0 - 2L_n} \quad \text{and} \quad \tan \bar{\theta} = \frac{y - y_0}{x + x_0 - 2L_n}. \quad (3)$$

The bending moments can be easily determined by

$$\frac{M_x + M_y}{2} = -\frac{D(1 + \sigma)}{2} \left( \frac{\partial^2 w}{\partial R^2} + \frac{1}{R} \frac{\partial w}{\partial R} \right) \quad (4)$$

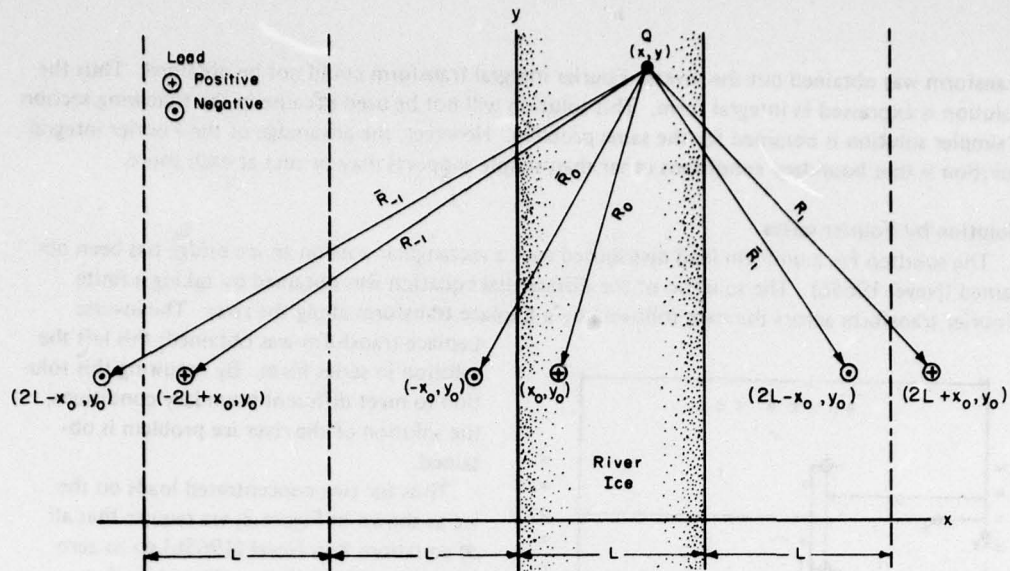


Figure 1. River ice by superposition.

$$\frac{M_y - M_x}{2} = -\frac{D(1-\sigma)}{2} \left( \frac{\partial^2 w}{\partial R^2} - \frac{1}{R} \frac{\partial w}{\partial R} \right) \cos 2\theta \quad (5)$$

$$M_{xy} = -\frac{D(1-\sigma)}{2} \left( \frac{\partial^2 w}{\partial R^2} - \frac{1}{R} \frac{\partial w}{\partial R} \right) \sin 2\theta \quad (6)$$

where  $R$  is any of the  $R_n$  or  $\bar{R}_n$  and the  $\theta$  the corresponding  $\theta$  or  $\bar{\theta}$ .

After the appropriate sum is obtained, the maximum moment may be obtained by using Mohr's circle. The maximum stress is then  $(6/h^2)M_{\max}$ .

If the point is under the load ( $r < b$ ), the corresponding term in the series should be replaced by

$$w(R) = \frac{P}{\pi k \ell^2} \left\{ \frac{\ell^2}{b^2} + \frac{\ell}{b} [\ker'(b/\ell) \operatorname{ber} R - \operatorname{kei}'(b/\ell) \operatorname{bei} R] \right\} \quad (7)$$

However, eq 7 is not valid for determining moments if the load is highly concentrated. A three dimensional theory of elasticity has been given (Nevel 1970) and evaluated for  $r = 0$ . Numerically these results agree with a formula proposed by Westergaard (1926). When  $b/h > 1.724$ , Westergaard says that eq 8, which is a direct result of eq 7, should be used.

$$\left. \begin{aligned} M_x &= M_y = \frac{P}{2\pi} (1+\sigma) \frac{\ell}{b} \operatorname{kei}'(b/\ell) \\ M_{xy} &= 0. \end{aligned} \right\} \quad (8)$$

When  $b/h < 1.724$ ,  $b$  in eq 8 should be replaced by  $a$  where  $a = (1.6b^2 + h^2)^{1/2} - 5h/8$ .

#### Solution by Fourier integral

The solution for a load uniformly distributed over a rectangular area on a river has been obtained (Nevel 1965a). The solution of the differential equation was obtained by taking a Fourier integral transform along the river followed by a Laplace transform across the river. The inverse Laplace

transform was obtained but the inverse Fourier integral transform could not be obtained. Thus the solution is expressed in integral form. This solution will not be used because in the following section a simpler solution is obtained for the same problem. However, the advantage of the Fourier integral solution is that boundary conditions other than simple supports may be met at each shore.

### Solution by Fourier series

The solution for a uniform load distributed over a rectangular area on an ice bridge has been obtained (Nevel 1965b). The solution of the differential equation was obtained by taking a finite Fourier transform across the river followed by a Laplace transform along the river. The inverse

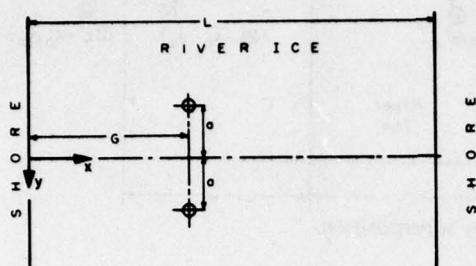


Figure 2. Concentrated loads.

Laplace transform was obtained; this left the solution in series form. By requiring this solution to meet different boundary conditions, the solution of the river ice problem is obtained.

Thus for two concentrated loads on the ice as shown in Figure 2, we require that all of equations 8 in Nevel (1965b) go to zero as  $\eta$  approaches infinity. This gives  $A = (e^{-\beta\alpha}/\epsilon^2)(\beta s + \gamma c)_\alpha$  and  $B = e^{-\beta\alpha}(\beta s - \gamma c)_\alpha$  [see Nevel (1965b) for all notations]. Thus the solution is

$$w = \frac{P}{k\ell^2} \frac{2}{\lambda} \sum_{i=1}^{\infty} Y \sin \phi \Gamma \sin \phi \xi \quad (9)$$

$$M_y = -P \frac{2}{\lambda} \sum_{i=1}^{\infty} (Y'' - \sigma \phi^2 Y) \sin \phi \Gamma \sin \phi \xi \quad (10)$$

$$M_x = -P \frac{2}{\lambda} \sum_{i=1}^{\infty} (\sigma Y'' - \phi^2 Y) \sin \phi \Gamma \sin \phi \xi \quad (11)$$

$$M_{xy} = -P(1 - \sigma) \frac{2}{\lambda} \sum_{i=1}^{\infty} \phi Y' \sin \phi \Gamma \cos \phi \xi \quad (12)$$

where

$$\begin{aligned} Y &= \frac{e^{-\beta(\eta+\alpha)}}{2\epsilon^2} (\beta s + \gamma c)_{\eta+\alpha} + H(\alpha - \eta) \frac{e^{-\beta(\alpha-\eta)}}{2\epsilon^2} (\beta s + \gamma c)_{\alpha-\eta} + \\ &\quad + H(\eta - \alpha) \frac{e^{-\beta(\eta-\alpha)}}{2\epsilon^2} (\beta s + \gamma c)_{\eta-\alpha} \\ Y' &= -\frac{e^{-\beta(\eta+\alpha)}}{2} s_{\eta+\alpha} + H(\alpha - \eta) \frac{e^{-\beta(\alpha-\eta)}}{2} s_{\alpha-\eta} - H(\eta - \alpha) \frac{e^{-\beta(\eta-\alpha)}}{2} s_{\eta-\alpha} \\ Y'' &= \frac{e^{-\beta(\eta+\alpha)}}{2} (\beta s - \gamma c)_{\eta+\alpha} + H(\alpha - \eta) \frac{e^{-\beta(\alpha-\eta)}}{2} (\beta s - \gamma c)_{\alpha-\eta} + \\ &\quad + H(\eta - \alpha) \frac{e^{-\beta(\eta-\alpha)}}{2} (\beta s - \gamma c)_{\eta-\alpha} \end{aligned}$$

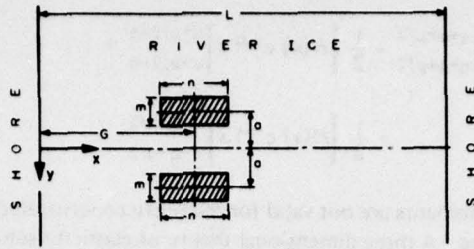


Figure 3. Distributed loads.

For two loads distributed over a rectangular area as shown in Figure 3, we require that all eq 10 in Nevel (1965b) go to zero as  $\eta$  approaches infinity. This gives

$$A_1 = -\frac{1}{\epsilon^4} \left[ e^{-\beta x} c \right]_{\alpha-\mu/2}^{\alpha+\mu/2} + \frac{\phi^2}{\epsilon^2} B_1$$

$$B_1 = -\frac{1}{\epsilon^2} \left[ e^{-\beta x} s \right]_{\alpha-\mu/2}^{\alpha+\mu/2}.$$

Thus the solution is

$$w = \frac{P}{k l^2} \frac{2}{\lambda} \sum_{t=1}^{\infty} Y_1 \frac{2}{\phi \nu \mu} \sin \frac{\phi \nu}{2} \sin \phi \Gamma \sin \phi \xi \quad (13)$$

$$M_y = -P \frac{2}{\lambda} \sum_{t=1}^{\infty} (Y_1'' - \sigma \phi^2 Y_1) \frac{2}{\phi \nu \mu} \sin \frac{\phi \nu}{2} \sin \phi \Gamma \sin \phi \xi \quad (14)$$

$$M_x = -P \frac{2}{\lambda} \sum_{t=1}^{\infty} (\sigma Y_1'' - \phi^2 Y) \frac{2}{\phi \nu \mu} \sin \frac{\phi \nu}{2} \sin \phi \Gamma \sin \phi \xi \quad (15)$$

$$M_{xy} = -P(1 - \sigma) \frac{2}{\lambda} \sum_{t=1}^{\infty} Y_1' \frac{2}{\nu \mu} \sin \frac{\phi \nu}{2} \sin \phi \Gamma \cos \phi \xi \quad (16)$$

where

$$\begin{aligned} Y_1 = & -\frac{1}{2\epsilon^4} \left[ e^{-\beta x} c \right]_{\eta+\alpha-\mu/2}^{\eta+\alpha+\mu/2} - \frac{1}{2\epsilon^4} \left[ H(x)(e^{-\beta x} c - 1) \right]_{\alpha-\mu/2-\eta}^{\alpha+\mu/2-\eta} + \\ & + \frac{1}{2\epsilon^4} \left[ H(x)(e^{-\beta x} c - 1) \right]_{\eta-\alpha+\mu/2}^{\eta-\alpha-\mu/2} + \frac{\phi^2}{\epsilon^4} Y_1'' \\ Y_1' = & \frac{1}{2\epsilon^2} \left[ e^{-\beta x} (\beta s + \gamma c) \right]_{\eta+\alpha-\mu/2}^{\eta+\alpha+\mu/2} - \frac{1}{2\epsilon^2} \left[ H(x) e^{-\beta x} (\beta s + \gamma c) \right]_{\alpha-\mu/2-\eta}^{\alpha+\mu/2-\eta} - \\ & - \frac{1}{2\epsilon^2} \left[ H(x) e^{-\beta x} (\beta s + \gamma c) \right]_{\eta-\alpha+\mu/2}^{\eta-\alpha-\mu/2} \end{aligned}$$

$$Y_1'' = -\frac{1}{2} \left[ e^{-\beta x} s \right]_{\eta+\alpha-\mu/2}^{\eta+\alpha+\mu/2} - \frac{1}{2} \left[ H(x) e^{-\beta x} s \right]_{\alpha-\mu/2-\eta}^{\alpha+\mu/2-\eta} + \\ + \frac{1}{2} \left[ H(x) e^{-\beta x} s \right]_{\eta-\alpha+\mu/2}^{\eta-\alpha-\mu/2}.$$

Here again the above moments are not valid for relatively concentrated loads if the point under discussion is under the load. A three dimensional theory of elasticity solution is not readily available for a rectangular load. Therefore, if the load is relatively concentrated, the rectangular load should be replaced with an equivalent circular load and Westergaard's formula should be used.

## APPLICATION AND RESULTS

The bearing capacity of river ice for the following vehicles is given in Appendix A, Figures A1-A11.

### Tracked vehicles

1. M48
2. M8
3. SP M109 How
4. PC M113
5. PC M577

### Wheeled vehicles

6. M51, 5-ton dump truck w/winch and 3½-ton Bolster trailer
7. M52, 5-ton tractor w/winch and M172A1, 25-ton low bed semi-trailer
8. 20-ton crane.

The detailed specifications of these vehicles are given in Appendix B.

For the tracked vehicles, it is obvious that the solution by Fourier series should be used since the contact area of the track is rectangular. Each vehicle was placed at the center of the river since this appears to be the most critical position. The moments were obtained under the center of the track and then the maximum stress was obtained. The results are shown in Figures A1-A4.

The M51 and M52 wheeled vehicles have 12-ply 11 x 20 tires inflated to 70 psi. For these tires Freitag and Green (1962) have shown that the contact area more nearly represents a rectangle than a circle. Hence, the Fourier series solution will be used for these vehicles. The Fourier series solution will also be used for the remaining wheeled vehicles for convenience since no contact area was available for the tires of these vehicles. The contact area of each tire was obtained by dividing the load on the tire by the inflation pressure. The ratio of length to width of the contact area of tires varies from about 1.5 to 2. The ratio of 1.5 was selected since this should produce higher stresses.

The stresses at the center of the contact area of each tire on each vehicle were determined as follows. The tire was placed at the center of the river since this seems to be the most critical position. Westergaard's formula was used to evaluate the moments due to the load of this tire since the contact area is relatively concentrated. The additional moments due to the remaining tire loads were evaluated by the Fourier series solution and added to the moments previously obtained from Westergaard's formula. The maximum stress under the tire being tested was then obtained by using Mohr's circle. A comparison of the maximum stress under each tire was made to determine the critical stress of the vehicle. The results are shown in Figures A5-A11. The Fortran II computer program for the computations is given in Appendix C.

For the vehicles considered, the stress in kilograms per square centimeter divided by the total load in tons (2000 lb) is presented in Figures A1-A11 as a function of the width of the river and the ice thickness. The total load means the weight of the vehicle plus the load that is being carried. For trucks, the graphs for completely loaded and unloaded conditions are presented. For a lightly loaded (up to 25%) truck, the graph for the unloaded truck should be used. For any other partially loaded (over 25%) truck the graph for the loaded truck should be used. The allowable stress for cold, clear, fresh water ice (as can be expected in Alaska) is 10 kg/cm<sup>2</sup>. If the ice is warm or partially

deteriorated, the allowable stress is  $5 \text{ kg/cm}^2$ . In the spring thaw, the allowable stress can drop as low as 1 to  $2 \text{ kg/cm}^2$ .

As an example, in using these graphs, suppose we have a 75-ft-wide river covered with 20-in.-thick ice. We desire to take across the ice an M51 truck that is half loaded. Referring to Figures A5 and A6, the truck weighs 11.3 tons and has a carrying capacity of 10.7 tons. This means that the total weight of the half loaded truck is 16.7 tons. We select Figure A6 because the truck is more than 25% loaded. For a 75-ft width and 20-in.-thick ice, we find that  $\text{Stress/Total Load} = 0.44$ . The stress in the ice will be  $0.44 \times (\text{Total Load}) = 0.44 \times 16.7 = 7.5 \text{ kg/cm}^2$ . This is a permissible value of stress for cold, clear, fresh water ice, but not for partially deteriorated ice.

In these graphs, the stress factor depends upon the ice thickness and the width of the river. In order to eliminate the width of the river, we choose the width of the river that gives the maximum stress factor. For these critical river widths, the stress factor depends upon the ice thickness as shown in Figure A12. If the river is very wide, the stress factor does not depend upon the river width but on the ice thickness as shown in Figure A13.

In summary, Figure A12 can be used as the basis for the bearing capacity of river ice. For a more exact analysis Figures A1-A11 can be used. A speed well below the critical velocity should be maintained.

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APPENDIX A: BEARING CAPACITY OF RIVER ICE  
FOR MILITARY VEHICLES

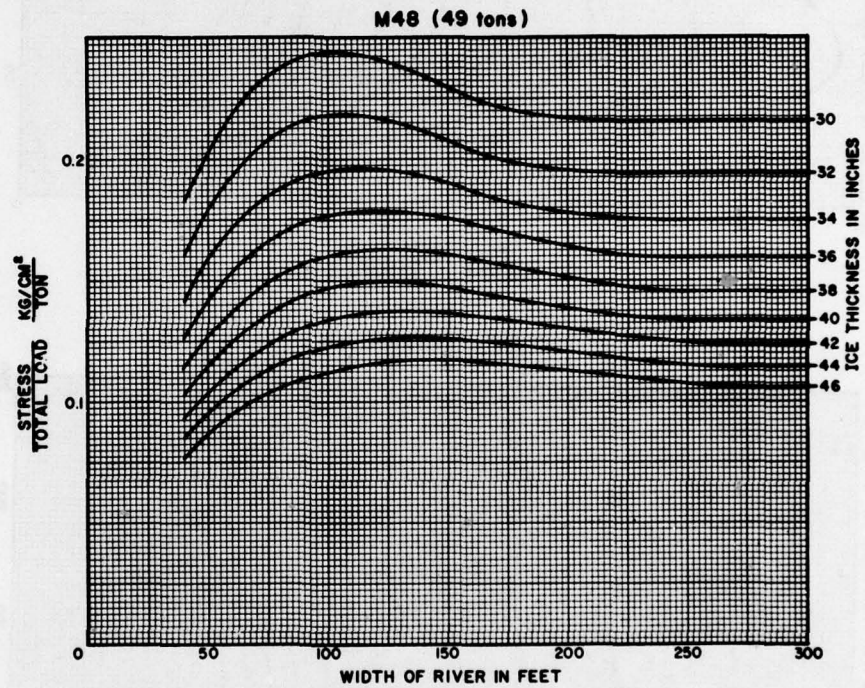


Figure A1.

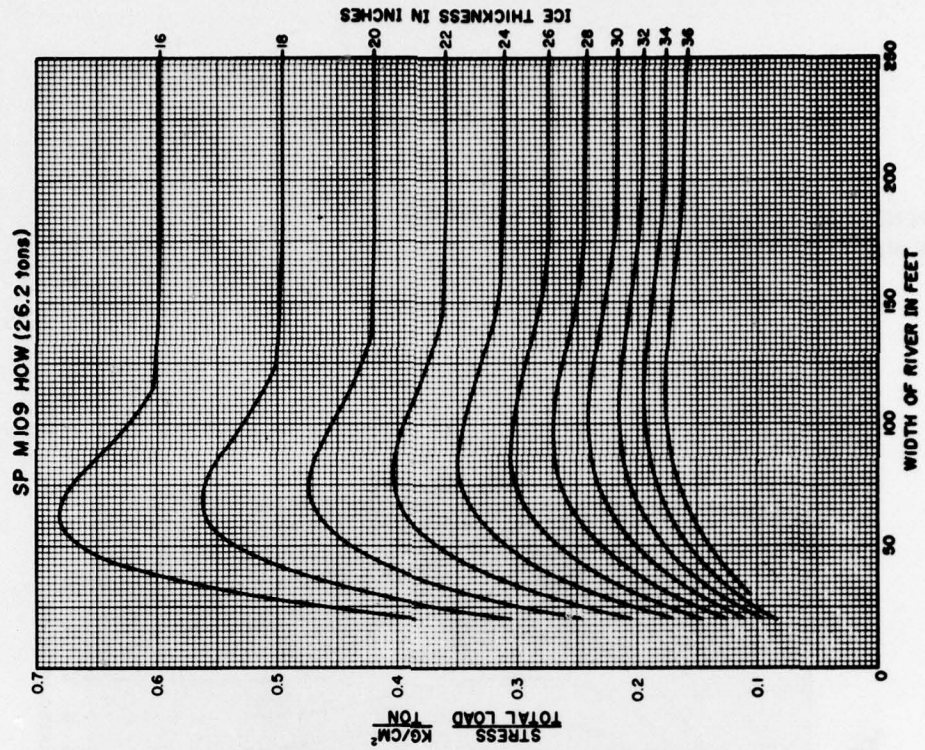


Figure A3.

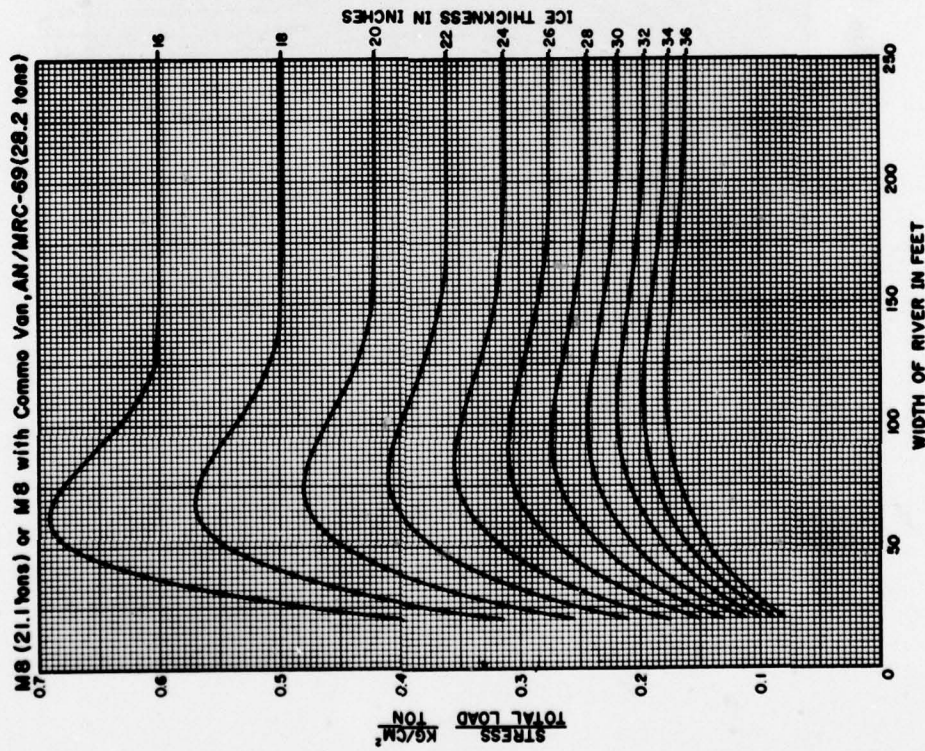
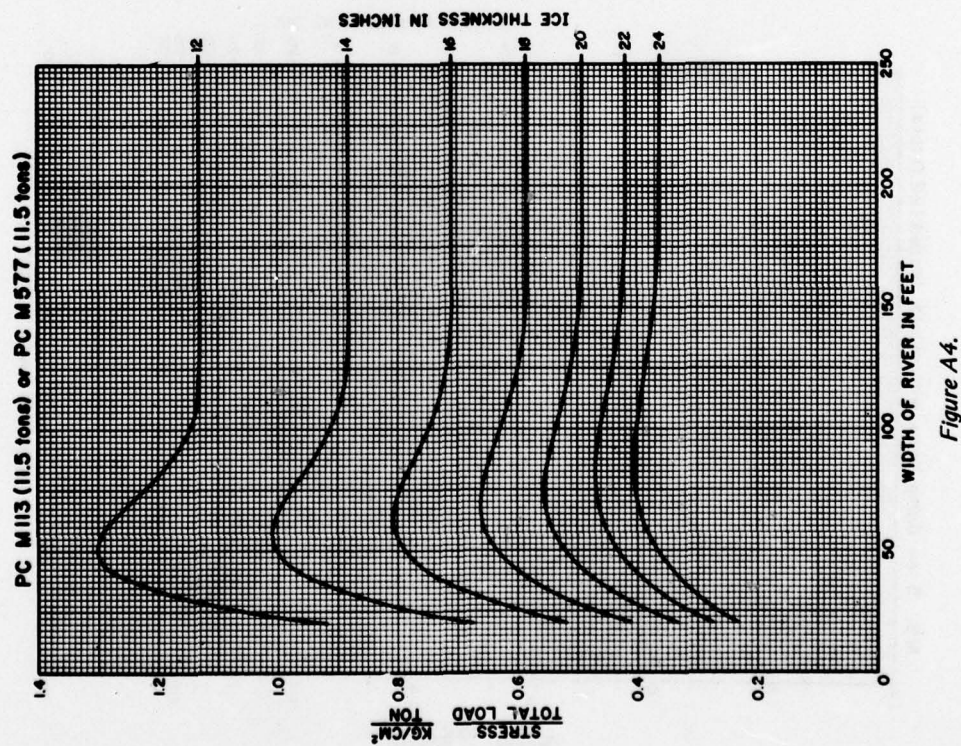
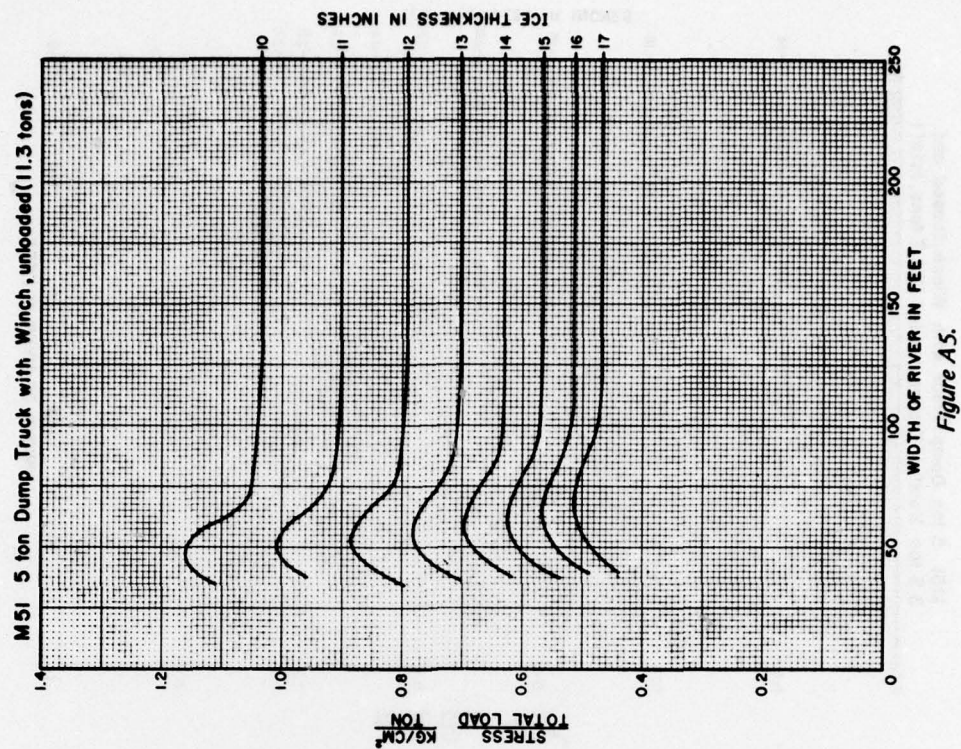


Figure A2.



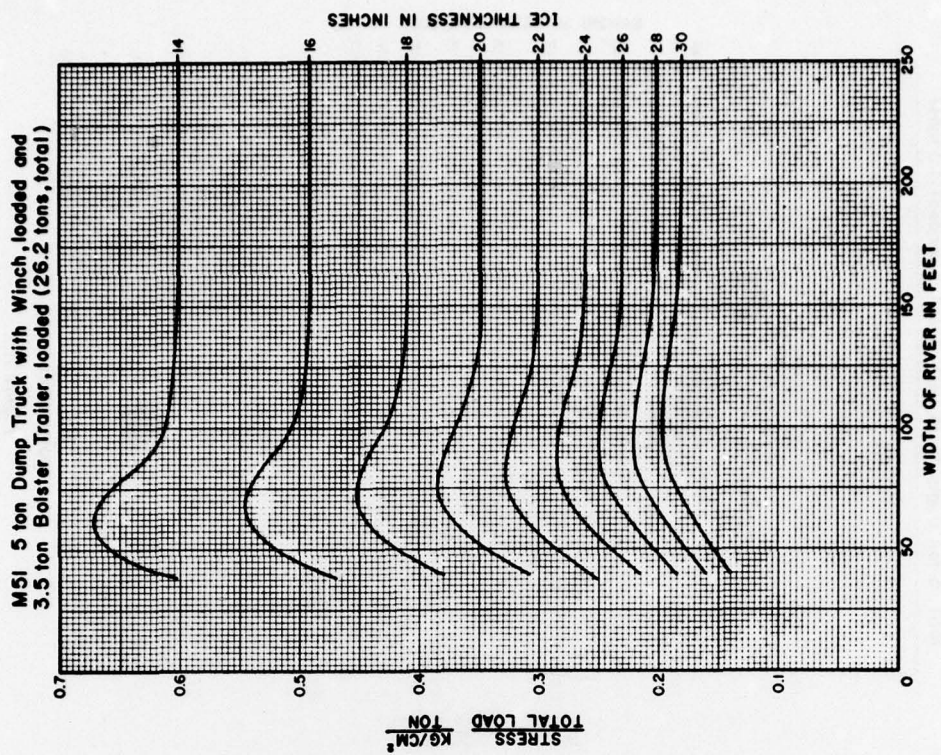


Figure A7.

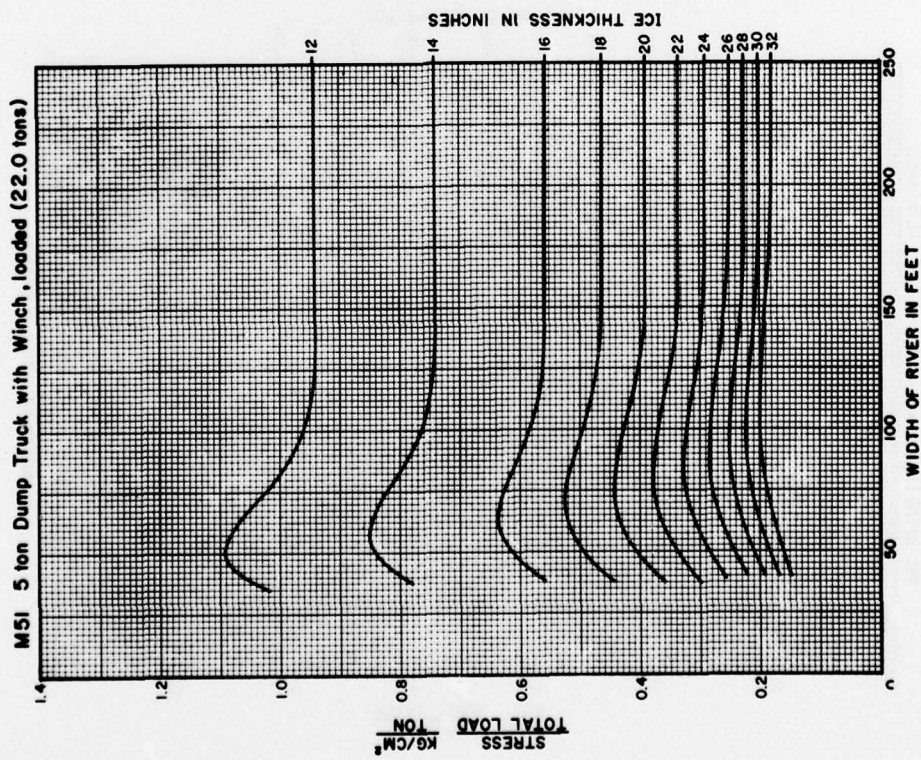


Figure A6.

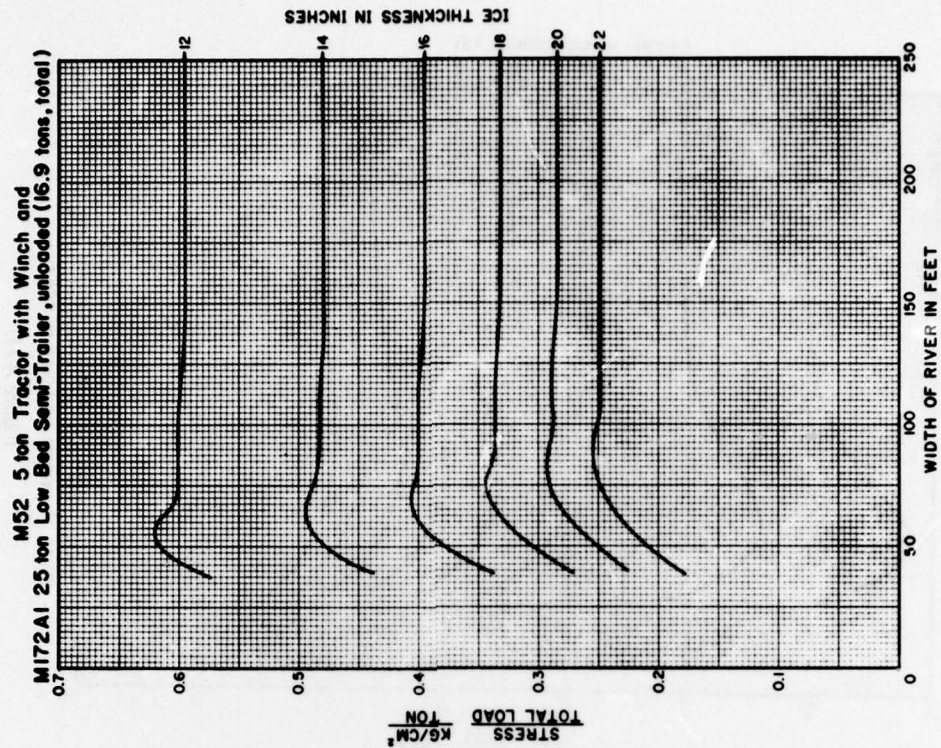


Figure A9.

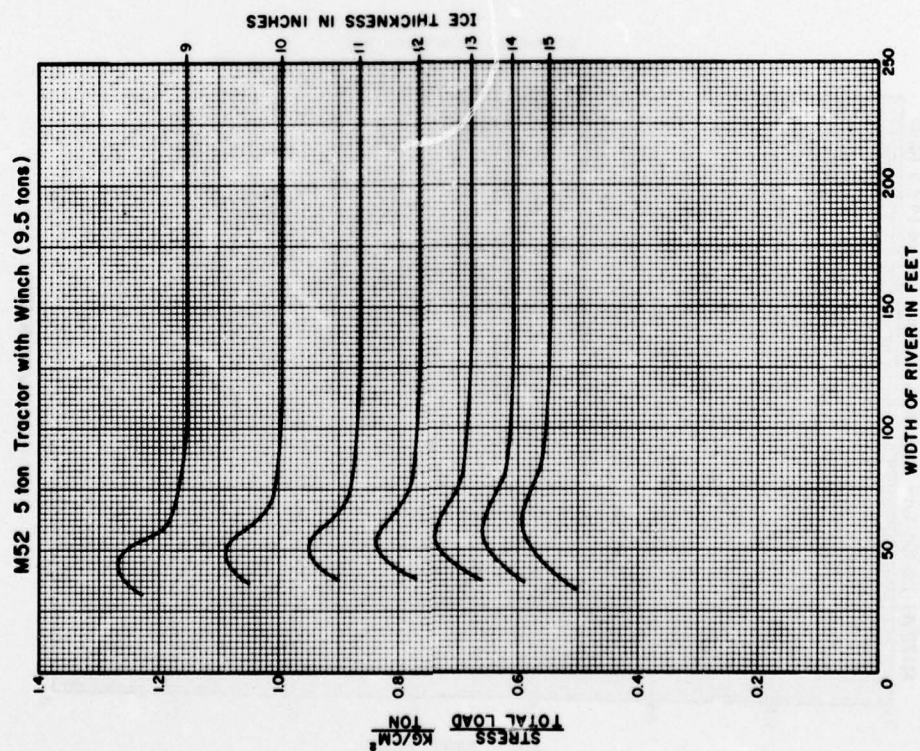


Figure A8.

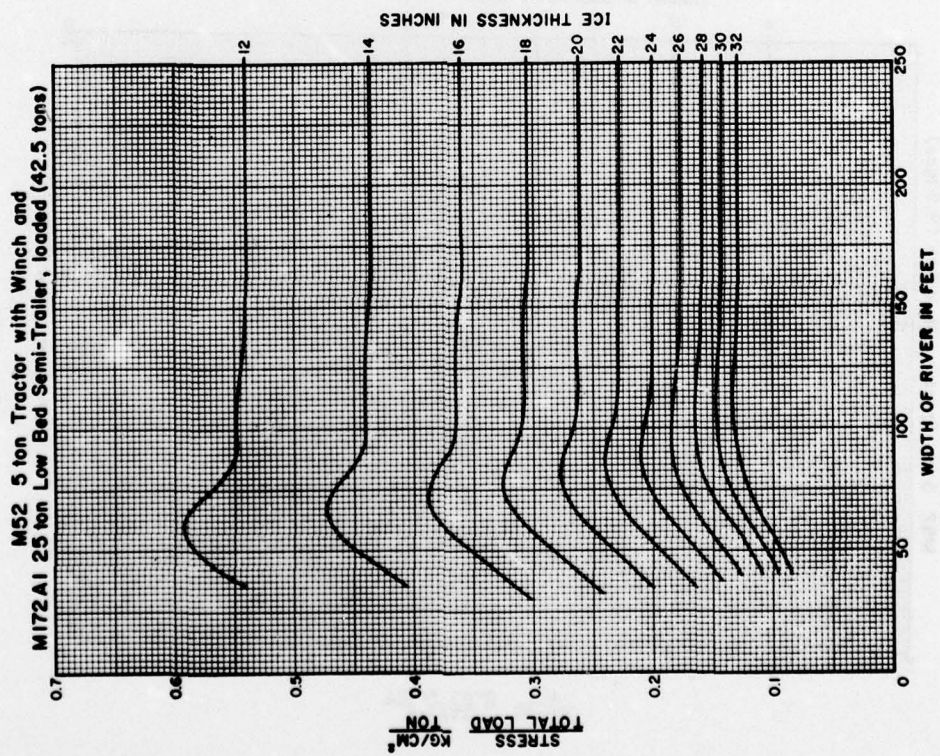


Figure A10.

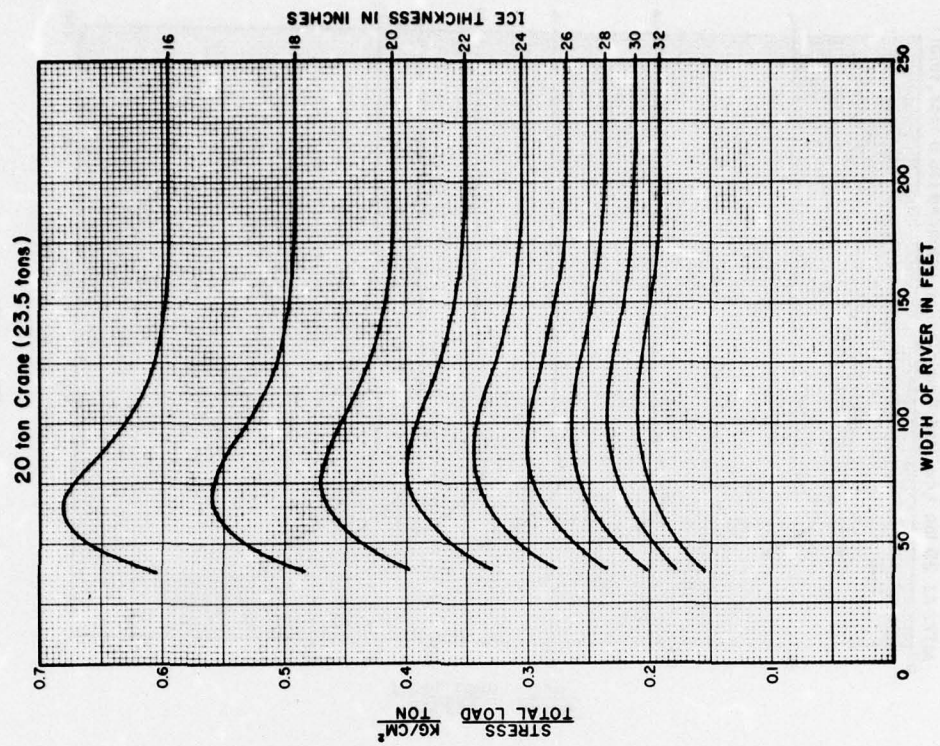
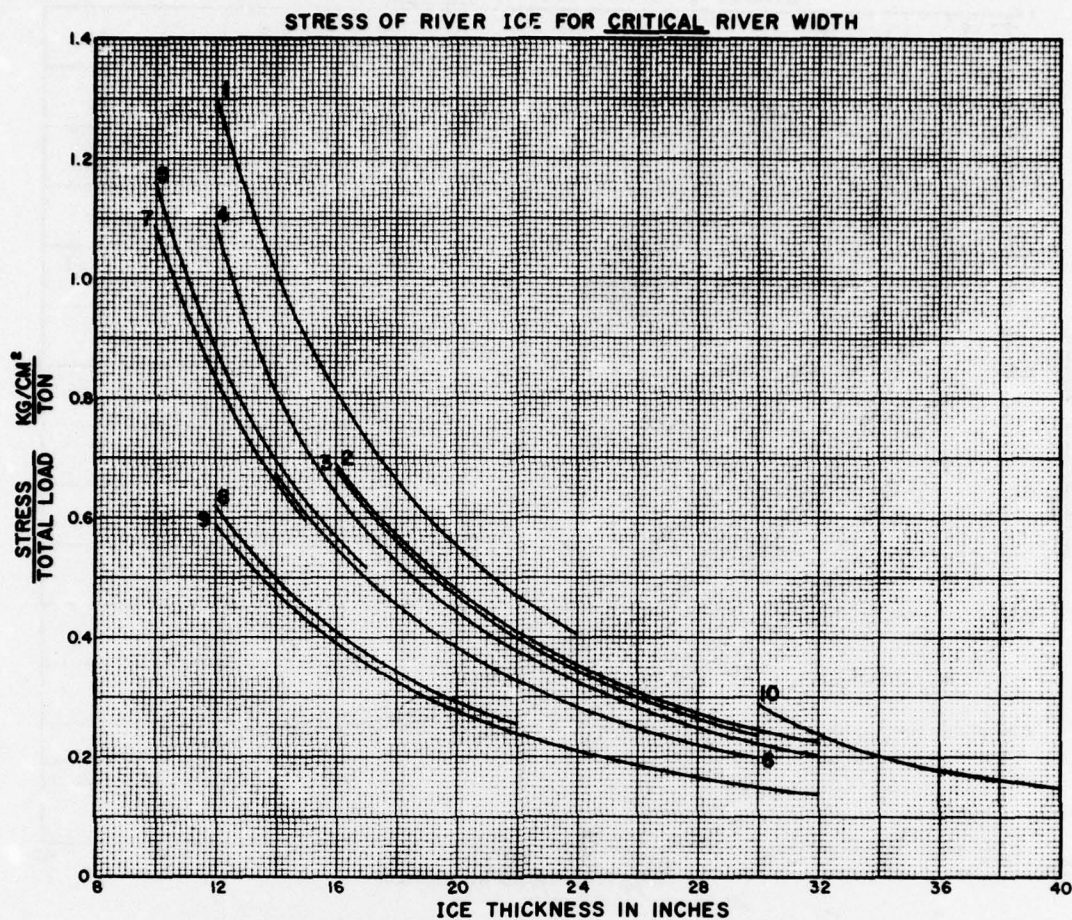
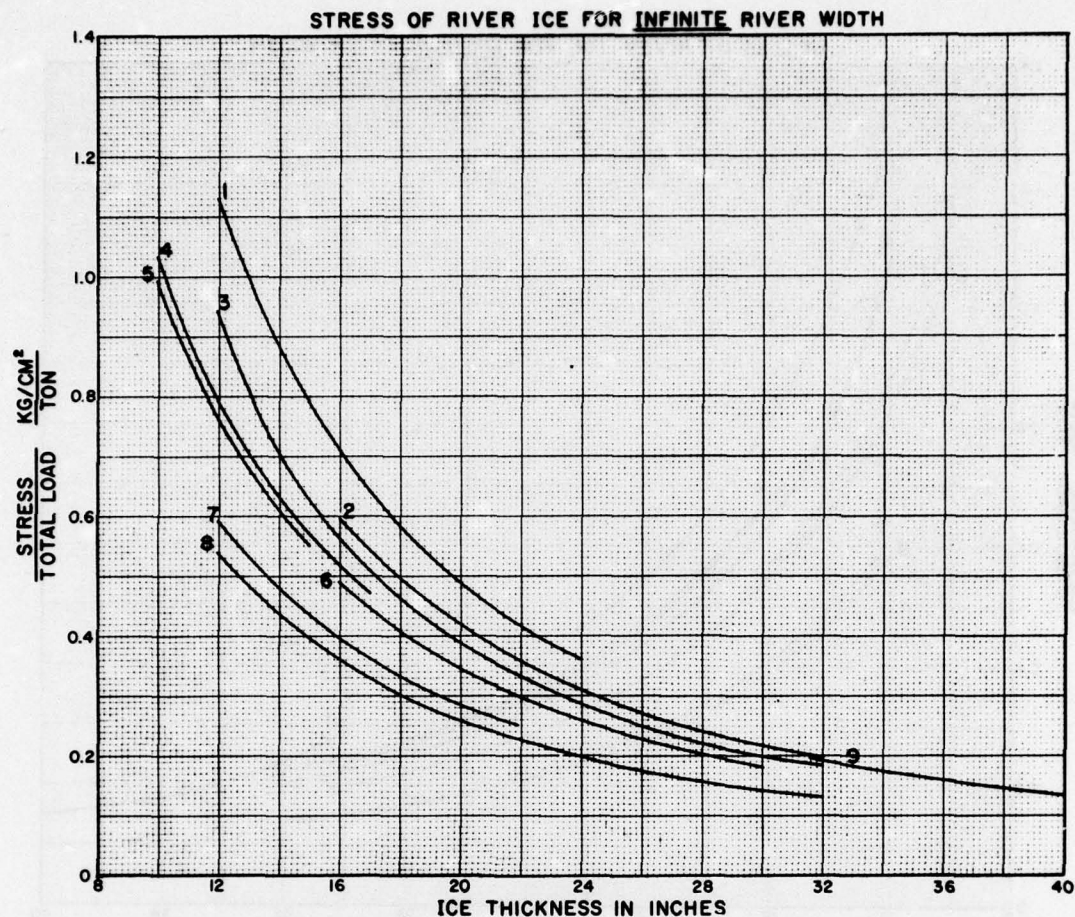


Figure A11.



	<i>Total weight (tons)</i>
1) PCM113 or PCM577	11.5
2) M8	21.1
M8 with commo van (AN/MRC-69)	28.2
3) 20-ton crane	23.5
SP M109 how	26.2
4) M51 5-ton dump truck with winch - loaded	22.0
5) M51 5-ton dump truck with winch - unloaded	11.3
6) M51 5-ton dump truck with winch - loaded and 3.5-ton bolster trailer - loaded	26.2
7) M52 5-ton tractor with winch	9.5
8) M52 5-ton tractor with winch and M172 A1 25-ton semi-trailer - unloaded	16.9
9) M52 5-ton tractor with winch and M172 A1 25-ton semi-trailer - loaded	42.5
10) M48 tank	49.0

Figure A12.



	<u>Total weight (tons)</u>
1) PCM113 or PCM577	11.5
2) M8	21.1
M8 with commo van (AN/MRC-69)	28.2
SP M109 how	26.2
20-ton crane	23.5
3) M51 5-ton dump truck with winch - loaded	22.0
4) M51 5-ton dump truck with winch - unloaded	11.3
5) M52 5-ton tractor with winch	9.5
6) M51 5-ton dump truck with winch - loaded and 3.5-ton bolster trailer - loaded	26.2
7) M52 5-ton tractor with winch and M172 A1 25-ton low bed semi-trailer - unloaded	16.9
8) M52 5-ton tractor with winch and M172 A1 25-ton low bed semi trailer - unloaded	42.5
9) M48 tank	49.0

Figure A13.

## APPENDIX B: VEHICLE SPECIFICATIONS

Axle no.	M51	M51	M51	M51	M52	M52	M52	M52	20-ton crane	M48	M577	M113	M8	M109
No. of axles	Empty	Loaded	Loaded w/trail	3	4	3	5	5	3	1	1	1	1	1
1 No. of tires/axle														
Axle load (lb)	9,304	9,912	9,912	1	1	9,008	9,166	1	5,000	98,000	23,900	22,900	4,225	52,461
Contact width (in.)	6.6	6.9	6.9			6.6	6.6		4.9	23	15	15	24.5	15
Contact length (in.)	10	10.3	10.3			9.8	9.8		7.3	154	105	105	157.1	166
Dist to tire 1 (in.)	36.9	36.9	36.9			36.9	36.9		37.5	55	42.5	42.5	53	54.5
2 No. of tires/axle														
Axle load (lb)	6,680	16,551	17,000	2	2	4,994	7,165	2	21,000					
Contact width (in.)	4	6.3	6.4			3.5	5.8		7.1					
Contact length (in.)	6	9.4	9.6			5.2	8.7		10.6					
Dist to tire 1 (in.)	29	29	29			29	29		32.5					
Dist to tire 2 (in.)	43	43	43			43	43		46.5					
3 No. of tires/axle														
Axle load (lb)	6,680	16,551	17,000	2	2	4,994	7,165	2	21,000					
Contact width (in.)	4	6.3	6.4			3.5	5.8		7.1					
Contact length (in.)	6	9.4	9.6			5.2	8.7		10.6					
Dist to tire 1 (in.)	29	29	29			29	29		32.5					
Dist to tire 2 (in.)	43	43	43			43	43		46.5					
4 No. of tires/axle														
Axle load (lb)			8,500		1		5,180	2	20,000					
Contact width (in.)			6.4				3.5	7						
Contact length (in.)			9.6				5.3	10.6						
Dist to tire 1 (in.)			36				34.2	34.2						
Dist to tire 2 (in.)							48.2	48.2						
5 No. of tires/axle														
Axle load (lb)							5,180	2	20,000					
Contact width (in.)							3.5	7						
Contact length (in.)							5.3	10.6						
Dist to tire 1 (in.)							34.2	34.2						
Dist to tire 2 (in.)							48.2	48.2						
Dist from axle 1-2 (in.)	140	140	140			140	140	140	150	0	0	0	0	0
Dist from axle 2-3 (in.)	54	54	54			55	55	55	50					
Dist from axle 3-4 (in.)			260				280	280						
Dist from axle 4-5 (in.)							42	42						

# APPENDIX C: FORTRAN II COMPUTER PROGRAM

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C UNWEEL VEHICLE ON RIVER ICE

C THIS PROGRAM CALCULATES THE MAXIMUM OVERALL STRESSES CAUSED BY A
C VEHICLE CROSSING A RIVER ICESHEET AND/OR THE MAXIMUMS UNDER THE
C INDIVIDUAL WHEELS DEPENDING UPON OUTPUT OPTIONS DESCRIBED BELOW.
C THE PROGRAM REQUIRES INPUT OF THE NUMBER OF WHEELS ON ONE SIDE OF
C THE VEHICLE CENTERLINE, THE CONTACT WIDTH AND THE LENGTH OF TIRES
C ON EACH AXLE, THE AXLE LOAD OF EACH AXLE, THE DISTANCES BETWEEN
C EACH WHEEL AND THE VEHICLE CENTERLINE AND THE DISTANCE BETWEEN
C AXLES.

C SET SENSE SWITCH 1, OFF FOR SIGMA = .33333 AND E = 50000. ON FOR
C ACCEPT SIGMA AND E.

C SET SENSE SWITCH 2 ON FOR PRINTOUT OF FINAL VALUES OF MY, MX AND
C MXY FOR INDIVIDUAL WHEELS.

C SET SENSE SWITCH 3 ON FOR PRINTOUT OF SERIES TERMS IN RIVER
C SURROUTINE.

C SET SENSE SWITCH 4 ON FOR PRINTOUT OF STRESSES DEVELOPED UNDER
C INDIVIDUAL WHEELS.

C SET SENSE SWITCH 5, ON FOR TAPE INPUT, OFF FOR TYPEWRITER INPUT.

C SET SENSE SWITCH 6 ON FOR TYPEOUT OF AXLE AND WHEEL NUMBER
C CURRENTLY BEING CALCULATED.

C TO RUN PROGRAM SET SENSE SWITCHES 1 AND 2 AS DESIRED. ENTER
C VALUES FOR RIVER PARAMETERS ON TYPEWRITER AS REQUESTED BY PROGRAM
C IN F FORMAT. IF SENSE SWITCH 5 IS ON BE CERTAIN THAT DATA TAPE
C HAS BEEN PREPARED AS DESCRIBED BELOW AND INSERTED IN TAPE READER.
C INSERT NUMBER OF VEHICLES TO BE RUN ON TAPE INPUT MODE IN I
C FORMAT WHEN REQUESTED BY PROGRAM.

C ALL DISTANCES AND WEIGHTS MUST BE IN THE F FORMAT FOR INPUT.
C DISTANCES MUST BE ENTERED IN INCHES AND WEIGHTS IN POUNDS. THE
C NO. OF AXLES, THE NO. OF WHEELS AND THE NUMBER OF TRUCKS MUST
C BE ENTERED IN THE I FORMAT.

C AXLES ARE NUMBERED FROM THE FRONT STARTING AT ONE WHILE WHEELS
C ON EACH AXLE ARE NUMBERED FROM THE VEHICLE CENTERLINE STARTING
C AT ONE. IF FOR EXAMPLE A VEHICLE HAD TWO TIRES, ONE ON EITHER SIDE
C OF THE VEHICLE CENTERLINE, THE CONFIGURATION WOULD BE DESCRIBED
C AS AXLE 1, WHEEL 1. IF THE AXLE HAD 4 WHEELS, THE INSIDE WHEEL
C WOULD BE DESIGNATED AS AXLE 1, WHEEL 1, WHILE THE OUTSIDE WHEEL
C WOULD BE DESIGNATED AXLE 1, WHEEL 2.

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```

110  FORMAT (21H FINAL H IN INCHES = )
    ACCEPT 90, HFINAL
    TYPE 120
120  FORMAT(20H FINAL WIDTH IN FEET)
    ACCEPT 90, WFINAL
    LFINWFINAL/10.
    IF(SENSE SWITCH 5)135,115
115  TYPE 116
116  FORMAT(47H NUMBER OF AXLES INCL. TRAILOR IF APPLICABLE = )
    ACCEPT 35, N0
    TLCA0=0.
    DO 220 I = 1, NC
    TYPE 140, I
140  FORMAT(10H AXLE NO = 15)
    TYPE 150
150  FORMAT(50H NO. OF WHEELS ON ONE SIDE OF VEHICLE CENTERLINE = )
    ACCEPT 35, J(1)
    TYPE 160
160  FORMAT(23H AXLE LOAD IN POUNDS = )
    ACCEPT 30, XLOAD(1)
    TLCA0 = TLOAD + XLOAD(1)
    TYPE 170
170  FORMAT(36H CONTACT WIDTH OF TIRES IN INCHES = )
    ACCEPT 90, ZN(1)
    TYPE 180
180  FORMAT(37H CONTACT LENGTH OF TIRES IN INCHES = )
    ACCEPT 90, ZN(1)
    TYPE 190
190  FORMAT(50H WHEEL DISTANCES MEASURED FROM VEHICLE CENTERLINE )
    DO 220 K = 1, J(1)
    TYPE 200, K
    TYPE 210
210  FORMAT(18H WHEEL DISTANCE = )
    ACCEPT 90, A(I,K)
    X(I) = 0.
    IF(N0-1)134,134,221
    DO 240 L0 = 2, NC
    L01 = L0 - 1
221  TYPE 230, L01, L0
    FORMAT(22H DISTANCE BETWEEN AXLE 15, 5H AND 15)
    ACCEPT 90, X(L0)
    X(L0) = X(L01)/12.
    GO TO 241
240  X(L0) = X(L0) + X(L01)
134  GO TO 241
135  TYPE 136
136  FORMAT(19H NUMBER OF TRUCKS = )
    ACCEPT 35, KTR
    DO 360 I1 = 1, KTR
    READ PAPER TAPE 35, N0
    TLCA0 = 0.
    DO 225 I = 1, NC
    READ PAPER TAPE 35, J(1)
    READ PAPER TAPE 36, XLOAD(1)
    TLCA0 = TLOAD + XLOAD(1)
    READ PAPER TAPE 90, ZM(1)
    READ PAPER TAPE 90, ZN(1)
    DO 225 K = 1, J(1)
    READ PAPER TAPE 90, A(I,K)
    X(I) = 0.
225  IF(N0-1)241,241,227
    DO 226 L0 = 2, NC
    L01 = L0 - 1
    READ PAPER TAPE 90, X(L0)
    X(L0) = X(L01)/12.
    X(L0) = X(L0) + X(L01)
    M=INITI
    L=(H+H+M*FCON)*.25/12.
    PRINT 242, M
    FORMAT(3M H=6.0//)
242  DO 246 I=6, LFIN
    DO 246 I=6, LFIN
    BIG(1)=0.
    DO 345 M = 1, NC
    DO 345 N = 1, J(M)
    IF(SENSE SWITCH 4)248,255
    PRINT 250, M,N
248  FORMAT(7H AXLE = 15, 9H WHEEL = 15)
    WIDTH=40.
    LAMBDA = WIDTH/L
    CEE = LAMBDA/2.
    STR=0.
    SMX=0.
    SMY=0.
    SMX=0.
    SMY=0.
    DO 335 M1 = 1, NC
    IF(ABS(X(M1)-X(M1))-WIDTH/2,1293,335,335
293  DO 335 N1 = 1, J(M1)
    XL=XLOAD(M1)
    MU = ZN(M1)/L/12.
    NU = ZN(M1)/L/12.
    ALPHA = A(M1,N1)/L/12.
    XJ=J(M1)
    IF(N0-1)300,300,294
    IF(M1-M)300,295,300
294  IF(N1-N)300,310,300
295  IF(N1-N)300,310,300
300  ETA=A(N,N1)/L/12.
    X1=X(M1)-X(M)
    GAM=CEE+X1/L
    CALL RIVER
    STRESS=0.
    GO TO 330
    ETA=2.*ALPHA
    MU=MU/2.
    ALPHA=MU/2.
    XL=XL/2.
    GAM = CEE
    CALL RIVER
    XPI = 3.1415926536
    XR = ZN(M1)*ZN(M1)/XPI
    XA = SORTF(1.6*XP+H+H)-.675*H
    XAA = XA/L/12.
    CONST = (1.+SIGMA)/XPI
    CALL XEIP(XAA,XAR)
    STRESS = CONST*XAR/XAA
    STRESS = STRESS*XL/H/H/XJ/TLOAD*2000.*.453592477/6.4516256
330  SMX=SMX+MX
    SMY=SMY+MY
    STR=STR+STRESS

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332 IF(SENSE SWITCH 61332.335
333 TYPE 333.M.N
334 FORMAT(7H AXLE =15.8H WHEEL =15)
335 CONTINUE
      SMAX=(SMX+SMY)/2.*SQRTF(SMXY*SMXY*(SMX-SMY)
      2 /2.)+2.1+STR
336 IF(SENSE SWITCH 41336.338
337 PRINT 332,SMAX,P+WIDTH
338 FORMAT(6H SMAX=F8.3,4X,3H H=F6.0,4X,7H WIDTH=F7.0)
339 IWIDTH=WIDTH
340 IF(SMAX-4IG(IWIDTH/10))341,341,339
341 WIDTH = WIDTH + 10.
342 IF(IWIDTH-UFINAL)250,290,345
343 CONTINUE
344 DO 347 I=4,LFIN
      IK=I*10
345 PRINT 347,BIG(I),H,IK
346 FORMAT(6H SPAX=F8.3,4X,3H H=F6.0,4X,7H WIDTH=F5)
347 PRINT 348
348 FORMAT(17H MAXIMUM STRESSES)
349 H = H + HINC
350 IF(H-HFINAL) 245,245,360
360 CONTINUE
      END
SUBROUTINES REQUIRED
FAT3 FSTY FSEL FSHD FSR5 FSR5
FST1 FST1 FSP FSH FSR5 FSR5
FSD FSD FSC FSR FSR RIVER
FSL FSL FSC FSR FSR
XRETP FST FST FST FST
      0
SUBROUTINE RIVER
      REAL LAMBDA,MU,NU,MX,MY,MXY
      COMMON LAMBDA,ALPHA,MU,NU,ETA,SIGMA,MY,MX,GAM,CEE,XJ,
      2 H,XL,TLOAD,MXY
      MXY=0.
      MX=0.
      MY=0.
      PI=3.1415926536
      J=0.
      T = -1.
      T = T+2.
      S = T+1.
      PHI=PI+T/LAMBDA
      PH12=PHI+PHI
      EPS14=1.+PHI2+PHI2
      EPS12=SQRTF(EPS14)
      RETA=SQRTF((EPS12+PHI2)/2.)
      GAMMA=SQRTF((EPS12-PHI2)/2.)
      PHIX=PI+S/LAMBDA
      PH12X=PHI+PHI
      EPS14X=1.+PHI2X+PHI2X
      EPS12X=SQRTF(EPS14X)
      RETAX=SQRTF((EPS12X+PHI2X)/2.)
      GAMMA=SQRTF((EPS12X-PHI2X)/2.)
      X=ETA+ALPHA+MU/2.
      Y2=Y2+EXPF(-RETA*X)*SINF(GAMMA*X)
      Y=Y+EXPF(-BETA*X)*COSF(GAMMA*X)
      V1=Y1-EXPF(-BETA*X)*(GAMMA+COSF(GAMMA*X))
      2 +BETA*SINF(GAMMA*X))
      U=ETA+ALPHA+MU/2.
      J=1
      IF(U) 10,10,30
      U=ALPHA+MU/2.-ETA
      IF(U) 20,20,30
      X=ALPHA+MU/2.-ETA
      Y=Y-EXPF(-BETA*X)*COSF(GAMMA*X)
      Y2=Y2-EXPF(-BETA*X)*SINF(GAMMA*X)
      V1=Y1-EXPF(-BETA*X)*(GAMMA+COSF(GAMMA*X)+BETA*
      2 *SINF(GAMMA*X)).
      X=ETA+ALPHA+MU/2.
      Y2=(Y2-EXPF(-BETA*X)*SINF(GAMMA*X))/2.
      Y=(2.+Y-EXPF(-BETA*X)*COSF(GAMMA*X))/2.
      V1=Y1+EXPF(-BETA*X)*(GAMMA+COSF(GAMMA*X)
      2 +RETA*SINF(GAMMA*X))
      GO TO 40
      X=MU+U
      Y=Y-EXPF(-BETA*X)*COSF(GAMMA*X)
      Y2=Y2-EXPF(-BETA*X)*SINF(GAMMA*X)
      X1=Y1-EXPF(-BETA*X)*(GAMMA+COSF(GAMMA*X)+BETA*
      2 *SINF(GAMMA*X))
      X=U
      Y=(Y+EXPF(-BETA*X)*COSF(GAMMA*X))/2.
      Y2=(Y2+EXPF(-BETA*X)*SINF(GAMMA*X))/2.
      X1=Y1-EXPF(-BETA*X)*(GAMMA+COSF(GAMMA*X)+BETA*
      2 *SINF(GAMMA*X))
      IF(J) 32,32,36
      V1=Y1+X1
      GO TO 40
      V1=Y1-X1
      Y=(Y+PHI2+Y2)/EPS14
      TS=2./PHI/LAMBDA/MU/NU*SINF(PHI+NU/2.)*COSF(PI+T*(GAM
      2 /LAMBDA-5))
      CON=6.*XL/H/HX/J/TLOAD+2000.*.4535924/6.4516258
      XMY=CON*(Y2-SIGMA*PHI2*Y)*TS
      MY=MY+XMY
      MX=CON*(SIGMA*Y2-PHI2*Y)*TS
      MX=MX+XMX
      TS=(1.-SIGMA)*2./LAMBDA/MU/NU*SINF(PHI+NU/2.)
      2 *SINF(PHI+GAM)*COSF(PHI+CEE)
      X1=CON*Y1*TS
      MXY=MX+X1
      24 IF(SENSE SWITCH 31 25,27
      25 PRINT 26,MV,MX,MXY,T
      26 FORMAT(4H MV=E20.8,4X,4H MX=E20.8,4X,5H MXY=E20.8,
      2 3X,3H T=F5.0)
      27 IF(ABSF(XMY)-.00001) 28,28,1
      28 IF(ABSF(XMX)-.00001) 29,29,1

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