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# A NEW TEST OF NORMALITY AND OF EXPONENTIALITY CALLED THE Q-TEST

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September 1977

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A new test statistic, denoted by Q and equal to the quotient of two unbiased estimates of the standard deviation of a normal distribution is proposed for testing the hypothesis that a given sample is drawn from a normal population or, alternatively, from an exponential population. The sampling distribtuions of Q have been computed and used for setting the limits of rejection regions corresponding to 1%, 2%, and 5% levels of significance and also for stating the decision power of Q used as a shape estimator.

## FOREWORD

The research work reported herein was conducted by Prof. Dr. Waloddi Weibull, Chemin Fontanettaz 15, 1012 Lausanne, Switzerland under USAF Contract No. F44620-73-C-0066. This contract, which was initiated under Project No. 7351, "Metallic Materials", Task 735106, "Behavior of Metals", was administered by the European Office of Aerospace Research. The work was monitored by the Metals and Ceramics Division, Air Force Materials Laboratory, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio, under the direction of Mr. W. J. Trapp, AFML/LL.

This report covers work conducted during the period 1 June 1973 to 1 August 1973. The manuscript was submitted by the author for publication in October 1973.

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## 1. INTRODUCTION

The log-normal function has frequently been proposed as the appropriate statistical model of fatigue life distributions. Also, the exponential distribution function has been advocated for this purpose, in particular by Epstein et al. (1,2).

In view of the necessity of using the correct distribution function when analyzing a given sample of test data, it is important to find sharp tests for deciding whether the assumed function is the correct one or not.

A new test which seems to be quite sensitive to outliers within the sample will now be presented.

## 2. THE TEST STATISTIC Q

Let $x_1, \ldots, x_N$ be the ordered elements of a sample drawn from a normal population with unknown parameters m and σ. From these elements an unbiased and asymptotically efficient estimate of σ is given by

$$\hat{\sigma}_1 = \sqrt{\Sigma(x_i - \bar{x})^2 / (N-1)} = \sqrt{(\Sigma x_i^2 - N.\bar{x}^2)/(N-1)} \qquad (1)$$

It is, however, possible to obtain another, also unbiased, estimate $c_2$ by using the best, linear, unbiased estimator, which consists in multiplying each observation by an appropriate coefficient $a_i$, thus arriving at

$$\hat{\sigma}_2 = \Sigma a_i . x_i \qquad (2)$$

The coefficients $a_i$ have been computed and tabulated for complete and censored samples, for instance, by Sarhan & Greenberg (3). Values for complete samples of the sizes N = 5(1)20 are listed in Table 1.

Even if these two estimates are unbiased, it is evident that their values will never be exactly equal for any given sample, because they will react differently for the same deviations of the elements. For instance, for any variation $\Delta x_i$ of the i:th element the variation of $\hat{\sigma}_1$ is independent of the order number i, whereas the variation of $\hat{\sigma}_2$, as being equal to $a_i \Delta x_i$, is much larger for the order numbers i=1 and i=N than for other order numbers and even equal to zero for i = (N+1)/2, as is easily read from Table 1.

It thus seemed plausible that the quotient

$$Q = \hat{c}_1/\hat{c}_2 \tag{3}$$

used as a test statistic, may provide a test which is sensitive to deviations in the extreme elements of the sample. This is a valuable property, in particular when testing samples of fatigue performance data, which are frequently composed of two or even three parts belonging to different populations.

Introducing (1) and (2) into (3) we arrive at the test statistic

$$Q = \sqrt{(\Sigma x_i^2 - N.\bar{x}^2)/(N-1)} \; / \; \Sigma a_i . x_i \tag{4}$$

where $\bar{x} = \Sigma x_i/N$ and the coefficients $a_i$ are given in Table 1.

The properties and the use of the statistic Q have been examined as indicated in the following.

## 3. PROPERTIES OF THE STATISTIC Q

It is easily proved that, due to the condition $\Sigma a_i = 0$, the sampling distribution of Q is both a scale and location invariant and depending only on the shape of the distribution function. Thus it can be used as a shape estimator and also for testing the hypothesis that the sample is drawn from the assumed population.

All relevant properties of Q are given by its sampling distribution. To this purpose these distributions have been computed by use of Program 8/73 for normal distributions and for Weibull distributions with the parameters $\alpha = 1/m = 0.1, 0.4, 0.7, 1.0$ and for sample sizes N = 5, 10, 15, 20. The number of generated random samples in this Monte-Carlo study were 10,000 for N=5 and 10 and 5,000 for N=15 and 20.

This program does also provide the percentiles of Q corresponding to the percentages 1%, 2%, 5%, 50%, 95%, 98%, and 99%. Computed values are given in Table 2.

By use of these sampling distributions also the decision power DP of Q has been computed for sample sizes N=10 and 20. The results are presented in Table 3.

## 4. THE Q-TEST OF NORMALITY

The percentiles $q_p$ from the normal distribution given in Table 2 can now be used as the limits of the rejection regions of the test statistic Q defined by (4). These limits are interpolated for all sample sizes between 5 and 20, as indicated in Figure 1.

## 5. THE Q-TEST OF EXPONENTIALITY

In the same way, the hypothesis that the sample is drawn from an exponential population (m=1) can be tested. The limits are given in Figure 2.

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3. Sarhan, A.E. & Greenberg, B.G.: "Contributions to Order Statistics". J. Wiley & Sons, Inc., New York. London, 1962.

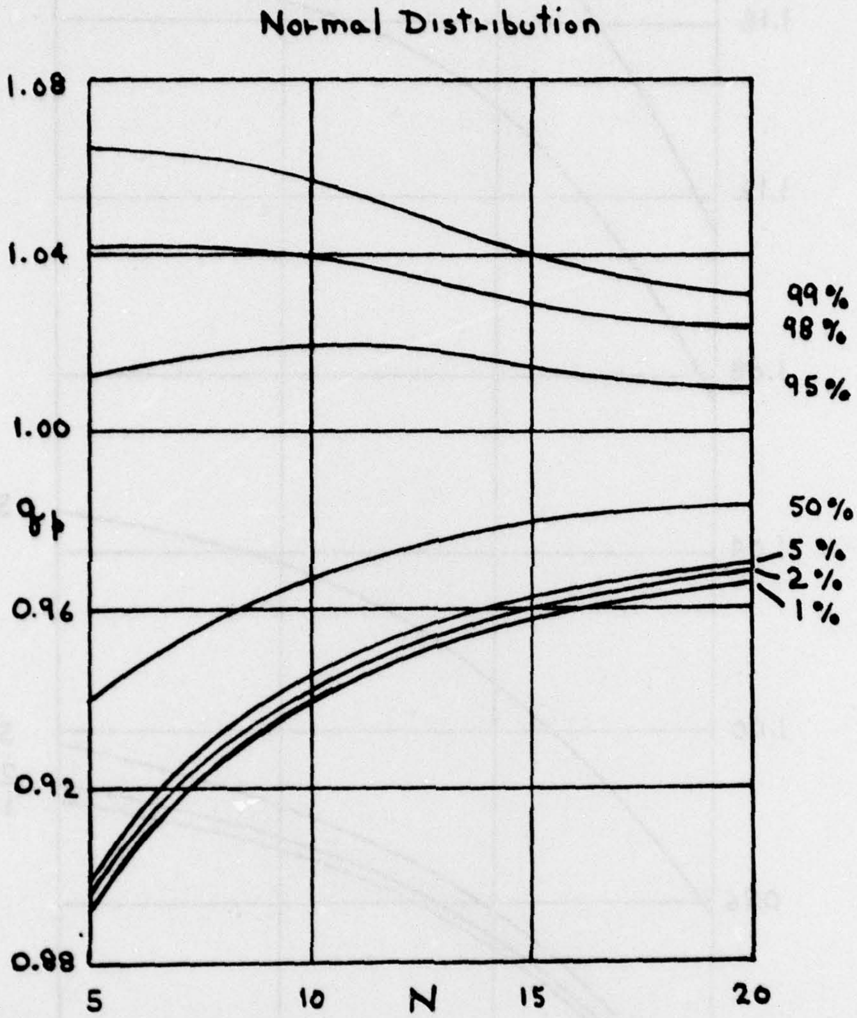


Figure 1. Percentiles of Q as Functions of the Sample Size N.

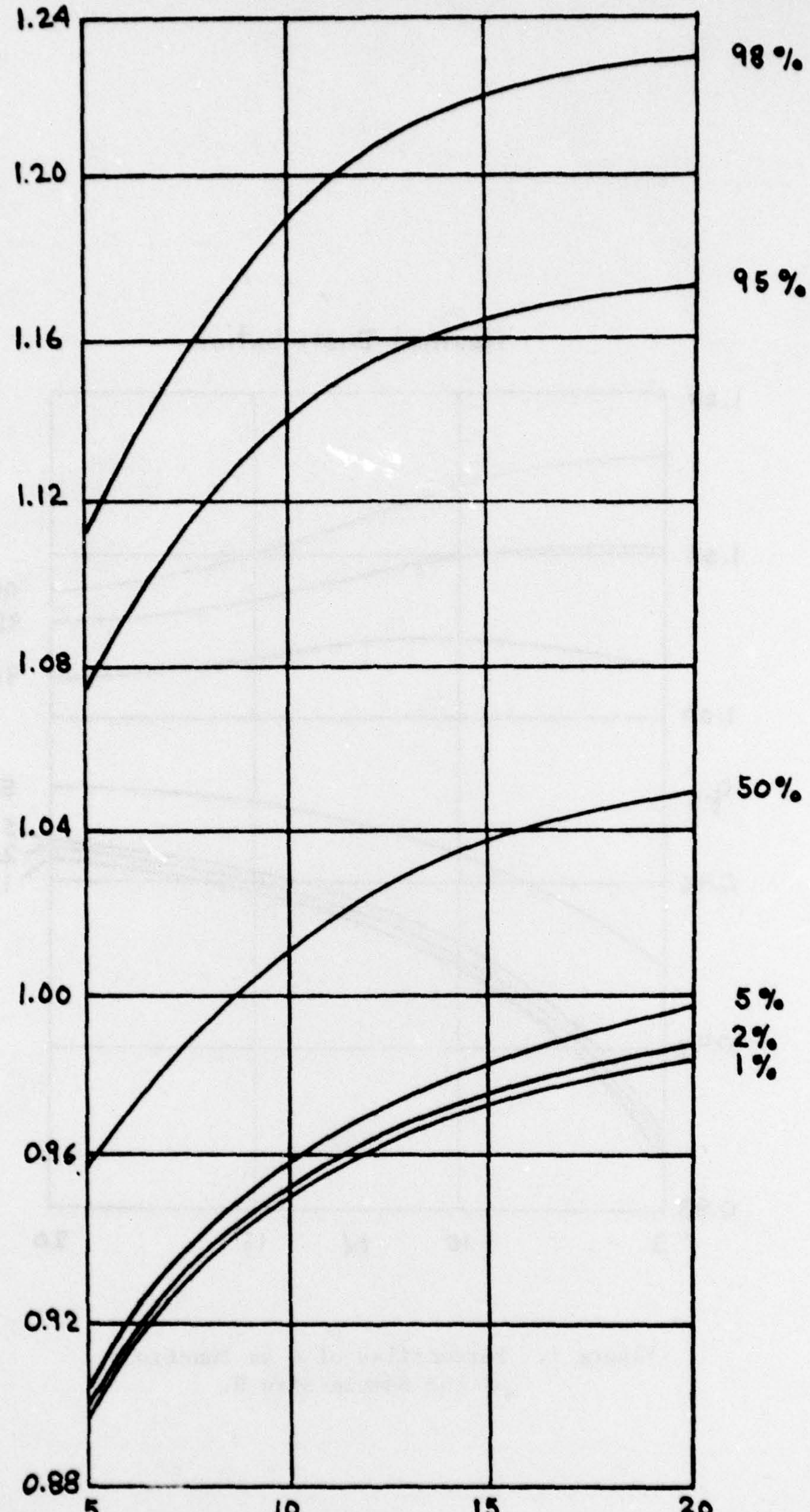


Figure 2. Percentiles of Q as Functions of N; Exponential dbn.

TABLE 1. VALUES OF THE COEFFICIENTS $a_i$

| i \ N | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|
| 1 | -.3724 | -.3175 | -.2778 | -.2476 | -.2237 | -.2044 | -.1883 | -.1748 |
| 2 | -.1352 | -.1386 | -.1351 | -.1294 | -.1233 | -.1172 | -.1115 | -.1061 |
| 3 | .0000 | -.0432 | -.0625 | -.0713 | -.0751 | -.0763 | -.0760 | -.0749 |
| 4 | .1352 | .0432 | .0000 | -.0230 | -.0360 | -.0436 | -.0481 | -.0506 |
| 5 | .3724 | .1386 | .0625 | .0230 | .0000 | -.0142 | -.0234 | -.0294 |
| 6 | - | .3175 | .1351 | .0713 | .0360 | .0142 | .0000 | -.0097 |
| 7 | - | - | .2778 | .1294 | .0751 | .0436 | .0234 | .0097 |
| 8 | - | - | - | .2476 | .1233 | .0763 | .0481 | .0294 |
| 9 | - | - | - | - | .2237 | .1172 | .0760 | .0506 |
| 10 | - | - | - | - | - | .2044 | .1115 | .0749 |
| 11 | - | - | - | - | - | - | .1883 | .1061 |
| 12 | - | - | - | - | - | - | - | .1748 |

| i \ N | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|
| 1 | -.1632 | -.1532 | -.1444 | -.1366 | -.1297 | -.1235 | -.1178 | -.1128 |
| 2 | -.1013 | -.0968 | -.0927 | -.0889 | -.0854 | -.0822 | -.0792 | -.0765 |
| 3 | -.0735 | -.0717 | -.0699 | -.0681 | -.0663 | -.0645 | -.0628 | -.0611 |
| 4 | -.0520 | -.0526 | -.0526 | -.0524 | -.0519 | -.0512 | -.0505 | -.0497 |
| 5 | -.0335 | -.0362 | -.0379 | -.0391 | -.0398 | -.0401 | -.0402 | -.0402 |
| 6 | -.0164 | -.0212 | -.0247 | -.0272 | -.0290 | -.0302 | -.0312 | -.0318 |
| 7 | .0000 | -.0070 | -.0122 | -.0160 | -.0189 | -.0211 | -.0228 | -.0241 |
| 8 | .0164 | .0070 | .0000 | -.0053 | -.0094 | -.0125 | -.0150 | -.0169 |
| 9 | .0335 | .0212 | .0122 | .0053 | .0000 | -.0041 | -.0074 | -.0101 |
| 10 | .0520 | .0362 | .0247 | .0160 | .0094 | .0041 | .0000 | -.0033 |
| 11 | .0735 | .0526 | .0379 | .0272 | .0189 | .0125 | .0074 | .0033 |
| 12 | .1013 | .0717 | .0526 | .0391 | .0290 | .0211 | .0150 | .0101 |
| 13 | .1632 | .0968 | .0699 | .0524 | .0398 | .0302 | .0228 | .0169 |
| 14 | - | .1532 | .0927 | .0681 | .0519 | .0401 | .0312 | .0241 |
| 15 | - | - | .1444 | .0889 | .0663 | .0512 | .0402 | .0318 |
| 16 | - | - | - | .1366 | .0854 | .0645 | .0505 | .0402 |
| 17 | - | - | - | - | .1297 | .0822 | .0628 | .0497 |
| 18 | - | - | - | - | - | .1235 | .0792 | .0611 |
| 19 | - | - | - | - | - | - | .1178 | .0765 |
| 20 | - | - | - | - | - | - | - | .1128 |

TABLE 2. PERCENTILES OF Q FOR VARIOUS SAMPLE SIZES.

| Sample size | Per-centage | Normal dbn. | Weibull distribution | | | |
|---|---|---|---|---|---|---|
| | | | 0.1 | 0.4 | 0.7 | 1.0 |
| 5 | 1 | 0.894 | 0.895 | 0.894 | 0.895 | 0.896 |
| 5 | 2 | 0.896 | 0.896 | 0.896 | 0.897 | 0.898 |
| 5 | 5 | 0.899 | 0.900 | 0.900 | 0.901 | 0.904 |
| 5 | 50 | 0.933 | 0.934 | 0.934 | 0.942 | 0.955 |
| 5 | 95 | 1.013 | 1.018 | 1.014 | 1.036 | 1.074 |
| 5 | 98 | 1.042 | 1.052 | 1.046 | 1.071 | 1.112 |
| 5 | 99 | 1.064 | 1.078 | 1.068 | 1.097 | 1.130 |
| 10 | 1 | 0.941 | 0.942 | 0.941 | 0.944 | 0.949 |
| 10 | 2 | 0.943 | 0.944 | 0.943 | 0.947 | 0.953 |
| 10 | 5 | 0.946 | 0.947 | 0.947 | 0.952 | 0.960 |
| 10 | 50 | 0.968 | 0.972 | 0.969 | 0.985 | 1.011 |
| 10 | 95 | 1.019 | 1.033 | 1.021 | 1.071 | 1.142 |
| 10 | 98 | 1.041 | 1.059 | 1.040 | 1.105 | 1.194 |
| 10 | 99 | 1.059 | 1.085 | 1.054 | 1.132 | 1.231 |
| 15 | 1 | 0.959 | 0.960 | 0.959 | 0.964 | 0.972 |
| 15 | 2 | 0.960 | 0.961 | 0.961 | 0.966 | 0.976 |
| 15 | 5 | 0.962 | 0.964 | 0.963 | 0.970 | 0.983 |
| 15 | 50 | 0.979 | 0.983 | 0.980 | 1.000 | 1.035 |
| 15 | 95 | 1.014 | 1.036 | 1.020 | 1.077 | 1.164 |
| 15 | 98 | 1.027 | 1.062 | 1.036 | 1.114 | 1.221 |
| 15 | 99 | 1.040 | 1.079 | 1.053 | 1.141 | 1.260 |
| 20 | 1 | 0.967 | 0.968 | 0.968 | 0.974 | 0.985 |
| 20 | 2 | 0.968 | 0.970 | 0.970 | 0.976 | 0.989 |
| 20 | 5 | 0.971 | 0.972 | 0.972 | 0.980 | 0.997 |
| 20 | 50 | 0.984 | 0.989 | 0.986 | 1.010 | 1.049 |
| 20 | 95 | 1.010 | 1.035 | 1.017 | 1.080 | 1.173 |
| 20 | 98 | 1.024 | 1.055 | 1.031 | 1.112 | 1.228 |
| 20 | 99 | 1.030 | 1.071 | 1.045 | 1.142 | 1.278 |

TABLE 3. DECISION POWER DP OF Q

N = 10

| α \ α | 0.1 | 0.4 | 0.7 | 1.0 | normal |
|---|---|---|---|---|---|
| 0.1 | - | 5.4 | 19.8 | 44.3 | 7.8 |
| 0.4 | 5.4 | - | 24.9 | 49.4 | 3.1 |
| 0.7 | 19.8 | 24.9 | - | 26.4 | 26.9 |
| 1.0 | 44.3 | 49.4 | 26.4 | - | 50.8 |
| normal | 7.9 | 3.1 | 26.9 | 50.8 | - |

N = 20

| α \ α | 0.1 | 0.4 | 0.7 | 1.0 | normal |
|---|---|---|---|---|---|
| 0.1 | - | 11.2 | 37.6 | 69.1 | 16.8 |
| 0.4 | 11.2 | - | 48.2 | 77.5 | 7.1 |
| 0.7 | 37.6 | 48.2 | - | 40.8 | 53.9 |
| 1.0 | 69.1 | 77.5 | 40.8 | - | 81.1 |
| normal | 16.8 | 7.1 | 53.9 | 81.1 | - |