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ANALYSIS OF MONOPOLE ANTENNA ARRAYS ON CYLINDERS BY THE GEOMETRICAL THEORY OF DIFFRACTION

THESIS

Presented to the Faculty of the School of Engineering of the Air Force Institute of Technology Air University in Partial Fulfillment of the Requirements for the Degree of Master of Science

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Bruce A. Thieman, B.S.

2nd Lt

Graduate Electrical Engineering

December 1977

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Preface

This investigation demonstrated the capability of the Geometrical Theory of Diffraction (GTD) to analyze the fields due to monopole arrays on cylinders. Although one of the principle arguments for the use of the GTD method to solve for the field equations was its requirement for less computer usage compared to alternative methods, a computer, as such, was not used.

In this regard I would like to thank Mr. Robert Barham for the use of his HP-67 programable calculator. I would also like to thank Professor Kouyoumjian at Ohio State for his comments and his help in obtaining many of the sources on GTD used herein. I would especially like to thank Professor Bill Davis for taking the time to go through most of the theory with me and for his comments on the grammatical problems that occured.

Bruce A. Thieman

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Abstract

Using the Geometrical Theory of Diffraction (GTD) the fields due to a monopole array mounted on an aircraft near the top or bottom were analyzed. The aircraft was modeled in its most basic form; an infinitely long elliptical cylinder. The aircraft crosssection at the location of the antenna was modeled as a 2 meter by 4 meter ellipse. The frequency band of operation was 250 mHz to 400 mHz with an average wavelength assumed of 1 meter. The three element array was found to have a beam broadening effect when compared to the array mounted on an infinite ground plane. The array exhibited a poor potential to form a single major lobe in its antenna pattern but had a high mobility of pattern nulls.

ANALYSIS OF ANTENNA ARRAYS ON CYLINDERS

BY THE

GEOMETRICAL THEORY OF DIFFRACTION

I. Introduction

One of the principal problems in the design of a reliable airborne antenna is the effect on volumetric antenna radiation patterns by the aircraft not acting as a flat ground plane. The particular antenna pattern problem to be considered here consists of an antenna array of quarter-wavelength, radial monopoles, which operate in the 250 mHz to 400 mHz range, mounted on an aircraft fuselage near the top or bottom. The array configuration is depicted in Fig. 1.



Fig. 1. Array Configuration

Since this is a general analysis, the aircraft is modeled in its most basic form. The fuselage is assumed to be an infinitely long perfectly conducting elliptical cylinder. The cylinder

solution employed in this analysis, when compared to the experimental case of one monopole on a cylinder, gives an accuracy of about+1 dB in pattern computation down to a level about 20 dB below the pattern maximum.

Antenna location on an aircraft is usually based primarily upon convenience with respect to the aircraft structure with no regard for system requirements. The antenna system performance is determined by analyzing antenna patterns of scale models. This approach to antenna design and location requires large amounts of engineering time and money. This investigation develops an analytic approach to determining the antenna system performance.

The first analytic solutions used to determine on-aircraft antenna patterns were the cylindrical harmonic solutions for infinitely long circular (Ref 1; Ref 2) and elliptic cylinders (Ref 3). These solutions modeled the fuselage by a cylinder whose elliptical cross-section approximated the fuselage cross-section at the antenna. location. These solutions failed for structures of many wavelengths in diameter unless expanded asymptotically for large dimensions. The expansion of these solutions required computers of very large memory.

A more recent approach is the use of the integral equation method via moment methods. By solving the boundary value problem for fields on aircraft structures, the surface currents and the resulting scattered fields can be found. Richmond (Ref 4; Ref 5), developed a moment solution based on a wire grid modelling structure. This solution required the determination of approximately 100 unknown currents per square wavelength in order that the grid

adequately simulate a perfectly conducting surface.

Another approach is the surface current model method (Ref 6). This method divides the surface of a conducting body into patches, with each patch having two orthogonal unknown complex currents. This reduces the number of unknown currents to about 20 per square wavelength allowing solutions for much larger surfaces. This solution, modal solution, and the moment method are restricted to lower frequencies due to the limitation of the size of matrices which modern computers can solve without unacceptable loss of accuracy.

Another approach that has obtained success at analyzing onaircraft antennas is the Geometrical Theory of Diffraction (referred to simply as the GTD henceforth). The GTD is a high frequency solution which is divided into two basic problems; wedge diffraction and curved surface diffraction. Both of these diffraction solutions have been applied in computing the patterns of slot antennas mounted on cylinders (Ref 7; Ref 8; Ref 9), rockets (Ref 10), and aircraft (Ref 11; Ref 12). Using this approach, one applies a ray optics technique to determine components of the field incident on the various scatterers. Components of the diffracted field are found using the GTD solutions in terms of rays which are summed with the geometrical optics terms in the far field.

The approach applied in this thesis is based on previous work by Burnside (Ref 13) and Yu (Ref 14), which demonstrated the capability of the numerical solutions to predict the radiation patterns of fuselage mounted antennas in an efficient and economical way. If the volumetric patterns were found directly by analyzing rays on complex three-dimensional surfaces as done in Reference 12, the

resulting numerical solution would be very complex, time-consuming, and uneconomical. Nervertheless, if certain assumptions can be made, the approach undertaken previously can be used to overcome these difficulties and simplify the problem a great deal.

The basic theoretical background on the geometrical theory of diffraction (GTD) for curved surface diffraction is presented in Chapter II. Expressions for the parameters and special functions involved are given in Chapter III. The equation for diffraction by a monopole on an infinitely long cylinder are presented in Chapter III, as is the mathematical model of the aircraft. In Chapter IV, the numerical results obtained for the single monopole case are compared with published results and measurements and in addition the field patterns of the three element array for different phases are presented. Chapter V contains the conclusions.

II. Theoretical Background

For scattering objects that are large in terms of the wavelength, such as for electrically large aircraft, the GTD is the optimum approach. The following development of the GTD assumes; that ray trajectories are determined by the generalized Fermat's principle, that the field amplitude variation along a ray path can be found from power conservation in a tube (or strip) of rays, and that the diffraction and excitation of waves can be treated as local phenomena. The validity of these assumptions and the GTD have been established in published materials.

According to geometrical optics the region exterior to a perfectly-conducting surface with a radial electric current source at point 0 is divided into an illuminated region and a shadow region by a plane tangent to the surface at Q_1 called the shadow boundary. A portion of these two regions adjacent to the shadow boundary is a transition region. The angular extent of the transition region is of the order $(k \rho_g)^{-1/3}$ when the source is on the surface; ρ_g being the radius of curvature of the surface at Q in the direction of propagation, and k being the wave number $2\pi/\lambda$. The three regions are depicted in Fig. 2. To solve for the fields due to the array, the derivation of the high-frequency, far-zone field due to a point source has to be determined for these three regions.

Diffraction by a Curved Surface

When an incident ray strikes a smooth, curved perfectly con-



ducting surface at grazing incidence, i.e., at the shadow boundary, a part of its energy is diffracted into the shadow region. This phenomena is described by a class of diffracted rays now know as creeping waves (Ref 15). These ray paths include the points Q_1 and Q_2 which form a curve on the diffracting surface, as illustrated in Fig. 3. The actual concept of creeping waves was introduced by Franz and Depperman (Ref 16; Ref 17). The basic concepts, as presented in the following discussion is taken in part form "Asymptotic High-Frequency Methods" by Kouyoumjian (Ref 18).

The diffraction by a smooth curved surface is shown in Fig. 3 in which O is the source point and P is the observation point in the shadow region. Diffracted rays are characterized by an extended form of Fermat's principal which is: The diffracted rays connecting two points 0 and P in the exterior of a perfectly conducting body are those curves joining 0 and P which have stationary (optical) length among all curves from 0 to P having an arc on the body surface. Applying this principle, the line OQ_1Q_2P is the shortest distance between 0 and P which does not penetrate the surface. In detail, a ray incident on the shadow boundary at Q, divides; one part of the incident energy continues straight on as predicted by geometrical optics, a second part follows the surface S into the shadow region as a surface ray shedding diffracted rays tangentially as it propagates where \hat{t} , \hat{n} , and \hat{b} are the unit vectors in the direction of incidence, normal to the surface S and binormal to the surfave $(b=t \times n)$. The incident field $\overline{E}^{i}(Q_{1})$ may be seperated into its normal and binormal components $(\hat{n} \cdot \tilde{E}^{i}(Q_{1}) \text{ and } \hat{b} \cdot \tilde{E}^{i}(Q_{1})).$ It is assumed that these two components induce surface ray fields



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Fig. 3. Diffraction by a smooth convex surface

which propagate independently of each other along the geodesic arc between Q_1 and Q_2 .

The binormal surface ray field at Q_1 is related to the binormal component of the incident field at Q_1 by

$$\mathbf{A}(\mathbf{Q}_1)\exp(\mathbf{j}\phi(\mathbf{Q}_1)) = \mathbf{D}^{\mathbf{S}}(\mathbf{Q}_1)(\mathbf{\hat{b}}_1 \cdot \mathbf{\tilde{E}}^{\mathbf{i}}(\mathbf{Q}_1))$$
(1)

where $D^{S}(Q_{1})$ is the scalar diffraction coefficient for a soft boundary condition. The terms hard and soft boundary conditions are terms from acoustics. A soft boundary is one where the preasure field is zero at the boundary ($\overline{E} = E_{z}\hat{z}$). A hard boundary condition is one in which the derivative in the direction normal to the boundary is zero ($\overline{E}/n = 0$; $\overline{H} = H_{z}\hat{z}$).

The amplitude of the surface ray is assumed to be governed by the conservation of energy between a pair of adjacent surface rays. This principal states that in time harmonic fields, the energy flux is the same through every cross section of a tube of rays, such as the astigmatic tube of rays in Fig. 4.



Fig. 4. Astigmatic Bundle of Rays

Let A^2 be the intensity of the field at S and A_o^2 , the intensity at the reference point O. For an isotropic, homogeneous medium

$$\mathbf{A}^2 \mathbf{d}\boldsymbol{\sigma} = \mathbf{A}^2 \mathbf{d}\boldsymbol{\sigma} \tag{2}$$

where do is the cross section of the tube of rays at S and do is the cross section at 0. Therefore

$$A = A_0 \sqrt{d\sigma_0/d\sigma}$$
(3)

If s is the distance from 0 to S, then

$$\frac{d\sigma_0}{d\sigma} = \frac{\rho_1 \rho_2}{(\rho_1 + s)(\rho_2 + s)}$$
(4)

where ρ_1 and ρ_2 are the principal radii of curvature of the wavefront through 0.

Applying the energy conservation principle to a narrow strip of rays on a curved surface (see Fig. 5) gives

$$A^{2}(\boldsymbol{l} + d\boldsymbol{l})d\boldsymbol{\eta}(\boldsymbol{l} + d\boldsymbol{l}) - A^{2}(\boldsymbol{l})d\boldsymbol{\eta}(\boldsymbol{l}) = -2\boldsymbol{q}(\boldsymbol{l})A^{2}(\boldsymbol{l})d\boldsymbol{\eta}(\boldsymbol{l})d\boldsymbol{l}$$
(5)

where $2\mathfrak{q}(\boldsymbol{l})$ is a proportionality factor relating the change of flux , to the energy radiated. Dividing by dland letting dl tend to zero gives

$$\frac{d}{d\chi}(A^2d\gamma) = -2\alpha(A^2d\gamma)$$
(6)



Fig. 5. A Band of Diffracted Rays on the Surface of a Diffracting Object

Upon integrating Eq. 6, the result is

$$A(Q_2) = A(Q_1) \sqrt{\frac{d\eta_1}{d\eta_2}} \exp\left[-\int_{Q_1}^{Q_2} d(\boldsymbol{k}) d\boldsymbol{k}\right]$$
(7)

where dq_1 and dq_2 = the seperation between a pair of rays at Q_1 and Q_2 respectively. $\alpha(l)$ = the attenuation constant which is a function of

> A and the coordinates along the surface and depends on the local radius of curvature and its derivatives.

From Fig. 3, Q_2 is a caustic of the diffracted field and the second coustic is located a distance ρ from Q_2 . The caustic of a given family of rays is defined as that surface or curve which is everywhere tangent to the ray surface. Thus the surface of a diffracting body is a caustic of the family of diffracted rays resulting from a given field being incident on the body. Thus the binormal component of the diffracted field which radiates from Q_2 towards P can be found using a caustic at Q_2 for a reference point as

$$\hat{b}_{2} \cdot \vec{E}^{d}(P) = D^{s}(Q_{2})A(Q_{2})exp(j Q_{2})\sqrt{\frac{\rho}{s(\rho + s)}}exp(-jks)$$
(8)

where $D^{S}(Q_{2})$ is a diffraction coefficient of the soft boundary. Based on the reciprocity principle--that a source at O produces the same field at P as a source at P produces at O-- $D^{S}(Q_{2})$ has the same form as $D^{S}(Q_{1})$. Combining Eqs. (1), (7), and (8) yields

$$\hat{\mathbf{b}}_{2} \cdot \mathbf{\bar{E}}^{d}(\mathbf{P}) = (\hat{\mathbf{b}}_{1} \cdot \mathbf{\bar{E}}^{i}(\mathbf{Q}_{1}))\mathbf{D}^{2}(\mathbf{Q}_{1})\mathbf{D}^{\mathbf{S}}(\mathbf{Q}_{2})\sqrt{\frac{d\eta_{1}}{d\eta_{2}}\sqrt{\frac{\rho}{s(\rho+s)}}}$$
$$\cdot \exp\left[-jk(\boldsymbol{l}+s) - \int_{\mathbf{Q}_{1}}^{\mathbf{Q}_{2}} \mathbf{d}^{s}(\boldsymbol{l})d\boldsymbol{l}\right]$$
(9)

The incident field $\hat{b}_1 \cdot \tilde{E}^i(Q_1)$ excites an infinity of surface ray modes, each with its own diffraction coefficient and attenuation coefficient. For example, in the canonical problem in Appendix A, if the field scattered from a cylinder given by Eq. (58) is approximated asymptotically, it is seen that there is a surface ray mode associated with each term of the series, i.e., with each value of m. Therefore, Eq. (9) is replaced by

$$\hat{\mathbf{b}}_{2} \cdot \tilde{\mathbf{E}}^{d}(\mathbf{P}) = (\hat{\mathbf{b}}_{1} \cdot \tilde{\mathbf{E}}^{i}(\mathbf{Q}_{1})) \sqrt{\frac{d\mathbf{\gamma}_{1}}{d\mathbf{\gamma}_{2}}} \sqrt{\frac{\mathbf{\rho}}{\mathbf{p}(\mathbf{\rho}+s)}} \exp(-jk(\mathbf{l}+s))$$
$$\cdot \sum_{m=1}^{\infty} p_{m}^{s}(\mathbf{Q}_{1}) p_{m}^{s}(\mathbf{Q}_{2}) \exp\left[-\int_{\mathbf{Q}_{1}}^{\mathbf{Q}_{2}} \mathbf{d}_{m}^{s}(\mathbf{l}) d\mathbf{l}\right]$$
(10)

This equation relates the diffracted field at P to the incident field at Q_1 for the soft boundary condition.

An expression similar to Eq. (10) is also obtained for the normal component of the incident field:

$$\hat{\mathbf{h}}_{2} \cdot \tilde{\mathbf{E}}^{d}(\mathbf{P}) = (\hat{\mathbf{h}}_{1} \cdot \tilde{\mathbf{E}}^{i}(\mathbf{Q}_{1})) \sqrt{\frac{d\gamma_{1}}{d\gamma_{2}}} \sqrt{\frac{\rho}{s(\rho+s)}} \exp(-jk(\boldsymbol{l}+s))$$
$$\cdot \sum_{m=1}^{\infty} p_{m}^{h}(\mathbf{Q}_{1}) p_{m}^{h}(\mathbf{Q}_{2}) \exp\left[-\int_{\mathbf{Q}_{1}}^{\mathbf{Q}_{2}} \mathbf{x}_{m}^{s}(\boldsymbol{l}) d\boldsymbol{l}\right]$$
(11)

In this case, the scalar diffraction coefficients and attenuation constants for the hard surface diffraction replace those of the soft surface. The vector diffracted field at P can now be written in terms of the electromagnetic field incident at Q_1 as

$$\overline{\mathbf{E}}(\mathbf{P}) = (\widehat{\mathbf{n}}_{2}\widehat{\mathbf{n}}_{1}\mathbf{v}(\mathbf{Q}_{1},\mathbf{Q}_{2}) + \widehat{\mathbf{b}}_{2}\widehat{\mathbf{b}}_{1}\mathbf{u}(\mathbf{Q}_{1},\mathbf{Q}_{2})) \cdot \overline{\mathbf{E}}^{i}(\mathbf{Q}_{1})\sqrt{\frac{\rho}{s(\rho+s)}}\exp(-jks)$$
(12)

where $v(Q_1,Q_2)$ and $u(Q_1,Q_2)$ are equal to

with the superscripts h and s, respectively added to D_m and σ_m . The $d\eta_1$, $d\eta_2$ and ρ are found by differential geometry involving the rays and surface; this is discussed at length in Levy and Keller (Ref 15).

To obtain the diffraction and attenuation constants, the exact canonical solution of the boundary value problem for the diffraction in question must be known. Such a solution is expanded asymptotically for ka large where a is the radius of curvature for cylindrical diffraction. By comparing this expansion with the GTD solution, the values of d_m and D_m can be determined. Appendix A contains a derivation of the generalized diffraction coefficient and attenuation coefficient for diffraction by curved surfaces.

The diffraction thus far discussed is applied to an open curved surface. For a closed surface, each surface ray mode produced at Q_1 encircles the surface an infinite number of times. The length of the surface ray path for the <u>nth</u> encirclement is l + nTwhere T is the distance traveled in one transversal of the closed surface. These multiple encircling rays may be summed to contribute

$$1 - \exp\left[-jkT - \int_{0}^{T} \mathcal{A}(\boldsymbol{l})d\boldsymbol{l}\right]$$
(14)

to the denominator of the diffracted field. Note for this derivation that the exp(jkz) convention has been adopted.

Direct Rays

The rays emanating from a point current element with current moment IAx determines the incident field for the diffracted and reflected rays as well as the field at all points where Fermat's principle produces a straight line from the source.

Referring to Fig. 6, A is defined as (Ref 19: 16)

$$A_{x} = \frac{I_{o} \Delta x \exp(-jks')}{4\pi s'}$$
(15)

Applying Maxwell's equations

$$\vec{H} = \nabla X A_{x} \hat{x} = \nabla A_{x} X \hat{x}$$
(16)

$$= \frac{I\Delta x}{4\pi} \nabla \frac{\exp(-jks')}{s'} \vec{x} \hat{x}$$
(17)

$$\overline{H} = \frac{I\Delta x}{4\pi} \left[\frac{-jk(exp(-jks'))}{s'} - \frac{1}{s'^2} exp(-jks') \right] \widehat{r} \chi \widehat{x}$$
(18)



Fig. 6. Geometry for current source at origin

Since

$$\hat{\mathbf{r}} \times \hat{\mathbf{x}} = \hat{\mathbf{r}} \times (\hat{\mathbf{r}}\cos\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}\sin\boldsymbol{\theta}) = -\hat{\boldsymbol{\theta}}\sin\boldsymbol{\theta}$$
 (19)

then

$$\overline{H} = \frac{I \Delta x}{4 \pi} \left[\frac{jk}{s'} + \frac{1}{s'^2} \right] \exp(-jks') \sin \Theta \phi \qquad (20)$$

In the far zone field the l/s' term is dominant, the l/s'² term tending to zero much faster than l/s' therefore it can be neglected. The electric field is obtained from

$$\overline{E} = \frac{1}{j\omega\epsilon} \nabla X \overline{H}$$
(21)

, to give

$$\overline{E} = \frac{I \Delta x}{4\pi} \exp(-jks') \left[\frac{j\omega\mu}{s'} + \sqrt{\frac{\mu}{\epsilon}} \frac{1}{s'^2} \right] \sin\theta \hat{\theta} + \sqrt{\frac{\mu}{\epsilon}} \left[\frac{\mu}{\epsilon} \frac{1}{s'^2} \right] \cos\theta \hat{r}$$
(22)

Again, in the far zone field the 1/s' term is dominant and the direct electric field becomes

$$\mathbf{E} = \frac{\mathbf{I} \Delta \mathbf{x}}{4\pi} \exp(-jks') \frac{j\omega\mu}{s'} \sin \Theta \hat{\Theta}$$
(23)

Launching Coefficients and Constants

The theory presented up to now has assumed a point source near a cylinder. Monopoles mounted on a cylinder could be modeled as a line of point sources with sinusoidally distributed amplitudes, using this theory. The far-zone field would then be the result of vectorially summing the fields from each source point at each field point. The result is a large number of ray tracings and field calculations for even a simple antenna array.

To simplify the calculations, the sum of the incident fields due to all of the point sources making a monopole model are approximated by a launching coefficient. To determine the launching coefficients, the radiation from an axial mounted electric current source on a perfectly conducting cylinder is analyzed. The asymptotic solution of this canonical problem is described in Appendix B, where it is found that

$$L_{m}^{s} = -(\frac{jk}{2})^{1/2} H_{m}^{(2)}(ka) D_{m}^{s}(Q_{1})$$
(23)
$$L_{m}^{h} = -j(\frac{jk}{2})^{1/2} H_{m}^{(2)}(ka) D_{m}^{h}(Q_{1})$$

where ν_m are the zeroes of the Hankel function in the first equation and the zeroes of the derivative of the Hankel function in the

second equation. Note that Q_1 is now the source location. These launching coefficients are valid for monopoles whose length is small in comparison to the dimensions of the scatterers.

Eq. (12) now becomes

$$\overline{E}(P) = (\widehat{n}_2 v(Q_1, Q_2) + \widehat{b}_2 u(Q_1, Q_2)) \sqrt{\frac{\rho}{s(\rho + s)}} exp(-jks)$$
(24)

where $v(Q_1,Q_2)$ and $u(Q_1,Q_2)$ are equal to

$$\sqrt{\frac{d\eta_1}{d\eta_2}} \exp(-jkl) \sum_{m=0}^{\infty} L_m(Q_1) D_m(Q_2) \exp\left[-\int_0^{Q_2} \mathcal{A}_m(\boldsymbol{\ell}) d\boldsymbol{\ell}\right]$$
(25)

with the superscripts h and s, respectively added to D_m , L_m , and d_m .

Diffraction Coefficients and Attenuation Constants

The diffraction coefficients and attenuation constants depend on the local geometry of the surface, the wave number k, and the nature of the surface, as described by the boundary conditions. Voltmer (Ref 20) has derived the diffraction coefficients and attnuation constants for general convex surfaces and are included in Table I. Appendix A contains the derivation of D_m and d_m for a cylinder.

To first order these functions are

$$\alpha_{m}^{s} = \frac{1}{\rho_{g}} \left(\frac{k\rho_{g}}{2}\right)^{1/3} q_{m}^{exp(j\pi/6)}$$
(26)

$$(D_{m}^{s})^{2} = \frac{\exp(-j\pi/12)}{2^{5/6}\pi^{1/2}(k\rho_{g})^{1/6}} \frac{\rho_{g}^{1/2}}{(A_{i}'(-q_{m}))^{2}}$$
(27)

TABLE, I

GEHERAL IZED DIFFRACTION COEFFICIENTS AND ATTENUATION CONSTANTS

ATTENUATION CONSTANT ame (Column C) · (Column D)	a Assult D. Correction Terms	$\left[\frac{1}{r^2} \right]^{1/3} \left[1 + \left(\frac{2}{k_0^2} \right)^{2/3} q_m \left(\frac{1}{60} - \frac{2}{45} \rho_g g_0 + \frac{4}{125} \rho_g^2 \right) e^{-\frac{1}{3}r/3} \right]$	$\frac{1}{a_{\rm A}} + \left(\frac{2}{b_{\rm B}^2}\right)^{2/3} \left(\overline{q_{\rm B}} \left(\frac{1}{b_{\rm B}} - \frac{2}{45} g_{\rm B} g_{\rm B} g_{\rm B} + \frac{4}{135} g_{\rm B}^2\right) + \frac{1}{\overline{q}_{\rm B}^2} + \frac{1}{\overline{q}_{\rm B}^2} \left(\frac{1}{10} + \frac{g_{\rm B}}{4\rho_{\rm LM}} - \frac{g_{\rm B}^2}{50}g_{\rm B}^2 + \frac{3}{50}g_{\rm B}^2\right) - \frac{1}{3} - \frac{1}{3}$	$ \left\{ \begin{array}{l} \left\{ \frac{2}{8\rho_0} \right\}^{2/3} \left\{ \overline{q}_{m} \left\{ \frac{1}{8\rho_0} - \frac{2}{45} \rho_{g} \mu_{g} + \frac{4}{135} \rho_{g}^{2} \right\} \\ \left. + \left(\frac{2}{8\rho_0} \right)^{2/3} \left\{ \overline{q}_{m} \left\{ \frac{1}{8\rho_0} - \frac{2}{4\rho_0} \eta_{g} + \frac{4}{90} \right\} \right\} \\ \left. + \frac{1}{\overline{q}_{m}^{2}} \left(1\overline{\eta} - \frac{2}{4\rho_0} \eta_{0} - \frac{2}{90} \eta_{g}^{2} + \frac{4}{90} \right) \right\} e^{-\frac{1}{3}\sigma/3} $
	C. Keller's	9	9 	
DIFFAACTION COEFFICIENT Column A) - (Column B)	B. Correction Terms	$1 + \left(\frac{2}{k_0 g}\right)^{2/3} q_m \left(\frac{1}{30} + \frac{p_q}{4^{p} t_0} + \cdots\right) e^{-\frac{1}{2} e/3}$	$1 + \left(\frac{2}{k_0}\right)^{2/3} \left(\overline{q}_{pq} \left(\frac{1}{30} + \frac{p_{q}}{4^{0}t_{11}} + \cdots + \frac{p_{q}}{10}\right) - \frac{1}{3^{1/3}}\right)$ $- \frac{1}{\overline{q}} \left(1\overline{0} + \frac{p_{q}}{4^{0}t_{11}} + \cdots + \frac{1}{3^{1/3}}\right) e^{-\frac{1}{3^{1/3}}}$	$1 \cdot \left(\frac{2}{k_{0}}\right)^{2/3} \left(\tilde{q}_{0}\left(\frac{1}{20} + \frac{2}{4}e_{10} + \cdots\right)\right)$ $- \frac{1}{\tilde{q}_{m}^{2}} \left(\frac{1}{10} - \frac{2}{4}e_{10} + \cdots\right) e^{-\frac{2}{3}e/3}$
square of D ² = (c	A. Keller's Result	-1/22-5/6 1/3e-1=/12 k ^{1/6} (A1'(-9 _n)) ²	51/2-6/1 _2/2	1/10
Surface		Soft Acoustic and Soft EM	Nard Acoustic	F 14

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e radius of curvature along the geodesic
 et radius of curvature perpendicular to the geodesic (transwerse curwe)
 buts indicate differentiation with respect to arc length variable

for the "soft" surface, where ρ_g is the radius of curvature of the surface along which the surface ray is propagating in the plane containing the normal and tangent to the surface ray, and

$$\boldsymbol{\alpha}_{\mathrm{m}}^{\mathrm{h}} = \frac{1}{\rho_{\mathrm{g}}} \left(\frac{\mathrm{k}\rho_{\mathrm{g}}}{2} \right)^{1/3} \bar{q}_{\mathrm{m}}^{\mathrm{exp}(\mathrm{j}\pi/6)}$$
(28)

$$(D_{\rm m}^{\rm h})^2 = \frac{\exp(-j\pi/12)}{2^{5/6}\pi^{1/2}(k\rho_{\rm g})^{1/6}\bar{q}_{\rm m}} \frac{\rho_{\rm g}^{1/2}}{(A_{\rm i}(-\bar{q}_{\rm m}))^2}$$
(29)

for the hard surface, where the Miller-type Airy function is given by

$$A_{i}(-x) = \frac{1}{\pi} \int_{0}^{\infty} \cos(\frac{1}{3}t^{3} - xt) dt$$
 (30)

$$\mathbf{A}_{i}(-\mathbf{q}_{m}) = 0 \tag{31}$$

$$\mathbf{A}_{i}^{\dagger}(-\mathbf{\bar{q}}_{m}) = 0 \tag{32}$$

where q_m and \bar{q}_m are the zeroes of A_i and A_i ' respectively with the prime denoting differentiating with respect to the argument of the function. These zeroes are calculated in Appendix C.

The terms "hard" and "soft" refer to the boundary conditions. A soft boundary condition is a Direchlet boundary (the field vanishes at the surface). A hard boundary is a Neumann boundary condition (the normal derivative of the field vanishes at the surface). This assumes two types of surface ray modes; one in which $\vec{E} = \hat{b}\vec{E}$, satisfying a soft boundary condition, and one in which $\vec{E} = \hat{n}\vec{E}$, satisfying the hard boundary condition. The corresponding coefficient approximations are essentially what Keller and Levy (Ref 15:166-169) derived. Table I contains Voltmer's

corrected results.

Transition Regions

The series representations are rapidly convergent when the field point is deep in the shadow region. When the local average radius of curvature of the surface is larger than a wavelength at the points of diffraction, only the <u>lst</u> terms are neccessary to achieve good accuracy. As the field diffraction point Q_2 approaches Q_1 (i.e. I becomes very small), more terms must be added to maintain accuracy and it becomes inconvenient to treat the excitation, propagation and diffraction of the surface ray seperately. The series representation can now be approximated by integral representations, which have been found to be proportional to Fock type functions. When the source is on the surface, the angular extent of the transition region from the shadow boundary is nearly $(k \rho_g)^{-1/3}$ radians. The expressions for the fields in the transition region are also deduced from the asymptotic solutions to appropriate canonical problems as described in Appendix E.

The well tabulated Fock functions (Ref 21) are

1) Hard Fock function

$$g(\boldsymbol{\xi}) = \frac{1}{\sqrt{\pi}} \int_{\Gamma_1} \frac{\exp(-j\boldsymbol{\tau}\boldsymbol{\xi})}{w_2'(\boldsymbol{\tau})} d\boldsymbol{\tau}$$
(33)

2) Soft Fock function

$$\widetilde{\mathcal{E}}(\boldsymbol{\xi}) = \frac{1}{\sqrt{\pi}} \int_{\boldsymbol{\Gamma}_1} \frac{\exp(-j\boldsymbol{\tau}\boldsymbol{\xi})}{w_2(\boldsymbol{\tau})} d\boldsymbol{\tau}$$
(34)

which are described in Appendix E and tabulated in Appendix G. The argument of the Fock function to the first order is

$$\boldsymbol{\xi} = \int_{Q_1}^{Q_2} \frac{1}{\rho_g} \left(\frac{k\rho_g}{2}\right)^{1/3} d\boldsymbol{k}$$
(35)

When the field point is in the shadow region, $\lambda > 0$ and when the field point is in the illuminated portion of the transition region, $\lambda < 0$, where λ is the distance traveled on the surface of the cylinder. Note that for these expressions that the surface ray divergence factor $d\eta_1/d\eta_2$ is equal to one in this region.

In the transition region, for sources very close to the surface the term

$$\sum_{m=1}^{\infty} L_m^h(Q_1) D_2^h(Q_2) \exp\left[-\int_{Q_1}^{Q_2} d_m(\boldsymbol{l}) d\boldsymbol{l}\right]$$
(36)

in Eq. (25) is replaced for $v(Q_1,Q_2)$ by

$$\begin{cases} g(\boldsymbol{\varsigma}), & \boldsymbol{\lambda} \leq 0 \\ g(\boldsymbol{\varsigma})\boldsymbol{\delta}, & \boldsymbol{\lambda} \geq 0 \end{cases}$$
(37)

and for $u(Q_1, Q_2)$ by

$$j(\hat{n}_{1} \cdot \hat{s}/\xi)g(\xi), \quad \ell \leq 0$$

-jg(\xi) $\delta/m, \quad \ell \geq 0$ (38)

$$m = \left(\frac{k\rho_{g}(Q_{1})}{2}\right)^{1/3}$$
(38)

 $\delta = (\rho_{g}(Q_{2})/\rho_{g}(Q_{1}))^{1/6}$ (39)

where

III. <u>Diffraction</u> by an <u>Infinitely Long Elliptic Cylinder</u>

The special case of the GTD solution treated here is one in which $1/4\lambda$ monopole antennas are mounted on the curved surface of an infinitely long, 2λ by 4λ elliptical cylinder. An average wavelength of 1 meter is assumed for this problem.

The particular array to be considered is depicted in Fig. 1. To solve the field problem, each monopole is approximated by a launch coefficient. The field at the field point P will be the sum of the fields produced by each source.

Ray Approximations

Previous work by Pathak and Luebbers (Ref 22) has shown that in the illuminated region (P_1 , see Fig. 7), the diffracted rays s_1 and s_2 have a negligable amplitude compared to the direct ray s_0 . For point sources on the cylinder, the field can be modeled as only ray s_0 and Eq. (23).

In the shadow region only the diffracted rays s_1 and s_2 contribute to the field at P_3 . Since the rays leaving Q_1 (or 0) and encircling the cylinder n times, cover a cylinder circumference of about nine meters, they are heavily attenuated and their effect is very small. The field at P_3 is then the sum of the rays $Q_1Q_2P_3$ and $Q_1Q_3P_3$ applied to Eq. (24).

In the transition region, the integral form in Eq. (36) and (37) are used to solve for the field at F_2 , again simplified by ignoring those rays which encircle the cylinder one or more times.



Note that for s_1 the diffraction field formula in Eq. (23) is used. For the equations in all of these regions, the spread factor $d\eta_1/d\eta_2$ is one.

Field Equations

The GTD solutions for the radiated electric field of a point source on an elliptical cylinder, as shown in Fig. 8, with all amplitudes combined into A and neglecting torsional effects are

Lit Region

$$\mathbf{E} = -\mathbf{A} \sin(\boldsymbol{\Theta}_{\mathbf{m}}) \exp(-\mathbf{j}(\mathbf{ks'} + \boldsymbol{\varphi}) \widehat{\boldsymbol{\Theta}}$$
(40)

Transition Region

a) Lit side

$$\mathbf{E} = \frac{A}{2} \sin(\boldsymbol{\Theta}_{\mathrm{m}}) \exp\left[-j(\mathbf{k}(\mathbf{s} - \boldsymbol{\lambda}) + \boldsymbol{\varphi}\right] \mathbf{g}^{*}(-\boldsymbol{\xi})$$
(41)

b) Shadow side

$$E = \frac{A}{2} \exp\left[-j(k(s + l) + q)\right] g^{*}(\xi) \left(\frac{f_{g}(Q_{2})}{f_{g}(Q_{1})}\right)^{1/6} \hat{n} \qquad (42)$$

Deep Shadow Region

1

$$E = \sum_{j} A \exp[-j(k(s + l) + \varphi)] \hat{n}_{j}$$

$$\cdot \sum_{m=1} \exp\left[-\int_{Q_{1}}^{Q_{2}} M_{m}(l) dl\right] L_{m}^{h}(Q_{1}) D_{m}^{h}(Q_{2}) \qquad (43)$$



Fig 8. Geometry of monopole antenna mounted on an infinitely long elliptic cylinder

Where	
g*(•)	complex conjugate of the Fock function (Ref 21)
n, b, t	normal, binormal and tangent unit vectors to
	the surface
ſs	longitudinal radius of curvature
5	defined in Eq. (35)
q	phase of source
A	amplitude of source
s '	distance from source to diffraction point or
	field point if cylinder is not intersected.
S	distance from diffraction point Q_2 to field
	point

Geometry of Elliptical Cylinder (Equations)

In order to analyze this problem by GTD, an efficient solution for the geodesic paths on the elliptic cylinder must be found. The coordinate system adopted here is illustrated in Fig. 9 and defined by

 $x = d \cosh u \cos v = a_{f} \cos v$ $y = d \sinh u \sin v = b_{f} \sin v$ z = z(44)

where 2d is the distance between the foci of the ellipse. To define an ellipse, u must equal $\tanh^{-1}(b_{f}/a_{f})$. The coordinate v then defines any point on the ellipse and varies from 0 to 2π .

Using the calculus of variations, the geodesic paths on an elliptic surface are given by

$$z = \frac{c}{\sqrt{1 - c^2}} \int_{v_i}^{v_f} \sqrt{a_f^2 \sin^2 v + b_f^2 \cos^2 v} \, dv$$
 (45)

where v_i and v_f are the equivalent of Q_1 and Q_2 respectively in this parametric form.



P. A.

Fig. 9. Elliptic Cylinder Coordinate System (Ref 14:26) If one defines the geodesic starting direction by the angle (\mathbf{x}_s) as shown in Fig. 8, then C = -cos \mathbf{x}_s . The parameters of the problem are

$$z = \frac{-\cos d_s}{|\sin d_s|} \int_{v_i}^{v_f} \sqrt{a_f^2 \sin^2 v + b_f^2 \cos^2 v \, dv}$$
(46)
$$I = \frac{1}{\sin \alpha_{s}} \int_{\mathbf{v}_{i}}^{\mathbf{v}_{f}} \sqrt{a_{f}^{2} \sin^{2} \mathbf{v} + b_{f}^{2} \cos^{2} \mathbf{v} d\mathbf{v}}$$
(47)

$$b_{1} = \frac{-a_{f} \sin v \hat{x} + b_{f} \cos v \hat{y}}{\sqrt{a_{f}^{2} \sin^{2} v + b_{f}^{2} \cos^{2} v}}$$
(48)
(48)

 $\hat{\mathbf{e}}_2 = \hat{\mathbf{z}}$ (49)

 $\hat{t} = \sin \hat{s} \hat{e}_1 - \cos \hat{s} \hat{e}_2$ (unit tangent (50) vector)

$$\hat{\mathbf{n}} = \frac{\mathbf{b}_{f} \cos \mathbf{v} \hat{\mathbf{x}} + \mathbf{a}_{f} \sin \mathbf{v} \hat{\mathbf{y}}}{\sqrt{\mathbf{a}_{f}^{2} \sin^{2} \mathbf{v} + \mathbf{b}_{f}^{2} \cos^{2} \mathbf{v}}} \qquad (unit normal (51))$$
vector)

$$\hat{\mathbf{b}} = \hat{\mathbf{t}} \times \hat{\mathbf{n}} = -\cos \boldsymbol{\mathscr{A}}_{\mathbf{s}} \hat{\mathbf{e}}_{1} - \sin \boldsymbol{\mathscr{A}}_{\mathbf{s}} \hat{\mathbf{e}}_{2}$$
 (unit binormal (52)
vector)

$$g = \frac{\left(a_{f}^{2} \sin^{2} v + b_{f}^{2} \cos^{2} v\right)^{3/2}}{a_{f}^{b} \sin^{2} \alpha_{s}}$$
(53)

radius of curvature)

Using the preceeding Eqs. (40)-(53), the total radiated field for a point source on a cylinder can be determined.

IV. Pattern Calculations

The field equations (40) - (53) are used in the following figures to calculate the field due to the array in Fig. 1. The tabulated fields due to each monopole are included in Appendix G.

To verify the general GTD equations derived for a monopole, the antenna pattern for a single monopole on the cylinder was calculated and compared to experimental results obtained from published literature (Ref 3:662). Fig. 10 contains a plot of both the experimental data and the calculated values. The variation of the experimental field is do to interference from the rays that encircle the cylinder more than once. It was assumed in the calculations that for fields not in the deep shadow region, that only the dominant term would be considered.

Fig. 11 and Fig. 12 (Ref 14:64) provide a means of comparison of the calculated values with those obtained from a scale model test of an aircraft. Fig. 10 and Fig. 11 are almost identical except that the theoretical case exhibits a stronger field in the deep shadow region. The effect of wings (Fig. 12) on the aircraft causes an even further degradation of field strength in the shadow region. The calculated antenna patterns are therefore valid as a good approximation for \bullet within the range of $\pm 105^{\circ}$ with respect to the antenna when the antenna is mounted near the top or bottom of the fuselage.

Fig. 13 is a plot of the antenna pattern of antenna 2 or 3. Do to symmetry, their patterns are reflections of each other. The effect of mounting the antenna off of the main axis of the cylinder is to shift the nulls of the pattern as well as cause sharper

changes in the pattern where the minima of the radius of curvature of the cylinder occur. Note that the null has shifted about 26° from its position in Fig. 10 although antenna 2 and 3 are mounted perpendicular to the cylinder surface at a point that is about 14° off of the primary axis of the ellipse ($v_i \sim 14^{\circ}$). This shift is caused by the antenna angle in space being about 26° from the verticle.

Fig. 14 is a plot of the antenna pattern with all three elements of the array driven with the same amplitude and phase. Comparing this pattern to that of a 3-element triangular array depicted in Fig. 15, driven under the same conditions, shows that the cylinder has the effect of broadening the lobes as well as increasing the angle • where the pattern maxima occurs. The result is a pattern more like that of a single monopole or dipole in space.

Fig. 16 is a plot of the antenna array pattern with element 1 driven 45° out of phase with elements 2 and 3. It takes very little phase change to drastically alter the pattern. The effect of the out of phase element is to focus more of the energy in the lit region of the cylinder. By changing elements 2 or 3 such that they are out of phase with each other should eliminate the null at e = 0 and also allow the lobes to be moved or combined into a narrower beam as well as move the nulls.



Fig 10. Antenna pattern of monopole on major axis of elliptical cylinder (calculated and experimental)

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Fig. 11. Roll plane pattern (E_4) for a 1/25 scale model of a KC-135 with a $\lambda/4$ monopole on the fuselage forward of the wings. (from Ref 14:64)



Fig. 12. Roll plane pattern (Eq) for a $\lambda/4$ monopole above the wings.(from Ref 12:99)



Fig. 13. Antenna pattern of monopole mounted at $\Theta = 14^{\circ}$ on a 2 λ by 4 λ elliptical cylinder.

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Fig. 14. Antenna pattern of array in Fig. 1 with all elements driven in phase

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Fig. 15. Antenna pattern of 3 element array on an infinite ground plane, all elements driven in phase.



and the second

Fig. 16. Antenna pattern of array in Fig. 1 with element 1 driven 45° out of phase with elements 2 and 3.

V. Conclusions

The geometrical theory of diffraction has been extended to treat monopoles on perfectly conducting cylinders of variable curvature. Following Pathak's approach (Ref 23:19-20) of defining a launch coefficient for a slot in a cylinder, a launch coefficient for a monopole was obtained. For the monopole case, the launch coefficients are the same as for an axially mounted and a length wise mounted slot superimposed on each other.

By carefully keeping track of the distance traveled by the diffracted rays an interference pattern for two or more rays of two or more sources was obtained. If the field pattern is analyzed in such a way as to let the observation distance approach infinity, then the phase could be approximated as the phase at the diffraction point for the ray leaving the cylinder (Q_2) . In the illuminated region the phase would be assumed to be that of the source. Although this approximation is valid it was assumed for this problem that since this antenna is to serve as an air to air communications link that the range was not infinite. A nominal distance between communication points of 300 meters was assumed such as to account for aircraft in close proximity to each other. The field further out will just have a smaller ripple on the lobe boundary due to the phases of the wavefronts being nearer the same as a function of their distance allowing the previous approximation to be valid.

The effect of the cylinder is to broaden the beamwidth in the principle plane. By mounting the antenna perpendicular to the surface, the condition of the antennas not being parallel in the x, y

plane resulted. Linear array theory was therefore not valid (i.e., an array factor is not present). If the antenna were mounted parallel to each other, the effect would be similar (in the lit region) to tilting a monopole on a flat ground plane so that one side of the beam is directed into instead of along the ground plane. Again, linear antenna theory would not apply. If the field point is in the deep shadow region of all three elements of the array then the result is an equivalent linear array summation at the diffraction point Q_2 but it is only valid for that field point producing a tangent at Q_2 . It can also be assumed that the angle with which the antenna is attached to the surface and not its angle with respect to the center of the cylinder controls the location of the nulls.

The array analyzed provides a broad beam for communication problems but with high mobility of nulls for a null steering use. Due to the creeping wave effect, it appears it would be very hard to create a single beam in one general direction. The symmetrical cylinder would tend to cause multiple lobes as in Fig. 16. An end fire or broadside pattern in the x, y plane is not readily obtainable because the elements are not parallel. However when analyzing the fields in the x, z plane ($\alpha_s = 0$, 180) the antenna elements appear to be alinear array with antenna 2 and 3 reduced in height. Planes of view in between ($0 < \alpha_s < 90$) would be a combination of the two approachs although the GTD can still be applied directly and obtain valid results.

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Appendix A

Derivation of
$$D_m$$
 and $\boldsymbol{\prec}_m$

The first step is to formulate and solve the boundary value problem of diffraction of a cylindrical wave by a circular cylinder of radius a. Then the solution will be expanded asymptotically for large values of ka.

In order to simplify computations and avoid repetition, a scalar field is assumed with conditions stated to allow for hard and soft boundary conditions. This scalar field $u(r,\Theta)$ must satisfy the equation

$$(\nabla^2 + k^2)u = \delta(r - \rho)\delta(\Theta)$$
 (54)

Two different boundary conditions at r = a will be considered. They correspond to different physical problems.

By expanding the diffracted field in cylindrical wave functions and imposing the boundary conditions (Ref 24:Ch. 5) the field for the ray OQ_1Q_2P as shown in Fig. 17 becomes

$$u = \frac{-j}{4} \sum_{m=-\infty}^{\infty} \exp(-jm\theta) \left[J_{m}(kr_{\zeta}) - \frac{GJ_{m}(ka)}{GH_{m}^{(2)}(ka)} H_{m}^{(2)}(kr_{\zeta}) \right] H_{m}^{(2)}(kr_{\zeta})$$
(55)

Here $r_{>}$ and $r_{<}$ are, respectively, the larger and smaller of the quantities r and ρ .



Fig. 17. Geometry for Diffraction

The operator G, which is determined by the boundary conditions as given in Table II.

Table II

The Operator G

A. A.

		G	Boundary condition		Field Equivalent
Case	I:	1	u = 0	S	u = E _z ; perfectly conducting cylinder
Case	11:	du dn	$\frac{\partial u}{\partial n} = 0$	h	u = H _z ; perfectly conducting cylinder

The representation of the field as given by Eq. (55) is valid only for small values of ka. A numerically useful result for large values of ka is obtained by employing the well-known Watson transformation which leads to an asymptotic expansion of the field.

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Accessible descriptions of the method are given in References 25 and 26.

Letting C be a contour encircling the real axis as shown in Fig. 18, the sum in Eq. (55) becomes

$$u = -\frac{1}{8} \int_{C} \frac{\exp(j\mathcal{V}(\Theta - \pi))}{\sin(\mathcal{V}\pi)} \frac{H_{\mathcal{V}}^{(2)}(kr_{\mathcal{V}})}{GH_{\mathcal{V}}^{(2)}(ka)}$$

$$(GH_{\mathcal{V}}^{(2)}(ka)J_{\mathcal{V}}(kr_{\mathcal{V}}) - GJ_{\mathcal{V}}(ka)H_{\mathcal{V}}^{(2)}(kr_{\mathcal{V}}))d\mathcal{V}$$
(56)

When γ is replaced by $-\gamma$ on that part of the contour for which $q_m < 0$, Eq. (56) becomes

$$u = -\frac{1}{4} \int_{D} \frac{\cos(\mathcal{V}(\boldsymbol{\theta} - \boldsymbol{\pi}))}{\sin(\mathcal{V}\boldsymbol{\pi})} \frac{H_{\mathcal{V}}^{(2)}(kr_{\mathcal{V}})}{GH_{\mathcal{V}}^{(2)}(ka)}$$

• $(GH_{\boldsymbol{\nu}}^{(2)}(ka)J_{\boldsymbol{\nu}}(kr_{\boldsymbol{\zeta}}) - GJ_{\boldsymbol{\nu}}(ka)H_{\boldsymbol{\nu}}^{(2)}(kr_{\boldsymbol{\zeta}}))d\boldsymbol{\nu}$ (57)





If the point (r, Θ) lies in the geometrical shadow, the contour D can be closed in the lower half-plane. The integral can now be evaluated by computing the residues at the zeroes $V_{\rm m}$ of ${\rm GH}_{V}^{(2)}({\rm ka})=0$. This yields

$$u = -\frac{\pi j}{4} \sum_{m=1}^{\infty} \frac{\cos(\mathcal{V}_{m}(\mathbf{0} - \pi))}{\sin(\mathcal{V}_{m}\pi)} \frac{GH_{\mathcal{V}_{m}}^{(1)}(ka)}{\frac{\partial}{\partial \mathcal{V}_{m}}GH_{\mathcal{V}_{m}}^{(2)}(ka)} \mathcal{V}_{m}^{(2)}(kr)H_{\mathcal{V}_{m}}^{(2)}(k\rho)$$
(58)

The values for \mathcal{V}_{m} are given in Table IV, Appendix D. Asymptotically expanding the Hankel function appearing in Eq. (58) (Ref 18:865-866):

$$\sim \frac{j \exp(-jk((r^{2} - a^{2})^{1/2} + (\rho^{2} - a^{2})^{1/2}))}{2k(r^{2} - a^{2})^{1/4}(\rho^{2} - a^{2})^{1/4}}$$

$$\cdot \sum_{m=1}^{\infty} \frac{\cos(\nu_{m}(\Theta - \pi))}{\sin(\nu_{m}\pi)}$$

$$\cdot \exp(j\nu_{m}(\cos^{-1}(\frac{a}{r}) + \cos^{-1}(\frac{a}{\rho}))) \frac{GH^{(1)}(ka)}{\nu_{m}} \qquad (59)$$

Using Euler's equation to replace $\frac{\cos(\boldsymbol{\nu}_{m}(\boldsymbol{\Theta}-\boldsymbol{\pi}))}{\sin(\boldsymbol{\nu}_{m}\boldsymbol{\Theta})}$ yields

u

$$u \sim \frac{j \exp(-jk((r^2 - a^2)^{1/2} + (\rho^2 - a^2)^{1/2}))}{2k(r^2 - a^2)^{1/4}(\rho^2 - a^2)^{1/4}}$$

$$\cdot \sum_{m=1}^{\infty} \frac{\exp(-j\boldsymbol{\nu}_{m}\boldsymbol{\Psi}_{1}) + \exp(-j\boldsymbol{\nu}_{m}\boldsymbol{\Psi}_{2})}{1 - \exp(-j2\boldsymbol{\pi}\boldsymbol{\nu}_{m})}$$
$$\cdot \exp(j\boldsymbol{\nu}_{m}(\cos^{-1}(\frac{a}{r}) + \cos^{-1}(\frac{a}{\rho}))) \frac{GH^{(1)}(ka)}{\frac{\boldsymbol{\nu}_{m}}{\boldsymbol{\nu}_{m}}}$$
(60)

where $\Psi_1 = \Theta$ and $\Psi_2 = 2\Pi - \Theta$ Eq. (60) contains the leading terms in the asymptotic expansion of the field in the shadow region.

To calculate the GTD field, Eq. (12) in the text of the thesis is used as a starting point. For a torsionless surface path on a cylinder, no E field exists in the \hat{b} direction so Eq. (10) is equal to zero and Eq.(12) may be written as

 $u_{i} = \frac{-jI\omega\mu}{\sqrt{8kR}} \exp(-j(kR + \varphi))$

$$u = u_{1}\sqrt{\frac{d\eta_{1}}{d\eta_{2}}}\sqrt{\frac{\rho}{s(\rho + s)}}exp(-jk(l + s))$$

$$\cdot \sum_{m=1}^{\infty} D_{m}(Q_{1})D_{m}(Q_{2})exp\left[-\int_{Q_{1}}^{Q_{2}} \sigma_{m}(l)dl\right]$$
(61)

(62)

with

Due to the symmetry of the problem it is clear that the geodesics are arcs of the generating circle of the cylinder. Furthermore, the quantity $d\eta_1/d\eta_2$ has the value unity since a band of diffracted rays does not spread on the cylinder. Also, the radius of curvature ρ in Eq. (61) is infinite so that $(\rho/(\rho + s)s)^{1/2}$ assumes its limiting value $s^{-1/2}$. Moreover, because the geodesic curvature of a ray is constant, the quantities $d_m(\ell)$ and $D_m(\cdot)$ are constants with $D_m(Q_1) = D_m(Q_2)$. Introducing these simplifications into Eq. (61) and using Eq. (62), noting that R at Q_1 is $(\rho^2 - a^2)^{1/2}$. Then Eq. (61) becomes

*A torsionless surface path is one where the direction of **b** does not change along a geodesic path on the cylinder.

$$u = \frac{-jI\omega\mu}{\sqrt{8ks}(\rho^2 - a^2)^{1/2}} \exp(-j(k((\rho^2 - a^2)^{1/2} + s) + \varphi))$$

$$\cdot \sum_{m=1}^{\infty} D_m^2(Q) \exp(-jk - \alpha_m) l \qquad (63)$$

Eq. (63) gives the field associated with any ray from 0 to P having an arc length \boldsymbol{l} on the cylinder. For the ray OQ_1Q_2P , the length \boldsymbol{l} for a circle has the value $\boldsymbol{l}_0 = a\boldsymbol{\Theta} - a\cos^{-1}(a/\rho) - a\cos^{-1}(a/r)$, while $s = (r^2 - a^2)^{1/2}$. In addition to this ray, all of the rays which are tangent to the cylinder at Q_1 and encircle the cylinder g times before emerging at Q_2 are also diffracted rays through P. For these rays \boldsymbol{l} has the value $\boldsymbol{l}_g = \boldsymbol{l}_0 + 2g\pi a$. Inserting this value of \boldsymbol{l} into Eq. (63) and summing over g, the diffracted field at P due to those rays which are tangent at Q_1 is given by

$$u = \frac{-jI\omega\mu}{\sqrt{8k(\rho^2 - a^2)^{1/4}(r^2 - a^2)^{1/4}}} \exp\left[-j(k((\rho^2 - a^2)^{1/2} + (r^2 - a^2)^{1/2}) + (r^2 - a^2)^{1/2}\right]} \cdot \sum_{m=1}^{\infty} D_m^2 \exp((-jk - \alpha_m) f_0)(1 - \exp(2\pi(-jka - a\alpha_m)))^{-1}}$$
(64)

Along with the family of rays described above, there exists other rays which are tangent to the cylinder at Q_3 , encircle the cylinder g times and then emerge tangentially at Q_4 and pass through P. Computing the field along these rays yields a result of the form of Eq. (64) with \oplus replaced by $2\pi - \oplus$ in \int_0 . Adding this result to Eq. (65), we obtain for the total diffracted field at P

$$u = \frac{-jI\omega\mu \exp(-jk((\rho^{2} - a^{2})^{1/2} + (r^{2} - a^{2})^{1/2}) + \varphi)}{\sqrt{8k}(\rho^{2} - a^{2})^{1/4}(r^{2} - a^{2})^{1/4}}$$

$$\cdot \sum_{m=1}^{\infty} \frac{D_{m}^{2}\exp((-jka - a\alpha_{m})\Psi_{1}) + \exp((-jka - a\alpha_{m})\Psi_{2})}{1 - \exp(2\pi(-jka - a\alpha_{m}))}$$

$$\cdot \exp((jka + a\alpha_{m})(\cos^{-1}(\frac{a}{r}) + \cos^{-1}(\frac{a}{r}))) \qquad (65)$$

where $\Psi_1 = \Theta$ and $\Psi_2 = 2\pi - \Theta$. Comparing Eq. (65) to Eq. (60) and ignoring the amplitude and the phase of the source gives

$$\mathbf{w}_{m} = -\mathbf{j}\mathbf{k} + \mathbf{j}\boldsymbol{\nu}_{m}\mathbf{a}^{-1}$$
$$D_{m}^{2} = -\left(\frac{\mathbf{j}\mathbf{k}}{2\pi}\right)^{-1/2} \frac{\mathbf{GH}_{\boldsymbol{\nu}_{m}}^{(1)}(\mathbf{k}\mathbf{a})}{\frac{\partial}{\partial\boldsymbol{\nu}}\mathbf{GH}_{\boldsymbol{\nu}_{m}}^{(2)}(\mathbf{k}\mathbf{a})}$$
(66)

Expansion of $GH_{\nu_m}^{(1)}(ka)/[(\partial/\partial\nu)GH_{\nu_m}^{(2)}(ka)]$ is done in Appendix D.

Voltmer's generalizations for surfaces of variable curvature are contained in Table I in the text, and are seen to be dependent on not only ρ_g , the radius of curvature of the surface with respect to are length along the ray trajectory, but also $\dot{\rho_g}$, $\ddot{\rho_g}$, and $\rho_{\rm tn}$, where the dot denotes a derivative with respect to are length and $\rho_{\rm tn}$ is the radius of curvature of the surface in the direction of the binormal to the ray.

Appendix B

The Launching Coefficient

The launching coefficinet is derived by asymptotically expanding a canonical problem for ka large and comparing this to the GTD solution as was done for the diffraction coefficients in Appendix A. Start with Eq. (58) in Appendix A.

$$\mathbf{u} = -\frac{j\mathbf{n}}{4}\sum_{m=1}^{\infty} \frac{\cos(\boldsymbol{\nu}_{m}(\boldsymbol{\theta} - \boldsymbol{\pi}))}{\sin(\boldsymbol{\nu}_{m}\boldsymbol{\pi})} \frac{GH^{(1)}(ka)}{\frac{\boldsymbol{\nu}_{m}}{\boldsymbol{\nu}_{m}} H^{(2)}(kr)H^{(2)}(kr)} (kr)H^{(2)}(kr)}{\frac{\boldsymbol{\nu}_{m}}{\boldsymbol{\nu}_{m}} \mathcal{L}^{(2)}(ka)} \mathcal{L}^{(2)}(kr)H^{(2)}(kr)}$$
(58)

Replacing $\frac{\cos(\boldsymbol{\gamma}_{m}(\boldsymbol{\Theta} - \boldsymbol{\pi}))}{\sin(\boldsymbol{\gamma}_{m}\boldsymbol{\pi})} = \frac{\exp(-j\boldsymbol{\gamma}_{m}\boldsymbol{\gamma}_{1}) + \exp(-j\boldsymbol{\gamma}_{m}\boldsymbol{\gamma}_{2})}{1 - \exp(-j\boldsymbol{z}\boldsymbol{\pi}_{m}\boldsymbol{\gamma})}$, and letting $(1 - \exp(-j\boldsymbol{z}\boldsymbol{\pi}_{m}\boldsymbol{\gamma}))^{-1} = 1$ for ka large gives

$$u = -\frac{j\pi}{4}\sum_{m=1}^{\infty} \frac{GH_{\boldsymbol{\mathcal{V}}_{m}}^{(1)}(ka)}{\frac{j}{\boldsymbol{\mathcal{V}}_{m}}GH_{\boldsymbol{\mathcal{V}}_{m}}^{(2)}(ka)} H_{\boldsymbol{\mathcal{V}}_{m}}^{(2)}(ka)H_{\boldsymbol{\mathcal{V}}_{m}}^{(2)}(k\rho)$$

$$\cdot \exp(-j\boldsymbol{\mathcal{V}}_{m}\boldsymbol{\mathcal{V}}_{m}) + \exp(-j\boldsymbol{\mathcal{V}}_{m}\boldsymbol{\mathcal{V}}_{2}) \qquad (67)$$

where $\Psi_1 = \Theta$ and $\Psi_2 = 2\pi - \Theta$. Using the definition of the diffraction coefficient from Appendix A

$$\frac{\mathcal{GH}^{(1)}(\mathbf{ka})}{\mathcal{V}_{m}^{\mathrm{GH}^{(2)}(\mathbf{ka})}} = -\left(\frac{j\mathbf{k}}{2\pi}\right)^{1/2} \mathbb{D}_{m}^{2}$$
(66)

and the asymptotic expansion

$$H_{\nu}^{(2)}(k\rho) \sim \sqrt{\frac{2}{\pi k \rho}} \exp(k_{\rho} - (2\nu + 1)\frac{\pi}{4})$$
(68)

and defining the boundary conditions as

$$\frac{\partial}{\partial n} \cdot u(\bar{\rho}', \bar{\rho}) = 0 \text{ for } \rho' = a \text{ (hard boundary)} \tag{69}$$

and

$$u(a, \hat{a}, \hat{b}) = 0$$
 (soft boundary) (70)

then for the hard boundary condition, noting the results for a point source in Eq. (23)

$$E_{\bullet}(\vec{\rho}) = I_{x} \frac{\partial}{\partial \rho} u(\vec{\rho}, \hat{x}a)$$
(71)

Then Eq. (67) becomes

$$E_{\Theta} = -\frac{jkI_{x}}{\sqrt{8\pi k j}} \sum_{m=1}^{\infty} \left[-j\pi \left(\frac{jk}{2}\right)^{1/2} H_{\mathcal{V}_{m}}^{(2)}(ka) D_{m}^{h} \right]$$

$$\cdot \left[exp(-j\mathcal{V}_{m}\Psi_{1}) + exp(-j\mathcal{V}_{m}\Psi_{2}) \right]$$

$$\cdot D_{m}^{h} \frac{exp(-jk/2)}{\sqrt{\rho}} \qquad (72)$$

Referring to Fig. 17, the GTD solution for E with a launching coefficient replacing the field incident at Q_1 is

$$E_{\theta} = CI_{x} \sum_{m=1}^{\infty} L_{m}^{h} D_{m}^{h} \left[exp((-jka - ad_{m}^{h})\Psi_{1}) + exp((-jka - ad_{m}^{h})\Psi_{2}) \right] \frac{exp(-jk\rho)}{V\rho}$$
(73)

Comparing Eqs. (72) and (73) gives

$$C = -\frac{\mathbf{k} \cdot \exp(j\pi/4)}{78\pi \mathbf{k}}$$
(74)

$$\mathbf{L}_{m}^{h} = -j\pi (\frac{jk}{2\pi})^{1/2} \mathbf{H}_{m}^{(2)} (ka) \mathbf{D}_{m}^{h}$$
(75)

$$a_{m}^{h} = ka - ja_{m}^{h}a$$
 (75)

Replacing the hard coefficients with the soft coefficients and differentiating $H_{\nu_m}^{(2)}(ka)$ with respect to ka will solve the problem for the soft boundary condition. $H_{\nu_m}^{(2)}(ka)$ and $H_{\nu_m}^{(2)}(ka)$ are expanded in Appendix D.

Comparing this to Pathak's work on slots in cylinders (Ref 23: 46-62) shows that the same launch coefficient results for a magnetic current source in the \hat{z} direction. The soft boundary condition for a monopole is equivalent to a launch coefficient of a magnetic line source briented in the \hat{e} direction (circumferential slot). The monopole can then be modeled as 2 magnetic line sources perpendicular to the direction of the monopole electric source for fields in the deep shadow region.

Appendix C

Airy Functions, Derivatives and Zeros-

The Airy differential equation,

$$\frac{\mathrm{d}^2 \mathbf{w}(\mathbf{z})}{\mathrm{d}\mathbf{z}^2} - \mathbf{z} \mathbf{w}(\mathbf{z}) = 0 \tag{76}$$

yields two independent solutions which assumes several forms. The form used in this thesis is defined by Miller as

Ai(z) =
$$\frac{1}{2\pi j} \int_{\Gamma_1} \exp(zt - t^3/3) dt$$
 (77)

which for z = x (x real) may be written as

Ai(x) =
$$\frac{1}{\pi} \int_{0}^{\infty} \cos(xt + t^{3}/3) dt$$
 (78)

 \mathbf{F} is shown in Figure 19. A second independent solution is given by

$$\operatorname{Ai}(ze^{-j2\pi/3}) = \frac{1}{2\pi j} \int_{\Gamma_1} \exp(ze^{-j2\pi/3}t - t^3/3) dt$$
$$= \frac{e^{j2\pi/3}}{2\pi j} \int_{\Gamma_2} \exp(zt - t^3/3) dt \qquad (79)$$

while the Wronskian relation satisfied by these solutions is



$$\frac{e^{j\pi/6}}{2\pi} = Ai(z)Ai'(ze^{-j2\pi/3})e^{-j2\pi/3} - Ai(ze^{-j2\pi/3})Ai'(z)$$
(80)

The derivatives are obtained by differentiating under the integrals of Eqs. (77) and (79).

The roots of the Airy function and its derivative which satisfy

Ai(z) = 0 and Ai'(z) = 0

are real and negative and are denoted as $-q_m$ and $-\overline{q_m}$, respectively, where m denotes the number of the root. The first two roots are presented in Table III as are the values of Ai $(-\overline{q_m})$ and Ai $'(-q_m)$.

Table.III.

Airy Functions, Derivatives, and Zeros

m	9 m	q _m	Ai!(-q _m)	$\operatorname{Ai}(-\overline{q}_m)$
1	+2.33811	+1.01879	+0.70121	+0.53566
2	+4.08795	+3.24820	-0.80311	-0.41902
			(from	Ref 20:149)

Appendix D

Hankel Functions

An evaluation of the integral resulting from the Watson transformation requires the calculation of the roots of the equation

$$GH_{\nu}^{(2)}(ka) = 0$$
 (81)

where

I Soft EM cylinder $G = \frac{\partial}{\partial X} |_{X=KQ}$ Hard EM cylinder

A closed-form evaluation of these zeros is not possible. However, an asymptotic representation of the roots may be obtained for large values of ka. The procedure is outlined in Reference 20, pages 127 to 133. The results are listed in Tables IV and V on the next page. The asymptotic approximations for the Hankel functions of the second order are

$$\mathfrak{V}_{\mathfrak{m}}^{(2)}(\mathbf{ka}) \sim 2 \ e^{j\frac{\pi}{3}} \left(\frac{2}{\mathbf{ka}}\right)^{1/3} \operatorname{Ai}(-\overline{q}_{\mathfrak{m}}) \left\{ 1 + \left(\frac{2}{\mathbf{ka}}\right)^{2/3} e^{j\frac{2\pi}{3}} \right\}$$
(82)

$$\mathfrak{L}_{\mathcal{V}_{m}}^{(2)}(\mathbf{k}a) \sim -2e^{-j\frac{\pi}{3}} \left(\frac{2}{\mathbf{k}a}\right)^{2/3} \operatorname{Ai}'(-\mathbf{q}_{m}) \left\{ 1 - \left(\frac{2}{\mathbf{k}a}\right)^{2/3} \frac{\mathbf{q}_{m}}{15} e^{j\frac{2\pi}{3}} \right\}$$
(83)

an alternative form is

$$H_{\nu}^{(2)}(k\rho) \sim \sqrt{\frac{2}{\pi k \rho}} \exp -j(k\rho - (2\nu + 1)\frac{\pi}{4})$$
(68)

T	a	b	1	e	I	V

$$\frac{\text{Hankel Function Zeros, } GH_{\nu}^{(2)}(\text{ka}) = 0}{\text{Surface}}$$
Surface
$$\frac{\text{Zeros}-\nu}{\text{Soft}}$$

$$\nu_{\text{m}} = \text{ka} + q_{\text{m}}e^{-j\pi/3}\left(\frac{\text{ka}}{2}\right)^{1/3} - \frac{q_{\text{m}}^{2}}{60}e^{j\pi/3}\left(\frac{\text{ka}}{2}\right)^{-1/3}$$

$$\frac{\text{Hard}}{\text{Cylinder}} \quad \overline{\nu}_{\text{m}} = \text{ka} + \overline{q}_{\text{m}}e^{-j\pi/3}\left(\frac{\text{ka}}{2}\right)^{1/3} - \frac{\overline{q}_{\text{m}}^{2}}{60}\left(1 + \frac{6}{\overline{q}_{\text{m}}^{3}}\right)\left(\frac{\text{ka}}{2}\right)^{-1/3}$$
(from Ref 20;145)

Table V

Hankel Function Quotient,
$$\frac{GH_{\nu}^{(1)}(ka)}{\frac{2}{\lambda\nu}GH_{\nu}^{(2)}(ka)}$$

 Surface	G-Operator	Quotient
Soft	1	$\frac{e^{-j5\pi/6} \frac{ka}{2}^{1/3}}{2\pi(\text{Ai'}(-q_m))^2} (1 + \left(\frac{2}{ka}\right)^{2/3} \frac{q_m}{30} e^{-j\pi/3}$
Hard	<u>d</u> d(ka)	$\frac{e^{-j5\pi/6}\left(\frac{ka}{2}\right)^{1/3}}{2\pi\bar{q}_{m}(Ai(-\bar{q}_{m}))^{2}}(1+\left(\frac{2}{ka}\right)^{2/3}\frac{\bar{q}_{m}}{30}e^{-j\pi/3}$

(from Ref 20:136)

Appendix E

The Field in the Transition Region

The material for this Appendix is taken in part from Reference 23, pages 63 to 70. In the transition region adjacent to the shadow boundary, the representation of the field radiated by the monopole described by the launching coefficient in Appendix B is poorly convergent so a different form is needed. It will be seen that a suitable asymptotic solution can be obtained in terms of integrals. The first order approximation is adequate for ka > 3, and since the added complexity is hard to justify, only the first order will approximation will be derived.

For the principle plane analysis of the cylinder (looking length-wise down the cylinder), only the hard boundary condition exists, therefore only it will be treated. The soft condition is solved identically as below except without taking the $\partial u/\partial \rho$.

The Watson transform is again applied to Eq. (55), but this time the resulting expression is left in the form of an integral. In the far zone, $H_{\nu}^{(2)}(k\rho)$ is approximated asymptotically as in Eq. (68) so that (see Ref 18:865-866)

$$\frac{1}{2\sqrt{8\pi}k_{j}} \frac{-k}{\sqrt{\rho}} \frac{\exp(-jk\rho)}{\sqrt{\rho}} \sum_{l=0}^{\infty} \int_{-\infty}^{\infty} d\nu$$

$$\cdot \frac{H^{(1)}_{\nu} \cdot (ka)H^{(2)}_{\nu} (ka) - H^{(2)}_{\nu} \cdot (ka)H^{(1)}_{\nu} (ka)}{H^{(2)}_{\nu} (ka)}$$

 $\exp(-j\nu(\Psi_1 + 2\pi \ell)) + \exp(-j\nu(\Psi_2 + 2\pi \ell))$ (84)

From the Wronskian relationship for the Hankel function,

$$H_{m}^{(1)}(ka)H_{m}^{(2)}(ka) - H_{m}^{(2)}(ka)H_{m}^{(1)}(ka) = \frac{4j}{\pi ka}$$
(85)

then

Ç.,

$$\frac{\partial u}{\partial \rho} \sim -\frac{2 j k \exp(-j k \rho)}{k a \sqrt{8 n k j}} \sum_{I=0}^{\infty} \int_{-\infty}^{dm} \frac{dm}{dm} \cdot \frac{(\exp(-j m (\Psi_1 + 2 \pi I)) + \exp(-j m (\Psi_2 + 2 \pi I)))}{H_m^{(2)}(ka)}$$
(86)

Replacing $H_m^{(2)}(ka)$ by its Watson approximations (for m-ka>>1) gives rise to the Fock function representations for the $\partial u/\rho$:

$$\frac{\partial \mathbf{u}}{\partial \rho} \sim \frac{2}{\pi k a} \frac{-jk}{\sqrt{3\pi k_j}} \int_{\mathbf{f}=0}^{\infty} \left[\exp(-jka(\Psi_1 + 2\pi \Psi)) \left(\frac{ka}{2}\right)^{2/3} \cdot \left(-j\Psi_{\overline{n}}\right) \int_{\mathbf{f}_1}^{\mathbf{f}} \frac{\exp(-j\xi_1 \mathbf{T})}{\Psi_2(\mathbf{T})} d\mathbf{T} + \exp(-jka(\Psi_2 + 2\pi \Psi)) \right] \cdot \left(\frac{ka}{2}\right)^{2/3} \left(-j\Psi_{\overline{n}}\right) \int_{\mathbf{f}_1}^{\mathbf{f}} \frac{\exp(-j\xi_2 \mathbf{T})}{\Psi_2(\mathbf{T})} d\mathbf{T} + \exp(-jka(\Psi_2 + 2\pi \Psi)) \right] \cdot \left(\frac{ka}{2}\right)^{2/3} \left(-j\Psi_{\overline{n}}\right) \int_{\mathbf{f}_1}^{\mathbf{f}} \frac{\exp(-j\xi_2 \mathbf{T})}{\Psi_2(\mathbf{T})} d\mathbf{T} + \exp(-jk(\Psi_2 + 2\pi \Psi)) \right) \cdot \left(\frac{ka}{2}\right)^{2/3} \left(-j\Psi_{\overline{n}}\right) \int_{\mathbf{f}_1}^{\mathbf{f}} \frac{\exp(-j\xi_2 \mathbf{T})}{\Psi_2(\mathbf{T})} d\mathbf{T} + \exp(-jk(\Psi_2 + 2\pi \Psi)) \right) \cdot \left(\frac{ka}{2}\right)^{2/3} \left(\frac{ka}{2}\right)^{2/3} \left(-j\Psi_{\overline{n}}\right) \int_{\mathbf{f}_1}^{\mathbf{f}} \frac{\exp(-j\xi_2 \mathbf{T})}{\Psi_2(\mathbf{T})} d\mathbf{T} + \exp(-jk(\Psi_2 + 2\pi \Psi)) \right) \cdot \left(\frac{ka}{2}\right)^{2/3} \left(\frac{ka}{$$

Here $W_2(7)$ is the Fock-type Airy function related to the Millertype Airy function by

 $w_2(\tau) = 2 \mathcal{V}_{\pi} \exp(-j\pi/6) A_i(-\tau \exp(j\pi/3))$ (88)

$$= \frac{1}{n} \int_{2} \exp(\tau z - z^{3}/3) dz$$
 (89)

OF

with the contour of integration shown in Appendix C.

Introducing the Fock functions

$$\widetilde{g}(\xi) = \frac{1}{\sqrt{\pi}} \int_{\Gamma_1} \frac{\exp(-j\xi\tau)}{w_2(\tau)} d\tau \qquad (90)$$

then Eq. (87) becomes

$$\frac{\partial u}{\partial \rho} \sim -jC(\frac{2}{ka})^{1/3} \widetilde{g}(\xi_1) \exp(-jka \psi_1) + \widetilde{g}(\xi_2) .$$

$$\cdot \exp(-jka \psi_2) \frac{\exp(-jk\rho)}{\sqrt{\rho}}$$
(91)

Where only the f=0 term is retained for large ka. Substituting this into Eq. (71), the far-zone electric fields for the line source $\mathbf{\hat{r}} \mathbf{I}_{\mathbf{x}}$ is

$$E_{\Theta} \sim C I_{x} \left\{ -j(\frac{2}{ka})^{1/3} \right\}.$$

. $(\hat{g}(\xi_{1}) \exp(-jka\Psi_{1}) + \tilde{g}(\xi_{2}) \exp(-jka\Psi_{2}))$.

$$\cdot \frac{\exp(-jk\rho)}{\gamma \rho'} \tag{92}$$

)

Comparing with Eq. (73) yields

-

Part in

$$\mathbf{v}(Q_1, Q_2) = -j \sqrt{\frac{d\eta_1}{d\eta_2}} \left(\frac{2}{ka}\right)^{1/3} \tilde{g}(\boldsymbol{\xi}_1) \exp(-jka\boldsymbol{\Psi}_1) \exp(-jk\boldsymbol{I})$$
(93)

Tabulation of $\hat{g}(\boldsymbol{\varsigma})$ is included in Appendix G.

Appendix F

Field Data

The following data consists of the field at P due to each antenna. Values were calculated for field points at a constant distance of 300 meters. The angle v is the angle the field radius vector makes with the verticle—the same as the parametric v used in the mathematical model of the cylinder. The field is calculated for v between 0 and 180° in steps of 5° . Due to symmetry the field due to antenna 2 in the 180° to 360° range is the same as for antenna 3 in the 0 to 180° range with the direction of the field vector turned around (shifted 180° spatially).

To calculate the total fields using these tables, add the source phase of the antenna in question to that in the chart for that antenna, and multiply the x and y quantities by the source amplitude. Now for each field point determined by v, an x,y,phase data point for each antenna is taken from the table. To obtain the total field these three vectors are added vectorially.

Also noted are the equations which were used to calculate the values in the chart. The equations are

Eq. #	Equations used
1	Lit region Eq. (40)
2	Lit side of transition region Eq.(41)
3	Shadow side of transition region Eq.(42)
4	Deep shadow region Eq. (43)

Ta	ble	VI

			F	field Data		
		Fo #	Magnitude	Antenna 1		Phaga [-i(.)]
		194 • 11	Magnit vauc	A	.	11030[-3(*)]
	0	1	0.000	0.000	0,000	0.00
	5	ī	2.944	-0.258	2 033	2.76
	10	ī	5.865	-1 024	5 766	11 01
	15	ī	8.740	-2 276	8 138	24 70
	20	ī	11.545	-3 057	10 708	13 70
	25	i	14.259	-6.062	12,906	67.89
	30	ī	16.861	-8.479	14.574	97.07
	35	ī	19.330	-11,147	15,791	131.00
	40	ī	21.646	-13.986	16.521	169.44
	45	ī	23.793	-16.903	16.744	212.09
	50	2	24.000	-18,463	15,337	258.61
	55	2	24.510	-22.623	15.659	306.28
	60	2	25.068	-21,781	12,409	344.02
	65	2	25.479	-23-166	10,627	28.17
	70	2	25.555	-23.907	8.589	76.30
	75	2	25.480	-24.654	6.436	127.84
	80	2	25.026	-24.674	4.182	182.32
	85	2	24.304	-22,965	1,958	239.15
	90		23.316	-23, 316	0.000	3.06
	95	3	22.223	-22.138	-1.937	79.95
]	00	3	21.082	-20.762	-3.661	156.54
]	105	3	19.826	-19,150	-5.131	231.73
]	10	3	18.771	-17.639	-6.420	303.32
]	115	3	17.618	-15,967	-7.446	8.05
]	20	3	16.366	-14.173	-8.183	73.22
]	125	• 3	15.310	-12.541	-8,781	137.67
]	30	4	14.466	-11.082	-9.299	195.93
]	.35	4	11.399	-8,060	-8.060	208.36
]	40	4	11.468	-7.372	-8,785	284.12
1	45	4	10.830	-6.212	-8.871	344.67
1	50	4	7.950	-3.975	-6.885	50.41
]	55	4	9.520	-4.023	-8.628	107.02
]	60	4	6.316	-2.160	-5.935	128.98
]	.65	4	8.580	-2.221	-8.288	164.79
]	70	4	3.831	-0.665	-3.773	145.14
]	.75	4	7.829	-0.682	-7.800	164.79
1	.80	4	0.000	0.000	0.000	0.00

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Table	VII
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Field Data Antenna 2

7	Eq. #	Magnitude	x	У	Phase $[-j(\cdot)]$
0	1	15.418	0.000	15.418	22.90
5	1	17.970	-1.591	17.899	33.46
10	1	20.382	-3.578	20.066	49.28
15	1	22.635	-5.912	21.849	70.24
20	1	24.712	-8.521	23.196	96.17
25	1	25.595	-10.897	23.159	126.87
30	2	25.222	-12.699	21.792	156.53
35	2	25.220	-14.555	20.596	186.01
40	2	25.546	-16.516	19.489	220.88
45	2	25.606	-18,201	18.012	260.81
50	2	30.279	-19.442	16.135	305.37
55	2	24.678	-20.296	14.038	344.71
60	2	23.830	-20.707	11.792	.36.77
65	3	22.866	-20.724	9.664	86.77
70	3	21.723	-20.413	7.430	132.00
75	3	20.575	-19.874	5.325	234.78
80	3	19.410	-19.115	3.371	7.94
85	3	18.229	-18.160	1.589	83.45
90	3	17.083	-17.083	0.000	196.85
95	3	15.960	-15.899	-1.391	274.39
100	4	14.854	-14.628	-2.579	352.03
105	4	10.921	-10.590	-2.827	296.08
110	4	10.639	-9.997	-3.639	18.86
115	4	10.469	-9.483	-4.424	110.64
120	4	9.859	-8.538	-4.930	190.99
125	4	8.484	-6.950	-4.866	272.01
130	4	8.214	-6.338	-5.318	347.50
135	4	6.569	-4.645	-4.645	19.13
140	4	1.111	-4.995	-5.953	144.93
145	4	0.130	-3.519	-5.026	189.05
150	4	3.825	-1.912	-3.313	200.93
155	4	7.305	-3.087	-0.021	315.70
160	4	1.370	-0.469	-1.20/	323.01
105	4	(.241	-1.0(4	-0.994	24.03
170	4	0.001	-1.140	-0.901	59.50
1/5	4	0.130	-0.109	-0.099	9.34
190	4	5.024	0.000	-2.025	117.30

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Table VIII

Field Data Antenna 3

•	Eq. #	Magnitude	x	у	Phase $[-j(\cdot)]$
0	1	15.418	0.000	-15.418	22.90
5	1	12.745	1.110	-12.697	17.67
10	1	9.973	1.732	-9.821	17.83
15	1	7.124	1.844	-6.881	23.36
20	1	4.440	1.443	-3.966	34.23
25	1	1.285	0.543	-1.165	50.35
30	1	1.658	-0.829	1.436	71.60
35	1	4.587	-2.631	3.758	97.81
40	1	7.478	-4.827	5.712	128.78
45	1	10.308	-7.319	7.260	164.27
50	1	13.056	-10.039	8.349	204.00
55	1	15.700	-12.905	8.943	247.67
.60	1	18.220	-15.827	9.027	294.94
65	1	20.596	-18.715	8.600	345.45
70	1	20.810	-21.480	7.678	38.80
75	1	24.846	-24.039	6.283	94.60
80	1	25.670	-25.313	4.300	152.40
85	2	25.797	-25.713	2.082	210.15
90	2	25.842	-25.841	0.000	268.30
95	2	25.972	-25.858	-2.431	327.97
100	2	26.147	-25.719	-4.707	11.99
105	2	25.405	-24.496	-6.734	89.95
110	2	24.700	-23.157	-8.594	150.92
115	2	22.770	-20.577	-9.749	212.65
120	3	21.622	-18.725	-10.811	289.60
125	3	21.618	-17.708	-12.400	302.50
130	3	20.368	-15.603	-13.092	0.72
135	3	19.219	-13.590	-13.590	58.52
140	3	18.288	-11.755	-14.009	108.95
145	3	16.926	-9.708	-13.865	161.14
150	3	15.697	-7.848	-13.594	206.92
155	4	14.397	-6.084	-13.048	273.65
160	4	11.630	-3.978	-10.929	304.10
105	4	11.518	-2.981	-11.126	347.01
1/0	4	0.214	-1.437	-0.140	329.34
115	4	10.593	-0.923	-10.373	345.40
100	4	3.329	0.000	-3.291	290.14

and the second



Table of the Hard Fock Function

The function is defined as

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 $g(\boldsymbol{\xi}) = \frac{1}{\sqrt{\pi}} \int \exp(-j\boldsymbol{\xi}t) \frac{1}{w_1'(t)} dt$

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The Hard Fock Function

Ę	Modulus	Argument
-1.00	1.861	15.460
-0.95	1.847	12.626
-0.90	1.833	10.077
-0.85	1.818	7.801
-0.80	1.802	5.785
-0.75	1.785	4.016
-0.70	1.766	2.482
-0.65	1.747	1.174
-0.60	1.726	0.079
-0.55	1.705	359.187
-0.50	1.682	358.487
-0.45	1.658	357.971
-0.40	1.633	357.627
-0.35	1.607	357.446
-0.30	1.580	357.419
-0.25	1.552	357.536
-0.20	1.523	357.790
-0.15	1.493	358.172
-0.10	1.463	358.672
-0.05	1.431	359.284
0.00	1.399	0.000
0.05	1.367	0.812
0.10	1.334	1.713
0.15	1.300	2.696
0.20	1.266	3.755
0.25	1.232	4.883
0.30	1.197	6.074
0.35	1.163	7.323
0.40	1.128	8.624
0.45	1.093	9.971
0.50	1.059	11.360
0.55	1.025	12.786

San Distances

	٤	Modulus	Argument
	0.60	0.991	14.244
	0.65	0.958	15.731
	0.70	0.924	17.243
	0.75	0.892	18.776
	0.80	0.860	20.326
	0.85	0.828	21.893
	0.90	0.798	23.474
	0.95	0.768	25.070
	1.00	0.738	26.683
		(f	rom Ref 21:ch. 4)
For more a	ccurate valu	ues see Ref 21.	

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A MONITORING AGENCY NAME & ADDRESS/II dillarant I	from Controlling Office)	15. SECURITY CLASS, (of this report)
	••••	Unclassified
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pattern nulls but with a relativly poor ability to form a single main lobe. The report contains a 26-item bibliography.

Bruce A. Thieman was born on 9 April 1954 in Dayton, Ohio. He graduted from high school in Mountain Home, Idaho in 1972 and attended the University of Idaho from which he received the degree of Bachelor of Science in Electrical in May 1976. Upon graduation, he received a commission in the USAF through the ROTC program. He was called to active duty in August 1976 and entered the School of Engineering, Air Force Institute of Technology.

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