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INFORMATION CONDITIONS, COMMUNICATION AND GENERAL EQUILIBRIUM

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INFORMATION CONDITIONS, COMMUNICATION AND GENERAL EQUILIBRIUM*

by

Pradeep Dubey and Martin Shubik

1. WHO KNOWS WHAT?

The mathematical treatment of general equilibrium theory has been somewhat vague on the specifics of information and communication conditions assumed in the proof of the existence of equilibrium points.

This vagueness comes about primarily because the description of the individual optimization problem is given--though very implicitly-- as an n-person simultaneous move game in strategic or normal form. Details concerning the way trade takes place and the specifics of information structure are suppressed in the normal form.

A rough verbal description of the assumptions implicit in the discussion of general equilibrium is that each individual knows his own preferences, endowments and the market prices of all items being traded. How information concerning prices, or for that matter how prices came into being in the first place, are not discussed.

In some recent publications Shubik [1], Shapley and Shubik [2] and Dubey and Shubik [3] have considered specific mechanisms for the formation of price in a market. These mechanisms make it easy to formulate trade explicitly as an n-person game in strategic form.

Recent concern with trade in markets with exogenous undertainty has resulted in considerable interest in the details of information and "who knows what." Arrow [4] and Debreu [5] were able to extend results on the existence of a competitive equilibrium to markets with many periods and with uncertainty by the introduction of futures markets and by the device of normalizing a multistage process into a game in normal form. Radner [6] and Dubey and Shubik [7] have been concerned explicitly with markets with nonsymmetric information conditions.

There are several questions concerning the general equilibrium model which need to be answered, given that it has been fully formulated as a playable game with all rules given. In particular what is the relationship between the noncooperative equilibria of the general equilibrium model solved as a game and its competitive equilibria? What happens to the noncooperative equilibria and the competitive equilibria when there are different levels of information possessed by the traders? In multistage markets what is the relationship between the set of noncooperative equilibria and the competitive equilibria?

For a market modelled as a simultaneously played one move game without exogenous uncertainty it has been shown that for a continuum of nonatomic players the noncooperative equilibria coincide* with the competitive equilibria [8].

For simultaneously played one move market games** with exogenous uncertainty and players with differing sets of information concerning

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*This is true given certain technical conditions [8].

**Shapley and Shubik [9] have used the term "market game" to refer to a class of games representing markets described in cooperative form. Possibly here we should use the phrase "strategic form market game" to indicate that the market game referred to here is somewhat different--and indeed would have a different cooperative form than the cooperative market game.

Nature's move it has been shown that a competitive equilibrium may not
exist [6], but noncooperative equilibria will exist and can be related
to a modified competitive equilibrium which can be naturally defined and
always exists [7].

The above results suggest that there is a relationship between
the noncooperative equilibria of a market game and its competitive equi-
libria. In particular we know that the noncooperative equilibrium solu-
tion is more general than that of the competitive equilibrium. It is easy to
construct games which have noncooperative equilibria but for which the competi-
tive equilibrium does not exist, but when a competitive equilibrium does exist
it can always be associated with a noncooperative equilibrium point of a market
game which yields the same prices and payoffs as the competitive equilibrium.

When we consider multistage markets, in which trade takes place
more than once we encounter an important divergence in the growth of the
multiplicity of noncooperative equilibria and competitive equilibria.
The former appear to proliferate faster than the latter. This phenomenon
also crops up in one-stage market-games when we refine information of
some of the traders. While the competitive equilibria of the markets do
not change, the set of noncooperative equilibria exhibits rapid growth.

The paper is organized as follows. In Section 2 we turn to games
in extensive form in general, with no limitation to those which represent
economic activities. We demonstrate an important relationship between
games which have identical moves but different levels of information.
Specifically it is shown that if the information sets of one game are a
refinement of the information sets of the other then the set of pure stra-
tegy equilibrium points of the game with less information is contained
within the set of pure strategy equilibrium points of the game with more
information.

In Section 3 we discuss the implication of the general results of Section 2 to market games. In Section 4 an example of a two stage market is provided to show the proliferation of noncooperative equilibria; and several questions are raised concerning the plausibility of certain equilibria as solutions. Some basic problems in the modelling of communication are noted. These relate directly to the development of criteria to judge the plausibility of noncooperative equilibria as solutions.

2. EQUILIBRIUM POINTS AND INFORMATION IN GAMES IN EXTENSIVE FORM

We begin with a definition of games in extensive form. This is due (in its present generality) to Kuhn [10]. We will eschew the general definition and restrict the length of the game to be finite, though the moves at any node and the number of nodes are allowed to be infinite-- indeed they are required to be so for the market game of Section 3. Moreover, we also assume throughout, unless specified otherwise, that there are no chance moves in the games.

Let Γ be a tree with a distinguished node A. We say that node C _follows_ node B if the sequence of arcs joining A to C passes through B; and that C _follows_ B _immediately_ if C follows B and there is an arc joining B to C. A node that has no followers is called a _terminal_ node. A _path_ in Γ is a sequence of nodes, starting with node A, such that each node in the sequence is an immediate follower of the previous node. The _length of a path_ is the number (possibly infinite) of nonterminal nodes that it contains. The _length of_ Γ is the maximum of the lengths of all paths in Γ. An n-_person game in extensive form_ (without chance moves) is a tree Γ of finite length endowed with the following structure:

(a) A distinguished node called the root (or the <u>starting point</u>) of Γ .

(b) A partition of the nonterminal vertices of Γ into n sets $P^1, \ldots, P^n$ called the <u>player</u> <u>sets</u>.

(c) For each $i = 1, \ldots, n$, a partition of P^i into subsets I^i_j , called <u>information</u> <u>sets</u>, such that (i) no node follows another node in the same information set; and (ii) there is for each I^i_j an indexing set A^i_j such that the arcs that issue from* any node in I^i_j are in one-to-one correspondence with the elements of A^i_j . [Note: these correspondences are part of the data of the game. If we change them, we change the game.]

(d) A real-valued payoff function defined for each player on the terminal nodes of Γ .

In the parlance of game theory, arcs that issue from a node are called <u>moves</u>; and a path is called a <u>play</u> of the game.

A <u>pure</u> <u>strategy</u> of player i is a function which maps each information set I^i_j of i into an element of A^i_j . Let S^i denote the <u>set of pure strategies</u> of i . Put $S = S^1 \times \ldots \times S^n$. By s^i we will denote an element of S^i . Any $(s^1, \ldots, s^n) \in S$, defines a unique play of the game in an obvious manner. Let $\Pi^i(s)$ denote the payoff to i at the terminal node of this play. Then a <u>pure</u> <u>strategy</u> <u>noncooperative</u> <u>equilibrium</u> (N.E.) of the game is an element ${}_*s = ({}_*s^1, \ldots, {}_*s^n)$ in S such that, for every i ,

$$\Pi^i({}_*s | s^i) \leq \Pi^i({}_*s)$$

---

*I.e. connect to an immediately following node.

for all $s^i \in S^i$, where $({}_*s|s^i)$ is the same as ${}_*s$ but with ${}_*s^i$ replaced by s^i.

Let Γ and $\hat{\Gamma}$ be games in extensive form on the player-set $N = \{1, \ldots, n\}$. We will say that $\hat{\Gamma}$ is a <u>refinement</u> of Γ (or Γ is a <u>coarsening</u> of $\hat{\Gamma}$), and denote it by $\Gamma \underset{\sim}{\prec} \hat{\Gamma}$, if $\hat{\Gamma}$ is obtainable from Γ only by forming partitions of the information sets in Γ. Calling $\mathcal{C}$ the collection of games in extensive form that arise by imposing all possible information sets on a fixed underlying tree,* it is clear that $\underset{\sim}{\prec}$ gives a partial ordering on $\mathcal{C}$. There is, in this ordering, a unique maximal element $\Gamma_R^{\mathcal{C}}$ and a unique minimal element $\Gamma_C^{\mathcal{C}}$ in $\mathcal{C}$. In $\Gamma_R^{\mathcal{C}}$ all information sets are singletons. The information sets of $\Gamma_C^{\mathcal{C}}$ are obtained by constructing for each player i the coarsest partition of his or her nodes under the conditions c-(i) and c-(ii). (We illustrate this in Figure I below.) For any $\Gamma \in \mathcal{C}$, $\Gamma_R^{\mathcal{C}}$ is sometimes called its <u>most refined form</u> and $\Gamma_C^{\mathcal{C}}$ its <u>most coarsened form</u>.

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*I.e. the nodes, the moves, the indexing sets and the payoffs are all held fixed and only the information sets are varied.
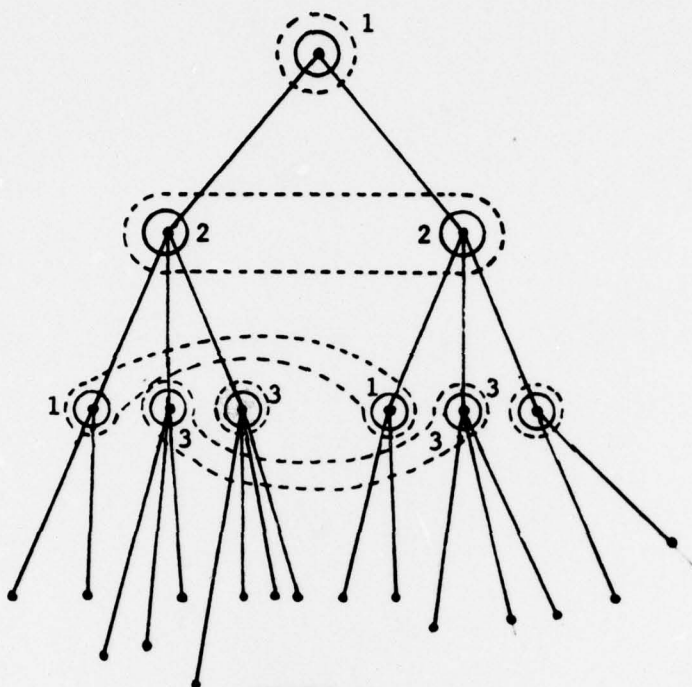
FIGURE I

The broken lines describe $\Gamma_C^{\mathcal{C}}$, and the unbroken lines describe $\Gamma_R^{\mathcal{C}}$.

Let $\Gamma \lesssim \hat{\Gamma}$. Though the strategy sets S^i and $\hat{S}^i$ of i in Γ and $\hat{\Gamma}$ are formally different, there is a natural inclusion $S^i \subset \hat{S}^i$. A strategy s^i in S^i is identified with $\hat{s}^i$ in $\hat{S}^i$, where $\hat{s}^i$ is as follows: the move chosen by $\hat{s}^i$ at any information set $\hat{I}_t^i$ of i in $\hat{\Gamma}$ is the same as the move chosen by s^i at I_j^i, where I_j^i is the <u>unique</u> information set of i in Γ for which $\hat{I}_t^i \subset I_j^i$.

For any game Γ we will denote by $\mathcal{N}(\Gamma)$ the set of all its <u>pure-strategy</u> noncooperative equilibrium points.

Our aim in this section is the following straightforward but striking result.

<u>Proposition</u>. Suppose $\Gamma \lesssim \hat{\Gamma}$. Then $\mathcal{N}(\Gamma) \subset \mathcal{N}(\hat{\Gamma})$.

<u>Proof</u>. If $\eta(\Gamma) = \emptyset$ there is nothing to prove. Let $s = (s^1, \ldots, s^n)$ be in $\eta(\Gamma)$. Put $\hat{s} = (\hat{s}^1, \ldots, \hat{s}^n)$ where $\hat{s}^1$ corresponds to s^1 as described earlier. We will show that $\hat{s}$ is in $\eta(\hat{\Gamma})$.

First observe that s and $\hat{s}$ lead to the same play P. Now if $\hat{s}$ is not in $\eta(\hat{\Gamma})$ there is some player i who can improve his pay-off by deviating from $\hat{s}^i$ to $\underset{\sim}{\hat{s}}^i$, provided that the others keep their strategies fixed according to $\hat{s}$. Take the path $\underset{\sim}{P}$ defined by $(\hat{s}^1, \ldots, \hat{s}^{i-1}, \underset{\sim}{\hat{s}}^i, \hat{s}^{i+1}, \ldots, \hat{s}^n)$, and let $\hat{I}^i_1, \ldots, \hat{I}^i_\ell$ be the information sets of i in $\hat{\Gamma}$ which contain nodes of $\underset{\sim}{P}$. Consider $I^i_{j1}, \ldots, I^i_{j\ell}$ where I^i_{jt} is the (unique) information set of i in Γ such that $\hat{I}^i_t \subset I^i_{jt}$. Since no two nodes of a play can lie in the same information set, the sets $\{I^i_{jt} : t = 1, \ldots, \ell\}$ are all disjoint. Construct the strategy $\underset{\sim}{s}^i$ for i in the game Γ as follows: the choice (of move) made by $\underset{\sim}{s}^i$ at I^i_{jt} is the same as the choice made by $\underset{\sim}{\hat{s}}^i$ at $\hat{I}^i_t$; the choice made by $\underset{\sim}{s}^i$ at information sets other than $I^i_{j1}, \ldots, I^i_{j\ell}$ is arbitrary. Clearly the play defined by $(s^1, \ldots, s^{i-1}, \underset{\sim}{s}^i, s^{i+1}, \ldots, s^n)$ is also $\underset{\sim}{P}$. Thus s is not in $\eta(\Gamma)$ since i can improve his payoff by deviating to $\underset{\sim}{s}^i$, a contradiction.

Q.E.D.

<u>Remark 1</u>. This proposition is not true for mixed strategies. Consider the simple game of matching pennies where Player 1 wins if they match. If both players move simultaneously the payoff matrix is 2×2 (Figure IIa); each player has two moves which coincide with his strategies. The only noncooperative equilibrium is in mixed strategies where each player uses a mixture over his two pure strategies of $(1/2, 1/2)$. The expected payoff to each is zero. A refinement of the information sets in this
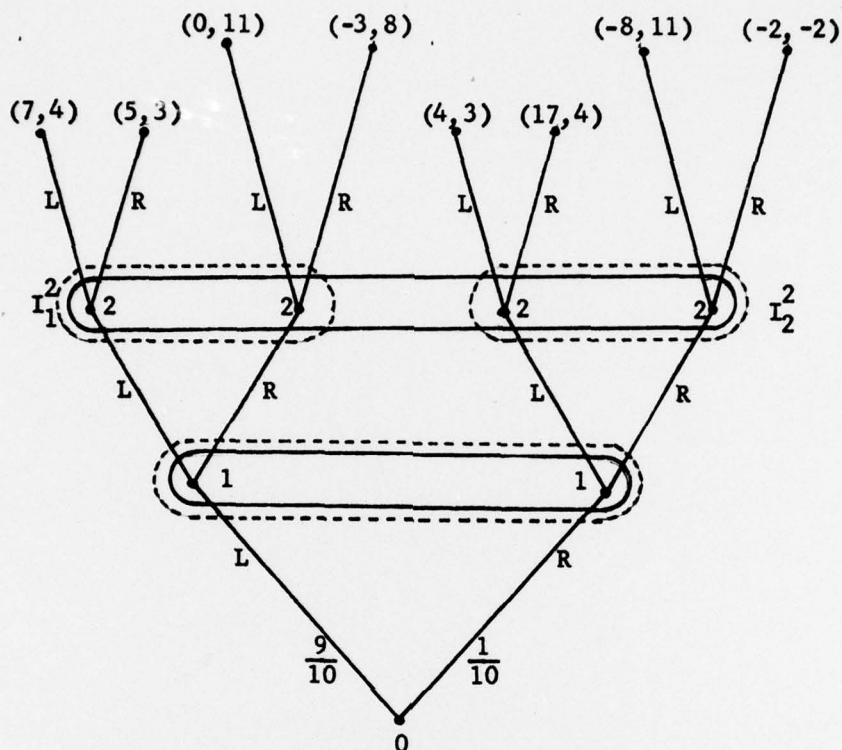
game is given if we assume that Player 2 is informed of Player 1's move before Player 2 is called upon to move.

In Figure IIb we observe that the noncooperative equilibrium in the 2x4 matrix game is given where Player 2 uses his strategy C.

	1	1
1	1	-1
2	-1	1

a

	A (1,1;2,1)	B (1,1;2,2)	C (1,2;2,1)	D (1,2;2,2)
1	1	1	-1	-1
2	-1	1	-1	1

b

FIGURE II

The notation (1,2;2,1) can be read as the sentence: "If Player 1 chooses his first strategy   choose move 2; if he chooses his second strategy   choose move 1."

Remark 2. If there are chance moves in the game, and if we vary information of the traders regarding each others' moves only, while their information about chance moves is held fixed, then the proposition continues to hold (with the obvious modifications in the proof). However, the following example shows that the result breaks down outside of this case.

FIGURE III

Here 0 represents a chance move which selects L (R) with probability 9/10 (1/10) . The first (second) component of the payoff vector is the payoff to 1 (2) .

In the game with unbroken information sets, the unique N.E. is: 1 chooses L ; 2 chooses L . In the game with broken information sets, the unique N.E. is: 1 chooses L ; 2 chooses L on I_1^2 , and R on I_2^2 .

Remark 3. The assumption c-(i), while it may be waived in the general description of a game, is critical. Without it the proposition does not hold. Consider the following example.
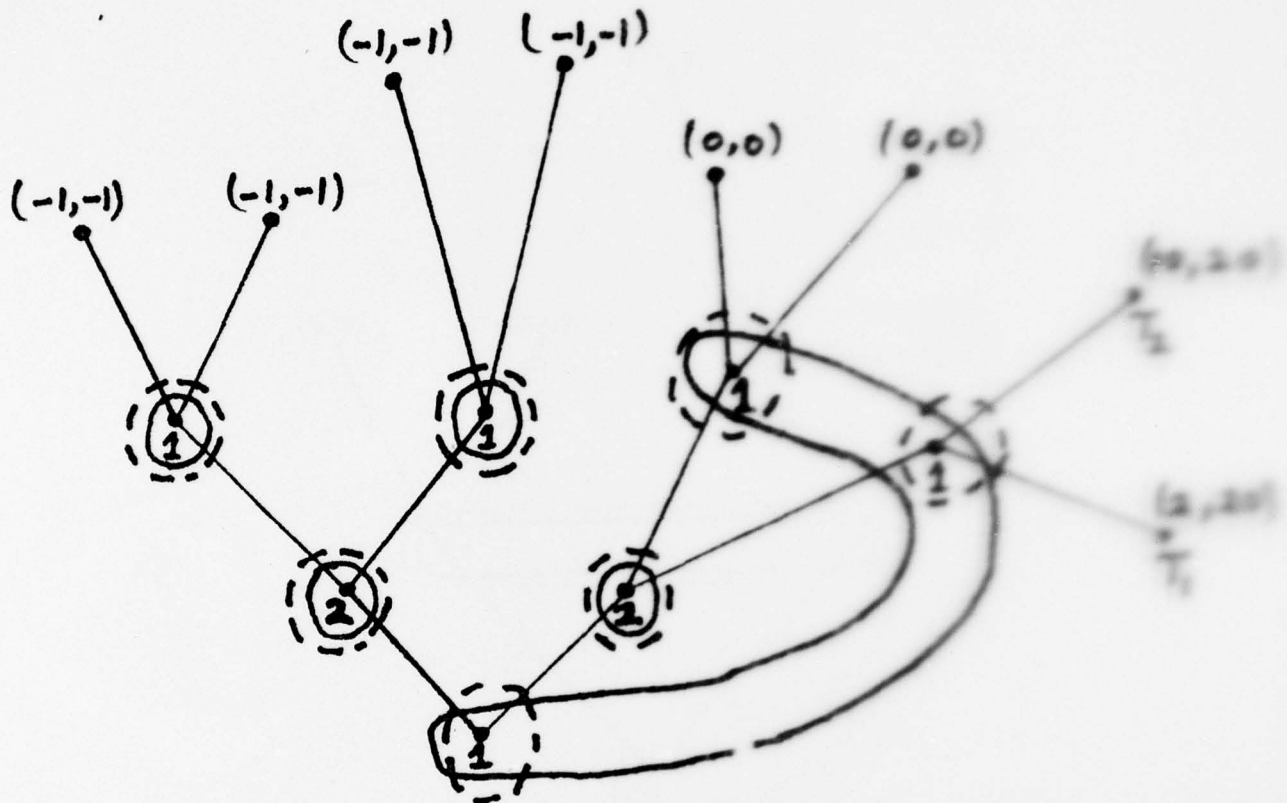

FIGURE IV

Any N.E. of the coarse game leads to a play terminating T_1

Any N.E. of the refined game leads to a play terminating at T_2

3. EQUILIBRIUM POINTS IN MARKET GAMES

Our approach to the competitive equilibrium solution is to repre-
sent a market as a game with a trading method fully defined and to show
that the competitive equilibria may be obtained as noncooperative equi-
libria of the appropriate game. There are many ways of doing this [1],
[2], [3], [11], [12]. Here we select one of these models, the "bid-offer
model" [3], though the results we will cite hold for the other models
as well. First let us quote from [3] a description of the game in stra-
tegic form.

For a positive integer r we shall denote by I_r the set
$\{1, \ldots, r\}$, by E^r the Euclidean space of dimension r, and by
Ω^r the non-negative orthant of E^r.

Let

I_n = the set of traders

I_{m+1} = the set of commodities.

The __initial__ __allocation__ of trader i is a vector $a^i \in \Omega^{m+1}$, where
a^i_j is the amount of commodity j available to i (for $j \in I_m$), and
a^i_{m+1} represents the money held by i. The traders' __utility functions__
are real-valued:

$$u^i : \Omega^{m+1} \longrightarrow \Omega^1, \quad i \in I_n$$

and are assumed to be concave, continuous and non-decreasing.

When we drop an index and use a bar it will indicate summation
over the indexing set. Thus for $x^i \in \Omega^m$, $\bar{x}^i$ means $\sum_{j \in I_m} x^i_j$, etc.
We assume that $\bar{a}_j > 0$ for all $j \in I_m$.

To cast the rules of the market in the form of a game we define the strategy set of trader i to be

$$S^i = \{(q^i, b^i) : q^i \in \Omega^m, b^i \in \Omega^m, q_j^i \leq a_j^i, \bar{b}^i \leq a_{m+1}^i\} .$$

Here q_j^i is the quantity of commodity j that trader i offers for sale. b_j^i is i's bid on good j .

The product $S^1 \times \ldots \times S^n$ will be denoted S . $\bar{S}^i$ denotes $S^1 \times \ldots \times S^{i-1} \times S^{i+1} \times \ldots \times S^n$. s (and, s^i , $\bar{s}^i$) will stand for elements of S (and, S^i , $\bar{S}^i$).

The outcome engendered by a particular $s \in S$ is determined in three simple steps. First we calculate a "price vector" $p \in \Omega^m$ by dividing the amount bid for each good by the total supply:

$$p_j = \frac{\bar{b}_j}{\bar{q}_j} , \quad \text{all } j \in I_m , \quad \text{where } s^i = (q^i, b^i) .$$

(If $\bar{q}_j = 0$, we set $p_j = 0$.) Next we calculate the final allocation that results when the bids are executed

$$\xi_j^i(s) = \begin{cases} b_j^i/p_j & , \text{ if } j \in I_m \text{ and } p_j > 0 \\ 0 & , \text{ if } j \in I_m \text{ and } p_j = 0 \\ a_j^i - \sum\limits_{j \in I_m} b_j^i + \sum\limits_{j \in I_m} q_j^i p_j , & \text{ if } j = m+1 \end{cases}$$

Finally we calculate the payoff to the traders:

$$P^i(s) = u^i(\xi^i(s)) .$$

In this way, we obtain an n-person game in the standard "strategic" (or "normal" form).

 This game can easily (like any game in strategic form) be represented as a game in extensive form. In fact for each of the $n!$ permutations of I_n, we get a tree (of length n) where the players move in succession and have each only one information set. [See Figure III for $I_n = \{1, 2, 3\}$ and the identity permutation.]
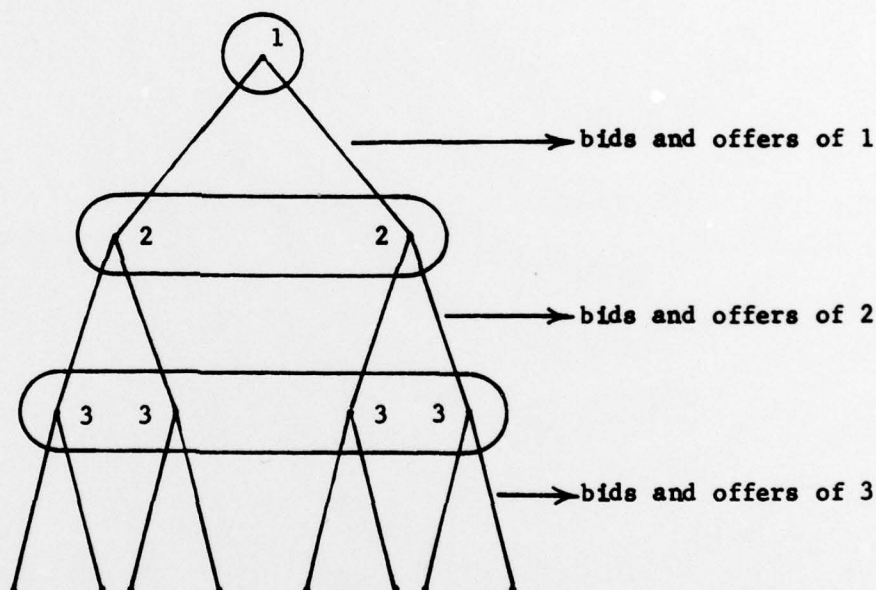

→ bids and offers of 1

→ bids and offers of 2

→ bids and offers of 3

FIGURE V

 Each of these $n!$ games is in its coarsest form. They are all equivalent so we may denote them by the single symbol Γ^*. We have shown in [3], [8], [13], that--under appropriate conditions--the N.E.'s of Γ^* "converge" to the C.E.'s of the market if the market itself "approaches" a nonatomic market. (See [13], [14] for rigorous definitions of "converge" and "approaches.") In this sense, the N.E.'s of Γ^* may be considered "associated" with the C.E.'s of the market.

By taking any one of the $n!$ representations of Γ^* and refin-
ing information sets we derive a large class $\mathcal{C}^*$ of games with non-
symmetric information conditions. However, by the Proposition of Section
2, for any game $\Gamma \in \mathcal{C}^*$, $\eta(\Gamma^*) \subset \eta(\Gamma)$. To put this in words: <u>the</u>
<u>C.E.-associated noncooperative equilibria are the only noncooperative</u>
<u>equilibria that are common to all information variants of the market game.</u>

4. <u>MULTISTAGE GAMES, INFORMATION AND COMMUNICATION</u>

<u>Forward Stability</u>

An Example

This last section is devoted to examining two examples to illustrate the increase in the number of equilibria with information and to indicate several difficulties modelling communication conditions.

Example 1. Consider a market with equal numbers of two types of traders. There are n traders of each type. Trader i of type 1 has a utility function of the form:

$$U^i = 2\min[x_1^i, \; y_1^i] + 2\min[x_2^i, \; y_2^i] + m^i$$

where $x_t^i \; (y_t^i)$ is the amount of the first (second) commodity consumed by i during period t (where $t = 1, 2$); and m^i is the amount of a commodity used for payment owned at the end of trade. Trader j of type 2 has the same utility function.

We assume that in each period traders of type 1 have as endowments $(2A, 0)$ units of the first and second goods and traders of type 2 have $(0, 2A)$ units. The goods are perishable hence no inventories are carried forward. Each trader is also assumed to own M units of the commodity used as a means of payment where M is assumed to be "large" ($M = 2A$ for example).

In each period all traders move simultaneously.* Following the bid-offer model described in Section 3, a move by a trader i in the first period is a vector of four numbers $(q_{11}^i, \; b_{11}^i, \; q_{21}^i, \; b_{21}^i)$ consisting of offers to sell and bids to buy for the two commodities. For trader i of type 1 $0 \leq q_{11}^i \leq A$; $q_{21}^i = 0$; $b_{11}^i + b_{21}^i \leq M$ and $b_{11}^i, \; b_{21}^i \geq 0$.

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*One does not need multiperiod games to generate many equilibria, just less than total ignorance suffices even in a game with one move per player.

We define three different games with varying amounts of informa-
tion which have different strategy sets (and even call for a slight modi-
fication of the meaning of a move during the second period). The three
models can be described as follows:

(a) total ignorance,

(b) aggregate market information,

(c) microeconomic information.

(a) Total Ignorance:

The two stage game in its most coarsened form*requires that a trader
when called upon to move the second time forgets what he did during the
first period. At any level when a trader is called upon to move all of
his choice points lie in a single information set. In particular this
means that a trader will not know how much of the means of payment he
has available for the second market because he does not know his income
and expenditures from the first. His move in the second period may be
regarded as a vector of four numbers $(q_{12}^i, b_{12}^i, q_{22}^i, b_{22}^i)$ where however
we assume that $b_{12}^i + b_{22}^i \leq 1$ and they can be regarded as percentages,
i.e. they represent the percentages of the money he has on hand (the amount
he has, he does not know, but the referee or his "banker" does) which
he wishes to spend on goods 1 and 2.

A strategy in this game is a vector of eight numbers (four for
each of the two moves).

(b) Aggregate Market Information:

After the traders have all selected their first moves (as above)
the market calculates prices and total volume of trade. All traders are

*This example includes "amnesia". We have seen from our example in Figure
IV that our theorem does not in general hold true for amnesia. However
it is easy to check that the result holds for this example. Indeed this
holds true for large trading market games in general.

(a) <u>Total Ignorance</u>:

This game has only one active equilibrium point which coincides with the unique competitive equilibrium where $P_{11} = P_{21} = P_{12} = P_{22} = 1$ and each trader has as his final allocation $(A, A; A, A, M)$ and gains $2A + 2A = 4A$ from trade.

(b) <u>Aggregate Market Information</u>:

Suppose that all traders of type 1 use the following strategy

$$q_{11}^i = tA , \quad b_{11}^i = 0 , \quad q_{21}^i = b_{21}^i = tA , \quad 0 < t < 2$$

then:

$$q_{12}^i = A \text{ if } P_{11} = 1 , \quad P_{21} = t/2-t , \quad q_{11} = ntA , \quad q_{21} = n(2-t)A$$

$$= 0 \text{ otherwise;}$$

$$b_{12}^i , \ q_{22}^i = 0 \text{ always}$$

$$b_{22}^i = A \text{ if } P_{11} = 1 , \quad P_{21} = t/2-t , \quad q_{11} = ntA , \quad q_{21} = n(2-t)A$$

$$= 0 \text{ otherwise.}$$

Traders of type 2 all employ the strategy:

$$q_{11}^j = 0 , \quad b_{11}^j = tA , \quad q_{21}^j = (2-t)A , \quad b_{21}^j = 0$$

then:

$$q_{12}^j , \ b_{22}^j = 0 \text{ always}$$

$$q_{22}^j = A \text{ if } P_{11} = 1 , \quad P_{21} = t/2-t , \quad q_{11} = ntA , \quad q_{21} = n(2-t)A$$

$$= 0 \text{ otherwise;}$$

informed of these statistics p_{11} , p_{21} , q_{11} and q_{21} . A
move in the second period is as before, however a strategy now depends
upon the information received. It is a vector of four numbers followed
by four functions where each function is a function of eight variables
--the four numbers selected by the trader at the first period and the
information obtained from the market at the end of the first period.
A strategy is: $(q_{11}^1, \; b_{11}^1, \; q_{21}^1, \; b_{21}^1; \; \phi_1^1, \; \phi_2^1, \; \phi_3^1, \; \phi_4^1)$ where:

$$q_{12}^1 = \phi_1^1(q_{11}^1, \; b_{11}^1, \; q_{21}^1, \; b_{21}^1; \; p_{11}, \; p_{21}, \; q_{11}, \; q_{21}) \; , \quad \text{etc.....,}$$

(c) Microeconomic Information:

We may assume that each trader obtains complete information, i.e.
he is informed of the bids and offers of all traders--he is given $4n$
numbers; the moves of a trader are four numbers, as before, however a
strategy is a vector of four numbers followed by four functions where
each function is a function of $4n$ variables. A strategy is:
$(q_{11}^1, \; b_{11}^1, \; q_{21}^1, \; b_{21}^1; \; \phi_1^1, \; \phi_2^1, \; \phi_3^1, \; \phi_4^1)$ where

$$q_{12}^1 = \phi_1^1(q_{11}^1, \; b_{11}^1, \; q_{21}^1, \; b_{21}^1; \; ..., \; q_{11}^n, \; b_{11}^n, \; q_{21}^n, \; b_{21}^n) \; , \quad \text{etc.....,}$$

4.1. Noncooperative Equilibrium Solutions

As the number of traders is increased in an appropriate manner,
as has been noted in Section 3 and proved elsewhere [13], certain nonco-
operative equilibria approach (in the sense of prices and distributions
associated with them) the competitive equilibria. In the examples here
we assume that n is large enough that the effect of any individual is
negligible (i.e. we treat the traders nonatomically).

similarly $b_{12}^j = A$ if $p_{11} = 1$, etc.

$\qquad\qquad = 0$ otherwise.

If these strategies formed an equilibrium then traders of type
1 would obtain $2(2-t)A + 2A$ and traders of type 2 would obtain $2tA + 2A$.

If any trader of either type departs from his strategy he will
obtain 0 in the second period, but for $0 < t < 2$ there is no way that
a trader of either type can gain enough in the first period to compensate
for the $2A$ he will lose in the second period.

Thus all strategies with $0 < t < 2$ define a class of equilibrium
points.

Without communication, coordination and enforcement among the traders
it is hard to believe that these equilibria would have much chance of
appearing in a multistage game.

It is both straightforward and tedious to show that if all individuals
have detailed microeconomic information there are more strategies and
more equilibria. With aggregate information only it is not possible to
identify which individual has "spoiled the market" hence action must be
directed against an aggregate. If details are known in some games, re-
prisals may be directed at individuals. It is difficult to do so in these
games because all are forced to deal through the market.

4.2. On Communication and Coordination

Suppose we actually tried to use the market structure (b) above
as a basis for an experimental game. The most natural way to run such
a game would be to have all traders make bids and offers in the first
period to some type of market mechanism; obtain aggregate information

then move again. This is especially true if say there were 100 or more individuals in the markets.

It is fairly clear that if we were attempting to run this experiment with any efficiency we would not have all the players writing messages containing complex descriptions of what they are going to do. Intent in an anonymous many trader market is essentially signalled by previous and current actions and not generally by all sending each other threat strategies.

Even if we were to build the possibility for sending communications into a game with contingency planning, the problem of assigning a degree of plausibility to different equilibria still remains.

We close with a simple example of a game which can be used as a two person experiment. Consider a 3 x 3 matrix game, as shown in Figure

	1	2	3
1	5, 5	-2, 11	-10, -10
2	11, -2	0, 0	-10, -10
3	-10, -10	-10, -10	-11, -11

FIGURE VI

which is to be played twice. After the first play the moves and payoffs of the players are announced; after which they play again and the final score is given by adding the score from both periods.

It is easy to see that there are two different equilibrium points in this game.

If both players select move 2 each period regardless of any information the payoff to each is 0 and neither can improve.

If both play as follows:

"I will select move 1 in period 1; if he does likewise

then I select move 2 in period 2; if he does anything

else I select move 3 in period 2."

then the payoff to each is 5.

If this game were actually played and the two players did not know each other and could not discuss matters or make bargains it is hard to believe that each could infer that they would always choose the second equilibrium.

Suppose however that the first player were permitted as his first move to precommit himself to a plan for play which is given to the referee who then randomizes and with probability p this information is given to the second player and with $1-p$ the information is not given. If $p = 1$ it is clear that with virtual certainty they can achieve an equilibrium with payoffs of 5 each. Although this game also has an equilibrium point with payoffs of 0.

If $p = 0$ this is the previous case. As p sweeps across the range of 0 to 1 it appears plausible that the behavior of player 2 when called upon to play without having received a message from player 1 will vary with p. In spite of the work on subjective probability and Harsanyi's recent work on the selection of a specific noncooperative equilibrium [15] it appears to us that neither current theory nor experimental or other knowledge of behavior provides us with an adequate answer to how player 2 will behave.

5. CONCLUDING REMARKS

Our goals in this paper have been threefold. First we wished to demonstrate that in games in extensive form in general there is a natural heirarchy among equilibrium points with those pure strategy equilibria of games with less information remaining as equilibria when information is increased. *It*

Second we wished to show that the competitive equilibria of a market game could be interpreted as equilibria which belong to the set of non-cooperative equilibria for <u>any level</u> of information in the market. As soon as there is more information than total ignorance (and in general there is) then in general the competitive equilibrium will not be the only noncooperative equilibrium of the market game.

Our third point concerns the plausibility of the occurrence of noncooperative equilibria and the relevance of communication and coordination. General equilibrium theory is essentially non-historical and non-institutional. It has no natural place for items such as custom, habit, friendship and other sociological variables. These items may easily convert highly implausible correlations of strategy into plausible possibilities. Thus our observation is that when information and communication are taken into account the competitive equilibrium solution is neither the only nor necessarily the most plausible of the noncooperative equilibria of a market game.

The competitive equilibrium has the desirable property that it belongs to all market games regardless of information refinements. But in actuality total ignorance is not usually the rule; furthermore the institutions in which trade takes place provide history and communication which

may make it plausible to have large groups of individuals who are not even well known to each other, correlate their strategies to produce other equilibria.

Elsewhere it has been suggested [16] that codes and traditions may be regarded as the means for providing enough correlation of strategies to enforce noncooperative equilibria other than the competitive equilibria.

It should be noted that once we consider games with groups of individuals using strategies which can be regarded as threats against other groups the distinction between cooperative and noncooperative game theory becomes difficult to maintain.

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