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AFFDL-TR-77-124

**MODAL INVESTIGATION
OF LIGHTWEIGHT AIRCRAFT STRUCTURES
USING DIGITAL TECHNIQUES**

*STRUCTURAL INTEGRITY BRANCH
STRUCTURAL MECHANICS DIVISION*

DECEMBER 1977

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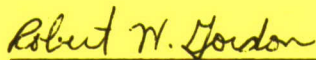
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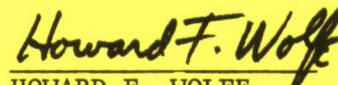
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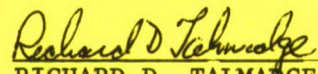
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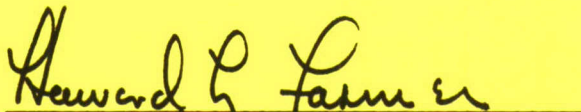

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Digital impact response test techniques were used to measure the dynamic properties of lightweight aircraft structures to include natural frequencies, mode shapes and modal damping. Two different types of structures were tested, honey-comb and skin-stiffened panels. The digital impact response method used consisted of applying a transient force pulse to the structure, measuring the structure's response at various points, digitizing, calculating the transfer functions using fast Fourier transforms, and deter-		

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mining the dynamic properties from these data. A second method was used on these same structures for direct comparison purposes. This method was an analog technique using sine sweep tests and accelerometer mapping. The comparison indicated close agreement in the results. The use of the digital technique has resulted in a considerable savings in the manhours required to obtain the dynamic properties of the structures.

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FOREWORD

This effort was performed by the Structural Integrity Branch, Structural Mechanics Division, Air Force Flight Dynamics Laboratory (AFFDL), in support of three development programs. The F-4 rudder tests were conducted under project 69CW0201. The weldbound program was initiated under work unit 14710118 and was continued with the adhesive bonded panel program under work unit 14710128. Both programs were completed under work unit 13670404.

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SECTION I
INTRODUCTION

The Air Force Flight Dynamics Laboratory (AFFDL) conducts research and exploratory development on the sonic fatigue phenomenon in military flight vehicles. The sonic fatigue life of flight vehicle structural components directly affects their reliability and life cycle costs. Sonic fatigue tests are conducted on structural components to determine their life. The dynamic properties of a structural component must be known to accurately measure its sonic fatigue response. The required dynamic properties, i.e., natural frequencies, mode shapes and modal damping of a structure, are useful in determining possible failure modes and high stress locations for strain gage placement.

The methods previously used by the AFFDL to determine the dynamic properties of sonic fatigue test structures were sine sweeps, sand patterns, and accelerometer mapping. These analog techniques produce adequate results but are time consuming and difficult to apply in many cases. The digital modal testing techniques introduced in recent years have greatly reduced the time and effort required to measure structural dynamic properties [1,2]. The digital methods employ the Fast Fourier Transform, digital minicomputer systems, and frequency domain techniques to extract modal information from a structure's transfer functions. Much greater flexibility is also available with the digital data capabilities of the new systems.

A very useful version of the basic digital technique is the digital impact technique which uses a small hammer to excite a structure. Digital impact testing reported in the literature has been limited mainly to relatively stiff, massive structures such as machine tools and automobile frames [3,4]. Few studies of the digital impact technique used on lightweight aircraft skin-stringer type structures have been reported [5].

This technical report describes a digital impact test technique used to determine the dynamic properties of three types of lightweight aircraft structures used in sonic fatigue tests. The first test structure was an F-4, boron-epoxy aircraft rudder, the second was a series of aluminum, skin-stringer panels with riveted and weld-bonded joints, and the third was an aluminum, skin-stringer panel with adhesively bonded joints. A brief discussion of digital fourier analysis is presented and the details of the test technique are described. The measured mode shapes of each of the test structures are presented and are compared directly with mode shapes obtained by conventional analog methods. Advantages of the digital impact technique are discussed along with the specific problems of applying the digital technique on lightweight, flexible aircraft structures.

SECTION II

DIGITAL FOURIER ANALYSIS

The digital impact modal analysis technique is based on a frequency domain characterization of dynamic systems. A frequency domain representation of a system offers no more information than a time domain representation but the information is in an easily applied form. The frequency response functions or "transfer functions" of the system provide this convenient data format. The transfer functions are determined from the fourier transforms of the system's time domain inputs and outputs. A system's transfer functions contain all the information required to characterize the system's dynamic behavior including natural frequencies, mode shapes and modal damping.

The digital Fast Fourier Transform (FFT), developed in recent years, is a powerful mathematical tool in implementing frequency domain dynamic analysis [6]. Before the development of the FFT, time to frequency domain transformations were very difficult to perform, thus making frequency domain analysis unfeasible. The FFT algorithm is used with a digital minicomputer and can perform time-frequency domain transformations in seconds. Since the time or frequency domain information is in digital form, mathematical manipulations are easily performed by the computer. The FFT is a discrete, finite fourier transform which was developed for use with digital processors. A time domain signal to be transformed is first dig-

itized to N discrete time samples of finite duration. The computed fourier spectrum is therefore not continuous, but contains magnitude and phase information only at a finite number of discrete frequencies. The frequency spectrum is a complex quantity and therefore requires two values to describe each frequency point. A time signal with N samples will therefore yield N/2 frequency domain points [6].

The transfer function of a dynamic system relates the response of one point in the system to the input excitation at the same point or any other point in the system. The transfer function is a complex valued function of frequency and can be presented in many forms for dynamic structural systems [1]. The form used in this effort is the dynamic compliance transfer function which relates the response displacement of a structure to a force input. A single transfer function is required to define a dynamic system with one input and one output. For a complex system with m inputs and n outputs, m x n transfer functions are required. Real dynamic structural systems with distributed loads and continuous surfaces possess an infinite number of inputs and outputs. However, transfer functions need not be measured at an infinite number of points on the structure. A reasonably small number of transfer functions measured at selected locations on a structure will adequately define its natural frequencies, mode shapes, and modal damping.

The transfer function, $H(f)$, may be expressed

$$H(f) = \frac{Y(f)}{X(f)} \quad (1)$$

where $X(f)$ and $Y(f)$ are the frequency spectra of the time domain input, $x(t)$, and output, $y(t)$, respectively. This expression is adequate to compute the transfer function of systems with no extraneous noise inputs, but this rarely occurs in real structures. When noise is present on either $x(t)$ or $y(t)$ it will appear as error in the transfer function and cannot be reduced by averaging techniques [6]. Another expression of the transfer function is commonly used involving power spectral density functions. The auto power spectral density of $x(t)$ is defined as:

$$G_{xx}(f) = X(f) \cdot X^*(f) \quad (2)$$

where the $*$ denotes a complex conjugate quantity. The cross power spectrum between the input and output is defined as:

$$G_{yx}(f) = Y(f) \cdot X^*(f) \quad (3)$$

Multiplying the numerator and denominator of equation (1) by $X^*(f)$:

$$H(f) = \frac{Y(f) \cdot X^*(f)}{X(f) \cdot X^*(f)} = \frac{G_{yx}(f)}{G_{xx}(f)} \quad (4)$$

There is an important advantage to expressing the transfer function this way. Averaging can be applied to the power spectral densities G_{xx} and G_{yx} which can eliminate noise from the transfer function [1]. Another frequency domain function is used to judge the quality of a measured transfer function. The coherence function, $\gamma^2(f)$, is defined:

$$\gamma^2(f) = \frac{|G_{yx}|^2}{G_{xx} G_{yy}}, \quad 0 \leq \gamma^2 \leq 1 \quad (5)$$

where $\overline{\quad}$ denotes averaging. The coherence function is a measure of the causal relationship between the input and the output. A value of γ^2 equal to 1 means that the measured response is due solely to the applied input. Values of γ^2 less than 1 indicate system nonlinearities or response due to extraneous force inputs (7). The coherence function is generated at the same time as the transfer function and gives a good indication of measured data quality. "Bad" data can be recognized and discarded while extraneous force inputs or noise can be discovered.

The information required to characterize the dynamic properties of a structure can be extracted from its transfer functions. The notes of Formenti [9] are used below to briefly explain methods of modal parameter extraction. Generally, structures can have both real and complex normal modes. Real normal modes have all points on the structure vibrating either in phase or 180° out of phase with each other. Complex normal modes can have relative phase angles between points of any value. Most structures with well separated, lightly damped resonances will have mode shapes that are real or nearly real, that is, the phase angles will be close to 0° or 180° . For real normal modes, a resonant frequency appears as a peak in the imaginary part (quadrature) of the transfer function with a zero value in the real part (coincidence). For any resonant frequency, the modal displacement at a specific point on a structure is the quadrature amplitude of the transfer function measured at that point. If the mode shape is complex, however, the modal displacement will

not be the transfer function quadrature amplitude. The modal displacements and phase angles of complex normal modes can be determined from a Laplace domain representation of the transfer function. The Laplace domain method works equally well in determining modal displacements from real normal modes as it does from complex normal modes and is thus the better method to use on structures.

The modal damping at a natural frequency can be determined from a Nyquist plot of the transfer function in the frequency range close to the natural frequency. The plot approximates a circle in the Nyquist plane. The damping factor, C/C_c , is calculated from a least squares circle fit of the plot by:

$$C/C_c = 2R/[f_n (\Delta f / \Delta S_{\max})] \quad (6)$$

where R is the circle radius, f_n is the natural frequency, Δf is the discrete frequency interval of the transfer function and ΔS_{\max} is the maximum arc length between frequency points (8). This method is valid for real and complex mode shapes, but is generally used in conjunction with quadrature amplitude for characterizing real modes only. The Laplace domain method of modal displacement measurement described above also determines modal damping and can be used on both real and complex mode shapes.

The structural mode shape at a specific natural frequency is formed by mapping the modal displacements of transfer functions measured at points on the structure. Transfer functions may be obtained by exciting the structure at points where modal displacement data

are desired and measuring the structural response at a single reference point. Or, the structure may be excited at a single point and the structural response measured at the desired locations. The same information will be acquired using either method due to the reciprocal property of the transfer function [9]. The type of structure to be tested will determine which method should be used. This is discussed in detail in Section III.

A digital smoothing technique known as "windowing" can be applied to transfer functions to enhance mode shape mapping [10]. Windowing is the multiplication of the system response time signal by an exponentially decaying time function. This artificially causes the response signal to "die out" more quickly and offers two advantages. Damping is added to transfer function peaks which smooths and broadens them (with a negligible effect on peak amplitudes). Wider transfer function peaks allow accurate measurement of modal displacements, even when resonant peaks shift slightly in frequency. Also, windowing can reduce digital time sampling leakage error by causing the response to approach zero more quickly in the sampling window [6,10]. Windowing must be applied with care, however, since the added damping effect may "smear" together closely spaced modal peaks. Modal damping cannot be measured from windowed transfer functions because of the added damping of the window.

SECTION III
IMPACT TESTING TECHNIQUE

The basic modal analysis technique employing the digital fourier transform can be applied with a variety of structural excitation methods [2,10]. These include hammer impact, point excitation by a shaker or magnetic transducer, and acoustic excitation of the whole structure. Various input spectra, including wide or narrow band random, can be used. Fast sine sweeps can also be used for excitation. The digital impact modal analysis technique uses a hammer impact for excitation, but the technique is basically the same for all the above types of excitation.

The digital impact technique, described here, consists of applying a transient force pulse at one point on a structure and measuring the structure's response at another point. The hammer impact force pulse simultaneously excites all modes of the structure within the frequency range of the impact spectrum. The pulse is generated by a small hammer instrumented with a force gage and the response is measured with a miniature accelerometer. The impact and response signals are then processed to form the transfer functions. The input energy spectrum of a hammer impact on a structure is dependent on hammer mass, hammer head stiffness and the local structural stiffness at the impact point [5,10]. Increasing the hammer mass or lowering the hammer head stiffness or local structural stiffness will lower the upper frequency cutoff of the energy spectrum.

The hammer used in this effort had low mass with a stiff head, so the structural stiffness was the limiting factor. Impacts on very stiff massive structures with this hammer produced relatively flat force spectra to over 5000 Hz, but the spectrum of an impact on thin flexible aircraft skin contained energy only up to 100 Hz. Thus, it is very important to insure that impacts on a structure to be tested will produce energy adequate to cover the desired frequency range. Impacts can be made at almost any location on a stiff structure without loss of frequency content. Transfer functions can be measured by impacting at desired locations and measuring response at a common point. However, on flexible skin type structures, impacts must be made at relatively stiff locations at or near a stiffener. Therefore, when testing a flexible structure it is more advantageous to apply the impacts at one stiff point while the response is measured at the desired points on the skin.

A Fourier Analyzer System, shown schematically in Figure 1, is used to process the force and acceleration signals. The signals are simultaneously digitized and stored. The acceleration signal is integrated twice then both the resulting displacement and force time records are fourier transformed using the FFT algorithm to form the dynamic compliance transfer function as detailed in Section II. A Hewlett-Packard Fourier Analyzer System, shown in Figure 2, was used for this effort.

A modal analysis computer "user" program in the minicomputer controls the system. Two different modal analysis programs were

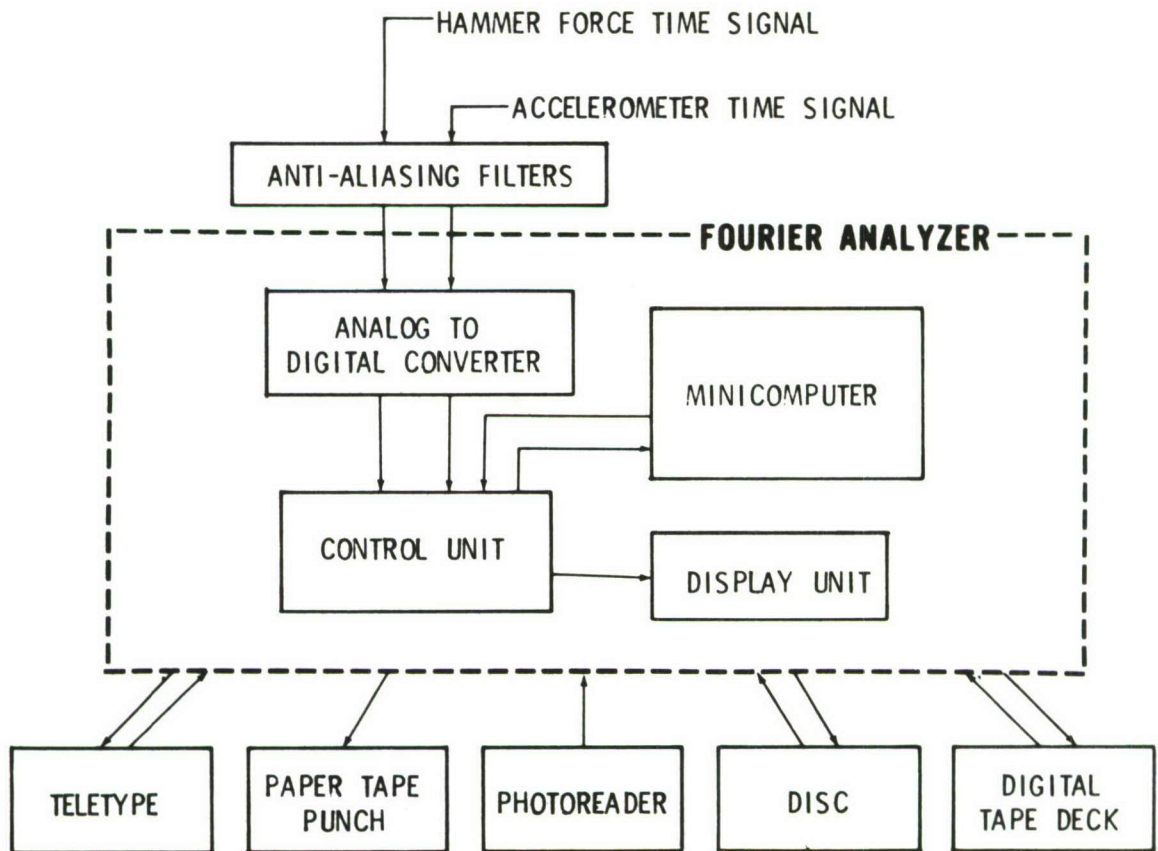


Figure 1 Block Diagram of Fourier Analyzer System

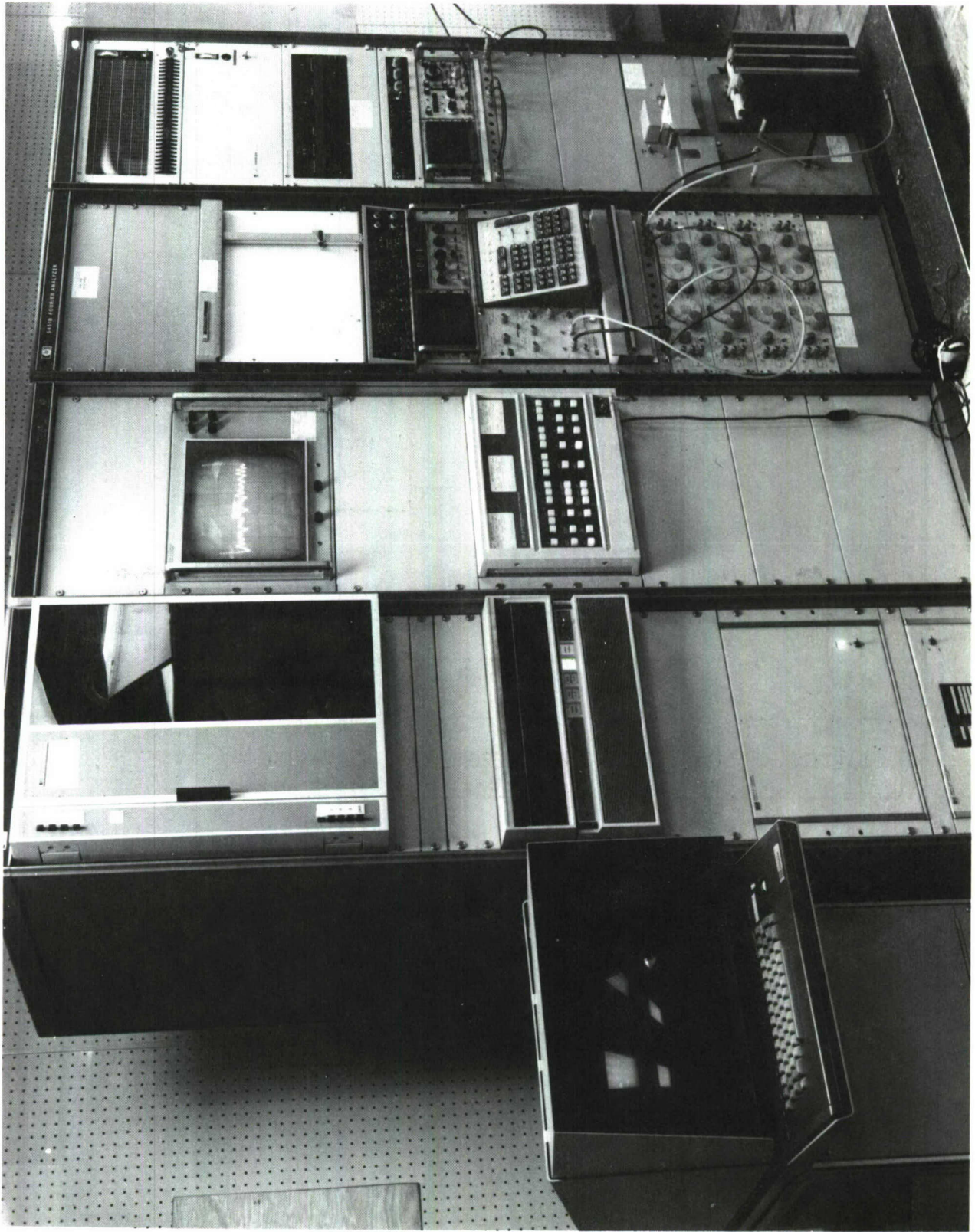


Figure 2 Fourier Analyzer System at AFFDL Sonic Fatigue Facility

used during the effort reported here. The first program, designated USER 30, was developed by the Structural Dynamics Research Corporation (SDRC) in 1973 [11]*. It is one of the first generation of modal analysis programs to be developed. USER 30 is capable of measuring and storing up to 50 transfer functions. It can generate up to eleven mode shapes at one time from a group of transfer functions. USER 30 extracts modal displacements as transfer function quadrature amplitude and therefore can only be used on structures with lightly damped, well separated modes. Another program, USER 6868, developed by the Air Force Materials Laboratory, must be used along with USER 30 to determine modal damping. The second program is designated USER 230 [12]. It is a second generation program and is much improved over USER 30. It can handle over 100 transfer functions and can compute over 15 modes at one time. Modal parameters can be determined by the quadrature amplitude for normal modes. USER 230 can also use a Laplace domain method to extract parameters from complex modes (Section II). Modal damping is calculated simultaneously with the mode shapes by USER 230. The program also includes a more flexible display package. The development of modal analysis user programs has gone on continuously since the first generation programs were written. Programs currently in general use possess even greater capabilities than those of USER 230.

* Since 1973, SDRC has developed more advanced modal analysis user programs which were not available for use during the effort reported here.

A grid of points, at which displacement information is desired, must be determined for each structure to be tested. The grid should be fine enough to pick up all expected mode shapes. If a single impact point is to be used, care must be taken to place it away from any node lines of expected modes. The same care must be taken in selecting a common response accelerometer location. After the grid is determined, the impacting is begun and the transfer functions for each point are measured and stored. The coherence function is carefully observed on the display unit for each transfer function to ensure good data quality. Next, a few transfer functions measured at high response locations are studied and natural frequency peaks are identified as described in Section II. The mode shape for each natural frequency is then generated by the Fourier Analyzer System. An animated isometric view of each mode is displayed on the system's CRT display unit. The slow motion display simulates the actual vibration of the structure in the mode of interest. Vibration speed and viewing angle can be controlled by the operator. The animated display provides valuable information in analyzing the mode shapes of any structure. The modal displacement information can also be printed out in digital form. This data was used in the tests reported here to generate contour maps of each mode shape on a large digital computer (CDC 6600) and a commercially available mapping routine (General Purpose Contouring Program, California Computer Products, Inc.).

SECTION IV

F-4 RUDDER TEST

The digital impact modal analysis technique was used to determine the dynamic properties of an F-4 aircraft boron-epoxy rudder. The rudder was previously installed on an operational F-4 aircraft for an in-service evaluation of composite technology. The modal analysis was performed prior to a sonic fatigue test of the rudder in the Air Force Flight Dynamics Laboratory (AFFDL), Sonic Fatigue Test Facility. The rudder consisted of laminated boron-epoxy skins over a full depth aluminum honeycomb core, a fiberglass spar and a titanium drive rib, as shown in Figure 3. For the modal analysis and the subsequent sonic fatigue test the rudder was mounted from its hinge points and torque tube assembly, shown in Figure 4, to simulate the boundary conditions of the aircraft mounting. Details of the sonic fatigue tests of the rudder can be found in Reference [13].

Before the digital modal analysis was performed, the dynamic properties of the rudder were determined using established analog techniques. These included sine sweep and accelerometer mapping techniques used at AFFDL on sonic fatigue test specimens. The results of these tests are compared with the digital technique results below. The rudder was excited acoustically by a large loudspeaker set-up next to the rudder. An accelerometer was used to measure the rudder's response. A discrete frequency signal to the

RUDDER WEIGHT	(LB)
METAL PART	11.85
FIBERGLASS SPAR	0.98
CORE	3.20
ADHESIVES AND PAINT	2.42
BORON-EPOXY SKINS	<u>5.50</u>
TOTAL WITHOUT BALANCE WEIGHT	23.95
UPPER BALANCE WEIGHT	6.90
TOTAL RUDDER	30.25

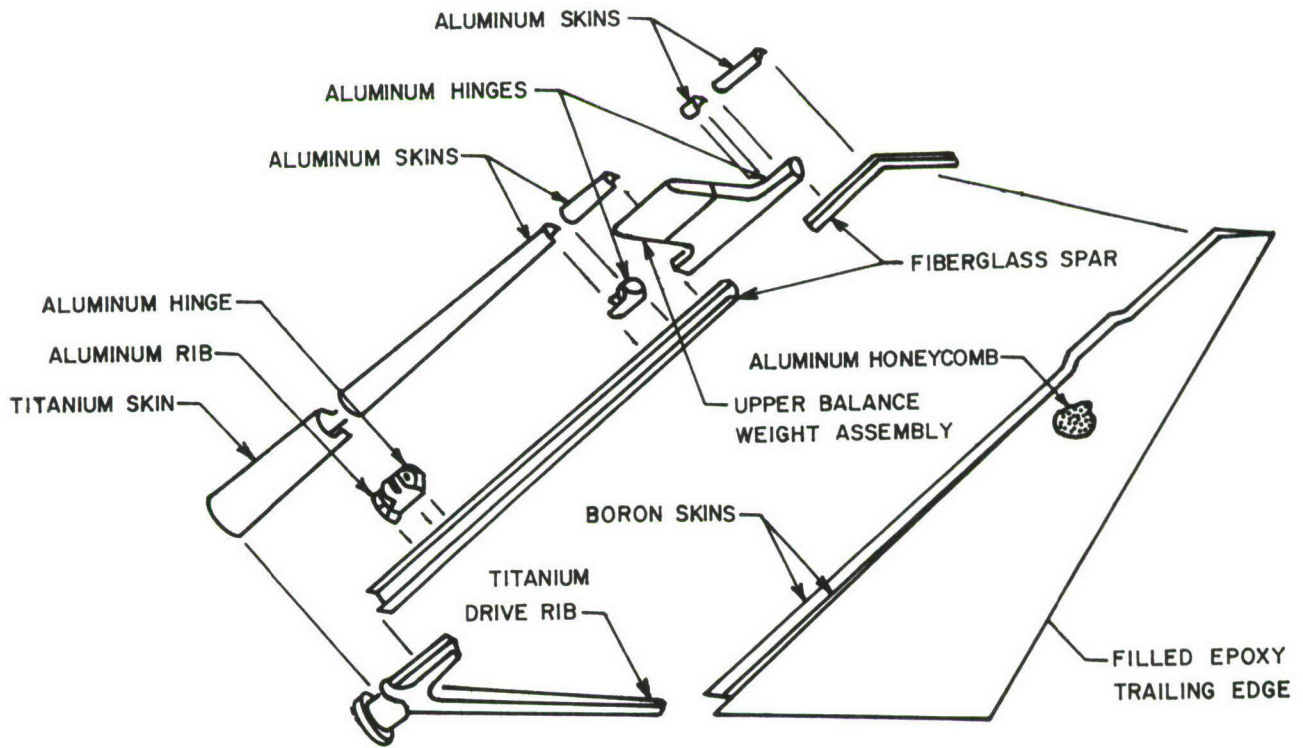
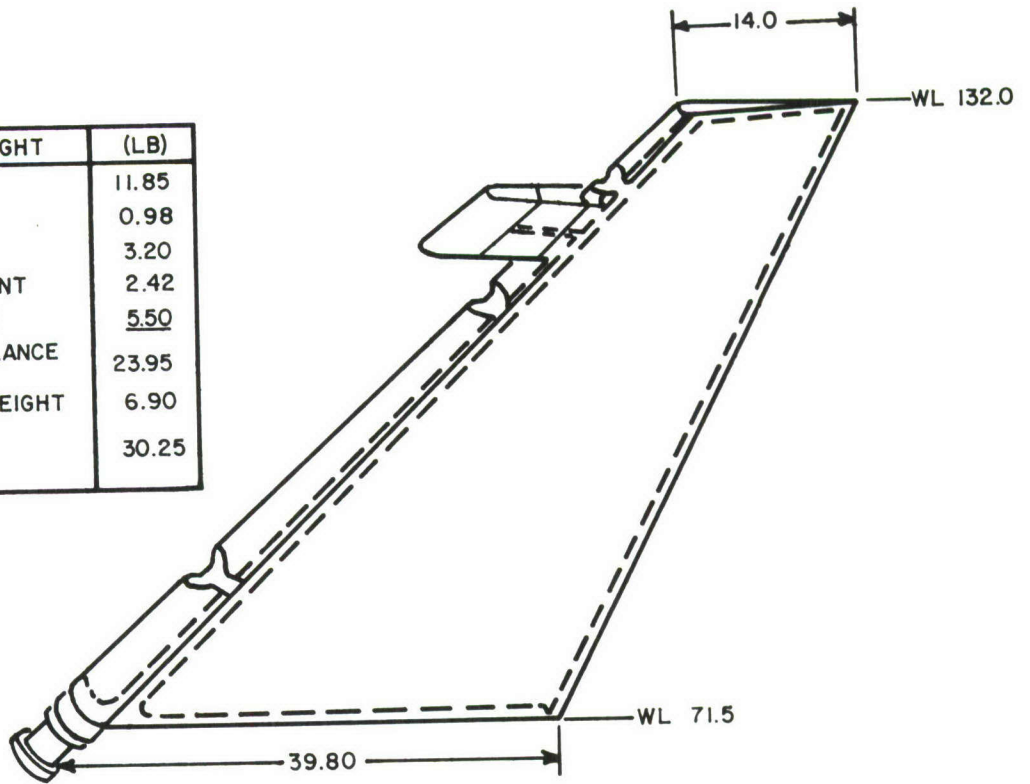


Figure 3 Schematic View of F-4 Boron/Epoxy Rudder

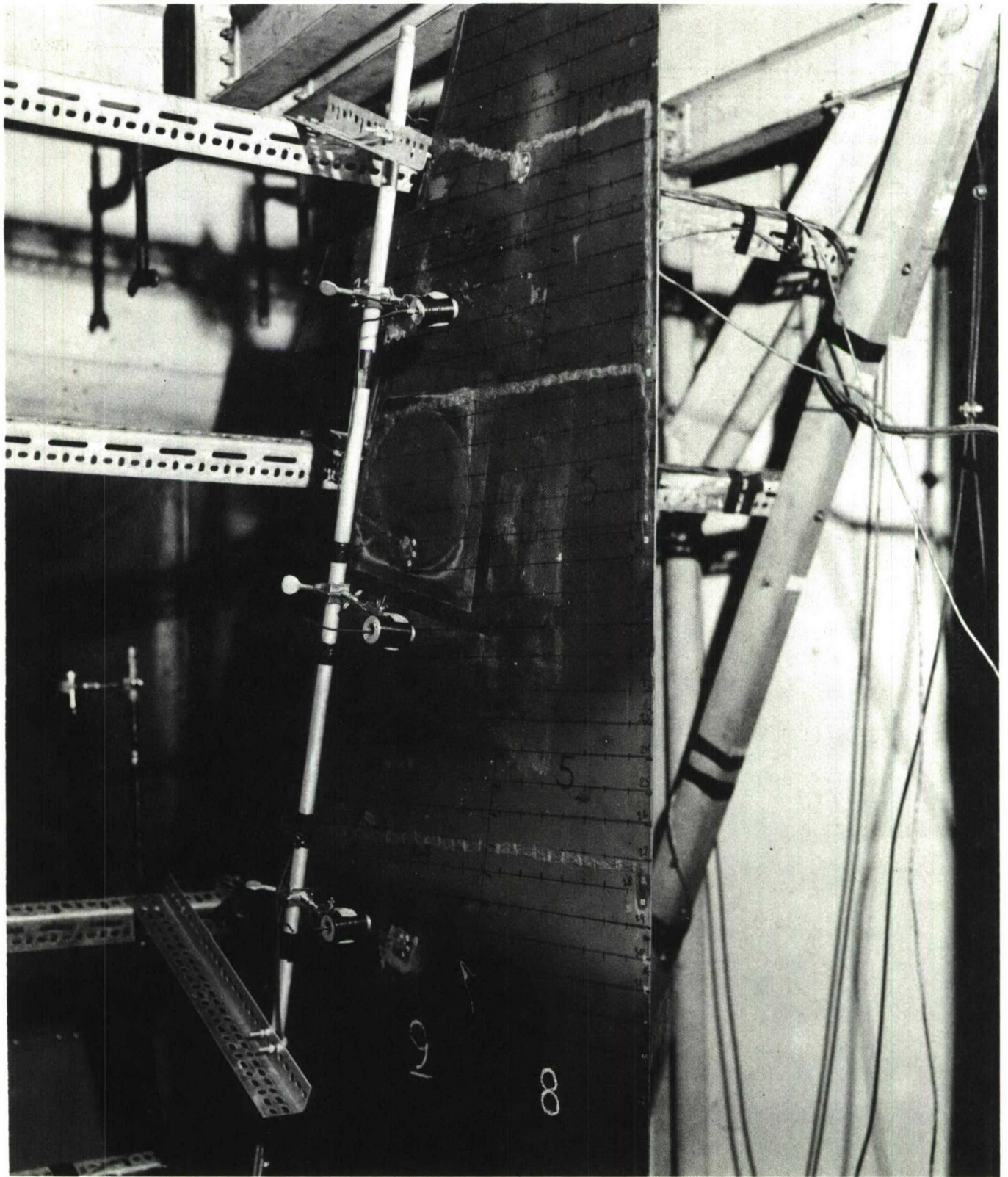


Figure 4 F-4 Rudder in Test Chamber at the AFFDL Sonic Fatigue Test Facility

loudspeaker was swept slowly in frequency until peaks in the accelerometer response were noted. From these peaks the following frequencies were selected for further investigation: 64, 122, 229, 269, 340, 460, and 560 Hz. The mode shape at each frequency was mapped by holding the acoustic excitation at the resonant frequency and measuring the acceleration response at 36 grid locations on one surface of the rudder, as shown in Figure 5. The phase angle of the response at each point was measured relative to a second accelerometer which remained at a reference location. From these measurements, the relative displacements were calculated, normalized, and plotted by hand in an isometric view. The most responsive modes to the acoustic excitation were found to be at 122 Hz, 229 Hz, 340 Hz, and 460 Hz in order of highest response. These four modes are shown in Figures 6 through 9. The half-power bandwidth method was used to measure the damping factors (C/C_c) from strain response plots for a few of the most responsive modes. The rudder was mounted in the sonic fatigue test chamber and excited by a discrete frequency sweep from 50 to 600 Hz at a constant overall sound pressure level of approximately 130 dB provided by a Wyle WAS 3000 air modulator. Strain response was measured from strain gages on the rudder for the sonic fatigue test. Resonant peaks too close in frequency which did not meet the necessary requirements for the bandwidth method were omitted from damping calculations. The damping factors determined ranged from 0.0045 to 0.0140 as shown in Table I.

The digital impact modal analysis of the rudder was performed

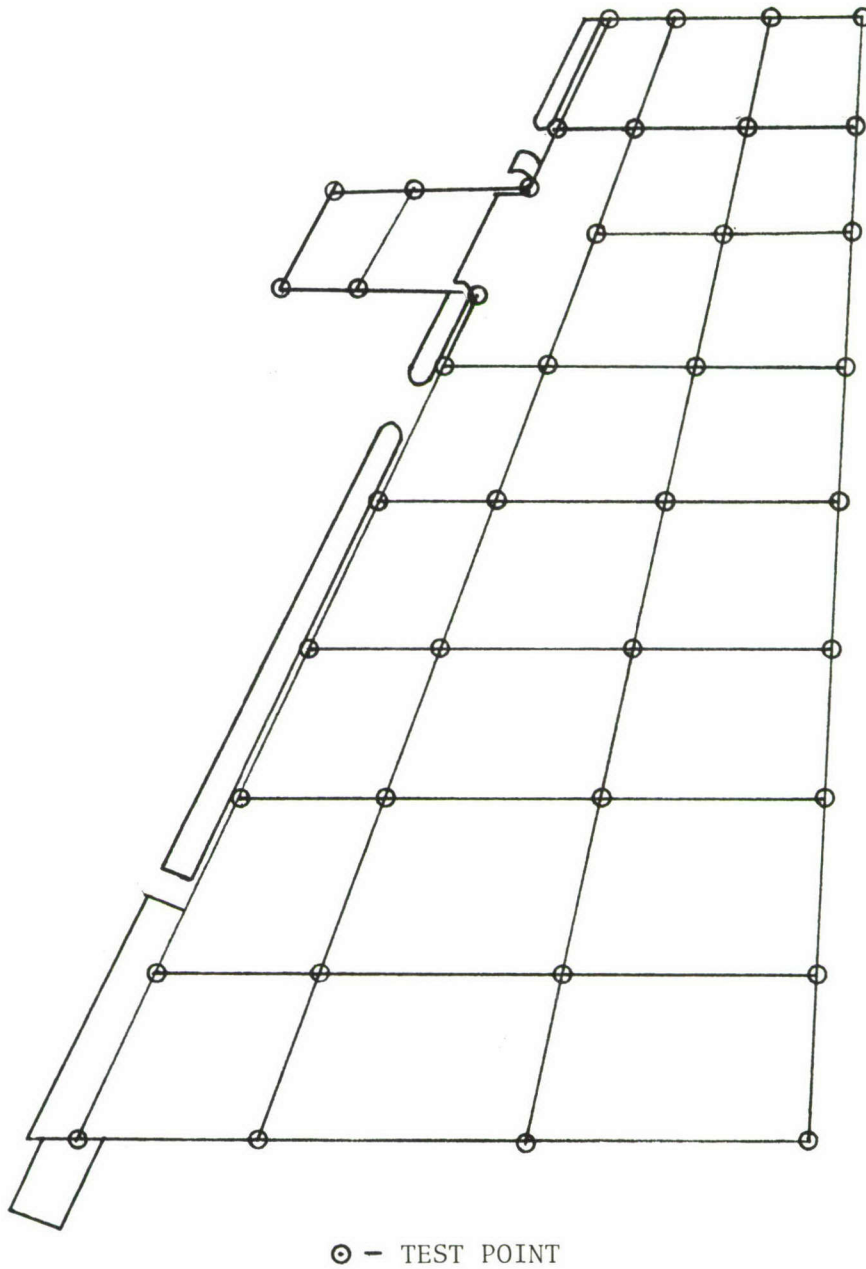


Figure 5 F-4 Rudder Test Point Grid

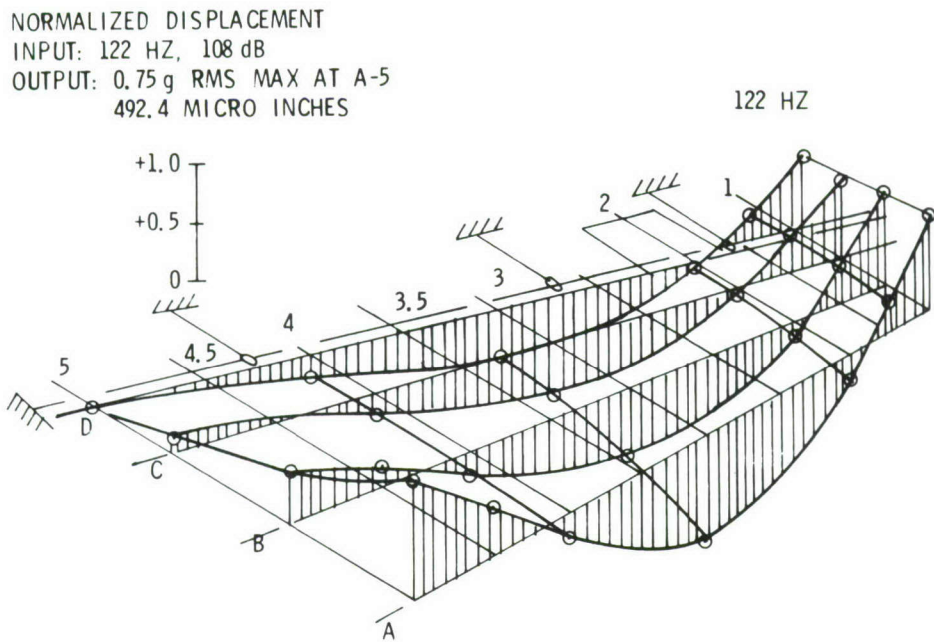


Figure 6 F-4 Rudder Mode Shape from Accelerometer Mapping, 122 Hz

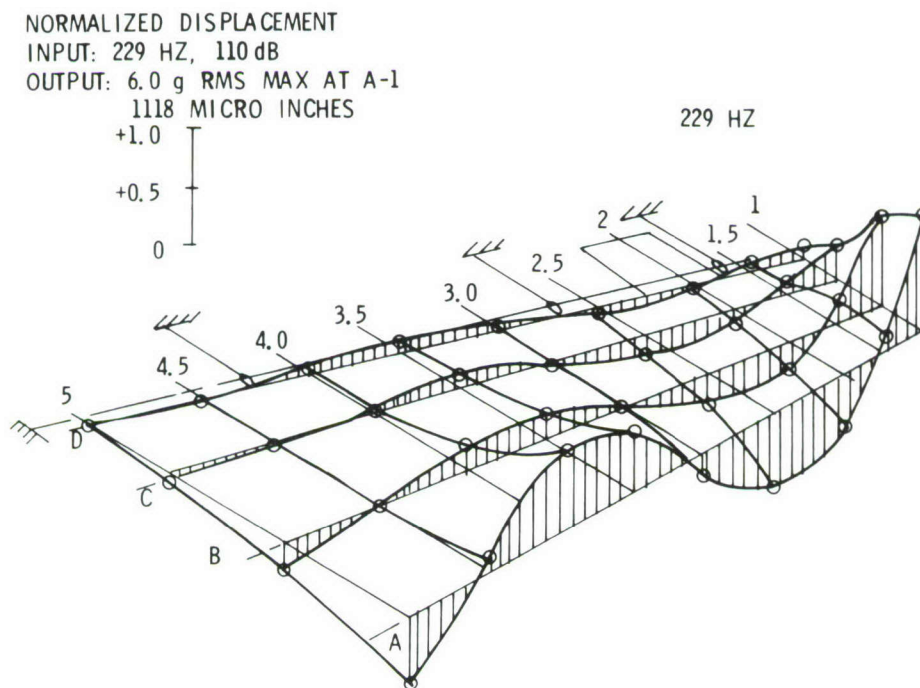


Figure 7 F-4 Rudder Mode Shape from Accelerometer Mapping, 229 Hz

NORMALIZED DISPLACEMENT
INPUT: 340HZ, 110dB
OUTPUT: 36G RMS AT A-5
304 MICRO INCHES

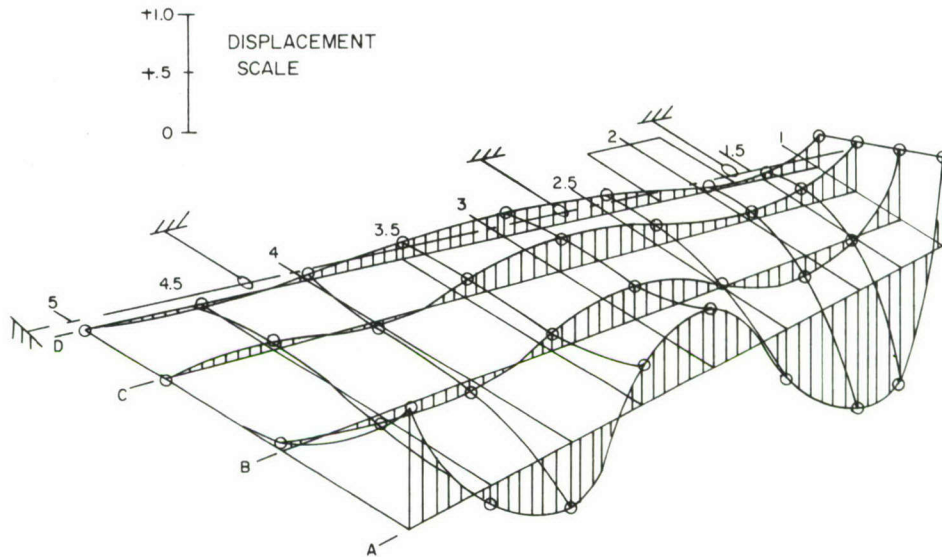


Figure 8 F-4 Rudder Mode Shape from Accelerometer Mapping, 340 Hz

NORMALIZED DISPLACEMENT
INPUT: 460HZ, 110dB
OUTPUT: 4.5G RMS MAX AT A-2.5
207 MICRO INCHES

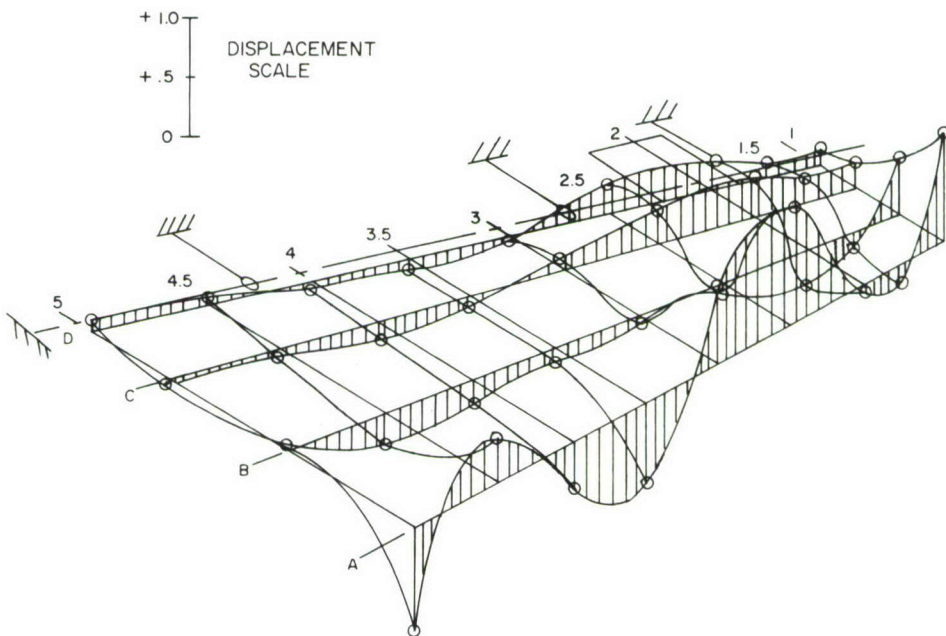


Figure 9 F-4 Rudder Mode Shape from Accelerometer Mapping, 460 Hz

TABLE I

F-4 RUDDER DAMPING COMPARISON

Analog Technique (Bandwidth Method)		Digital Technique (Transfer Function Method)	
Modal Frequency (Hz)	Damping Factor (C/C _c)	Modal Frequency (Hz)	Damping Factor (C/C _c)
220.5	.0079	58.2	.0012
223.5	.0100	121.5	.0061
224.5	.0134	162.5	.0124
225	.0140	223.5	.0070
226.2	.0077	303.5	.0054
	<u>.011</u> AVG	319.5	.0059
		395.5	.0022
440	.0045	440.5	.0047
441	.0052	448.5	.0022
	<u>.0048</u> AVG		
521	.0053		
RANGE - .0045 to .0140		RANGE - .0012 to .0124	

at the Air Force Materials Laboratory (AFML) using an AFML Hewlett-Packard Fourier Analyzer System. The USER 30 version of the modal analysis user program, described in Section III, was used for the rudder tests.

The measurement grid from the analog tests was also used for the digital measurements. The stiff, honeycomb rudder allowed hammer impacts with spectral content up to 500 Hz which was adequate for the rudder test. Therefore, the rudder could be impacted at each grid point and the response measured at a single reference location on the rudder's surface with an accelerometer. Transfer functions were measured from a single impact at each grid point and resonant frequencies were selected from a few of the transfer functions showing the highest response. A few transfer functions were also measured with a different response point. This helped identify any resonant modes which may have had a node line near the original response point and thus appeared low in amplitude. Resonant frequencies were noted at 28, 59, 121, 161, 181, 216, 224, 303, 320, 396, and 448 Hz. Mode shapes were generated for each natural frequency and each was studied on the animated display. Contour maps of the mode shapes were generated and plotted, as described in Section III. Figures 10 through 20 show the contour plots. Modal damping was measured from two of the transfer functions with high response using the Nyquist plane technique described in Section II. The range of values of the damping factor was 0.0012 - 0.0124 as shown in Table I.



Figure 10 F-4 Rudder Mode Shape from Digital Impact Method, 28 Hz

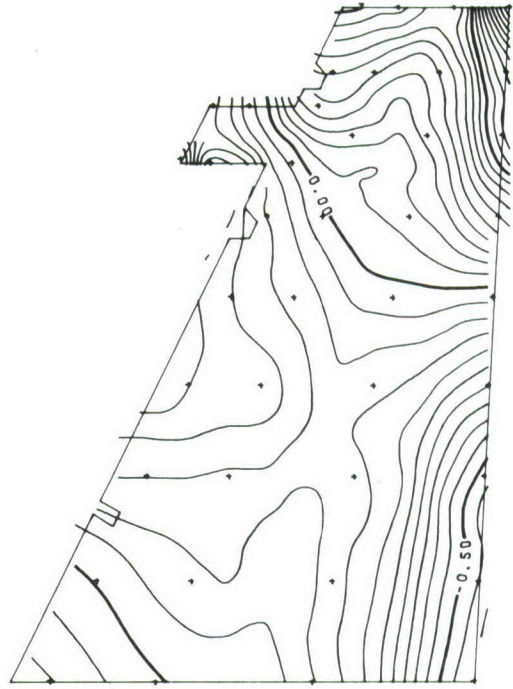


Figure 11 F-4 Rudder Mode Shape from Digital Impact Method, 59 Hz

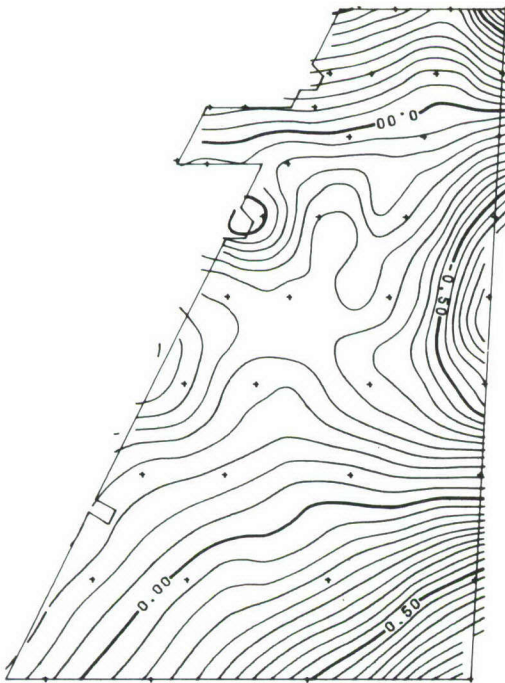


Figure 12 F-4 Rudder Mode Shape from Digital Impact Method, 121 Hz

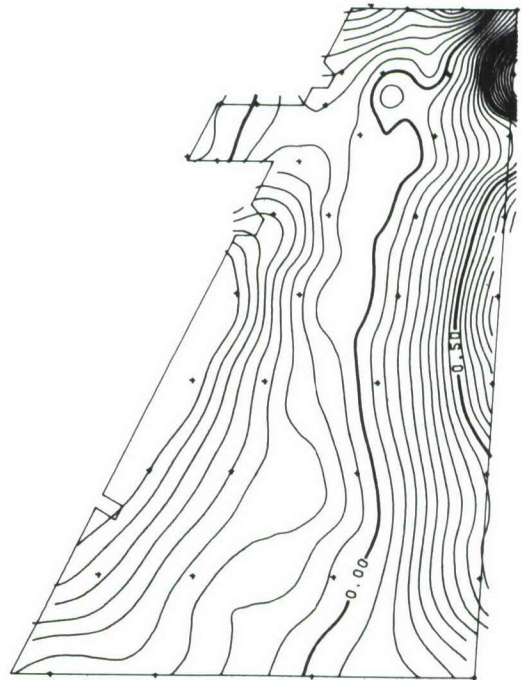


Figure 13 F-4 Rudder Mode Shape from Digital Impact Method, 161 Hz

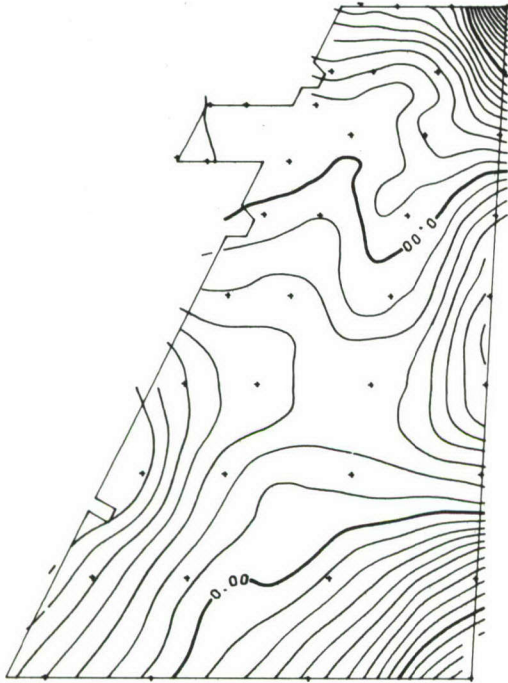


Figure 14 F-4 Rudder Mode Shape from Digital Impact Method, 181 Hz

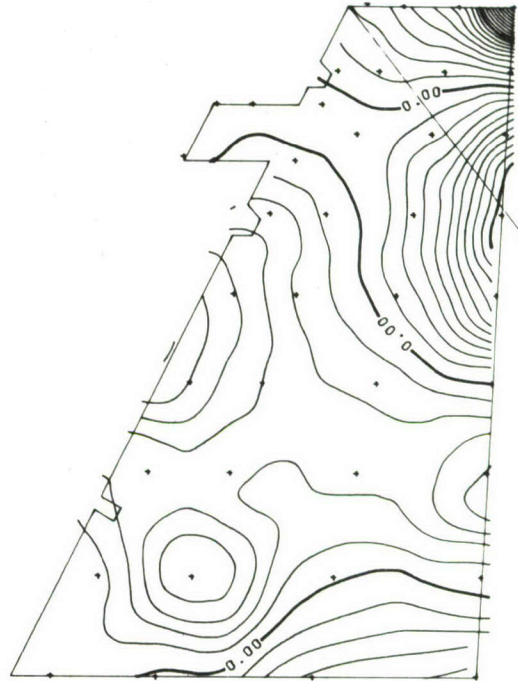


Figure 15 F-4 Rudder Mode Shape from Digital Impact Method, 216 Hz

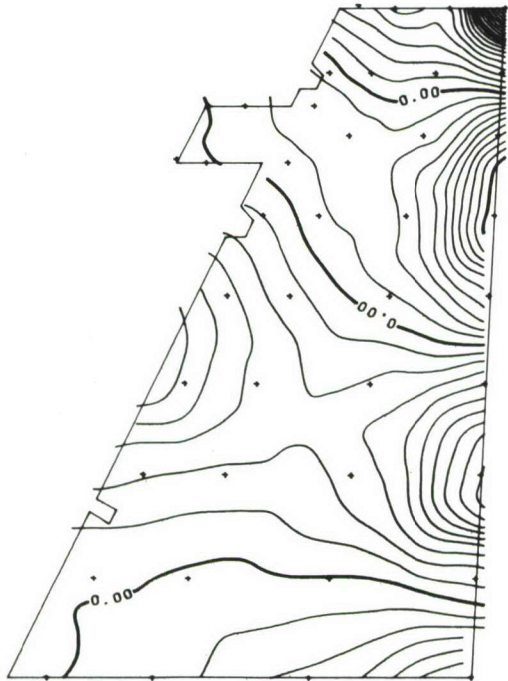


Figure 16 F-4 Rudder Mode Shape from Digital Impact Method, 224 Hz

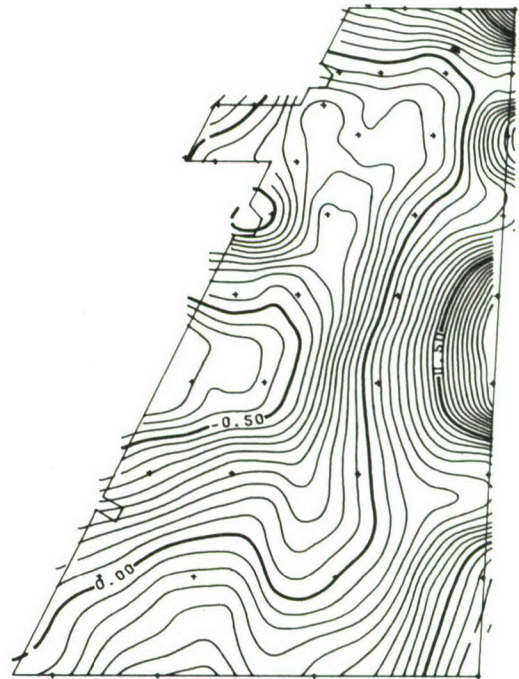


Figure 17 F-4 Rudder Mode Shape from Digital Impact Method, 303 Hz

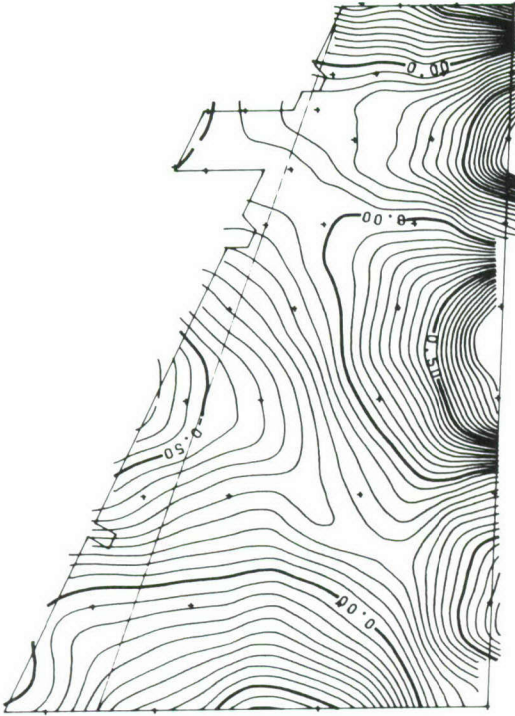


Figure 18 F-4 Rudder Mode Shape from Digital Impact Method, 320 Hz

Figure 19 F-4 Rudder Mode Shape from Digital Impact Method, 396 Hz

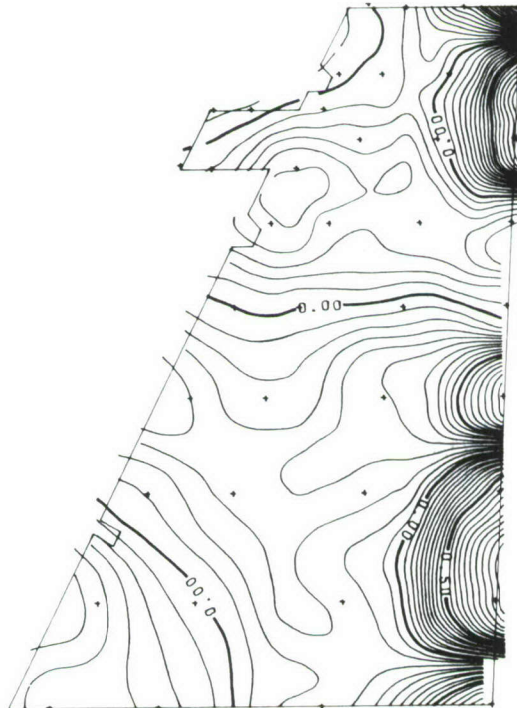


Figure 20 F-4 Rudder Mode Shape from Digital Impact Method, 448 Hz

A direct comparison was made of the mode shapes obtained from the analog and digital techniques. The digital and analog data are plotted together in an isometric view for the two mode shapes at 121 Hz and 224 Hz in Figures 21 and 22. The two techniques compare closely. Small variations in frequency and amplitude can be expected when any technique is repeated on a structure. Damping factors measured using both techniques also compare favorably. Table I shows a damping factor of .011 for the 223 Hz mode from the analog method which compares reasonably well with the .007 digital value. The mode at 400 Hz has an average value of .0048 from the analog data and a .0047 value from the digital technique.

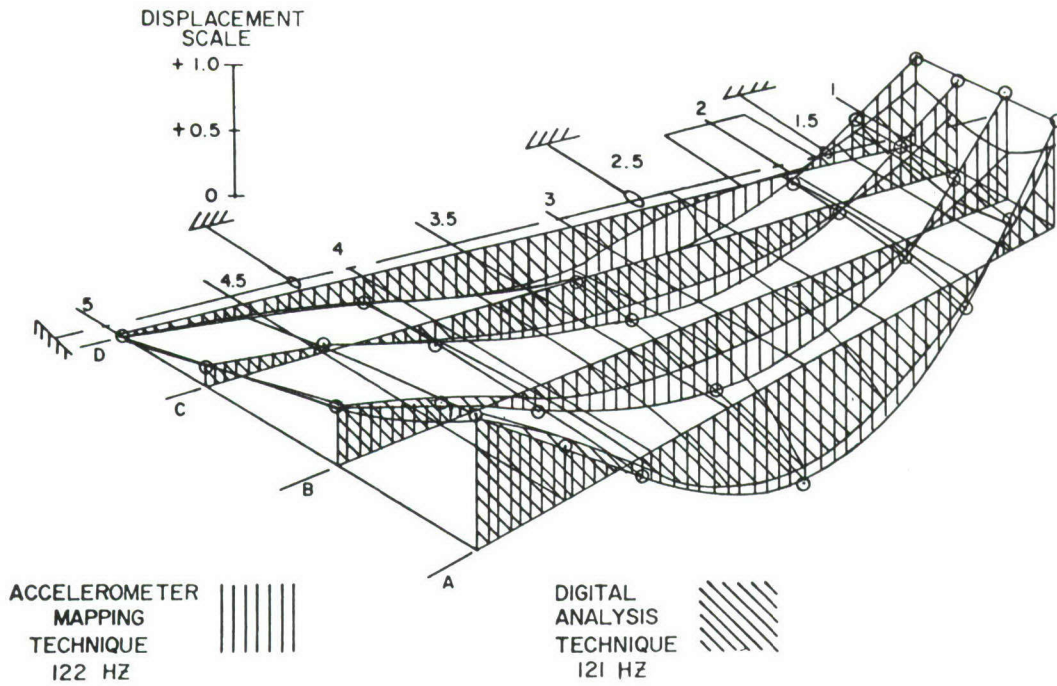


Figure 21 F-4 Rudder Mode Shape Comparison, 121 and 122 Hz

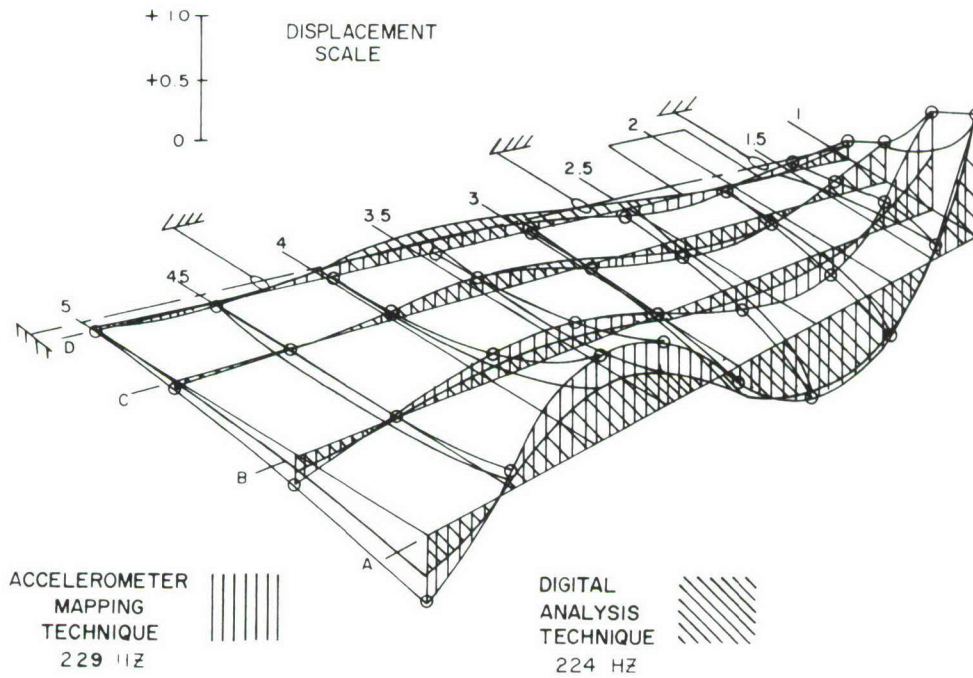


Figure 22 F-4 Rudder Mode Shape Comparison, 224 and 229 Hz

SECTION V

WELDBOND PANEL TEST

The dynamic properties of a group of aluminum skin-stringer test panels were determined using the digital impact method. The panels were part of a test program to determine the sonic fatigue resistivity of weldbonded skin to stringer attachment. The test panels were modeled from sections of the C-140 aircraft fuselage but had thinner gage skins. Half of the panels had weldbonded stringer attachments and half had conventional rivet joints. Both flat and curved panels with three or four bays were tested. Details of the 16 panel configurations are listed in Table II. The modal analysis described here was performed on panels #3 through #16.

The Fourier Analyzer System located at the Air Force Materials Laboratory (AFML) was used to perform the modal analysis of weld-bond panels #3 through #14. The weldbond panel tests were performed with the USER 30 modal analysis user program (see Section III). Measurement grids of 48 and 50 points were used on the three and four bay panels, respectively, since the modal program could handle a maximum of 50 transfer functions. Weldbond panels #15 and #16 were tested on-sight at the Air Force Flight Dynamics Laboratory (AFFDL). A Fourier Analyzer System identical to the AFML system was used. These two panels were tested with the USER 230 program, described in Section III which had just become available for use. A 66 point grid was used on these two panels.

TABLE II
WELDBOND PANEL CONFIGURATIONS

PANEL No.	TYPE JOINT*	SKIN THICKNESS	NUMBER OF BAYS	MID BAY	RADIUS OF CURVATURE
		(inches)		(inches)	(inches)
1	R	0.032	3	7 X 21	∞
2	WB	"	3	"	"
3	R	0.040	3	"	"
4	WB	"	3	"	"
5	R	0.032	4	5.25 X 20.75	"
6	WB	"	4	"	"
7	R	0.040	4	"	"
8	WB	"	4	"	"
9	R	0.032	3	7 x 21	42.5
10	WB	"	3	"	"
11	R	0.040	3	"	"
12	WB	"	3	"	"
13	R	0.032	4	5.25 X 20.75	"
14	WB	"	4	"	"
15	R	0.040	4	"	"
16	WB	"	4	"	"

* R = Riveted
WB = Weldbonded

The panels were mounted in their sonic fatigue test frames and supported in a horizontal position. The panel tests were performed with the same impacting hammer used on the F-4 rudder. The panels were impacted at one point on the surface and the response was measured at each grid point to form the transfer functions. The impact point was chosen near a stringer where the local structural stiffness was adequate. This method was used since impacts at the grid points on unsupported skin gave frequency spectra with an upper frequency limit of 200 Hz or less. Impacting at or near a stringer provided energy up to 500 Hz, which was sufficient to cover the desired frequency range. Prior estimates of the panel natural frequencies indicated that 500 Hz was an adequate upper limit. Figures 23, 24, and 25 show force spectra for impacts at various panel locations, and illustrate the necessity of impacting at a stiff location on the structure. A trial and error method was used on each panel to find the impact location which produced the best input force spectrum. Care was taken to place the impact point away from any node lines of expected modes.

The transfer functions were computed from the average of five hammer impacts. Averaging of more than five impacts showed no significant improvement in data quality. The coherence function was observed as each transfer function was computed. Any questionable data was measured again. A typical transfer function and coherence function from one of the weldbond panels are shown in Figures 26 and 27. An exponential decaying time window of the form e^{-at} was

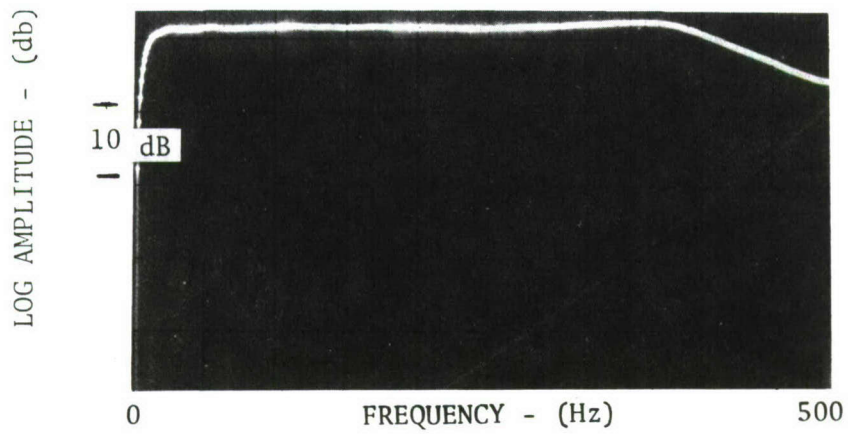


Figure 23 Force Spectrum of Hammer Impact at Stiff Frame of Weldbond Panel

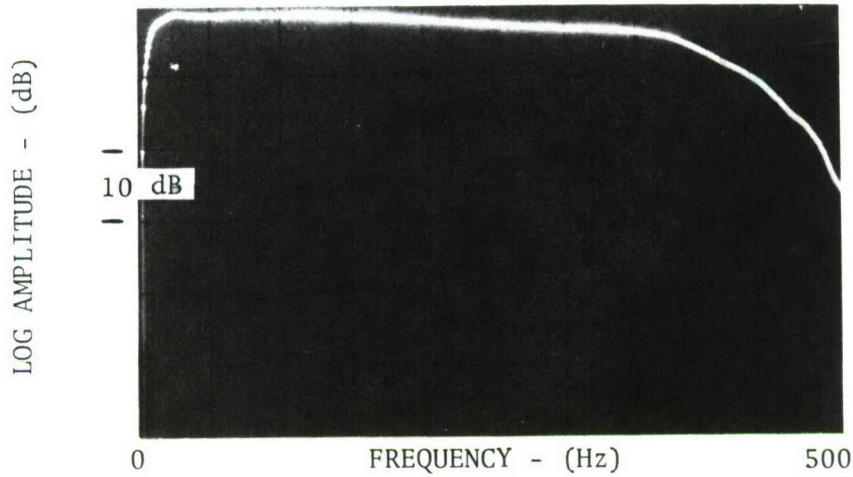


Figure 24 Force Spectrum of Hammer Impact on Weldbond Panel Skin Near a Stiffener

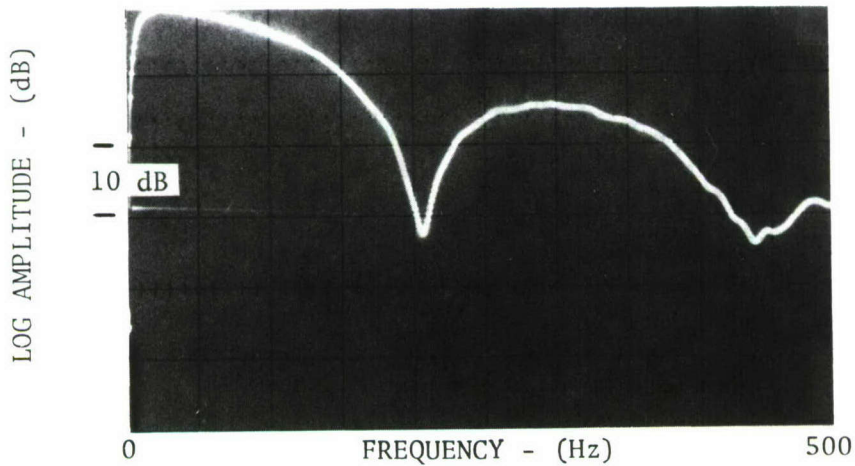


Figure 25 Force Spectrum of Hammer Impact on Weldbond Panel Skin Away from Stiffeners

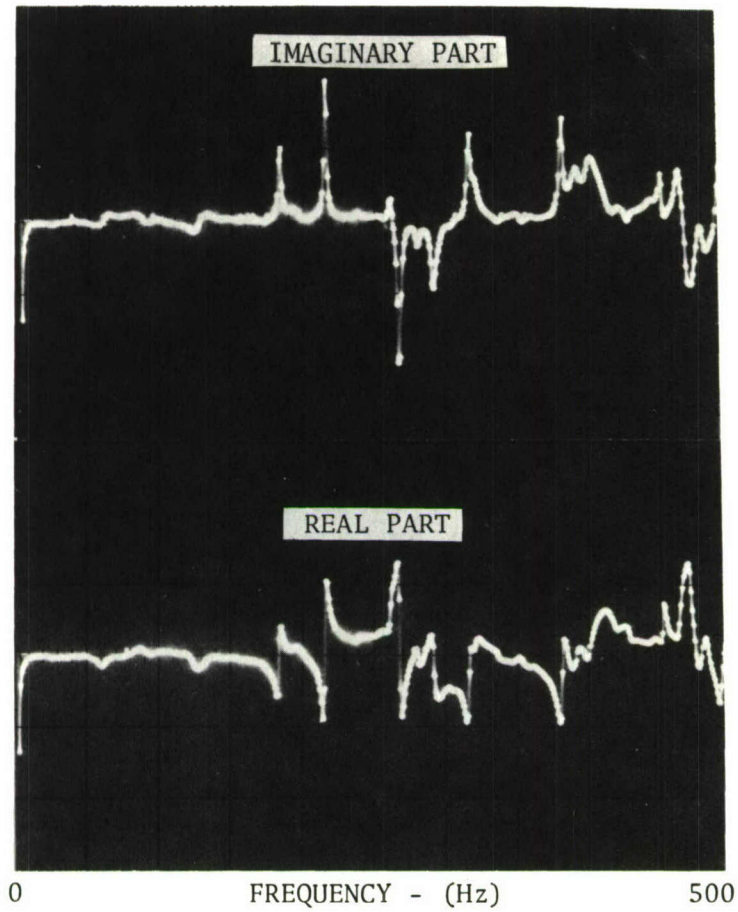


Figure 26 Typical Weldbond Panel Transfer Function, Real and Imaginary Parts

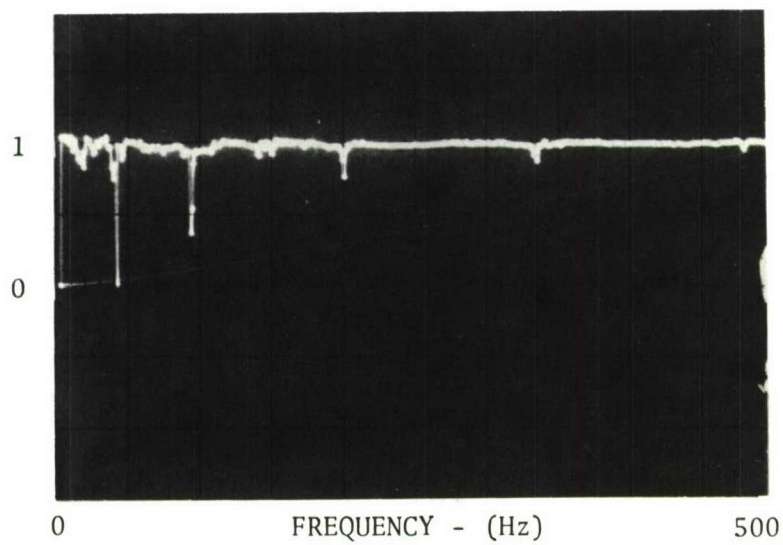


Figure 27 Typical Weldbond Panel Coherence Function

generated by the Fourier Analyzer System and used to smooth the transfer functions. This windowing added approximately .007 to the damping factor, C/C_c , for all modes. A few transfer functions were measured without windowing and were used to identify natural frequencies and compute modal damping. Mode shapes were generated from the windowed transfer functions and each was studied on the animated display. Figures 28 and 29 show animated display sequences of two weldbond panel modes. Contour plots of each mode were made as described in Section III. Resonant frequencies were observed in the group of test panels from 80 Hz to the 500 Hz upper limit. Mode shapes ranged from whole panel in-phase motion to multiple bay modes to individual bay modes as frequency increased. Contour plots of a few typical modes can be seen in Figures 30 through 35.

Modal damping factors for panels #3 through #14 were calculated using the Nyquist plane circle fit method, USER 6868, described in Section II. Figure 36 shows a single transfer function peak. A Nyquist plot of this peak is shown in Figure 37. Damping factors for panels #15 and #16 were calculated by the modal analysis user program, USER 230. Values of the damping factor, C/C_c , ranged from .024 for low frequency whole panel modes to .003 for higher frequency modes with five or six node lines per bay. The damping of half of the panels in the weldbond program was also measured by analog methods. The bandwidth method was applied to panel strain response plots generated from a discrete frequency sweep of acoustic excitation similar to that described for the F-4 rudder (Section IV). Only the

panels with rivet joints were tested by this method. Table III lists the damping data measured from these seven riveted panels by both analog and digital techniques. The two methods compare favorably.

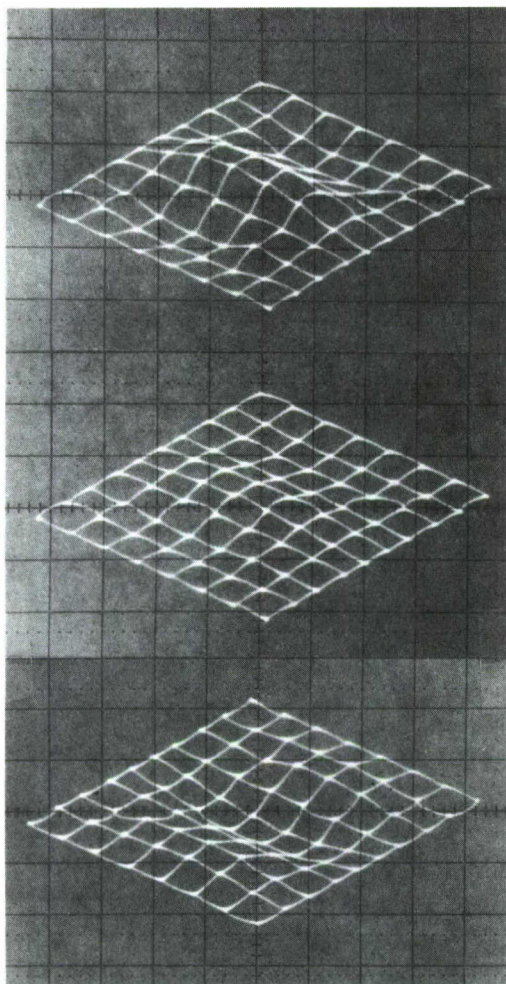


Figure 28 Weldbond Panel Mode
Shape Display, 123 Hz

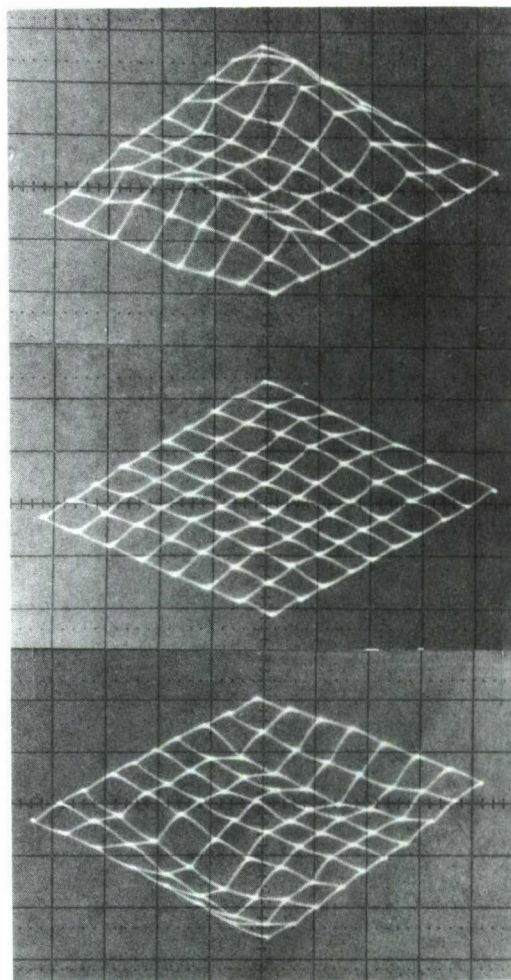


Figure 29 Weldbond Panel Mode
Shape Display, 97 Hz

Figure 30 Weldbond Panel Mode Shape from Digital Impact Technique, Panel 16 (curved), 202 Hz

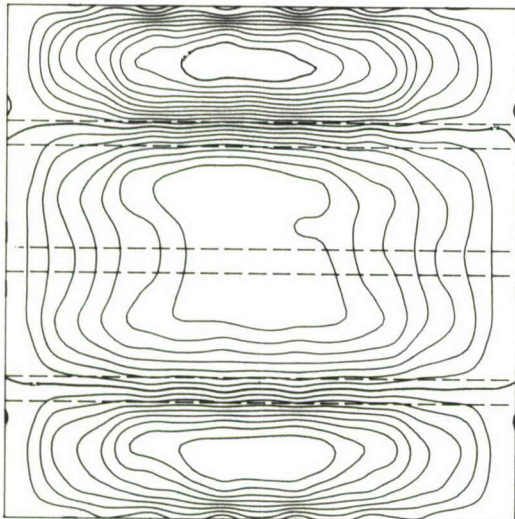
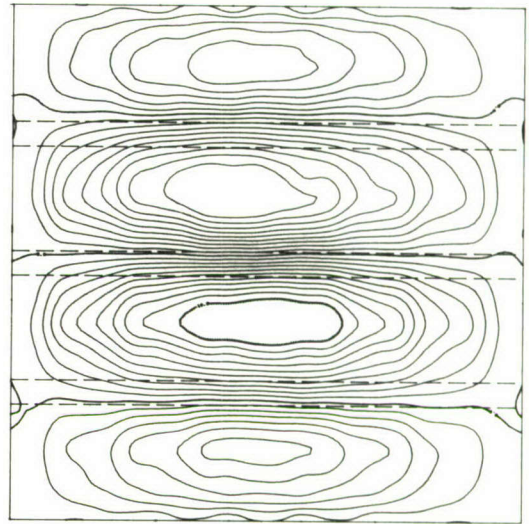


Figure 31 Weldbond Panel Mode Shape from Digital Impact Technique, Panel 16 (curved), 223 Hz

Figure 32 Weldbond Panel Mode Shape from Digital Impact Technique, Panel 6 (flat), 198 Hz

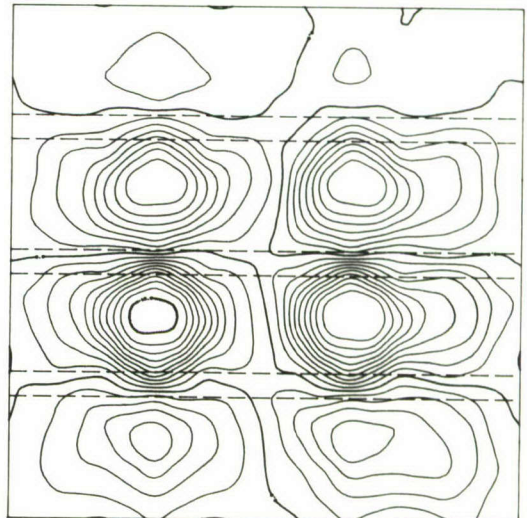


Figure 33 Weldbond Panel Mode Shape from Digital Impact Technique, Panel 10 (curved), 231 Hz

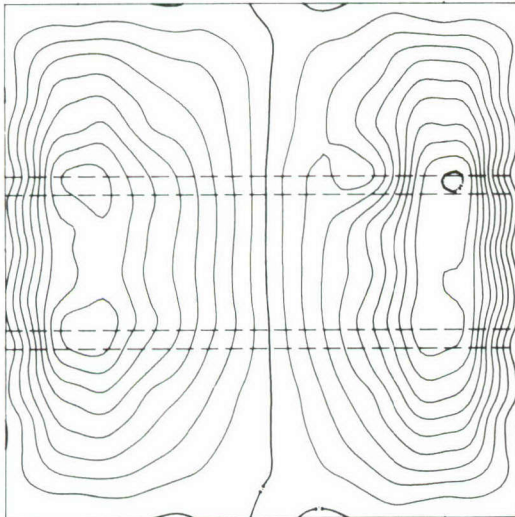
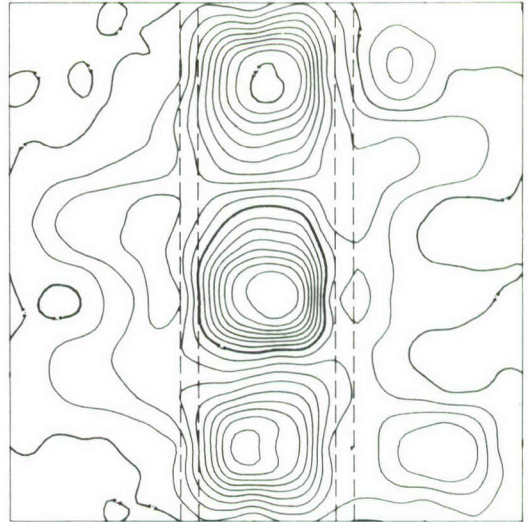
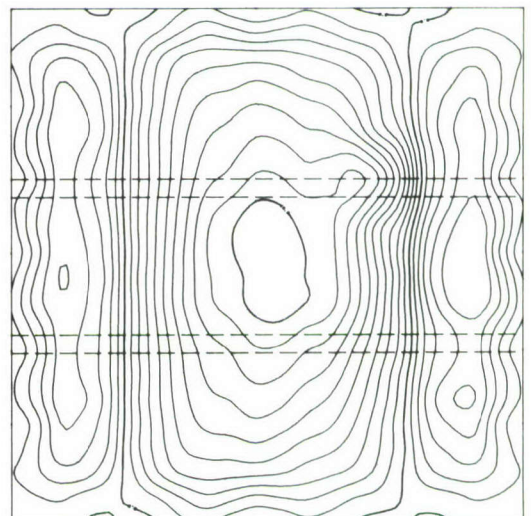


Figure 34 Weldbond Panel Mode Shape from Digital Impact Technique, Panel 11 (curved), 135 Hz

Figure 35 Weldbond Panel Mode Shape from Digital Impact Technique, Panel 11 (curved), 153 Hz



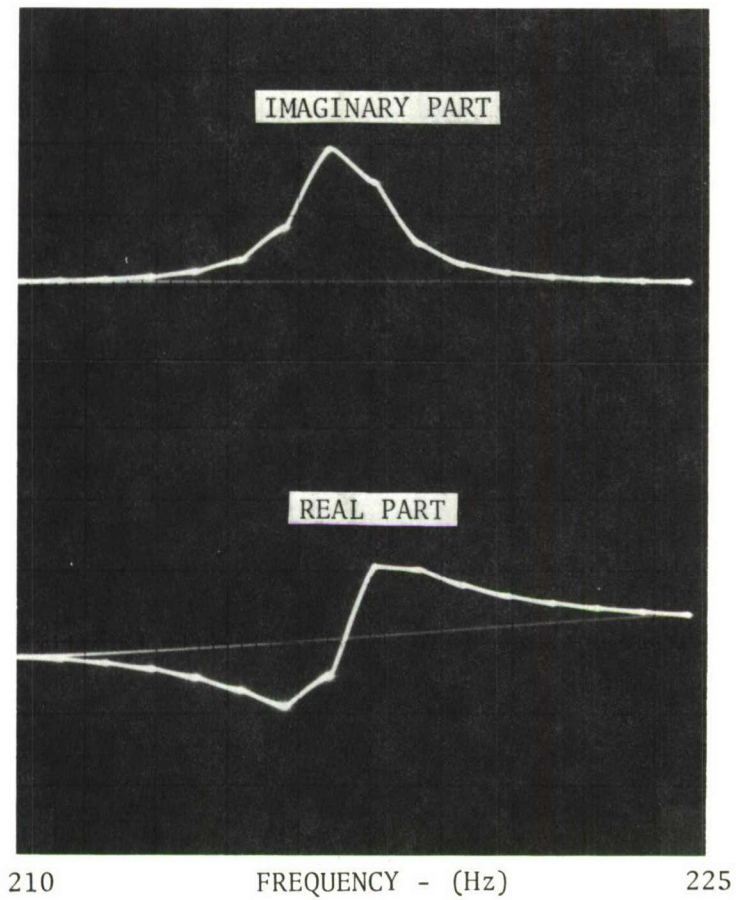


Figure 36 Weldbond Panel Transfer Function in Frequency Range Near a Resonant Peak, Real and Imaginary Parts

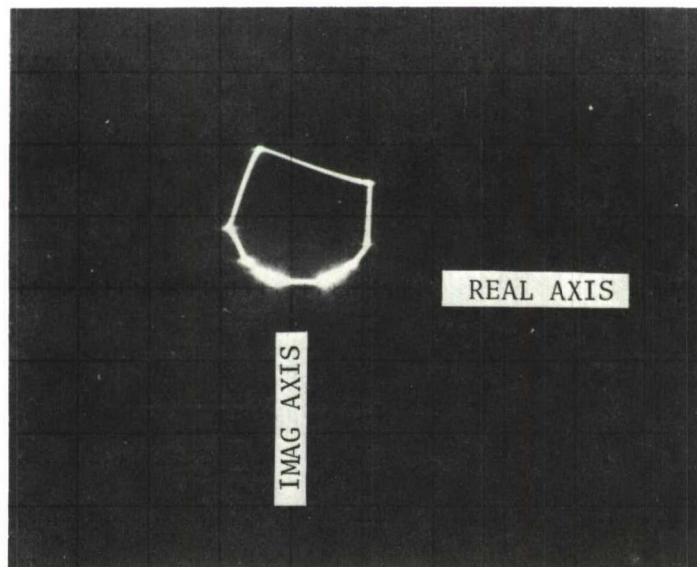


Figure 37 Weldbond Panel Transfer Function in Frequency Range Near a Resonant Peak, Nyquist Plot

TABLE III
WELDBOND PANEL DAMPING COMPARISON

WELDBOND PANEL NR.	ANALOG TECHNIQUE (BANDWIDTH METHOD)		DIGITAL TECHNIQUE (TRANSFER FUNCTION METHOD)	
	RESONANT FREQ. (Hz)	DAMPING FACTOR (C/C_c)	RESONANT FREQ (Hz)	DAMPING FACTOR (C/C_c)
3	99	.027	111	.017
3	135	.006	140	.006
5	147	.006	150	.006
5	253	.014		
5	257	.007	268	.004
5	260	.005		
7	168	.013	163	.004
7	176	.009	164	.011
7	178	.008		
9	425	.003	414	.002
11	275	.009	270	.009
11	311	.004	306	.007
11	407	.002	405	.005
13	220	.004	224	.008
15	216	.004	213	.004
15	242	.006	242	.005

SECTION VI

BONDED PANEL TEST

A third test was performed with the digital impact modal analysis technique to determine the dynamic properties of an aluminum skin-stringer panel with adhesively bonded stringer attachments. For comparison purposes, the panel was also tested with conventional analog techniques. The digital and analog results are compared directly.

The test panel was one of eight test specimens in an AFFDL program to determine the sonic fatigue resistivity of adhesively bonded structures. The test panel was essentially the same as panels #1 and #2 of the weldbonded program (see Table I), 3 bay, flat, .032" skins, except the skin-to-substructural joints were adhesively bonded without rivets or spotwelds.

The bonded panel was mounted in the sonic fatigue test frame for both digital and analog tests. The analog methods used to determine the panel's dynamic properties were similar to those used on the F-4 rudder (see Section IV). The resonant frequencies were determined from Chladni patterns or sand patterns. The panel was supported horizontally and excited by a loudspeaker from below. Colored sand was sprinkled on the panel and a discrete frequency input to the loudspeaker was slowly swept in frequency. Natural frequencies were observed when the panel vibration caused the sand to align in a definite pattern, collecting along the node lines of the

particular mode. Well defined modes were found at 108, 126, 262, 307, and 391 Hz. These modal sand patterns are shown in Figures 38, 39, and 40. Two of these frequencies were selected for the comparison; 126 and 307 Hz. An accelerometer mapping technique was then used to obtain the detailed mode shapes. The mode shapes are shown in the contour plots of Figures 41 and 42. The contour plots were generated by the same computer plotting routine described in Section III. Modal damping was not determined using analog methods.

The digital impact modal analysis method was applied to the bonded panel with the AFFDL Fourier Analyzer System and USER 230 modal program. The user program is described briefly in Section III. A grid of 80 points was used on the panel and is shown in Figure 43. The test procedure was nearly the same as that used on the weldbond panels. Impacts were applied at a single point near a stiffener and the response was measured at each of the grid points. Five impacts were averaged to form the transfer functions. One of the panels in the bonded program is shown being tested in Figure 44. Windowing was used to smooth the transfer functions as described in Section II. Modes were observed at 81, 97, 123, 258, 305, 377, 390, 403, and 436 Hz. The modes at 123 and 305 Hz are shown in Figures 45 and 46. They correspond to the 126 and 307 Hz modes measured by the analog methods. The analog and digital results are compared in the isometric plots of Figures 47 and 48. Close agreement between the two methods can be seen.

Figure 38 Bonded Panel Sand Pattern, 126 Hz

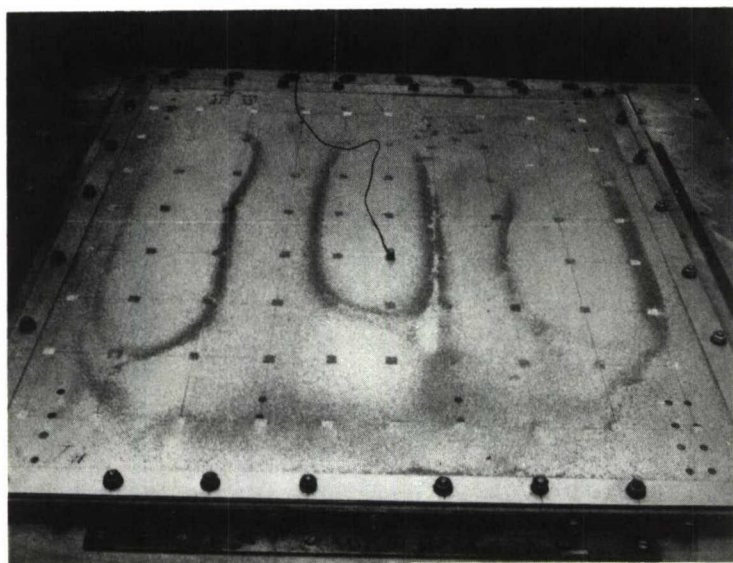
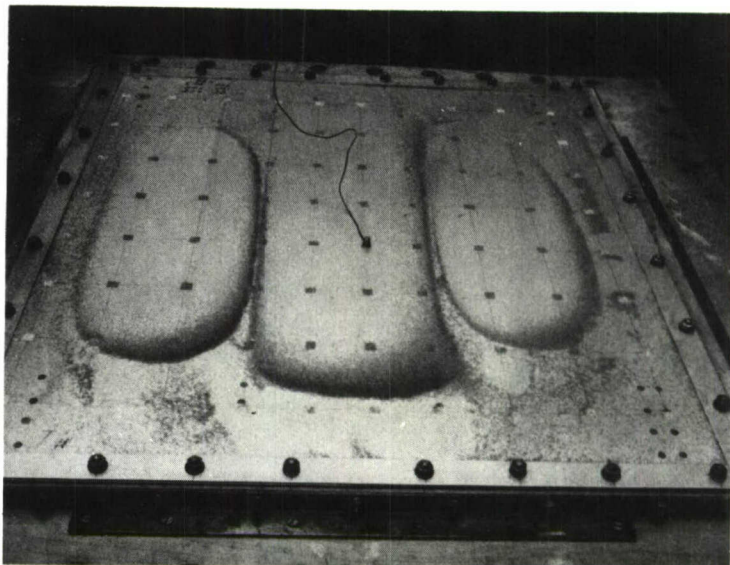


Figure 39 Bonded Panel Sand Pattern, 262 Hz

Figure 40 Bonded Panel Sand Pattern, 307 Hz



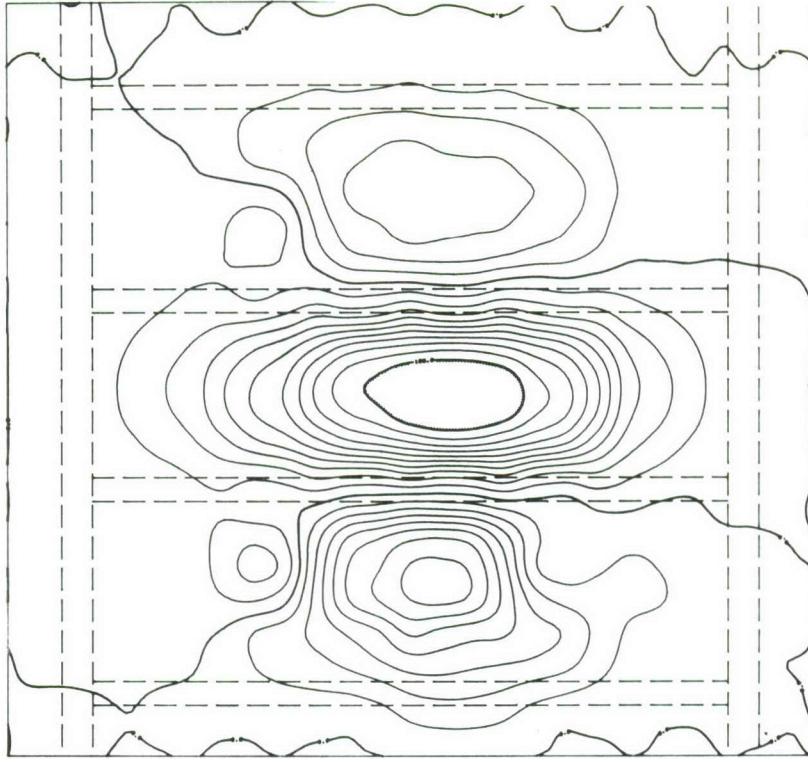


Figure 41 Bonded Panel Mode Shape from Accelerometer Mapping, 126 Hz

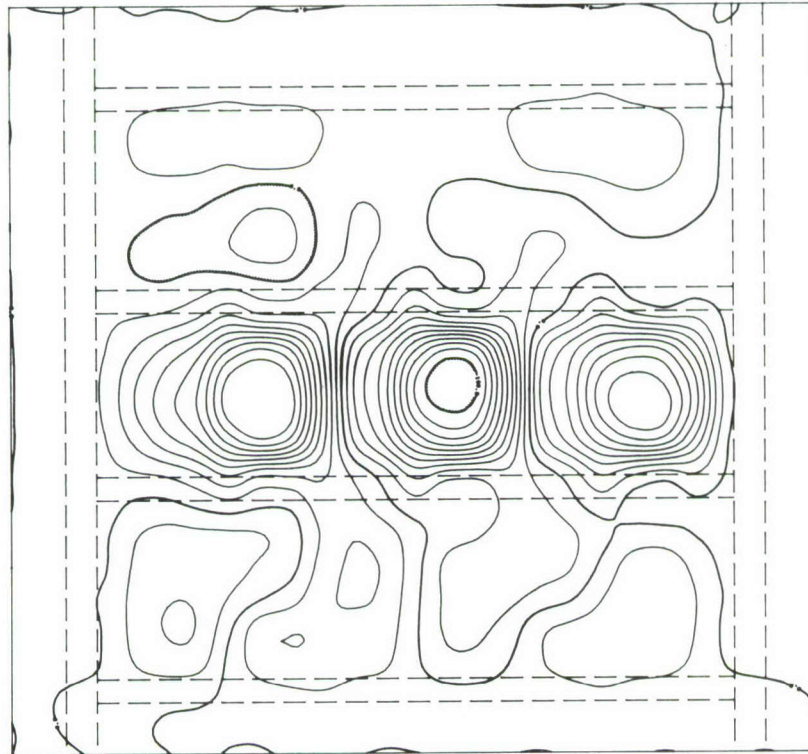


Figure 42 Bonded Panel Mode Shape from Accelerometer Mapping, 307 Hz

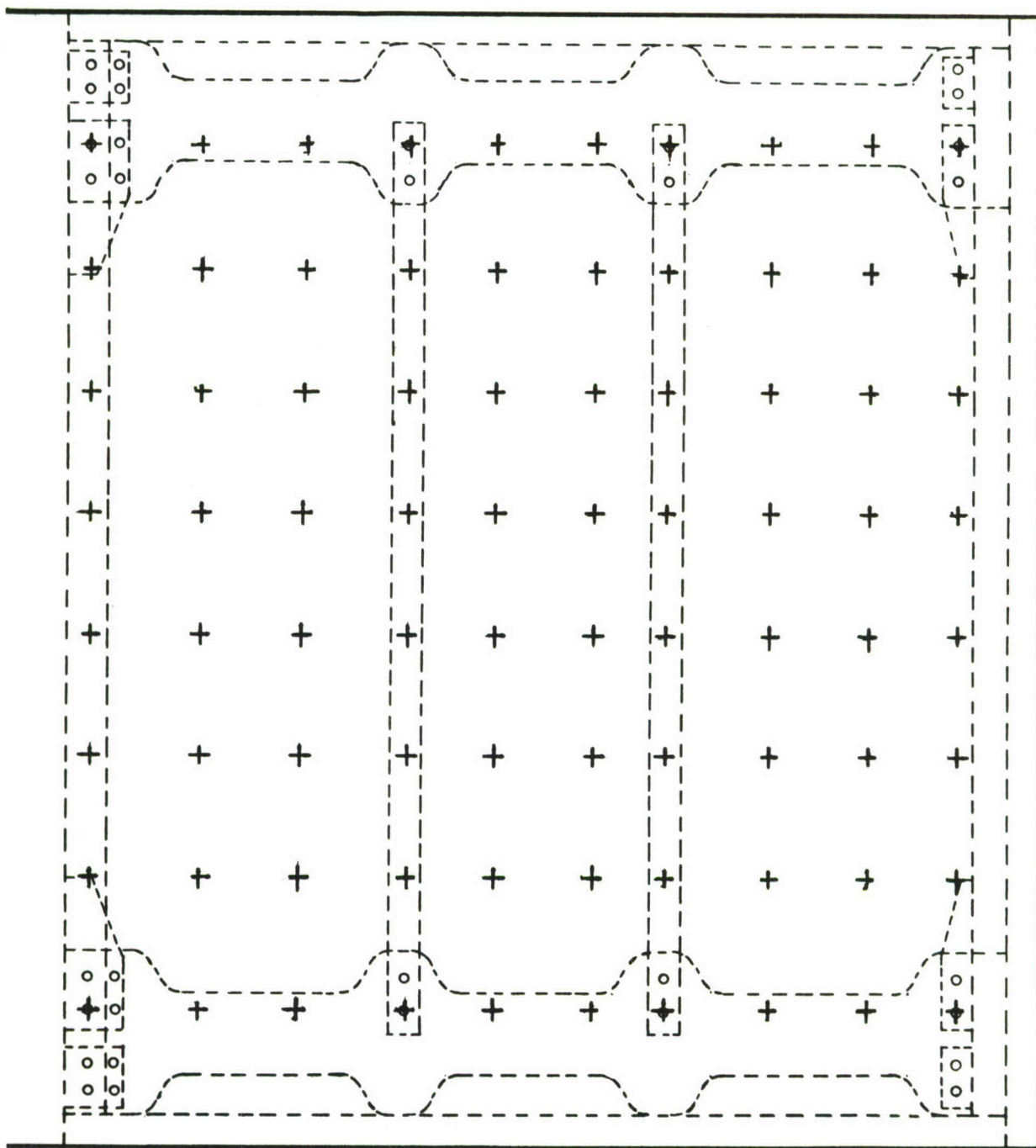


Figure 43 Bonded Panel Test Point Grid



Figure 44 Bonded Panel Test with the Digital Impact Technique

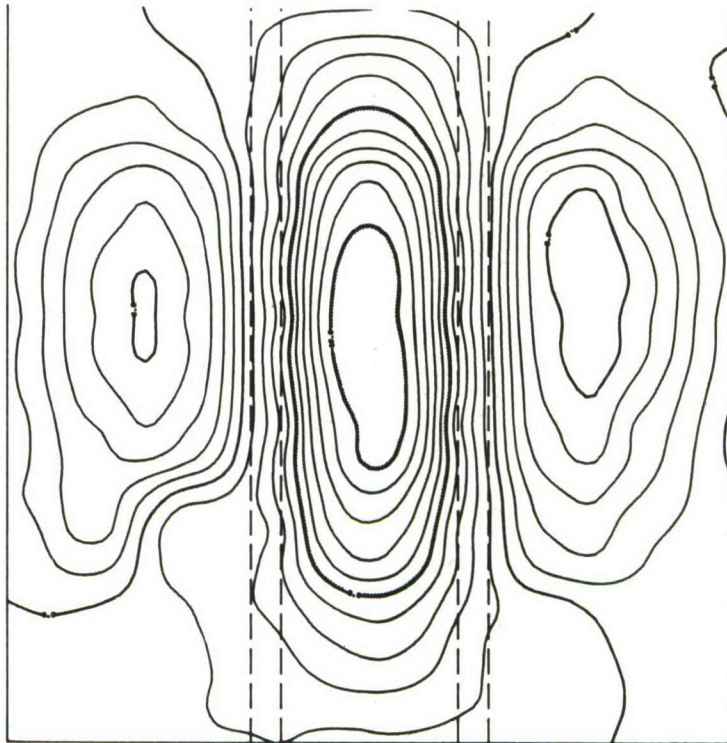


Figure 45 Bonded Panel Mode Shape from Digital Impact Technique, 123 Hz

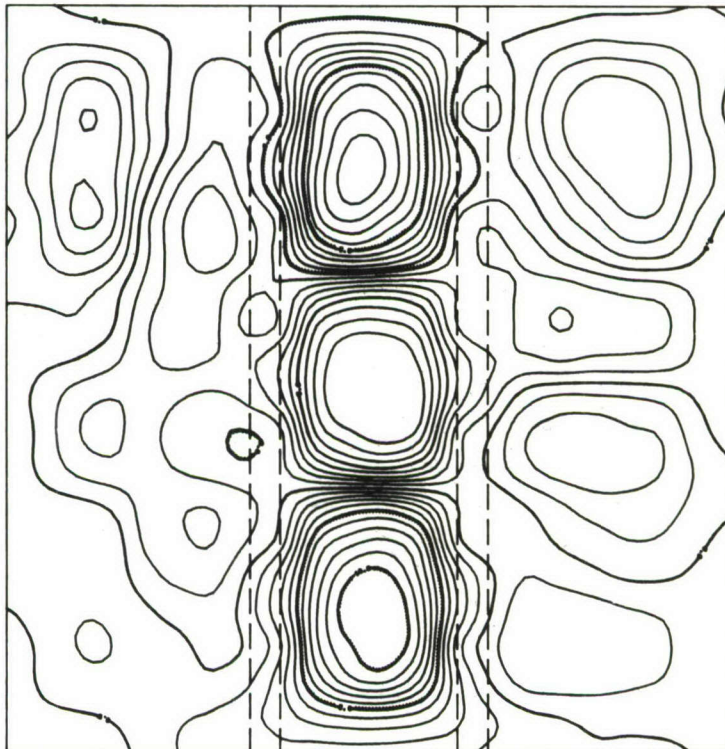


Figure 46 Bonded Panel Mode Shape from Digital Impact Technique, 305 Hz

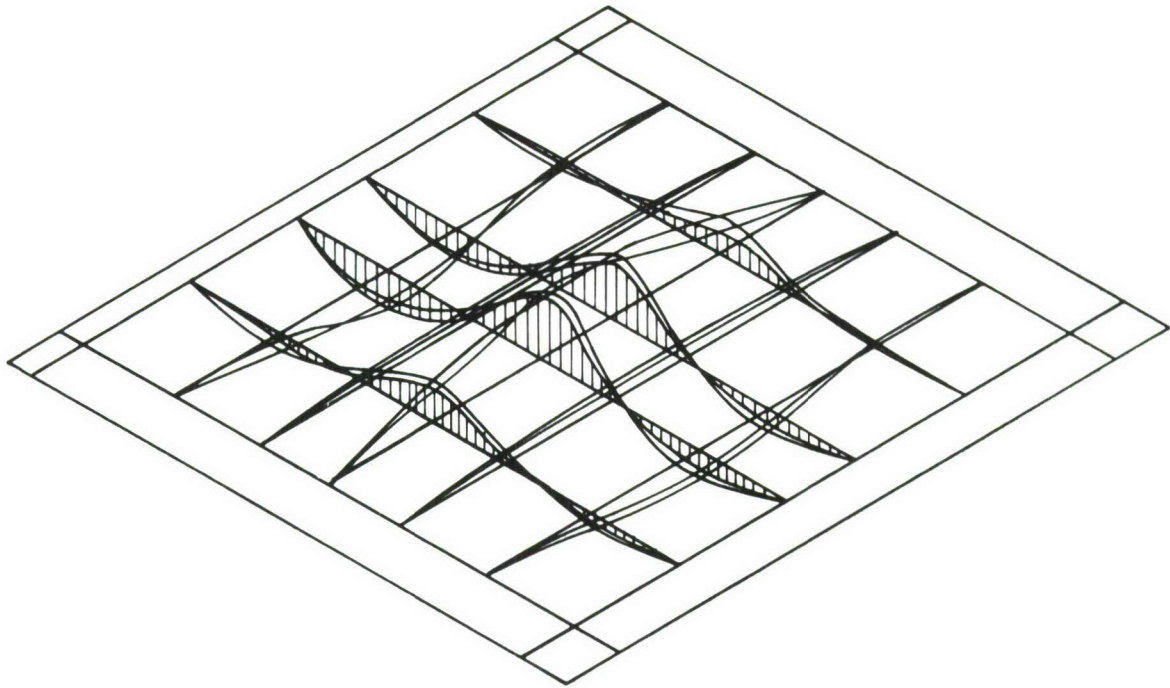


Figure 47 Bonded Panel Mode Shape Comparison, 123 and 126 Hz

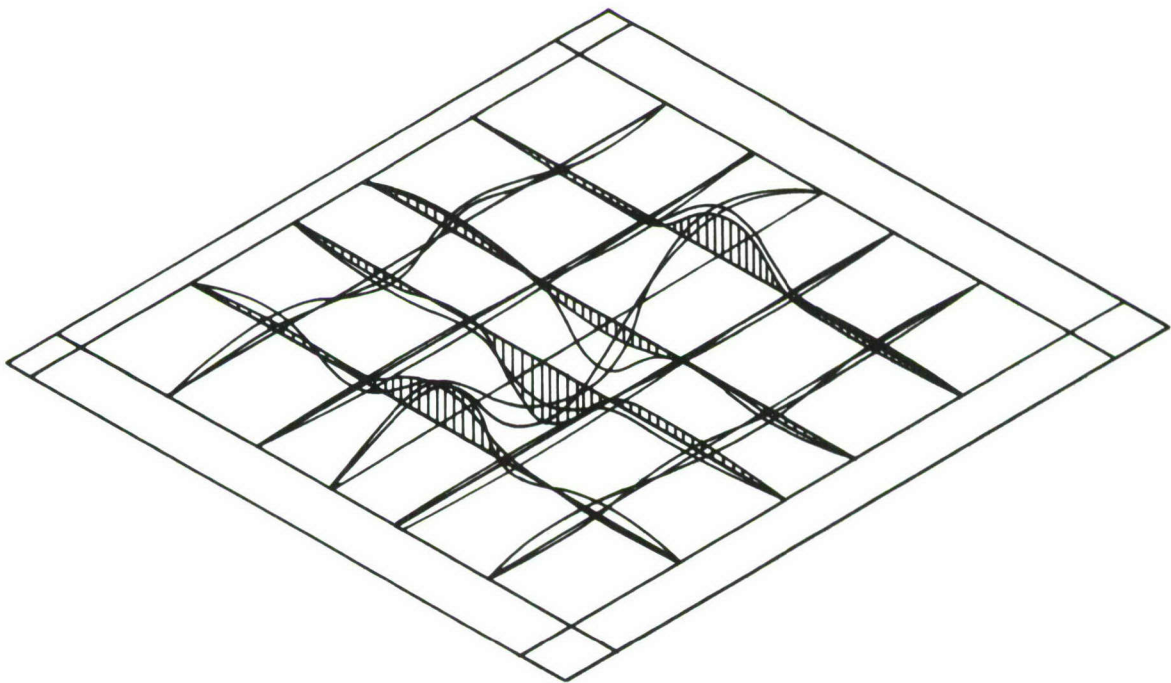


Figure 48 Bonded Panel Mode Shape Comparison, 305 and 307 Hz

SECTION VII

DISCUSSION OF RESULTS

The digital impact modal analysis technique described above has been used to measure the dynamic properties of sonic fatigue test structures with equal accuracy as the analog methods previously used. Natural frequencies, mode shapes and modal damping measured by both techniques were compared directly and showed good agreement. More importantly, the digital technique has exhibited several advantages in application over the current analog techniques. The analog techniques required considerable time and effort to set up the test equipment while the digital technique required only set up of the hammer force gage and response accelerometer and a few simple commands on the Fourier Analyzer System. The digital technique excited and measured all modes of the test structures simultaneously with a simple series of hammer impacts. The analog technique required each mode to be measured separately. Thus, a considerable savings in manhours resulted using the digital technique, especially on lightweight panel-type structures with many resonant modes. Overall sound pressure levels of approximately 110 dB were required to excite panels for the sand pattern analog technique. This level was hazardous to test personnel and precautions had to be taken to ensure their protection. The hammer impacts of the digital technique posed no safety problem. The sand pattern technique only indicated node lines of resonant modes. No amplitude or phase information

could be determined. The accelerometer mapping technique measured both amplitude and phase data, but required each mode to be plotted by hand or by a contour mapping computer program before it could be visualized. On the other hand, the digital impact technique with its animated display measured both amplitude and phase information and displayed all modes in simulated motion a few minutes after the impacts were applied. Thus, the digital technique combined the best features of each of the analog techniques, i.e., the amplitude and phase measurement capability of accelerometer mapping with the real time mode shape display of the sand patterns. The animated display offers a unique format for visualizing the actual motion of vibrational modes. Finally, the digital method offers the flexibility and storage features of a digital mini-computer system. Transfer functions measured on a structure can be stored in memory and used to generate mode shapes at any future time.

The digital impact method as applied in this program to lightweight, flexible aircraft structures showed certain disadvantages over the analog techniques. The force spectra of the hammer impacts on the weldbond and bonded panels were difficult to control. The upper frequency cutoff of the input spectrum was dependent almost solely on the local stiffness of the structure at the impact point. Nothing could be done to extend the upper frequency cutoff of the spectrum beyond the stiffness limiting value. This necessitated impacting the structures at stiff locations near the boundaries or on stiffeners to provide input energy as high in frequency as possible.

This was an inefficient way of exciting the structure since most resonant modes have node lines at these locations. No problem was encountered in the weldbond and bonded panel tests since impacts at stiffeners produced adequate spectral content to excite the frequency range to be investigated. However, no modal information could be determined if the frequency range to be tested would have been increased. Thus, in many cases, the measurement of higher frequency modes of lightweight aircraft structures may not be possible using a hammer impact excitation.

Another disadvantage of the digital impact test technique compared to the conventional analog techniques was found in predicting which modes of a test structure would be the most responsive to the sonic fatigue test loading. The "most responsive modes" which produce the largest displacements are potentially the most damaging under sonic fatigue loading. This information is needed prior to testing for adequate strain gage placement on a structure. Previous efforts have shown that the modes of a test structure which respond highly to the discrete frequency acoustic excitation of the analog sand pattern and accelerometer mapping techniques usually respond highly to the broadband sonic fatigue test spectrum. Thus, the analog techniques could be used to predict the modes which were potentially the most damaging under sonic fatigue loading. The digital impact technique, however, uses a single point excitation which is substantially different in spatial characteristics from the sonic fatigue excitation. The force pulse of the digital method will ex-

cite all the modes of a structure below the upper frequency cutoff of the impact spectrum but the relative response amplitudes of these modes will not necessarily correspond to the relative amplitudes of modes excited by the broadband sonic fatigue excitation. Therefore, the results of the digital impact mode shape tests cannot be used to predict the "most responsive modes" to the sonic fatigue test environments.

SECTION VIII

CONCLUSIONS AND RECOMMENDATIONS

The digital impact technique of measuring the dynamic properties of lightweight, flexible aircraft structures has produced results in good agreement with established analog techniques. The digital technique has shown several advantages over the analog techniques:

1. Greatly reduced set-up and test time.
2. No noise hazard.
3. Unique output format for visualizing structural vibration on-line. (Animated display.)
4. Flexibility of digital data processing and storage.

The digital technique has also shown two disadvantages which may limit its application to lightweight, flexible aircraft structures to be tested for sonic fatigue life:

1. Limited range of excitation spectrum.
2. Most responsive modes cannot be determined.

A broadband single point excitation instead of the hammer impact is recommended for use with the basic digital modal technique to eliminate the disadvantage of limited input spectrum range. A non-contracting magnetic driver could be used for this application. The input spectrum could then be easily controlled by electronically shaping the spectrum of the driver input signal. The usable range of the input spectrum would depend on the limits of the electronic signal generating equipment and not on the mass and stiffness of the

impacting hammer and structure to be tested. This would increase the upper limit of the excitation spectrum on skin-stringer structures to the 5,000 to 10,000 Hz range instead of the upper limit of 500 to 1,000 Hz for hammer impacts. It is also recommended that a variation of the basic digital modal testing technique be used with broadband acoustic excitation as a modal measurement technique for use on sonic fatigue test specimens. The transfer function of the basic digital technique would be replaced by a "normalized response function" for generating mode shapes. A Fourier Analyzer System would be used to compute the normalized response functions in much the same way as it is used to compute transfer functions. Response spectra would be measured at desired points on the structure and each would be divided by a normalizing response spectrum measured at a common reference location. The resulting normalized response functions would then be used to generate mode shapes. A broadband acoustic input with the same spectrum shape as the sonic fatigue test environment and a lower overall level would be used for excitation. The reduced level would prevent any damage prior to sonic fatigue testing. This proposed method will eliminate both of the disadvantages discussed above. The broadband acoustic input would be easily controlled by the signal generation electronics. And, since the same acoustic spectrum shape would be used for both tests, the relative response amplitudes of modes measured during modal testing would give a good indication of which modes would be the most responsive to the high level sonic fatigue test environment.

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