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COMPUTER PROGRAM FOR RELAXATION SOLUTIONS OF THE NONLINEAR SMALL-DISTURBANCE EQUATIONS FOR TRANSONIC FLOW IN AN AXIAL COMPRESSOR BLADE ROW

William J. Rae

Calspan Report No. AB-5487-A-3
APRIL 1978

AD No.
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Prepared For:

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH
BOLLING AIR FORCE BASE, D.C. 20332

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INTERIM SCIENTIFIC REPORT

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

(29) REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER
(18) AFOSR-TR- 78-0855

2. GOVT ACCESSION NO.

3. RECIPIENT'S CATALOG NUMBER
(9)

4. TITLE (and Subtitle)
(6) Computer Program for Relaxation Solutions of the Nonlinear Small-Disturbance Equations for Transonic Flow in an Axial Compressor Blade Row.

5. TYPE OF REPORT & PERIOD COVERED
Interim Scientific Rep[illegible]t,

6. PERFORMING ORG. REPORT NUMBER
AB-5487-A-3

7. AUTHOR(s)
(10) William J. Rae

8. CONTRACT OR GRANT NUMBER(s)
(15) F44620-74-C-0059

9. PERFORMING ORGANIZATION NAME AND ADDRESS
Calspan Corporation
4455 Genesee Street, P. O. Box 235
Buffalo, New York 14221

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
2307A4
61102F

11. CONTROLLING OFFICE NAME AND ADDRESS
Air Force Office of Scientific Research/NA
Bolling Air Force Base
Washington, DC 20332

12. REPORT DATE
(11) Apr[illegible] [illegible]78

13. NUMBER OF PAGES
77 (12) 78p.

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)

15. SECURITY CLASS. (of this report)
UNCLASSIFIED

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

(16) 2307 (17) A4 (14) CALSPAN-AB-5487-A-3

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

TURBOMACHINERY
COMPRESSORS
TRANSONIC FLOW
FINITE-DIFFERENCE SOLUTIONS

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report contains a description of a computer program for calculating the three-dimensional transonic flow through a compressor blade row, in the nonlinear small-distrubance approximation, and for subsonic values of the inlet relative Mach number at the blade tip.

The problem formulation is reviewed briefly; following this, a description of the program is given together with a listing and sample case.

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407 727

ABSTRACT

This report contains a description of a computer program for calculating the three-dimensional transonic flow through a compressor blade row, in the nonlinear small-disturbance approximation, and for subsonic values of the inlet relative Mach number at the blade tip.

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INTRODUCTION

Three-dimensional transonic flows through a compressor blade row represent a very complex problem. Some of the complications that make the problem very difficult to solve are the presence of adjacent zones of subsonic and supersonic flow, and the communication between these zones that takes place through radial pressure gradients.

As a first step in applying recent computational techniques to this problem, a study of the nonlinear small-disturbance approximation has been made(1-4). This report contains a description of the computer program used to obtain the numerical results presented in these papers. For convenience, the basic equations used are summarized in Section 1. The program itself is described in Section 2, and a sample case is discussed in Section 3. A listing of the program and a guide to preparing the input are given in the Appendices.

The finite-difference equations used are the set on which the results of Reference 4 were based. They supersede, in a few small details, the earlier sets used in References 1-3.

It should be stressed that the program cannot handle cases where the inlet relative Mach number is supersonic at the tip, because the farfield boundary condition used is not a radiation condition. Thus, Mach waves, originating at the blade row, reflect from the grid boundaries instead of escaping as they should. As a result, the solution will contain a set of waves which pass back and forth between the upstream and downstream edges of the grid.

It should also be pointed out that some of the program options have not been used extensively. In particular, only a few runs have been made in the design mode, where the loading and thickness distributions are prescribed. For this reason, there are unknown limits on the usable ranges for some of the parameters such as step size and relaxation factors.

Section 1

SUMMARY OF BASIC EQUATIONS

The coordinate system and rotor geometry used are shown in Figure 1, where the dimensionless variables are defined as

$$z = \frac{\omega x}{U_\infty} \quad , \quad \rho = \frac{\omega r}{U_\infty} \quad , \quad \zeta = \theta - z \quad ; \quad \phi = \frac{\omega \varphi}{U_\infty^2} \tag{1}$$

The velocity components seen by a blade-fixed observer are

$$W_x = U_\infty + u \quad , \quad W_r = v \quad , \quad W_\theta = \omega r + w \tag{2}$$

These dimensional perturbation velocities are related to the velocity potential by

$$\begin{aligned} \bar{u} &= \frac{u}{U_\infty} = \left.\frac{\partial \phi}{\partial z}\right)_{\rho,\theta} = \left.\frac{\partial \phi}{\partial z}\right)_{\rho,\zeta} - \left.\frac{\partial \phi}{\partial \zeta}\right)_{z,\rho} \; ; \\ \bar{v} &= \frac{v}{U_\infty} = \left.\frac{\partial \phi}{\partial \rho}\right)_{z,\theta} = \left.\frac{\partial \phi}{\partial \rho}\right)_{z,\zeta} \\ \bar{w} &= \frac{w}{U_\infty} = \left.\frac{1}{\rho}\frac{\partial \phi}{\partial \theta}\right)_{z,\rho} = \left.\frac{1}{\rho}\frac{\partial \phi}{\partial \zeta}\right)_{z,\rho} \end{aligned} \tag{3}$$

The velocity potential satisfies the equation

$$\begin{aligned} &\left\{1 - M_\infty^2(1+\rho^2) - (\gamma+1)M_\infty^2\,\phi_z\right\}\phi_{zz} + \rho^2\phi_{zz} - 2(1+\rho^2)\,\phi_{\zeta z} \\ &\qquad + \frac{(1+\rho^2)^2}{\rho^2}\phi_{\zeta\zeta} + (1+\rho^2)\left(\phi_{\rho\rho} + \frac{1}{\rho}\phi_\rho\right) = 0 \end{aligned} \tag{4}$$

Subscripts denote derivatives in the z, ρ, ζ coordinate system; thus the symbol ϕ_z stands for $\left.\frac{\partial \phi}{\partial z}\right)_{\rho,\zeta}$. There are B blades in the row, and they are taken to lie in the helical surfaces defined by U_∞ and ωr. The axial projection of their chord is a constant, c_a. Thus, they are located at

$$0 \le z \le \frac{\omega c_a}{U_\infty} \quad ; \quad \rho_H \le \rho \le \rho_T \quad ; \quad \zeta = \frac{2j\pi}{B} \quad , \quad j = 0, 1, 2, \ldots, B-1 \tag{5}$$

If the program is run in the off-design mode, the blade shapes are given by (see Figure 2)

$$n_{\substack{u\\L}}(s,r) = h(s,r) \pm \frac{1}{2}\, t(s,r) - \alpha(r)\cdot s \tag{6}$$

where h, t and α are the camber, thickness, and angle of incidence. The blade-surface boundary condition is

$$\frac{u_n}{W_o} = \frac{dn}{ds} \tag{7}$$

When the program is run in the design mode, the prescribed quantities are the loading and thickness distributions:

$$\Delta C_p(s,r) \equiv \frac{p_L - p_u}{\frac{1}{2}\,\rho_\infty\, U_\infty^2} = -2\,(1+\rho^2)\left[\left.\frac{u_s}{W_o}\right)_L - \left.\frac{u_s}{W_o}\right)_u\right] \tag{8}$$

$$t'(s,r) = \frac{dn_u}{ds} - \frac{dn_L}{ds} \tag{9}$$

The blade-surface boundary conditions are applied in the helical surfaces $\zeta = 0$ and $\zeta = 2\pi/B$.

Far upstream of the blades, the perturbation potential is set equal to zero; far downstream, it is expressed as $\phi = Cz$, where

$$C = \frac{-B}{\pi\,(1-M_\infty^2)(\rho_T^2-\rho_H^2)} \int_{\rho_H}^{\rho_T} \rho\,\Delta\phi\; d\rho \tag{10}$$

Section 2

PROGRAM DESCRIPTION

A guide to the preparation of the input is given in Appendix A, while the listing of the program itself and a dictionary of the FORTRAN variables are given in appendices B and C.

The program begins by reading and printing input values. The finite-difference grid is then calculated, in subroutine GRID. The region in which the solution is to be found is divided into a grid, with the indices L , K and N used to number points in the ζ , z and ρ directions, respectively. Equal spacing is used in the ζ- and ρ- directions:

$$\zeta(L) = (L-1)\Delta\zeta \ , \quad L = 1,2,\ldots,LMX \ ; \quad \Delta\zeta = \frac{2\pi}{B}\Big/(LMX-1) \tag{11}$$

$$\rho(N) = \rho_H + (N-1)\Delta\rho \ , \quad N = 1,2,\ldots,NMX \ ; \quad \Delta\rho = \frac{\rho_T - \rho_H}{NMX-1} \tag{12}$$

The spacing in the z- direction is nonuniform; the z- coordinate is taken as

$$z = z(\tau) \quad ; \quad \frac{\partial}{\partial z} = f\,\frac{\partial}{\partial \tau} \ , \quad f \equiv \frac{d\tau}{dz} \tag{13}$$

The variable τ is then allowed to vary from -1 to +1, as z varied from $-\infty$ to $+\infty$.

$$\tau = -1 + K\Delta\tau \ , \quad K = 1,2,\ldots,KMX \ ; \quad \Delta\tau = \frac{2}{KMX+1} \tag{14}$$

The particular dependence $z(\tau)$ used here is

$$z(K) = z_M + \frac{1}{2\alpha_1}\,\ln\frac{1+\tau(K)}{1-\tau(K)} \tag{15}$$

where

$$\begin{aligned} z_M &= \frac{1}{2}(z_I + z_B) \\ z_I &= FXI\cdot\frac{\omega c_a}{u_\infty} \ , \quad z_B = FXB\cdot\frac{\omega c_a}{u_\infty} \\ 2\alpha_1 &= \frac{\ln\left[\Delta\tau/(2-\Delta\tau)\right]}{\frac{1}{2}(z_B - z_I)} \end{aligned} \tag{16}$$

The locations of the upstream and downstream edges of the grid are set by the input values for FXB and FXI.

Next, the blade-surface boundary conditions are calculated, in subroutine BVAL. These conditions depend on IBC: for IBC = 1 (the off-design case), the blade-shape parameters in the array BV are used to calculate the surface slopes DNDS on the suction and pressure sides, at each of the axial and radial grid points on the blades.

For IBC = 2 (the design case) the assigned loading and thickness distributions are used to generate the following quantities, which are used in applying blade-surface boundary conditions at L = 1 and L = LMX, respectively:

$$DNDS(KB, N, 1) = \int_0^z \frac{\Delta C_p}{2}\, dz \tag{17}$$

$$DNDS(KB, N, 2) = \frac{\rho^2}{1+\rho^2}\,\frac{\Delta C_p}{2} + \rho t'(s) \tag{18}$$

The program version listed here uses a parabolic-arc blade shape, for IBC = 1:

$$t(s, r) = \frac{4 t_{MAX}}{[c(r)]^2}\left\{s\,[c(r) - s]\right\} \tag{19}$$

$$h(s, r) = \frac{4 h_{MAX}}{[c(r)]^2}\left\{s\,[c(r) - s]\right\} \tag{20}$$

The maximum thickness and camber, which occur at $s/c = 1/2$ for this shape, are allowed to vary in the radial direction according to:

$$t_{MAX} = C_a\left\{BV(1) + BV(2)\,\frac{\rho_T}{\rho} + BV(3)\,\frac{\rho}{\rho_T}\right\} \tag{21}$$

$$t_{MAX} = C_a \left\{ BV(4) + BV(5)\,\frac{\rho_T}{\rho} + BV(6)\,\frac{\rho}{\rho_T} \right\} \tag{22}$$

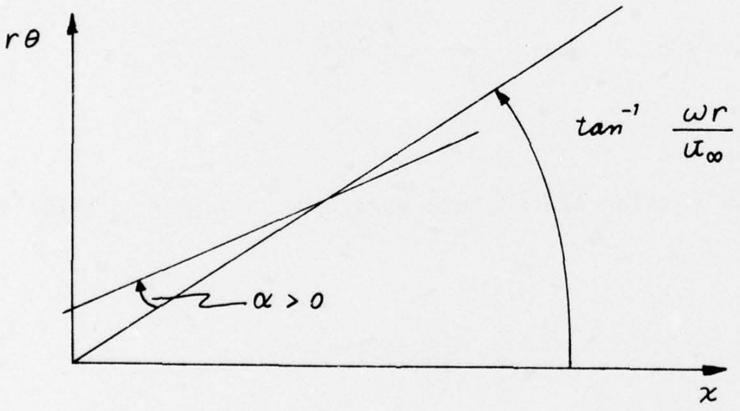
Note that

$$\frac{C_a}{C(r)} = 1 \Big/ \sqrt{1+\rho^2} \tag{23}$$

In addition, the angle of incidence varies radially as:

$$\alpha = BV(7) + BV(8)\,\frac{\rho_T}{\rho} + BV(9)\,\frac{\rho}{\rho_T} \tag{24}$$

The angle of incidence is measured with respect to the helical direction, as shown below:



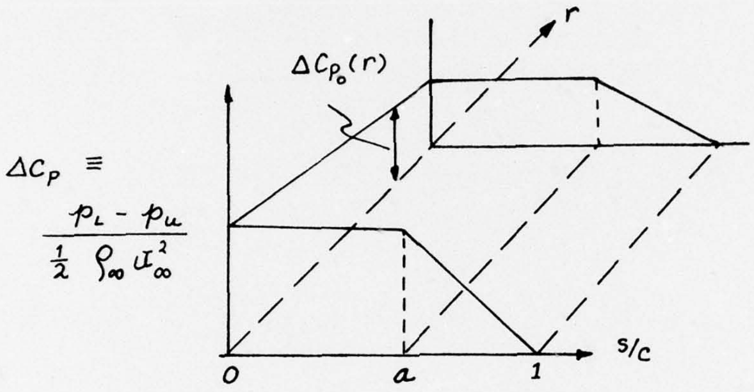
For IBC = 2 (the design case) the program version listed here uses a parabolic-arc thickness distribution:

$$\begin{aligned} t'(s) &= 4\,\frac{t_{MAX}}{C(r)} \left[1 - 2\,\frac{S}{C} \right] \\ &= 4\,\frac{t_{MAX}}{C_a}\,\frac{1}{\sqrt{1+\rho^2}} \left[1 - 2\,\frac{Z(K)}{\omega C_a / U_\infty} \right] \end{aligned} \tag{25}$$

The maximum thickness is taken as a constant:

$$\frac{t_{MAX}}{C_a} = BV(3) \tag{26}$$

The loading distribution had the form



where the value of $\Delta C_{p_o}(r)$ varies linearly between prescribed values at the hub and tip, and a is constant. These parameters are read in by:

$$BV(2) = a \;, \; BV(1) = \Delta C_{p_o})_{HUB} \;, \; BV(4) = \Delta C_{p_o})_{TIP} \tag{27}$$

As noted in Reference 3, this distribution is inconsistent with the condition $v = 0$ at the hub and tip; the loading $\Delta\phi(r)$ should have zero radial gradient at the hub and tip, if v is to equal zero there.

Other blade shapes and loading distributions can be used, by making appropriate changes in this subroutine, and in the array BV. The sole function of BVAL is to return the arrays DNDS (KB, N, J), J = 1, 2 as defined in Equations (7), (17) and (18) above. Use of other blade-shape or loading distributions does not require changes elsewhere in the program.

Following this calculation of blade-surface boundary conditions, the main iteration loop begins. The outer loop (DO 300 N=...) is in the radial direction. Within this loop, the line on which the solution is being updated is swept downstream in the loop DO 5 K=... . The innermost loop (DO 7 L=...) carries out the relaxation by the standard recursive algorithm, described below.

The finite-difference equation solved in the innermost loop is

$$
\begin{aligned}
&\left\{V_K^L+\rho^2\right\}(1-\mu_K^L)\left\{f_{K+\frac{1}{2}}\,{}^{N}\phi^{L}_{K+1}-2f_K\left[\frac{{}^{N}\overset{+}{\phi}{}^{L}_{K}}{\omega_e}+\left(1-\frac{1}{\omega_e}\right){}^{N}\phi^{L}_{K}\right]+f_{K-\frac{1}{2}}\,{}^{N}\overset{+}{\phi}{}^{L}_{K-1}\right\}\\
&+V^L_{K-1}\,\mu^L_{K-1}\left\{f_{K-\frac{1}{2}}\left(2\,{}^{N}\overset{+}{\phi}{}^{L}_{K}-{}^{N}\phi^{L}_{K}-{}^{N}\overset{+}{\phi}{}^{L}_{K-1}\right)-f_{K-\frac{3}{2}}\left({}^{N}\overset{+}{\phi}{}^{L}_{K-1}-{}^{N}\phi^{L}_{K-2}\right)\right\}\\
&+\rho^2\mu^L_K\left\{f_{K+\frac{1}{2}}\left({}^{N}\phi^{L}_{K+1}-{}^{N}\phi^{L}_{K}\right)-f_{K-\frac{1}{2}}\left({}^{N}\overset{+}{\phi}{}^{L}_{K}-{}^{N}\overset{+}{\phi}{}^{L}_{K-1}\right)\right\}\\
&-2\,\frac{(1+\rho^2)\Delta\tau}{2\Delta\zeta}\left\{-{}^{N}\overset{+}{\phi}{}^{L+1}_{K-1}+{}^{N}\phi^{L+1}_{K}+{}^{N}\overset{+}{\phi}{}^{L}_{K-1}-2\,{}^{N}\phi^{L}_{K}+{}^{N}\phi^{L}_{K+1}+{}^{N}\phi^{L-1}_{K}-{}^{N}\phi^{L-1}_{K+1}\right\}\\
&+\frac{(1+\rho^2)^2}{\rho^2 f_K}\left(\frac{\Delta\tau}{\Delta\zeta}\right)^2\left\{{}^{N}\overset{+}{\phi}{}^{L+1}_{K}-2\,{}^{N}\overset{+}{\phi}{}^{L}_{K}+{}^{N}\overset{+}{\phi}{}^{L-1}_{K}\right\}\\
&+\frac{1+\rho^2}{f_K}\left(\frac{\Delta\tau}{\Delta\rho}\right)^2\left\{\left(1+\frac{\Delta\rho}{\rho}\right){}^{N+1}\phi^{L}_{K}+\left(1-\frac{\Delta\rho}{\rho}\right){}^{N-1}\phi^{L}_{K}-2\,{}^{N}\phi^{L}_{K}\right\}=0
\end{aligned}
\tag{28}
$$

where

$$V_K^L = 1-M_\infty^2(1+\rho^2)-(\gamma+1)M_\infty^2 f_K\,\frac{{}^{N}\phi^{L}_{K+1}-{}^{N}\overset{+}{\phi}{}^{L}_{K-1}}{2\Delta\tau}$$

$$\mu_K^L = \begin{matrix}0\\1\end{matrix}\quad,\quad \text{for } V_K^L \lessgtr 0 \tag{29}$$

This equation is rewritten as

$$A_K^L \, {}^N\overset{+}{\phi}{}_K^{L+1} + B_K^L \, {}^N\overset{+}{\phi}{}_K^{L} + C_K^L \, {}^N\overset{+}{\phi}{}_K^{L-1} = D_K^L \, , \quad L = 2, 3, \ldots, LMX-1 \tag{30}$$

The solution is found by

$${}^N\overset{+}{\phi}{}_K^{L} = EE(L) \, {}^N\overset{+}{\phi}{}_K^{L+1} + FF(L) \tag{31}$$

where $EE(L)$ and $FF(L)$ obey the recursion relations

$$\begin{aligned} EE(L) &= -A_K^L \,/\, [B_K^L + C_K^L \, EE(L-1)] \\ FF(L) &= [D_K^L - C_K^L \, FF(L-1)] \,/\, [B_K^L + C_K^L \, EE(L-1)] \end{aligned} \tag{32}$$

Subroutine BVALO uses the blade-surface or periodicity conditions to set EE(1) and FF(1). Then the recursion formulas above are used to find EE(L) and FF(L) for L = 2, 3, ..., LMX - 1. Subroutine BVALI then sets the value of the potential at LMX, by again using the blade-surface or periodicity conditions. Equation (31) is then used to find the values of the potential, from LMX - 1 down to L = 1. These tentative values of ϕ are stored in the array SV; the updated values are found by

$${}^N\overset{+}{\phi}{}_K^{L} = \omega \, SV(L) + (1-\omega) \, {}^N\phi_K^L \tag{33}$$

The specific formulas used in subroutine BVALO are:

(a) Periodicity condition, for $z < 0$

$$EE(1) = 0 \, , \quad FF(1) = {}^N\phi_K^{LMX} \tag{34}$$

(b) Periodicity condition, for $z > \omega c_a / U_\infty$

$$EE(1) = 0 \quad , \quad FF(1) = {}^{N}\phi^{LMX}_{K} + \Delta\phi(N) \tag{35}$$

(c) Blade-surface condition for IBC = 1: the derivative $\phi_{\zeta\zeta}$ is approximated by

$$\begin{aligned}\phi_{\zeta\zeta}\Big)_{L=1} &= \frac{6}{\Delta\zeta}\left\{\frac{-\phi^{3}_{K} + 8\phi^{2}_{K} - 7\phi^{1}_{K}}{12\,\Delta\zeta} - \frac{1}{2}\,\phi_{\zeta}\Big)_{L=1}\right\} \\ &= \frac{6}{\Delta\zeta}\left\{\frac{-\phi^{3}_{K} + 8\phi^{2}_{K} - 7\phi^{1}_{K}}{12\,\Delta\zeta} - \frac{\rho}{2}\,\frac{dn}{ds}\Big)_{L=1}\right. \\ &\quad \left. - \frac{\rho^{2} f_{K}}{2(1+\rho^{2})}\,\frac{2\phi^{1}_{K+1} + 3\phi^{1}_{K} - 6\phi^{1}_{K-1} + \phi^{1}_{K-2}}{6\,\Delta\zeta}\right\}\end{aligned} \tag{36}$$

This expression is then used to write the potential equation at L = 1 as

$$\begin{aligned}&\left\{V^{1}_{K} + \rho^{2}\right\}(1-\mu^{1}_{K})\left\{f_{K+\frac{1}{2}}\,{}^{N}\phi^{1}_{K+1} - 2f_{K}\left[\frac{{}^{N}\overset{+}{\phi}{}^{1}_{K}}{\omega_e} + \left(1-\frac{1}{\omega_e}\right){}^{N}\phi^{1}_{K}\right] + f_{K-\frac{1}{2}}\,{}^{N}\overset{+}{\phi}{}^{1}_{K-1}\right\} \\ &+ V^{1}_{K-1}\,\mu^{1}_{K-1}\left\{f_{K-\frac{1}{2}}\left(2\,{}^{N}\overset{+}{\phi}{}^{1}_{K} - {}^{N}\phi^{1}_{K} - {}^{N}\overset{+}{\phi}{}^{1}_{K-1}\right) - f_{K-\frac{3}{2}}\left({}^{N}\overset{+}{\phi}{}^{1}_{K-1} - {}^{N}\phi^{1}_{K-2}\right)\right\} \\ &+ \rho^{2}\mu^{1}_{K}\left\{f_{K+\frac{1}{2}}\left({}^{N}\phi^{1}_{K+1} - {}^{N}\phi^{1}_{K}\right) - f_{K-\frac{1}{2}}\left({}^{N}\overset{+}{\phi}{}^{1}_{K} - {}^{N}\overset{+}{\phi}{}^{1}_{K-1}\right)\right\} \\ &- 2(1+\rho^{2})\,\frac{\Delta\tau}{4\Delta\zeta}\left\{{}^{N}\phi^{3}_{K-2} - {}^{N}\phi^{3}_{K} + 8\,{}^{N}\overset{+}{\phi}{}^{2}_{K} - 8\,{}^{N}\phi^{2}_{K-1} - \left[4\,{}^{N}\overset{+}{\phi}{}^{1}_{K} + 3\,{}^{N}\phi^{1}_{K}\right] + 8\,{}^{N}\phi^{1}_{K-1} - {}^{N}\phi^{1}_{K-2}\right\} \\ &+ \frac{(1+\rho^{2})^{2}}{\rho^{2} f_{K}}\left(\frac{\Delta\tau}{\Delta\zeta}\right)^{2}\left\{-\frac{1}{2}\,{}^{N}\phi^{3}_{K} + 4\,{}^{N}\overset{+}{\phi}{}^{2}_{K} - \frac{3}{2}\,{}^{N}\overset{+}{\phi}{}^{1}_{K} - 3\rho\Delta\zeta\,\frac{dn}{ds}\Big)_{L=1}\right. \\ &\qquad\qquad \left. - \frac{\rho^{2} f_{K}}{2(1+\rho^{2})}\,\frac{\Delta\zeta}{\Delta\tau}\left[2\,{}^{N}\phi^{1}_{K+1} + 3\,{}^{N}\phi^{1}_{K} - 6\,{}^{N}\phi^{1}_{K-1} + {}^{N}\phi^{1}_{K-2}\right]\right\} \\ &+ \frac{1+\rho^{2}}{f_{K}}\left(\frac{\Delta\tau}{\Delta\rho}\right)^{2}\left\{\left(1+\frac{\Delta\rho}{\rho}\right){}^{N+1}\phi^{1}_{K} + \left(1-\frac{\Delta\rho}{\rho}\right){}^{N-1}\phi^{1}_{K} - 2\,{}^{N}\phi^{1}_{K}\right\} = 0\end{aligned} \tag{37}$$

This is then written as

$$AE\ {}^{N}\overset{+}{\phi}{}_{K}^{1} + BE\ {}^{N}\overset{+}{\phi}{}^{2} + CE = 0 \tag{38}$$

which leads to

$$EE(1) = -BE/AE\ , \quad FF(1) = -CE/AE \tag{39}$$

(d) Blade-surface condition for IBC = 2:

$$EE(1) = 0\ , \quad FF(1) = {}^{N}\phi_{K}^{LMX} + \int_{0}^{z} \frac{\Delta C_{p}}{2}\, dz \tag{40}$$

The latter integral is calculated in BVAL before the iterations begin, and is stored in the array DNDS (K-KLEP, N, 1), where KLEP is the K-value of the station just upstream of the leading edge.

The specific formulas used in subroutine BVALI are:

(a) Periodicity condition, for $z < 0$: points along LMX are treated as field points, with values of the potential on LMX + 1 set equal to their values on L = 2. Thus, Equation (30) is written as

$$B_{K}^{LMX}\ {}^{N}\overset{+}{\phi}{}_{K}^{LMX} + C_{K}^{LMX}\ {}^{N}\overset{+}{\phi}{}_{K}^{LMX-1} = D_{K}^{LMX} - A_{K}^{LMX}\ {}^{N}\phi_{K}^{2} \tag{41}$$

comparison with Eq (31) then leads to the explicit expression:

$${}^{N}\overset{+}{\phi}{}_{K}^{LMX} = SV(LMX) = \frac{D_{K}^{LMX} - A_{K}^{LMX}\ {}^{N}\phi_{K}^{2} - C_{K}^{LMX}\ FF(LMX-1)}{B_{K}^{LMX} + C_{K}^{LMX}\ EE(LMX-1)} \tag{42}$$

(b) Periodicity condition, for $z > \omega c_{a}/U_{\infty}$: here the same formulas are used, except that ${}^{N}\phi_{K}^{LMX+1}$ is replaced by ${}^{N}\phi_{K}^{2} - \Delta\phi(N)$, and there is a contribution to ϕ_{RR} from the radial derivative of the circulation:

$$
\begin{aligned}
\phi_{\zeta\zeta}^{LMX} = &- \frac{\rho^2}{2(1+\rho^2)} \left\{ \left[\frac{\Delta\phi(N+1) - 2\Delta\phi(N) + \Delta\phi(N-1)}{(\Delta\rho)^2} \right.\right. \\
&\left.\left. + \frac{\Delta\phi(N+1) - \Delta\phi(N-1)}{2\rho\Delta\rho} \right] \right\} + \frac{{}^N\phi_K^{2} + {}^N\phi_K^{LMX-1} - 2\,{}^N\phi_K^{LMX} - \Delta\phi(N)}{(\Delta\zeta)^2}
\end{aligned}
\tag{43}
$$

It should be noted that the evaluation of the mixed derivative at LMX, and at the stations immediately off the blades (K = KLEP and KTEO) uses differences across the blade surface.

(c) Blade-surface condition for IBC = 1: the derivative $\phi_{\zeta\zeta}$ is approximated by

$$
\begin{aligned}
\phi_{\zeta\zeta}\Big)_{LMX} &= \frac{6}{\Delta\zeta} \left\{ \left[\frac{-\,{}^N\phi_K^{LMX-2} + 8\,{}^N\phi_K^{LMX-1} - 7\,{}^N\phi_K^{LMX}}{12\Delta\zeta} + \frac{1}{2}\,\phi_\zeta\Big)_{LMX} \right] \right\} \\
&= \frac{6}{\Delta\zeta} \left\{ \left[\frac{-\,{}^N\phi_K^{LMX-2} + 8\,{}^N\phi_K^{LMX-1} - 7\,{}^N\phi_K^{LMX}}{12\Delta\zeta} + \frac{\rho}{2}\,\frac{dn}{ds}\Big)_{LMX} \right.\right. \\
&\left.\left. + \frac{\rho^2 f_K}{2(1+\rho^2)}\,\frac{2\,{}^N\phi_{K+1}^{LMX} + 3\,{}^N\phi_K^{LMX} - 6\,{}^N\phi_{K-1}^{LMX} + {}^N\phi_{K-2}^{LMX}}{6\Delta\tau} \right] \right\}
\end{aligned}
\tag{44}
$$

Using this expression, the potential equation at LMX is written as:

$$
\begin{aligned}
&\Big\{V_K^{LMX}+\rho^2\Big\}(1-\mu_K^{LMX})\Big\{f_{K+\frac{1}{2}}\,{}^{N}\phi_{K+1}^{LMX}-2f_K\Big[\frac{{}^{N}\overset{+}{\phi}{}^{LMX}}{\omega_e}+\Big(1-\frac{1}{\omega_e}\Big){}^{N}\phi_K^{LMX}\Big]+f_{K-\frac{1}{2}}\,{}^{N}\overset{+}{\phi}{}_{K-1}^{LMX}\Big\}\\
&+V_{K-1}^{LMX}\,\mu_{K-1}^{LMX}\Big\{f_{K-\frac{1}{2}}\big(2\,{}^{N}\overset{+}{\phi}{}_K^{LMX}-{}^{N}\phi_K^{LMX}-{}^{N}\overset{+}{\phi}{}_{K-1}^{LMX}\big)-f_{K-\frac{3}{2}}\big({}^{N}\overset{+}{\phi}{}_{K-1}^{LMX}-{}^{N}\phi_{K-2}^{LMX}\big)\Big\}\\
&+\rho^2\mu_K^{LMX}\Big\{f_{K+\frac{1}{2}}\big({}^{N}\phi_{K+1}^{LMX}-{}^{N}\phi_K^{LMX}\big)-f_{K-\frac{1}{2}}\big({}^{N}\overset{+}{\phi}{}_K^{LMX}-{}^{N}\overset{+}{\phi}{}_{K-1}^{LMX}\big)\Big\}\\
&-2(1+\rho^2)\,\frac{\Delta\tau}{4\Delta\zeta}\Big\{{}^{N}\phi_{K+2}^{LMX-2}-{}^{N}\phi_K^{LMX-2}+8\,{}^{N}\overset{+}{\phi}{}_K^{LMX-1}-8\,{}^{N}\phi_{K+1}^{LMX-1}\\
&\qquad\qquad+4\,{}^{N}\phi_{K+1}^{LMX}-{}^{N}\overset{+}{\phi}{}_K^{LMX}-4\,{}^{N}\phi_{K-1}^{LMX}+{}^{N}\phi_{K-2}^{LMX}\Big\}\\
&+\frac{(1+\rho^2)^2}{\rho^2 f_K}\Big(\frac{\Delta\tau}{\Delta\zeta}\Big)^2\Big\{-\frac{1}{2}\,{}^{N}\phi_K^{LMX-2}+4\,{}^{N}\overset{+}{\phi}{}_K^{LMX-1}-\frac{7}{2}\,{}^{N}\overset{+}{\phi}{}_K^{LMX}+3\rho\Delta\zeta\,\frac{d\eta}{ds}\Big)_{LMX}\\
&\qquad+\frac{\rho^2 f_K}{2(1+\rho^2)}\,\frac{\Delta\zeta}{\Delta\tau}\Big[2\,{}^{N}\phi_{K+1}^{LMX}+3\,{}^{N}\phi_K^{LMX}-6\,{}^{N}\phi_{K-1}^{LMX}+{}^{N}\phi_{K-2}^{LMX}\Big]\Big\}\\
&+\frac{1+\rho^2}{f_K}\Big(\frac{\Delta\tau}{\Delta\rho}\Big)^2\Big\{\Big(1+\frac{\Delta\rho}{\rho}\Big)\,{}^{N+1}\phi_K^{LMX}+\Big(1-\frac{\Delta\rho}{\rho}\Big)\,{}^{N-1}\phi_K^{LMX}-2\,{}^{N}\phi_K^{LMX}\Big\}=0
\end{aligned}
\tag{45}
$$

This is written in the form

$$
AE\,{}^{N}\overset{+}{\phi}{}_K^{LMX}+BE\,{}^{N}\overset{+}{\phi}{}_K^{LMX-1}+CE=0 \tag{46}
$$

and is solved, along with Equation (31), to give

$$ {}^{N}\overset{+}{\phi}{}_{K}^{LMX} = SV(LMX) = -\frac{CE + BE \cdot FF(LMX-1)}{AE + BE \cdot EE(LMX-1)} \tag{47} $$

(d) Blade-surface condition for IBC = 2: here the derivative $\phi_{\zeta\zeta}$ is approximated by using

$$ \phi_{\zeta}\big)_{LMX} = \phi_{\zeta}\big)_{L=1} - \frac{\rho^2}{1+\rho^2}\,\frac{\Delta C_p}{2} - \rho t'(s) \tag{48} $$

This equation, substituted into the first form of Equation (44), leads to the following form of the potential equation

$$
\begin{aligned}
&\left\{V_K^{LMX} + \rho^2\right\}\left(1-\mu_K^{LMX}\right)\left\{f_{K+\frac{1}{2}}\,{}^{N}\phi_{K+1}^{LMX} - 2f_K\left[\frac{{}^{N}\overset{+}{\phi}{}_{K}^{LMX}}{\omega_e} + \left(1-\frac{1}{\omega_e}\right){}^{N}\phi_{K}^{LMX}\right] + f_{K-\frac{1}{2}}\,{}^{N}\overset{+}{\phi}{}_{K-1}^{LMX}\right\} \\
&+ V_{K-1}^{LMX}\,\mu_{K-1}^{LMX}\left\{f_{K-\frac{1}{2}}\left(2\,{}^{N}\overset{+}{\phi}{}_{K}^{LMX} - {}^{N}\phi_{K}^{LMX} - {}^{N}\overset{+}{\phi}{}_{K-1}^{LMX}\right) - f_{K-\frac{3}{2}}\left({}^{N}\overset{+}{\phi}{}_{K-1}^{LMX} - {}^{N}\phi_{K-2}^{LMX}\right)\right\} \\
&+ \rho^2\,\mu_K^{LMX}\left\{f_{K+\frac{1}{2}}\left({}^{N}\phi_{K+1}^{LMX} - {}^{N}\phi_{K}^{LMX}\right) - f_{K-\frac{1}{2}}\left({}^{N}\overset{+}{\phi}{}_{K}^{LMX} - {}^{N}\overset{+}{\phi}{}_{K-1}^{LMX}\right)\right\} \\
&- 2\left(1+\rho^2\right)\frac{\Delta\tau}{4\Delta\zeta}\left\{{}^{N}\phi_{K+2}^{LMX-2} - {}^{N}\phi_{K}^{LMX-2} + 8\,{}^{N}\overset{+}{\phi}{}_{K}^{LMX-1} - 8\,{}^{N}\phi_{K+1}^{LMX-1}\right. \\
&\qquad\qquad \left. + 4\,{}^{N}\phi_{K+1}^{LMX} - {}^{N}\overset{+}{\phi}{}_{K}^{LMX} - 4\,{}^{N}\phi_{K-1}^{LMX} + {}^{N}\phi_{K-2}^{LMX}\right\} \\
&+ \frac{\left(1+\rho^2\right)^2}{\rho^2 f_K}\left(\frac{\Delta\tau}{\Delta\zeta}\right)^2\left\{-\frac{1}{2}\,{}^{N}\phi_{K}^{LMX-2} + 4\,{}^{N}\overset{+}{\phi}{}^{LMX-1} - \frac{7}{2}\,{}^{N}\overset{+}{\phi}{}_{K}^{LMX}\right. \\
&\qquad\qquad \left. + \frac{1}{2}\left[-6\Delta\zeta\left\langle -\frac{\rho^2}{1+\rho^2}\,\frac{\Delta C_p}{2} - \rho t'(s)\right\rangle - 11\,{}^{N}\phi_{K}^{1} + 18\,{}^{N}\phi_{K}^{2} - 9\,{}^{N}\phi_{K}^{3} + 2\,{}^{N}\phi_{K}^{4}\right]\right\} \\
&+ \frac{1+\rho^2}{f_K}\left(\frac{\Delta\tau}{\Delta\rho}\right)^2\left\{\left(1+\frac{\Delta\rho}{\rho}\right){}^{N+1}\phi_{K}^{LMX} + \left(1-\frac{\Delta\rho}{\rho}\right){}^{N-1}\phi_{K}^{LMX} - 2\,{}^{N}\phi_{K}^{LMX}\right\} = 0
\end{aligned}
\tag{49}
$$

The loading and thickness values are calculated in BVAL, and are stored in:

$$DNDS(K-KLEP,N,2) = \frac{\rho^2}{1+\rho^2}\frac{\Delta C_p}{2} + \rho t'(s) \tag{50}$$

This line relaxation procedure is carried out up to KMX - 1. Then values of the potential at KMX are set equal to the value required by mass conservation:

$${}^{N}\overset{+}{\phi}{}^{L}_{KMX} = \frac{C - DAF \cdot {}^{N}\overset{+}{\phi}{}^{L}_{KMX-2} - DBF \cdot {}^{N}\overset{+}{\phi}{}^{L}_{KMX-1}}{DCF} \tag{51}$$

where the following coefficients are calculated in GRID

$$\begin{aligned} DAF &= (Z_{KMX} - Z_{KMX-1})/(Z_{KMX-1} - Z_{KMX-2})(Z_{KMX} - Z_{KMX-2}) \\ DBF &= (Z_{KMX} - Z_{KMX-2})/(Z_{KMX-1} - Z_{KMX-2})(Z_{KMX-1} - Z_{KMX}) \\ DCF &= (2Z_{KMX} - Z_{KMX-1} - Z_{KMX-2})/(Z_{KMX} - Z_{KMX-2})(Z_{KMX} - Z_{KMX-1}) \end{aligned} \tag{52}$$

Next, the circulation is updated, using

$$\overset{+}{\Delta}\phi(N) = OBV\,\Delta\phi^{*} + (1-OBV)\,\Delta\phi(N) \tag{53}$$

where

$$\begin{aligned} \Delta\phi^{*} &= \frac{\left\{-(Z_{TE}-Z_b)^2\Delta\phi_a + (Z_{TE}-Z_a)^2\Delta\phi_b\right\}}{(Z_b - Z_a)(2Z_{TE} - Z_a - Z_b)} \\ Z_{TE} &= \omega c_a / u_\infty \\ \Delta\phi_{a,b} &= \overset{+}{\phi}(Z_{a,b}, \rho, 0) - \overset{+}{\phi}\left(Z_{a,b}, \rho, \frac{2\pi}{\beta}\right) \end{aligned} \tag{54}$$

This completes a sweep in the z- direction. The iteration counter ITK is then incremented, and compared with ITKMX, the maximum number of z - sweeps for each radial one. After ITKMX sweeps are done, the solution moves to the next radial station, where the process is repeated for N = 2, 3, ... NMX - 1. The iteration counter ITR is used to number the radial sweeps; a total of ITRMX is allowed.

After each radial sweep, values of ϕ and $\Delta\phi$ are updated at N = 1 and N = NMX according to

$$ {}^{1}\phi_K^L = \frac{4}{3}\,{}^{2}\phi_K^L - \frac{1}{3}\,{}^{3}\phi_K^L $$

$$ {}^{NMX}\phi_K^L = \frac{4}{3}\,{}^{NMX-1}\phi_K^L - \frac{1}{3}\,{}^{NMX-2}\phi_K^L \qquad (55) $$

$$ \Delta\phi(1) = 2\,\Delta\phi(2) - \Delta\phi(3) $$

$$ \Delta\phi(NMX) = 2\,\Delta\phi(NMX-1) - \Delta\phi(NMX-2) \qquad (56) $$

Also updated at the end of each radial sweep is the quantity C, used [see Equation (10)] in enforcing mass conservation at the downstream edge of the grid. This quantity, called CONST, is evaluated by the trapezoidal rule.

The OUTPUT subroutine can be called at intervals of JPRT in the ITK iterations and at intervals of NPRT in the ITR iterations. The former choice is intended for diagnostic purposes. It would normally be used only for checkout purposes, and even then for very few iterations and very few radial stations, since it generates many lines of output. To avoid this diagnostic output, set JPRT > ITKMX.

Several options are available in the output; these are controlled by the indicator IOP, and are described in Appendix A. The usual sequence is to run the solution for a certain number of iterations, write the solution on tape (ISAVE = 1), examine the output, and then restart the solution (ISTART = 1) with adjusted values of the relaxation factors, iteration-count parameters, etc. Some of the output options allow a minimum number of lines to be printed during these intermediate stages.

One special option is IOP = 4, which calculates Mach number contours. This option can only be used with a solution found on a previous run: no iterations are made, and the Mach number calculation is done on the values read in from the tape.

The quantities calculated and displayed by subroutine OUTPUT include the perturbation velocities u_s/W_0 , u_n/W_0 , v/U_∞ and the local Mach number. These are defined as follows:

$$\frac{u_s}{W_0} = \frac{\partial\phi/\partial z}{1+\rho^2} = f_K \frac{({}^N\phi^L_{K+1} - {}^N\phi^L_{K-1})}{(1+\rho^2)\, 2\Delta\tau} \tag{57}$$

$$\frac{v}{U_\infty} = \frac{{}^{N+1}\phi^L_K - {}^{N-1}\phi^L_K}{2\Delta\rho} \tag{58}$$

$$\frac{u_n}{W_0} = \frac{1}{\rho}\frac{\partial\phi}{\partial\zeta} - \frac{\rho}{1+\rho^2}\frac{\partial\phi}{\partial z}$$

$$= \frac{{}^N\phi^{L+1}_{K-1} - {}^N\phi^L_{K-1} + {}^N\phi^L_{K+1} - {}^N\phi^{L-1}_{K+1}}{2\rho\Delta\zeta} - \rho\frac{u_s}{W_0} \tag{59}$$

The first term in this expression is equal to $\partial\phi/\partial\zeta$, with a truncation error $O(\Delta\zeta)^2$. This particular form is shown so that:

$$\frac{u_n}{W_o} = \frac{{}^N\phi^{L+1}_{K-1} - {}^N\phi^{L-1}_{K+1}}{2\rho\Delta\zeta} = \frac{{}^N\phi^{L+1}_{K-1} - {}^N\phi^{L-1}_{K+1}}{2\sqrt{1+\rho^2}\,\Delta\hat{n}} \tag{60}$$

when

$$\frac{\rho\Delta\zeta}{\Delta Z} = \frac{1+\rho^2}{\rho} \tag{61}$$

An alternate expression for $\partial\phi/\partial\zeta$, having the same truncation error, was used in References 1-4, namely:

$$\frac{1}{\rho}\,\frac{\partial\phi}{\partial\zeta} = \frac{\phi^{L+1}_K - \phi^{L-1}_K}{2\rho\Delta\zeta} \tag{62}$$

This formula tends to give erratic results when shock waves are present, since it maximizes the amount of differencing done across the shock.

The local Mach number is defined by equating the coefficient of ϕ_{ss} in Equation (4) to $1-M^2$:

$$1-M^2 = 1-M_\infty^2(1+\rho^2) - (\gamma+1)M_\infty^2\phi_Z$$

or

$$M^2 = M_{rel}^2\left[1+(\gamma+1)\,u_s/W_o\right]$$

Thus

$$M = M_{rel}\sqrt{1+(\gamma+1)\,u_s/W_o} \approx M_{rel}\left[1+\frac{\gamma+1}{2}\,\frac{u_s}{W_o}\right] \tag{63}$$

The square-root formula was used in References 1-4; the present report uses the linearized version.

At the very end of every calculation, the output subroutine is called with IOP = 5. This displays the following performance data at each radius:

the values of N, $\rho(N)$, $\Delta\phi(N)$

$$w/u_\infty\big)_{z\to\infty} = \frac{-B}{2\pi\rho}\,\Delta\phi \tag{64}$$

$$u/u_\infty\big)_{z\to\infty} = C - \rho\,\frac{w}{u_\infty}\Big)_{z\to\infty} \tag{65}$$

the turning angle, defined as $\tan^{-1}\dfrac{\left[\dfrac{w}{u_\infty} - \rho\,\dfrac{u}{u_\infty}\right]_{z\to\infty}}{1+\rho^2}$ (66)

the total pressure ratio $\dfrac{p_{02}}{p_{01}} = 1 + \dfrac{\gamma M_\infty^2}{1 + \dfrac{\gamma-1}{2}M_\infty^2}\,\dfrac{B}{2\pi}\,\Delta\phi$ (67)

The convergence of the solution can be monitored by calculating the residuals. This is done, at ITR intervals of IRXP, in subroutine RESID. This subroutine evaluates all five terms in the potential equation at the interior points of the grid, i.e., for N = 2, 3, ..., NMX - 1, K = 2, 3, ..., KMX - 1, and L = 2, 3, ..., LMX - 1. The five terms (called Ⓐ through Ⓔ) are evaluated as follows [compare with Equation (4)]:

$$Ⓐ = (\Delta\tau)^2\left\{1 - M_\infty^2(1+\rho^2) - (\gamma+1)M_\infty^2\,\phi_z\right\}\phi_{\tau\tau} \tag{68}$$

$$\textcircled{B} = \rho^{2} (\Delta\tau)^{2} \phi_{\tau\tau} \tag{69}$$

$$\textcircled{C} = -2(1+\rho^{2})(\Delta\tau)^{2} \phi_{\xi\tau} \tag{70}$$

$$\textcircled{D} = \frac{(1+\rho^{2})^{2}}{\rho^{2} f} (\Delta\tau)^{2} \phi_{\xi\xi} \tag{71}$$

$$\textcircled{E} = \frac{1+\rho^{2}}{\rho f} \frac{\partial}{\partial\rho} (\rho \phi_{\rho}) \tag{72}$$

The residual at each point is defined as

$${}^{N}R^{L}_{K} = \left| \textcircled{A} + \textcircled{B} + \textcircled{C} + \textcircled{D} + \textcircled{E} \right| \tag{73}$$

At each value of N, the value of the maximum residual is printed, along with the K and L values of the point where it occurs. Also shown is the average residual, i.e., the mean of ${}^{N}R^{L}_{K}$ over the range N = 2, NMX - 1, K = 2, KMX - 1; L = 2, LMX - 1. The magnitudes of the residuals themselves are only meaningful in relation to the sizes of the individual terms whose sum they represent. Thus the mean value of ϕ and of the sum of the absolute values of the five terms are also shown, i.e.:

$$\text{AVG PHI} \equiv \frac{1}{P} \sum_{\substack{N=2,\,NMX-1\\ K=2,\,KMX-1\\ L=2,\,LMX-1}} \left| {}^{N}\phi^{L}_{K} \right| \tag{74}$$

$$AVG\ SUM \equiv \frac{1}{P} \sum_{\substack{N=2,NMX-1 \\ K=2,KMX-1 \\ L=2,LMX-1}} \left\{ |Ⓐ| + |Ⓑ| + |Ⓒ| + |Ⓓ| + |Ⓔ| \right\} \tag{75}$$

where $$P = (NMX - 2)\ (KMX - 2)\ (LMX - 2) \tag{76}$$

A second way to monitor the convergence of the solution is to observe the perturbation velocities at a number of selected points. Our experience has been that these will usually converge to three decimal places when AVG RES/ AVG SUM $\approx$ 3 x 10^{-2}.

If IRXP is prefixed by a minus sign, subroutine RESID will display the values of all five terms and the residual, at each of the interior grid points. This option, which produces many lines of output, can be used for diagnostic purposes.

If IBC = 2, subroutine OUTPUT also calculates the blade shape, using the formula

$$\frac{n_{u,L}}{c_a} = \frac{1+\rho^2}{\omega c_a / U_\infty} \int_0^z \left.\frac{u_n}{W_0}\right)_{u,L} dz \tag{77}$$

The US array is used to store this integral, which is done by the trapezoidal rule. The quantities printed are

$$z,\ \frac{z}{\frac{\omega c_a}{U_\infty}},\ \frac{n_u}{c_a},\ \frac{n_L}{c_a},\ \frac{h}{c_a},\ \frac{t}{c_a},\ \frac{n_u}{c(r)},\ \frac{n_L}{c(r)},\ \frac{h}{c(r)},\ \frac{t}{c(r)}$$

Section 3

SAMPLE CASE

To illustrate the operation of the program, some details are presented of the two-dimensional case shown in Figure 7 of Reference 4. This cascade had a stagger angle of 45°, an inlet relative Mach number of 0.9, a solidity C_a/L_T of 0.411, six percent thickness-to-chord ratio, and no camber or thickness. The input data were as follows:

$B = 10$, $\hbar = 0.9$, $C_a/L_T = 0.41073$, $M_x = 0.636396$, $M_{\theta_{tip}} = 0.669891$

$\gamma = 1.4$, FXB = -2 , FXI = +2 , RXE = 1 , RXH = 0.9

OBV = 0.05

KMX = 60 , LMX = 30 , NMX = 3 , IBC = 1 , IDM = 2 , ITKMX = 360 ,

JPRT = 400 , ISTART = 0 , ISAVE = 0 , IOP = 1 , ITRMX = 1 ,

NPRT = 0 , ITPR = 1 , IRXP = 1

BV(1) = 0.0848528 , BV(2) through BV(10) = 0.

Figures 3a - 3c are the first three pages of the output, showing the run conditions, coordinate system, and blade geometry. Figures 4a - 4d are portions of the pages on which the potential, streamwise, normal and radial velocity components appear. The last of these is zero for the case shown; it would be non-zero if NMX were greater than 3, i.e., if two-dimensional strip-theory calculations were being done at a series of radial stations. Figure 5 is the last page of the output, showing the most recent value of the circulation, the residuals, and the performance data. Note that the average residual is already down to less than 1/100 of the average sum of terms, even at this early stage of the iterations.

This case used 3.5 minutes of CPU time on an IBM 360/65.

REFERENCES

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2. Rae, W.J. "Relaxation Solutions for Three-Dimensional Transonic Flow Through a Compressor Blade Row, in the Nonlinear Small-Disturbance Approximation" AFOSR TR-76-1081, AD-A032553 (August 1976).

3. Rae, W.J. "Finite-Difference Calculations of Three-Dimensional Transonic Flow Through a Compressor Blade Row, Using the Small-Disturbance Nonlinear Potential Equation" pp. 228-252 of Transonic Flow Problems in Turbomachinery ed. by T.C. Adamson & M.F. Platzer Hemisphere Publishing Corp., Washington (1977).

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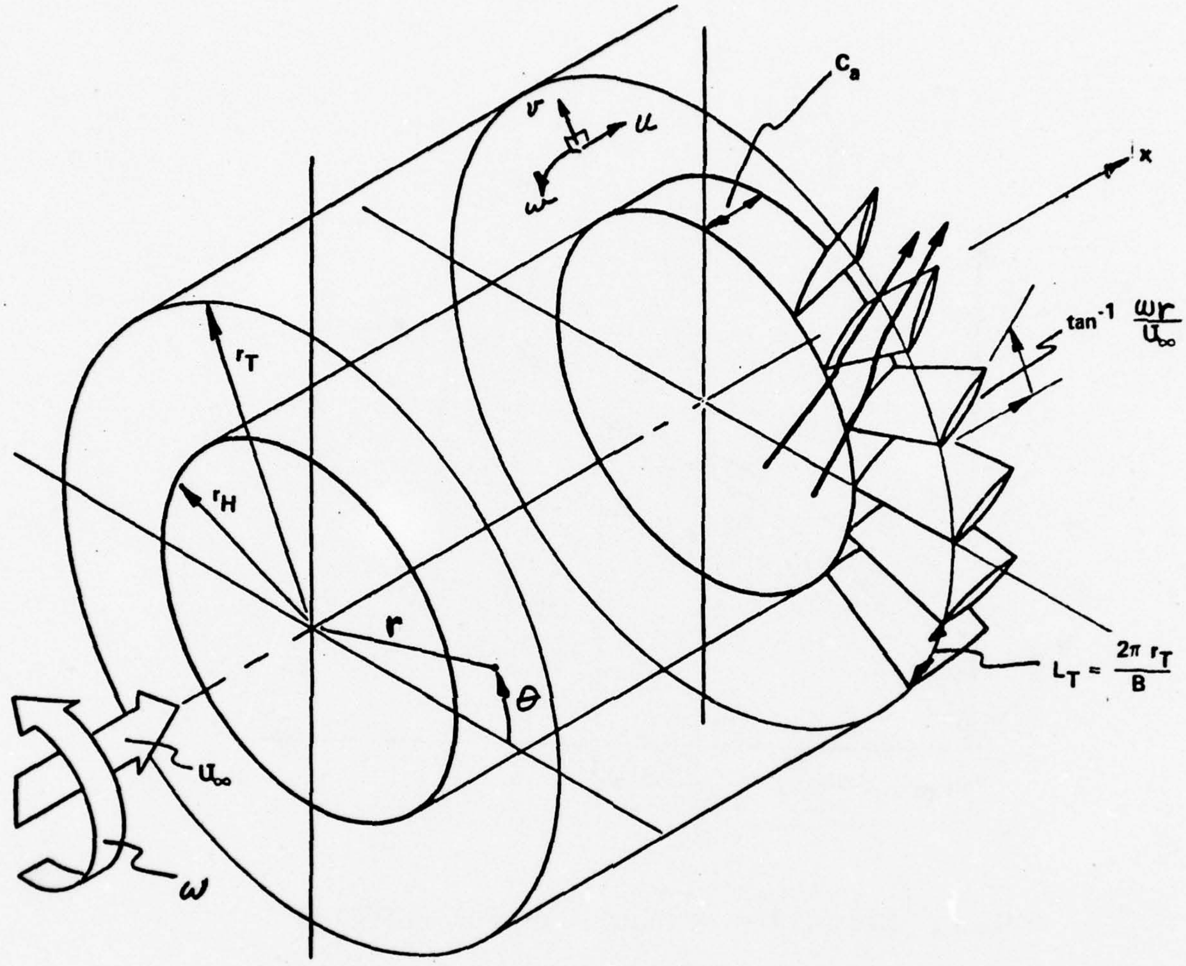

Figure 1 BLADE-FIXED COORDINATES

ROTOR IS STATIONARY IN A
HELICAL APPROACH FLOW

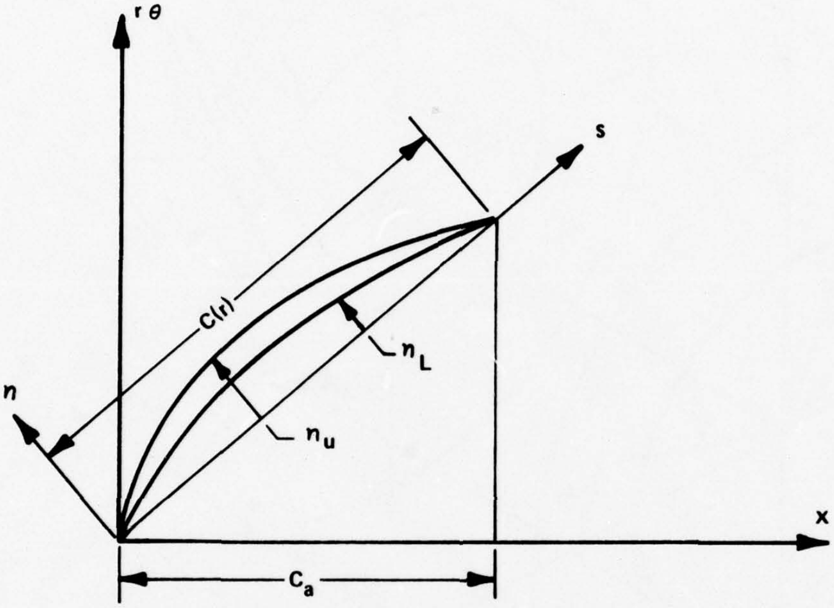

Figure 2 DEFINITIONS OF BLADE-SURFACE GEOMETRY

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                           SAMPLE CASE FOR USERS GUIDE REPORT
                           CASE IS THAT OF FIGURE 7, JOUNAL OF ENERGY, P. 289, SEPT/OCT 1977

THIS BLADE ROW HAS  10 BLADES, WITH A HUB-TO-TIP RATIO OF 0.900 AND SOLIDITY CA/LT = 0.411

AXIAL MACH NO. = 0.636,  TANGENTIAL MACH NO  AT THE TIP = 0.670,  TOTAL MACH NO. AT THE TIP = 0.924

SPECIFIC HEAT RATIO = 1.400

THIS IS A 2-DIMENSIONAL CALCULATION, WITH GRID SIZE KMX/LMX/NMX =  60/ 30/  3

RELAXATION FACTORS FOR ELLIPTIC AND HYPERBOLIC POINTS ARE LISTED BELOW AS RXE AND RXH

THE BLADES LIE BETWEEN Z = 0  AND 0.271652

  K       Z           X/CA        RXE        RXH

  1   -0.543304     -2.0000     1.0000     0.9000
  2   -0.425544     -1.5665     1.0000     0.9000
  3   -0.355454     -1.3085     1.0000     0.9000
  4   -0.304851     -1.1222     1.0000     0.9000
  5   -0.264902     -0.9752     1.0000     0.9000
  6   -0.231672     -0.8528     1.0000     0.9000
  7   -0.203059     -0.7475     1.0000     0.9000
  8   -0.177810     -0.6546     1.0000     0.9000
  9   -0.155114     -0.5710     1.0000     0.9000
 10   -0.134416     -0.4948     1.0000     0.9000
 11   -0.115323     -0.4245     1.0000     0.9000
 12   -0.097539     -0.3591     1.0000     0.9000
 13   -0.080842     -0.2976     1.0000     0.9000
 14   -0.065058     -0.2395     1.0000     0.9000
 15   -0.050047     -0.1842     1.0000     0.9000
 16   -0.035696     -0.1314     1.0000     0.9000
 17   -0.021913     -0.0807     1.0000     0.9000
 18   -0.008619     -0.0317     1.0000     0.9000
 19    0.004253      0.0157     1.0000     0.9000
 20    0.016758      0.0617     1.0000     0.9000
 21    0.028946      0.1066     1.0000     0.9000
 22    0.040862      0.1504     1.0000     0.9000
 23    0.052544      0.1934     1.0000     0.9000
 24    0.064027      0.2357     1.0000     0.9000
 25    0.075343      0.2773     1.0000     0.9000
 26    0.086521      0.3185     1.0000     0.9000
 27    0.097589      0.3592     1.0000     0.9000
 28    0.108573      0.3997     1.0000     0.9000
 29    0.119498      0.4399     1.0000     0.9000
 30    0.130387      0.4800     1.0000     0.9000
```

FIGURE 3a

```
SAMPLE CASE FOR USERS GUIDE REPORT
CASE IS THAT OF FIGURE 7, JOUNAL OF ENERGY, P. 289, SEPT/OCT 1977 (Cont.)

  K        Z          X/CA       RXE        RXH
 31     0.141264     0.5200     1.0000     0.9000
 32     0.152154     0.5601     1.0000     0.9000
 33     0.163079     0.6003     1.0000     0.9000
 34     0.174063     0.6408     1.0000     0.9000
 35     0.185131     0.6815     1.0000     0.9000
 36     0.196309     0.7226     1.0000     0.9000
 37     0.207625     0.7643     1.0000     0.9000
 38     0.219108     0.8066     1.0000     0.9000
 39     0.230790     0.8496     1.0000     0.9000
 40     0.242705     0.8934     1.0000     0.9000
 41     0.254894     0.9383     1.0000     0.9000
 42     0.267399     0.9843     1.0000     0.9000
 43     0.280270     1.0317     1.0000     0.9000
 44     0.293564     1.0807     1.0000     0.9000
 45     0.307348     1.1314     1.0000     0.9000
 46     0.321698     1.1842     1.0000     0.9000
 47     0.336709     1.2395     1.0000     0.9000
 48     0.352494     1.2976     1.0000     0.9000
 49     0.369191     1.3591     1.0000     0.9000
 50     0.386974     1.4245     1.0000     0.9000
 51     0.406068     1.4948     1.0000     0.9000
 52     0.426766     1.5710     1.0000     0.9000
 53     0.449461     1.6545     1.0000     0.9000
 54     0.474711     1.7475     1.0000     0.9000
 55     0.503323     1.8528     1.0000     0.9000
 56     0.536554     1.9752     1.0000     0.9000
 57     0.576502     2.1222     1.0000     0.9000
 58     0.627105     2.3085     1.0000     0.9000
 59     0.697193     2.5665     1.0000     0.9000
 60     0.814954     3.0000     1.0000     0.9000
```

FIGURE 3a (Cont.)

```
L    ZETA

 1   0.0
 2   0.0217
 3   0.0433
 4   0.0650
 5   0.0867
 6   0.1083
 7   0.1300
 8   0.1517
 9   0.1733
10   0.1950
11   0.2167
12   0.2383
13   0.2600
14   0.2817
15   0.3033
16   0.3250
17   0.3467
18   0.3683
19   0.3900
20   0.4117
21   0.4333
22   0.4550
23   0.4767
24   0.4983
25   0.5200
26   0.5417
27   0.5633
28   0.5850
29   0.6067
30   0.6283

OPTIMUM SHOCK CAPTURING OCCURS WHEN THE GRID-SIZE RATIO RHO*(DELTA ZETA)/(DELTA Z) = (1+RHO**2)/RHO
THE TABLE BELOW LISTS THESE OPTIMUM VALUES, COMPARED TO THE VALUES ACTUALLY USED AT K =30

N    RHO       US/WO CRIT     ARCTAN(RHO),     M REL            GRID-SIZE RATIO
                               DEGREES                     OPTIMUM    ACTUAL(AT K = 30)

1   0.9474    1.2552E-01      43.452         0.8766         2.0029        1.8871
2   1.0000    9.7737E-02      45.000         0.9000         2.0000        1.9920
3   1.0526    7.1374E-02      46.469         0.9240         2.0026        2.0968
```

FIGURE 3b

```
BLADE GEOMETRY AND ANGLE OF ATTACK SPECIFIED ARE
                T MAX/CAX = TA + TB/R + TC*R, R = RO(N)/RTIP
                     WHERE TA =    8.485E-02 TB =     0.0         TC =     0.0

                H MAX/CAX = HA + HB/R + HC*R
                     WHERE HA =     0.0         HB =     0.0         HC =     0.0

                ALPHA = AA + AB/R + AC*R    (DEGREES)
                     WHERE AA =     0.0         AB =     0.0         AC =     0.0
```

FIGURE 3c

```
VALUES OF THE POTENTIAL AFTER ITR =    1, ITK =  360  AT RHO( 2) =  1.0000

 K   L=  1          2           3           4           5           6           7           8           9          10

 1    0.0         0.0         0.0         0.0         0.0         0.0         0.0         0.0         0.0         0.0
 2  -1.470E-03  -1.503E-03  -1.529E-03  -1.547E-03  -1.556E-03  -1.557E-03  -1.548E-03  -1.531E-03  -1.506E-03  -1.474E-03
 3  -2.653E-03  -2.649E-03  -2.624E-03  -2.579E-03  -2.518E-03  -2.441E-03  -2.354E-03  -2.260E-03  -2.163E-03  -2.068E-03
 4  -3.414E-03  -3.299E-03  -3.163E-03  -3.010E-03  -2.849E-03  -2.687E-03  -2.531E-03  -2.390E-03  -2.269E-03  -2.175E-03
 5  -3.630E-03  -3.396E-03  -3.159E-03  -2.930E-03  -2.719E-03  -2.538E-03  -2.395E-03  -2.296E-03  -2.246E-03  -2.248E-03
 6  -3.389E-03  -3.090E-03  -2.822E-03  -2.600E-03  -2.433E-03  -2.328E-03  -2.291E-03  -2.323E-03  -2.423E-03  -2.585E-03
 7  -2.896E-03  -2.621E-03  -2.418E-03  -2.298E-03  -2.263E-03  -2.314E-03  -2.450E-03  -2.664E-03  -2.947E-03  -3.290E-03
 8  -2.379E-03  -2.222E-03  -2.169E-03  -2.222E-03  -2.377E-03  -2.626E-03  -2.961E-03  -3.370E-03  -3.838E-03  -4.348E-03
 9  -2.028E-03  -2.057E-03  -2.206E-03  -2.469E-03  -2.834E-03  -3.292E-03  -3.828E-03  -4.424E-03  -5.056E-03  -5.694E-03
10  -1.954E-03  -2.202E-03  -2.573E-03  -3.055E-03  -3.636E-03  -4.301E-03  -5.028E-03  -5.789E-03  -6.543E-03  -7.240E-03
11  -2.195E-03  -2.670E-03  -3.266E-03  -3.970E-03  -4.767E-03  -5.634E-03  -6.534E-03  -7.415E-03  -8.209E-03  -8.850E-03
12  -2.744E-03  -3.450E-03  -4.276E-03  -5.209E-03  -6.225E-03  -7.282E-03  -8.309E-03  -9.213E-03  -9.905E-03  -1.034E-02
13  -3.591E-03  -4.536E-03  -5.608E-03  -6.785E-03  -8.021E-03  -9.226E-03  -1.026E-02  -1.101E-02  -1.142E-02  -1.151E-02
14  -4.731E-03  -5.941E-03  -7.291E-03  -8.730E-03  -1.016E-02  -1.137E-02  -1.218E-02  -1.256E-02  -1.256E-02  -1.224E-02
15  -6.187E-03  -7.713E-03  -9.397E-03  -1.111E-02  -1.257E-02  -1.344E-02  -1.377E-02  -1.366E-02  -1.319E-02  -1.244E-02
16  -8.014E-03  -9.958E-03  -1.205E-02  -1.387E-02  -1.482E-02  -1.507E-02  -1.482E-02  -1.419E-02  -1.327E-02  -1.214E-02
17  -1.034E-02  -1.292E-02  -1.537E-02  -1.635E-02  -1.643E-02  -1.605E-02  -1.524E-02  -1.414E-02  -1.282E-02  -1.136E-02
18  -1.347E-02  -1.719E-02  -1.809E-02  -1.802E-02  -1.738E-02  -1.635E-02  -1.505E-02  -1.355E-02  -1.191E-02  -1.016E-02
19  -1.977E-02  -2.012E-02  -1.971E-02  -1.880E-02  -1.754E-02  -1.602E-02  -1.432E-02  -1.249E-02  -1.058E-02  -8.617E-03
20  -2.246E-02  -2.157E-02  -2.032E-02  -1.879E-02  -1.704E-02  -1.514E-02  -1.312E-02  -1.102E-02  -8.899E-03  -6.788E-03
21  -2.359E-02  -2.196E-02  -2.013E-02  -1.813E-02  -1.600E-02  -1.379E-02  -1.151E-02  -9.216E-03  -6.948E-03  -4.721E-03
22  -2.369E-02  -2.154E-02  -1.927E-02  -1.691E-02  -1.451E-02  -1.203E-02  -9.570E-03  -7.141E-03  -4.757E-03  -2.444E-03
23  -2.303E-02  -2.048E-02  -1.788E-02  -1.528E-02  -1.260E-02  -9.962E-03  -7.370E-03  -4.812E-03  -2.333E-03   2.313E-05
24  -2.174E-02  -1.889E-02  -1.609E-02  -1.320E-02  -1.039E-02  -7.616E-03  -4.887E-03  -2.241E-03   2.894E-04   2.550E-03
25  -1.995E-02  -1.694E-02  -1.384E-02  -1.084E-02  -7.882E-03  -4.982E-03  -2.167E-03   5.376E-04   3.016E-03   4.646E-03
26  -1.781E-02  -1.450E-02  -1.130E-02  -8.164E-03  -5.091E-03  -2.106E-03   7.713E-04   3.464E-03   5.383E-03   5.931E-03
27  -1.517E-02  -1.178E-02  -8.457E-03  -5.209E-03  -2.053E-03   9.959E-04   3.896E-03   6.141E-03   6.752E-03   6.779E-03
28  -1.226E-02  -6.753E-03  -5.329E-03  -2.000E-03   1.219E-03   4.316E-03   6.893E-03   7.632E-03   7.652E-03   7.299E-03
29  -9.044E-03  -5.442E-03  -1.940E-03   1.449E-03   4.732E-03   7.624E-03   8.585E-03   8.578E-03   8.222E-03   7.534E-03
30  -5.536E-03  -1.862E-03   1.697E-03   5.158E-03   8.338E-03   9.612E-03   9.556E-03   9.200E-03   8.499E-03   7.495E-03
31  -1.754E-03   1.974E-03   5.607E-03   9.044E-03   1.070E-02   1.058E-02   1.024E-02   9.529E-03   8.487E-03   7.196E-03
32   2.294E-03   6.095E-03   9.756E-03   1.182E-02   1.166E-02   1.133E-02   1.063E-02   9.548E-03   8.188E-03   6.656E-03
33   6.640E-03   1.049E-02   1.295E-02   1.277E-02   1.249E-02   1.179E-02   1.068E-02   9.250E-03   7.618E-03   5.911E-03
34   1.127E-02   1.404E-02   1.392E-02   1.370E-02   1.304E-02   1.190E-02   1.039E-02   8.642E-03   6.812E-03   5.007E-03
35   1.507E-02   1.510E-02   1.497E-02   1.436E-02   1.321E-02   1.162E-02   9.735E-03   7.757E-03   5.824E-03   4.004E-03
36   1.630E-02   1.630E-02   1.577E-02   1.463E-02   1.294E-02   1.091E-02   8.750E-03   6.658E-03   4.724E-03   2.961E-03
37   1.770E-02   1.727E-02   1.615E-02   1.438E-02   1.217E-02   9.796E-03   7.506E-03   5.434E-03   3.585E-03   1.930E-03
38   1.887E-02   1.779E-02   1.596E-02   1.355E-02   1.090E-02   8.360E-03   6.124E-03   4.177E-03   2.470E-03   9.563E-04
39   1.956E-02   1.769E-02   1.507E-02   1.207E-02   9.205E-03   6.775E-03   4.723E-03   2.961E-03   1.427E-03   7.679E-05
40   1.959E-02   1.677E-02   1.334E-02   1.001E-02   7.360E-03   5.201E-03   3.391E-03   1.840E-03   4.932E-04  -6.785E-04
```

FIGURE 4a

```
VALUES OF THE POTENTIAL AFTER ITR =     1, ITK =   360   AT RHO( 2) =   1.0000   (Cont.)

 K    L=  1          2           3           4           5           6           7           8           9          10

41   1.871E-02   1.475E-02   1.072E-02   7.833E-03   5.584E-03   3.740E-03   2.184E-03   8.484E-04  -3.017E-04  -1.283E-03
42   1.645E-02   1.120E-02   8.129E-03   5.837E-03   3.990E-03   2.446E-03   1.134E-03   1.416E-05  -9.333E-04  -1.712E-03
43   1.107E-02   8.153E-03   5.921E-03   4.118E-03   2.615E-03   1.344E-03   2.643E-04  -6.413E-04  -1.378E-03  -1.939E-03
44   7.836E-03   5.806E-03   4.109E-03   2.681E-03   1.470E-03   4.453E-04  -4.086E-04  -1.096E-03  -1.611E-03  -1.940E-03
45   5.495E-03   3.959E-03   2.638E-03   1.509E-03   5.550E-04  -2.360E-04  -8.653E-04  -1.327E-03  -1.606E-03  -1.685E-03
46   3.678E-03   2.490E-03   1.464E-03   5.944E-04  -1.222E-04  -6.840E-04  -1.083E-03  -1.307E-03  -1.336E-03  -1.149E-03
47   2.249E-03   1.340E-03   5.680E-04  -6.319E-05  -5.480E-04  -8.770E-04  -1.036E-03  -1.009E-03  -7.739E-04  -3.094E-04
48   1.149E-03   4.843E-04  -5.194E-05  -4.509E-04  -7.012E-04  -7.888E-04  -6.970E-04  -4.075E-04   9.825E-05   8.355E-04
49   3.555E-04  -7.796E-05  -3.829E-04  -5.468E-04  -5.552E-04  -3.929E-04  -4.408E-05   5.047E-04   1.261E-03   2.219E-03
50  -1.272E-04  -3.313E-04  -4.019E-04  -3.252E-04  -8.788E-05   3.219E-04   9.112E-04   1.678E-03   2.608E-03   3.654E-03
51  -2.799E-04  -2.522E-04  -8.676E-05   2.265E-04   6.940E-04   1.315E-03   2.080E-03   2.965E-03   3.930E-03   4.928E-03
52  -8.184E-05   1.716E-04   5.559E-04   1.071E-03   1.710E-03   2.454E-03   3.276E-03   4.140E-03   5.003E-03   5.828E-03
53   4.569E-04   8.996E-04   1.445E-03   2.079E-03   2.781E-03   3.523E-03   4.274E-03   5.003E-03   5.680E-03   6.284E-03
54   1.245E-03   1.795E-03   2.399E-03   3.036E-03   3.682E-03   4.312E-03   4.905E-03   5.440E-03   5.904E-03   6.285E-03
55   2.088E-03   2.634E-03   3.185E-03   3.721E-03   4.226E-03   4.685E-03   5.086E-03   5.420E-03   5.682E-03   5.869E-03
56   2.737E-03   3.185E-03   3.604E-03   3.985E-03   4.319E-03   4.601E-03   4.825E-03   4.992E-03   5.102E-03   5.158E-03
57   2.996E-03   3.303E-03   3.574E-03   3.804E-03   3.992E-03   4.139E-03   4.246E-03   4.317E-03   4.357E-03   4.372E-03
58   5.602E-03   5.888E-03   6.181E-03   6.476E-03   6.766E-03   7.046E-03   7.310E-03   7.555E-03   7.776E-03   7.973E-03
59   5.841E-03   6.088E-03   6.329E-03   6.559E-03   6.778E-03   6.981E-03   7.169E-03   7.340E-03   7.493E-03   7.631E-03
60   5.996E-03   6.218E-03   6.424E-03   6.613E-03   6.785E-03   6.940E-03   7.077E-03   7.200E-03   7.310E-03   7.410E-03
```

FIGURE 4a (Cont.)

```
VALUES OF US/WO AFTER ITR =     1, ITK =  360  AT RHO( 2) =  1.0000          US/WO CRIT =    9.774E-02

 K   L=  1          2           3           4           5           6           7           8           9          10

  2  -7.735E-03  -7.722E-03  -7.650E-03  -7.520E-03  -7.340E-03  -7.118E-03  -6.863E-03  -6.588E-03  -6.306E-03  -6.030E-03
  3  -8.356E-03  -7.724E-03  -7.024E-03  -6.291E-03  -5.557E-03  -4.858E-03  -4.226E-03  -3.693E-03  -3.282E-03  -3.015E-03
  4  -5.503E-03  -4.210E-03  -3.014E-03  -1.972E-03  -1.136E-03  -5.465E-04  -2.314E-04  -2.060E-04  -4.709E-04  -1.013E-03
  5   1.721E-04   1.449E-03   2.353E-03   2.838E-03   2.879E-03   2.478E-03   1.657E-03   4.585E-04  -1.062E-03  -2.835E-03
  6   5.982E-03   6.320E-03   6.036E-03   5.153E-03   3.723E-03   1.826E-03  -4.480E-04  -2.996E-03  -5.715E-03  -8.497E-03
  7   9.435E-03   8.108E-03   6.098E-03   3.525E-03   5.224E-04  -2.778E-03  -6.251E-03  -9.773E-03  -1.322E-02  -1.646E-02
  8   9.098E-03   5.910E-03   2.224E-03  -1.792E-03  -5.990E-03  -1.025E-02  -1.444E-02  -1.845E-02  -2.210E-02  -2.519E-02
  9   4.914E-03   2.265E-04  -4.663E-03  -9.626E-03  -1.456E-02  -1.937E-02  -2.391E-02  -2.797E-02  -3.127E-02  -3.345E-02
 10  -2.104E-03  -7.732E-03  -1.336E-02  -1.892E-02  -2.435E-02  -2.950E-02  -3.410E-02  -3.766E-02  -3.972E-02  -3.976E-02
 11  -1.074E-02  -1.696E-02  -2.315E-02  -2.928E-02  -3.519E-02  -4.051E-02  -4.459E-02  -4.654E-02  -4.570E-02  -4.211E-02
 12  -2.028E-02  -2.711E-02  -3.402E-02  -4.089E-02  -4.728E-02  -5.218E-02  -5.419E-02  -5.224E-02  -4.670E-02  -3.872E-02
 13  -3.063E-02  -3.840E-02  -4.647E-02  -5.437E-02  -6.070E-02  -6.309E-02  -5.970E-02  -5.163E-02  -4.093E-02  -2.927E-02
 14  -4.221E-02  -5.165E-02  -6.162E-02  -7.036E-02  -7.387E-02  -6.853E-02  -5.702E-02  -4.302E-02  -2.871E-02  -1.511E-02
 15  -5.596E-02  -6.849E-02  -8.123E-02  -8.759E-02  -7.934E-02  -6.301E-02  -4.492E-02  -2.769E-02  -1.210E-02   1.695E-03
 16  -7.387E-02  -9.261E-02  -1.062E-01  -9.315E-02  -6.961E-02  -4.647E-02  -2.609E-02  -8.549E-03   6.479E-03   1.930E-02
 17  -1.009E-01  -1.337E-01  -1.116E-01  -7.661E-02  -4.732E-02  -2.370E-02  -4.348E-03   1.174E-02   2.518E-02   3.648E-02
 18  -1.803E-01  -1.377E-01  -8.310E-02  -4.685E-02  -2.023E-02   6.338E-04   1.755E-02   3.143E-02   4.302E-02   5.244E-02
 19  -1.772E-01  -8.626E-02  -4.399E-02  -1.529E-02   6.555E-03   2.398E-02   3.806E-02   4.983E-02   5.931E-02   6.656E-02
 20  -7.742E-02  -3.726E-02  -8.450E-03   1.358E-02   3.111E-02   4.510E-02   5.694E-02   6.640E-02   7.350E-02   7.895E-02
 21  -2.561E-02   6.188E-04   2.185E-02   3.901E-02   5.259E-02   6.438E-02   7.372E-02   8.057E-02   8.597E-02   9.016E-02
 22   1.194E-02   3.138E-02   4.768E-02   6.048E-02   7.210E-02   8.123E-02   8.776E-02   9.337E-02   9.784E-02   1.006E-01
 23   4.218E-02   5.716E-02   6.879E-02   8.011E-02   8.898E-02   9.540E-02   1.011E-01   1.056E-01   1.090E-01   1.078E-01
 24   6.747E-02   7.758E-02   8.850E-02   9.741E-02   1.035E-01   1.093E-01   1.142E-01   1.174E-01   1.174E-01   1.014E-01
 25   8.743E-02   9.763E-02   1.065E-01   1.121E-01   1.178E-01   1.225E-01   1.258E-01   1.269E-01   1.133E-01   7.518E-02
 26   1.074E-01   1.161E-01   1.211E-01   1.265E-01   1.311E-01   1.344E-01   1.363E-01   1.260E-01   8.400E-02   4.795E-02
 27   1.259E-01   1.304E-01   1.354E-01   1.398E-01   1.431E-01   1.457E-01   1.383E-01   9.453E-02   5.146E-02   3.103E-02
 28   1.400E-01   1.446E-01   1.488E-01   1.520E-01   1.549E-01   1.513E-01   1.071E-01   5.563E-02   3.355E-02   1.724E-02
 29   1.541E-01   1.580E-01   1.611E-01   1.641E-01   1.632E-01   1.214E-01   6.106E-02   3.597E-02   1.943E-02   4.501E-03
 30   1.675E-01   1.704E-01   1.734E-01   1.745E-01   1.370E-01   6.800E-02   3.795E-02   2.185E-02   6.102E-03  -7.768E-03
```

FIGURE 4b

```
VALUES OF US/WO AFTER ITR =    1, ITK =  360  AT RHO( 2) =  1.0000          US/WO CRIT =   9.774E-02   (Cont.)

 K    L=  1         2          3          4          5          6          7          8          9          10


31   1.799E-01  1.828E-01  1.852E-01  1.530E-01  7.626E-02  3.951E-02  2.459E-02  7.991E-03 -7.153E-03 -1.929E-02
32   1.925E-01  1.953E-01  1.683E-01  8.543E-02  4.101E-02  2.776E-02  1.027E-02 -6.394E-03 -1.992E-02 -2.947E-02
33   2.049E-01  1.815E-01  9.502E-02  4.294E-02  3.154E-02  1.307E-02 -5.398E-03 -2.068E-02 -3.141E-02 -3.764E-02
34   1.912E-01  1.046E-01  4.593E-02  3.613E-02  1.655E-02 -4.038E-03 -2.153E-02 -3.385E-02 -4.071E-02 -4.324E-02
35   1.132E-01  5.081E-02  4.171E-02  2.086E-02 -2.150E-03 -2.239E-02 -3.685E-02 -4.461E-02 -4.695E-02 -4.602E-02
36   5.855E-02  4.827E-02  2.615E-02  4.709E-04 -2.313E-02 -4.048E-02 -4.958E-02 -5.166E-02 -4.978E-02 -4.613E-02
37   5.635E-02  3.257E-02  4.069E-03 -2.352E-02 -4.482E-02 -5.591E-02 -5.763E-02 -5.443E-02 -4.946E-02 -4.398E-02
38   4.019E-02  8.911E-03 -2.320E-02 -4.989E-02 -6.414E-02 -6.523E-02 -6.011E-02 -5.340E-02 -4.661E-02 -4.001E-02
39   1.524E-02 -2.155E-02 -5.554E-02 -7.509E-02 -7.506E-02 -6.697E-02 -5.794E-02 -4.956E-02 -4.190E-02 -3.466E-02
40  -1.763E-02 -6.103E-02 -9.031E-02 -8.801E-02 -7.515E-02 -6.298E-02 -5.270E-02 -4.385E-02 -3.587E-02 -2.822E-02
41  -6.367E-02 -1.130E-01 -1.055E-01 -8.457E-02 -6.828E-02 -5.581E-02 -4.571E-02 -3.698E-02 -2.890E-02 -2.093E-02
42  -1.506E-01 -1.300E-01 -9.463E-02 -7.326E-02 -5.853E-02 -4.726E-02 -3.784E-02 -2.937E-02 -2.122E-02 -1.294E-02
43  -1.647E-01 -1.031E-01 -7.687E-02 -6.037E-02 -4.819E-02 -3.827E-02 -2.951E-02 -2.123E-02 -1.296E-02 -4.360E-03
44  -1.031E-01 -7.752E-02 -6.068E-02 -4.821E-02 -3.808E-02 -2.919E-02 -2.088E-02 -1.267E-02 -4.220E-03  4.697E-03
45  -7.398E-02 -5.899E-02 -4.706E-02 -3.711E-02 -2.832E-02 -2.009E-02 -1.200E-02 -3.749E-03  4.890E-03  1.406E-02
46  -5.533E-02 -4.464E-02 -3.528E-02 -2.681E-02 -1.880E-02 -1.093E-02 -2.917E-03  5.416E-03  1.419E-02  2.346E-02
47  -4.112E-02 -3.261E-02 -2.465E-02 -1.699E-02 -9.413E-03 -1.703E-03  6.281E-03  1.462E-02  2.332E-02  3.227E-02
48  -2.920E-02 -2.186E-02 -1.466E-02 -7.456E-03 -1.112E-04  7.464E-03  1.530E-02  2.334E-02  3.137E-02  3.899E-02
49  -1.854E-02 -1.185E-02 -5.084E-03  1.825E-03  8.910E-03  1.614E-02  2.336E-02  3.031E-02  3.646E-02  4.109E-02
50  -8.634E-03 -2.367E-03  4.025E-03  1.051E-02  1.698E-02  2.322E-02  2.887E-02  3.343E-02  3.628E-02  3.681E-02
51   5.715E-04  6.337E-03  1.207E-02  1.760E-02  2.265E-02  2.687E-02  2.980E-02  3.101E-02  3.019E-02  2.727E-02
52   8.519E-03  1.332E-02  1.771E-02  2.142E-02  2.414E-02  2.553E-02  2.537E-02  2.356E-02  2.023E-02  1.569E-02
53   1.390E-02  1.701E-02  1.931E-02  2.059E-02  2.066E-02  1.947E-02  1.706E-02  1.362E-02  9.432E-03  4.785E-03
54   1.524E-02  1.620E-02  1.625E-02  1.533E-02  1.349E-02  1.085E-02  7.577E-03  3.900E-03  2.060E-05 -3.881E-03
```

FIGURE 4b (Cont.)

```
 59   1.150E-03   9.608E-04   7.069E-04   4.006E-04   5.646E-05  -3.089E-04  -6.732E-04  -1.034E-03  -1.360E-03  -1.642E-03


VALUES OF UN/WO AFTER ITR =    1, ITK =  360  AT RHO( 2) =  1.0000

 K   L=  1          2           3           4           5           6           7           8           9          10


  2   7.503E-03   7.820E-03   8.224E-03   8.545E-03   8.767E-03   8.880E-03   8.879E-03   8.764E-03   8.538E-03   8.213E-03
  3   9.831E-03   9.775E-03   9.765E-03   9.594E-03   9.268E-03   8.800E-03   8.212E-03   7.532E-03   6.791E-03   6.027E-03
  4   1.091E-02   1.018E-02   9.511E-03   8.689E-03   7.751E-03   6.742E-03   5.715E-03   4.718E-03   3.801E-03   3.004E-03
  5   1.004E-02   8.616E-03   7.340E-03   6.019E-03   4.723E-03   3.516E-03   2.455E-03   1.584E-03   9.364E-04   5.331E-04
  6   7.408E-03   5.504E-03   3.924E-03   2.488E-03   1.261E-03   2.894E-04  -3.994E-04  -7.893E-04  -8.720E-04  -6.413E-04
  7   3.653E-03   1.693E-03   2.448E-04  -8.866E-04  -1.678E-03  -2.122E-03  -2.216E-03  -1.958E-03  -1.339E-03  -3.481E-04
  8  -3.205E-04  -1.911E-03  -2.883E-03  -3.458E-03  -3.646E-03  -3.449E-03  -2.859E-03  -1.855E-03  -4.055E-04   1.507E-03
  9  -3.755E-03  -4.746E-03  -5.109E-03  -5.061E-03  -4.600E-03  -3.704E-03  -2.322E-03  -3.925E-04   2.125E-03   5.184E-03
 10  -6.328E-03  -6.696E-03  -6.449E-03  -5.768E-03  -4.603E-03  -2.862E-03  -4.325E-04   2.770E-03   6.678E-03   1.095E-02
 11  -8.078E-03  -7.880E-03  -7.035E-03  -5.658E-03  -3.601E-03  -6.637E-04   3.322E-03   8.277E-03   1.364E-02   1.847E-02
 12  -9.189E-03  -8.465E-03  -6.962E-03  -4.669E-03  -1.256E-03   3.614E-03   9.901E-03   1.669E-02   2.238E-02   2.636E-02
 13  -9.864E-03  -8.584E-03  -6.198E-03  -2.423E-03   3.393E-03   1.142E-02   2.020E-02   2.688E-02   3.097E-02   3.285E-02
 14  -1.030E-02  -8.298E-03  -4.429E-03   2.244E-03   1.257E-02   2.437E-02   3.216E-02   3.614E-02   3.739E-02   3.681E-02
 15  -1.073E-02  -7.529E-03  -5.044E-04   1.271E-02   2.962E-02   3.852E-02   4.198E-02   4.228E-02   4.070E-02   3.798E-02
 16  -1.150E-02  -5.799E-03   1.012E-02   3.693E-02   4.643E-02   4.865E-02   4.754E-02   4.474E-02   4.100E-02   3.672E-02
 17  -1.365E-02  -5.561E-04   4.886E-02   5.659E-02   5.628E-02   5.319E-02   4.890E-02   4.404E-02   3.888E-02   3.361E-02
 18   4.374E-03   7.314E-02   6.980E-02   6.492E-02   5.918E-02   5.316E-02   4.707E-02   4.100E-02   3.507E-02   2.907E-02
 19   1.059E-01   8.605E-02   7.443E-02   6.542E-02   5.746E-02   5.007E-02   4.307E-02   3.651E-02   2.994E-02   2.358E-02
 20   1.046E-01   8.432E-02   7.172E-02   6.170E-02   5.299E-02   4.505E-02   3.791E-02   3.078E-02   2.404E-02   1.758E-02
 21   9.417E-02   7.781E-02   6.579E-02   5.577E-02   4.692E-02   3.925E-02   3.157E-02   2.449E-02   1.777E-02   1.105E-02
 22   8.358E-02   6.966E-02   5.842E-02   4.870E-02   4.058E-02   3.235E-02   2.497E-02   1.801E-02   1.078E-02   3.388E-03
 23   7.330E-02   6.088E-02   5.036E-02   4.187E-02   3.312E-02   2.547E-02   1.791E-02   1.026E-02   2.804E-03  -5.337E-03
 24   6.314E-02   5.183E-02   4.302E-02   3.379E-02   2.553E-02   1.748E-02   9.828E-03   2.257E-03  -5.793E-03  -1.660E-02
 25   5.322E-02   4.342E-02   3.395E-02   2.522E-02   1.710E-02   9.338E-03   1.634E-03  -6.327E-03  -1.682E-02  -3.034E-02
 26   4.317E-02   3.373E-02   2.489E-02   1.660E-02   8.710E-03   9.094E-04  -6.982E-03  -1.699E-02  -3.228E-02  -3.547E-02
 27   3.362E-02   2.438E-02   1.594E-02   7.927E-03   5.418E-05  -7.792E-03  -1.723E-02  -3.318E-02  -3.835E-02  -3.861E-02
 28   2.389E-02   1.510E-02   6.966E-03  -9.597E-04  -8.780E-03  -1.767E-02  -3.307E-02  -4.171E-02  -4.116E-02  -4.113E-02
 29   1.428E-02   5.805E-03  -2.158E-03  -9.967E-03  -1.839E-02  -3.258E-02  -4.531E-02  -4.370E-02  -4.375E-02  -4.315E-02
 30   4.694E-03  -3.562E-03  -1.137E-02  -1.944E-02  -3.218E-02  -4.843E-02  -4.612E-02  -4.641E-02  -4.601E-02  -4.434E-02
```

FIGURE 4c

```
59   1.150E-03    9.608E-04    7.069E-04    4.006E-04    5.646E-05   -3.089E-04   -6.782E-04   -1.034E-03   -1.360E-03   -1.642E-03

VALUES OF UN/WO AFTER ITR =     1, ITK =   360   AT RHO( 2) =   1.0000

 K   L=  1          2            3            4            5            6            7            8            9           10

31  -4.886E-03   -1.299E-02   -2.083E-02   -3.205E-02   -5.056E-02   -4.833E-02   -4.908E-02   -4.903E-02   -4.740E-02   -4.444E-0
32  -1.451E-02   -2.256E-02   -3.229E-02   -5.134E-02   -5.019E-02   -5.171E-02   -5.222E-02   -5.076E-02   -4.753E-02   -4.333E-0
33  -2.420E-02   -3.293E-02   -5.036E-02   -5.171E-02   -5.427E-02   -5.554E-02   -5.443E-02   -5.102E-02   -4.617E-02   -4.108E-0
34  -3.354E-02   -4.720E-02   -5.302E-02   -5.672E-02   -5.899E-02   -5.846E-02   -5.499E-02   -4.945E-02   -4.333E-02   -3.797E-0
35  -4.269E-02   -5.373E-02   -5.904E-02   -6.255E-02   -6.285E-02   -5.953E-02   -5.327E-02   -4.591E-02   -3.933E-02   -3.448E-0
36  -5.366E-02   -6.122E-02   -6.617E-02   -6.763E-02   -6.475E-02   -5.782E-02   -4.891E-02   -4.078E-02   -3.488E-02   -3.108E-0
37  -6.315E-02   -6.985E-02   -7.278E-02   -7.080E-02   -6.336E-02   -5.251E-02   -4.226E-02   -3.513E-02   -3.064E-02   -2.807E-0
38  -7.333E-02   -7.825E-02   -7.779E-02   -7.028E-02   -5.698E-02   -4.368E-02   -3.507E-02   -2.992E-02   -2.701E-02   -2.557E-0
39  -8.365E-02   -8.576E-02   -7.919E-02   -6.292E-02   -4.476E-02   -3.445E-02   -2.875E-02   -2.565E-02   -2.410E-02   -2.354E-0
40  -9.404E-02   -9.088E-02   -7.176E-02   -4.483E-02   -3.283E-02   -2.693E-02   -2.387E-02   -2.238E-02   -2.182E-02   -2.185E-0
41  -1.041E-01   -8.750E-02   -4.199E-02   -2.951E-02   -2.417E-02   -2.158E-02   -2.036E-02   -1.994E-02   -2.000E-02   -2.033E-0
42  -1.111E-01   -3.030E-02   -2.350E-02   -2.026E-02   -1.869E-02   -1.802E-02   -1.788E-02   -1.807E-02   -1.843E-02   -1.880E-0
43  -1.105E-02   -1.456E-02   -1.517E-02   -1.523E-02   -1.536E-02   -1.565E-02   -1.605E-02   -1.650E-02   -1.689E-02   -1.710E-0
44  -5.488E-03   -9.445E-03   -1.141E-02   -1.250E-02   -1.329E-02   -1.397E-02   -1.454E-02   -1.498E-02   -1.519E-02   -1.507E-0
45  -4.781E-03   -7.588E-03   -9.589E-03   -1.090E-02   -1.186E-02   -1.258E-02   -1.308E-02   -1.329E-02   -1.315E-02   -1.255E-0
46  -4.882E-03   -6.829E-03   -8.569E-03   -9.732E-03   -1.064E-02   -1.119E-02   -1.141E-02   -1.124E-02   -1.059E-02   -9.372E-0
47  -5.058E-03   -6.405E-03   -7.796E-03   -8.749E-03   -9.328E-03   -9.533E-03   -9.320E-03   -8.619E-03   -7.339E-03   -5.379E-0
48  -5.095E-03   -5.955E-03   -6.943E-03   -7.514E-03   -7.676E-03   -7.396E-03   -6.614E-03   -5.249E-03   -3.203E-03   -3.593E-0
49  -4.844E-03   -5.236E-03   -5.750E-03   -5.832E-03   -5.454E-03   -4.562E-03   -3.083E-03   -9.283E-04    1.999E-03    5.776E-0
50  -4.148E-03   -4.031E-03   -3.988E-03   -3.475E-03   -2.441E-03   -8.241E-04    1.448E-03    4.431E-03    8.122E-03    1.237E-0
51  -2.826E-03   -2.118E-03   -1.430E-03   -2.274E-04    1.540E-03    3.910E-03    6.879E-03    1.036E-02    1.411E-02    1.779E-0
52  -6.715E-04    7.158E-04    2.102E-03    4.003E-03    6.408E-03    9.250E-03    1.237E-02    1.552E-02    1.841E-02    2.080E-0
53   2.430E-03    4.549E-03    6.519E-03    8.849E-03    1.142E-02    1.405E-02    1.653E-02    1.866E-02    2.029E-02    2.140E-0
54   6.505E-03    8.984E-03    1.109E-02    1.324E-02    1.529E-02    1.707E-02    1.848E-02    1.945E-02    1.998E-02    2.008E-0
55   1.063E-02    1.294E-02    1.455E-02    1.596E-02    1.706E-02    1.781E-02    1.818E-02    1.820E-02    1.786E-02    1.720E-0
56   1.356E-02    1.517E-02    1.592E-02    1.639E-02    1.655E-02    1.642E-02    1.600E-02    1.532E-02    1.440E-02    1.328E-0
```

FIGURE 4c (Cont.)

```
NOTE: THE FOLLOWING VALUES OF V RADIAL ARE BASED ON THE 2D STRIP-THEORY APPROXIMATION

VALUES OF V RADIAL/U INFINITYAFTER ITR =     1, ITK =  360  AT RHO( 2) =  1.0000

 K   L=  1          2          3          4          5          6          7          8          9         10

 2   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
 3   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
 4   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
 5   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
 6   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
 7   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
 8   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
 9   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
10   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
11   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
12   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
13   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
14   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
15   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
16   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
17   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
18   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
19   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
20   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
21   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
22   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
23   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
24   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
25   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
26   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
27   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
28   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
29   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
30   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
```

FIGURE 4d

```
NOTE: THE FOLLOWING VALUES OF V RADIAL ARE BASED ON THE 2D STRIP-THEORY APPROXIMATION  (Cont.)

VALUES OF V RADIAL/U INFINITYAFTER ITR =    1, ITK =  360  AT RHO( 2) =  1.0000

 K   L=  1          2          3          4          5          6          7          8          9          10


31   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
32   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
33   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
34   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
35   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
36   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
37   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
38   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
39   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
40   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
41   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
42   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
43   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
44   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
45   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
46   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
47   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
48   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
49   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
50   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
51   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
52   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
53   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
54   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
55   0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0        0.0
```

FIGURE 4d (cont.)

```
TRAILING-EDGE VELOCITIES ARE US(1,TE) = -1.638E-01  US(LMX,TE) = -1.695E-01  DELTA US =  5.776E-03  DPHI( 2) = -5.134E-03

AFTER ITR =  1 MAXIMUM RESIDUALS ARE

N        RESIDUAL    K    L        AVG RES        AVG PHI        AVG SUM

2       2.281E-04   37   29       1.672E-05      6.098E-03      3.193E-03


                               PERFORMANCE DATA

      N      RHO(N)       DPHI(N)        W/U INF       U/U INF     ARCTAN(UN/WO)     P02/P01
                                                                    (DEGREES)

      1      0.9474     -5.134E-03       0.0086        0.0056          0.10           0.996
      2      1.0000     -5.134E-03       0.0082        0.0056          0.07           0.996
      3      1.0526     -5.134E-03       0.0078        0.0056          0.05           0.996
```

FIGURE 5

APPENDIX A

GUIDE TO PREPARING THE INPUT

Cards 1 and 2: FORMAT 18A4: These cards are for run identification; any alphameric characters may be used. Neither card can be omitted.

Card 3: FORMAT 5E15:

Columns 1-15: B, the number of blades. For a two-dimensional cascade solution, use B=10 (and see comments below regarding IDM=2).

Columns 16-30: h, the hub-to-tip radius ratio. For a two-dimensional cascade solution, use h = 0.9.

Columns 31-35: Solidity, $C_a / L_T = C_a / \frac{2\pi r_T}{B}$

Columns 46-60: Axial Mach number at the inlet, u_∞ / a_∞

Columns 61-75: Tangential Mach number at the tip, $\omega r_T / a_\infty$. The present version of the program was developed for inlet relative Mach numbers $\sqrt{u_\infty^2 + (\omega r_T)^2} / a_\infty$ less than one. The program cannot handle values greater than one, because the boundary conditions imposed at the upstream and downstream edges of the grid are not radiation conditions.

Card 4: FORMAT 5E15

Columns 1-15: γ, the specific-heat ratio.

Columns 16-30: FXB: The upstream edge of the grid will be located at FXB times the axial projection of the chord C_a. Values of FXB = -2 or -3 have been used. FXB must be negative.

Columns 31-45: FXI: The downstream edge of the grid will be located at FXI • C_a downstream of the trailing edge. FXI = -FXB will place the greatest concentration of grid points near the mid-chord of the blades.

Columns 46-60: RXE is the relaxation factor used at subsonic points. Values up to 1.2 (and "tapered", i.e., ITPR=1 - see below) have been used for the off-design problem (IBC=1), but very little is known about the limitations on this parameter. For the design problem (IBC=2), it may be necessary to use values as low as 0.1, with ITPR=0, early in the iterations, increasing to 0.8 (and ITPR=0) at the later stages. Again, very little is known about these limitations.

Columns 61-75: RXH is the relaxation factor used at supersonic points. RXH=0.9 has been used successfully. The ITPR parameter does not affect RXH - a constant value is used, at all supersonic points in the grid.

Card 5: FORMAT 5E15

Columns 1-15: OBV, the relaxation factor used in updating the circulation. Values of 0.05 and 0.1 have been used, but the limits on this parameter and their coupling to the other relaxation factors are unknown.

Card 6: FORMAT 16I5

Columns 1-5/6-10/11-15: Grid size parameters KMX/LMX/NMX. These are limited to 60/30/10, by a COMMON statement in the version listed below. The number of calculations done in a given z-sweep is proportional to KMX, and the number of z-sweeps required for convergence is also proportional to KMX (since it takes KMX iterations before information at the downstream edge of the grid affects the solution at the upstream edge). Thus, the total time required for a given calculation is likely to vary as the square of KMX, and values as low as possible should be used.

If shock capturing is important, LMX should then be chosen so as to make $\rho\Delta\zeta/\Delta z$ approximately equal to $(1+\rho^2)/\rho$ at a radius in the center of the region where shock definition is desired. For the particular stretching of the z-coordinates used here, and for the special case where FXI = -FXB, the value of $\rho\Delta\zeta/\Delta z$ at midchord is

$$\left.\frac{\rho\Delta\zeta}{\Delta z}\right)_{x=c_a/2} = \frac{\rho}{\rho_T}\cdot\frac{L_T}{c_a}\cdot\frac{1}{2(1+2\,FXI)}\cdot\frac{(KMX+1)\ln KMX}{LMX-1} \qquad \text{(A-1)}$$

This equation can be solved for LMX, once the other parameters are chosen.

The quantity NMX must be three or greater; for a two-dimensional cascade solution, set it equal to 3.

Column 20: IBC=1 for the off-design case, IBC=2 for the design case.

Column 25: IDM=3 for a three-dimensional solution. For a two-dimensional cascade solution, set IDM=2 and NMX=3. For this case, the hub/tip ratio h and the number of blades B are meaningless, and can be read in as any arbitrary numbers; the values h = 0.9 and B = 10 are recommended. The desired cascade parameters are actually the inlet relative Mach number M_0 and the stagger angle. These determine ρ, which is the ratio of the pitchwise to axial velocity components upstream of the cascade:

$$\rho = \omega r / u_\infty$$

The hub and tip radii are then determined by the fact that, for NMX=3, the value of ρ lies half-way between ρ_H and ρ_T :

$$\rho = \frac{\rho_H + \rho_T}{2} = \frac{h+1}{2} \rho_T \; ;$$

Thus,

$$\rho_T = \frac{2}{1+h} \rho \; , \qquad \rho_H = \frac{2h}{1+h} \rho \qquad \text{(A-2)}$$

The desired value of the pitchwise component of the inlet Mach number at station ρ is

$$M_\theta = \rho M_x = \frac{\rho}{\rho_T} M_{\theta, tip}$$

$$= \frac{\rho}{\rho_T} \cdot \rho_T M_x = \frac{\rho_T}{\sqrt{1+\rho^2}} M_0 = \frac{2}{1+h} \frac{\rho}{\sqrt{1+\rho^2}} M_0 \qquad \text{(A-3)}$$

For a two-dimensional strip-theory calculation of a three-dimensional case, set IDM=2 and $3 < \text{NMX} \leq 10$. In the latter case, all radial derivatives are set equal to zero, and each radial station is treated as though it were a two-dimensional section.

Columns 26-30: ITKMX is the maximum number of iterations that will be made in the z-direction, on each radial sweep.

Columns 31-35: JRPT: Output is printed at intervals of JRPT in the z-direction iterations on every radial sweep. It is used only for looking at intermediate results, and gives a large amount of output. Normally, results are needed only after a given number of radial iterations (see NRPT below). To suppress these intermediate results, set JRPT > ITKMX.

Columns 36-40: ISTART = 0 if you are starting from scratch; ISTART = 1 if you are reading in values of ϕ (on tape) from a previous run.

Columns 41-45: ISAVE=1 will write all values of $\Delta\phi(N)$ and $\phi(L,K,N)$ on tape at the end of the calculation. ISAVE=0 writes no tape.

Columns 46-50: IOP controls what is printed on output. See the comment cards in subroutine OUTPUT for details.

Columns 51-55: ITRMX gives the maximum number of radial iterations. On each radial iteration, ITKMX axial iterations are done, at radial stations N=2, NMX-1.

Columns 56-60: NRPT: Output will occur at intervals of NRPT in the radial iterations.

Columns 61-65: ITPR=1 will cause the relaxation factor used at elliptic points to be "tapered", according to

$$\omega = 1 + (RXE - 1)\ \exp\left\{-\left(\frac{Z - \frac{1}{2}\frac{\omega c_a}{U_\infty}}{\omega c_a / U_\infty}\right)^2\right\} \qquad \text{(A-4)}$$

ITPR=0 uses ω=RXE at all elliptic points.

Columns 66-70: Residuals are calculated and printed at intervals of IRXP in the radial iterations. If IRXP is preceded by a negative sign, the parameter ISHO is internally set equal to 1, and absolute values of all terms in the potential equation are displayed as well.

Cards 7 and 8: These contain the input data for blade shape and loading - see comments in subroutine BVAL for details. Both cards must be used, with dummy zeros, if necessary.

Card 8: If IOP=4, this card specifies the locations and Mach-number values at which Mach-number contours are calculated.

APPENDIX B

PROGRAM LISTING

```
LEVEL 21.7 ( DEC 72 )                                    OS/360  FORTRAN H

          COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,XREF
                   C
                   C        PROGRAM 3DNSDA - THREE-DIMENSIONAL NONLINEAR SMALL-DISTURBANCE
                   C        (VERSION A) PROGRAM FOR CALCULATING THE FLOW THROUGH AN
                   C        ISOLATED COMPRESSOR BLADE ROW.  DOCUMENTATION IS GIVEN IN:
                   C              W. J. RAE, NONLINEAR SMALL-DISTURBANCE EQUATIONS FOR
                   C                         THREE-DIMENSIONAL TRANSONIC FLOW THROUGH A
                   C                         COMPRESSOR BLADE ROW, CALSPAN CORPORATION
                   C                         REPORT AB-5487-A-1, AFOSR TR-76-1082,
                   C                         AD A031234 (AUG 1976)
                   C              - - -      RELAXATION SOLUTIONS FOR THREE-DIMENSIONAL
                   C                         TRANSONIC FLOW THROUGH A COMPRESSOR BLADE ROW,
                   C                         IN THE NONLINEAR SMALL-DISTURBANCE APPROXIMA-
                   C                         TION, CALSPAN CORPORATION REPORT AB-5487-A-2,
                   C                         AFOSR TR-76-1081, AD A032553 (AUG 1976)
                   C              - - -      CALCULATIONS OF THREE-DIMENSIONAL TRANSONIC
                   C                         COMPRESSOR FLOWFIELDS BY A RELAXATION METHOD,
                   C                         AIAA JOURNAL OF ENERGY, VOL. 1, NO. 5,
                   C                         SEPT - OCT 1977, PP 284 - 296
                   C
                   C          USERS ARE URGED TO COMMUNICATE WITH THE AUTHOR, CONCERNING
                   C          PROBLEMS, RESULTS, AND SUGGESTED CHANGES -
                   C          AERODYNAMIC RESEARCH DEPARTMENT, CALSPAN CORPORATION
                   C          PO BOX 235
                   C          BUFFALO, NY 14221
                   C          TELEPHONE 716/632-7500
                   C
   ISN 0002              COMMON ZT(30),XI(60),RO(10),F(30,60,10),A(10),B(10),D(10),E(10),
                        *   EE(30),FF(30),SV(30).DPHI(10),BV(10),US(30,60),UN(30,60),
                        *   DNDS(30,10,2),FAK(60),FHW(60),UTA(30),UTB(30),UTC(30).BBV(30),
                        *   BSA(30),BSB(30),BSC(30),VSA(30),VSB(30),BAV(30),LOP(30)
                        *   ,SONIC(10)
   ISN 0003              COMMON DAF,DBF,DCF,AKLKE,RX,OX,CONST
   ISN 0004              COMMON H,DRO,DZT,TDZ,TPB,XIC,XIB,XID,TDR,AKLK,DRO2,DZT2
   ISN 0005              COMMON OMCU,RHUB,RTIP,R2,RXE,RXH,UA,UB,TRDZ,ERA,ERB,CKLKE,SB,
                        *   BTR,FXB,FXI,EM2,AMA,AMB,DM,DKA,DKR,DQ,DQS.TZ,TTZ,TZS,TRS,TDQ
   ISN 0006              COMMON K,N,IDM,     KMX,LMX,NMX,KMXM1,KMXM2,LMXM1,LMXM2,NMXM1,
                        *   NMXM2,KTEO,IRX,    ITK,ITR,ITRMX,KLEP,ITPR,KUP,KDN
   ISN 0007              COMMON ID(36)
                   C
                   C      THE FOLLOWING CALSPAN ROUTINE PLACES A ZERO IN THE LOCATIONS
                   C      FROM ZT(1) THROUGH ID(36)
                   C
   ISN 0008              CALL CLEAR(ZT(1),ID(36))
                   C
   ISN 0009              READ(5,102) (ID(I),I=1,36)
   ISN 0010          102 FORMAT(18A4)
   ISN 0011              WRITE(6,200)
   ISN 0012          200 FORMAT(1H1)
   ISN 0013              WRITE(6,201) (ID(I),I=1,36)
   ISN 0014          201 FORMAT(30X,18A4)
   ISN 0015              READ(5,100) BN,H,CALT,EMX,EMTG,GMA,FXB,FXI,RXE,RXH,OBV
   ISN 0016          100 FORMAT(5E15.5)
   ISN 0017              READ(5,101) KMX,LMX,NMX,IBC,IDM,ITKMX,JPRT,ISTART,ISAVE,IOP
                        *   ,ITRMX,NPRT,ITPR,IRXP
   ISN 0018          101 FORMAT(16I5)
   ISN 0019              READ(5,100) (BV(I),I=1,10)
```

```
      IBN = INT(BN+0.01)
      WRITE(6,202) IBN,H,CALT
  202 FORMAT(//5X,'THIS BLADE ROW HAS',I4,' BLADES, WITH A HUB-TO-TIP',
     *   ' RATIO OF ',F5.3,' AND SOLIDITY CA/LT = ',F5.3)
      RTIP = EMTG/EMX
      RHUB = H*RTIP
      EM2 = EMX*EMX
      REL = SQRT(EM2+EMTG*EMTG)
      WRITE(6,203) EMX,EMTG,REL,GMA
  203 FORMAT( /5X,'AXIAL MACH NO. = ',F5.3,',  TANGENTIAL MACH NO  AT',
     *   ' THE TIP = ',F5.3,',  TOTAL MACH NO. AT THE TIP = ',F5.3,
     *   //5X,'SPECIFIC HEAT RATIO =',F6.3)
      IF(REL.GT.1.) WRITE(6,207)
  207 FORMAT(//15X,'WARNING - INLET RELATIVE MACH NUMBER AT THE TIP ',
     *   'IS SUPERSONIC',
     *   /15X,'BOUNDARY CONDITIONS UPSTREAM AND DOWNSTREAM DO NOT ',
     *   'CONTAIN THE REQUIRED RADIATION CONDITIONS',
     *   /15X,'SOLUTION UPSTREAM MAY BE INVALID',
     *   /15X,'SOLUTION DOWNSTREAM MAY NOT CONVERGE, ESPECIALLY IF',
     *   ' THE OUTLET RELATIVE MACH NUMBER IS SUPERSONIC',/)
      IF(ISTART.NE.1) GO TO 98
C
C  READ STARTING VALUES FROM TAPE
C
      KS = KMX
      LS = LMX
      NS = NMX
      READ(1) LMX,KMX,NMX,DPHI
      DO 97 N=1,NMX
      READ(1) ((F(L,K,N),L=1,LMX),K=1,KMX)
   97 CONTINUE
      IF(KS.NE.KMX) GO TO 33
      IF(LS.NE.LMX) GO TO 33
      IF(NS.NE.NMX) GO TO 33
      GO TO 98
   33 WRITE(6,210) KS,LS,NS,KMX,LMX,NMX
  210 FORMAT(//5X,'WARNING - GRID SIZE ON CARD INPUT DIFFERS FROM',
     *   ' THAT ON TAPE',
     *   //10X,'CARD KMX = ',I4,' LMX = ',I4,' NMX = ',I4,
     *   //10X,'TAPE KMX = ',I4,' LMX = ',I4,' NMX = ',I4,)
C
C  END OF TAPE INPUT
C
   98 WRITE(6,204) IDM,KMX,LMX,NMX
  204 FORMAT( /5X,'THIS IS A',I2,'-DIMENSIONAL CALCULATION, WITH GRID',
     *   ' SIZE KMX/LMX/NMX = ',I3,'/',I3,'/',I3)
      WRITE(6,206)
  206 FORMAT(/5X,'RELAXATION FACTORS FOR ELLIPTIC AND HYPERBOLIC',
     *   ' POINTS ARE LISTED BELOW AS RXE AND RXH')
      SB = (GMA + 1.)*EM2
      TPI = 2.0*3.14159265
      TPB = TPI/BN
      OMCU = TPB*RTIP*CALT
      LMXM1 = LMX -1
      LMXM2 = LMX - 2
      KMXM1 = KMX - 1
      KMXM2 = KMX -2
      NMXM1 = NMX - 1
      NMXM2 = NMX - 2
      CALL GRID
      DO 99 I = 1,LMX
   99 LOP(I) = I
      IF(IOP.NE.4) GO TO 15
      READ(5,104) AMA,AMB,DM,KUP,KDN
  104 FORMAT(3E15.5,3I5)
```

```
ISN 0071              DO 21 N = 1,NMX
ISN 0072              UA = RO(N)
ISN 0073              UB =(1./(1.+RO(N)*RO(N)))/TDQ
ISN 0074              TRDZ = TDZ*RO(N)
ISN 0075              CALL OUTPUT(IOP,IBC)
ISN 0076           21 CONTINUE
ISN 0077              GO TO 30
                C
ISN 0078           15 CALL BVAL(IBC)
ISN 0079            9 OBM = 1. - OBV
ISN 0080              ITR = 1
ISN 0081              NPT = 0
ISN 0082              IRXPT = 0
ISN 0083              ISHO = 0
ISN 0084              IF(IRXP.GE.0) GO TO 44
ISN 0086              IRXP = -IRXP
ISN 0087              ISHO = 1
ISN 0088           44 BGL = RO(1)*DPHI(1)/2.
ISN 0089              DO 45 N = 2,NMXM1
ISN 0090           45 BGL = BGL + RO(N)*DPHI(N)
ISN 0091              BGL = (BGL + RO(NMX)*DPHI(NMX)/2.)*DRO
ISN 0092              CONST =-2.*BGL/(TPB*(1.-EM2)*(RTIP*RTIP-RHUB*RHUB))
                C
                C   THE FOLLOWING STATEMENTS ARE USED TO SUPPRESS EXTRANEOUS OUTPUT
                C   WHEN THE SOLUTION IS BEING FOUND AT ONLY A SINGLE RADIAL STATION.
                C
ISN 0093              NA = 1
ISN 0094              NB = NMX
ISN 0095              IF(NMX.NE.3) GO TO 304
ISN 0097              NA = 2
ISN 0098              NB = 2
                C
                C  BEGINNING OF RHO - SWEEP
                C
ISN 0099          304 DO 300 N = 2,NMXM1
ISN 0100              R2 = RO(N)*RO(N)
ISN 0101              BTR = 1.0 - EM2*(1.0+R2)
ISN 0102              AKLK = D(N)*TZS
ISN 0103              UA = RO(N)
ISN 0104              UB =(1./(1.+RO(N)*RO(N)))/TDQ
ISN 0105              TRDZ = TDZ*RO(N)
ISN 0106              DKA = -B(N)*TTZ
ISN 0107              ERA = 1. + 0.5*DRO/RO(N)
ISN 0108              ERB = 1. - 0.5*DRO/RO(N)
ISN 0109              ITK = 1
ISN 0110              IPRT = 0
ISN 0111              IF(IDM.NE.2) GO TO 1
ISN 0113              CONST =-DPHI(N)/(TPB*(1.-EM2))
                C
                C  BEGINNING OF XI - SWEEP
                C
ISN 0114            1 DO 2 L = 1,LMX
ISN 0115              UTA(L) = 0.0
ISN 0116              UTB(L) = 0.0
ISN 0117              VSA(L) = 0.
ISN 0118              BAV(L) = BTR
ISN 0119            2 CONTINUE
ISN 0120           38 DO 5 K = 2,KMXM1
ISN 0121              RX = 1. + (RXE-1.)*EXP(-((XI(K)-0.5*XID)/XID)**2)
ISN 0122              IF(ITPR.EQ.0) RX = RXE
ISN 0124              OX = 1. - 1./RX
ISN 0125              RXM = 1.0 - RX
ISN 0126              IRX = 1
```

```
                      C
                      C   BEGINNING OF ZETA - SWEEP
                      C
ISN 0127                   4 AKLKE = AKLK/FAK(K)
ISN 0128                     DKR = TRS*E(N)/FAK(K)
ISN 0129                     CKLKE = AKLKE
ISN 0130                  29 DO 3 L = 1,LMX
ISN 0131                     UTC(L) = UTB(L)
ISN 0132                     UTB(L) = UTA(L)
ISN 0133                     UTA(L) = F(L,K,N)
ISN 0134                     BBV(L) = BAV(L)
ISN 0135                     BAV(L) = BTR - SB*FAK(K)*(F(L,K+1,N)-F(L,K-1,N))/TDQ
ISN 0136                     VSB(L) = VSA(L)
ISN 0137                     VSA(L) = 0.
ISN 0138                     IF(BAV(L).GT.0.) GO TO 8
ISN 0140                     VSA(L) = 1.
ISN 0141                     IRX = 2
ISN 0142                   8 BSA(L) = (1.-VSA(L))*(BAV(L)+A(N))
ISN 0143                     BSB(L) = VSB(L)*BBV(L)
ISN 0144                     BSC(L) = VSA(L)*A(N)
ISN 0145                   3 CONTINUE
                      C
                      C   UTA,UTB, AND UTC NOW CONTAIN DATA, FROM THE PREVIOUS ITERATION,
                      C   FOR F(L,K,N),F(L,K-1,N), AND F(L,K-2,N), RESPECTIVELY
                      C
ISN 0146                  14 CALL BVAL0(IBC)
ISN 0147                     DO 7 L = 2,LMXM1
ISN 0148                     BKL = -2.*AKLKE - 2.*FAK(K)*BSA(L)/RX + 2.*FHW(K-1)*BSB(L)
                            *    - BSC(L)*FHW(K-1)
ISN 0149                     DKL = -BSA(L)*(FHW(K)*F(L,K+1,N)-2.*FAK(K)*OX*UTA(L)
                            *    + FHW(K-1)*F(L,K-1,N)) + BSB(L)*(FHW(K-1)*(UTA(L)+F(L.K-1,N))
                            *    + FHW(K-2)*(F(L,K-1,N)-UTC(L))) - BSC(L)*(FHW(K)*(F(L,K+1,N)
                            *    -UTA(L)) + FHW(K-1)*F(L,K-1,N))
                            *    + 2.*DKA*(-F(L+1,K-1,N)+F(L,K-1,N)+F(L+1,K,N)-2.*F(L,K,N)
                            *    + F(L-1.K,N)+F(L,K+1,N)-F(L-1,K+1,N))
                            *    -DKR*(ERA*F(L,K,N+1)+ERB*F(L,K,N-1)-2.*UTA(L))
ISN 0150                     DNM = BKL + CKLKE*EE(L-1)
ISN 0151                     EE(L) = -AKLKE/DNM
ISN 0152                  16 FF(L) = (DKL - CKLKE*FF(L-1))/DNM
ISN 0153                   7 CONTINUE
ISN 0154                     CALL BVAL1(IBC)
                      C
ISN 0155                 406 DO 19 J = 1,LMXM1
ISN 0156                     L = LMX-J
ISN 0157                     SV(L) = EE(L)*SV(L+1) + FF(L)
ISN 0158                  19 CONTINUE
                      C
ISN 0159                  70 IF(IRX.EQ.1) GO TO 6
ISN 0161                  11 IF(IRX.EQ.2) RX = RXH
ISN 0163                     RXM = 1.0 - RX
                      C
ISN 0164                   6 DO 12 L = 1,LMX
ISN 0165                  12 F(L,K,N) = RX*SV(L) + RXM*F(L,K,N)
                      C
ISN 0166                   5 CONTINUE
                      C
ISN 0167                     DO 41 L = 1,LMX
ISN 0168                  41 F(L,KMX,N) = (CONST-DAF*F(L,KMXM2,N)-DBF*F(L,KMXM1,N))/DCF
ISN 0169                 307 IF(IBC.EQ.2) GO TO 43
                      C
                      C   RESET DPHI
                      C
ISN 0171                  47 X1 = XI(KTEO-2)
ISN 0172                     X2 = XI(KTEO-1)
ISN 0173                     X3 = XID
ISN 0174                     CD = (X2-X1)*(2.*X3-X1-X2)
```

```
ISN 0175            EBD = -(X3-X2)*(X3-X2)/CD
ISN 0176            FBD = (X3-X1)*(X3-X1)/CD
ISN 0177            DFY = EBD*(F(1,KTEO-2,N) - F(LMX,KTEO-2,N))
                   *   + FBD*(F(1,KTEO-1,N) - F(LMX,KTEO-1,N))
ISN 0178            DPHI(N) = OBV*DFY + OBM*DPHI(N)
              C
ISN 0179         43 IPRT = IPRT + 1
ISN 0180            IF(IPRT.LT.JPRT) GO TO 42
ISN 0182            IPRT = 0
ISN 0183            CALL OUTPUT(IOP,IBC)
ISN 0184         42 ITK = ITK + 1
ISN 0185            IF(ITK.GT.ITKMX) GO TO 300
ISN 0187            GO TO 1
ISN 0188        300 CONTINUE
              C
ISN 0189        306 IF(NMX.EQ.3) GO TO 310
ISN 0191            DO 301 K = 1,KMX
ISN 0192            DO 302 L = 1,LMX
ISN 0193            F(L,K,1) = (4.*F(L,K,2)-F(L,K,3))/3.
ISN 0194            F(L,K,NMX) = (4.*F(L,K,NMXM1)-F(L,K,NMXM2))/3.
ISN 0195        302 CONTINUE
ISN 0196        301 CONTINUE
ISN 0197            DPHI(1) = 2.*DPHI(2) - DPHI(3)
ISN 0198            DPHI(NMX) = 2.*DPHI(NMXM1) - DPHI(NMXM2)
ISN 0199            GO TO 305
ISN 0200        310 DO 311 K = 1,KMX
ISN 0201            DO 312 L = 1,LMX
ISN 0202            F(L,K,1) = F(L,K,2)
ISN 0203            F(L,K,3) = F(L,K,2)
ISN 0204        312 CONTINUE
ISN 0205        311 CONTINUE
ISN 0206            DPHI(1) = DPHI(2)
ISN 0207            DPHI(3) = DPHI(2)
ISN 0208        305 NPT = NPT + 1
ISN 0209            IF(NPT.LT.NPRT) GO TO 62
ISN 0211            NPT = 0
ISN 0212            ITK = ITK - 1
ISN 0213            DO 63 N = NA,NB
ISN 0214            UA = RO(N)
ISN 0215            UB =(1./(1.+RO(N)*RO(N)))/TDQ
ISN 0216            TRDZ = TDZ*RO(N)
ISN 0217            CALL OUTPUT(IOP,IBC)
ISN 0218         63 CONTINUE
ISN 0219         62 CONTINUE
ISN 0220            IRXPT = IRXPT + 1
ISN 0221            IF(IRXPT.NE.IRXP) GO TO 32
ISN 0223            IRXPT = 0
ISN 0224            CALL RESID(ISHO)
ISN 0225         32 CONTINUE
ISN 0226            IF(IBC.EQ.2) GO TO 34
ISN 0228            BGL = RO(1)*DPHI(1)/2.
ISN 0229            DO 46 N = 2,NMXM1
ISN 0230         46 BGL = BGL + RO(N)*DPHI(N)
ISN 0231            BGL = (BGL + RO(NMX)*DPHI(NMX)/2.)*DRO
ISN 0232            CONST =-2.*BGL/(TPB*(1.-EM2)*(RTIP*RTIP-RHUB*RHUB))
ISN 0233         34 CONTINUE
ISN 0234            ITR = ITR + 1
ISN 0235            IF(ITR.LE.ITRMX) GO TO 304
              C
```

```
ISN 0237          92 IF(ISAVE .NE.1) GO TO 30
               C
               C  SAVE VALUES ON TAPE
               C
ISN 0239             WRITE(2) LMX,KMX,NMX,DPHI
ISN 0240             DO 95 N=1,NMX
ISN 0241             WRITE(2) ((F(L,K,N),L=1,LMX),K=1,KMX)
ISN 0242          95 CONTINUE
               C
               C  END OF TAPE WRITE
               C
ISN 0243          30 CONTINUE
ISN 0244             CALL OUTPUT(5,IBC)
ISN 0245             STOP
ISN 0246             END
```

```
LEVEL 21.7 ( DEC 72 )                               OS/360  FORTRAN H

          COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                            SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,XREF
                C
  ISN 0002               SUBROUTINE GRID
  ISN 0003               COMMON ZT(30),XI(60),RO(10),F(30,60,10),A(10),B(10),D(10),E(10),
                       *    EE(30),FF(30),SV(30),DPHI(10),BV(10),US(30,60),UN(30,60),
                       *    DNDS(30,10,2),FAK(60),FHW(60),UTA(30),UTB(30),UTC(30),BBV(30),
                       *    BSA(30),BSB(30),BSC(30),VSA(30),VSB(30),BAV(30),LOP(30)
                       *    ,SONIC(10)
  ISN 0004               COMMON DAF,DBF,DCF,AKLKE,RX,OX,CONST
  ISN 0005               COMMON H,DRO,DZT,TDZ,TPB,XIC,XIB,XID,TDR,AKLK,DRO2,DZT2
  ISN 0006               COMMON OMCU,RHUB,RTIP,R2,RXE,RXH,UA,UB,TRDZ,ERA,ERB,CKLKE,SB,
                       *    BTR,FXB,FXI,EM2,AMA,AMB,DM,DKA,DKR,DQ,DQS,TZ,TTZ,TZS,TRS,TDQ
  ISN 0007               COMMON K,N,IDM,     KMX,LMX,NMX,KMXM1,KMXM2,LMXM1,LMXM2,NMXM1,
                       *    NMXM2,KTEO,IRX,    ITK,ITR,ITRMX,KLEP,ITPR,KUP,KDN
  ISN 0008               COMMON ID(36)
  ISN 0009               XIC = 0.0
  ISN 0010               XID = OMCU
  ISN 0011               XIB =  FXB*XID
  ISN 0012               XII = XID + FXI*XID
  ISN 0013               DQ = 2.0/FLOAT(KMX+1)
  ISN 0014               TDQ = 2.*DQ
  ISN 0015               DQS = DQ*DQ
  ISN 0016               XIMP = (XIB + XII)/2.
  ISN 0017               TALF = ALOG(DQ/(2.0-DQ))/( FXB*XID-XIMP)
  ISN 0018               ALF = TALF/2.
  ISN 0019               DO 1 K = 1,KMX
  ISN 0020               Q = -1.0 + FLOAT(K)*DQ
  ISN 0021               FAK(K) = ALF*(1. - Q*Q)
  ISN 0022               BKT = DQ*(0.5+FLOAT(K))-1.
  ISN 0023               FHW(K) = ALF*(1. - BKT*BKT)
  ISN 0024             1 XI(K) = XIMP + ALOG((1.0+Q)/(1.0-Q)) / TALF
  ISN 0025             4 DZT = TPB/FLOAT(LMXM1)
  ISN 0026               DZT2 = DZT*DZT
  ISN 0027               TDZ = 2.0*DZT
  ISN 0028               DO 2 L = 1,LMX
  ISN 0029             2 ZT(L) = FLOAT(L-1)*DZT
  ISN 0030               DRO = (RTIP-RHUB)/FLOAT(NMXM1)
  ISN 0031               TDR = 2.*DRO
  ISN 0032               DRO2 = DRO*DRO
  ISN 0033               DO 3 N = 1,NMX
  ISN 0034             3 RO(N) = FLOAT(N-1)*DRO  + RHUB
  ISN 0035               TZ = DQ/DZT
  ISN 0036               TZS = TZ*TZ
  ISN 0037               TTZ = DQ/TDZ
  ISN 0038               TRS = DQS/DRO2
  ISN 0039               DAF = (XI(KMX)-XI(KMXM1))/((XI(KMXM1)-XI(KMXM2))*(XI(KMX)
                       *    -XI(KMXM2)))
  ISN 0040               DBF = (XI(KMX)-XI(KMXM2))/((XI(KMXM1)-XI(KMXM2))*(XI(KMXM1)
                       *    -XI(KMX)))
  ISN 0041               DCF = (2.*XI(KMX)-XI(KMXM1)-XI(KMXM2))/((XI(KMX)-XI(KMXM2))*
                       *    (XI(KMX)-XI(KMXM1)))
  ISN 0042               IF(IDM.EQ.2) GO TO 13
  ISN 0044               DO 12 N = 1,NMX
  ISN 0045               R2 = RO(N)*RO(N)
  ISN 0046               E(N) = 1.0 + R2
  ISN 0047               D(N) = (E(N)*E(N))/R2
  ISN 0048               B(N) = -E(N)
```

```
ISN 0049          12 A(N) = R2
ISN 0050             GO TO 14
ISN 0051          13 DO 15 N = 1,NMX
ISN 0052             R2 = RO(N)*RO(N)
ISN 0053             E(N) = 0.0
ISN 0054             D(N) = (1.+R2)*(1.+R2)/R2
ISN 0055             B(N) = -1.-R2
ISN 0056             A(N) = R2
ISN 0057          15 CONTINUE
ISN 0058          14 CONTINUE
ISN 0059             DO 40 K = 1,KMX
ISN 0060             IF(XI(K).GT.XID) GO TO 41
ISN 0062          40 CONTINUE
ISN 0063          41 KTEO = K
ISN 0064             WRITE(6,201) XID
ISN 0065         201 FORMAT(/5X,'THE BLADES LIE BETWEEN Z = 0  AND',F9.6)
ISN 0066             WRITE(6,202)
ISN 0067         202 FORMAT(//4X,'K',7X,' Z',10X,' X/CA ',8X,'RXE', 9X,'RXH',/)
ISN 0068             DO 11 K = 1,KMX
ISN 0069             PCT = XI(K)/XID
ISN 0070             RX = 1. + (RXE-1.)*EXP(-((XI(K)-0.5*XID)/XID)**2)
ISN 0071             IF(ITPR.EQ.0) RX = RXE
ISN 0073             WRITE(6,203) K,XI(K),PCT,RX,RXH
ISN 0074          11 CONTINUE
ISN 0075         203 FORMAT(3X,I2,4X,F9.6,5X,F7.4,5X,F7.4,5X,F7.4)
ISN 0076             WRITE(6,204)
ISN 0077         204 FORMAT(//4X,'L     ZETA',/)
ISN 0078             DO 50 L = 1,LMX
ISN 0079             WRITE(6,205) L,ZT(L)
ISN 0080          50 CONTINUE
ISN 0081         205 FORMAT(3X,I2,F9.4,8X,F9.6)
ISN 0082             KHW = INT(FLOAT(KMX)/2.)
ISN 0083             WRITE(6,208) KHW
ISN 0084         208 FORMAT(//4X,'OPTIMUM SHOCK CAPTURING OCCURS WHEN THE GRID-SIZE',
                    *   ' RATIO RHO*(DELTA ZETA)/(DELTA Z) = (1+RHO**2)/RHO',
                    *   /4X,'THE TABLE BELOW LISTS THESE OPTIMUM VALUES, COMPARED ',
                    *   'TO THE VALUES ACTUALLY USED AT K =',I2,)
ISN 0085             WRITE(6,206) KHW
ISN 0086         206 FORMAT(//4X,'N    RHO',7X,'US/WO CRIT',  5X,' ARCTAN(RHO),',
                    *   6X,'M REL',16X,'GRID-SIZE RATIO',
                    *   /36X,'DEGREES',
                    *   26X,'OPTIMUM    ACTUAL(AT K = ',I2,')',/)
ISN 0087             DO 10 N = 1,NMX
ISN 0088             BTR = 1.0 - EM2*(1.0+RO(N)*RO(N))
ISN 0089             IF(SB.EQ.0.) GO TO 51
ISN 0091             USCRIT = BTR/(SB*(1.+RO(N)*RO(N)))
ISN 0092          51 PSI = 57.29578*ATAN(RO(N))
ISN 0093             SONIC(N) = USCRIT
ISN 0094             REL = SQRT(EM2*(1.+A(N)))
ISN 0095             OPT = (1.+RO(N)*RO(N))/RO(N)
ISN 0096             ACT = RO(N)*DZT*FHW(KHW)/DQ
ISN 0097             WRITE(6,207) N,RO(N),USCRIT,PSI,REL,OPT,ACT
ISN 0098          10 CONTINUE
ISN 0099         207 FORMAT(3X,I2,F9.4,1PE15.4,6X,0PF8.3,9X,F6.4,11X,F6.4,9X,F6.4)
ISN 0100             RETURN
ISN 0101             END
```

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LEVEL 21.7 ( DEC 72 )                              OS/360  FORTRAN H

          COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,XREF
                 C
   ISN 0002            SUBROUTINE BVALO(I)
   ISN 0003            COMMON ZT(30),XI(60),RO(10),F(30,60,10),A(10),B(10),D(10),E(10),
                      *   EE(30),FF(30),SV(30),DPHI(10),BV(10),US(30,60),UN(30,60),
                      *   DNDS(30,10,2),FAK(60),FHW(60),UTA(30),UTB(30),UTC(30),BBV(30),
                      *   BSA(30),BSB(30),BSC(30),VSA(30),VSB(30),BAV(30),LOP(30)
                      *   ,SONIC(10)
   ISN 0004            COMMON DAF,DBF,DCF,AKLKE,RX,OX,CONST
   ISN 0005            COMMON H,DRO,DZT,TDZ,TPB,XIC,XIB,XID,TDR,AKLK,DRO2,DZT2
   ISN 0006            COMMON OMCU,RHUB,RTIP,R2,RXE,RXH,UA,UB,TRDZ,ERA,ERB,CKLKE,SB,
                      *   BTR,FXB,FXI,EM2,AMA,AMB,DM,DKA,DKR,DQ,DQS,TZ,TTZ,TZS,TRS,TDQ
   ISN 0007            COMMON K,N,IDM,     KMX,LMX,NMX,KMXM1,KMXM2,LMXM1,LMXM2,NMXM1,
                      *   NMXM2,KTEO,IRX,    ITK,ITR,ITRMX,KLEP,ITPR,KUP,KDN
   ISN 0008            COMMON ID(36)
   ISN 0009            IF(XI(K).LT.XIC) GO TO 1
   ISN 0011            IF(XI(K).LT.XID) GO TO 2
   ISN 0013            GO TO 3
   ISN 0014          1 EE(1) = 0.
   ISN 0015            FF(1) = F(LMX,K,N)
   ISN 0016            GO TO 10
   ISN 0017          2 GO TO (100,200),I
   ISN 0018        200 EE(1) = 0.
   ISN 0019            KB = K - KLEP
   ISN 0020            FF(1) = F(LMX,K,N) + DNDS(KB,N,1)
   ISN 0021            GO TO 10
   ISN 0022        100 SLP = DNDS(K-KLEP,N,1)
   ISN 0023            AE = -BSA(1)*2.*FAK(K)/RX + 2.*BSB(1)*FHW(K-1) -3.5*AKLKE
                      *   - BSC(1)*FHW(K-1) +4.*DKA
   ISN 0024            BE = -8.*DKA + 4.*AKLKE
   ISN 0025            CE = BSA(1)*(FHW(K)*F(1,K+1,N)-2.*FAK(K)*OX*UTA(1)+FHW(K-1)
                      *   *F(1,K-1,N)) + BSB(1)*(-FHW(K-1)*(UTA(1)+F(1,K-1,N))
                      *   -FHW(K-2)*(F(1,K-1,N)-UTC(1))) + BSC(1)*(FHW(K)*(F(1,K+1,N)
                      *   -UTA(1))+FHW(K-1)*F(1,K-1,N))
                      *   -DKA*(UTC(3)-UTA(3)-8.*UTB(2)+8.*UTB(1)-UTC(1)-3.*UTA(1))
                      *   +AKLKE*(-.5*UTA(3)-3.*RO(N)*DZT*SLP-A(N)*DZT*UB*(UTC(1)
                      *   -6.*UTB(1)+3.*UTA(1)+2.*F(1,K+1,N))*FAK(K))
                      *   +DKR*(ERA*F(1,K,N+1)+ERB*F(1,K,N-1)-2.*F(1,K,N))
   ISN 0026            EE(1) = -BE/AE
   ISN 0027         12 FF(1) = -CE/AE
   ISN 0028            GO TO 10
   ISN 0029          3 EE(1) = 0.0
   ISN 0030            Z = F(LMX,K,N)
   ISN 0031          6 FF(1) = Z + DPHI(N)
   ISN 0032         10 RETURN
   ISN 0033            END
```

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LEVEL 21.7 ( DEC 72 )                                   OS/360  FORTRAN H

          COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,XREF
              C
  ISN 0002          SUBROUTINE BVAL1(I)
  ISN 0003          COMMON ZT(30),XI(60),RO(10),F(30,60,10),A(10),B(10),D(10),E(10),
                   *   EE(30),FF(30),SV(30),DPHI(10),BV(10),US(30,60),UN(30,60),
                   *   DNDS(30,10,2),FAK(60),FHW(60),UTA(30),UTB(30),UTC(30),BBV(30),
                   *   BSA(30),BSB(30),BSC(30),VSA(30),VSB(30),BAV(30),LOP(30)
                   *   ,SONIC(10)
  ISN 0004          COMMON DAF,DBF,DCF,AKLKE,RX,OX,CONST
  ISN 0005          COMMON H,DRO,DZT,TDZ,TPB,XIC,XIB,XID,TDR,AKLK,DRO2,DZT2
  ISN 0006          COMMON OMCU,RHUB,RTIP,R2,RXE,RXH,UA,UB,TRDZ,ERA,ERB,CKLKE,SB,
                   *   BTR,FXB,FXI,EM2,AMA,AMB,DM,DKA,DKR,DQ,DQS,TZ,TTZ,TZS,TRS,TDQ
  ISN 0007          COMMON K,N,IDM,     KMX,LMX,NMX,KMXM1,KMXM2,LMXM1,LMXM2,NMXM1,
                   *   NMXM2,KTEO,IRX,    ITK,ITR,ITRMX,KLEP,ITPR,KUP,KDN
  ISN 0008          COMMON ID(36)
  ISN 0009          FPA = F(2,K-1,N)
  ISN 0010          FPB = F(2, K ,N)
  ISN 0011          FPC = F(2,K+1,N)
  ISN 0012          DKL = 0.
  ISN 0013          IF(XI(K).LT.XIC) GO TO 1
  ISN 0015          IF(XI(K).LT.XID) GO TO 2
  ISN 0017          FPA = FPA - DPHI(N)
  ISN 0018          FPB = FPB - DPHI(N)
  ISN 0019          FPC = FPC - DPHI(N)
  ISN 0020          IF(IDM.EQ.2) GO TO 1
  ISN 0022          DKL = (1.+RO(N)*RO(N))*
                   *        0.5*TRS*(ERA*DPHI(N+1) + ERB*DPHI(N-1) -2.*DPHI(N))/FAK(K)
  ISN 0023        1 L = LMX
  ISN 0024          BKL = -2.*AKLKE - 2.*FAK(K)*BSA(L)/RX + 2.*FHW(K-1)*BSB(L)
                   *    - BSC(L)*FHW(K-1)
  ISN 0025          DKL = DKL - BSA(L)*(FHW(K)*F(L,K+1,N)-2.*FAK(K)*OX*UTA(L)
                   *    + FHW(K-1)*F(L,K-1,N)) + BSB(L)*(FHW(K-1)*(UTA(L)+F(L,K-1,N))
                   *    + FHW(K-2)*(F(L,K-1,N)-UTC(L))) - BSC(L)*(FHW(K)*(F(L,K+1,N)
                   *    -UTA(L)) + FHW(K-1)*F(L,K-1,N))
                   *    +2.*DKA*(-FPA+F(L,K-1,N)+FPB-2.*F(L,K,N)+F(L-1,K,N)
                   *    +F(L,K+1,N)-F(L-1,K+1,N))
                   *    - DKR*(ERA*F(L,K,N+1)+ERB*F(L,K,N-1)-2.*UTA(L))
  ISN 0026       61 AKL = AKLKE
  ISN 0027          CKL = CKLKE
  ISN 0028       25 SV(LMX) = (DKL-AKL*FPB      -CKL*FF(LMXM1))/(BKL+CKL*EE(LMXM1))
  ISN 0029          GO TO 10
  ISN 0030        2 CONTINUE
  ISN 0031          GO TO (100,200),I
  ISN 0032      200 SLP = DNDS(K-KLEP,N,2)
  ISN 0033          AE = -2.*FAK(K)*BSA(LMX)/RX + 2.*BSB(LMX)*FHW(K-1) - 7.*AKLKE/2.
                   *    -BSC(LMX)*FHW(K-1) + DKA
  ISN 0034          BE = -8.*DKA + 4.*AKLKE
  ISN 0035          CE = BSA(LMX)*(FHW(K)*F(LMX,K+1,N)-2.*FAK(K)*UTA(LMX)*OX
                   *    + FHW(K-1)*F(LMX,K-1,N)) + BSB(LMX)*(-FHW(K-1)*(UTA(LMX)
                   *    + F(LMX,K-1,N)) - FHW(K-2)*(F(LMX,K-1,N)-UTC(LMX)))
                   *+BSC(LMX)*(FHW(K)*(F(LMX,K+1,N)-UTA(LMX))+FHW(K-1)*F(LMX,K-1,N))
                   *    -DKA*(F(LMXM2,K+2,N)-UTA(LMXM2)-8.*F(LMXM1,K+1,N)+4.*(
                   *    F(LMX,K+1,N)-UTB(LMX)) + UTC(LMX))
                   *    +AKLKE*(-F(LMXM2,K,N)-6.*DZT*SLP          -11.*UTA(1)+18.*
                   *    UTA(2)-9.*UTA(3)+2.*UTA(4))/2.
                   *    +DKR*(ERA*F(LMX,K,N+1)+ERB*F(LMX,K,N-1)-2.*F(LMX,K,N))
  ISN 0036          GO TO 11
```

```
100 SLP = DNDS(K-KLEP,N,2)
    L = LMX
    AE = -2.*FAK(K)*BSA(LMX)/RX + 2.*BSB(LMX)*FHW(K-1)
   *    -BSC(LMX)*FHW(K-1)  + DKA  - 3.5*AKLKE
    BE = 4.*AKLKE   -8.*DKA
    CE = BSA(LMX)*(FHW(K)*F(LMX,K+1,N)-2.*FAK(K)*UTA(LMX)*OX
   *    +FHW(K-1)*F(LMX,K-1,N))+BSB(LMX)*(-FHW(K-1)*(UTA(LMX)+F(LMX,
   *    K-1,N))-FHW(K-2)*(F(LMX,K-1,N)-UTC(LMX)))+BSC(LMX)*(FHW(K)*
   *    (F(LMX,K+1,N)-UTA(LMX))+FHW(K-1)*F(LMX,K-1,N))
   *    -DKA*(F(LMXM2,K+2,N)-UTA(LMXM2)-8.*F(LMXM1,K+1,N)+4.*F(LMX,
   *    K+1,N)-4.*UTB(LMX)+UTC(LMX))+AKLKE*(-0.5*UTA(LMXM2)+3.*RO(N)
   *    *DZT*SLP+A(N)*DZT*UB*(UTC(LMX)-6.*UTB(LMX)+3.*UTA(LMX)+2.*
   *    F(LMX,K+1,N))*FAK(K))
   *    +DKR*(ERA*F(LMX,K,N+1)+ERB*F(LMX,K,N-1)-2.*F(LMX,K,N))
 11 SV(LMX) = -(CE+BE*FF(LMXM1))/(AE+BE*EE(LMXM1))
 10 RETURN
    END
```

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LEVEL 21.7 ( DEC 72 )                         OS/360  FORTRAN H

      COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                        SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,XREF
         C
 ISN 0002       SUBROUTINE OUTPUT(IOP,IBC)
 ISN 0003       COMMON ZT(30),XI(60),RO(10),F(30,60,10),A(10),B(10),D(10),E(10),
               *   EE(30),FF(30),SV(30),DPHI(10),BV(10),US(30,60),UN(30,60),
               *   DNDS(30,10,2),FAK(60),FHW(60),UTA(30),UTB(30),UTC(30),BBV(30),
               *   BSA(30),BSB(30),BSC(30),VSA(30),VSB(30),BAV(30),LOP(30)
               *   ,SONIC(10)
 ISN 0004       COMMON DAF,DBF,DCF,AKLKE,RX,OX,CONST
 ISN 0005       COMMON H,DRO,DZT,TDZ,TPB,XIC,XIB,XID,TDR,AKLK,DRO2,DZT2
 ISN 0006       COMMON OMCU,RHUB,RTIP,R2,RXE,RXH,UA,UB,TRDZ,ERA,ERB,CKLKE,SB,
               *   BTR,FXB,FXI,EM2,AMA,AMB,DM,DKA,DKR,DQ,DQS,TZ,TTZ,TZS,TRS,TDQ
 ISN 0007       COMMON K,N,IDM,     KMX,LMX,NMX,KMXM1,KMXM2,LMXM1,LMXM2,NMXM1,
               *   NMXM2,KTEO,IRX,    ITK,ITR,ITRMX,KLEP,ITPR,KUP,KDN
 ISN 0008       COMMON ID(36)
 ISN 0009       DIMENSION UR(30,60)
 ISN 0010       IF(IOP.EQ.2) GO TO 2
 ISN 0012       IF(IOP.EQ.3) GO TO 20
 ISN 0014       IF(IOP.EQ.4) GO TO 20
 ISN 0016       IF(IOP.EQ.5) GO TO 90
 ISN 0018       IF(IOP.EQ.6) GO TO 96
         C
         C    IOP = 1 GIVES VALUES OF THE POTENTIAL AND ALL VELOCITY COMPONENTS
         C    AT ALL GRID POINTS.
         C    IOP = 2 GIVES ONLY VELOCITY COMPONENTS, AND ONLY AT ZETA = 0. AND 2.*PI/B
         C    IOP = 3 IS THE SAME AS IOP = 1, BUT WITH VALUES OF THE POTENTIAL
         C    OMITTED.
         C    IOP = 4 GIVES MACH NUMBER CONTOURS.
         C    IOP = 5 CALCULATES PERFORMANCE DATA
         C    IOP = 6 GIVES THE LOCAL MACH NUMBER AT ALL POINTS
         C    IOP = 7 PRINTS ONLY THE POTENTIAL (AT ALL POINTS)
         C
         C    PRINT PHI
         C
 ISN 0020       LA = 1
 ISN 0021       LB = 10
 ISN 0022       WRITE(6,250) ITR,ITK,N,RO(N)
 ISN 0023   250 FORMAT(1H1,
              *         //3X,'VALUES OF THE POTENTIAL AFTER ITR = ',I4,
              *    ', ITK = ',I4,
              *    '  AT RHO(',I2,') = ',F7.4)
 ISN 0024    10 WRITE(6,200) (LOP(L),L=LA,LB)
 ISN 0025   200 FORMAT(/4X,'K    L=',I3,9I12,//)
 ISN 0026       DO 80 K = 1,KMX
 ISN 0027       WRITE(6,251)  K,(F(L,K,N),L=LA,LB)
 ISN 0028    80 CONTINUE
 ISN 0029   251 FORMAT(I5,1P10E12.3)
 ISN 0030       LA = LA + 10
 ISN 0031       IF(LA.GT.LMX) GO TO 20
 ISN 0033       LB = LB + 10
 ISN 0034       IF(LB.GT.LMX) LB=LMX
 ISN 0036       GO TO 10
 ISN 0037    20 CONTINUE
 ISN 0038       IF(IOP.EQ.7) RETURN
         C
         C    CALCULATE US AND UN
         C
```

```
ISN 0040           DO 60 K = 2,KMXM1
ISN 0041           DO 61 L = 2,LMXM1
ISN 0042           US(L,K) = UB*FAK(K)*(F(L,K+1,N) - F(L,K-1,N))
ISN 0043        61 UN(L,K) = ((F(L+1,K-1,N)-F(L,K-1,N))+(F(L,K+1,N)-F(L-1,K+1,N)))
                  *   /TRDZ - UA*US(L,K)
ISN 0044        60 CONTINUE
ISN 0045           DO 63 K = 2,KMXM1
ISN 0046           US(1,K) = UB*FAK(K)*(F(1,K+1,N)-F(1,K-1,N))
ISN 0047           US(LMX,K) = UB*FAK(K)*(F(LMX,K+1,N)-F(LMX,K-1,N))
ISN 0048           IF(K.LE.KLEP) GO TO 62
ISN 0050           IF(K.GE.KTEO) GO TO 62
ISN 0052           UN(1,K) = (-3.*F(1,K,N)+4.*F(2,K-1,N)-F(3,K-2,N)
                  *   +F(1,K+1,N)-F(1,K-1,N))/TRDZ - UA*US(1,K)
ISN 0053           UN(LMX,K) = (3.*F(LMX,K,N)-4.*F(LMXM1,K+1,N)+F(LMXM2,K+2,N)
                  *   + F(LMX,K+1,N)-F(LMX,K-1,N))/TRDZ - UA*US(LMX,K)
ISN 0054           GO TO 63
ISN 0055        62 FZTA = (F(2,K-1,N)-F(1,K-1,N)+F(LMX,K+1,N)-F(LMXM1,K+1,N))/TRDZ
ISN 0056           UN(1,K) = FZTA - UA*US(1,K)
ISN 0057           UN(LMX,K) = FZTA - UA*US(LMX,K)
ISN 0058        63 CONTINUE
             C
             C    FIND MACH NUMBER CONTOURS
             C
ISN 0059           IF(IOP.NE.4) GO TO 11
ISN 0061           TGTM = AMA
ISN 0062           RELSQ = EM2*(1.+A(N))
ISN 0063           GMAP = SB/EM2
ISN 0064           REL = SQRT(RELSQ)
ISN 0065           CF = GMAP/2.
ISN 0066       504 CONTINUE
ISN 0067           CALL CLEAR (UR(1,1),UR(30,60))
ISN 0068           CALL CLEAR (UN(1,1),UN(30,60))
             C
             C    THE FOLLOWING FORMULA WAS USED IN CALCULATING THE MACH NUMBER
             C    CONTOURS IN AIAA PAPER 77-199
             C      TGT = ((TGTM*TGTM/RELSQ)-1.)/GMAP
             C
             C    THE FOLLOWING IS THE LINEARIZED FORMULA
             C
ISN 0069           TGT =-(1.-TGTM/REL)/CF
             C
ISN 0070           KWMX = 0
ISN 0071           DO 500 L = 1,LMX
ISN 0072           SGN = (US(L,KUP) - TGT)/ABS(US(L,KUP)-TGT)
ISN 0073           KW = 1
ISN 0074           DO 501 K = KUP,KDN
ISN 0075           IF(SGN.GT.0.) GO TO 502
ISN 0077           IF((US(L,K)-TGT).GT.0.) GO TO 503
ISN 0079           GO TO 501
ISN 0080       502 IF((US(L,K)-TGT).LT.0.) GO TO 503
ISN 0082           GO TO 501
ISN 0083       503 X2 = XI(K)
ISN 0084           X1 = XI(K-1)
ISN 0085           Y2 = US(L,K)
ISN 0086           Y1 = US(L,K-1)
ISN 0087           Z = X1 + (X2-X1)*(TGT-Y1)/(Y2-Y1)
ISN 0088           UR(L,KW) = Z/XID
ISN 0089           UN(L,KW) = RO(N)*(ZT(L)+Z)/OMCU
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```
ISN 0090          IF(KW.GT.KWMX) KWMX = KW
ISN 0092          KW = KW + 1
ISN 0093          SGN = -SGN
ISN 0094      501 CONTINUE
ISN 0095      500 CONTINUE
ISN 0096          IF(KWMX.EQ.0) GO TO 510
ISN 0098          WRITE(6,205) N,RO(N),TGTM
ISN 0099      205 FORMAT(/5X,'FOR RO(',I2,') = ',F7.4,' THE LOCAL MACH NUMBER',
                 *    ' EQUALS',F7.4,' AT THE FOLLOWING VALUES OF X/CA AND ',
                 *    'R*THETA/CA',/)
ISN 0100          LA = 1
ISN 0101          LB = 20
ISN 0102          IF(LMX.LT.20) LB=LMX
ISN 0104      511 WRITE(6,230) (LOP(L),L=LA,LB)
ISN 0105      230 FORMAT(/1X,'KW L=',I2,19I6,//)
ISN 0106      231 FORMAT(1X,I2,20F6.2)
ISN 0107          DO 512 KW = 1,KWMX
ISN 0108          WRITE(6,231) KW,(UR(L,KW),L=LA,LB)
ISN 0109          WRITE(6,206) (UN(L,KW),L=LA,LB)
ISN 0110          WRITE(6,207)
ISN 0111      512 CONTINUE
ISN 0112      520 CONTINUE
ISN 0113      206 FORMAT(3X,20F6.2)
ISN 0114      207 FORMAT(1X)
ISN 0115          LA = LA + 10
ISN 0116          IF(LA.GT.LMX) GO TO 510
ISN 0118          LB = LB + 10
ISN 0119          IF(LB.GT.LMX) LB=LMX
ISN 0121          GO TO 511
ISN 0122      510 TGTM = TGTM + DM
ISN 0123          IF(TGTM.LE.AMB) GO TO 504
ISN 0125          RETURN
            C
            C   PRINT US
            C
ISN 0126       11 LA = 1
ISN 0127          LB = 10
ISN 0128          WRITE(6,253) ITR,ITK,N,RO(N),SONIC(N)
ISN 0129      253 FORMAT(///3X,'VALUES OF US/WO AFTER ITR = ',I4,', ITK = ',I4,
                 *    ' AT RHO(',I2,') = ',F7.4,10X,'US/WO CRIT = ',1PE12.3)
ISN 0130       21 WRITE(6,200) (LOP(I),I=LA,LB)
ISN 0131          DO 81 K = 2,KMXM1
ISN 0132          WRITE(6,251)  K,(US(L,K),L=LA,LB)
ISN 0133       81 CONTINUE
ISN 0134          LA = LA + 10
ISN 0135          IF(LA.GT.LMX) GO TO 40
ISN 0137          LB = LB + 10
ISN 0138          IF(LB.GT.LMX) LB = LMX
ISN 0140          GO TO 21
            C
            C   PRINT UN
            C
ISN 0141       40 WRITE(6,254) ITR,ITK,N,RO(N)
ISN 0142          LA = 1
ISN 0143          LB = 10
ISN 0144      254 FORMAT(///3X,'VALUES OF UN/WO AFTER ITR = ',I4,', ITK = ',I4,
                 *    ' AT RHO(',I2,') = ',F7.4)
ISN 0145       41 WRITE(6,200) (LOP(I),I=LA,LB)
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ISN 0146            DO 82 K = 2,KMXM1
ISN 0147            WRITE(6,251)  K,(UN(L,K),L=LA,LB)
ISN 0148         82 CONTINUE
ISN 0149            LA = LA + 10
ISN 0150            IF(LA.GT.LMX) GO TO 31
ISN 0152            LB = LB + 10
ISN 0153            IF(LB.GT.LMX) LB = LMX
ISN 0155            GO TO 41
ISN 0156         31 IF(IDM.EQ.2) WRITE(6,210)
ISN 0158        210 FORMAT(//10X,'NOTE: THE FOLLOWING VALUES OF V RADIAL ARE BASED',
               *    ' ON THE 2D STRIP-THEORY APPROXIMATION')
          C
          C   CALCULATE V RADIAL
          C
ISN 0159            IF(N.EQ.1) GO TO 52
ISN 0161            IF(N.EQ.NMX) GO TO 52
ISN 0163            DO 50 K = 2,KMXM1
ISN 0164            DO 51 L = 1,LMX
ISN 0165            UR(L,K) = (F(L,K,N+1) - F(L,K,N-1))/TDR
ISN 0166         51 CONTINUE
ISN 0167         50 CONTINUE
ISN 0168            GO TO 53
ISN 0169         52 DO 54 K = 2,KMXM1
ISN 0170            DO 55 L = 1,LMX
ISN 0171            UR(L,K) = 0.
ISN 0172         55 CONTINUE
ISN 0173         54 CONTINUE
          C
          C   PRINT V RADIAL
          C
ISN 0174         53 WRITE(6,255) ITR,ITK,N,RO(N)
ISN 0175        255 FORMAT(///3X,'VALUES OF V RADIAL/U INFINITY',
               *                                     'AFTER ITR = ',I4,', ITK = ',I4,
               *    '  AT RHO(',I2,') = ',F7.4)
ISN 0176            LA = 1
ISN 0177            LB = 10
ISN 0178         56 WRITE(6,200) (LOP(I),I=LA,LB)
ISN 0179            DO 83 K = 2,KMXM1
ISN 0180            WRITE(6,251)  K,(UR(L,K),L=LA,LB)
ISN 0181         83 CONTINUE
ISN 0182            LA = LA + 10
ISN 0183            IF(LA.GT.LMX) GO TO 30
ISN 0185            LB = LB + 10
ISN 0186            IF(LB.GT.LMX) LB = LMX
ISN 0188            GO TO 56
          C
          C   CALCULATE AND PRINT VELOCITIES ON ZETA = 0 AND ZETA1
          C
ISN 0189          2 WRITE(6,201) ITR,ITK,N,RO(N),SONIC(N)
ISN 0190        201 FORMAT(//4X,'SURFACE VELOCITIES AFTER ITR = ',I4,', ITK = ',I4,
               *    '  AT RHO(',I2,') = ',F7.4,10X,'US/WO CRIT = ',1PE12.3,
               *    /4X,'K',4X,'US(1,K)',4X,'US(LMX,K)',4X,'UN(1,K)',4X,
               *    'UN(LMX,K)',4X,'UR(1,K)',4X,'UR(LMX,K)',/)
ISN 0191            DO 65 K = 2,KMXM1
ISN 0192            US(1,K) = UB*FAK(K)*(F(1,K+1,N)-F(1,K-1,N))
ISN 0193            US(LMX,K) = UB*FAK(K)*(F(LMX,K+1,N)-F(LMX,K-1,N))
ISN 0194            IF(K.LE.KLEP) GO TO 64
ISN 0196            IF(K.GE.KTEO) GO TO 64
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ISN 0198           UN(1,K) = (-3.*F(1,K,N)+4.*F(2,K-1,N)-F(3,K-2,N)
                  *   +F(1,K+1,N)-F(1,K-1,N))/TRDZ - UA*US(1,K)
ISN 0199           UN(LMX,K) = (3.*F(LMX,K,N)-4.*F(LMXM1,K+1,N)+F(LMXM2,K+2,N)
                  *   + F(LMX,K+1,N)-F(LMX,K-1,N))/TRDZ - UA*US(LMX,K)
ISN 0200           GO TO 66
ISN 0201        64 FZTA = (F(2,K-1,N)-F(1,K-1,N)+F(LMX,K+1,N)-F(LMXM1,K+1,N))/TRDZ
ISN 0202           UN(1,K) = FZTA - UA*US(1,K)
ISN 0203           UN(LMX,K) = FZTA - UA*US(LMX,K)
ISN 0204        66 IF(N.EQ.1) GO TO 70
ISN 0206           IF(N.EQ.NMX) GO TO 70
ISN 0208           UR(1,K) = (F(1,K,N+1) - F(1,K,N-1))/TDR
ISN 0209           UR(LMX,K) = (F(LMX,K,N+1) - F(LMX,K,N-1))/TDR
ISN 0210           GO TO 72
ISN 0211        70 UR(1,K) = 0.0
ISN 0212           UR(LMX,K) = 0.0
ISN 0213        72 WRITE(6,251) K,US(1,K),US(LMX,K),UN(1,K),UN(LMX,K),
                  *   UR(1,K),UR(LMX,K)
ISN 0214        65 CONTINUE
ISN 0215        30 CONTINUE
ISN 0216           K1 = KTEO
ISN 0217           K2 = KTEO - 1
ISN 0218           K3 = KTEO - 2
ISN 0219           X1 = XI(K1)
ISN 0220           X2 = XI(K2)
ISN 0221           X3 = XI(K3)
ISN 0222           Y1 = US(1,K1)
ISN 0223           Y2 = US(1,K2)
ISN 0224           Y3 = US(1,K3)
ISN 0225           X = XID
ISN 0226           Z = (X-X2)*(X-X3)*Y1/((X1-X2)*(X1-X3))
                  *   +(X-X1)*(X-X3)*Y2/((X2-X1)*(X2-X3))
                  *   +(X-X1)*(X-X2)*Y3/((X3-X1)*(X3-X2))
ISN 0227           U1 = Z
ISN 0228           Y1 = US(LMX,K1)
ISN 0229           Y2 = US(LMX,K2)
ISN 0230           Y3 = US(LMX,K3)
ISN 0231           Z = (X-X2)*(X-X3)*Y1/((X1-X2)*(X1-X3))
                  *   +(X-X1)*(X-X3)*Y2/((X2-X1)*(X2-X3))
                  *   +(X-X1)*(X-X2)*Y3/((X3-X1)*(X3-X2))
ISN 0232           U2 = Z
ISN 0233           DU = U1 - U2
ISN 0234           WRITE(6,202) U1,U2,DU,N,DPHI(N)
ISN 0235       202 FORMAT( /5X,'TRAILING-EDGE VELOCITIES ARE US(1,TE) = ',1PE10.3,
                  *   '  US(LMX,TE) = ',E10.3,'  DELTA US = ',E10.3,
                  *   '  DPHI(',I2,') = ',E10.3)
ISN 0236           IF(IBC.NE.2) RETURN
             C
             C   CALCULATE THE BLADE SHAPE
             C
ISN 0238           IF(IOP.NE.6) GO TO 22
ISN 0240           K1 = KTEO
ISN 0241           K2 = KTEO - 1
ISN 0242           K3 = KTEO - 2
ISN 0243           X1 = XI(K1)
ISN 0244           X2 = XI(K2)
ISN 0245           X3 = XI(K3)
ISN 0246           K0 = KLEP - 3
ISN 0247           K4 = KTEO + 3
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ISN 0248          DO 23 K = K0,K4
ISN 0249          US(1,K) = UB*FAK(K)*(F(1,K+1,N)-F(1,K-1,N))
ISN 0250          US(LMX,K) = UB*FAK(K)*(F(LMX,K+1,N)-F(LMX,K-1,N))
ISN 0251          IF(K.LE.KLEP) GO TO 24
ISN 0253          IF(K.GE.KTE0) GO TO 24
ISN 0255          UN(1,K) = (-3.*F(1,K,N)+4.*F(2,K-1,N)-F(3,K-2,N)
                 *   +F(1,K+1,N)-F(1,K-1,N))/TRDZ - UA*US(1,K)
ISN 0256          UN(LMX,K) = (3.*F(LMX,K,N)-4.*F(LMXM1,K+1,N)+F(LMXM2,K+2,N)
                 *   + F(LMX,K+1,N)-F(LMX,K-1,N))/TRDZ - UA*US(LMX,K)
ISN 0257          GO TO 23
ISN 0258       24 FZTA = (F(2,K-1,N)-F(1,K-1,N)+F(LMX,K+1,N)-F(LMXM1,K+1,N))/TRDZ
ISN 0259          UN(1,K) = FZTA - UA*US(1,K)
ISN 0260          UN(LMX,K) = FZTA - UA*US(LMX,K)
ISN 0261       23 CONTINUE
ISN 0262       22 ORT = SQRT(1.+RO(N)*RO(N))/OMCU
ISN 0263          Y1 = UN(1,K1)
ISN 0264          Y2 = UN(1,K2)
ISN 0265          Y3 = UN(1,K3)
ISN 0266          X = XID
ISN 0267          Z = (X-X2)*(X-X3)*Y1/((X1-X2)*(X1-X3))
                 *   +(X-X1)*(X-X3)*Y2/((X2-X1)*(X2-X3))
                 *   +(X-X1)*(X-X2)*Y3/((X3-X1)*(X3-X2))
ISN 0268          UTEU = Z
ISN 0269          Y1 = UN(LMX,K1)
ISN 0270          Y2 = UN(LMX,K2)
ISN 0271          Y3 = UN(LMX,K3)
ISN 0272          Z = (X-X2)*(X-X3)*Y1/((X1-X2)*(X1-X3))
                 *   +(X-X1)*(X-X3)*Y2/((X2-X1)*(X2-X3))
                 *   +(X-X1)*(X-X2)*Y3/((X3-X1)*(X3-X2))
ISN 0273          UTEL = Z
ISN 0274          K1 = KLEP + 1
ISN 0275          K2 = KLEP + 2
ISN 0276          K3 = KLEP + 3
ISN 0277          Y1 = UN(1,K1)
ISN 0278          Y2 = UN(1,K2)
ISN 0279          Y3 = UN(1,K3)
ISN 0280          X1 = XI(K1)
ISN 0281          X2 = XI(K2)
ISN 0282          X3 = XI(K3)
ISN 0283          X = 0.
ISN 0284          Z = (X-X2)*(X-X3)*Y1/((X1-X2)*(X1-X3))
                 *   +(X-X1)*(X-X3)*Y2/((X2-X1)*(X2-X3))
                 *   +(X-X1)*(X-X2)*Y3/((X3-X1)*(X3-X2))
ISN 0285          ULEU = Z
ISN 0286          Y1 = UN(LMX,K1)
ISN 0287          Y2 = UN(LMX,K2)
ISN 0288          Y3 = UN(LMX,K3)
ISN 0289          Z = (X-X2)*(X-X3)*Y1/((X1-X2)*(X1-X3))
                 *   +(X-X1)*(X-X3)*Y2/((X2-X1)*(X2-X3))
                 *   +(X-X1)*(X-X2)*Y3/((X3-X1)*(X3-X2))
ISN 0290          ULEL = Z
ISN 0291          US(1,K1) = 0.5*XI(K1)*(UN(1,K1)+ULEU)
ISN 0292          US(LMX,K1) = 0.5*XI(K1)*(UN(LMX,K1)+ULEL)
ISN 0293          KA = KLEP+2
ISN 0294          KB = KTE0-1
ISN 0295          DO 75 K = KA,KB
ISN 0296          US(1,K) = US(1,K-1)+0.5*(XI(K)-XI(K-1))*(UN(1,K)+UN(1,K-1))
ISN 0297          US(LMX,K) = US(LMX,K-1)+0.5*(XI(K)-XI(K-1))*
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     *    (UN(LMX,K)+UN(LMX,K-1))
   75 CONTINUE
      US(1,KTEO) = US(1,KTEO-1)+0.5*(XID-XI(KTEO-1))*
     *    (UTEU+UN(1,KTEO-1))
      US(LMX,KTEO) = US(LMX,KTEO-1)+0.5*(XID-XI(KTEO-1))*
     *    (UTEL+UN(LMX,KTEO-1))
      WRITE(6,203)
  203 FORMAT(//10X,'BLADE SHAPE',/ 8X,'XI(K)',8X,'PCT',6X,'NU/CA',5X,
     *    'NL/CA',5X,'H/CA',6X,'T/CA',6X,'NU/C',6X,'NL/C',6X,
     *    'H/C',7X,'T/C')
      K1 = KLEP + 1
      OPR = ORT*OMCU
      DO 76 K = K1,KTEO
      US(1,K) = ORT*US(1,K)
      US(LMX,K) = ORT*US(LMX,K)
      HCA = (US(1,K) + US(LMX,K))/2.
      TCA = US(1,K) - US(LMX,K)
      PCT = XI(K)/XID
      OU = US(1,K)/OPR
      OL = US(LMX,K)/OPR
      HBC = HCA/OPR
      TBC = TCA/OPR
      IF(K.EQ.KTEO) GO TO 76
      WRITE(6,204) XI(K),PCT,US(1,K),US(LMX,K),HCA,TCA,OU,OL,HBC,TBC
   76 CONTINUE
      PCT = 1.
      WRITE(6,204) XID,PCT,US(1,KTEO),US(LMX,KTEO),HCA,TCA,
     *    OU,OL,HBC,TBC
  204 FORMAT(1PE15.3,0P9F10.4)
      RETURN
C
C     CALCULATE PERFORMANCE DATA
C
   90 CONTINUE
      WRITE(6,220)
      GMA = (SB/EM2) - 1.
      EMF = 0.5*EM2*(GMA-1.)
      EMF = GMA*EM2/(TPB*(1.+EM2*EMF))
      DO 91 N = 1,NMX
      W = -DPHI(N)/(TPB*RO(N))
      U = CONST - W*RO(N)
      DB = 57.296*ATAN((W-U*RO(N))/(1.+RO(N)*RO(N)))
      SPR = 1. + EMF*DPHI(N)
      WRITE(6,221) N,RO(N),DPHI(N),W,U,DB,SPR
   91 CONTINUE
  220 FORMAT(///38X,'PERFORMANCE DATA',
     *    ///10X,'N',5X,'RHO(N)',6X,'DPHI(N)',7X,'W/U INF',6X,'U/U INF',
     *    4X,'ARCTAN(UN/WO)',4X,'P02/P01',
     *    /68X,'(DEGREES)',/)
  221 FORMAT(8X,I3,4X,F7.4,4X,1PE10.3,
     *                                   5X,0PF8.4,5X,F8.4,6X,F6.2,9X,F6.3)
      RETURN
C
C  PRINT MACH NUMBERS
C
   96 REL = SQRT(EM2*(1.+A(N)) )
      CF = SB/(2.*EM2)
      DO 92 K = 2,KMXM1
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ISN 0341          DO 93 L = 1,LMX
ISN 0342          US(L,K) = UB*FAK(K)*(F(L,K+1,N) - F(L,K-1,N))
ISN 0343          UN(L,K) = REL*(1.+CF*US(L,K))
ISN 0344       93 CONTINUE
ISN 0345       92 CONTINUE
ISN 0346          WRITE(6,290) ITR,ITK,N,RO(N)
ISN 0347      290 FORMAT(1H1,3X,'VALUES OF THE LOCAL MACH NUMBER AFTER ITR = ',I4,
                 *    ', ITK = ',I4,'  AT RHO(',I2,') = ',F7.4)
ISN 0348          LA = 1
ISN 0349          LB = 20
ISN 0350          IF(LMX.LT.20) LB = LMX
ISN 0352       97 WRITE(6,291) (LOP(L),L=LA,LB)
ISN 0353      291 FORMAT(/1X,'K  L=',I2,19I6)
ISN 0354          WRITE(6,293)
ISN 0355      293 FORMAT(/)
ISN 0356          DO 94 K = 2,KMXM1
ISN 0357          WRITE(6,294) K,(UN(L,K),L=LA,LB)
ISN 0358       94 CONTINUE
ISN 0359      294 FORMAT(1X,I2,20F6.2)
ISN 0360          LA = LA + 20
ISN 0361          IF(LA.GT.LMX) GO TO 30
ISN 0363          LB = LB + 20
ISN 0364          IF(LB.GT.LMX) LB = LMX
ISN 0366          GO TO 97
ISN 0367          END
```

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LEVEL 21.7 ( DEC 72 )                                    OS/360  FORTRAN H

          COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,XREF
                C
  ISN 0002            SUBROUTINE BVAL(IBC)
  ISN 0003            COMMON ZT(30),XI(60),RO(10),F(30,60,10),A(10),B(10),D(10),E(10),
                     *   EE(30),FF(30),SV(30),DPHI(10),BV(10),US(30,60),UN(30,60),
                     *   DNDS(30,10,2),FAK(60),FHW(60),UTA(30),UTB(30),UTC(30),BBV(30),
                     *   BSA(30),BSB(30),BSC(30),VSA(30),VSB(30),BAV(30),LOP(30)
                     *   ,SONIC(10)
  ISN 0004            COMMON DAF,DBF,DCF,AKLKE,RX,OX,CONST
  ISN 0005            COMMON H,DRO,DZT,TDZ,TPB,XIC,XIB,XID,TDR,AKLK,DRO2,DZT2
  ISN 0006            COMMON OMCU,RHUB,RTIP,R2,RXE,RXH,UA,UB,TRDZ,ERA,ERB,CKLKE,SB,
                     *   BTR,FXB,FXI,EM2,AMA,AMB,DM,DKA,DKR,DQ,DQS,TZ,TTZ,TZS,TRS,TDQ
  ISN 0007            COMMON K,N,IDM,     KMX,LMX,NMX,KMXM1,KMXM2,LMXM1,LMXM2,NMXM1,
                     *   NMXM2,KTEO,IRX,    ITK,ITR,ITRMX,KLEP,ITPR,KUP,KDN
  ISN 0008            COMMON ID(36)
  ISN 0009            DO 1 K = 1,KMX
  ISN 0010            IF(XI(K).GE.XIC) GO TO 2
  ISN 0012          1 CONTINUE
  ISN 0013          2 KLE = K
  ISN 0014            KLEP = KLE - 1
  ISN 0015            K2 = KTEO - 1
                C
                C  KLEP IS THE LAST POINT UPSTREAM OF THE L. E., AND KTEO IS THE FIRST
                C  POINT DOWNSTREAM OF THE T. E.
                C
  ISN 0016            GO TO (100,200),IBC
  ISN 0017        200 DO 20 N = 1,NMX
  ISN 0018            R2 = RO(N)*RO(N)
                C
                C   FOR IBC = 2, TAU = TMAX(R)/C(R).
                C   THE BLADE GEOMETRY USED HERE HAS TMAX = CONSTANT.  BV(3) CONTAINS
                C   TMAX/CAX, BV(2) CONTAINS SMALL A, BV(1) AND BV(4) CONTAIN DELTA CP
                C  AT THE HUB AND TIP, RESPECTIVELY.  DELTA CP ZERO VARIES LINEARLY WITH
                C  RHO BETWEEN THESE TWO VALUES.
                C
  ISN 0019            TAU = BV(3)/SQRT(1.+R2)
  ISN 0020            AOM = BV(2)*OMCU
  ISN 0021            ACP = (BV(4)-BV(1))/(RO(NMX)-RO(1))
  ISN 0022            BCP = (BV(1)-H*BV(4))/(1.-H)
  ISN 0023            DCP = ACP*RO(N) + BCP
  ISN 0024            DO21 K = KLE,K2
  ISN 0025            KB = K - KLEP
  ISN 0026            HCP = DCP/2.
  ISN 0027            IF(XI(K).GT.AOM) GO TO 22
  ISN 0029            DCP2 = HCP
  ISN 0030            DNDS (KB,N,1) =  HCP*XI(K)
  ISN 0031            GO TO 23
  ISN 0032         22 DNDS(KB,N,1) = HCP*(AOM+(XI(K)-AOM)*(1.-0.5*(BV(2)+XI(K)/OMCU))
                     *   /(1.-BV(2)))
  ISN 0033            DCP2 = HCP*(1.-XI(K)/OMCU)/(1.-BV(2))
  ISN 0034         23 TPS = 4.*TAU*(1.-2.*XI(K)/OMCU)
  ISN 0035            DNDS(KB,N,2) = R2*DCP2/(1.+R2)+RO(N)*TPS
  ISN 0036            DPHI(N) = HCP*(AOM + OMCU*(1.-BV(2))/2.)
  ISN 0037         21 CONTINUE
  ISN 0038         20 CONTINUE
  ISN 0039            WRITE(6,220) BV(1),BV(4),BV(2),BV(3)
  ISN 0040        220 FORMAT(//5X,'BLADE LOADING SPECIFIED FOR THIS CASE HAS DELTA CP',
```

```
                       *     ' =',F8.4,' AT THE HUB,',F8.4,' AT THE TIP',
                       *     //10X,'LOADING IS CONSTANT OVER THE FIRST',2PF6.2,' PERCENT',
                       *     ' OF THE CHORD, AND THEN VARIES LINEARLY TO ZERO AT THE ',
                       *     //10X,'TRAILING EDGE',
                       *     //10X,'THICKNESS DISTRIBUTION IS THAT OF A DOUBLE PARABOLIC',
                       *     ' ARC, WITH TMAX/CAX = ',0PF8.4,//)
ISN 0041                     GO TO 10
ISN 0042                 100 CONTINUE
                   C
                   C   THE BLADE GEOMETRY AND ANGLE OF ATTACK ARE
                   C   T MAX/CAX = BV(1) + BV(2)/R + BV(3)*R
                   C   H MAX/CAX = BV(4) + BV(5)/R + BV(6)*R
                   C   ALPHA = BV(7) + BV(8)/R + BV(9)*R
                   C   WHERE R = RO(N)/RTIP
                   C
ISN 0043                     WRITE(6,210) (BV(JJ),JJ=1,9)
ISN 0044                 210 FORMAT(//5X,'BLADE GEOMETRY AND ANGLE OF ATTACK SPECIFIED ARE',
                       *     /20X,'T MAX/CAX = TA + TB/R + TC*R, R = RO(N)/RTIP',
                       *     /25X,'WHERE TA = ',1PE12.3,' TB = ',E12.3,' TC = ',E12.3,
                       *     //20X,'H MAX/CAX = HA + HB/R + HC*R',
                       *     /25X,'WHERE HA = ',1PE12.3,' HB = ',E12.3,' HC = ',E12.3,
                       *     //20X,'ALPHA = AA + AB/R + AC*R   (DEGREES)',
                       *     /25X,'WHERE AA = ',1PE12.3,' AB = ',E12.3,' AC = ',E12.3,//)
ISN 0045                     BV(7) = BV(7)/57.29578
ISN 0046                     BV(8) = BV(8)/57.29578
ISN 0047                     BV(9) = BV(9)/57.29578
ISN 0048                     DO 3 N = 2,NMXM1
ISN 0049                     R = RO(N)/RTIP
ISN 0050                     CBC = 1./SQRT(1.+RO(N)*RO(N))
ISN 0051                     HMXCA = BV(4) + BV(5)/R + BV(6)*R
ISN 0052                     TMXCA = BV(1) + BV(2)/R + BV(3)*R
ISN 0053                     ALPH  = BV(7) + BV(8)/R + BV(9)*R
ISN 0054                     DO 4 K = KLE,K2
ISN 0055                     KB = K - KLEP
ISN 0056                     DNDS(KB,N,1) = (4.*HMXCA + 2.*TMXCA)*CBC*(1.-2.*XI(K)/OMCU) - ALPH
ISN 0057                     DNDS(KB,N,2) = (4.*HMXCA - 2.*TMXCA)*CBC*(1.-2.*XI(K)/OMCU) - ALPH
ISN 0058                   4 CONTINUE
ISN 0059                   3 CONTINUE
ISN 0060                  10 RETURN
ISN 0061                     END
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LEVEL 21.7 ( DEC 72 )                          OS/360  FORTRAN H

         COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                            SOURCE,EBCDIC,NOLIST,NODECK,LOAD,MAP,NOEDIT,ID,XREF
               C
  ISN 0002              SUBROUTINE RESID(ISHO)
  ISN 0003              COMMON ZT(30),XI(60),RO(10),F(30,60,10),A(10),B(10),D(10),E(10),
                      *    EE(30),FF(30),SV(30),DPHI(10),BV(10),US(30,60),UN(30,60),
                      *    DNDS(30,10,2),FAK(60),FHW(60),UTA(30),UTB(30),UTC(30),BBV(30),
                      *    BSA(30),BSB(30),BSC(30),VSA(30),VSB(30),BAV(30),LOP(30)
                      *    ,SONIC(10)
  ISN 0004              COMMON DAF,DBF,DCF,AKLKE,RX,OX,CONST
  ISN 0005              COMMON H,DRO,DZT,TDZ,TPB,XIC,XIB,XID,TDR,AKLK,DRO2,DZT2
  ISN 0006              COMMON OMCU,RHUB,RTIP,R2,RXE,RXH,UA,UB,TRDZ,ERA,ERB,CKLKE,SB,
                      *    BTR,FXB,FXI,EM2,AMA,AMB,DM,DKA,DKR,DQ,DQS,TZ,TTZ,TZS,TRS,TDQ
  ISN 0007              COMMON K,N,IDM,      KMX,LMX,NMX,KMXM1,KMXM2,LMXM1,LMXM2,NMXM1,
                      *    NMXM2,KTEO,IRX,    ITK,ITR,ITRMX,KLEP,ITPR,KUP,KDN
  ISN 0008              COMMON ID(36)
  ISN 0009              DIMENSION ERMX(10),KER(10),LER(10)
  ISN 0010              WRITE(6,200) ITR
  ISN 0011          200 FORMAT(//5X,'AFTER ITR = ',I2,' MAXIMUM RESIDUALS ARE',
                      *    //4X,'N',7X,'RESIDUAL',4X,'K',4X,'L',
                      *    7X,'AVG RES',8X,'AVG PHI',8X,'AVG SUM',/)
  ISN 0012              DO 1 N = 2,NMXM1
  ISN 0013              SUM = 0.
  ISN 0014              SNORM = 0.
  ISN 0015              SBOT = 0.
  ISN 0016              KOUNT = 0
  ISN 0017              ERMX(N) = 0.
  ISN 0018              BTR = 1. - EM2*(1. + RO(N)*RO(N))
  ISN 0019              AKLK = D(N)*TZS
  ISN 0020              DKA = -B(N)*TTZ
  ISN 0021              ERA = 1. + 0.5*DRO/RO(N)
  ISN 0022              ERB = 1. - 0.5*DRO/RO(N)
  ISN 0023              DO 4 L = 1,LMX
  ISN 0024              VSA(L) = 0.
  ISN 0025              BSB(L) = 0.
  ISN 0026              BAV(L) = BTR
  ISN 0027            4 CONTINUE
  ISN 0028              DO 2 K = 2,KMXM1
  ISN 0029              AKLKE = AKLK/FAK(K)
  ISN 0030              DKR = TRS*E(N)/FAK(K)
  ISN 0031              DO 3 L = 2,LMXM1
  ISN 0032              BBV(L) = BAV(L)
  ISN 0033              BAV(L) = BTR - SB*FAK(K)*(F(L,K+1,N)-F(L,K-1,N))/TDQ
  ISN 0034              VSB(L) = VSA(L)
  ISN 0035              VSA(L) = 0.
  ISN 0036              IF(BAV(L).GT.0.) GO TO 8
  ISN 0038              VSA(L) = 1.
  ISN 0039            8 BSC(L) = BSB(L)
  ISN 0040              BSB(L) = FHW(K)*(F(L,K+1,N)-F(L,K,N)) -FHW(K-1)*(F(L,K,N)-F(L,
                      *    K-1,N))
  ISN 0041              BSA(L) = FHW(K)*F(L,K+1,N)-2.*FAK(K)*F(L,K,N)+FHW(K-1)*F(L,K-1,N)
  ISN 0042              TMA = BAV(L)*(1.-VSA(L))*BSA(L)+BBV(L)*VSB(L)*BSC(L)
  ISN 0043              TA = ABS(TMA)
  ISN 0044              TMB = A(N)*((1.-VSA(L))*BSA(L)+VSA(L)*BSB(L))
  ISN 0045              TB = ABS(TMB)
  ISN 0046              TMC = -2.*DKA*(-F(L+1,K-1,N)+F(L,K-1,N)+F(L+1,K,N)-2.*F(L,K,N)
                      *    +F(L-1,K,N)+F(L,K+1,N)-F(L-1,K+1,N))
  ISN 0047              TC = ABS(TMC)
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ISN 0048            TMD = AKLKE*(F(L+1,K,N)-2.*F(L,K,N)+F(L-1,K,N))
ISN 0049            TD = ABS(TMD)
ISN 0050            TME = DKR*(ERA*F(L,K,N+1)+ERB*F(L,K,N-1)-2.*F(L,K,N))
ISN 0051            TE = ABS(TME)
ISN 0052            TOP = TMA+TMB+TMC+TMD+TME
ISN 0053            BOT = TA +TB +TC +TD +TE
ISN 0054            RES = ABS(TOP)
ISN 0055            SUM = SUM + RES
ISN 0056            SNORM = SNORM + ABS(F(L,K,N))
ISN 0057            SBOT = SBOT + BOT
ISN 0058            KOUNT = KOUNT + 1
ISN 0059            IF(ISHO.EQ.0) GO TO 22
ISN 0061            WRITE(6,205) N,K,L,TMA,TMB,TMC,TMD,TME,TOP,RES
ISN 0062        205 FORMAT(3I5,1P7E13.3)
ISN 0063         22 IF(RES.LT.ERMX(N)) GO TO 3
ISN 0065            KER(N) = K
ISN 0066            LER(N) = L
ISN 0067            ERMX(N) = RES
ISN 0068          3 CONTINUE
ISN 0069          2 CONTINUE
ISN 0070            SUM = SUM/FLOAT(KOUNT)
ISN 0071            SNORM = SNORM/FLOAT(KOUNT)
ISN 0072            SBOT = SBOT/FLOAT(KOUNT)
ISN 0073            WRITE(6,210) N,ERMX(N),KER(N),LER(N),SUM,SNORM,SBOT
ISN 0074          1 CONTINUE
ISN 0075        210 FORMAT(I5,1PE15.3,2I5,3E15.3)
ISN 0076         10 RETURN
ISN 0077            END
```

APPENDIX C

DICTIONARY OF VARIABLES

FORTRAN SYMBOL	ALGEBRAIC EQUIVALENT	DEFINITION, USE, COMMENTS
A(N)	ρ^2	
AE		See Equations (38), (46)
ALF	α_1	Equation (16)
AKL	A_K^L	Equation (30)
AKLK	$\left(\frac{1+\rho^2}{\rho}\ \frac{\Delta\tau}{\Delta\zeta}\right)^2$	
AKLKE	$\left(\frac{1+\rho^2}{\rho}\ \frac{\Delta\tau}{\Delta\zeta}\right)^2 / f_K$	
AMA, AMB		Mach number limits for which contours are calculated if IOP=4
B(N)	$-(1+\rho^2)$	
BAV(L)	V_K^L	
BBV	V_{K-1}^L	
BE		See Equations (38), (46)
BGL	$\int_{\rho_H}^{\rho_T} \rho \Delta\phi \, d\rho$	
BKL	B_K^L	Equation (30)
BN	B	Number of blades
BSA(L)	$(1-\mu_K^L)(V_K^L+\rho^2)$	
BSB(L)	$\mu_{K-1}^L \ V_{K-1}^L$	
BSC(L)	$\rho^2 \mu_K^L$	
BTR	$1-\frac{W_0^2}{a_\infty^2}$	
BV(I)		Parameters for blade-surface boundary conditions
CALT	c_a / L_T	
CE		See Equations (38), (46)

FORTRAN SYMBOL	ALGEBRAIC EQUIVALENT	DEFINITION, USE, COMMENTS
CKL	C_K^L	Equation (30)
CKLKE	$= AKLKE$	
CONST	C	See Equation (10)
D(N)	$(1+\rho^2)^2/\rho^2$	
DB	$\tan^{-1}(u_n/W_0)_{z\to\infty}$	Used in Subroutine OUTPUT
DAF,DBF,DCF		See Equation (52)
DKA	$(1+\rho^2)\Delta\tau/2\Delta\zeta$	
DKL	D_K^L	Equation (30)
DKR	$\frac{1+\rho^2}{f_K}\left(\frac{\Delta\tau}{\Delta\rho}\right)^2$	
DM		Mach number interval for which contours are calculated if IOP=4
DNDS(K,N,J)		Surface slopes for IBC=1, loading and thickness parameters for IBC=2, see Equations (17) and (18)
DPHI(N)	$\Delta\phi(\rho)$	
DQ	$\Delta\tau$	
DQS	$(\Delta\tau)^2$	
DRO	$\Delta\rho$	
DRO2	$(\Delta\rho)^2$	
DZT	$\Delta\zeta$	
DZT2	$(\Delta\zeta)^2$	
E(N)	$1+\rho^2$	E(N) is defined to be zero for IDM=2
EMTG	$\omega r_T/a_\infty$	
EMX	M_∞	
EM2	M_∞^2	
ERA	$1+\Delta\rho/2\rho$	

FORTRAN SYMBOL	ALGEBRAIC EQUIVALENT	DEFINITION, USE, COMMENTS
ERB	$1 - \Delta\rho / 2\rho$	
F(L,K,N)	$\phi(\zeta, z, \rho)$	
FAK(K)	f_K	
FHW(K)	$f_{K+\frac{1}{2}}$	
FPA,FPB,FPC	${}^N\phi^2_{K-1}, {}^N\phi^2_K, {}^N\phi^2_{K+1}$	
FXB		Used to locate upstream and downstream edges of the grid - see Equation (16)
FXI		
GMA	γ	
H	$h = \rho_H / \rho_T$	
IBC		Indicator for blade-surface boundary conditions
IBN	B	
IDM		= 2, 3, for two, three-dimensional calculations
IOP		Used to select output option
IRPT		Print control
IRX		Equals 2 or 1 if there are or are not any hyperbolic points on the line being relaxed
IRXP		Controls intervals at which residuals are printed
IRXPT		Print control
ISAVE		= 1, 0 if results are, are not to be saved on tape
ISHO		= 1 if residuals are to be shown at every grid point
ISTART		Selects start option
ITK		Counter for iteration in z-direction
ITKMX		Maximum number of iterations in z-direction

FORTRAN SYMBOL	ALGEBRAIC EQUIVALENT	DEFINITION, USE, COMMENTS
ITPR		= 1 if elliptic relaxation factor is tapered, see Equation (A-4)
ITR		Counter for iterations in ρ-direction
ITRMX		Maximum number of iterations in ρ-direction
JPRT		Print control
KDN, KUP		Downstream, upstream limits within which Mach number contours are calculated if IOP=4
KHW	$K + \frac{1}{2}$	
KMX		Number of grid points in the z-direction
KMXM1	KMX - 1	
KMXM2	KMX -2	
KLEP		Last K-station upstream of leading edge
KTEO		First K-station downstream of trailing edge
KW, KWMX		Counters for locations of Mach number contours
LMX		Number of grid points in the ζ-direction
LMXM1	LMX -1	
LMXM2	LMX -2	
NMX		Number of grid points in the ρ-direction
NMXM1	NMX -1	
NMXM2	NMX - 2	
NPRT		Print control
NPT		Print control
OBM	1 - OBV	
OBV		Relaxation factor for circulation, see Equation (53)

FORTRAN SYMBOL	ALGEBRAIC EQUIVALENT	DEFINITION, USE, COMMENTS
OMCU	$\omega c_a / U_\infty$	Value of Z at trailing edge of blades
OPR	$\sqrt{1+\rho^2}$	
OPT	$\rho \Delta \zeta / \Delta Z)_{OPITMUM}$	See Equation (A-1)
ORT	$\sqrt{1+\rho^2} / \frac{\omega c_a}{U_\infty}$	
OX	$1 - 1/\omega$	
Q	τ	
REL	$\sqrt{M_\infty^2 + (\omega r / a_\infty)^2} = \frac{W_o}{a_\infty}$	Inlet relative Mach number at r
RELSQ	$(W_o / a_\infty)^2$	
RES	${}^{N}R_{K}^{L}$	See Equation (73)
RHUB	$\rho_H = \omega r_H / U_\infty$	
RO(N)	$\rho = \omega r / U_\infty$	
RTIP	$\rho_T = \omega r_T / U_\infty$	
RX	ω	Relaxation factor, see Equation (33)
RXE		Relaxation factors used at elliptic, hyperbolic points
RXH		
RXM	$1 - \omega$	
R2	ρ^2	
SB	$(\gamma + 1) M_\infty^2$	
SPR	p_{o_2} / p_{o_1}	
SV(L)		Temporary storage for ϕ
TA-TE		See Equations (68)-(72)
TALF	$2\alpha_1$	See Equation (16)
TDQ	$2\Delta\tau$	
TDR	$2\Delta\rho$	
TDZ	$2\Delta\zeta$	

FORTRAN SYMBOL	ALGEBRAIC EQUIVALENT	DEFINITION, USE, COMMENTS
TMA-TME		Absolute values of TA through TE
TPB	$2\pi / B = \zeta_1$	
TPI	2π	
TRS	$(\Delta\tau / \Delta\rho)^2$	
TRDZ	$2\rho\Delta\zeta$	
TTZ	$\Delta\tau / 2\Delta\zeta$	
TZ	$\Delta\tau / \Delta\zeta$	
TZS	$(\Delta\tau / \Delta\zeta)^2$	
U	$(u/U_\infty)_{Z \to \infty}$	Used in Subroutine OUTPUT
UA	ρ	
UB	$[2\Delta\tau\ (1+\rho^2)]^{-1}$	
ULEL	$\frac{u_n}{W_o}\ (\zeta = \frac{2\pi}{B}, Z=0, \rho)$	
ULEU	$\frac{u_n}{W_o}\ (\zeta = 0\ , Z = 0, \rho)$	
UN	u_n / W_o	
UR	v / U_∞	
US	u_s / W_o	
USCRIT		Sonic value of u_s / W_o
UTA,UTB,UTC		Values of ${}^{N}\phi^{L}_{K}$, ${}^{N}\phi^{L}_{K-1}$, ${}^{N}\phi^{L}_{K-2}$ from previous Z-iteration
UTEL	$\frac{u_n}{W_o}(\zeta = \frac{2\pi}{B}, Z = \frac{\omega c_a}{U_\infty}, \rho)$	
UTEU	$\frac{u_n}{W_o}(\zeta = 0, Z = \frac{\omega c_a}{U_\infty}, \rho)$	
VSA	μ^{L}_{K}	
VSB	μ^{L}_{K-1}	
W	$\omega / U_\infty)_{Z \to \infty}$	Used in Subroutine OUTPUT
XI (K)	$Z = \omega x / U_\infty$	
XIB	Z_B	See Equation (16)

FORTRAN SYMBOL	ALGEBRAIC EQUIVALENT	DEFINITION, USE, COMMENTS
XID	$\omega c_a / u_\infty$	
XII	z_I	See Equation (16)
XIMP	z_M	See Equation (16)
ZT(L)	ζ	