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CONTROL OF DYNAMICAL SYSTEMS

for the period

September 1, 1977 - March 1, 1978

Brown University
Lefschetz Center for Dynamical Systems
Division of Applied Mathematics
Providence, Rhode Island 02912

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Principal Investigators:
H. T. Banks
J. K. Hale
J. P. LaSalle

March 27, 1978

Report prepared by:
H. T. Banks

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I. Numerical Methods for Optimal Control of Equations with Delays (H. T. Banks)

1. Spline-type techniques.

During the summer of 1977 Banks and F. Kappel (a visitor from Austria) made significant progress on a conceptual and theoretical framework for the use of spline-based approximations to compute solutions of systems of delay equations. These efforts were in the general direction of finding alternatives to and, hopefully, improvements on the "averaging" approximation ideas developed earlier by Banks and J. Burns. During the fall Banks has carried out numerical experiments with techniques based on linear spline approximations. The results of these efforts suggest that our original expectations for the spline-type schemes were perhaps too conservative. For example, our original convergence estimates predicted first order convergence for the linear spline techniques. However, a careful analysis of our results from numerous test examples reveals that for integration (i.e., numerical solution) of delay equations the method is actually second order convergent. Banks is currently in the process of following up on these findings in two ways. First, he is developing software packages to use the linear spline approximations in solving optimal control problems with delay systems. Our conjecture here is that the method will also turn out to be numerically second order convergent. The second line of investigation, which is also prompted by our numerical findings this fall, is theoretical in nature. The phenomenon that N^{th} order splines give $(N+1)^{st}$ order convergence in actual computations is one that is found in the use of finite-element (i.e., spline) methods for solution of strongly coercive ordinary and partial differential equation boundary value problems. (One can even give an improved convergence and error analysis in special cases.) Banks is pursuing investigations in an attempt to prove conclusively that a similar phenomenon is found in use of the spline based methods for certain classes of optimal control problems.

2. Difference equation type techniques.

D. Reber, under the direction of Banks, has completed his Ph.D. thesis on modifications (of the Banks-Burns theoretical framework) which result in difference equation approximations (as opposed to the finite systems of ordinary differential equations in the Banks-Burns approach). Both theoretical and computational efforts to study general linear nonautonomous (time-varying) system control problems (via the "averaging" type approximation scheme) were made. The results indicate that while the methods developed by Reber offer a viable alternative, one cannot expect to make significant gains here unless one employs something better than a simple first order differencing scheme on derivatives in the problems. Banks and Reber are currently developing a framework to handle higher-order differencing schemes. In addition, they have begun efforts to extend some of these ideas to treat rather general nonlinear delay system control problems. Schemes involving approximations other than "averaging" type (e.g., splines) are being investigated in connection with higher order difference methods.

II. Optimal Control of Diffusion-Reaction Systems. (H.T. Banks)

Banks, J.P. Kernevez and M. Duban (an assistant to Kernevez at Université de Technologie de Compiégne) have begun their efforts on methods for optimization of diffusion-reaction problems

$$\frac{\partial s}{\partial t} - \frac{\partial^2 s}{\partial x^2} + \frac{a}{1+a} v(s) = 0$$

$$\frac{\partial a}{\partial t} - \alpha\frac{\partial^2 a}{\partial x^2} = 0$$

through boundary controls. (Here v is a nonlinear reaction velocity approximation determined by the specific reactions considered.) Several different (as yet we don't know which will prove "best" from a practical standpoint) theoretical formulations (along with necessary conditions for optimization) have been studied. Some initial numerical computations have been made, but it is still too early in our efforts to make any definitive statements regarding practical feasibility and comparisons of different approaches.

III. Nonlinear Oscillations (J. K. Hale)

Suppose the equation

$$\ddot{x} + G(x) = 0$$

has a 2π-periodic solution $p(t)$ for which the linear variational equation

$$\ddot{y} + G'(p(t))y = 0$$

has two linearly independent solutions. If Γ is the periodic orbit in $(x,\dot{x})$-space generated by p, then a fundamental problem in nonlinear oscillations is to study the existence of 2π-periodic solutions near Γ of the equation

$$\ddot{x} + G(x) = \lambda\dot{x} + \mu f(t)$$

where λ,μ are small real parameters and $f(t)$ is 2π-periodic. Under an additional generic hypothesis on G'' and f, Hale and Táboas have given a complete solution to this problem by specifying the bifurcation curves in (λ,μ)-space across which the number of 2π-periodic solutions changes.

IV. Qualitative Theory for Ordinary and Partial Differential Equations (J. K. Hale)

Two students of Hale have essentially completed their Ph.D. dissertations. Gene Cooperman has given a definitive answer for when a mapping T on a metric space has a maximal compact invariant set. The stability properties of this set are also discussed. These results have application to stability theory and the existence of periodic solutions of ordinary differential equations, partial differential equations and functional differential equations. The other student, Nicholas Alikakos has completed research on the asymptotic behavior of the solutions of reaction diffusion equations of the form

$$\frac{du_i}{dt} = \nu_i \Delta u_i + f(u_1, \dots, u_n), \quad i = 1,2,\dots n$$

with normal boundary conditions. Under certain conditions on f he has shown that the solutions asymptotically approach solutions of the ordinary differential equation. This is true for all values of the $\nu_i > 0$.

V. Control Systems (J. P. LaSalle)

1. Stable feedback generators.

An initial effort on this problem by LaSalle and J. Palmer (a student of LaSalle's) has been discontinued. Some progress was made on attempting to extend the concept of solutions of differential equations with discontinuous right-hand sides. Earlier work of Fillipov, Hermes, Hájek, and others was extended. However, we recently learned of work of Aizerman and one of his colleagues at the Institute of Control Sciences, Moscow, in this same direction and concluded that while interesting it does not seem to provide the necessary mathematical framework for the problem of investigating stable feedback generators. Lacking any fresh ideas, LaSalle decided to assign Palmer another problem.

2. Learning, identification and adaptation.

The concept of eventual stability for nonautonomous ordinary differential equations was introduced by LaSalle around 1958 and studied by R. Rath in his Notre Dame Ph.D. dissertation, 1962. An application was made in 1963 (R. Rath and J. P. LaSalle, "Eventual Stability", Proceedings of Second IFAC Congress, Basel, 1963) to prove the convergence of a scheme for adaptive control. It is clear today that the concept of eventual stability is, and should be, related to the limiting equations of nonautonomous systems.

This fall LaSalle and Zvi Artstein, Weizmann Institute of Science, Israel, began a joint investigation of the relationship of questions of stability, adaptation, identification, and learning to the limiting equations of control systems. This research is discussed in more detail in our proposal for research for continuation of this grant.

3. Stability.

a. Functional differential equations.

Under the direction of LaSalle, Palmer has completed an extension of Liapunov's direct method for the study of the stability of nonautonomous functional (delay-differential) equations.

b. Stability of nonautonomous difference equations.

M. Latina, R. I. Jr. College, under the direction of LaSalle has almost completed an extension of the invariance principle and Liapunov's direct method for nonautonomous discrete systems (time-varying difference equations). LaSalle gave in outline form the development of this theory in his lectures at an NSF-CBMS Regional Conference on Applied Mathematics, Mississippi State, August 1975. Latina is filling in the details of the outline and has made improvements in the theory. This summer they plan to work at applications.

4. Stability and control of discrete processes.

LaSalle is working on a two volume work on the control and stability of discrete processes. A first draft of Volume I on linear systems was completed last summer. Since then LaSalle has worked on organizing Volume II, which will be directed towards the more general nonlinear theory of both stability and control. It seemed desirable to see the exposition as a whole before undertaking the rewriting of Vol. I. One of the reasons for this is that Vol. I will contain statements of results and illustrations of how they are applied with the underlying theory deferred to Volume II.

PUBLICATIONS

September 1, 1977 - March 1, 1978

[1] H.T. Banks and J.A. Burns, "Approximation techniques for control systems with delays", Proc. Intl. Conf. on Math. Programming, Zakopane, Poland, September 1977, to appear.

[2] H.T. Banks, "Approximation of nonlinear functional differential equation control systems", to appear in the Journal of Optimization Theory and Applications.

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[6] J.K. Hale, "Generic bifurcation with applications", Heriot-Watt University Symposium on Nonlinear Analysis and Mechanics, Pitman, November 1977.

[7] J.K. Hale, Theory of Functional Differential Equations, Springer-Verlag, Vol. 3, 1977.

[8] J.K. Hale, "Topics in local bifurcation theory", Annals, New York Academy of Sciences, Conference on Bifurcations, to appear.

[9] J.K. Hale and P. Taboas, "Bifurcation near degenerate families", SIAM J. Applied Mathematics, to appear.

[10] J.P. LaSalle, "New stability results for nonautonomous systems", Bednarek and Cesari: Dynamical Systems, pp. 175-184, Academic Press, Inc., 1977.

[11] J.P. LaSalle, "Stability theory for difference equations", MAA Studies in Math., Vol. 14, pp. 1-31, 1977 (ed. J.K. Hale).

[12] D. Reber, "Approximation and optimal control of linear hereditary systems" Ph.D. Dissertation, Brown University, June 1978.

[13] D. Reber, "A finite difference technique for solving optimization problems governed by linear functional differential equations", submitted to the Journal of Differential Equations.

APPROXIMATION TECHNIQUES FOR CONTROL SYSTEMS WITH DELAYS

H. T. Banks
and
J. A. Burns

Abstract

We present a theoretical framework for approximation techniques for nonlinear system optimal control problems. Two particular approximation schemes that may be used in the context of this framework are discussed and typical numerical results for two examples to which we have applied these schemes are given. We conclude with a brief survey of related investigations.

PHASE SPACE FOR RETARDED EQUATIONS WITH INFINITE DELAY

Jack K. Hale and Junji Kato

Abstract

It is the purpose of this paper to examine initial data from a general Banach space. We develop a theory of existence, uniqueness, continuous dependence, and continuation by requiring that the space B only satisfies some general qualitative properties. Also, we impose conditions of B which at least indicate the feasibility of a qualitative theory as presently available for retarded equations with finite delay, the space of continuous functions. In particular, this will imply that the essential spectrum of the solution operator for a linear autonomous equation should be inside the unit circle for $t > 0$.

A few remarks at the beginning will assist the reader in understanding why our axioms prevent the norm in the space from imposing any differentiability properties on the initial functions. In the applications, it is certainly convenient at times to require the initial functions to belong to a Banach space of functions which have some derivative satisfying specified properties. However, if one considers all differential equations whose right hand sides are continuous or continuously differentiable in such a space, then the equations will be of neutral type; that is, the derivatives of the independent variable will also contain delays. The theory for such systems certainly should be developed but it will require much more sophistication than the one described in the text.

Our axioms are imposed to ensure that only retarded equations will be considered. This does not mean that one cannot consider these retarded equations with the initial data restricted to certain Banach spaces which impose conditions on the derivatives. In fact, one can consider such spaces which can be continuously imbedded (completely continuously embedded) in a space satisfying the axioms in the text to obtain existence and uniqueness. The other properties could be investigated using the differential equation itself and the fact that everything is known in the larger space. These remarks are not meant to imply that the task is trivial but are merely suggestions as to how one could possibly proceed.

APPROXIMATION OF NONLINEAR FUNCTIONAL DIFFERENTIAL EQUATION CONTROL SYSTEMS

H. T. Banks

Abstract

We develop a general approximation framework for use in optimal control problems governed by nonlinear functional differential equations. Our approach entails only the use of linear semigroup approximation results while the nonlinearities are treated as perturbations of a linear system. Numerical results are presented for several simple nonlinear optimal control problem examples.

BIFURCATION NEAR FAMILIES OF SOLUTIONS

Jack K. Hale

Summary: Many investigations in bifurcation theory are concerned with the following problem. If $M(0,0) = 0$ and $\partial M(0,0)/\partial x$ has a nontrivial null space, find all solutions of the equation

$$M(x,\lambda) = 0 \qquad (1.1)$$

for (x,λ) in a neighborhood of $(0,0) \in X \times \Lambda$.

If $\dim \Lambda = 1$; that is, there is only one parameter involved then the existence of more than one solution in a neighborhood of zero can be proved by making assumptions only about $\partial M(0,0)/\partial x$ and $\partial M(0,0)/\partial x \partial \lambda$. However, if $\dim \Lambda \geq 2$, then the problem is much more difficult and more detailed information is needed about the function M. A careful examination of the existing literature for $\dim \Lambda \geq 2$ reveals that the additional conditions imposed on M imply, in particular, that the solution $x = 0$ of the equation

$$M(x,0) = 0 \qquad (1.2)$$

is isolated (see, for example, the papers on catastrophe theory). These hypotheses eliminate the possibility that Equation (1.2) has a family of solutions containing $x = 0$. Such a situation occurs, for example, for $M(x,\lambda) = Ax + N(x,\lambda)$, where A is linear with a nontrivial null space and $N(x,0) = 0$ for all x. There also are interesting applications where Equation (1.2) is nonlinear and there exists a family of solutions. For example, Equation (1.2) could be an autonomous ordinary differential equation with a nonconstant periodic orbit of period 2Π with the family of solutions being

Summary (continued)

obtained by a phase shift. When the differential equation in the latter situation is a Hamiltonian system, the parameters (λ_1, λ_2) could correspond to a small damping term and a small forcing term of period 2Π. To the author's knowledge, the first complete investigations of special problems of each of these latter types are contained in papers by Hale, Táboas and Rodrigues.

It is the purpose of this paper to begin the investigation of the abstract problem for Equation (1.1), especially to extend the results in the paper by Hale and Táboas.

INTERACTION OF DAMPING AND FORCING IN A SECOND ORDER EQUATION

by

Jack K. Hale and Plácido Táboas

Synopsis. Suppose λ,μ are real parameters, f is a scalar function which is 2π-periodic, $xg(x) > 0$ for $x \neq 0$ and consider the equation

$$\ddot{x} + g(x) = -\lambda\dot{x} + \mu f(t). \tag{1}$$

For $\lambda = \mu = 0$, every solution has the form $x(t) = \phi(\omega(a)t + \alpha, a)$ for some constants a,α and $\phi(\theta+2\pi,a) = \phi(\theta,a)$. If there is an a_0 such that $\omega(a_0) = 1$ (i.e., there is a 2π-periodic orbit Γ in $(x,\dot{x})$-space) and $\omega'(a_0) \neq 0$, the problem is to characterize the number of 2π-periodic solutions of Equation (1) which lie in a neighborhood of Γ for (λ,μ) in a small neighborhood of $(0,0)$. A complete solution of this problem is given under the hypothesis that the function $h(\alpha) = \int_0^{2\pi} [\partial\phi(t,a_0)/\partial t] f(t-\alpha)dt \Big/ \int_0^{2\pi} [\partial\phi(t,a_0)/\partial t]^2 dt$ has a nonzero derivative except at a finite number of points α_j and $h''(\alpha_j) \neq 0$. The bifurcation curves in (λ,μ)-space are determined by the α_j and are tangent to the straight lines $\lambda = h(\alpha_j)\mu$ at $(\lambda,\mu) = (0,0)$. In general, the 2π-periodic solutions of (1) are not continuous at $(\lambda,\mu) = (0,0)$. The nature of this discontinuity is discussed in detail. It is also shown that a necessary and sufficient condition for a 2π-periodic solution $x(\lambda,\mu)$ to be continuous at $(\lambda,\mu) = (0,0)$ is that $\lambda/\mu \to$ constant as $\lambda \to 0$, $\mu \to 0$.

GENERIC BIFURCATION WITH APPLICATIONS

J. K. Hale

Abstract: This paper is a set of lecture notes on generic bifurcation and its applications with the emphasis on equations involving more than one independent parameter. The general theory is discussed for problems which are degenerate to order one or two. Applications are given primarily to the buckling of plates and shells with the parameter representing external forces, loading, imperfections, curvature and dimension.

GENERIC BIFURCATION WITH APPLICATIONS

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TOPICS IN LOCAL BIFURCATION THEORY

by

Jack K. Hale

Abstract

Suppose Λ, X, Z are Banach spaces, $M: \Lambda \times X \to Z$ is a mapping continuous together with derivatives up through some order r. A bifurcation surface for the equation (1) $M(\lambda, x) = 0$ is a surface in parameter space Λ for which the number of solutions x of (1) changes as λ crosses this surface. Under certain generic hypotheses on M, the author and his colleagues have shown that one can systematically determine the bifurcation surfaces by elementary scaling techniques and the implicit function theorem. This talk gives a summary of these results for the case of bifurcation near an isolated solution or families of solutions of the equation $M(\lambda_0, x) = 0$. The results have applications to the buckling theory of plates and shells under the effect of external forces, imperfections, curvature and variations in shape. The results on bifurcation near families has applications in nonlinear oscillations and the theory of homoclinic orbits.

BIFURCATION NEAR DEGENERATE FAMILIES

by

Jack K. Hale and Placido Táboas

Abstract: Suppose X, Λ, Z are Banach space, $M: X \times \Lambda \to Z$ is a smooth function and the equation

$$M(x,\lambda) = 0, \qquad (1)$$

for $\lambda = 0$, has a one parameter family of smooth solutions $p(s)$, $0 \leq s \leq 1$, $p(0) = p(1)$ with $dp(s)/ds \neq 0$. If $\Gamma = \{p(s), 0 \leq s \leq 1$ the objective is to discuss the solutions of (1) near Γ for λ in a neighborhood of zero. Assuming $A(s) = \partial M(p(s),0)/\partial s$ is Fredholm of index zero and $\dim \mathcal{N}(A(s)) = 1$, for $0 \leq s \leq 1$, the authors have previously given a solution to this problem under natural hypotheses on M. In this paper, the case where $\dim \mathcal{N}(A(s)) = 2$ is considered. Applications are given to the effect of small damping and small forcing on the existence of periodic solutions near a periodic solution of an autonomous Hamiltonian system.

NEW STABILITY RESULTS FOR NONAUTONOMOUS SYSTEMS

by

J.P. LaSalle

Abstract

The new invariance properties that have been established for nonautonomous ordinary differential equations greatly extend the range and power of Liapunov's direct method for the study of the stability of time-varying systems. An essential feature of the method is the establishment of a relationship between Liapunov functions and the location of the positive limit sets of solutions. The principal contribution of this paper is a theorem connecting Liapunov functions and positive limit sets of sufficient generality to close a gap in the present theory.

STABILITY THEORY FOR DIFFERENCE EQUATIONS

J. P. LaSalle

Abstract

This article is designed to give through the study of difference equation (discrete dynamical systems) a view of and an introduction to the general theory of the stability of dynamical systems in its most modern aspect. Much of what is presented here is known, although not perhaps as well known as it should be, and there are some things that are new. One of these has to do with a connectedness property of the positive limit sets of the solutions of difference equations which provides a means through the use of Liapunov functions of establishing the existence of equilibrium points (fixed points) and oscillations (periodic points). Another is the generalization of the usual concept of a vector Liapunov function, and this leads to a possible method of designing control systems where the measure of the error or the performance criterion is a vector rather than a scalar. Applications of the theory are illustrated by simple examples.

The article was written for undergraduate teachers of mathematics but it should also serve as a good introduction for engineers and scientists to the latest results in the theory of the stability of dynamical systems.

Abstract of APPROXIMATION AND OPTIMAL CONTROL OF LINEAR HEREDITARY SYSTEMS

by Douglas Christian Reber, Ph.D., Brown University, June 1978.

Our concern in this investigation is with the approximation and optimal control of systems governed by linear retarded functional differential equations (FDE). In chapter I we establish the existence, uniqueness and continuous dependence of solutions of FDE. We further demonstrate that certain FDE are equivalent to corresponding abstract ordinary differential equations (ODE). Such ODE are also known as abstract evolution equations (AEE). Chapter II details the manner in which these AEE may be approximated by difference equations in finite-dimensional spaces.

The optimal control problem for systems governed by FDE is then reduced to a sequence of mathematical programming problems in chapter III. In chapter IV we discuss numerical results for two systems, having applied standard techniques of numerical analysis to compute the solutions of the approximating problems. The first of these systems was chosen for its simplicity, so that an analytical solution would be readily available; the second system is associated with a biochemical process.

A FINITE DIFFERENCE TECHNIQUE FOR SOLVING OPTIMIZATION PROBLEMS GOVERNED BY LINEAR FUNCTIONAL DIFFERENTIAL EQUATIONS

by

Douglas C. Reber

Abstract: Aspects of the approximation and optimal control of systems governed by linear retarded nonautonomous functional differential equations (FDE) are considered. First, certain FDE are shown to be equivalent to corresponding abstract ordinary differential equations (ODE). Next, it is demonstrated that these abstract ODE may be approximated by difference equations in finite dimensional spaces. The optimal control problem for systems governed by FDE is then reduced to a sequence of mathematical programming problems. Finally, numerical results for two examples are presented and discussed.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

During the referenced period of the grant 13 publications were prepared. The papers deal with numerical methods for optimal control of equations with delays, optimal control of diffusion-reaction systems, nonlinear oscillations quatitative theory for ordinary and partial differential equations, and control systems. For additional details refer to the work unit file for the referenced grant.

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