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Some Qualitative Considerations on the Numerical Determination of Minimum Mass Structures with Specified Natural Frequencies

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Some Qualitative Considerations on the Numerical Determination of Minimum Mass Structures with Specified Natural Frequencies[1,2]

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Abstract. The problem of the axial vibration of a cantilever beam is investigated analytically. The range of values of the frequency parameter having technical interest is determined.

Key Words. Structural optimization, cantilever beams, axial vibrations, fundamental frequency constraint.

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[2] The authors are indebted to Dr. V.B. Venkayya, Wright-Patterson AFB, Ohio, for suggesting the topic.

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Notation

E	Modulus of elasticity, lb ft^{-2}
L	Length of the beam, ft
m	Normalized mass per unit length, $m = ML/M_o$
M	Mass per unit length, lb ft^{-2} sec^2
M_o	Reference mass, lb ft^{-1} sec^2 (Sections 2-3)
M_o	Tip mass, lb ft^{-1} sec^2 (Sections 4-5)
M_*	Total mass of the beam, lb ft^{-1} sec^2
x	Normalized axial coordinate, $x = X/L$
X	Axial coordinate, ft
u	Normalized axial displacement, $u = Y(X)/Y(L)$
Y	Axial displacement, ft
β	Frequency parameter, $\beta = \omega L\sqrt{(\rho/E)}$
ρ	Density, lb ft^{-4} sec^2
ω	Natural frequency, sec^{-1}

Superscript

'	Derivative with respect to the normalized axial coordinate x (for example, $u' = du/dx$)

1. Introduction

In this memorandum, we consider the problem of the axial vibration of a cantilever beam. With reference to a constant-section beam, we determine the range of values of the frequency parameter β having technical interest. This range of values of the frequency parameter is important in the solution of a subsequent problem: the determination of the mass distribution that minimizes the total mass of a beam for a given fundamental frequency constraint.

2. Nonoptimal Beam without a Concentrated Mass

Let m denote the normalized mass per unit length, u the normalized axial displacement, and β the frequency parameter. Let x denote the axial coordinate, normalized so that x=0 at the base of the beam and x=1 at the tip of the beam. Let the prime denote total derivative with respect to the axial coordinate x. With this understanding, the fundamental equation to be solved is the following:

$$(mu')' + \beta^2 mu = 0 . \tag{1}$$

In this equation, the frequency parameter β is related to the natural frequency ω, the length L, the density ρ, and the modulus of elasticity E by the relation

$$\beta = \omega L\sqrt{(\rho/E)} . \tag{2}$$

In the absence of a concentrated mass attached at the tip of the beam, the boundary conditions for Eq. (1) are as follows:[5]

$$u(0) = 0, \quad m(1)u'(1) = 0 . \tag{3}$$

If the mass distribution

$$m = m(x) \tag{4}$$

is prescribed a priori, then (1) is a second-order differential

[5]Equations (3) must be completed by the normalization condition $u(1) = 1$.

equation, to be solved in conjuction with the boundary conditions (3).

<u>Constant Section.</u> Next, we consider the particular case of a constant-section structure, that is, a structure with a constant mass per unit length:

$$m = \text{const} . \tag{5}$$

For this particular case, the differential equation (1) and the boundary conditions (3) simplify as follows:

$$u'' + \beta^2 u = 0 , \tag{6}$$

$$u(0) = 0, \quad u'(1) = 0 . \tag{7}$$

The solution of (6) consistent with the initial condition (7-1) is the following:[6]

$$u = A \sin(\beta x) , \tag{8}$$

with the implication that

$$u' = A\beta \cos(\beta x) . \tag{9}$$

From (9) and the final condition (7-2), we conclude that

$$\cos \beta = 0 , \tag{10}$$

so that

$$\beta = (2n+1)\pi/2, \quad n = 0,1,2,\ldots \, . \tag{11}$$

[6]The constant A has the value $A = 1/\sin\beta$.

Therefore, for this problem, the smallest nontrivial value of the frequency parameter is

$$\beta = \pi/2 . \tag{12}$$

3. Optimal Beam without a Concentrated Mass

Now, suppose that a constant-section structure has been studied in accordance with Section 2. Suppose that the frequency parameter β which allows satisfaction of the boundary conditions (7) has been determined, namely, $\beta=\pi/2$. The total mass of the structure studied in Section 2 is given by[7]

$$M_*/M_o = \int_0^1 m dx , \qquad m = \text{const} . \tag{13}$$

Therefore, it is natural to pose the following question: for the same value of the frequency parameter $\beta=\pi/2$, is there a better beam, that is, one having a smaller total mass? In particular, is there a beam which yields the smallest total mass for the given value of β? This question leads to the following variational problem: Minimize the total mass

$$M_*/M_o = \int_0^1 m dx , \qquad m = m(x) , \tag{14}$$

with the understanding that the following constraints must be satisfied:[8]

$$(mu')' + \beta^2 mu = 0 , \tag{15}$$

$$u(0) = 0 , \quad m(1)u'(1) = 0 , \tag{16}$$

and with the further understanding that $\beta=\pi/2$. Owing to the fact that the problem (15)-(16) is homogenous, the obvious solution under the physical constraint

$$m(x) \geq 0 \tag{17}$$

is

$$m(x) = 0 , \tag{18}$$

with the implication that

$$M_*/M_o = 0 . \tag{19}$$

In order to avoid the occurrence of the above trivial solution, Ineq. (17) could be changed as follows:

$$m(x) \geq m_o . \tag{20}$$

Then, the solution would become

$$m = m_o . \tag{21}$$

To arrive at solutions other than constant mass solutions, it is necessary to postulate some different physical situation (e.g., a concentrated mass attached at the end of the beam). In turn, this results in a change in the boundary condition (16-2), and this change makes it unnecessary to employ inequality constraints of the form (17) or (20).

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[7] The symbol M_o denotes a reference mass.

[8] Equations (16) must be completed by the normalization condition $u(1) = 1$.

4. Nonoptimal Beam with a Concentrated Mass

In this section, we assume that a concentrated mass M_o is attached at the tip of the beam. Using the same terminology as in Section 2, we see that the governing differential equation (1) still holds:

$$(mu')' + \beta^2 mu = 0 . \tag{22}$$

On the other hand, the boundary conditions (3) are modified as follows:[9]

$$u(0) = 0, \quad m(1)u'(1) = \beta^2 . \tag{23}$$

Constant Section. Again, we consider the particular case of a constant-section structure. Under condition (5) and after observing that

$$M_*/M_o = m , \tag{24}$$

then problem (22)-(23) becomes

$$u'' + \beta^2 u = 0 , \tag{25}$$

$$u(0) = 0, \quad u'(1) = (M_o/M_*)\beta^2 . \tag{26}$$

The solution of (25) consistent with the initial condition (26-1) is the following:

[9] Equations (23) must be completed by the normalization condition $u(1) = 1$.

$$u = A \sin(\beta x), \tag{27}$$

with the implication that

$$u' = A\beta \cos(\beta x) . \tag{28}$$

From (28) and the final condition (26-2), we conclude that

$$A \cos \beta = (M_o/M_*)\beta . \tag{29}$$

Owing to the fact that

$$u(1) = A \sin \beta , \tag{30}$$

elimination of A from (29)-(30) leads to the following transcendental equation:

$$\beta \tan \beta = (M_*/M_o)u(1), \tag{31}$$

which, for $u(1)=1$, reduces to

$$\beta \tan \beta = M_*/M_o . \tag{32}$$

This equation supplies the frequency parameter β in terms of the mass ratio (ratio of beam mass M_* to tip mass M_o).

In order to understand the significance of (32), let us consider two limiting cases: (i) negligible mass ratio and (ii) infinite mass ratio. If $M_*/M_o = 0$, then the solution of (32) is

$$\beta = n\pi, \quad n = 0,1,2,\ldots . \tag{33}$$

On the other hand, if $M_*/M_o = \infty$, then the solution of (32) is

$$\beta = (2n+1)\pi/2 , \quad n = 0,1,2,\ldots , \tag{34}$$

which is identical with (11). Since the first natural frequency corresponds to $n = 0$, we conclude that, for mass ratios in the range

$$0 \leq M_*/M_o \leq \infty , \tag{35}$$

the smallest frequency parameter β consistent with the trascendental equation (32) lies in the range

$$0 \leq \beta \leq \pi/2 . \tag{36}$$

5. Optimal Beam with a Concentrated Mass

As in Section 3, we can formulate the problem of finding the optimal mass distribution. The problem is as follows: Minimize the total mass

$$M_*/M_o = \int_0^1 m dx, \qquad m = m(x), \tag{37}$$

with the understanding that the following constraints must be satisfied:[10]

$$(mu')' + \beta^2 mu = 0, \tag{38}$$

$$u(0) = 0, \quad m(1)u'(1) = \beta^2, \tag{39}$$

and with the further understanding that the frequency parameter β has some fixed value in the range

$$0 \leq \beta \leq \pi/2. \tag{40}$$

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[10] Equations (39) must be completed by the normalization condition $u(1) = 1$.

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