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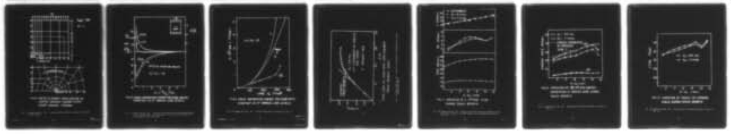

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# Final Scientific Report
# of
# Research on
# "Finite Element Elastic-Plastic Analysis of Cracks"

Supported by
AFOSR Grant 74-2667
February 1, 1974 - February 1, 1978
(Dr. William J. Walker, Program Manager)

Submitted by

Satya N. Atluri
School of Engineering Science and Mechanics
Georgia Institute of Technology
Atlanta, Georgia 30332

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| 2-D HYBRID CRACK ELEMENT | 3-D HYBRID CRACK ELEMENT |
| CURVED CRACKS | DIRECT SOLUTION FOR $K_I, K_{II}, K_{III}$ |
| ANISOTROPIC MATERIALS | SEMI-ELLIPTICAL SURFACE FLAWS IN PLATES |
| FREE-SURFACE EFFECTS | TENSION AND BENDING |
| CONVERGENCE OF HYBRID ELEMENT CRACK SOLUTIONS | |

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This is a summary of research performed in the areas: (a) development of two-dimensional hybrid finite element procedures to calculate stress intensity factors corresponding to general $r^{-\alpha}$ stress singularities in isotropic as well as anistropic materials; (b) study of the convergence of the assumed displacement hybrid finite element procedure in fracture mechanics problems; (c) development of a three-dimensional hybrid finite element procedure to calculate the elastic combined mode stress-intensity factors $K_I^*$, $K_{II}^*$ and $K_{III}^*$ that vary along an arbitrarily curved three-dimensional crack front; (d) hybrid finite element solutions of fundamental three-dimensional crack problems; (e) development of a

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two-dimensional finite element procedure for analyzing plane problems of fracture involving large-scale yielding conditions and under cyclic loading; (f) study of the J-integral as a ductile fracture initiation condition; (g) studies of finite deformation effects near the crack-tip; (h) analysis of stable crack growth under rising load and study of criteria for loss of stability of growth in ductile materials; and (i) elastic-plastic analysis of effects of crack closure on fatigue crack growth rates.

Key Words (cont)

SEMI-ELLIPTICAL SURFACE FLAWED CYLINDERS
CORNER CRACKED FASTENER HOLES
CRACKS IN ADHESIVELY BONDED METALLIC LAMINATES
DUCTILE FRACTURE
LARGE-SCALE PLASTIC YIELDING
PLASTIC HYBRID CRACK ELEMENT
FRACTURE INITIATION
J-INTEGRAL
FINITE DEFORMATION EFFECTS
CRACK-TIP BLUNTING
COMPACT-TENSION SPECIMEN
3-PT BEND SPECIMEN
CENTER-CRACK SPECIMEN
PLANE STRESS VS. PLANE STRAIN
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ELASTIC-PLASTIC ENERGY RELEASE RATE TO CRACK-TIP
PROCESS ZONE ENERGY RELEASE RATE
TRANSLATION OF SINGULARITIES
STABILITY CRITERIA
CRACK CLOSURE
FATIGUE CRACK GROWTH

# Final Scientific Report
of
Research on
# "Finite Element Elastic-Plastic Analysis of Cracks"

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Atlanta, Georgia 30332

March 1978

## 1. INTRODUCTION

Reported herein is a summary of research on elastic-plastic analysis of cracks, during the period February 1, 1974 to February 1, 1978, under the support of AFOSR Grant 74-2667.

The research described has been performed at the School of Engineering Science and Mechanics, Georgia Institute of Technology, under the direction of Satya N. Atluri. Three doctoral theses were written, by Drs. Nakagaki, Kathiresan, and Chen, based on research performed under this grant; these are referenced at the end of this report. Dr. Nakagaki later participated in this research as a Post-Doctoral Fellow and made significant contributions.

Dr. W. J. Walker of AFOSR provided invaluable encouragement and has aided immensely in defining the practical objectives of the basic research.

## 2. AREAS OF RESEARCH

The studies under the present grant included: (a) development of two-dimensional hybrid finite element procedures to calculate stress intensity factors corresponding to general $r^{-\alpha}$ stress singularities in isotropic as well as anistropic materials; (b) study of the convergence of the assumed displacement hybrid finite element procedure in fracture mechanics problems; (c) development of a three-dimensional hybrid finite element procedure to calculate the elastic combined mode stress-intensity factors $K_I$, $K_{II}$, and $K_{III}$ that vary along an arbitrarily curved three-dimensional crack front; (d) hybrid finite element solutions of fundamental three-dimensional crack problems; (e) development of a two-dimensional finite element procedure for analyzing plane problems of fracture involving large-scale yielding conditions and under cyclic loading; (f) study of the J-integral as a ductile fracture initiation condition; (g) studies of finite deformation effects near the crack-tip; (h) analysis of stable crack growth under rising load and study of criteria for loss of stability of growth in ductile materials; and (i) elastic-plastic analysis of effects of crack closure on fatigue crack growth rates.

A brief report on the salient results obtained in each of the above studies; a listing of publications that have resulted from this research; papers presented at professional conferences; and seminars given at other organizations, are given in the following.

## 3. SUMMARY OF RESEARCH ACCOMPLISHMENTS

### A. Two-Dimensional Linear Elastic Fracture Analysis.

A basic procedure, based on an assumed displacement hybrid finite element method, to calculate the stress intensity factors for mixed mode behaviour of arbitrary shaped curved cracks in plane stress and plane strain problems involving anisotropic, nonhomogeneous, but linearly elastic materials has been developed [Refs. 1, 4]. The attendant finite element program has been used to study: (i) unsymmetric boundary conditions and configurations such as oblique edge cracks in tension plates; (ii) general plane fracture problems with stress, displacement, or mixed conditions; (iii) curved cracks through the thickness of tension specimens; (iv) problems involving material inhomogeneity such as cracks intersecting bi-material interfaces in tension plates, wherein stress singularities of the general $r^{-\alpha}$ type may arise; (v) cracks meeting a free surface such as single-edge-notched tension plates with cracks of varying aspect ratio to study the effects of free surfaces on crack-tip stress intensity factors; (vi) cracks in orthotropic sheets, with the crack line located either symmetrically or unsymmetrically with respect to directions of principal axes of orthotropy, and for various values of Ex/Ey to obtain stress intensity factors for laboratory test specimens. These results are documented in Refs. 5-8.

### B. Study of Convergence of the Hybrid Element Procedure for Crack Problems.

A convergence study was conducted for the assumed displacement hybrid finite element method for solving stress intensity factors in linear fracture mechanics. Also methods to improve the satisfaction of stress-free conditions on a boundary segment of an element adjoining the crack surface were developed. It was found that a proper way to obtain convergent solutions for K-factors is to keep the size of the hybrid crack elements near the crack-tip at constant, pre-determined optimum size, while the size of the surrounding regular elements is progressively decreased. These results are documented in Refs. 1, 5, 6 and 7.

### C. Three-Dimensional Hybrid Crack Element for Linear Elastic Fracture Analysis.

A finite element procedure was developed for the calculation of modes I, II and III stress intensity factors, which vary along an arbitrarily curved three-dimensional crack front in a structural component. The present finite element model is based on a modified variational principle of potential energy with relaxed continuity requirements for displacements at the interelement boundary. The variational principle is a three-field principle, with the arbitrary interior displacements for the elements, interelement boundary displacements, and element boundary tractions as variables. The unknowns in the final algebraic system of equations, in the present model, are the nodal displacements and the three elastic stress intensity factors. Special elements, which contain proper square root and inverse square root variations in displacements and stresses, respectively, are used in a fixed region near the crack front. Interelement displacement compatibility is satisfied, by assuming an independent interelement boundary displacement field, and using a Lagrangian multiplier technique to enforce such interelement compatibility. These Lagrangian multipliers, which are physically the boundary tractions, are assumed from an equilibrated stress field derived from three-dimensional Beltrami (or Maxwell-Morera) stress functions that are complete. However, considerable care had to be exercised in the use of these stress functions such that the stresses produced by any of these function components are not linearly dependent.

Since the method is based on a rigorous variational principle, which enforces at least on an average the conditions of interelement displacement continuity when $\sqrt{r}$ type displacements are included in the near-tip region, the convergence of the finite element solution for nodal displacements as well as the stress intensity factors is established mathematically.

The geometry of the "basic element" used presently is a 20-node "isoparametric" brick element, with 60 degrees of freedom per element. Two options were developed: (a) a singular element for use near the crack front, wherein the stress intensity factors $K_I$, $K_{II}$ and $K_{III}$ are constant, and (b) a singular element where in the intensity factors $K_I$, $K_{II}$ and $K_{III}$ themselves vary quadriatically. It is believed that this option will reduce the number of elements to be used near the crack front, in order to obtain an accurate variation of the intensity factors along the crack front.

The relevant matrices were evaluated numerically, using non-product type quadrature formulae, with proper mathematical transformations being used when singular type functions are encountered in stresses and strains in the near-tip region.

The mathematical development of the 3-D hybrid crack element and related computational details are documented in Refs. 2, 10 and 13.

## D. Hybrid Finite Element Solutions of Fundamental Three-Dimensional Crack Problems.

The basic development for 3-D hybrid crack element discussed in (C) above has been exhaustively tested in analyzing various basic problems of 3-dimensional fracture that are of current interest. These include: (i) semi-elliptical surface flaws of various aspect ratios and various crack-depth ratios in plates under tension as well as bending; (ii) semi-elliptical surface flaws of various aspect ratios and various depth ratios on the inner and outer surface of pressurized thick shells; (iii) semi-elliptical surface and quarter-elliptical corner cracks near fastener holes in plates, typical of aircraft structural configurations. Moreover, the method was completely checked as to its accuracy and convergence in several other problems such as Sneddon's embedded penny shaped crack, buried elliptical cracks, and several other problems, for which analytical solutions exist. Also, the question of the optimum size of a singular element near the crack front has been studied thoroughly. All these results, along with the detailed mathematical derivations, have been presented in a recent Ph. D. thesis by Kathiresan [2], and are submitted to AFOSR for approval for publication as an AFOSR Technical Report [21]. The results for various problems have also been published in open literature [13, 17, 19, 24, 26, 28, 31, 32].

More recently, the 3-D hybrid crack element procedure has been applied to study the crack growth behaviour in adhesively bonded structure, wherein, each layer was treated in a three-dimensional fashion. Specifically results were obtained for a crack through the thickness of only the outer layer, near a hole through the thickness of the entire stack of layers. These are believed to be the only fully three-dimensional analysis results for the problem under consideration; and are presented in Ref. [32] along with a discussion as to the validity of the obtained results.

## E. Two-Dimensional Hybrid Element Procedure for Elastic-Plastic Analysis of Large Scale Yielding Fracture:

To study the problem of ductile fracture under large-scale yielding conditions, research was conducted in formulating and developing a second generation finite element procedure with the objectives: (i) developing circular-sector shaped embedded-singularity finite elements near the crack-tip. The correct r dependence of the diminant singular solution, corresponding to the nonlinear material model (Ramberg-Osgood law), was embedded in these near tip elements, where as the $\theta$-dependence was approximated in each sector element and solved for in the sense of the finite element method; (ii) maintaining continuity of displacements and tractions, between near-tip elements with singular stress/strain assumptions and the far field elements with regular stress/strain assumptions, through a hybrid displacement finite element model, (iii) using a $J_2$-flow theory of plasticity and arbitrary kinematic hardening which will accurately model the Bauschinger effect under fully reversed and cyclic loading, (iv) using an incremental finite element solution procedure that will be suitable in the limiting case of elastic-perfect-plastic materials, as well as cyclic loading situations. The "initial-stress" iteration approach was used for this purpose, (v) developing a more accurate finite element method for incremental analysis of elastic-plastic problems. The more common approach in leterature is to use "constant-stress" elements, and based on stress level in each element, the whole element either yields or stays elastic. Thus in problems such as the present, where the yield zones near the crack-tip play a dominant role in the analysis and its interpretation, in order to obtain a reasonably accurate description of the yield zone, a very fine finite element mesh is needed. However, if higher order elements are used, and if "plasticity-correction" iterations are performed at several points within the element, it then becomes possible to give a smoother definition of the yield zone. Thus, a portion of the element, in the present formulation, can yield while the rest of the element can remain elastic.

To start with, crack-tip blunting and other finite geometry changes near the crack-tip were ignored, and thus the analysis was restricted to a small-deformation assumption. This procedure was used to solve a hypothetical 3-point bend test specimen which was suggested, as a standard test for analytical solutions, by the ASTM Committee E-24 Round Robin Group on Elasto-Plastic Fracture Criteria. A detailed analysis of these results, with various fracture criteria in mind, was presented in (Atluri and Nakagaki [12], and Atluri and Nakagaki [14]. However, no other independent solution, from members of the Round Robin, with which to compare our results is available at the time of this writing.

## F. Study of the J-integral as a Fracture Initiation Criterion Under Large-Scale Yielding.

The above small-deformation analysis procedure was used to analyze a compact tension cracked specimen, of A533B steel, for which experimental fracture data was reported by the Westinghouse Research Laboratories. The thickness of the experimental specimen was roughly 1/2-1/3 of the characteristic inplane dimensions. Thus it was not a priori certain whether the conditions near the crack-tip can be characterized mathematically as those of plane stress or of plane strain. Thus the problem was analyzed using two types of two-dimensional approximations. Also, the experimental uniaxial stress/strain curve supplied to us by the Westinghouse Laboratories was seen to possess a yield point instability, which the numerical procedure could not account for. Thus, two different, smooth, Ramberg Osgood type curves were used to fit the actual experimental stress-strain curve. From these analyses, it was found that: (i) the mathematical characterization of the material-property data had no significant effect on the results, (ii) the empirical formula of Rice et al. for estimating J was in significant error in the case of the compact tension specimen, and (iii) the computed <u>plane-stress</u> values for J were much closer to the experimental data, as compared to the <u>plane-strain</u> values which differed by a factor of about 3. Thus it was concluded that the <u>plane-stress</u> conditions more accurately characterize the plastic flow near the crack tip in "small" specimens. These results are documented in detail in (Atluri and Nakagaki [15].

However, even the computed plane-stress values for J, based on the present analysis with a "non-growing stationary crack" assumption, differed by about 15-20% from the cited experimental results for J at fracture. From subsequent private communications with Begley and Landes, who conducted the experiments, it was learned that, in fact, there may have been stable crack growth, which was not monitored, in the experiments.

Taking into account the effect of this stable crack growth, and the results of a refined and consistent finite deformation analysis of the problem, as discussed in (G) below, it was concluded that J is a computationally advantageous criterion for the onset of crack growth in ductile materials.

### G. Studies of Finite Deformation Effects near the Crack-tip.

To delineate the effects of crack-tip blunting, and the attendant finite geometry changes near the crack-tip, under large scale plastic yielding conditions, a finite deformation, embedded singularity, elastic plastic incremental finite element procedure was developed [16]. The test problem was that of a 3-point bend fracture test specimen, the experimental data for which was reported and made available to us by Westinghouse. A detailed comparison of the results, for stresses, strains, displacements, and J, from this finite deformation analysis, and those obtained using the above small-deformation analysis was made. Significant conclusions of the above comparison were: (i) not with-standing the crack-tip blunting, J still remains as a valid parameter to characterize the severity of the near-tip condition, (ii) since the well-known Rice's definition of J is invalid for finite-deformation, a modified definition must be derived. This new definition involves the strain energy density function that is dependent on Piola-Lagrange (unsymmetric) stress tensor, and the traction term appearing in the J-integral should be interpreted as Piola Lagrange tractions in the deformed geometry as referred to the undeformed configuration. When this modified definition for J is employed, a more accurate path-dependence (within ± 1.5%) was noted for J, as compared to the small-deformation case wherein the J value on paths closest to the crack-tip was about 15% lower than that on paths in the far field. A detailed discussion of these results was presented in (Atluri, Nakagaki, and Chen [16].

One interesting observation was that at all load-point displacement levels, including that at fracture, there was an excellent correlation (within ± 4%) between the presently computed results for J and those in the Westinghouse experiments. This suggested that no appreciable stable crack growth may have been present in the experiment. Subsequent private communications with Dr. Landes at Westinghouse appeared to confirm this. Thus because of the particular specimen geometry and loading conditions, 3-point bend specimen may offer the advantage of precisely measuring $J_{IC}$ for crack growth initiation for a given ductile material.

Further refinements, such as the incorporation of a "knee-correction", to achieve better convergence of plastically adjusted stresses, and a more accurate incremental elastic-plastic law, were made in the above finite deformation analysis. The details of the nature and form of singularity of actually computed strains and stresses, and their variation in the angular coordinate near the crack tip were critically reviewed and compared with the small-deformation analysis, using $J_2$ deformation plasticity theory, of Hutchinson, Rice and Rosengren. These results are discussed in detail in (Atluri and Nakagaki [20]). Because of the finite root radius of the blunted notch, there can only be a strain concentration, however large, at the tip of the notch. The effect of this strain concentration, as opposed to the strain-singularity at the blunted crack tip, was properly accounted for in the analysis of [20].

The refined finite deformation analysis procedure was also ued to analyze three fracture test specimens, of 3-point bend, center-cracked, and compact tension types, of configurations identical to those in the experiments of Westinghouse Laboratories. The obtained numerical data was analyzed with both the J and COD criteria in mind, and the following conclusions were drawn: (i) the computed J correlated well with the cited experimental results for all the specimens; (ii) J was found to be directly correlated with COD for these test specimens. The above data indicated that, for these specimens, $J = 1.44\sigma_y$ (COD) for moderately hardening materials, where

$\sigma_y$ is the yield stress. Thus it can be seen that COD may also be used as a fracture criterion; (iii) however, it was found that the computation of COD, as essentially a near-tip geometric quantity, for arbitrary plane problems of ductile fracture, is not well defined. Attempts to correlate COD with such near-tip geometrical quantities such as the crack-opening at points where the elastic-plastic boundary intersects the crack profile were found to be discouraging; (iv) the empirical formula of Rice et al. for J was once again found to be inaccurate for the compact tension specimens. These results are elaborated upon in (Atluri, Nakagaki, and Chen [18]).

Once again, the computed J for the compact tension specimen, based on the finite element modeling of a non-growing crack, were found to be about 10-15% less than in the experiments. Again this suggested that there may, in fact, have been a subcritical crack growth prior to fracturing in the experiments, thus providing motivation for our study (H) on stable crack growth, described below.

## H. Analysis of Stable Crack-Growth and Study of Criteria Governing Loss of Stability.

A finite element methodology was developed to study the phenomena of stable crack growth in two-dimensional problems involving ductile materials. Crack growth is simulated by (i) the translation in steps, of a core of sector elements, with embedded singularities of Hutchinson-Rice-Rosengran type by an arbitrary amount, $\Delta a$ in each step, in the desired direction; (ii) reinterpolation of the requisite data in the new finite element mesh; and (iii) incremental relaxation of tractions in order to create a new crack face of length $\Delta a$. Steps (i) and (ii) were followed by corrective equilibrium-check iterations. A finite deformation analysis, based on the incremental updated Lagrangean formulation of the hybrid-displacement finite element method, is used.

The present procedure is used to simulate available experimental data on stable crack growth, and thus study the variation, during crack growth, of certain physical parameters that may govern the stability of such growth and the subsequent onset of rapid fracture. Attention is focussed in this study on the parameters: $G^{*\Delta}$ the energy release to the crack-tip per unit crack growth, for growth in finite steps $\Delta a$, calculated from global energy balance considerations; $G^{\Delta}_{pz}$ the energy release to a finite "process zone" near the crack-tip per unit crack growth, for growth in finite steps $\Delta a$, calculated again from global energy balance considerations; and the crack opening angles.

However, only the first phase of our study, viz., the simulation of available experimental data, has been completed.

It is recognized that formulating any criterion or criteria, governing the loss of stability of crack-growth, based on numerical simulation of a few experimental data is, at best, a risky proposition. Thus, we defer any conclusions regarding such criteria until the completion of second phase of our research, to be conducted. However, the results obtained so far, can lead to the following conclusions that may be germane to the problem of stable crack growth in ductile materials: 1) A direct numerical proof is provided for the original hypothesis of Rice that $G^{*\Delta} \rightarrow o$ as $\Delta a \rightarrow o$ for those materials for which the flow stress saturates at a finite value of large strain. Thus, for any meaningful numerical study of stable crack growth, a finite growth step must be postulated. Results obtained during this research and reported in [27,30] may be useful in providing guidelines for choosing $\Delta a$ such that the numerically computed $G^{*\Delta}$ is not sensitive to the errors inherent in numerical processes such as in the finite element method. 2) Since the magnitudes of $G^{*\Delta}$ and $G_{\Gamma}^{*\Delta}$ are clearly shown [27, 30] to depend on the postulated magnitudes of "finite" growth steps $\Delta a$, in the finite element modeling, it is clear that any criteria governing the loss of stability of crack growth cannot be based on the absolute magnitudes of these quantities. Any such criteria can only be based on the relative qualitative behaviour of $G^{*\Delta}$ and $G_{\Gamma}^{*\Delta}$ for the postulated growth step $\Delta a$. 3) In view of the previous observation and the fact $G^{*\Delta}$ and $G_{\Gamma}^{*\Delta}$ vary substantially during the crack extension process, it is clear that the generalizations of Griffith's approach, in the sense that a ductile material has some characteristic work of separation, per unit new crack area, and that this is to be equated at the critical condition to the rate of surplus of work done on the material, cannot be made in situations of stable growth under large-scale yield conditions. 4) The only discernible trends near the points of fracture as observed in experiment, for both the cases studied, is the marked change in the behaviour of $G^{*\Delta}$, CTOA, and to an extent in $G_{\Gamma}^{*\Delta}$, near these points, when these quantities reverse their nomotonically increasing trend during the prior extension process. While a theoretical argument explaining this is lacking, it yet remains to be seen whether these observations can be used to numerically predict loss of stability of growth in different specimens of the same material. This is the object of our work yet to be conducted.

A detailed discussion of the present analysis of stable crack growth and results leading to the above conclusions, have been presented in (Atluri and Nakagaki [27], and Nakagaki, Chen, and Atluri [30]).

## I. Analysis of Effects of Crack-Closure on Fatigue Crack Growth Rates.

The crack growth simulation procedure described in (G) above was also used to study the crack-closure and opening stresses under Mode I type constant amplitude fatigue loading. The values of closure and opening stresses for a center cracked Aluminum panel subject to constant amphitude cyclic loading, which causes a "small-scale" yielding near the crack-tip, were found to correlate well with existing results in literature for identical problems. However, the number of finite element degrees of freedom used in the present study was found to be an order of magnitude lower than in current literature for comparable accuracy. These results are yet to be published [30].

Considerations of general spectrum loading to understand the crack growth acceleration and retardation effects still remains and is the object of our research yet to be conducted.

## 4. CONCLUDING REMARKS AND ACKNOWLEDGEMENTS

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## 5. PUBLICATIONS

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25. S. N. Atluri, "Hybrid Crack Elements," Proceedings of Joint AFOSR/AFRPL Conference on Propellants, Edwards Air Force Base, California, March 1974 (Extended abstract only).

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27. S. N. Atluri and M. Nakagaki, "An Analysis of Stable Crack Growth in Ductile Materials", in Materials and Processes - In Service Performance, National SAMPE Technical Conference Series, Vol. 9, October 1977, pp. 48-57.

28. K. Kathiresan, and S. N. Atluri, "Surface Flaws in Thick Plates," in Proceedings of 14th Annual Society of Engineering Science Meeting, Lehigh University, Bethlehem, PA, November 1977, pp. 351-358.

29. S. N. Atluri, M. Nakagaki, K. Kathiresan, H. C. Rhee, and W. H. Chen, "On Hybrid Finite Element Models for Linear and Nonlinear Fracture Analyses", in Numerical Methods in Fracture Mechanics (Editors: Luxmore and Owen) Proceedings of International Conference on Numerical Methods in Fracture Mechanics, Swansea, UK, January 9-13, 1978) pp. 52-67, 1978.

30. M. Nakagaki, W. H. Chen, and S. N. Atluri, "A Finite Element Analysis of Stable Crack Growth: Formulation of Criteria", to appear in an ASTM STP, Proceedings of ASTM National Symposium on Elastic-Plastic Fracture, Atlanta, GA, November 1977.

31. S. N. Atluri and K. Kathiresan, "Surface Flaws in Pressure Vessels", Invited paper, Nuclear Engineering and Design, 1978. (to appear).

32. S. N. Atluri and K. Kathiresan, "Stress Analysis of Typical Flaws in Aerospace Structural Components Using a Three-Dimensional Hybrid Element Procedure", in Proceedings of 19th AIAA/ASME/SAE Structure, Structural Dynamics and Materials Conference, Bethesda, MD, April 3-8, 1978.

33. S. N. Atluri, "A Review of Numerical Methods for Engineering Fracture Analysis", to be presented and published at 2nd International Conference on Fracture Mechanics, Bangalore, India, March 1979.

## 5.3 PAPERS PRESENTED AT PROFESSIONAL MEETINGS

34. AIAA/ASME/SAE 15th Structures, Structural Dynamics and Materials Conference, Las Vegas, Navada, April 1974, paper title same as cited in Ref. 1.

35. 7th U.S. National Congress of Applied Mechanics, Boulder, Colorado, June 1974, paper title same as cited in Ref. 2.

36. 1974 International Conference on Finite Element Methods in Engineering, University of New South Wales, Sydney, Australia, August 29-31, 1974, paper title same as cited in Ref. 6.

37. NATO Advanced Study Institute on Continuum Mechanics Aspects of Rock Fracture and Geodynamics, Reykjavik, Iceland, August 1974, presented an invited paper, "Finite Element Analysis of Cracks Between Dissimilar Media".

38. ASTM Symposium on Fracture of High Modulus Fibers and Their Composites, National Bureau of Standards, Gaithersburg, MD, paper title same as cited in Ref. 4.

39. Conference on Fundamental Aspects of Deformation and Fracture of Composite Materials, Battelle Seattle Research Center, Seattle, Washington, February 1975, presented an invited paper, "Stress Intensity Factors of Cracked Orthotropic Plates", (Presented by A. S. Kobayashi).

40. ASME Joint Western and National Applied Mechanics Conference, University of Hawaii, Honolulu, March 1975, paper title same as cited in Ref. 8. (Presented by A. S. Kobayashi).

41. AICA International Symposium on Computer Methods for Partial Differential Equations, Lehigh University, Bethlehem, PA, June 1975, paper the same title as cited in Ref. 5.

42. AFRPL/Edwards AFB Contractors Meeting, California Institute of Technology, Pasadena, California, May 1975, invited presentation on "3-D Cracked Elements".

43. ASTM Committee E-24 Meeting, 9th U.S. National Symposium on Fracture Mechanics, Pittsburgh, PA, August 1975, invited presentation, "Analysis of Large-Scale Yielding Fracture Problems".

44. Third International Conference on Structural Mechanics in Reactor Technology, University of London, September 1975, paper title same as cited in Ref. 7.

45.-46. 10th Annual Meeting of Society of Engineering Science, University of Texas, Austin, Texas, October 1975, paper titles same as cited in Ref. 9 and 10.

47. "Workshop on 3-D Fracture Analysis", Battelle Columbus Lab., Columbus, Ohio, April 1976, invited presentation on "3-D Hybrid Cracked Elements".

48. 8th South Eastern Conference on Theoretical and Applied Mechanics, VPI and SU, Blacksburg, VA, April 1976, paper title same as cited in Ref. 11.

49. 17th AIAA/ASME SDM Conference, Valley Forge, PA, May 1976, paper title same as cited in Ref. 12.

50. 10th U.S. National Symposium on Fracture, Philadelphia, August 1976, title same as cited in Ref. 13.

51. 13th Annual Meeting of Society of Engineering Science, NASA-Langley, Hampton, VA, November 1976. (Ref. 16).

52. 18th AIAA/ASME/SAE SDM Conference, San Diego, California, March 1977, title same as cited in Ref. 22, (presented by J. S. Cheng).

53. "Crack Elements for Bi-Material Fracture," JANNAF Propulsion Conference, Edwards Air Force Base, CA, March 1977.

54. "Inner Surface Flaws in a Thick Walled Pressure Vessel," 3rd International Conference on Pressure Vessel Technology, ASME/JSME, Tokyo, Japan, April 1977.

55. "Fracture Initiation in Plane Ductile Fracture Problems", 3rd International Conference on Pressure Vessel Technology, ASME/JSME, Tokyo, Japan, April 1977.

56. "Fracture Analysis Under Mixed Mode Conditions", 2nd ASCE Engineering Mechanics Specialty Conference, Raleigh, NC, May 1977.

57. "Stress Analysis of Cracks in Elasto-Plastic Range", 4th International Conference on Fracture, University of Waterloo, Canada, June 1977.

58. "Outer Surface Flaws in Pressure Vessels," 4th International Conference on Structural Mechanics in Reactor Technology, San Francisco, CA, August 1977.

59. "Analysis of Stable Crack Growth in Ductile Materials", 9th SAMPE National Conference, Atlanta, GA October 1977.

60. "Criteria for Stable Crack Growth", ASTM National Symposium on Elastic-Plastic Fracture, Atlanta, GA November 1977.

61. "Surface Flaws in Plates", 14th SES Conference, Lehigh University, Bethlehem, PA November 1977.

62. "Stress Analysis of Typical Flaws in Aerospace Structural Components", 19th AIAA/ASME, Structures, Structural Dynamics Conference, Bethsda, MD April 1978.

63. "Finite Element Analysis of Elastic-Plastic 3-D Cracks", Joint AFOSR/AFRPL Rocket Propulsion Research Meeting, Edwards Air Force Bse, April 1978.

## 5.4 SEMINARS AT UNIVERSITIES

1. "On Elastic-Plastic Fracture Mechanics", at Department of Aeronautical Engineering, Indian Institute of Science, Bangalore, India, August 21, 1974.

2. "Computational Methods in Fracture Mechanics", National Aeronautical Lab, Bangalore, India, August 22, 1974.

3. "Computational Fracture Mechanics", College of Engineering, Boston University, Boston, Mass., January 1976.

4. "Hybrid Finite Elements for Fracture Analysis", four lectures at Short Course on Advances in Finite Element Methods, University of Tennessee Space Institute, Tullahoma, TN March 1978.

5. "3-D Crack Analysis by Finite Elements", University College, Cork, Ireland, January 14, 1978.

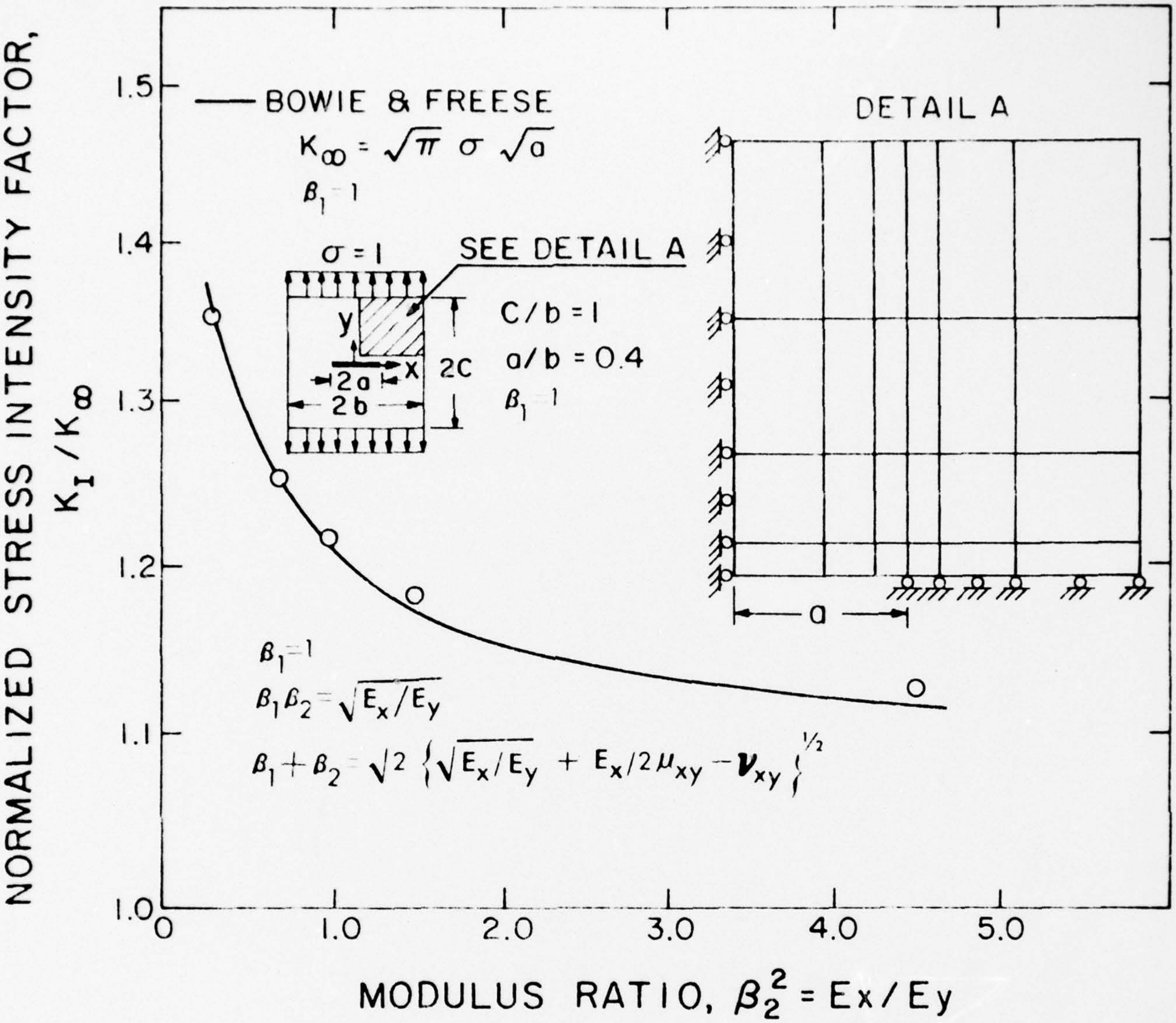


FIG. 1 (Taken From Ref. 7): Typical Result for a center cracked orthotropic sheet; crack symmetric w.r.t axes of orthotropy.

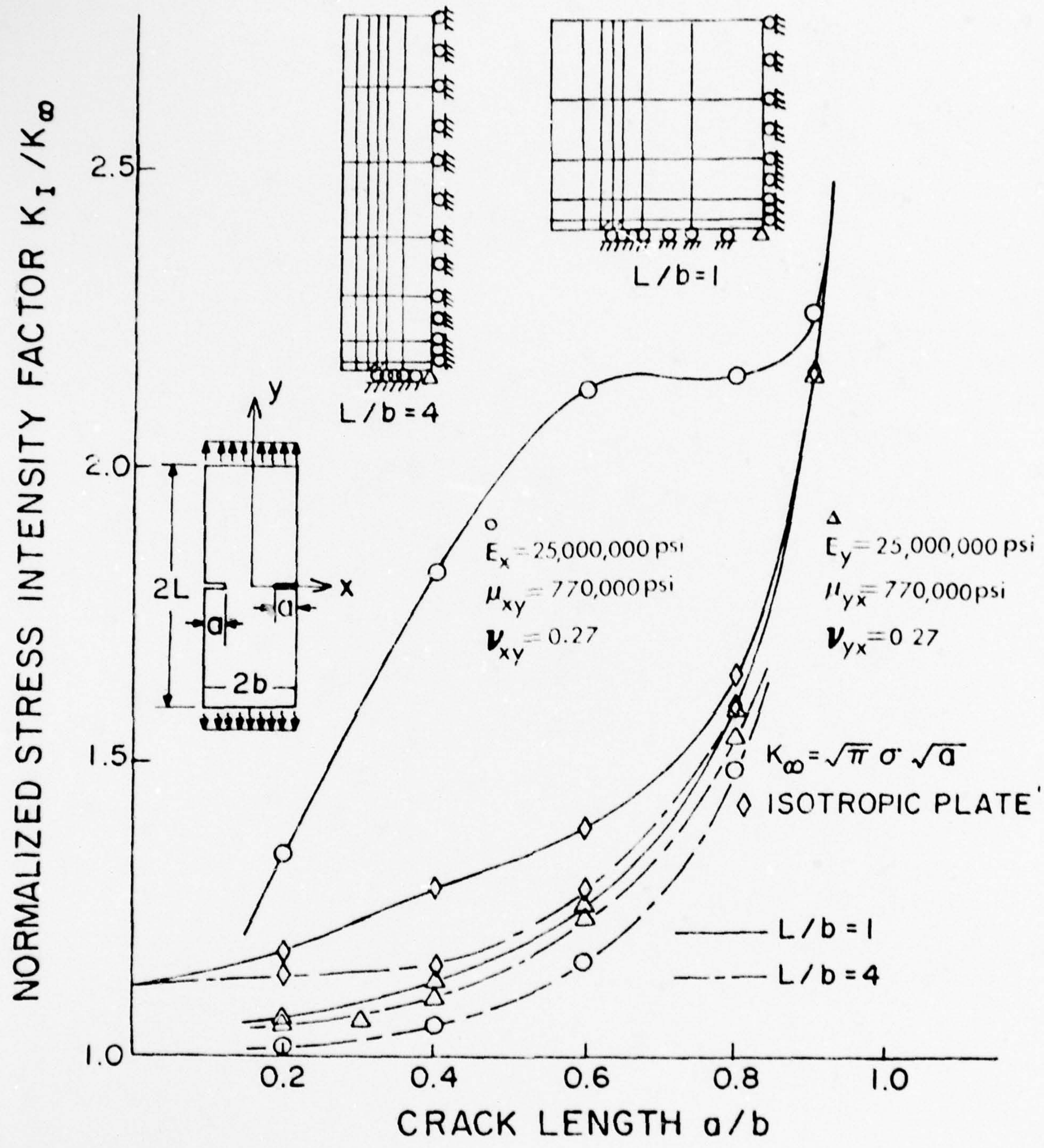


FIG. 2 (Taken From Ref. 7): Typical Result for an edge-cracked orthotropic sheet; crack symmetric w.r.t axes of orthotropy; Results for various Laboratory specimens.

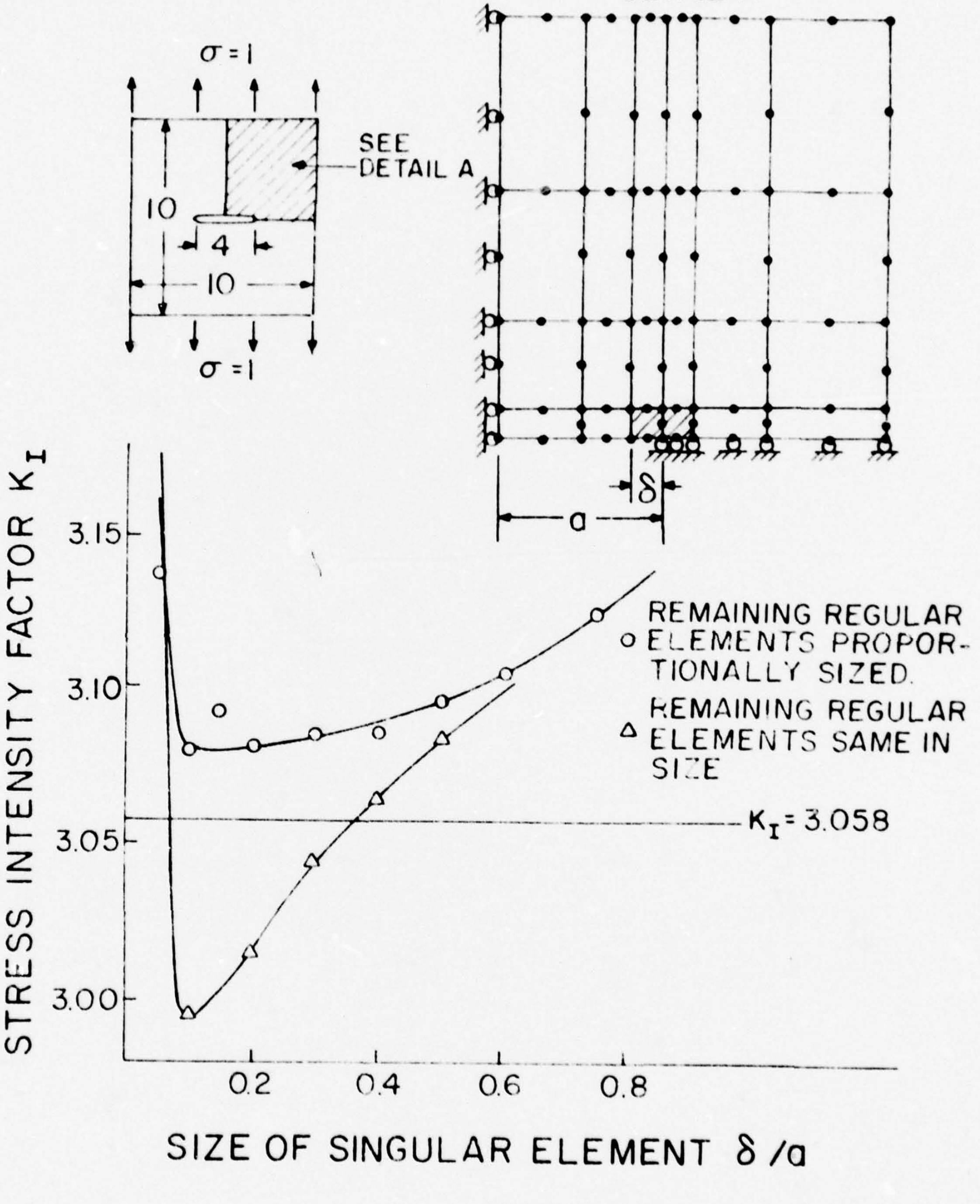


FIG. 3 (From Ref. 6): A Typical convergence study.

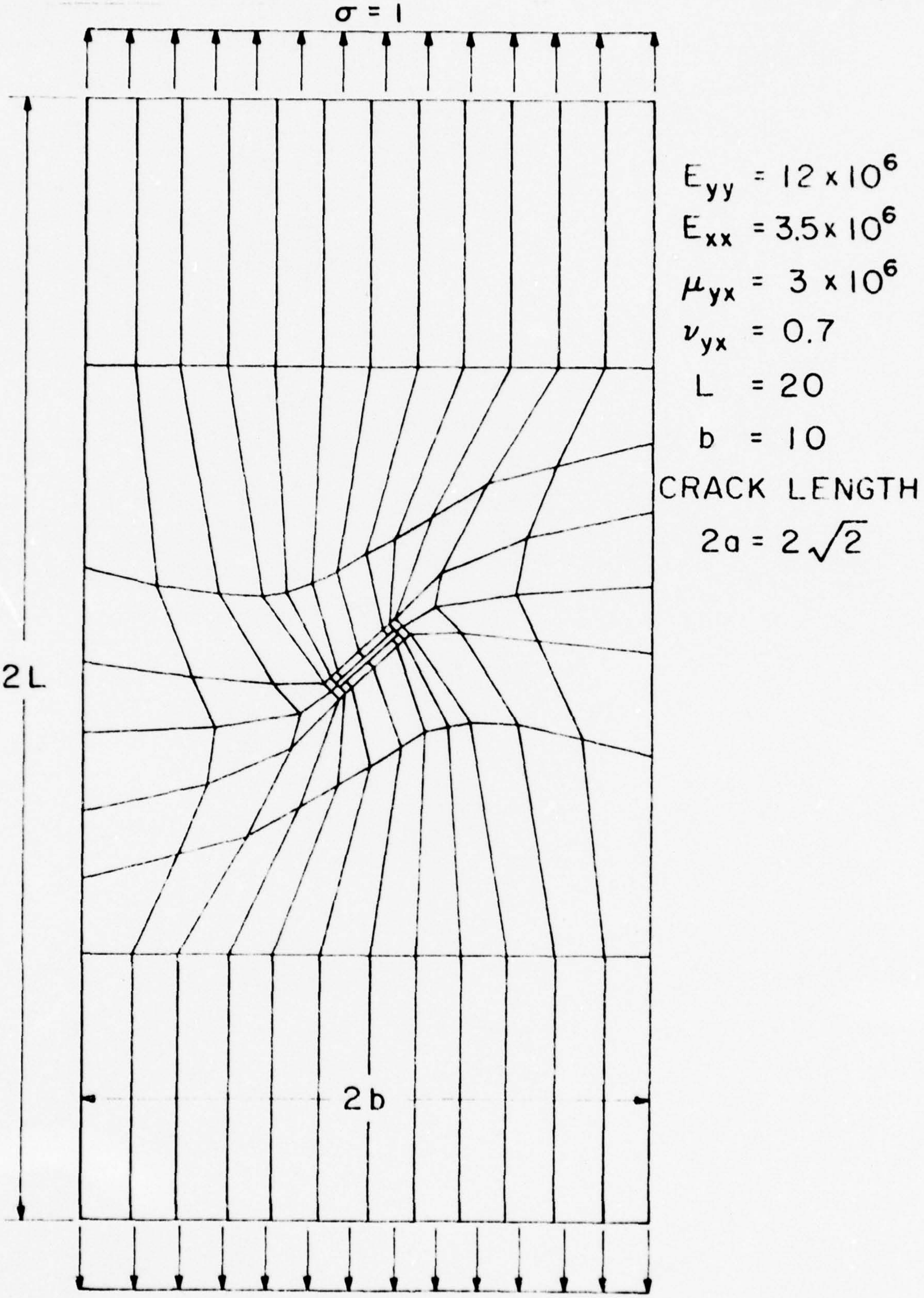


FIG. 4 (From Ref. 6): A Typical Mesh for analysing the problem of a crack unsymmetrically located w.r.t axes of orthotropy.

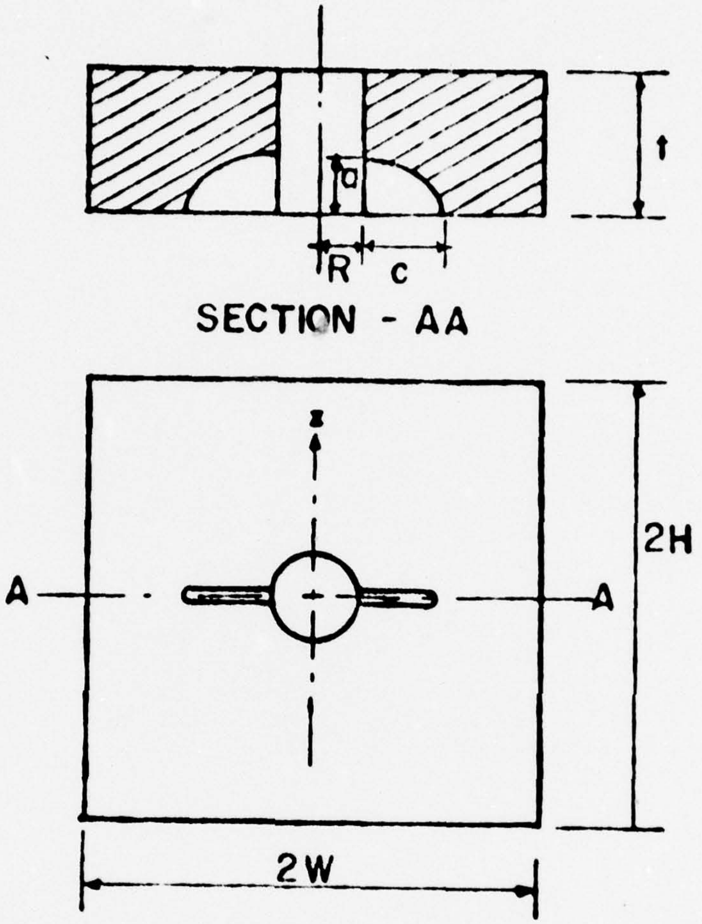


FIG. 45: Corner Cracks Emanating from Holes in Plates

FIG. 5 (From Refs. 2 and 21): A Typical 3-D Crack Problem.

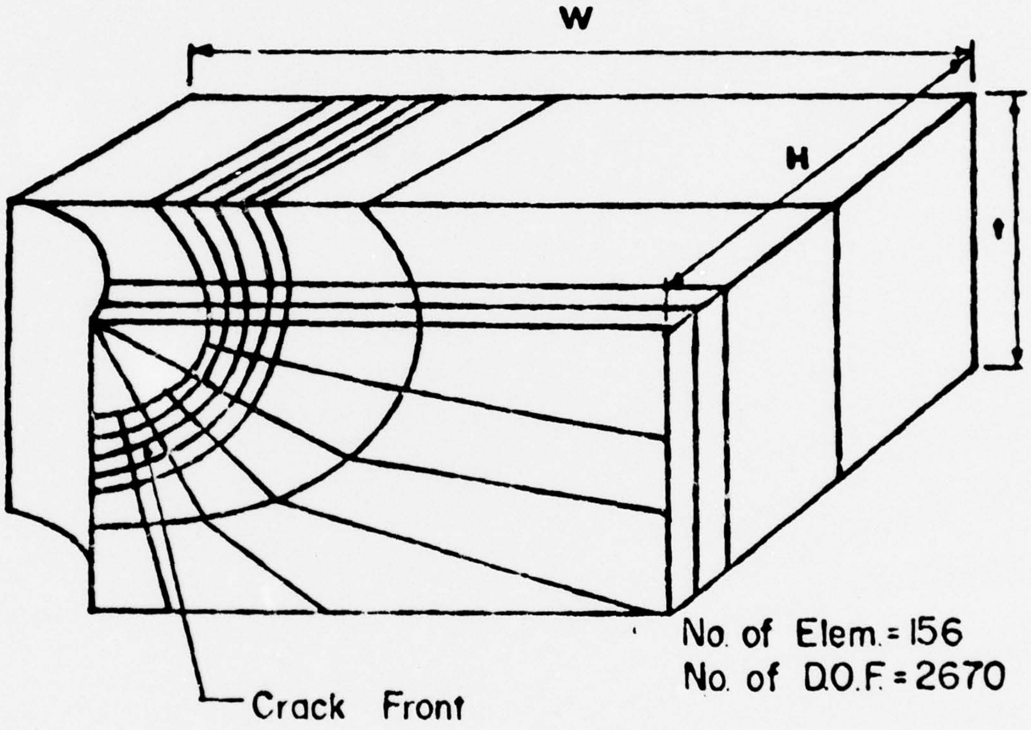


**FIG. 46: Finite Element Breakdown of Quarter of the Corner Cracked Hole Problem**

FIG. 6 (From Ref. 21 and 32): Typical Finite Element Mesh for Problem in Fig. 5.

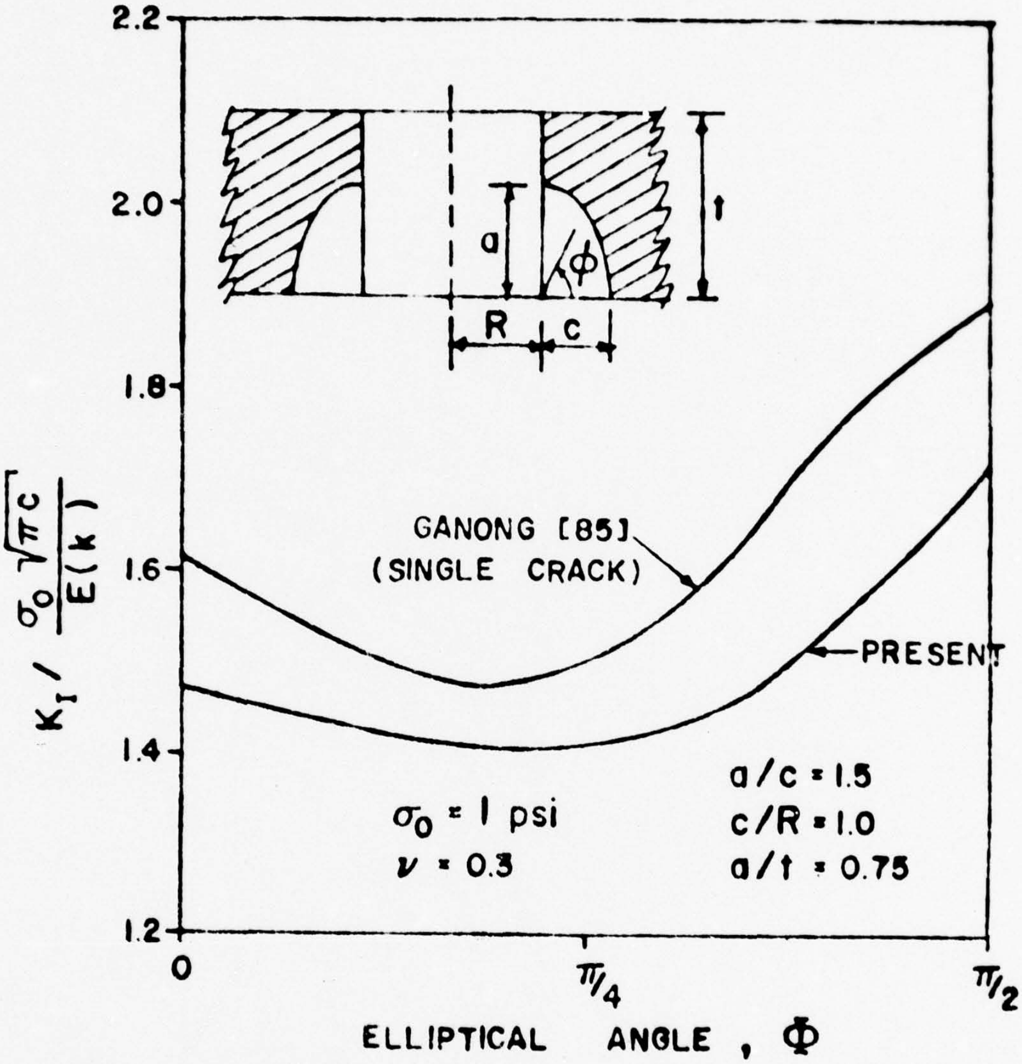


FIG. 51: Variation of Stress Intensity Factor for Corner Cracks Emanating from Hole

(a/c = 1.5; c/R = 1.0; a/t = 0.75)

FIG. 7 (Taken From Ref. 32): Typical Result for a corner crack near a Fastener Hole.

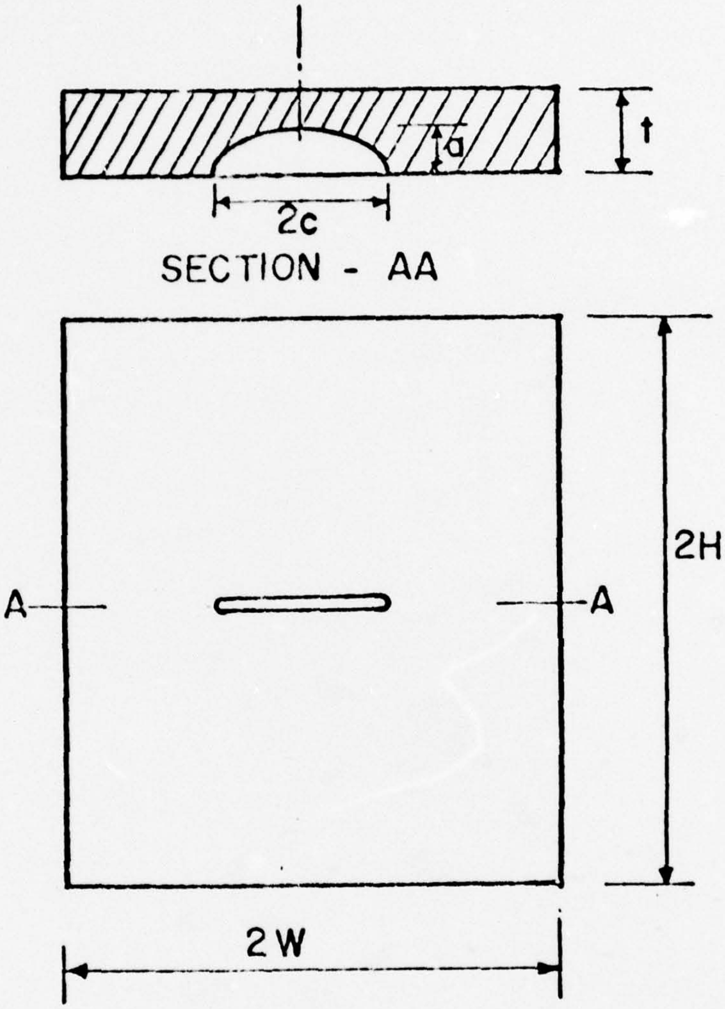
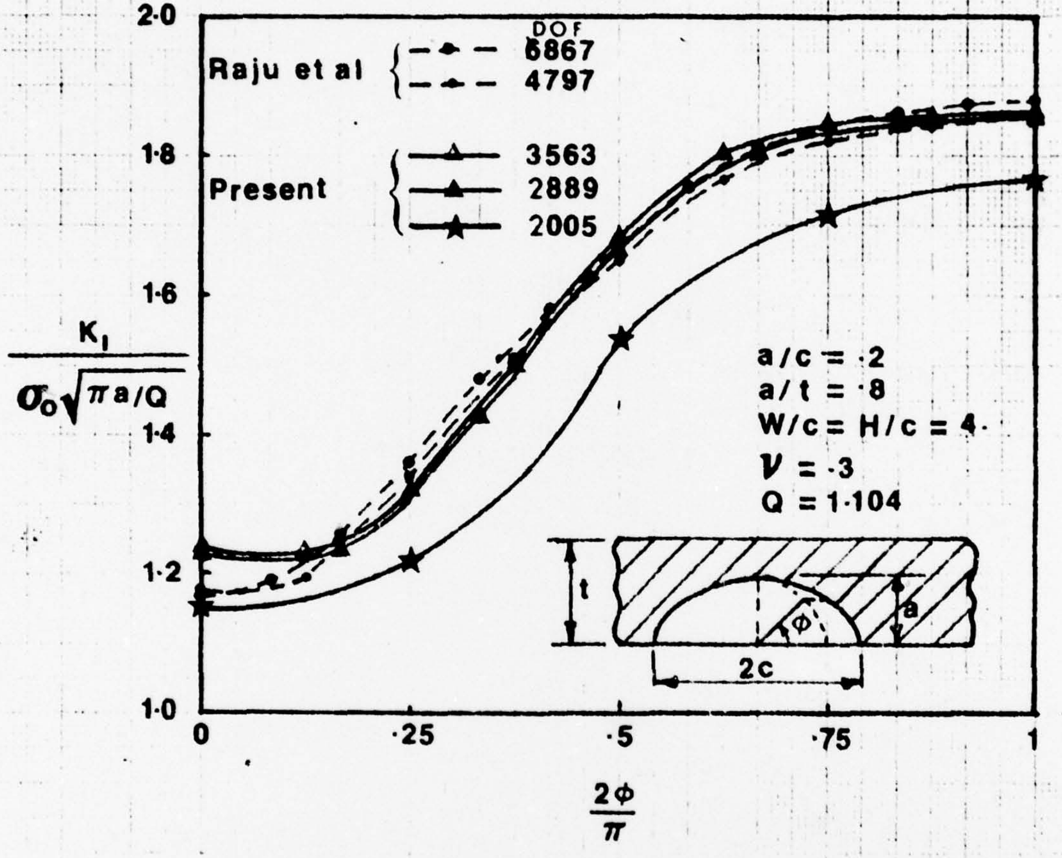


FIG. 2. Semi-Elliptical Surface Flaws in Plates in Tension and Bending

FIG. 8 (Taken From Ref. 28): A Typical 3 crack Problem of a surface flawed plate.



**Variation of Stress Intensity Factors for a Semi-Elliptical Surface Crack in a Thin Plate (a/t = ·8)**

FIG. 9 (Taken from Ref. 33): A Typical 3-D Results for surface-flawed Tension Plate: Comparison of Hybrid Element Results with an alternate Finite Element Result.

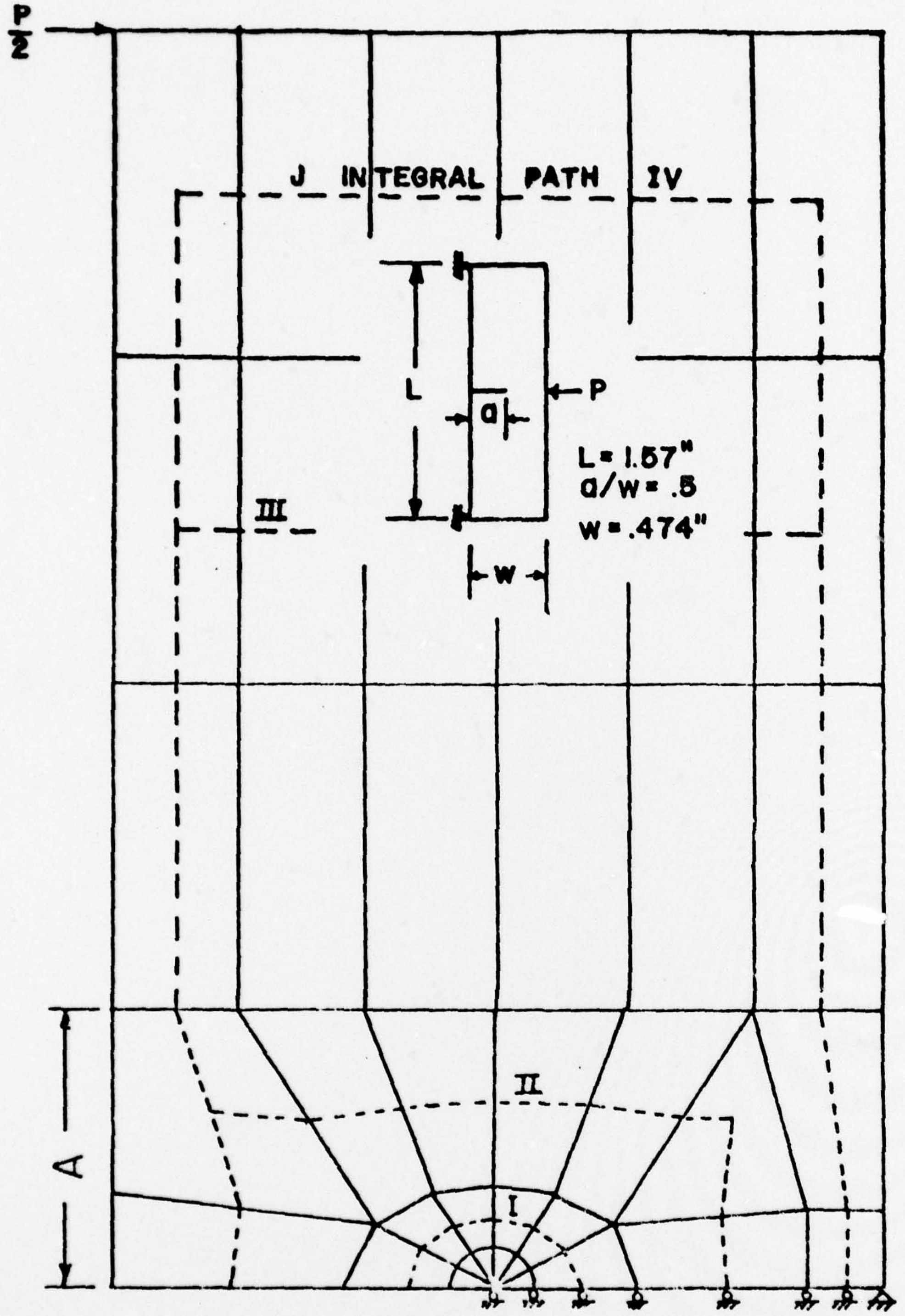


FIG. 2: Finite-Element Model of 3-Point Bend Bar (shown in insert) and J-Integral Paths

FIG. 10 (Taken From Ref. 16): A Typical Specimen: Analysis of Large-Scale Yielding Fracture.

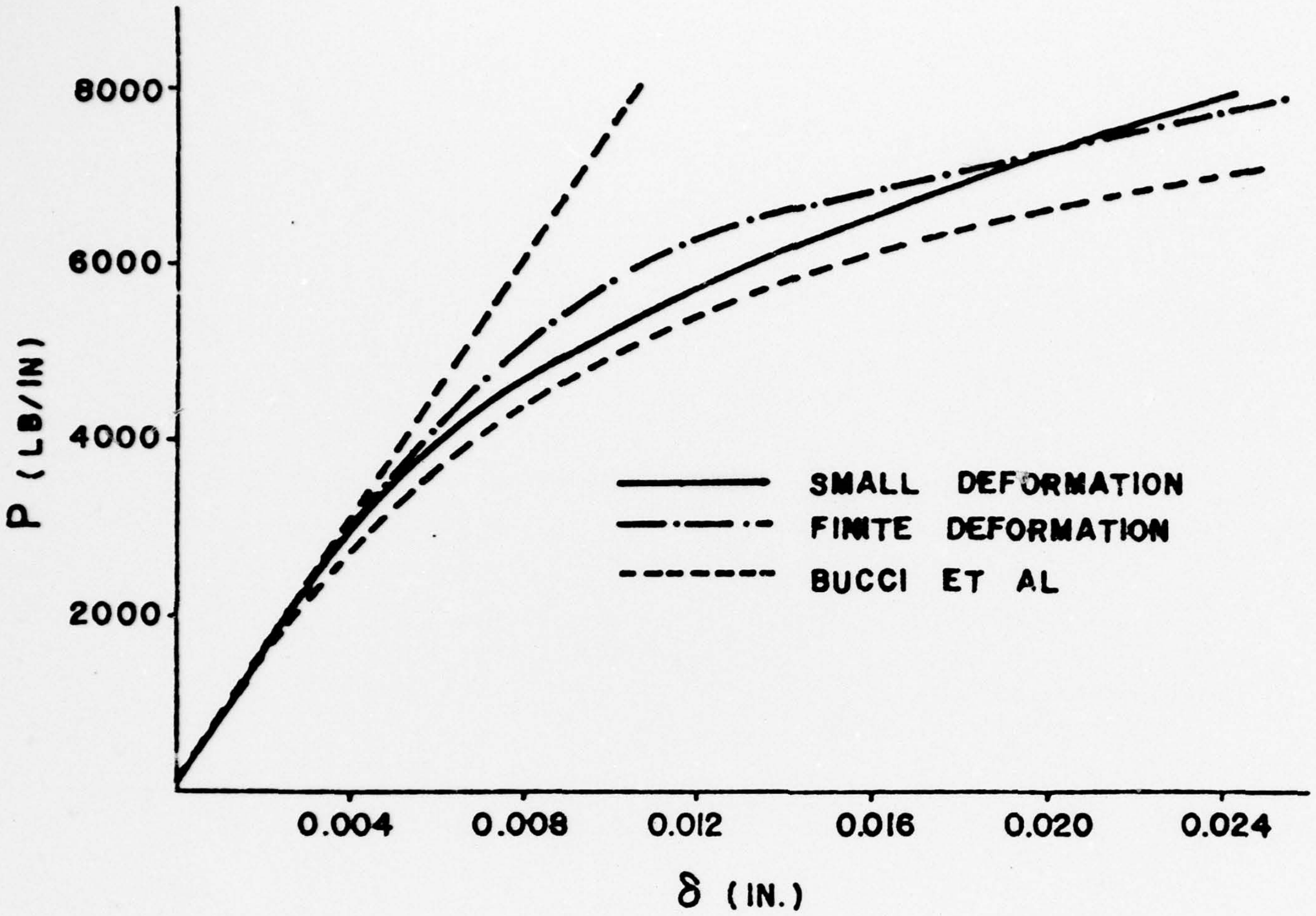


FIG. 4: Load vs. Load-Point Displacement for 3-Point Bend Bar

FIG. 11 (Taken from Ref. 16): Typical Load-Gauge Point Displacement Curves for Large-Scale Plastic Yielding Specimen.

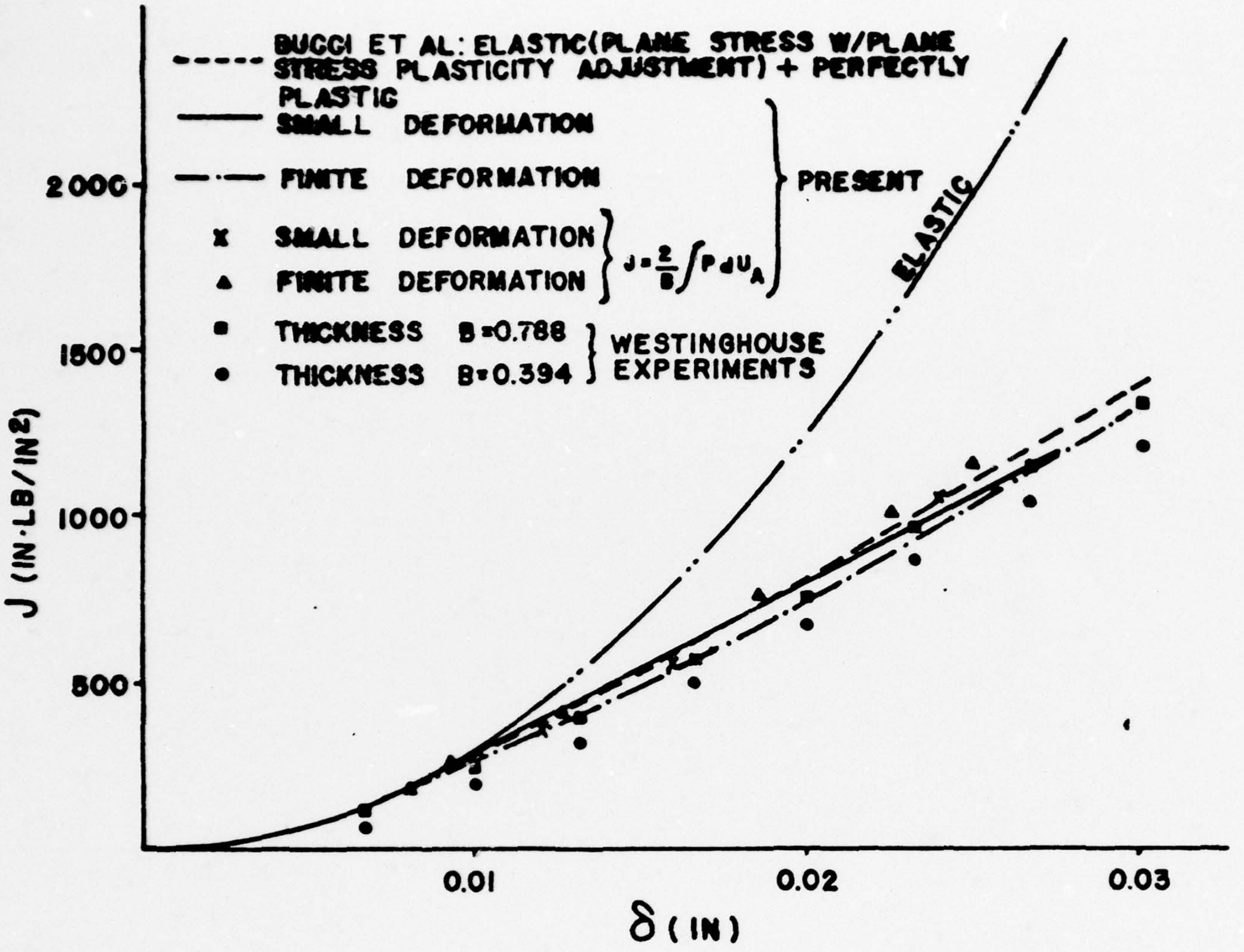


FIG.5: J-Integral vs. δ Curve for 3-Point Bend Bar

FIG. 12 (*Taken from* Ref. 16): Typical Results for J vs. Gauge Point Displacement: Comparison of Various Approximations with experimental data.

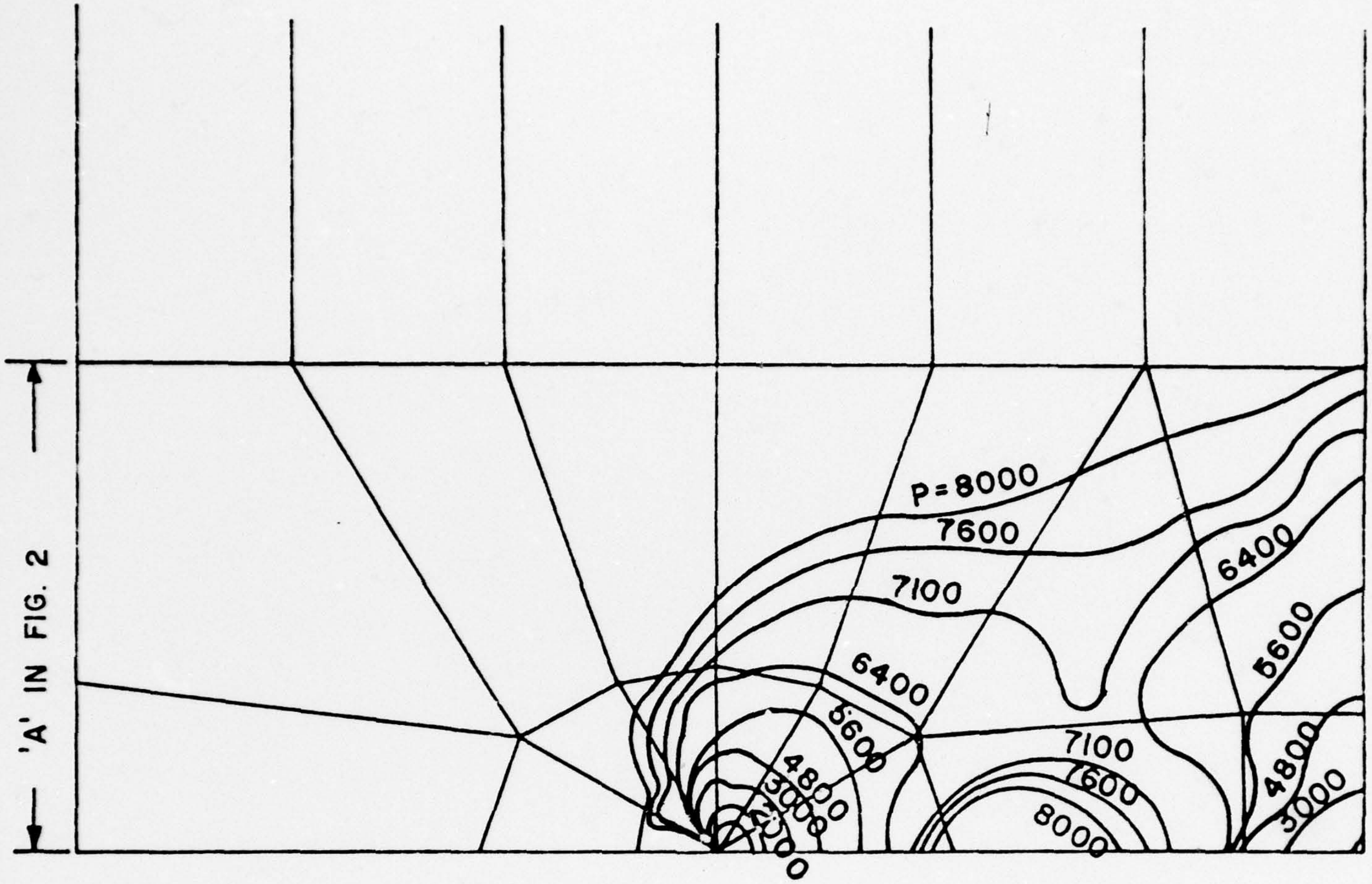


FIG. 6: Yield-Zones at Various Load Levels
(Finite Deformation Analysis)

FIG. 13 (Taken From Ref. 18): Typical Yield zones in a Large-Scale Yielding Fracture Test Specimen.

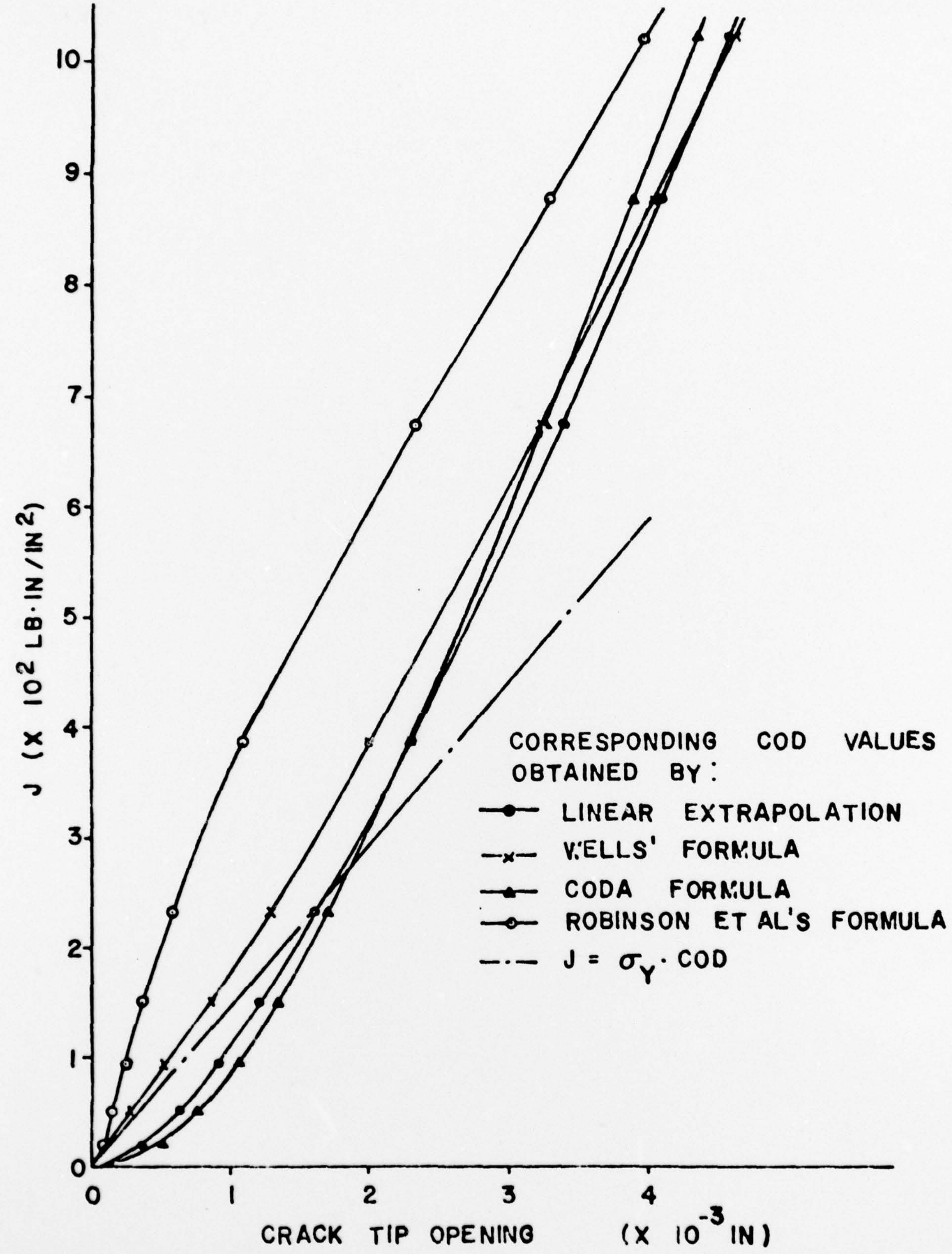


FIG.14 J vs COD Relation for a 3-Point Bend Bar.

FIG. 14 (Taken From Ref. 18): Correlation of Different Fracture-Initiation-Characterizing Parameters.

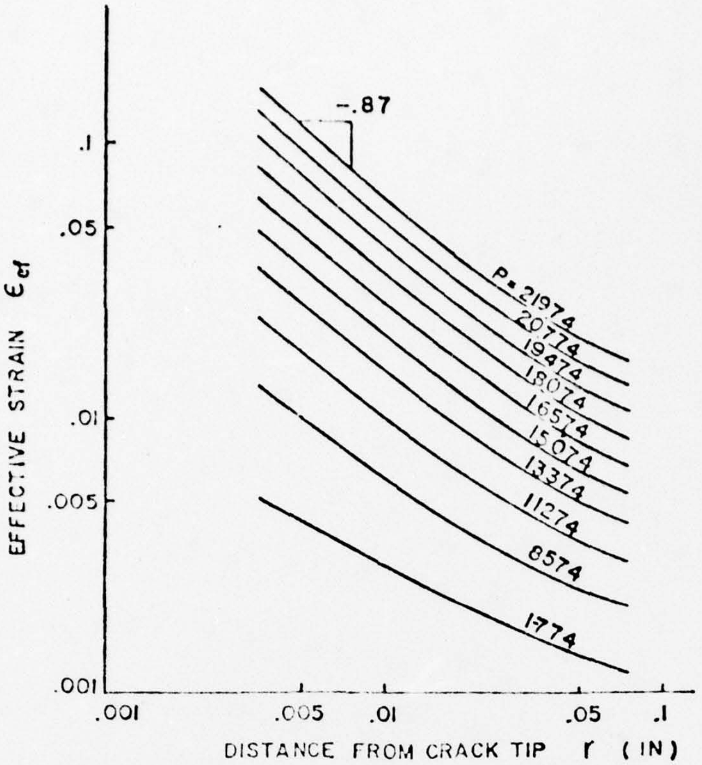


FIG. 11 BEHAVIOR OF STRAIN SINGULARITY NEAR THE CRACK TIP FOR A CT SPECIMEN.

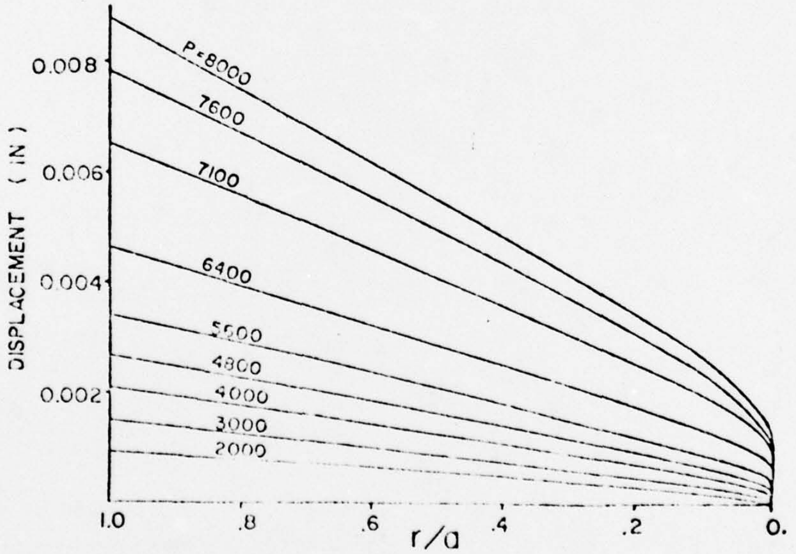


FIG. 12 CRACK-SURFACE DEFORMATION PROFILES AT VARIOUS LOAD LEVELS FOR 3-POINT BEND BAR.

FIG. 15 (Taken From Ref. 20): Nature of Near-field solutions at the crack-tip.

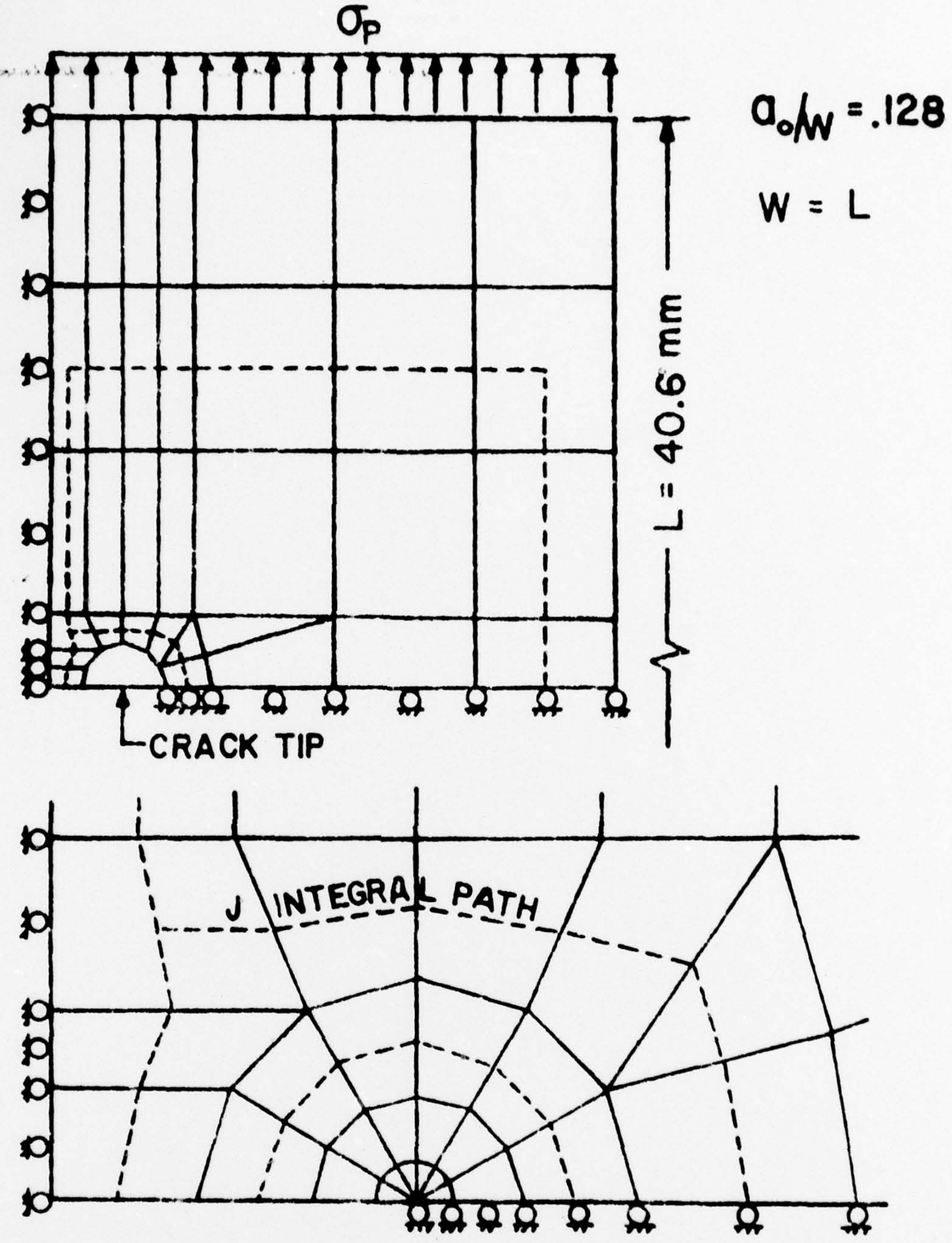


FIG 2. FINITE ELEMENT IDEALIZATION OF CENTER CRACKED SQUARE PLATE UNDER UNIAXIAL TENSION.

FIG. 16 (Taken from Ref. 30): A Typical Model Problem in Analysis of Stable Crack Growth.

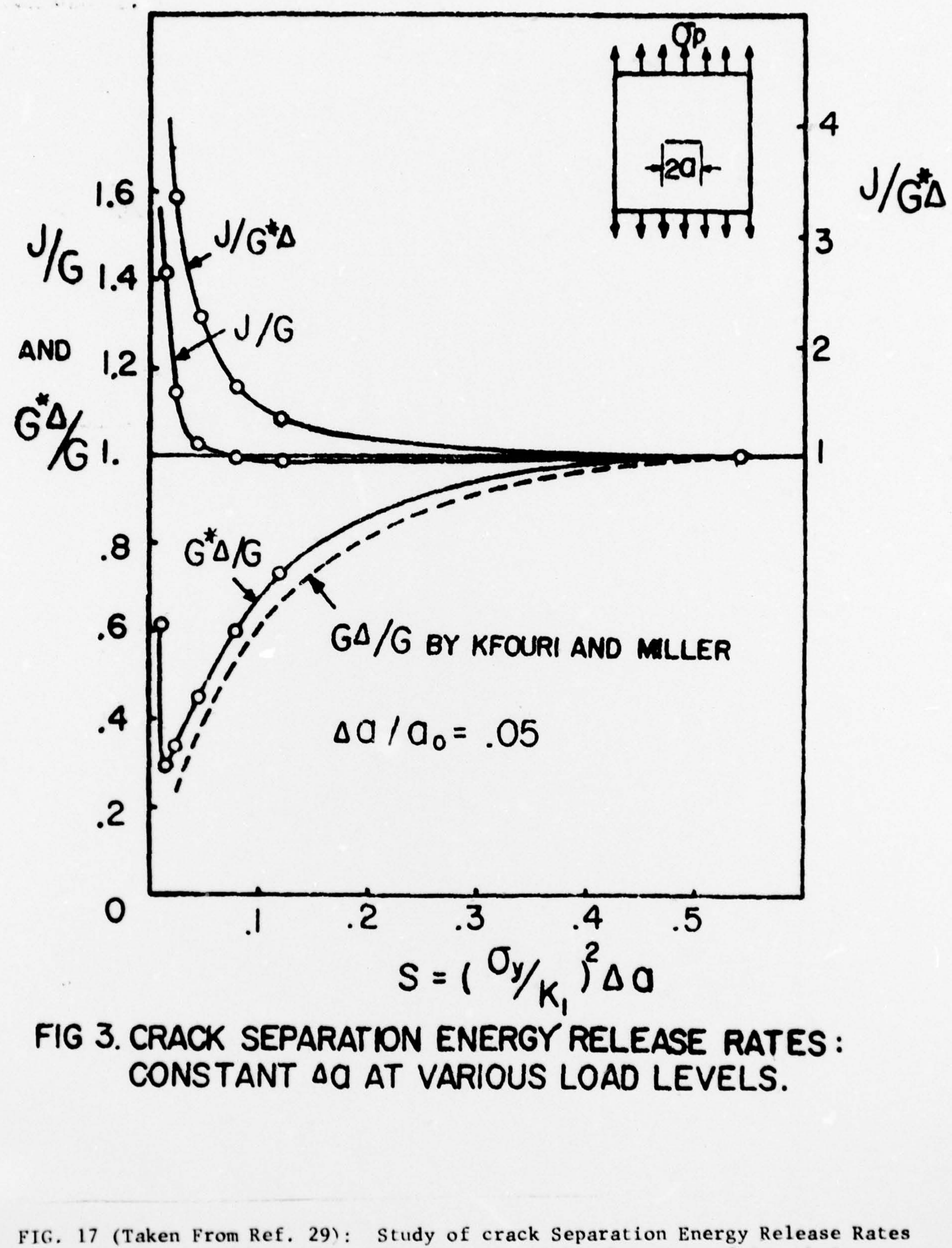


FIG 3. CRACK SEPARATION ENERGY RELEASE RATES: CONSTANT $\Delta a$ AT VARIOUS LOAD LEVELS.

FIG. 17 (Taken From Ref. 29): Study of crack Separation Energy Release Rates in Ductile Materials: The parameter S is the ratio of Growth Step to the Plastic Yield zone size.

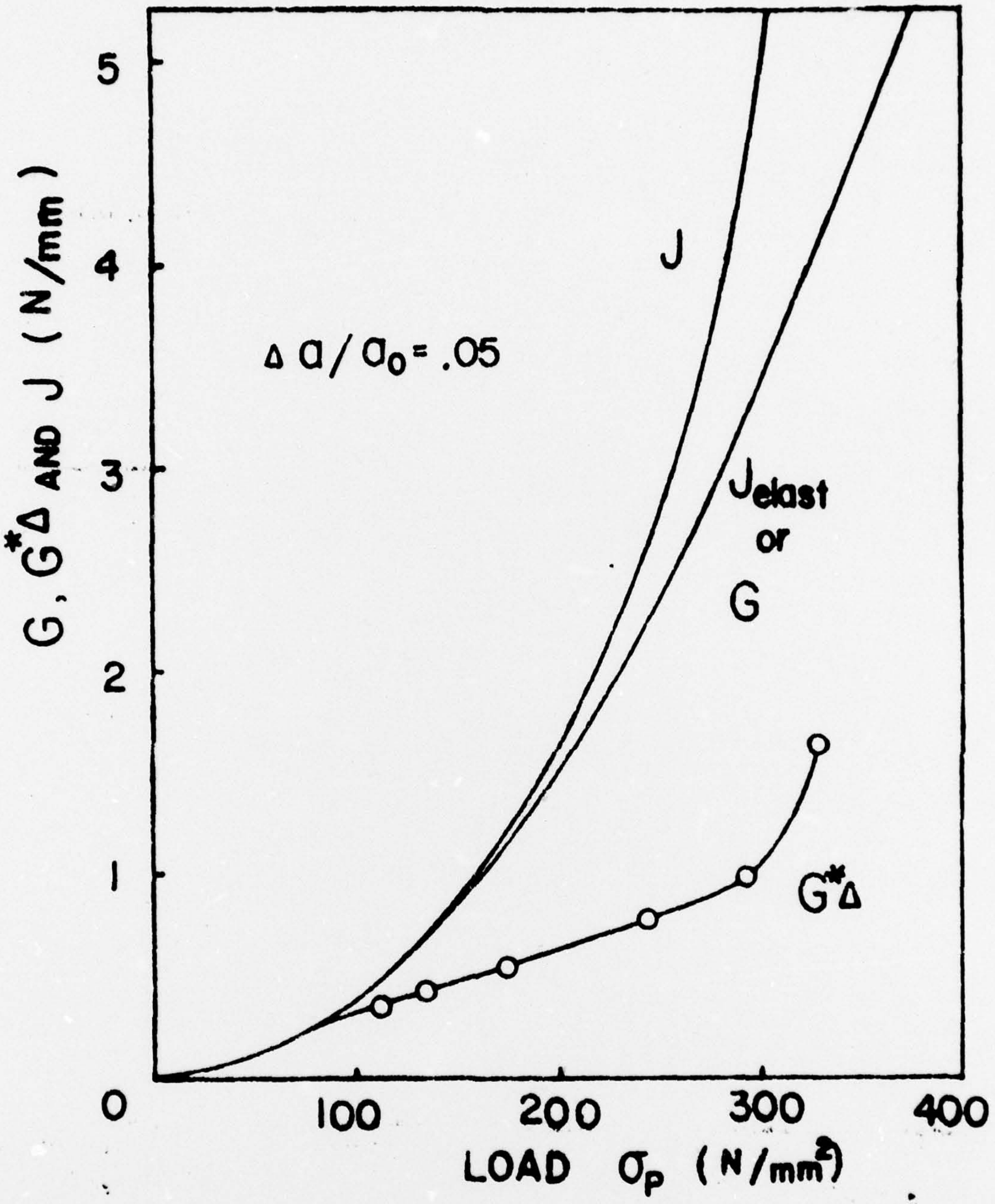


FIG 4. CRACK SEPARATION ENERGY RELEASE RATE: CONSTANT Δa AT VARIOUS LOAD LEVELS.

FIG. 18 (Taken From Ref. 30): Further Elaboration of Results in Fig. 17.

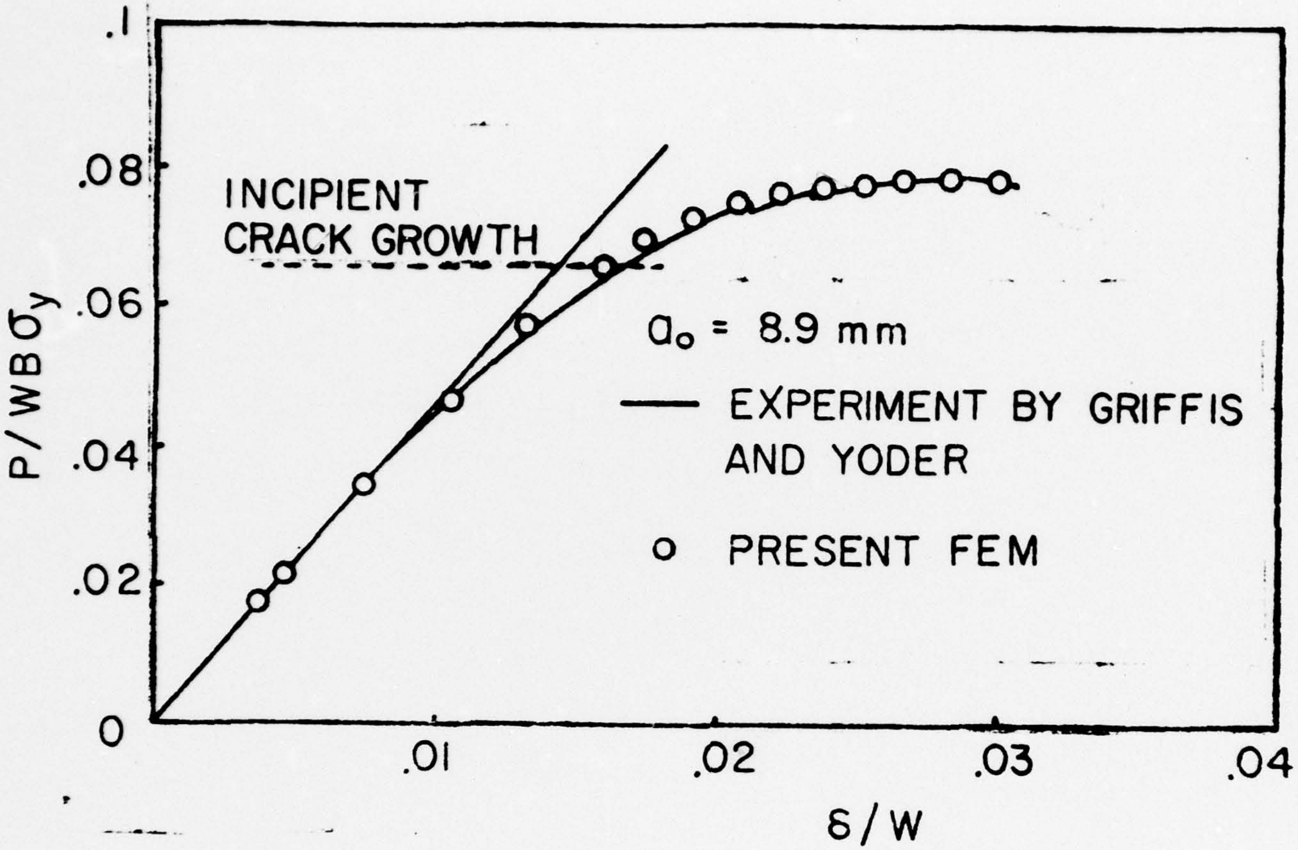


FIG 6. LOAD VS. LOAD POINT DISPLACEMENT CRACK GROWTH CURVE.

FIG. 19 (Taken From Ref. 30): Results of Simulation of Experimental Stable Crack Growth Data.

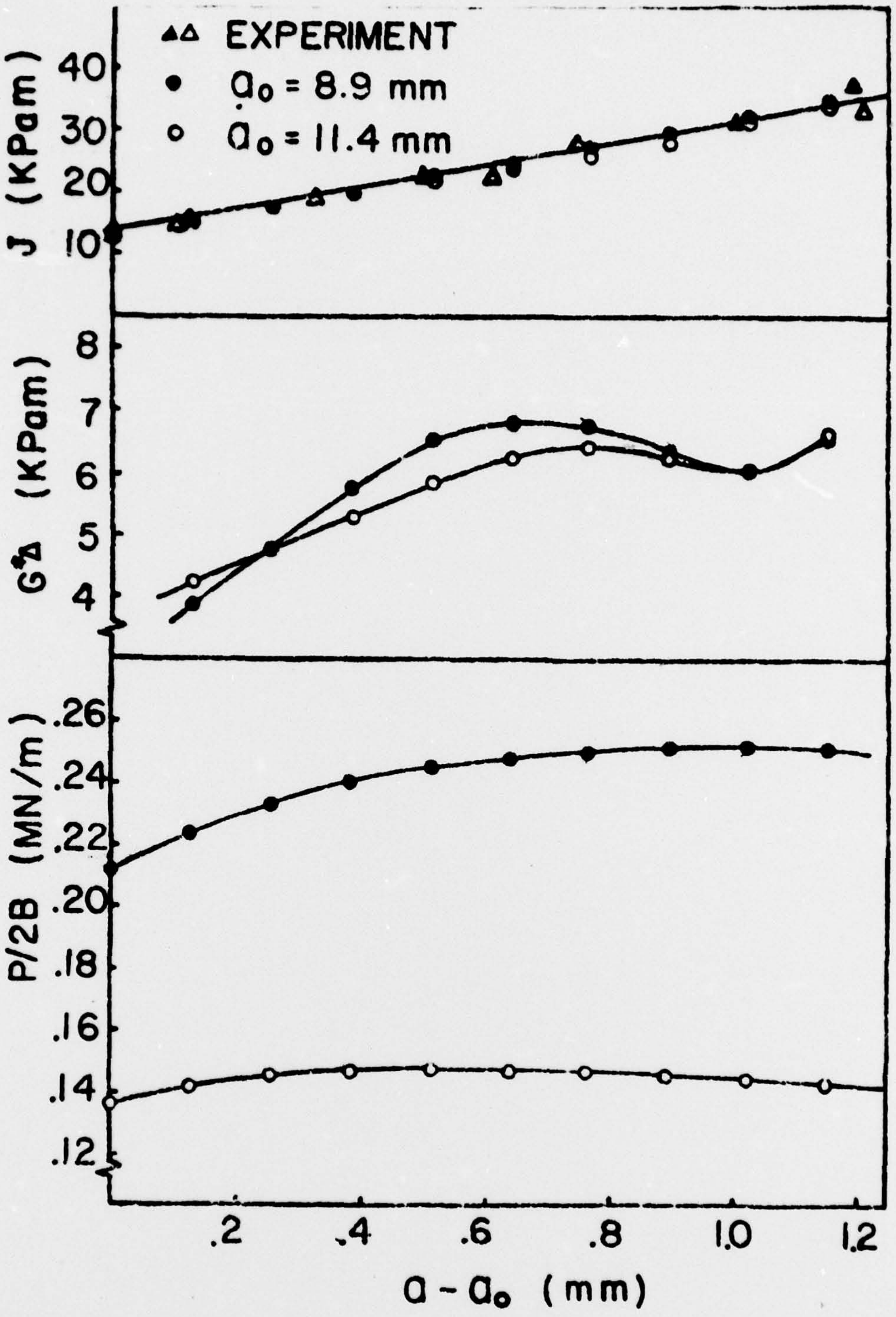


FIG 7. VARIATION OF J, $G^{*\Delta}$, AND P/2B DURING CRACK GROWTH.

FIG. 20 (Taken From Ref. 30): Results for Various Physical Parameters that may govern stability of Crack Growth, in the Simulation of Data in Fig. 19.

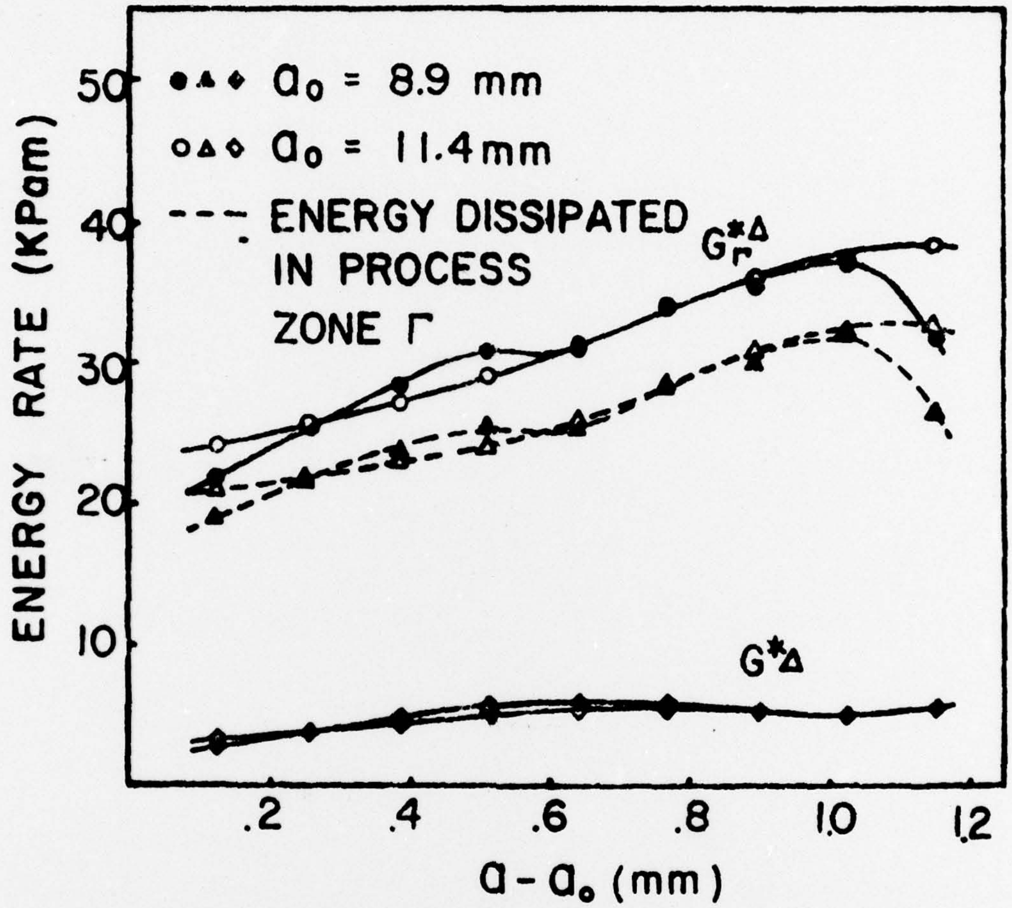


FIG 9. VARIATION OF $G^{*\Delta}_{\Gamma}$, $G^{*\Delta}$, AND ENERGY DISSIPATION IN PROCES ZONE DURING CRACK GROWTH.

FIG. 21 (Taken Fig. Ref. 30): Continuation of Results Similar to those in Fig. 19.

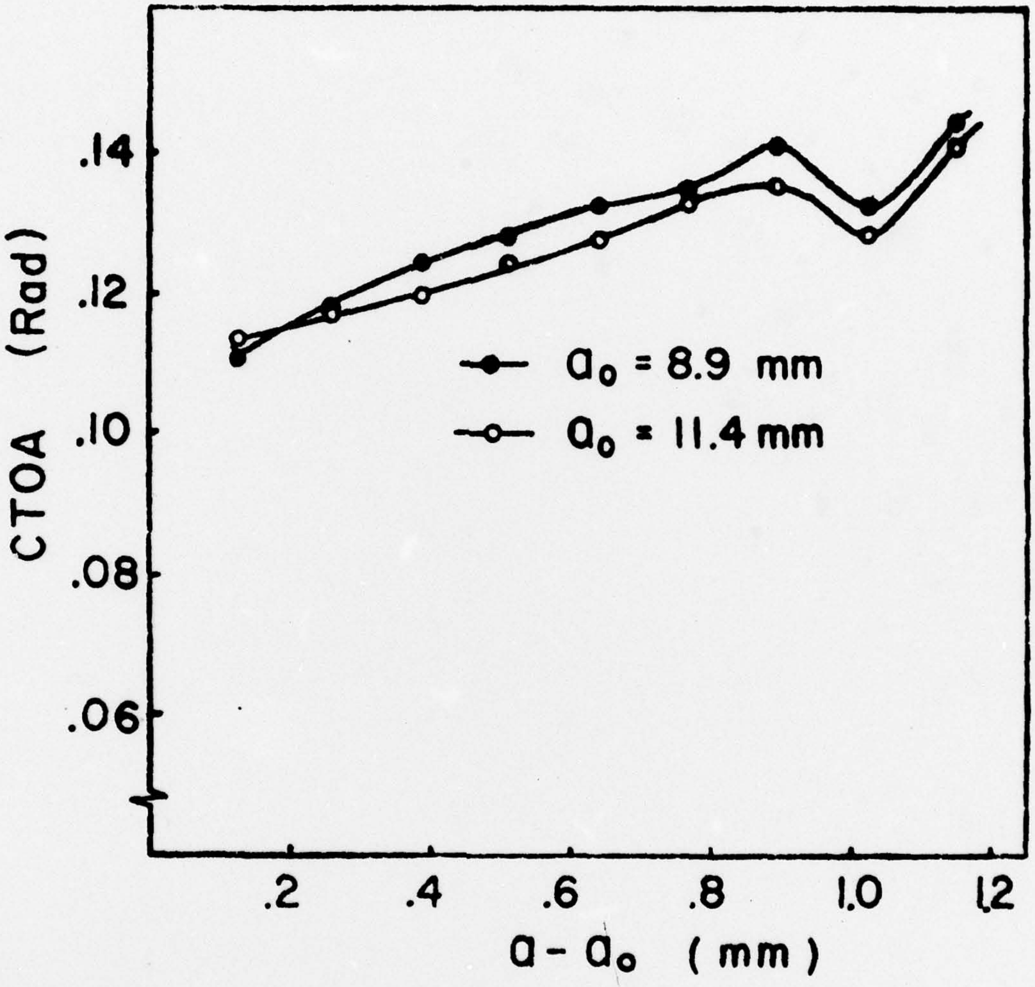


FIG 10. VARIATION OF CRACK-TIP OPENING ANGLE DURING CRACK GROWTH.

FIG. 22 (Taken From Ref. 30): Continuation of Results Similar to those in Fig. 19.