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ON A METHODOLOGY FOR THE ANALYSIS OF MULTI-ARRIVAL QUEUEING SYS--ETC(U)  
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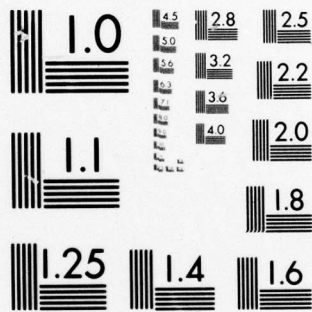


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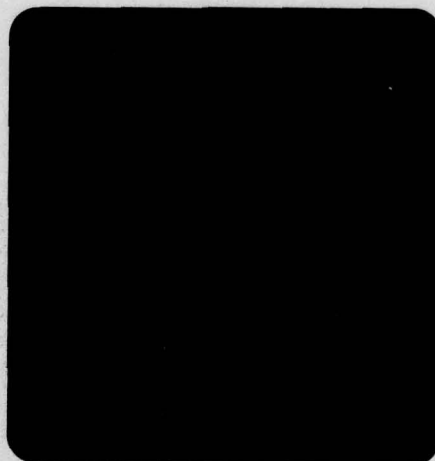
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ONR Contract ~~NO~~ 0014-75-C-0517 NR 042-324

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On a Methodology for the Analysis of  
Multi-Arrival Queueing Systems.

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FINAL REPORT. 1 Jan-76-31 Mar 78.

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31 Mar 78

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12 p.

Department of Operations Research  
and Engineering Management  
Southern Methodist University  
Dallas, Texas 75275

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ONR Contract N00014-75-C-0517, NR 042-324

On a Methodology for the Analysis of Multi-Arrival Queueing Systems

Final Report

This final report covers two consecutive periods in which this project has been in force; (i) Jan 1 - Dec 31, 1976 (original term of project) and (ii) Jan 1, 1977 - March 31, 1978 (no-cost extension). During these periods work has been in progress on the following problems:

1. A Generalized Measure of Performance - Properties and Optimization in a Class of Queueing Systems. (Investigators: Bhat & Shalaby)
2. Queueing Systems with Varying Number of Servers. (Investigators: Bhat & Kim)
3. Approximations in Queueing Systems. (Investigators: Bhat, Raju & Shalaby)
4. Queueing Theory Applications in Computer Systems. (Investigators: Bhat & Nance)

In addition to the principal investigator the following individuals have participated during the project period.

1. Mohamed Shalaby - Graduate Student
2. Sung Shick Kim - Graduate Student
3. Dr. Sagi N. Raju - University of Wisconsin - Whitewater (former graduate student).
4. Dr. Richard E. Nance, Virginia Polytechnic Institute & State University - Co-research worker. (No payment from the grant.)

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5. Dr. S. Subba Rao, Indian Institute of Management, Bangalore,  
India, consultant for a two-day period.

A brief description of research carried out in each of the problems follows.

#### Description of Research

##### 1. A Generalized Measure of Performance

A generalized measure of performance is defined as a weighted combination of the ergodic queue length distribution where the weights are general functions of system parameters. The objective is to choose the best system that optimizes this general performance measure. For the class of finite queueing systems studied extensively by Sagi N. Raju (work carried out under the previous phase of the project), a sequence of upper and lower bounds for the performance measure is obtained. The bounds are used to derive a sequence of approximations with bounded errors. The bounds are shown to converge to their corresponding exact values. An optimization model with a nonlinear objective function and constraints is developed for the class of systems identified as G/M/1/N. The bounds developed earlier are extended to the optimal measure of performance. Using these bounds a gradient search algorithm is used to derive the optimal arrival and service rates. Research work on the optimization model is continuing.

One paper prepared for publication (copy enclosed) "Bounds and Approximations to a Generalized Measure of Performance in the G/M/1/N Queue" (Shalaby & Bhat) Tech. Report OREM 78005, Feb 1978.

Another paper under preparation: "Optimizing the G/M/1/N Queue" (Shalaby).

## 2. Queueing Systems with Varying Number of Servers

In a multiserver queueing system with Poisson arrivals and exponential service times the number of servers varies at each service completion epoch based on a specified Markovian transition structure. Solving matrix equations of probability-generating functions general solution to the steady state behavior of this system has been obtained.

The significance of this problem is in the solution of a queueing system with heterogeneous classes of customers, in which customers of some class demand the concurrent use of more than one server (e.g., the use of more than one communication line for transmission of information). We believe that a system with a varying number of servers can be used as a good approximation for the system described above. The major problem to be resolved now is the determination of the appropriate transition probability matrix for the variations in the number of servers. Work is continuing in this direction.

A paper is being prepared for publication (Bhat & Kim).

## 3. Approximation in Queueing Systems

The work under this topic is a continuation of the investigations carried out by the principal investigator and Sagi N. Raju (Chapter IV of "On Certain Finite Queueing Systems", Ph.D. dissertation by S. N. Raju, 1976).

Using heuristic approximation techniques, complex multiserver priority-systems with heterogeneous classes of customers are analyzed to provide relevant performance measures. The proposed approximation scheme separates

the classes of customers as well as the resource and determines the measures of performance through an iterative scheme.

A research paper "An Approximate Analysis of a Multi-Server Finite Queue with Heterogeneous Customers" (Bhat and Raju, copy enclosed)

- Presented as an invited paper at the ORSA/TIMS meeting at Atlanta, Nov, 1977.

A survey paper "Approximation Techniques in the Solution of Queueing Problems" (Bhat, Fischer and Shalaby, copy enclosed)

- Presented at the ORSA/TIMS meeting in Miami, Nov. 1976
- Revised for publication in Naval Research Logistics Quarterly, March 1978

A research paper "Recursive Relations in the computation of the Equilibrium Results of Finite Queues" (Raju and Bhat, copy enclosed)

- Published in TIMS Studies in Management Sciences 7, North Holland Pub. Co., (1977) 247-270.

A research paper "The M/G/1 and G/M/1 Finite and Infinite Queues: An Analytical Approach to Numerical Results" (Raju and Bhat)

- Being prepared for publication.

#### 4. Queueing Theory Applications in Computer Systems

This phase of the investigation involves substantial revisions made for two papers submitted for publication by the principal investigator and his co-research worker Dr. Richard E. Nance of Virginia Polytechnic Institute and State University.



"A Processor Utilization Model for a Multi-Processor Computer System"

- to appear in Operations Research, 1978.

"An Evaluation of CPU Efficiency under Dynamic Quantum Allocation"

- Submitted for publication in J. ACM after revision.

#### Relevance of Research

The funded proposal set the following objectives for investigations under the project.

(1) Development of a composite measure of performance of queueing systems occurring in an integrated telecommunications network and its use in comparing different operating rules.

(2) Further investigations of the operating rule for traffic requiring multiple channel capacities.

(3) Efficiency of various approximation techniques in obtaining measures of performance.

The first three items under Description of Research address the three objectives identified above. It is the view of the principal investigator that more substantial work has been done under item (1) than anticipated at the time of the proposal. The research carried out so far is of a fundamental value with a potential for at least two journal articles. Research carried out under item (2) attacks the problem in an indirect manner since direct solution seems to be too cumbersome. Further work is needed for a complete solution of the problem. On the third topic, viz, approximations, the work done was more exploratory and heuristic in nature.

The heuristic iterative procedure suggested by Bhat and Raju for approximating heterogeneous systems has good potential for further refinement and wide use. The survey article, revised for publication in Naval Research Logistics Quarterly contains a bibliography of 101 papers and is going to be a useful source for researchers and practitioners in this area.

Thus, on the whole, taking into account the nature of investigations, it is the belief of the principal investigator that significant work has been carried out under this project. Furthermore, three graduate students have received training in doing research which is both fundamental and relevant in practice.

A summary of significant activities by the investigators of this project follows.

### Summary of Activities

ONR Contract N00014-75-C-0517, NR 042-324

(Jan 1, 1976 - March 31, 1978)

#### A. Ph.D. Dissertations:

Sagi N. Raju:

"On Certain Finite Queueing Systems" (last phase of work)

Mohamed Shalaby

"On the Optimization of Finite Queues"

Sung-Shick Kim:

"Multi-server Queueing Systems with Varying Number of Servers"  
(first phase of work).

#### B. Technical Papers on Project Work

##### Published

1. "Multi-Channel Queueing Systems with Heterogeneous Classes of Arrivals", U. N. Bhat and M. J. Fischer, Naval Res. Log. Qtly, Vol. 23, No. 2, 1976, 271-282.
2. "Recursive Relations in the Computation of the Equilibrium Results of Finite Queues", S. N. Raju and U. N. Bhat, TIMS Studies in the Management Sciences, Vol. 7, 1977, pp. 247-270.

##### Accepted for Publication

1. "A Processor Utilization Model for a Multi-Processor Computer System", R. E. Nance and U. N. Bhat, Operations Research, 1978.

Submitted After Revision (Likely to be Accepted)

1. "An Evaluation of CPU Efficiency under Dynamic Quantum Allocation",  
U. N. Bhat and R. E. Nance, J. ACM.
2. "Approximation Techniques in the Solution of Queueing Problems",  
U. N. Bhat, M. J. Fischer, M. Shalaby, Naval Res. Log. Quarterly.

Being Prepared for Publication (Final Draft)

1. "Bounds and Approximations to a Generalized Performance Measure in  
the G/M/1/N/ Queue", M. A. Shalaby and U. N. Bhat.
2. "The M/G/1 and G/M/1 Finite and Infinite Queues: An Analytical  
Approach to Numerical Results", S. N. Raju and U. N. Bhat.
3. "Queueing Systems with Varying Number of Servers", U. N. Bhat and  
S. S. Kim.

Reports (Working Papers)

1. "On Certain Finite Queueing Systems", S. N. Raju, Tech. Report IEOR  
76006, Dept. of IE/OR, SMU, Dallas, May 1976 (abstract of Ph.D.  
Dissertation).
2. "An Approximate Analysis of a Multi-Server Finite Queue with  
Heterogeneous Customers", U. N. Bhat and S. N. Raju, Tech. Rep.  
IEOR 77020, Dept. of IE/OR, SMU, Dallas, Oct. 1977.

C. Other Scholarly Activities of the Principal Investigator

1. Textbook: "Introduction to Operations Research Models", W. B.  
Saunders, 1977. (Co-authors: L. Cooper and L. LeBlanc)



2. Research Paper: "Use of Non-Linear Programming and Stochastic Modeling in the Medical Evaluation of Normal-Abnormal Liver Function", IEEE Transactions on Biomedical Engineering, 23(3), 220-207, 1976. Co-authors: S. I. Saffer, C. E. Mize, and S. A. Szygenda.
3. Letter to the Editor: "The Value of Queueing Theory - A Rejoinder", Interfaces, 1978, to appear.
4. Technical Report: "The Poisson Queue M/M/1: A Unified Theory", Tech. Rep. #IEOR 77021, Oct. 1977.
5. Professional Society Activities
  - ORSA/TIMS meeting, Philadelphia, March-April 1976
    - Session chairman
  - ORSA/TIMS meeting, Miami, Nov. 1976
    - Session chairman, Presented paper
  - ORSA/TIMS meeting, San Francisco, May 1977
    - Panel discussion, Session chairman
  - ORSA/TIMS meeting, Atlanta, Nov 1977
    - Presented paper
  - Elected Vice-Chairman - Chairman-Elect of the ORSA Interest Group/TIMS College on Applied Probability, 1977-78.
6. Editorial Activities
  - Referee for:
    - Operations Research
    - Applied Probability
    - SIAM Jour. on Appl. Math.
    - Math Reviews
    - NSF Proposals



D. Professional Advancement, etc.

U. Narayan Bhat:

Acting Dean: March - Sept. 1976

Associate Dean: Oct. 1976 - .

Sagi N. Raju (Ph.D. 1976)

Appointed Assistant Professor, University of Wisconsin,  
Whitewater.